

# Long Run Impact of Increased Wage Pressure\*

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## Abstract

An unanticipated permanent increase in wage pressure is analysed in a dynamic general equilibrium model combining standard theory of capital accumulation and monopolistic wage setting. The long run (steady state) implications are identical percentage reduction in employment, consumption, and capital stock whereas wages and the real interest rate are unchanged. The reduction in employment on impact is larger than the steady state reduction whereas wages rise and the real interest rate declines on impact.

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# 1 Introduction

Increases in wage pressure are normally analysed in static general equilibrium frameworks although the results often have a dynamic interpretation. This paper tries to clarify whether such interpretations concerning equilibrium wages and employment are correct by making a natural dynamic extension of these frameworks.

In the static analyses, it is common to illustrate the macroeconomic outcome in a wage-price-setting diagram similar to Figure 1 (e.g., Blanchard & Kiyotaki 1987, Layard, Nickell & Jackman 1991 ch. 1 and 8, or Booth 1995 ch. 8).<sup>1</sup> The equilibrium is determined by the intersection of an upward-sloping wage-setting curve ( $WS$ ) and a downward sloping or horizontal price-setting curve ( $PS$ ) in a wage-employment diagram. The  $WS$  curve describes the aggregate wage-setting behaviour of unions and is a mark-up on the labour supply ( $LS$ ) whereas the  $PS$  curve describes the aggregate labour demand of firms.<sup>2</sup> The consequences of unionization of the labour force are then either (a) an increase in wages and a relatively moderate reduction in employment (**A** to **B**) if the  $PS$  curve slopes downwards ( $PS_a$ ) or (b) unchanged wages and a large reduction in employment (**A** to **C**) if the  $PS$  curve is horizontal ( $PS_b$ ).<sup>3</sup> The same qualitative results apply to a shift upwards in the  $WS$  curve because of increased wage pressure (i.e., movements from **C** to either **D** or **E**). This paper considers two standard reasons for this; less elastic labour demands and higher replacement ratios.

< Figure 1 >

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<sup>1</sup>This is only possible because of symmetry assumptions on both sides of the market.

<sup>2</sup>It is called a price setting curve as it depends on the price setting behaviour in the output market. A high degree of imperfect competition in the output market reduces the labour demand and moves the curve downwards. The issue of this paper is independent on the degree of imperfect competition in the output market. Thus, to simplify matters the analysis assumes perfect competition in the output market. In that respect it is a little misleading to name the curve price setting.

<sup>3</sup>In both cases, the equilibrium is Pareto inoptimal due to a 'coordination failure' among the unions. E.g., because each union does not take into consideration that a larger wage claim increases the price level which reduces the real wages of other unions.

The slope of the  $PS$  curve is determined by the production function which only contains labour input. The horizontal  $PS$  curve arises when the production function is linear in labour whereas the downward sloping  $PC$  curve occurs when there is decreasing returns to labour.<sup>4</sup> The exclusion of capital from the production function simplifies the analysis considerably by leaving out capital accumulation and intertemporal considerations of agents. However, it may be natural to think of result (a) as the short run outcome corresponding to a fixed capital stock, and result (b) as the long run outcome corresponding to an endogenous capital stock and a fixed marginal product of capital (say, equal to the rate of time preference of consumers).<sup>5</sup> Or to be more precise, to interpret result (a) as the impact effect of an unanticipated permanent increase in wage pressure and result (b) as the steady state effect in a model with capital accumulation. This paper embeds a standard static general equilibrium analysis into a Ramsey model with endogenous labour supply in order to explore this issue. The results do confirm the above assertion concerning wages but not concerning employment: On impact higher wage pressure increases wages and afterwards wages converge back to the same steady state level but the impact reduction on employment is *larger* than the steady state reduction.<sup>6</sup> Thus, employment overshoots the new permanent level. To understand this, it is necessary to look at the macroeconomic implications.

There are many ways of modelling the interaction between unions and investment in capital equipment even in partial static models. The pioneering paper on the topic (Grout 1984) shows that unions decrease firm investments because of the so-called 'hold-up' problem; i.e., the incentive to make an irreversible investment is low in a firm with organized workers because the

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<sup>4</sup>It is presumed that all agents have correct expectations. It is also possible to have a downward sloping PS curve with constant returns if the expectations of the agents are not fulfilled (see Layard *et.al.* 1991).

<sup>5</sup>Layard *et.al.* (1991) does in fact make such interpretation on p. 107.

<sup>6</sup>The paper analyses increased wage pressure in a closed economy. In a small open economy with free capital movements and no capital adjustment costs the economy would jump directly from **A** to **C** in Figure 1; the wage at the macro level would be determined uniquely by the exogenous international real interest rate.

firm knows that the union increases the wage-claim after the investment is made.<sup>7</sup> I abstract from this effect by assuming that capital goods are perfectly mobile between industry sectors and by assuming that households can convert capital goods into consumption goods at no costs. Thus, the incorporation of capital does not add any new 'frictions' to the standard static general equilibrium analysis. Despite this, unions still reduce the capital stock due to (what may be called) a 'hold-up' problem at the aggregate level: An unanticipated permanent increase in wage pressure increases wage-claims and reduces employment. This reduces the lifetime income of households who reduce savings (and consumption) and thus capital accumulation.

The percentage fall in consumption and capital stock from the old steady state to the new steady state equals the percentage fall in employment (which approximately equals the percentage point increase in unemployment) whereas the real interest rate is unchanged. The economy would jump immediately to this new steady state if households consumed the difference between the capital stock at the old steady state and the new steady state. However, this does not occur because of households preferences for consumption smoothing. Instead, consumption falls on impact and both consumption and capital decrease over time towards their new steady state levels. The development of the capital stock implies that the real interest rate is increasing over time which is the reason for the overshooting of employment: It is, *ceteris paribus*, better to work at times where the interest rate is high which implies that unions adjust wages over time such that employment is increasing over time.

The next section describes the model. Section 3 solves the model for aggregate variables and describes the dynamics. The main results, established in Section 4, are finally discussed in Section 5.

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<sup>7</sup>The under-investment result in Grout (1984) is unambiguous because firm and workers bargain over both employment and wages after the investment is made. Hoel (1990) shows that there may be over-investment if the firms has the 'right-to-manage' employment and there is only bargaining over the wage. Anderson & Devereux (1988) obtains similar results in a non-cooperative setting. Devereux & Lockwood (1991) shows that Grout's 'hold-up' result may be reversed in an OLG-model because of agents intertemporal considerations.

## 2 The Model

The economy consists of  $m$  sectors each producing an intermediate good and one sector using the intermediate goods to produce a final good. The final good is chosen as numeraire. Households buy the final good which may be used either for consumption or savings in new capital equipment. Each intermediate good is produced using capital and sector specific labour. All markets are perfectly competitive except the labour markets where each type of labour is controlled by one household/union. The household uses this power to set a wage per unit of labour that is above the opportunity costs of employment. Employment enters directly into the utility function due to increasing disutility from work and into the budget constraint due to increased wealth from working. All agents have perfect foresight.

### 2.1 The Representative Household

Each household  $i$  consists of  $\bar{L}_i$  members. The household maximizes discounted lifetime utility defined as (time subscripts are omitted to ease notation)

$$U_s = \int_{t=s}^{\infty} (\ln C_i - \beta L_i^\gamma) e^{\rho(s-t)} dt \quad , \quad \gamma > 1, \quad (1)$$

where  $C_i$  is household consumption,  $L_i$  is the number of employed household members which depends on the wage claim of the household,  $\rho$  is the rate of time preference, and  $\gamma$  is a parameter that determines the change in marginal disutility from increased work. Because of analytical tractability the analysis is confined to this utility function which is a special case of a more general class of utility functions that have reasonable properties (cf. Barro & Sala-i-Martin 1995 ch. 9).<sup>8</sup>

The flow budget constraint equals

$$\dot{A}_i = W_i L_i + R A_i - C_i + \kappa W (\bar{L}_i - L_i) + \tau, \quad (2)$$

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<sup>8</sup>E.g., employment does not converge towards zero or infinity but is constant along the balanced growth path.

where  $A_i$  is holdings of capital by the household,  $R$  is the rate of return on capital,  $W_i$  is the wage claim of the household,  $\kappa W$  is unemployment benefits which are linked to the overall wage-level, and  $\tau$  is a lump-sum tax that finances unemployment benefits. All households have the same initial holding of capital. In what follows, the upper limit on household employment,  $\bar{L}_i$ , is assumed not binding and the household is assumed to have negligible influence on the overall wage-level. Maximization of (1) subject to (2), a labour demand relationship  $L_i(W_i)$ , a non-negative consumption condition, a non-negative employment condition, and a no-Ponzi game condition yields the usual Keynes-Ramsey rule for consumption

$$\frac{\dot{C}_i}{C_i} = R - \rho, \quad (3)$$

and the wage equation

$$W_i = \frac{C_i \beta \gamma [L_i(W_i)]^{\gamma-1} + \kappa W}{1 - 1/\eta_i}, \quad (4)$$

where  $\eta_i \equiv -L'_i(W_i) W_i / L_i(W_i)$  is the (numerical) wage elasticity of labour demand. The last equation states that the wage is set as a mark-up on the (marginal) opportunity costs of employment equal to the forgone utility of leisure measured in money terms and the forgone unemployment benefits. Equation (4) may be interpreted as standard wage-curve changing over time because of capital accumulation. To derive  $\eta_i$ , it is necessary to look at the behaviour of firms.

## 2.2 The Final Good Sector

Final goods,  $Y$ , are produced using  $m$  intermediate goods according to the constant elasticity production function

$$Y = m^{\frac{1}{1-\varepsilon}} \left( \sum_{i=1}^m X_i^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{\varepsilon}{\varepsilon-1}},$$

where  $\varepsilon$  is the elasticity of substitution between inputs. Cost minimization implies that the demand for intermediate good  $i$  equals

$$X_i = \left( \frac{P_i}{P} \right)^{-\varepsilon} \frac{Y}{m} = \left( \frac{P_i}{P} \right)^{-\varepsilon} \frac{C + \dot{A}}{m}, \quad C \equiv \sum_{i=1}^m C_i, \quad A \equiv \sum_{i=1}^m A_i, \quad (5)$$

where  $C + \dot{A}$  equals the aggregate demand for final goods and where

$$P = \left( \frac{1}{m} \sum_{i=1}^m P_i^{1-\varepsilon} \right)^{\frac{1}{1-\varepsilon}} = 1, \quad (6)$$

is the marginal (and average) costs of producing  $Y$ . The last equality in (6) follows from the assumption of perfectly competitive output markets.

### 2.3 The Intermediate Good Sectors

Each intermediate good sector  $i$  is characterized by many identical firms each optimizing an intertemporal profit function. The production of good  $i$  is given by Cobb-Douglas technology

$$X_i = F(K_i, ZL_i) = \alpha K_i^{1-\nu} (ZL_i)^\nu, \quad (7)$$

where  $K_i$  and  $L_i$  are input of capital and labour, respectively.  $Z$  represents the current state of knowledge which grows at the exogenous rate  $g$ . There is no adjustment costs associated with changing inputs, and so firms use inputs according to the marginal conditions

$$F_{L_i}(K_i, L_i) = \nu \alpha K_i^{1-\nu} Z^\nu L_i^{\nu-1} = \frac{W_i}{P_i}, \quad (8)$$

$$F_{K_i}(K_i, L_i) = (1-\nu) \alpha K_i^{-\nu} Z^\nu L_i^\nu = \frac{R}{P_i}, \quad (9)$$

where the final good price,  $P_i$ , and the nominal wage,  $W_i$ , are sector specific while the rental rate on capital goods,  $R$ , is identical in all sectors. The labour demand in sector  $i$  is obtained from (5), (7), (8), and (9) (see Appendix)

$$L_i = W_i^{-(\nu\varepsilon - \nu + 1)} R^{(1-\nu)(1-\varepsilon)} Z^{\nu(\varepsilon-1)} \left[ \frac{C + \dot{A}}{m} \right]^{\nu^{1+\nu(\varepsilon-1)} \alpha^{\varepsilon-1} (1-\nu)^{(1-\nu)(\varepsilon-1)}, \quad (10)$$

which states that labour demand is decreasing in the wage and increasing in aggregate demand for final goods (consumption and investment) treated as exogenous by each household and firm both too small to influence the overall economy. Since the capital stock in sector  $i$  is endogenous, the labour demand

also depends on the rental price  $R$ ; an increase in  $R$  reduces the capital stock which reduce the marginal product of labour. The assumption of perfect capital mobility across sectors influences the wage responsiveness of labour demand; the households must take into consideration that a wage rise reduces employment, which reduces the marginal product of capital and thereby the amount of capital, which decreases employment further. Equation (10) yields

$$\eta_i = \nu\varepsilon - \nu + 1 \equiv \eta, \quad (11)$$

which relates the (numerical) wage elasticities of labour demand to the degree of substitution between intermediate goods in the production of final goods. The elasticity of substitution between intermediate goods,  $\varepsilon$ , is also an indirect measure of the degree of competition in the wage-setting. Equation (4), (10), and (11) determine employment and wage in each sector as function of the aggregate variables  $C$ ,  $R$ , and  $X$ .

### 3 General Equilibrium and the Dynamic System

Symmetric households and firms implies that sector related variables (with subscript  $i$ ) are identical across sectors, e.g.,  $P_i = P$  ( $= 1$ ). We are interested in the development of aggregate employment, consumption, and capital stock equal to  $L = mL_i$ ,  $C = mC_i$ , and  $K = mK_i$ , respectively. In order to express the dynamics in a two dimensional system, it is useful to define  $k \equiv K/(ZL) = K_i/(ZL_i)$  and  $c \equiv C/(ZL) = C_i/(ZL_i)$ . Aggregate employment is derived as a function of  $k$  and  $c$  using (4), (8), and (11):

$$L = m \left( \frac{\nu\alpha k^{1-\nu} (1 - 1/\eta - \kappa)}{\beta\gamma c/m} \right)^{\frac{1}{\gamma}}. \quad (12)$$

Equation (3), (5), (7), (9), and (12) yield (see Appendix)

$$\hat{k} \equiv \frac{\dot{k}}{k} = \alpha k^{-\nu} - \frac{(\gamma - 1)c/k + \rho + \gamma g}{\gamma - \nu}, \quad (13)$$



$$\hat{c} \equiv \frac{\dot{c}}{c} = (1 - \nu) \alpha k^{-\nu} + \frac{(1 - \nu) c/k - (\gamma - \nu + 1) \rho - \gamma g}{\gamma - \nu}. \quad (14)$$

The path of the economy is characterized by (13), (14), and a conventional transversality condition. Figure 2 illustrates the phase-diagram corresponding to (13) and (14). The economy follows the saddle path towards the unique non-trivial steady state  $(k^*, c^*)$  (see Appendix); point **C** in Figure 2. At the balanced growth path, the growth rates of aggregate consumption,  $\hat{C}$ , and capital,  $\hat{K}$ , equal the growth rate of knowledge,  $g$ , whereas aggregate employment is constant (cf. (12)).

< Figure 2 >

Both (13) and (14) are independent of the two unions related parameters  $\varepsilon$  and  $\kappa$ . Thus, the dynamics of  $k$  and  $c$  are identical for two economies having identical initial value of  $k$  but different values of  $\varepsilon$  and  $\kappa$ . Note though, that  $k$  depends both on the state variables  $K$  and  $Z$  and the control variable  $L$  which implies that  $k$  jumps if  $L$  do so (in response to some unanticipated change). Hence, to use the phase diagram, it is important to recognize that the state variable  $\tilde{K} \equiv K/Z$  is below its steady state value,  $\tilde{K}^*$ , whenever  $k$  is below its steady state value,  $k^*$ , and vice versa.<sup>9</sup>

## 4 The Impact of Monopolistic Wage Setting

This section study the implications of monopolistic wage setting in two ways. First, we compare the wage setting economy to the corresponding competitive economy that have identical capital intensity. In the competitive economy households take wages as given and unemployment benefits do not exists.<sup>10</sup> Second, we analyse the consequences of changes in wage pressure caused by changes in the (indirect) degree of competition among wage setters,  $\varepsilon$ , or in the replacement ratio,  $\kappa$ .

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<sup>9</sup>See Lemma 6 in the proof of Proposition 2.

<sup>10</sup>The aggregate variables in the competitive economy are independent of  $\varepsilon$ . However, the competitive solution may be found from the previous equations by letting  $\varepsilon$  converge towards infinity and setting  $\kappa$  equal to zero.

Let  $u$  denote the steady state unemployment rate defined as the relative difference in employment between the wage setting economy and the corresponding competitive economy in steady state.<sup>11</sup> Then (11) and (12) yield

$$u = 1 - \left(1 - \frac{1}{\nu\varepsilon - \nu + 1} - \kappa\right)^{\frac{1}{\gamma}}. \quad (15)$$

As usual the unemployment rate is an increasing function of the replacement ratio,  $\kappa$ , and a decreasing function of the degree of competition among wage setters,  $\varepsilon$ .

**Proposition 1** *Compare a wage setting economy and a competitive economy that have identical capital intensity  $k$  and identical values of all parameters except  $\varepsilon$ . (i) The employment,  $L$ , consumption,  $C$ , and capital stock,  $K$ , are  $u\%$  smaller in the union economy. (ii) The two economies have identical growth rates in employment,  $\hat{L}$ , consumption,  $\hat{C}$ , and capital stock,  $\hat{K}$ . (iii) The real wage,  $W$ , and real interest rate,  $R$ , are identical.*

**Proof.** (ii) Both (13) and (14) are independent of  $\varepsilon$ .  $\hat{L}$  is also independent of  $\varepsilon$  for a given value of  $k$  according to (12). This implies that the two economies have identical values of  $\hat{K}$  and  $\hat{C}$ . (i) The two economies have identical steady state values of  $k$  and  $c$ . It then follows from (15) that  $K$  and  $L$  are  $u\%$  smaller in the steady state of the wage setting economy. It follows from the identical growth rates that this also holds outside steady state. (iii) follows directly from (8) and (9). ■

In general, it is not likely that a wage setting economy has the same capital-labour ratio as a corresponding competitive economy since parameter changes that influence the rate of unemployment (i.e.,  $\varepsilon, \kappa, \nu, \gamma$ ) have different effects on the capital-labour ratio in the two types of economies. Proposition 1 does though describe the long run consequences of monopolistic wage setting as both economies converge to the same long run (steady

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<sup>11</sup>This is in accordance with the standard definition of unemployment equal to the difference between employment and the number of persons willing to work at the going wage rate as equation (8) shows that the steady state wage level is identical in the two economies.

state) capital intensity. This is clear from the phase diagram in Figure 2 which applies both to the wage setting economy and the competitive economy. Outside steady state, the main difference between the wage setting economy and the competitive economy is not the position of the saddle path in Figure 2 but the position on the path. This position change in the wage setting economy when  $\varepsilon$  or  $\kappa$  changes.

**Proposition 2** *Consider an unanticipated permanent reduction in  $\varepsilon$  or rise in  $\kappa$ . (i) On impact employment,  $L$ , and consumption,  $C$ , fall whereas the capital stock,  $K$ , is unchanged. (ii) Present and future growth rates of consumption,  $\hat{C}$ , and capital,  $\hat{K}$ , decrease whereas growth rates of employment,  $\hat{L}$ , increase. (iii) Present and future levels of real wages,  $W$ , increase whereas the levels of the real interest rate,  $R$ , decrease.<sup>12</sup>*

**Proof.** See Appendix. ■

Both a reduction in the degree of competition among wage setters,  $\varepsilon$ , or rise in the replacement ratio,  $\kappa$ , increase the wage claims of the households and reduce employment.<sup>13</sup> This causes an immediate rise in the capital-labour ratio (as capital cannot jump). Two examples of that are illustrated in Figure 2: If the economy is in the steady state initially then it moves from **C** to **D**. The movement from **A** to **B** illustrates a case where the economy is below its steady state initially. It is clear from Figure 2 that  $c$  always increases. This is due to the reduction in employment as aggregate consumption  $C$  declines as explained below. The increase in the capital-labour ratio raises the overall wage level and reduces the return on capital.

The wage rise of one household increases the aggregate real income of the household members but has negative externalities (similar to the static

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<sup>12</sup>Note, that the proposition holds for any initial value (not only the steady state value) of the state variable  $\bar{K} \equiv K/Z$  and for all finite changes (not only marginal changes) in  $\varepsilon$  and  $\kappa$ .

<sup>13</sup>In fact, this is not as straightforward as in static models since (4) depends on consumption. The fall in consumption increases the marginal utility of employment which counteracts the direct effect. The direct effect does, however, dominate as shown in the proof of Proposition 2.

models) on the real income of other households.<sup>14</sup> In total the negative externalities dominate and therefore all households expect lower real income in the future. The households respond by reducing both present and future consumption and savings. The reduction in savings reduces the capital stock over time which makes real wages a decreasing function of time and real interest rates an increasing function of time. This implies that the opportunity costs of consumption are an increasing function of time making the reduction of consumption an increasing function of time, i.e., the growth rates of consumption decreases. The development of real interest rates increases the marginal gain of working over time making the wage claims of the households a decreasing function of time and employment an increasing function of time, i.e., employment overshoots the new steady state level.

The economy converges towards a new steady state characterized by larger unemployment, lower capital stock, and lower consumption but unchanged wages and real interest rate (cf. Proposition 1). The difference between the long run (steady state) consequences in Proposition 1 and the short- and medium run consequences in Proposition 2 are caused by households desire for consumption smoothing. After the parameter change it is in principle possible for the households to instantly consume the difference between the two steady state capital stocks after which the economy would be in the new steady state with a permanently lower consumption level. However, such consumption path conflicts with the households desire for a smooth consumption path. Instead, households reduce consumption at all points in time after the parameter change and run down the capital stock smoothly.<sup>15</sup>

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<sup>14</sup>There are 3 negative externalities: (i) The costs of producing final goods increase which reduces real wages by increasing the consumer price index. (ii) The costs of producing capital goods increase which reduces capital demand and labour demand. (iii) The reduction in aggregate employment reduces the household's return on their capital stock. However, there are also 2 positive externalities: (i) The reduction in the employment of one sector increases the price of that particular intermediate good. Final goods producers substitute towards the other intermediate goods which increase labour demand in other sectors. (ii) Capital tends to flow to the other sectors which also increases the demand for labour in these sectors.

<sup>15</sup>Mathematically, the jump down to the new permanent level after the increase on

## 5 Concluding Remarks

It is common to analyse changes in wage pressure in static general equilibrium models. The macroeconomic outcome depends on the assumptions on the production function: Increased wage pressure (*a*) increases wages and reduces employment with decreasing returns to labour but (*b*) has a larger impact on employment and no impact on wages with constant returns to labour (cf. Figure 1). It seems natural to expect (*a*) to be the impact effect and (*b*) to be the steady state effect of an unanticipated permanent increase in wage pressure in a model with capital accumulation. This paper encompasses the static general equilibrium analysis into a standard dynamic framework with capital accumulation. The results confirm the above conjecture on the development of wages but not on the development of employment: Wages rise but converge back to the previous level whereas employment decreases most on impact and then increases towards the new lower steady state level. Consider the special case of increased wage pressure that corresponds to unionization of the labour force (i.e., when  $\varepsilon$  goes from infinity to a finite value). The consequences according to the above conjecture from the static models are a movement in Figure 1 from **A** to **B** on impact and a steady state movement from **A** to **C**. The interpretation of Figure 1 using the dynamic model is that the economy moves from **A** to **F** on impact, and afterwards both the *WS* curve and the *PS* curve moves downwards towards **C**.<sup>16</sup>

The model uses specific functional forms but does have reasonable properties; i.e., the economy converges towards a balanced growth path where the aggregate employment and the labour share of income are untrended. In a more general setting many things are possible. For instance, multiple equilibria and sunspots may arise if mark-ups are not constant (see Galí 1994).

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impact would violate (3).

<sup>16</sup>A more detailed description is provided in the Appendix.

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Figure 1. Macroeconomic outcome.

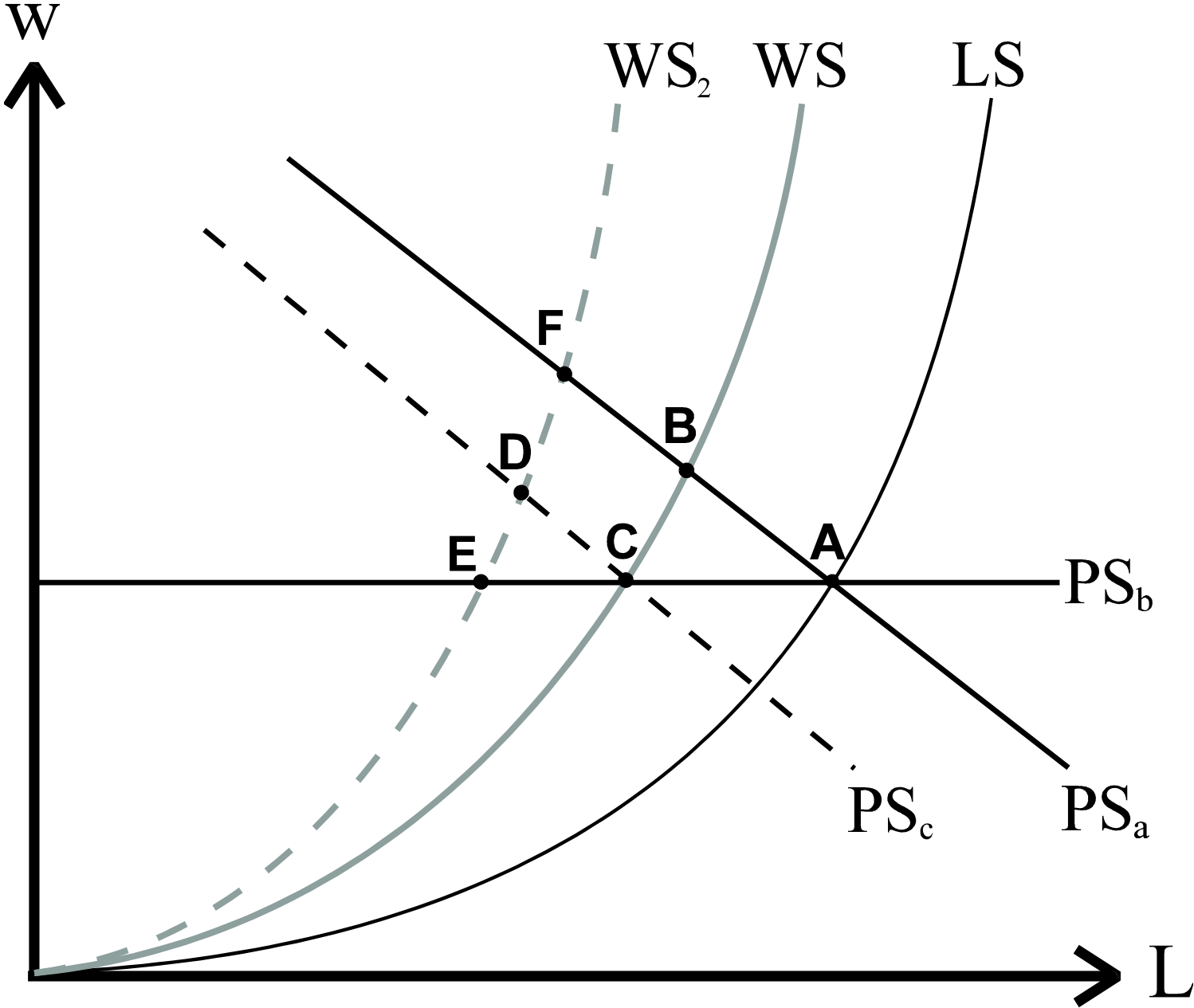
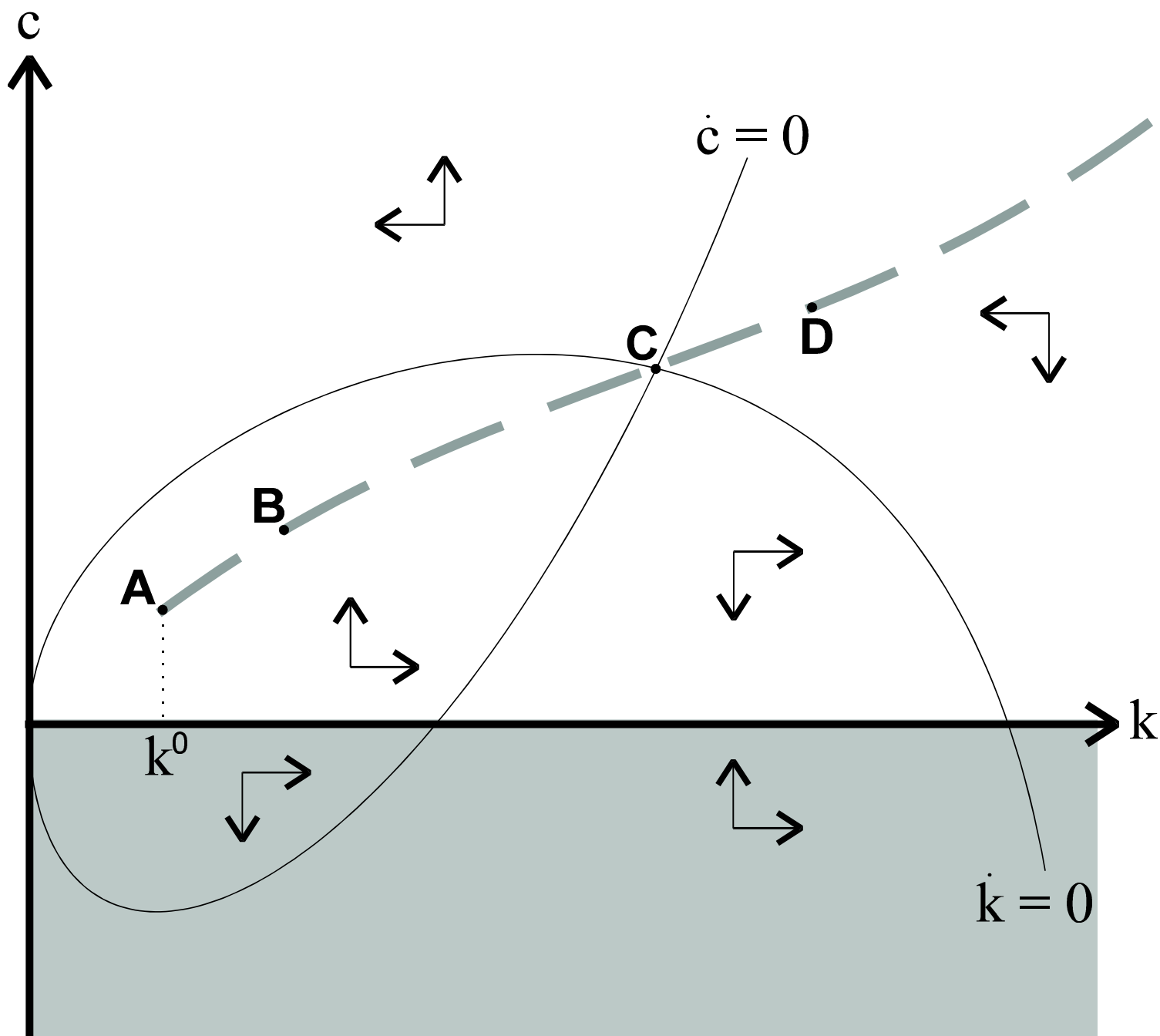


Figure 2. Phase diagram.





## A Appendix

### A.1 The optimization of households

This section derives (3) and (4). The optimal solution is obtained by setting up the present value Hamiltonian

$$H = (\log C_i - \beta [L_i(W_i)]^\gamma) e^{-\rho t} + \lambda [W_i L_i(W_i) + R A_i - C_i + \kappa W (\bar{L}_i - L_i(W_i))],$$

where  $\lambda$  is the costate variable associated with the state variable  $A_i$ .  $C_i$  and  $W_i$  are the control variables. Differentiation with respect to  $C_i$  yields

$$H_{C_i} = \frac{e^{-\rho t}}{C_i} - \lambda = 0.$$

Differentiation with respect to  $A_i$  and inserting the above equation yields

$$H_{A_i} = \lambda R = -\dot{\lambda} = \rho \frac{e^{-\rho t}}{C_i} + \frac{e^{-\rho t}}{C_i^2} \dot{C}_i$$

$\Rightarrow$

$$\frac{\dot{C}_i}{C_i} = R - \rho,$$

which is identical to (3). Differentiation with respect to  $W_i$  yields

$$-\beta \gamma [L_i(W_i)]^{\gamma-1} e^{-\rho t} L_i'(W_i) + \lambda [L_i(W_i) + W_i L_i'(W_i) - \kappa W L_i'(W_i)] = 0$$

$\Rightarrow$

$$W_i = \frac{C_i \beta \gamma [L_i(W_i)]^{\gamma-1} + \kappa W}{1 - 1/\eta_i}, \quad \eta_i = -\frac{L_i'(W_i) W_i}{L_i(W_i)}.$$

### A.2 Derivation of the labour demand

This section derives (10).  $P_i$  is isolated from (7) and (5) and inserted into (8)

$$\nu \alpha K_i^{1-\nu} Z^\nu L_i^{\nu-1} = \frac{W_i}{\left[ \alpha K_i^{1-\nu} Z^\nu L_i^\nu \right]^{-\frac{1}{\varepsilon}} \left[ \frac{C+\dot{A}}{m} \right]^{\frac{1}{\varepsilon}} P}$$

$\Leftrightarrow$

$$\frac{\nu}{L_i} \left[ \alpha K_i^{1-\nu} Z^\nu L_i^\nu \right]^{\frac{\varepsilon-1}{\varepsilon}} \left[ \frac{C+\dot{A}}{m} \right]^{\frac{1}{\varepsilon}} P = W_i$$

$$\begin{aligned}
&\Leftrightarrow \nu \alpha^{\frac{\varepsilon-1}{\varepsilon}} K_i^{\frac{(1-\nu)(\varepsilon-1)}{\varepsilon}} Z^{\frac{\nu(\varepsilon-1)}{\varepsilon}} L_i^{-1+\frac{\nu(\varepsilon-1)}{\varepsilon}} \left[ \frac{C + \dot{A}}{m} \right]^{\frac{1}{\varepsilon}} P = W_i \\
&\Leftrightarrow \nu \alpha^{\frac{\varepsilon-1}{\varepsilon}} K_i^{\frac{(1-\nu)(\varepsilon-1)}{\varepsilon}} Z^{\frac{\nu(\varepsilon-1)}{\varepsilon}} L_i^{\frac{\nu(\varepsilon-1)-\varepsilon}{\varepsilon}} \left[ \frac{C + \dot{A}}{m} \right]^{\frac{1}{\varepsilon}} P = W_i \quad (16)
\end{aligned}$$

Dividing (8) with (9) gives

$$\frac{\nu}{1-\nu} \frac{K_i}{L_i} = \frac{W_i}{R}$$

$\Leftrightarrow$

$$K_i = \frac{1-\nu}{\nu} \frac{W_i}{R} L_i$$

Inserting this into (16) gives

$$\nu \alpha^{\frac{\varepsilon-1}{\varepsilon}} \left( \frac{1-\nu}{\nu} \frac{W_i}{R} L_i \right)^{\frac{(1-\nu)(\varepsilon-1)}{\varepsilon}} Z^{\frac{\nu(\varepsilon-1)}{\varepsilon}} L_i^{\frac{\nu(\varepsilon-1)-\varepsilon}{\varepsilon}} \left[ \frac{C + \dot{A}}{m} \right]^{\frac{1}{\varepsilon}} P = W_i$$

$\Leftrightarrow$

$$\nu^{\frac{1+\nu(\varepsilon-1)}{\varepsilon}} \alpha^{\frac{\varepsilon-1}{\varepsilon}} (1-\nu)^{\frac{(1-\nu)(\varepsilon-1)}{\varepsilon}} R^{-\frac{(1-\nu)(\varepsilon-1)}{\varepsilon}} Z^{\frac{\nu(\varepsilon-1)}{\varepsilon}} L_i^{-\frac{1}{\varepsilon}} \left[ \frac{C + \dot{A}}{m} \right]^{\frac{1}{\varepsilon}} P = W_i^{\frac{1+\nu(\varepsilon-1)}{\varepsilon}}$$

$\Leftrightarrow$

$$L_i = W_i^{\nu-1-\nu\varepsilon} R^{(1-\nu)(1-\varepsilon)} Z^{\nu(\varepsilon-1)} \left[ \frac{C + \dot{A}}{m} \right] \nu^{1+\nu(\varepsilon-1)} \alpha^{\varepsilon-1} (1-\nu)^{(1-\nu)(\varepsilon-1)},$$

where we have also used  $P = 1$  from equation (6).

### A.3 Derivation of the dynamic system

This section derives (13) and (14). Equation (7), equation (5), and the assumption of clearing in the capital market yield

$$\dot{K} = \alpha K^{1-\nu} (ZL)^\nu - C, \quad (17)$$

$\Rightarrow$

$$\frac{\dot{k}}{k} = \frac{\dot{K}}{K} - \frac{\dot{L}}{L} - \frac{\dot{Z}}{Z} = \alpha k^{-\nu} - c/k - \frac{\dot{L}}{L} - g, \quad (18)$$

and (3) and (9) give

$$\frac{\dot{C}}{C} = (1 - \nu) \alpha K^{-\nu} (ZL)^\nu - \rho.$$

$\Rightarrow$

$$\frac{\dot{c}}{c} = \frac{\dot{C}}{C} - \frac{\dot{L}}{L} - \frac{\dot{Z}}{Z} = (1 - \nu) \alpha k^{-\nu} - \rho - \frac{\dot{L}}{L} - g. \quad (19)$$

The growth rate of employment is derived by differentiation of (12) and inserting  $\dot{k}/k$  and  $\dot{c}/c$ :

$$\frac{\dot{L}}{L} = \frac{1}{\gamma} \left( (1 - \nu) \frac{\dot{k}}{k} - \frac{\dot{c}}{c} \right),$$

$\Rightarrow$

$$\frac{\dot{L}}{L} = -\frac{(1 - \nu) c/k}{\gamma - \nu} + \frac{\rho + \nu g}{\gamma - \nu}. \quad (20)$$

Inserting this into (18) and (19) gives

$$\frac{\dot{k}}{k} = \alpha k^{-\nu} - \frac{\gamma - 1}{\gamma - \nu} c/k - \frac{\rho}{\gamma - \nu} - \frac{\gamma}{\gamma - \nu} g,$$

$$\frac{\dot{c}}{c} = (1 - \nu) \alpha k^{-\nu} - \rho + \frac{(1 - \nu)}{\gamma - \nu} c/k - \frac{\rho}{\gamma - \nu} - \frac{\gamma}{\gamma - \nu} g.$$

$\Rightarrow$

$$\frac{\dot{k}}{k} = \alpha k^{-\nu} - \frac{(\gamma - 1) c/k + \rho + \gamma g}{\gamma - \nu},$$

$$\frac{\dot{c}}{c} = (1 - \nu) \alpha k^{-\nu} + \frac{(1 - \nu) c/k - (\gamma - \nu + 1) \rho - \gamma g}{\gamma - \nu}.$$

#### A.4 Construction of the phase diagram

Setting  $\dot{k}/k$  equal to zero gives

$$c(k)|_{\dot{k}=0} = \frac{(\gamma - \nu) \alpha k^{1-\nu} - (\rho + \gamma g) k}{\gamma - 1} \quad (21)$$

Differentiation of (21) yields

$$c'(k)|_{\dot{k}=0} = \frac{(1 - \nu) (\gamma - \nu) \alpha k^{-\nu} - (\rho + \gamma g)}{\gamma - 1}$$

⇒

$$\lim_{k \rightarrow 0} c'(k)|_{\dot{c}=0} = \infty \quad , \quad \lim_{k \rightarrow \infty} c'(k)|_{\dot{c}=0} = -\frac{\rho + \gamma g}{\gamma - 1}.$$

Setting  $\dot{c}/c$  equal to zero gives

$$c(k)|_{\dot{c}=0} = -(\gamma - \nu) \alpha k^{1-\nu} + \frac{(\gamma - \nu + 1) \rho + \gamma g}{1 - \nu} k \quad (22)$$

Differentiation of (22) yields

$$c'(k)|_{\dot{c}=0} = -(1 - \nu) (\gamma - \nu) \alpha k^{-\nu} + \frac{(\gamma - \nu + 1) \rho + \gamma g}{1 - \nu}$$

⇒

$$\lim_{k \rightarrow 0} c'(k)|_{\dot{c}=0} = -\infty \quad , \quad \lim_{k \rightarrow \infty} c'(k)|_{\dot{c}=0} = \frac{(\gamma - \nu + 1) \rho + \gamma g}{1 - \nu}.$$

The non trivial steady state is characterized by

$$k^* = \left( \frac{\alpha (1 - \nu)}{\rho + g} \right)^{1/\nu} \quad , \quad c^* = \nu \alpha (k^*)^{1-\nu} + \rho k^*$$

Using the equations above, it is seen that (21) and (22) have the graphic from outlined in Figure 1. The assumption of non-negative consumption and employment excludes the economy from being in the shaded area. The dashed curve is the saddle path. On the saddle path,  $\chi \equiv c/k$  is a decreasing function of  $k$  when  $k < k^*$  and an increasing function when  $k > k^*$  (see the proof of Proposition 2, Lemma 1). Note, that the two curves may intersect to the right of the maximum of (21) as illustrated in Figure 1 due to endogenous employment. This happens if the labour supply is sufficiently elastic:

$$c'(k)|_{\dot{c}=0} = 0 \quad \Leftrightarrow \quad \bar{k} = \left[ \frac{\alpha (1 - \nu) (\gamma - \nu)}{\rho + \gamma g} \right]^{1/\nu}$$

⇒

$$k^* > \bar{k} \quad \Leftrightarrow \quad 1 + \nu + \nu g/\rho > \gamma.$$

## A.5 Proof of saddle point stability

(13) and (14) are linearized around steady state. The determinant of the Jacobian yields:

$$J = \begin{vmatrix} (1 - \nu) \alpha (k^*)^{-\nu} - \frac{\rho + \gamma g}{\gamma - \nu} & -\frac{\gamma - 1}{\gamma - \nu} \\ -\nu (1 - \nu) \alpha (k^*)^{-\nu - 1} c^* - \frac{1 - \nu}{\gamma - \nu} (c^*/k^*)^2 & (1 - \nu) \alpha (k^*)^{-\nu} + 2 \frac{1 - \nu}{\gamma - \nu} c^*/k^* - \frac{(\gamma + 1 - \nu) \rho + \gamma g}{\gamma - \nu} \end{vmatrix}$$

$\Rightarrow$

$$J = \begin{vmatrix} \rho + g - \frac{\rho + \gamma g}{\gamma - \nu} & -\frac{\gamma - 1}{\gamma - \nu} \\ -\nu(\rho + g) \left( \nu \frac{\rho + g}{1 - \nu} + \rho \right) - \frac{1 - \nu}{\gamma - \nu} \left( \nu \frac{\rho + g}{1 - \nu} + \rho \right)^2 & \rho + g + 2 \frac{1 - \nu}{\gamma - \nu} \left( \nu \frac{\rho + g}{1 - \nu} + \rho \right) - \frac{(\gamma + 1 - \nu)\rho + \gamma g}{\gamma - \nu} \end{vmatrix}$$

$\Rightarrow$

$$\det J = -\frac{\nu\gamma(\nu g + \rho)(\rho + g)}{(1 - \nu)(\gamma - \nu)} < 0,$$

where the last inequality follows from the assumption  $\gamma > 1$ . It is clear from Figure 1 that the two trivial steady states are unstable. Thus, point **C** is globally stable.

## A.6 Proof of Proposition 2

Let a "hat" over a variable denote the growth rate of the variable. The proof is divided into 10 lemmas:

**Lemma 1** *The consumption-capital ratio,  $\chi \equiv c/k$ , is decreasing over time if  $k < k^*$  and increasing if  $k > k^*$ .*

**Proof.** From (18) and (19), we get

$$\hat{\chi} \equiv \frac{\dot{\chi}}{\chi} = \frac{\dot{c}}{c} - \frac{\dot{k}}{k} = -\nu\alpha k^{-\nu} - \rho + \chi$$

$\Rightarrow$

$$\frac{d\hat{\chi}}{dt} = \nu^2\alpha k^{-\nu}\hat{k} + \dot{\chi}.$$

The term  $\nu^2\alpha k^{-\nu}\hat{k}$  is positive when  $k < k^*$ . The above equation implies that  $\dot{\chi}$  will continue to increase if  $\hat{\chi} > 0$  contradicting the fact that the economy will ever approach a steady state. Thus,  $\hat{\chi} < 0$  if  $k < k^*$ . The opposite argument applies when  $k > k^*$  implying that  $\hat{\chi} > 0$  if  $k > k^*$ . ■

**Lemma 2**  *$\hat{k} \equiv \dot{k}/k$  is decreasing over time when  $k < k^*$  and increasing when  $k > k^*$ .*

**Proof.** (13) yields

$$\frac{d\hat{k}}{dt} = -\nu\alpha k^{-\nu}\frac{\dot{k}}{k} - \frac{\gamma - 1}{\gamma - \nu}\dot{\chi}$$

$\Rightarrow$

$$\frac{d\hat{k}}{dt} = -\nu\alpha k^{-\nu} \left( \alpha k^{-\nu} - \frac{(\gamma-1)\chi + \rho + \gamma g}{\gamma - \nu} \right) - (-\nu\alpha k^{-\nu} - \rho + \chi) \frac{\gamma-1}{\gamma-\nu} \chi$$

The first term is negative whereas the second is positive when  $k < k^*$ .  $\dot{k}/k > 0$  implies that

$$\Gamma \equiv \alpha k^{-\nu} - \frac{\rho + \gamma g}{\gamma - \nu} > \frac{\gamma-1}{\gamma-\nu} \chi$$

$\Rightarrow$

$$\frac{d\hat{k}}{dt} < -\nu\alpha k^{-\nu} \left( \alpha k^{-\nu} - \frac{(\gamma-1)\chi + \rho + \gamma g}{\gamma - \nu} \right) - (-\nu\alpha k^{-\nu} - \rho + \chi) \left( \alpha k^{-\nu} - \frac{\rho + \gamma g}{\gamma - \nu} \right)$$

$\Leftrightarrow$

$$\frac{d\hat{k}}{dt} < \nu\alpha k^{-\nu} \frac{(\gamma-1)\chi}{\gamma-\nu} - (\chi - \rho) \left( \alpha k^{-\nu} - \frac{\rho + \gamma g}{\gamma - \nu} \right)$$

$\Leftrightarrow$

$$\frac{d\hat{k}}{dt} < \frac{\nu\alpha k^{-\nu} (\gamma-1)\chi - (\chi - \rho) ((\gamma - \nu)\alpha k^{-\nu} - \rho - \gamma g)}{\gamma - \nu}$$

$\Leftrightarrow$

$$\frac{d\hat{k}}{dt} < \frac{\gamma\alpha k^{-\nu} (\rho + \nu g - \chi(1 - \nu)) + (\rho + \gamma g) (\chi - \nu\alpha k^{-\nu} - \rho)}{\gamma - \nu}$$

Lemma 1 implies that  $\hat{\chi} = \chi - \nu\alpha k^{-\nu} - \rho < 0$  when  $k < k^*$ . Lemma 1 and  $\chi^* \equiv c^*/k^* = (\rho + \nu g) / (1 - \nu)$  imply that  $\rho + \nu g - \chi(1 - \nu) < 0$  when  $k < k^*$ . Thus,  $d\hat{k}/dt < 0$  when  $k < k^*$ . Using the same method, it is straightforward to prove the opposite, i.e.  $d\hat{k}/dt > 0$  when  $k < k^*$ . ■

**Lemma 3**  $\hat{K} \equiv \dot{K}/K$  is decreasing over time when  $k < k^*$  and increasing when  $k > k^*$ .

**Proof.** From (17), we have  $\dot{K} = \alpha K^{1-\nu} (ZL)^\nu - C \Rightarrow$

$$\hat{K} = \frac{\dot{K}}{K} = \alpha k^{-\nu} - c/k$$

$\Rightarrow$

$$\frac{d\hat{K}}{dt} = -\nu\alpha k^{-\nu} \frac{\dot{k}}{k} - \dot{\chi}$$

$\Rightarrow$

$$\frac{d\hat{K}}{dt} = -\nu\alpha k^{-\nu} \left( \alpha k^{-\nu} - \frac{(\gamma-1)\chi + \rho + \gamma g}{\gamma - \nu} \right) - (-\nu\alpha k^{-\nu} - \rho + \chi)\chi.$$

The first term is negative whereas the second term is positive. Lemma 1 implies that  $\hat{\chi} = -\nu\alpha k^{-\nu} - \rho + \chi < 0 \Leftrightarrow \nu\alpha k^{-\nu} > \chi - \rho$ . Using this inequality to substitute the term outside the first bracket yields

$$\frac{d\hat{K}}{dt} < -(\chi - \rho) \left( \alpha k^{-\nu} - \frac{(\gamma-1)\chi + \rho + \gamma g}{\gamma - \nu} \right) - (-\nu\alpha k^{-\nu} - \rho + \chi)\chi$$

$\Leftrightarrow$

$$\frac{d\hat{K}}{dt} < -(\chi - \rho) \left( \alpha k^{-\nu} - \frac{\rho + \gamma g}{\gamma - \nu} \right) + \nu\alpha k^{-\nu}\chi - (\chi - \rho)\chi \left( 1 - \frac{\gamma-1}{\gamma-\nu} \right)$$

$\Leftrightarrow$

$$\frac{d\hat{K}}{dt} < \frac{-(\chi(1-\nu) - \rho)(\gamma - \nu)\alpha k^{-\nu} + (\chi - \rho)(\rho + \gamma g) - (\chi - \rho)\chi(1-\nu)}{\gamma - \nu}$$

$\Leftrightarrow$

$$\frac{d\hat{K}}{dt} < \frac{(\rho + \nu g - \chi(1-\nu))\gamma\alpha k^{-\nu} + (\chi - \nu\alpha k^{-\nu} - \rho)(\rho + \gamma g - \chi(1-\nu))}{\gamma - \nu}$$

$\Leftrightarrow$

$$\frac{d\hat{K}}{dt} < \frac{(\chi + (\gamma - \nu)\alpha k^{-\nu} - \rho)(\rho + \nu g - \chi(1-\nu)) + (\gamma - \nu)g(\chi - \nu\alpha k^{-\nu} - \rho)}{\gamma - \nu}.$$

Lemma 1 implies that  $\hat{\chi} = \chi - \nu\alpha k^{-\nu} - \rho < 0$  when  $k < k^*$ . Lemma 1 and  $\chi^* \equiv c^*/k^* = (\rho + \nu g) / (1 - \nu)$  imply that  $\rho + \nu g - \chi(1 - \nu) < 0$ . Thus,  $d\hat{K}/dt < 0$  if

$$\chi > \rho - (\gamma - \nu)\alpha k^{-\nu}.$$

This is fulfilled as

$$\chi > \chi^* = \frac{\rho + \nu g}{1 - \nu} > \frac{\rho + \nu g - \gamma(\rho + g)}{1 - \nu} = \rho - (\gamma - \nu)\alpha(k^*)^{-\nu} > \rho - (\gamma - \nu)\alpha k^{-\nu},$$

holds whenever  $k < k^*$ . Applying the same method for the case  $k > k^*$  gives a corresponding inequality that has to be fulfilled:

$$\frac{d\hat{K}}{dt} > \frac{(\chi + (\gamma - \nu)\alpha k^{-\nu} - \rho)(\rho + \nu g - \chi(1-\nu)) + (\gamma - \nu)g(\chi - \nu\alpha k^{-\nu} - \rho)}{\gamma - \nu}.$$

Lemma 1 implies that  $\hat{\chi} = \chi - \nu\alpha k^{-\nu} - \rho > 0$  when  $k > k^*$ . Lemma 1 and  $\chi^* \equiv c^*/k^* = (\rho + \nu g)/(1 - \nu)$  imply that  $\rho + \nu g - \chi(1 - \nu) > 0$ . Thus,  $d\hat{K}/dt > 0$  if

$$\chi > \rho - (\gamma - \nu)\alpha k^{-\nu}.$$

Lemma 1 implies that  $\hat{\chi} = \chi - \nu\alpha k^{-\nu} - \rho > 0 > -\gamma\alpha k^{-\nu}$  when  $k > k^*$ . ■

**Lemma 4**  $\hat{C} \equiv \dot{C}/C$  is decreasing over time when  $k < k^*$  and increasing when  $k > k^*$ .

**Proof.** Equation (3) and (9) yield

$$\hat{C} = (1 - \nu)\alpha k^{-\nu} - \rho$$

$\Rightarrow$

$$\frac{d\hat{C}}{dt} = -\nu(1 - \nu)\alpha k^{-\nu}\hat{k}.$$

It follows from Lemma 2 that  $\hat{k} > 0$  when  $k < k^*$  implying that  $\frac{d\hat{C}}{dt} < 0$ . The opposite is true when  $k > k^*$ . ■

**Lemma 5**  $\hat{L} \equiv \dot{L}/L$  is increasing over time when  $k < k^*$  and decreasing when  $k > k^*$ .

**Proof.** Equation (20) yields

$$\hat{L} = -\frac{(1 - \nu)\chi}{\gamma - \nu} + \frac{\rho + \nu g}{\gamma - \nu} \quad \Rightarrow \quad \frac{d\hat{L}}{dt} = -\frac{(1 - \nu)}{\gamma - \nu}\dot{\chi},$$

which is positive when  $k < k^*$  and negative when  $k > k^*$  according to Lemma 1. ■

**Lemma 6**  $k^* - k$  is an increasing function of  $\tilde{K}^* - \tilde{K}$ .

**Proof.** Lemma 3 and

$$\frac{\dot{\tilde{K}}}{\tilde{K}} = \frac{\dot{K}}{K} - g,$$

implies that  $\dot{\tilde{K}}/\tilde{K}$  is decreasing over time when  $k < k^*$  and increasing when  $k > k^*$ . This implies together with Lemma 2 that both  $k - k^*$  and  $\tilde{K} - \tilde{K}^*$  increases over time when  $k < k^*$  and decreases over time when  $k > k^*$ . ■



**Lemma 7** *A reduction in  $\varepsilon$  or rise in  $\kappa$  increases  $k$  on impact.*

**Proof.** Denote by  $\tilde{K}_1^*$  the steady state value of the capital-knowledge ratio before the parameter change and  $\tilde{K}_2^*$  the steady state value after the parameter change. Proposition 1 implies that  $\tilde{K}_2^* < \tilde{K}_1^*$ . Let  $\tilde{K}^0$  be the initial value of the capital-knowledge ratio. We know from Lemma 6 that  $k^* - k^0$  is an increasing function of  $\tilde{K}^* - \tilde{K}^0$ . The fact that  $k^*$  is unchanged whereas  $\tilde{K}_2^* - \tilde{K}^0 < \tilde{K}_1^* - \tilde{K}^0$  implies that  $k_2^0 > k_1^0$ , i.e. the value of the capital intensity immediately after the parameter change,  $k_2^0$ , is larger than the capital intensity just before the parameter change,  $k_1^0$ . ■

**Lemma 8** *Part (i) of Proposition 2: On impact a reduction in  $\varepsilon$  or rise in  $\kappa$  reduces employment,  $L$ , and consumption,  $C$ , whereas the capital stock,  $K$ , is unchanged.*

**Proof.**  $K$  is the state variable and is therefore unchanged.  $k = \frac{K}{ZL}$  increases on impact according to Lemma 7 which is only possible if  $L$  decreases as  $Z$  is also a state variable. Lemma 1 implies that  $\chi = C/K$  decreases when  $k$  increases. This is only possible if  $C$  decreases as  $K$  is the state variable. ■

**Lemma 9** *Part (ii) of Proposition 2: A reduction in  $\varepsilon$  or rise in  $\kappa$  reduces present and future growth rates of consumption,  $\hat{C}$ , and capital,  $\hat{K}$ , whereas the growth rates of employment,  $\hat{L}$ , increase.*

**Proof.** Lemma 3, 4, and 5 imply that  $\hat{C}$  and  $\hat{K}$  is a decreasing function of  $k$  whereas  $\hat{L}$  is an increasing function of  $k$ . It follows from Lemma 7 that  $k$  increases on impact implying that  $\hat{C}$  and  $\hat{K}$  decrease whereas  $\hat{L}$  increases. This also applies to future values of  $\hat{C}$ ,  $\hat{K}$ , and  $\hat{L}$  as future values of  $k$  increase because (13) is independent of  $\varepsilon$  and  $\kappa$ . ■

**Lemma 10** *Part (iii) of Proposition 2: A reduction in  $\varepsilon$  or rise in  $\kappa$  reduces present and future levels of the real interest rate,  $R$ , and increases present and future levels of the real wage,  $W$ .*

**Proof.** Equation (8) and (9) yield

$$W = \nu \alpha k^{1-\nu} Z \quad , \quad R = (1 - \nu) \alpha k^{-\nu} ,$$

showing that  $W$  is an increasing function of  $k$  whereas  $R$  is a decreasing function of  $k$ . It follows from Lemma 2 and Lemma 7 that present and future values of  $k$  increase establishing the result. ■

## A.7 Unionization in Figure 1 using the dynamic model

On the balanced growth path the real wage grows with the exogenous rate  $g$  due to labour augmenting technological progress. Therefore, we will look at the real wage per effective units of labour input:  $w \equiv W/Z$ . The  $PS$  curve is found from (8)

$$w = \frac{W}{Z} = \nu \alpha (k)^{1-\nu} = \nu \alpha \left( \frac{K}{ZL} \right)^{1-\nu} , \quad (23)$$

which is a downwards sloping curve for given values of the state variables  $K$  and  $Z$ . The wage curve is determined from (4) giving

$$w = \frac{W}{Z} = \frac{c\beta\gamma L^\gamma}{m^{1+\gamma} (1 - 1/\eta - \kappa)} , \quad (24)$$

which is an upward sloping curve for a given value of  $c$ . Point **A** in Figure 1 corresponds to the intersection between the two curves at the steady state  $(k^*, c^*)$  when  $\eta \rightarrow \infty$ . Point **C** corresponds to the intersection between two new set of curves at the same steady state  $(k^*, c^*)$  after unionization; i.e., when  $\eta$  takes some finite value. The impact effect of unionization is a movement from **A** to **F** corresponding to the new  $WS_2$  curve: The  $PS$  curve is unchanged according to (23) whereas the wage curve moves both because of the reduction in  $\eta$  and the increase in  $c$ . From Proposition 2, we know that it has to move sufficiently to the left of the  $WS$  curve in order to get an intersection to the left of point **C**. After the impact effect to point **F**,  $k$  and  $c$  decline over time which moves the  $PS$  curve to the left and the  $WS$  curve to the right over time such that the intersection between the curves converges towards the final steady state **C**.