

Optimal taxation with household production¹

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Abstract

This paper suggests that the optimal tax system should favour market-produced services which are close substitutes for home-produced services. First, we modify the classical Corlett-Hague rule for optimal commodity taxation by showing that it may be optimal to impose a relatively low tax rate on consumer services even if such services are complements to leisure. Second, we find that when services and other goods are equally substitutable for leisure, so that uniform commodity taxation would be optimal in the absence of home production, the optimal tax structure will certainly involve a relatively low tax rate on consumer services.

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1. Introduction

Ever since the classical contribution by Corlett and Hague (1953), tax theorists have recognized that a commodity tax system which minimizes the excess burden of taxation involves relatively high rates of tax on goods that are complementary to leisure, in order to offset the distortionary effect of taxes on labour supply. As shown by Sandmo (1974) and Sadka (1977), uniform commodity taxation is optimal only if preferences are homothetic in goods and weakly separable in goods and leisure.

The practical applicability of the Corlett-Hague rule has so far been limited by the fact that little is known about the degree of complementarity between leisure and the various goods and services. The present paper suggests that optimal tax theory offers more guidance for tax policy once it is recognized that many services supplied from the market may also be produced within the household sector where they cannot be taxed. We are thinking of 'consumer services' such as house repair, repair of other consumer durables, cleaning and window-cleaning, garden care, housekeeping, cooking and dish-washing, haircuts, etc.

The analysis below indicates that it is optimal to have a relatively low tax burden on such consumer services. In our model economy, taxes distort the choice between the consumption of services and the consumption of other goods, the choice between labour and leisure, and the allocation of labour between home production and market production. The optimal tax system balances these distortions against each other. We find that the degree of substitutability between leisure and the various goods and services still plays an important role for optimal taxation, but our main result is a modified Corlett-Hague rule showing that the inevitable tax distortion in favour of household production may call for a relatively low tax rate on consumer services even if these services are complementary to pure leisure. The point is that a high tax on complements to leisure may not be an efficient way of stimulating tax-discouraged labour supply to the market if such a tax encourages substitution of home production for market production. To put it another

way, taxes should distort the pattern of market activity as little as possible, and since untaxed home production tends to reduce market production of services relative to market production of other goods, there is a presumption in favour of a lenient tax treatment of services.

We also show that if utility is homothetic in services and other goods, and if goods and services are weakly separable from leisure, so that uniform commodity taxation would be optimal in the absence of home production, the optimal tax structure will certainly involve a relatively low tax rate on those market produced services which could alternatively be produced within the household sector.

Our paper is a direct descendant of the article by Sandmo (1990). In our benchmark model we are more restrictive than Sandmo by abstracting from taxable non-labour inputs to home production, assuming like Piggott and Whalley (1998) that labour time is the only significant input. On the other hand, whereas Sandmo studies optimal commodity taxation with household production for an exogenously given income tax rate, we allow for optimization of the entire tax system, including the income tax. Our benchmark model is essentially identical to the one set up in the first part of the paper by Piggott and Whalley (op. cit.), but while they simulate their model to substantiate their hypothesis that a VAT base broadening to include consumer services would be welfare-reducing, we offer an explicit theoretical analysis of the optimal taxation of such services within a fully optimized tax system.

The next section describes our model. Sections 3 and 4 set up the optimal tax problem and derive our modified Corlett-Hague rule for optimal taxation. In section 5 we consider optimal taxation in the benchmark case where all goods and services are equally substitutable for leisure. Section 6 discusses the more general case with a taxable commodity input to household production, and section 7 concludes the paper.

2. Taxation and resource allocation with household production

We consider an economy inhabited by a representative consumer deriving utility from consumption of consumer services (S), from consumption of other goods (C), and from leisure (L). Total service consumption is the sum of services bought in the market (S^m) and services produced in the home (S^h), and consumption of leisure is found by subtracting time spent on home production (H) and time spent on market production (N) from the total time endowment of unity. Hence we have

$$U = U(C, S, L), \quad S = S^m + S^h, \quad L = 1 - H - N \quad (1)$$

where U is a well-behaved utility function. Services bought in the market are seen to be perfect substitutes for home-produced services which are given by the concave household production function

$$S^h = h(H), \quad h' > 0, \quad h'' < 0 \quad (2)$$

Choosing the C -good as the numeraire, denoting the consumer price of services by P and the after-tax wage rate by W , the consumer must obey the budget constraint

$$C + PS^m = WN, \quad P = p + a, \quad W = w(1 - t) \quad (3)$$

where p is the producer price of market-produced services, a is the indirect tax rate on consumer services, w is the pre-tax wage rate, and t is the proportional tax rate on labour income. Notice that since C is the numeraire, the specification in (3) is fully consistent with the existence of

indirect taxes on C -goods, and a should be interpreted only as the *additional* indirect tax burden (positive or negative) on consumer services.²

Using (2) and the definitions of S and L stated in (1), the budget constraint (3) may be rewritten as

$$C + PS + WL = W + Ph(H) - WH \quad (4)$$

The consumer maximises (1) subject to (4), yielding the first-order conditions

$$\frac{U_S}{U_C} = P = p + a \quad (5)$$

$$\frac{U_L}{U_C} = W = w(1 - t) \quad (6)$$

$$Ph'(H) = W \Leftrightarrow (p + a)h'(H) = w(1 - t) \quad (7)$$

where U_S , U_C and U_L are the marginal utilities of consumer services, other goods and services, and leisure, respectively. These optimality conditions illustrate how taxes distort the choice between consumer services and other goods, the choice between leisure and material consumption, and the allocation of labour between home production and market production. Notice from (7) that time spent on home production is determined independently of the consumption of C , S and L , depending only on W and P . It is as if the consumer maximises the shadow profits $Ph(H) - WH$ in a separate ‘home production department’ and then subsequently allocates these profits plus his potential net labour income W optimally across the consumption of goods, services and leisure in

accordance with (4) through (6).³ A higher market price of services or a lower market wage rate will stimulate home production of services, since (7) implies that

$$H_P \equiv \partial H / \partial P = -h' / Ph'' > 0, \quad H_W \equiv \partial H / \partial W = 1 / Ph'' < 0 \quad (8)$$

Eqs (6) and (7) show that the shadow prices of leisure and of time spent on home production are identical and equal to $W=w(I-t)$, i.e., the consumer is indifferent at the margin between spending an extra hour working in the market, taking an extra hour of leisure, or allocating an extra hour to home production. At first glance one might therefore think that home production could be subsumed under the heading of 'leisure' and could hence be ignored for the purpose of optimal tax analysis. This would be wrong, however: since home production competes directly with market production of services (but not with market production of other goods), it distorts the pattern of demand for market-produced goods and services, and the optimal tax policy must account for this.

3. The optimal tax problem

The government's optimal tax problem is to choose a and t so as to maximise the indirect utility of the representative consumer subject to the requirement that taxes must yield an exogenous amount of revenue. The solution to this problem is identical to the solution to the dual problem of maximising total tax revenue for a given level of indirect utility. In specifying the latter problem, it will be useful to introduce the consumer's expenditure function which is derived by solving the problem

$$\begin{aligned}
& \text{Minimize } C + PS + WL \quad \text{w.r.t. } C, S, L \\
& \text{s.t. } U(C, S, L) = \bar{V}
\end{aligned} \tag{9}$$

The solution to (9) yields an expenditure function $e(P, W, \bar{V})$ with properties which follow from standard consumer theory:

$$\begin{aligned}
e_P &= S = S^m + S^h & e_W &= L = 1 - H - N \\
e_{PP} &= S_P < 0 & e_{WW} &= L_W < 0 & S_W &= e_{PW} = e_{WP} = L_P
\end{aligned} \tag{10}$$

As before, subscripts refer to partial derivatives, but S , S^m , L and N are now indicating *compensated* demands and supplies. Using (10) as well as identities from conventional demand theory, we can write the optimal tax problem in the following way,

$$\begin{aligned}
& \text{Maximize } aS^m + twN = \\
& a[e_P(P, W, \bar{V}) - h(H(P, W))] + tw[1 - H(P, W) - e_W(P, W, \bar{V})] \\
& \text{w.r.t. } a, t \quad \text{s.t. } e(P, W, \bar{V}) = W + Ph(H(P, W)) - WH(P, W)
\end{aligned} \tag{11}$$

where the right-hand side of the constraint is the full income of the consumer given in (4), and where we recall that $P = p + a$ and $W = w(I - t)$. The market sector of the economy is characterized by perfect competition and a linear production technology so that producer prices and wages p and w can be taken to be fixed. Like Sandmo (1990), we thus assume constant returns in the market

sector although household production is subject to diminishing returns. This asymmetry of technologies seems plausible since the returns to household production are typically constrained by the fixity of the physical infrastructure of the household (e.g. the house and the garden).

4. The optimal tax rule

The first-order conditions for the solution to the problem in (11) are

$$(1+\mu)(e_p - h) + a(e_{pp} - h'_{H_p}) - tw(e_{wp} + H_p) - \mu(Ph'_{H_p} - WH_p) = 0 \quad (12)$$

$$(1+\mu)(1 - H - e_w) + a(h'_{H_w} - e_{pw}) + tw(H_w + e_{ww}) + \mu(Ph'_{H_w} - WH_w) = 0 \quad (13)$$

where μ is the shadow price associated with the constraint in (11). From (7) we see that the last term on the left-hand side of (12) as well as (13) is zero. Using (7), (8) and (10), and eliminating μ by substituting (13) into (12), we find that an optimal tax policy implies

$$\frac{a(S_p - h'_{H_p}) - tw(S_w - h'_{H_w})}{S^m} = \frac{tw(L_w + H_w) - a(L_p + H_p)}{N} \quad (14)$$

The left-hand side of (14) indicates the relative change in the compensated demand for *market-produced* services induced by taxation, while the right-hand side measures the relative change in the compensated supply of labour to the *market* generated by direct and indirect taxes. Eq. (14) is therefore a Ramsey-type rule stating that tax rates should be chosen so as to generate an equal proportional change in the consumer's compensated market demand and supply. Recalling our assumption of linear production technologies in the market sector with labour as the only

input, eq. (14) implies that the optimal tax structure will change the compensated demand for market-produced consumer services and the compensated demand for other goods and services in the same proportion.⁴ In this sense tax rates should be chosen so as to distort the pattern of market activity as little as possible. Since some forms of market production can easily be replaced by home production whereas others cannot, a uniform commodity tax is likely to cause a greater contraction in some market sectors than in others. Thus, eq. (14) already suggests that it may be optimal to have relatively low tax rates on those services which are (near) perfect substitutes for home-produced services. To elaborate on this possibility, we will rewrite (14) by introducing the definitions

$$\begin{aligned}\sigma_W^S &\equiv \frac{W}{S} S_W, & \sigma_W^L &\equiv \frac{W}{L} L_W < 0, & \sigma_P^S &\equiv \frac{P}{S} S_P < 0, \\ \sigma_P^L &\equiv \frac{P}{L} L_P, & \epsilon_W^H &\equiv \frac{W}{H} H_W < 0, & \epsilon_P^H &\equiv \frac{P}{H} H_P > 0\end{aligned}\tag{15}$$

The sigmas in (15) indicate the compensated wage and price elasticities of total service demand S and of leisure demand L and the epsilons represent wage and price elasticities of time spent on home production (the signs of which follow from (8)). Using (15), we find after some manipulations that the optimal tax rule (14) may be rewritten as

$$\frac{a}{P} = \frac{t}{1-t} \cdot \left(\frac{\sigma_W^S + (S^m/S)(L/N)\sigma_W^L - (H/N)(C/PS)\epsilon_W^H}{\sigma_P^S + (S^m/S)(L/N)\sigma_P^L - (H/N)(C/PS)\epsilon_P^H} \right)\tag{16}$$

The classical Corlett-Hague rule for optimal taxation requires that goods and services which are complementary to leisure should carry a relatively high indirect tax rate in order to offset the tendency of the tax system to induce substitution towards leisure. Although it may seem most

natural to assume that consumer services and leisure are substitutes, some services may in fact be complementary to leisure. Let us therefore consider the case where consumer services and leisure are complements, implying $\sigma_w^S < 0$ and $\sigma_p^L < 0$, and let us assume realistically that the optimal income tax rate t is positive. Combining these assumptions with the signs of the elasticities in (15), we see from (16) that services should indeed be heavily taxed ($a > 0$) when home production is insignificant, i.e. when H is close to zero. In these circumstances our analysis thus confirms the conventional Corlett-Hague rule. However, if a large fraction of service demand is satisfied through home production, so that S^m/S is low and H/N is large, eq. (16) implies that it may well be optimal to impose a relatively low tax rate on consumer services ($a < 0$) even if they are complements to leisure. The same result holds unambiguously if returns to scale in household production are close to being constant, since the elasticities ϵ_w^H and ϵ_p^H will then become numerically very large relative to the other terms in (16).

The intuition for our finding is not difficult to grasp: the optimal tax system must minimize the distortionary substitution away from market activities towards untaxed leisure and home production activities. Although a high tax rate on complementary services will certainly reduce the consumption of leisure, there is no guarantee that it will also stimulate the supply of labour to the market, since a high service tax rate will also induce the consumer to spend more time on home production of services. If the latter effect is sufficiently strong, the need to offset the tax-induced discouragement of market activity may call for a relatively low tax rate on services. In other words, in the presence of home production the degrees of complementarity with leisure are no longer the only relevant determinants of the optimal commodity tax structure.

When consumer services are substitutes for rather than complements to leisure, it seems even more likely that the optimal tax system will involve a low tax rate on services. Thus, when $\sigma_w^S > 0$ and $\sigma_p^L > 0$ and home production constitutes a large fraction of total service production (S^m/S low and H/N large), the numerator on the right-hand side of (16) will certainly be positive whereas

the denominator will be negative. A relatively low tax on services ($a < 0$) will then be optimal, given a positive income tax rate.

5. Optimal taxation with separable preferences

A priori we have no basis for assuming that the aggregate of 'consumer services' is either more or less substitutable for leisure than the aggregate of 'other goods and services'. It is therefore natural to consider the benchmark case where preferences are weakly separable in consumption and leisure, so services and other goods are equally substitutable for leisure. The utility function (1) may then be specified as

$$U = U(u(C, S), L) \quad (17)$$

where $u(C, S)$ is a quasi-concave subutility function. Adding the assumption that the u -function is homothetic, we have a preference structure which is well-known to imply optimality of *uniform* commodity taxation ($a=0$) in the absence of home production, as demonstrated by Sandmo (1974) and Sadka (1977). Once one allows for home production crowding out market-based service production, one would then expect the balance to tip in favour of a relatively low tax rate on services. We shall now show that this conjecture is indeed correct.

Our first step is to rewrite the optimal tax rule (14), using (3), (7) and (10):

$$a \cdot \left(\frac{C}{PS^m} \cdot H_P - \frac{N}{S^m} \cdot e_{PP} - e_{WP} \right) = t_W \cdot \left(\frac{C}{PS^m} \cdot H_W - \frac{N}{S^m} \cdot e_{PW} - e_{WW} \right) \quad (18)$$

We wish to show that, given the preference structure assumed in (17), the two square

brackets in (18) must have opposite signs, implying optimality of a low tax on services ($a < 0$) when the income tax rate is positive. To prove this, we observe that when the subutility function $u(C, S)$ is homothetic and quasi-concave, the ratio C/S will depend only on the relative consumer price P , with C/S being higher the higher the value of P . Hence we have $C/S = f(P)$, $f' > 0$, so the expenditure function may be written as

$$\begin{aligned} e(P, W, \bar{V}) &= C + PS + WL = [P + f(P)]S + WL \quad \Leftrightarrow \\ e(P, W, \bar{V}) &= [P + f(P)]e_P + We_W \end{aligned} \quad (19)$$

Combining (4) and (19), we then get

$$[P + f(P)]e_P + We_W = W + Ph(H) - WH \quad (20)$$

Differentiating (20) w.r.t. P and using (7), we find after some manipulations that

$$-(N/S^m)e_{PP} - e_{WP} = (1/W)[S^m + f'S - (S^h/S^m)f e_{PP}] > 0 \quad (21)$$

We may also differentiate the last eq. in (19) w.r.t. W and use (20) to eliminate $P + f$ from the resulting expression, yielding

$$e_{WW} = -\left(\frac{P+f}{W}\right)e_{PW} = -\left(\frac{WN + PS^h}{WS}\right)e_{PW} \quad (22)$$

Substituting (21) and (22) into (18), our optimal tax rule may now be written as

$$\begin{aligned}
 aX &= twZ, \\
 X &\equiv (C/PS^m)H_p + (1/W)[S^m + Sf' - (S^h/S^m)fe_{pp}] > 0 \\
 Z &\equiv (C/PS^m)[H_w + \left(\frac{PS^h}{C+PS}\right)e_{ww}] < 0
 \end{aligned} \tag{23}$$

In the limiting case with no household production - where $S^h = H_w = 0$ - the expression for Z in (23) will be zero whereas X will always be positive. In this special case our model reproduces the standard result in the literature that uniform commodity taxation ($a=0$) is optimal when preferences are separable in consumption and leisure and Engel curves are straight lines through the origin. However, in the presence of household production we have $Z < 0$, and according to (23) it is then optimal to impose a relatively low indirect tax rate on such services ($a < 0$), whenever the optimal income tax rate is positive. This finding is quite intuitive: in the absence of home production there is no efficiency case for non-uniform commodity taxation when all goods and services are equally substitutable for leisure. But when home production is present, a uniform commodity tax will be particularly discouraging for those market activities which can most easily be replaced by household production. It then becomes optimal to alleviate this distortion of the pattern of market activities through lenient taxation of market-produced services which are perfect substitutes for home-produced services, even if this implies some tax bias against the consumption of other goods.

6. Allowing for taxable commodity inputs into household production

In an appendix of this paper we follow Sandmo (1990) who analyses optimal taxation in a more

general model incorporating taxable non-labour inputs into home production. Allowing for an optimization of the entire tax system (see appendix B), as opposed to Sandmo (op.cit.) who takes the labour income tax rate to be fixed, we obtain several results regarding the optimal tax structure. Let a and b denote additional tax rates on consumer services and non-labour inputs, respectively, and let t denote the labour income tax rate. On the assumptions that (i) labour and non-labour inputs are technological complements in household production, and (ii) the degree of complementarity between services and leisure is not too strong, we can derive the following propositions on the optimal tax system: (i) Uniform commodity taxation ($a=b=0$) is never optimal. (ii) If the optimal value of t is non-negative, it will never be optimal to impose a surtax on market-produced services, $a>0$, combined with a relatively low tax on non-labour inputs to home production, $b<0$. In other words, the tax system should never discriminate systematically in favour of home production. (iii) If the optimal t covers (almost) all of the government's revenue needs, the optimal tax system will always involve $a<0$ and $b>0$.

It is interesting to consider these results in the light of the famous production efficiency theorem of Diamond and Mirrlees (1971). According to this theorem input choices should not be distorted by taxes if there are constant returns to scale or if all pure profits can be taxed away. In our setting there are decreasing returns to home production, and the resulting shadow profits⁵ cannot be captured by direct taxation. For this reason the production efficiency theorem does not apply. Instead it becomes second-best optimal to use the available tax instruments to tax shadow profits in an indirect manner. In our basic model the government can cut into shadow profits only by subsidising market services ($a<0$), or by subsidising market work ($t<0$), but obviously these instruments cannot be applied simultaneously, given the government budget constraint. Hence it is not surprising that market services should be subsidised when the optimal labour income tax rate is positive. With non-labour inputs to home production a tax on these inputs ($b>0$) is an additional indirect means of taxing shadow profits. In this case we saw that if the optimal labour income tax

rate is positive, thereby boosting shadow profits, then at least one of the commodity tax rates a or b should be used to tax shadow profits in home production (result (ii) above). Furthermore, if t is large, thus boosting shadow profits a lot, the social planner should use all other available instruments ($a < 0$ and $b > 0$) to appropriate part of these profits (result (iii) above).

In more general circumstances one cannot unambiguously sign the optimal values of a and b , as Sandmo (1990) pointed out. This suggests that preferential tax treatment is optimal only for market-produced *labour* services - as argued by Richter (1997) - and not necessarily for goods the production of which requires other taxable inputs. Yet, in practice it may be very difficult to differentiate the tax treatment of inputs to home production from the tax treatment of business inputs and final consumption goods, since many goods may be used for all of these purposes. If administrative obstacles prevent a special tax treatment of inputs to home production, it can be shown that the results derived in sections 4 and 5 will continue to apply.

7. Conclusions

This paper studied optimal taxation in an economy where some market-produced commodities (denoted 'consumer services') are close substitutes for commodities produced within the household sector. Our analysis led to a revision of two well-known results in the theory of optimal taxation. First, we modified the classical Corlett-Hague rule for optimal commodity taxation by showing that it may be efficient to impose a relatively low rate of tax on consumer services even if such services are complementary to leisure. Second, we showed that a preference structure which would call for uniform commodity taxation in the absence of home production unambiguously warrants a relatively low tax rate on consumer services in the presence of household production. The basis for these results is that the optimal tax system must not only seek to minimize the distortion of the labour-leisure choice; it must also alleviate the distortion of the pattern of market

activity caused by untaxed household service production.

The analysis in this paper may thus be seen as a theoretical underpinning of the numerical simulations reported in Frederiksen et al. (1995), Sørensen (1997) and Piggott and Whalley (1998). Using more elaborate models of the interaction between the formal and the informal economy, these authors all find that consumer welfare can be improved by reducing the indirect tax rate on consumer services below the tax rate on other goods and services.

Of course, one might argue that the results of this paper have the same flavour as the original Corlett-Hague analysis:⁶ if we include time spent on household production under the heading of 'leisure', an increased consumption of 'leisure' will reduce the demand for market-produced services. Thus 'leisure' and market-produced consumer services will be close substitutes, and according to the standard Corlett-Hague rule these services should therefore be subsidised. From this perspective the main contribution of this paper is to give some guidance on how to apply the Corlett-Hague rule in a real world setting: whereas we generally know very little about the degree of substitutability between leisure and the various goods and services, it is fairly easy to point out those goods which can easily be replaced by home-produced alternatives.

In order to focus on the pure efficiency aspects of the taxation of services, we have disregarded equity concerns by neglecting heterogeneity across households. Inclusion of distributional considerations would seem to have ambiguous implications for the optimal taxation of services. On the one hand, there may be a tendency for some domestic services to weigh more heavily in the consumer budgets of high-income households, providing an equity case for a relatively high tax rate on these services. On the other hand, since production of consumer services tends to be intensive in the use of unskilled labour, a low tax rate on such services might improve the distribution of factor incomes by raising the relative demand for low-paid labour via a rise in the relative demand for services. For this reason our neglect of equity goals does not necessarily bias our results in favour of a low tax burden on services.

Appendix: optimal taxation with a taxable commodity input into household production

A. Optimal commodity taxation with an exogenous labour income tax

Sandmo (1990) considers a representative consumer with the quasi-concave utility function

$$U = U(C, S, L), \quad S = S^m + S^h, \quad L = 1 - H - N \quad (\text{A1})$$

The household production of services, S^h , is given by the strictly concave production function

$$\begin{aligned} S^h &= h(H, Z), & h_H &> 0, & h_Z &> 0 \\ h_{HH} &< 0, & h_{ZZ} &< 0, & h_{HH} \cdot h_{ZZ} - (h_{HZ})^2 &> 0 \end{aligned} \quad (\text{A2})$$

where Z is input of goods bought in the market, and H is the input of household time. We assume that H and Z are technological complements; i.e. $h_{ZH} = h_{HZ} \geq 0$. The budget constraint of the household is given by

$$\begin{aligned} C + PS^m + QZ &= WN, \\ P &= p + a, \quad Q = q + b, \quad W = w(1 - t) \end{aligned} \quad (\text{A3})$$

q is the producer price of Z -goods, and b is the (additional) indirect tax rate on Z -goods. The consumer maximises (A1) subject to (A2) and (A3), yielding a indirect utility function

$$\begin{aligned}
V &= V(p+a, q+b, w(1-t)) , \\
V_a &= -\ddot{e}S^m, \quad V_b = -\ddot{e}Z, \quad V_t = -\ddot{e}wN
\end{aligned}
\tag{A4}$$

with \ddot{e} indicating the consumer's marginal utility of lump sum income. The indirect utility function, V , is the objective function of the social planner. Sandmo assumes that the income tax rate, t , is positive and fixed. Thus, the optimal tax problem is to maximize V with respect to a and b subject to the revenue constraint

$$aS^m + bZ + twN = R \tag{A5}$$

where R is an exogenous amount of revenue measured in terms of the numeraire. By solving the optimal tax problem and using the Slutsky equations as well as the symmetry properties of the compensated cross-price effects, one can show that the optimal tax system must obey the following first-order conditions

$$\frac{a}{P} = \left(\tilde{a} + \frac{t}{1-t} \cdot \sigma_W^{S^m} - \frac{b}{Q} \cdot \sigma_Q^{S^m} \right) / \sigma_P^{S^m} \tag{A6}$$

$$\frac{b}{Q} = \left(\tilde{a} + \frac{t}{1-t} \cdot \sigma_W^Z - \frac{a}{P} \cdot \sigma_P^Z \right) / \sigma_Q^Z \tag{A7}$$

$$\tilde{a} \equiv \frac{\ddot{e} - \mu}{\mu} + a \cdot \frac{\partial S^m}{\partial Y} + tw \cdot \frac{\partial N}{\partial Y}$$

where μ is the shadow price associated with the government budget constraint, Y is (hypothetical)

lump sum income, and the \acute{o} 's refer to compensated elasticities. (A6) and (A7) correspond to equations (26) and (27) in Sandmo (1990) except that Sandmo simplifies his formulae by assuming that $\acute{o}_Q^{S^m} = \acute{o}_P^Z = 0$. From these tax formulae Sandmo correctly concludes that the optimal signs and the relative magnitude of a and b are in principle ambiguous. However, one can gain more insight into the optimal tax structure by inserting (A7) into (A6) so as to eliminate \tilde{a} :

$$\frac{a}{P} = \frac{b}{Q} \cdot \left(\frac{\acute{o}_Q^Z - \acute{o}_Q^{S^m}}{\acute{o}_P^{S^m} - \acute{o}_P^Z} \right) + \frac{t}{1-t} \cdot \left(\frac{\acute{o}_W^{S^m} - \acute{o}_W^Z}{\acute{o}_P^{S^m} - \acute{o}_P^Z} \right) \quad (\text{A8})$$

Equations (A8) and (A5) solve the optimal tax problem. Given the assumption that H and Z are technological complement, the signs of the elasticities are as follows:

$$\begin{aligned} \acute{o}_P^Z &> 0, & \acute{o}_Q^Z &< 0, & \acute{o}_W^Z &\leq 0 \\ \acute{o}_P^{S^m} &< 0, & \acute{o}_Q^{S^m} &> 0, & \acute{o}_W^{S^m} &0 \end{aligned} \quad (\text{A9})$$

From (A9) we see that the first bracketed expression on the RHS of (A8) is always positive. The second bracketed expression will be negative if $\acute{o}_W^{S^m} > \acute{o}_W^Z$; i.e. if $\acute{o}_W^{S^m}$ is not too negative. It is easy to show that $\acute{o}_W^{S^m}$ will be positive whenever services and leisure are substitutes and may be positive even when they are complements, as long as the degree of complementarity is not too strong. The condition that $\acute{o}_W^{S^m} > \acute{o}_W^Z$ allows for an even higher degree of complementarity. Given that this condition is met, we can say the following about the optimal tax system. If the income tax rate t covers the entire revenue R , it follows from the government budget

constraint (A.5) that a and b would either have to be zero or have opposite signs. In (A8) the second term on the RHS is negative, so the optimal tax rule could never be fulfilled for $a=b=0$. Furthermore, since the first bracketed expression is positive, a tax system satisfying (A8) will always be characterized by $a < 0$ and $b > 0$, implying that inputs into home production should carry a surtax whereas market-produced services should be lightly taxed or even subsidized.

B. A more general optimal tax problem

We now go beyond Sandmo (1990) by considering the case where the labour income tax rate is determined endogenously. Let the social planner maximize the indirect utility function (A4) with respect to a , b , and t subject to the government budget constraint (A5). It can be shown that the solution to this problem is given by (A5), (A8) and

$$\frac{b}{Q} = \frac{a}{P} \cdot \left(\frac{\hat{\sigma}_P^N - \hat{\sigma}_P^Z}{\hat{\sigma}_Q^Z - \hat{\sigma}_Q^N} \right) + \frac{t}{1-t} \cdot \left(\frac{\hat{\sigma}_W^Z - \hat{\sigma}_W^N}{\hat{\sigma}_Q^Z - \hat{\sigma}_Q^N} \right) \quad (\text{A10})$$

The signs of the compensated elasticities of Z are given in (A9), and the signs of the compensated labour supply elasticities are

$$\hat{\sigma}_P^N < 0, \quad \hat{\sigma}_Q^N \geq 0, \quad \hat{\sigma}_W^N > 0 \quad (\text{A11})$$

where $\hat{\sigma}_P^N$ will be negative if services and leisure are either substitutes or weak complements.

Assuming that the degree of complementarity between services and leisure is not too strong; i.e. $\hat{\sigma}_P^N < \hat{\sigma}_P^Z$ and $\hat{\sigma}_W^{S^m} > \hat{\sigma}_W^Z$, we can write (A8) and (A10) in the form

$$\frac{a}{P} = \hat{\sigma}_1 \cdot \frac{b}{Q} - \hat{\sigma}_2 \cdot \frac{t}{1-t} \quad (\text{A8'})$$

$$\frac{b}{Q} = \hat{\sigma}_3 \cdot \frac{a}{P} + \hat{\sigma}_4 \cdot \frac{t}{1-t} \quad (\text{A10'})$$

where $\hat{\sigma}_i > 0$, $i = 1, 2, 3, 4$. Equations (A5), (A8') and (A10') do not support strong conclusions regarding the optimal tax structure. Nevertheless one can make a few interesting remarks on the properties of the optimal tax system. First, uniform commodity taxation ($a=b=0$) can never be optimal. Second, if the optimal income tax covers the entire revenue need, it must be the case that the optimal tax rate on inputs into home production is positive ($b>0$), while the optimal tax rate on market-produced services is negative ($a<0$). Third, if the optimal income tax rate is non-negative, the optimal tax system could never involve a negative b combined with a positive a .

References

Corlett, W.J. and D.C. Hague (1953), 'Complementarity and the excess burden of taxation', *Review of Economic Studies*, **21**, 21-30.

- Diamond, P. and J. Mirrlees** (1971), 'Optimal taxation and public production I-II', *American Economic Review*, **61**, 8-27, 261-78.
- Frederiksen, N.K., P.R. Hansen, H. Jacobsen, and P.B. Sørensen** (1995), 'Subsidising consumer services - Effects on employment, welfare and the informal economy', *Fiscal Studies*, **16**, 71-93.
- Piggott, J. and J. Whalley** (1998), 'VAT base broadening, self supply, and the informal sector', NBER Working Paper 6349, National Bureau of Economic Research, Cambridge, MA.
- Richter, W.F.** (1997), 'An efficiency case for exempting basic services from tax', Unpublished Working Paper, Faculty of Economics, University of Dortmund.
- Sadka, E.** (1977), 'A theorem on uniform taxation', *Journal of Public Economics*, **7**, 387-91.
- Sandmo, A.** (1974), 'A note on the structure of optimal taxation', *American Economic Review*, **64**, 701-06.
- Sandmo, A.** (1990), 'Tax distortions and household production', *Oxford Economic Papers*, **42**, 78-90.
- Sørensen, P.B.** (1997), 'Public finance solutions to the European unemployment problem?', *Economic Policy*, **25**, October 1997.

Notes

1 . In preparing this paper we have benefited greatly from discussions with Agnar Sandmo and from suggestions by two anonymous referees. All remaining shortcomings are our own responsibility. Part of the work on the paper was carried out while Peter Birch Sørensen was a visiting scholar at the University of California at Santa Cruz. The hospitality of the Economics Department at UCSC and financial support from the Danish Social Science Research Council are gratefully acknowledged. The activities of the Economic Policy Research Unit are supported by a grant from the Danish National Research Foundation.

2 . We follow standard practice in the optimal tax literature by specifying the commodity tax rate a in additive terms, since this is analytically more convenient. If in reality government levies an ad valorem tax rate t^s on consumer services, an ad valorem tax rate t^c on other goods and services, and a proportional tax rate t^w on labour income, our normalization procedure implies the conversion formulae $a/p = (t^s - t^c)/(1 + t^c)$ and $t = (t^w + t^c)/(1 + t^c)$. Our 'income tax rate' t thus captures the distorting effect on labour supply of the ordinary income tax as well as the effect of the indirect tax on C -goods.

3 . By definition there can be no trade in services produced within the household. Therefore, we have a non-negativity constraint on market production of consumer services, i.e. $S^m \geq 0$ or equivalently $h(H) \leq S$. The characterization of the consumer's optimum is defined by the eqs (4)-(7) only under the assumption that there exists an interior solution where $h(H) < S$. Since we consider a representative consumer, this is clearly a reasonable assumption.

4 . If taxes reduce the demand for market-produced services by, say, 25 percent, and if labour supply to the market is likewise reduced by 25 percent, it follows from the assumption of linear production technologies that the output of other goods must also fall by 25 percent.

5 . If Z is the taxable non-labour input to home production and q is its producer price, the shadow profits in household production are given by $(p+a)h(H,Z) - (q+b)Z - w(1-t)H$.

6 . This point was suggested to us by a referee.