

# Environmental Policy in a Two Sector Endogenous Growth Model \*

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April 23, 1998

## Abstract

This paper analyses policy measures within a two sector endogenously growing economy with pollution. Pollution is either generated by production or by the use of physical capital in production, and can be reduced by public abatement activities. In this generalized Uzawa–Lucas model, the effects of fiscal policy are derived for all core variables and ratios. In addition, the optimal taxation rules are derived. If a pollution tax is not available it turns out that a first best solution may be reached by use of factor income taxation. Additionally, the effects and the possibility of environmental policy are complemented for a small open economy.

*JEL classification:* D62, O41, E62, Q28, F43

*Keywords:* Endogenous growth, environmental externality, optimal taxation, international capital mobility

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\*Financial assistance by the Danish Social Science Research Council and the Deutsche Forschungsgemeinschaft are gratefully acknowledged. We are indebted to Bernd Genser, Lucas Bretschger, Knud Jørgen Munk, and Peter Birch Sørensen for helpful comments and discussions. All deficiencies and errors are ours.

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## 1. Introduction

By analyzing the effects of environmental policy on growth this paper considers an increasingly important topic throughout the industrialized world. It contributes to the literature by analysing the effects of environmental policy in a fairly general two sector endogenous growth model. This is important, since the results crucially depend on the model specification. Our model is an extension of the generalized Uzawa–Lucas model<sup>1</sup> used by [10] King & Rebelo (1990), which additionally allows for disutility of pollution and public abatement activities that are financed by tax revenues. To keep the analysis general, we investigate two sources of gross pollution, namely pollution generated by final goods production and pollution generated by the use of physical capital in final goods production. The emission of gross pollution can be reduced by public abatement activities. The positive analysis in this paper not only examines the long term growth effects of taxes on consumption, gross pollution, capital and labour income and their channels, but also their influence on all core variables and ratios. The normative analysis addresses the question of the optimal taxation scheme. Furthermore, we investigate the effects and the possibility of environmental policies in a closed economy and a small open economy setting. The analysis of the closed economy is a dynamic general equilibrium analysis, whereas the investigation of the small open economy starts out from conventional assumptions in international trade models, thereby simplifying the analysis by an exogenously given world interest rate, but broadening the analysis by assuming perfect international capital mobility.

In the following, the literature on fiscal policy, environmental policy, and international capital mobility in human capital growth models is reviewed. Several authors, [10] King & Rebelo (1990), [21] Rebelo (1991), [6] Devereux & Love (1994), and [16] Milesi–Ferretti & Roubini (1995), analyse the consequences of tax rate changes for economic growth with production specifications similar to this paper. However, they do not consider the environment and redistribute the tax revenues lump sum to consumers. By incorporating the environment, this paper is able to analyse a broader set of tax instruments, namely a tax on gross pollution in addition to the value added tax and the taxes on labour and capital income. Like in [17] Nielsen et al. (1995), the tax revenues are used to finance public abatement activities.

The effects of environmental policy on economic growth are ambiguous in the

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<sup>1</sup>The reproducible input factors human capital and physical capital are used as input factors both in the education sector and in the final good sector.

literature of endogenous growth with human capital.<sup>2</sup> In the simplest endogenous growth model, the *AK* model, the growth effect of environmental policy is negative. This is shown both by [8] Gradus & Smulders (1993) for a centrally planned economy by varying the weight on pollution in the utility function and by [11] Ligthart & van der Ploeg (1994) for a decentralised economy. However, [8] Gradus & Smulders (1993) show in a variant of the two sector Uzawa–Lucas model that environmental policy does not affect long term growth. Furthermore, [9] Hettich (1998) shows in a Uzawa–Lucas model with pollution and leisure that the growth effect of a tighter environmental policy depends on the pollution specification. If pollution is caused by the physical capital stock, then higher abatement activities stimulate growth<sup>3</sup>, whereas if pollution is complementary to the output level, higher abatement activities does not affect long term growth. This paper shows that a tighter environmental policy has a negative effect on growth in a generalized Uzawa–Lucas model irrespective of the pollution specification.

Finally, [10] King & Rebelo (1990) and [15] Milesi–Ferretti & Roubini (1994) consider the effects of tax rates on growth in small open economies with international capital mobility.<sup>4</sup> In their models, international growth rate differentials can be explained in a residence–based capital tax system, but not in a source–based system.<sup>5</sup> The production processes are similar to those in this paper, but these contributions do not consider an environmental externality.

Irrespective of the chosen pollution specification the major conclusions of our paper are firstly that factor income taxes and the pollution tax are growth re-

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<sup>2</sup>In the following, we formally neglect that a better environment in principle could have a positive effect on the productivity of inputs. Environmental quality in such cases not only acts as a public consumption good, but also as a productive public capital good. It has been shown that a better environmental quality may have a stimulating growth effect in the presence of positive environmental spillovers in production, see [3] Bovenberg & Smulders (1995), [7] van Ewijk & van Wijnbergen (1995), and [25] Smulders & Gradus (1996). Such productivity spillovers are certainly conceivable in the field of agriculture and tourism, but it is questionable whether they exist in an aggregate production function.

<sup>3</sup>A higher pollution tax gives rise to more abatement which crowds out consumption. This makes leisure less scarce which stimulates studying and therefore growth.

<sup>4</sup>[22] Rebelo (1992) and [12] Long & Wong (1997) survey the literature on endogenous growth in open economies.

<sup>5</sup>In overlapping generation models of the Uzawa–Lucas type [5] Buiter & Kletzer (1995) explain persistent growth rate differentials independently of the international taxation system. In a variant of the Uzawa–Lucas model augmented with endogenous population [20] Razin & Yuen (1996) find that the growth rate effect of taxes depends on the principle of international taxation and the preference bias between child quantity and child quality.

ducing, whereas the consumption tax is a lump sum tax. Secondly, the market allocation without government intervention is inefficient due to the presence of the environmental externality. Thus, there is too much pollution, too little abatement and excessive economic growth in an unregulated market economy. The first best solution can be achieved by setting an optimal pollution tax equal to the marginal damage of pollution. It turns out that an optimal solution instead may be reached by certain combinations of factor income taxes, if the pollution tax is unavailable as an instrument. Thirdly, under a residence-based tax system, a small open economy can choose its own fiscal policy and hence determine its long term growth rate. As long as the environmental problem is a national one, the government can choose the level of pollution and hence the environmental quality. However, under a source-based tax system, a small open economy is partially restricted in its fiscal policy and can as a consequence only lead a second best environmental policy.

The paper is organized as follows. The model is laid out in section 2 and the first order conditions of the representative agent problem and central planner problem are derived. Section 3 determines the optimal tax rates. Furthermore, the balanced growth rate is derived and the effects of tax and parameter changes on all core variables and ratios are determined. Finally, the small open economy version of the model is analysed. Section 4 considers an alternative pollution specification and section 5 concludes the paper.

## 2. The model

This section presents a two sector endogenous growth model of a closed economy and derives the first order conditions of the representative agent problem and the social planner problem. The first sector produces universal goods which can be used for consumption, abatement activities, and physical capital accumulation, while the second sector is an education sector in which human capital is accumulated.<sup>6</sup> Human capital is assumed to be embodied in people and is consequently a private good, which is both rival and excludable. Both sectors use physical and human capital as input factors and factors move freely between the two sectors. The production of the final good causes a negative environmental externality in form of pollution. The government is assumed to engage in public abatement activities, which are solely financed by tax revenues. The economy consists of a

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<sup>6</sup>The specification of the two production sectors is similar to [10] King & Rebelo (1990) and [21] Rebelo (1991).

large number of identical and infinitely lived households that own the factors of production and rent them to firms. Households are assumed to maximise their discounted lifetime utility and firms are assumed to maximise their market value.

## 2.1. Technology

In the first sector, final goods  $Y$  are produced with a constant returns to scale technology using physical capital  $K$  and human capital  $H$  as inputs. The technology is assumed to take the Cobb–Douglas form

$$Y_t = A (v_t K_t)^\alpha (u_t H_t)^{1-\alpha}, \quad (2.1)$$

where  $A, H, K, Y > 0$ ,  $0 < \alpha, u, v < 1$ . Parameter  $A$  reflects the exogenous level of the technology,  $\alpha$  is the exogenous physical capital share in final goods production, and  $v$  and  $u$  are the endogenous shares of physical and human capital devoted to final goods production, respectively. Human capital is assumed to be a private good, which is embodied in people.<sup>7</sup> In the following,  $(uH)$  is referred to as the effective labour force.

The flow resource constraint of the closed economy is<sup>8</sup>

$$Y_t = C_t + \dot{K}_t + Z_t, \quad (2.2)$$

where  $C, Z > 0$ . Final output  $Y$  is a universal good since it can be used either for consumption  $C$ , investment in physical capital  $\dot{K}$ , or for public abatement activities  $Z$ .<sup>9</sup>

The second sector is an education sector that produces human capital by use of physical capital representing facilities like school buildings and human capital representing education time and knowledge. The human capital accumulation function is assumed to be a constant returns to scale technology

$$\dot{H}_t = B [(1 - v_t) K_t]^\beta [(1 - u_t) H_t]^{1-\beta}, \quad (2.3)$$

where  $B > 0$  reflects the exogenous level of the technology and  $0 < \beta < 1$  is the exogenous physical capital share in education. The specification of the

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<sup>7</sup>Unlike [13] Lucas (1988), we neglect the possibility of an externality from the average stock of human capital to final goods production.

<sup>8</sup>For simplicity, human and physical capital are assumed to depreciate at a zero rate. However, this assumption does not change the qualitative results.

<sup>9</sup>A dot above a variable indicates its derivative with respect to time.

final goods production function (2.1) and the human capital accumulation function (2.3) ensure that diminishing returns do not arise, when physical and human capital growth at the same rate. Since both inputs  $H$  and  $K$  can be accumulated infinitely, the rates of return remain constant along a balanced growth path and unlimited growth is in principle possible.

Final goods production is assumed to cause a negative environmental externality as a side product, which harms utility.<sup>10</sup> The externality is assumed to affect individual utility only and not to harm the production processes, i.e. there is no positive spillover of a better environment to production of goods or human capital accumulation. Of course, it is conceivable that pollution directly affects the productivity in the final goods or the education sector. However, this is an aspect not being analysed formally in this paper, but there is a discussion of the issue at the end of section 3.2. Aggregate pollution  $P$  is a public ‘bad’, which can be reduced by means of public abatement activities  $Z$  that consume a part of output, in line with the flow resource constraint (2.2). Public abatement can be interpreted as knowledge about clean production methods. Both  $P$  and  $Z$  are modelled as flow quantities, which is justified as long as the balanced growth path is analysed. If public abatement  $Z$  increases, the output pollutes less. The net pollution function  $P$  is assumed to be given by the following functional form

$$P_t = \left( \frac{Y_t}{Z_t} \right)^\chi, \quad (2.4)$$

where  $\chi > 0$  is the exogenous elasticity of pollution  $P$  with respect to the output–abatement ratio  $Y/Z$ . According to equation (2.4) pollution is increasing in final goods production and decreasing in public abatement activities. Section 4 analyses an alternative specification of the pollution function, where pollution instead is generated by the use of physical capital in final goods production.

## 2.2. Firms

The economy consists of a large number of identical and competitive firms. They rent physical capital from households at the interest rate  $r$  and hire human capital at the wage rate  $w$ . Firms use these input factors to produce final goods with

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<sup>10</sup>Like [19] van der Ploeg & Withagen (1991), [8] Gradus & Smulders (1993) and [11] Ligthart & Ploeg (1994) pollution is modelled as an ‘output’ factor in this paper. Alternatively, pollution could be modelled as an input factor, see [17] Nielsen et al. (1995) and [2] Bovenberg & de Mooij (1997). Both modelling approaches are equivalent, see [23] Siebert et al. (1980).

the technology described by equation (2.1). Firms must pay a pollution tax  $\tau_p$  according to their gross pollution. Since pollution is complementary to output, a pollution tax is equivalent to a tax on output. Firms are assumed to maximise their cash flow by choosing  $uH$  and  $vK$ , given the pollution tax  $\tau_p$  levied on total production  $Y$ . The representative firm's cash flow is given by:

$$\pi_t = (1 - \tau_p) Y_t - w_t (u_t H_t) - r_t (v_t K_t) \quad (2.5)$$

The cash flow is maximised when the marginal cost of each factor equals its after-tax marginal product:

$$w_t = (1 - \tau_p) (1 - \alpha) A \left( \frac{v_t K_t}{u_t H_t} \right)^\alpha = (1 - \tau_p) (1 - \alpha) \frac{Y_t}{u_t H_t} \quad (2.6)$$

$$r_t = (1 - \tau_p) \alpha A \left( \frac{v_t K_t}{u_t H_t} \right)^{\alpha-1} = (1 - \tau_p) \alpha \frac{Y_t}{v_t K_t} \quad (2.7)$$

### 2.3. Households

Households are assumed to be identical, atomistic agents with perfect foresight over an infinite time horizon. Preferences are restricted to ensure the existence of a sustainable balanced growth path. The necessary conditions imply a specific functional form of the utility function, where consumption is multiplicatively separable from net pollution.<sup>11</sup> For simplicity, we assume an additively separable instantaneous utility function.<sup>12</sup> The corresponding discounted life time utility is given by

$$U_0 = \int_0^\infty (\ln C_t - \eta \ln P_t) e^{-\rho t} dt, \quad (2.8)$$

where  $C$  is consumption,  $P$  is aggregate net pollution,  $\eta$  is the positive exogenous marginal disutility of pollution, and  $\rho$  is the positive exogenous rate of time preference. Utility is seen to be increasing in consumption at a decreasing rate,  $U_C > 0$  and  $U_{CC} < 0$ , while it is decreasing in aggregate pollution at an increasing

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<sup>11</sup>For the necessary conditions on the utility function, see [25] Smulders & Gradus (1996).

<sup>12</sup>A more general utility function is  $U = \frac{(CP^{-\eta})^{1-\theta} - 1}{1-\theta}$ , where  $\theta$  is the inverse of the intertemporal elasticity of substitution. Since this function does not change the qualitative results concerning the growth rate effects, but complicates the analysis, we use the simplified function (2.8), where the intertemporal elasticity of substitution is equal to one.

rate,  $U_P < 0$  and  $U_{PP} > 0$ .<sup>13</sup> Households choose consumption and the allocation of human and physical capital between the two sectors in order to maximise their life time utility (2.8) subject to the human capital accumulation constraint (2.3) and the flow budget constraint

$$\dot{K}_t = (1 - \tau_k) r_t v_t K_t + (1 - \tau_h) w_t u_t H_t - (1 + \tau_c) C_t, \quad (2.9)$$

where  $\tau_c$  is the tax on consumption (value added tax),  $\tau_h$  is the flat-rate tax on labour income,  $\tau_k$  is the flat-rate tax on capital income,  $w$  is the wage rate, and  $r$  is the rate of return to physical capital. Education is assumed to be a non-market sector which does not give rise to direct income, it can consequently not be taxed by means of income taxes.<sup>14</sup>

## 2.4. Government

The government levies a pollution tax<sup>15</sup> on final goods production, a capital income tax, a labour income tax and a consumption tax in order to finance public abatement activities. Public spending has to be financed solely by taxes, since the government does not issue bonds. Hence, the government is assumed to run a balanced budget in every period, which is given by:

$$\tau_k r_t v_t K_t + \tau_h w_t u_t H_t + \tau_c C_t + \tau_p Y_t = Z_t \quad (2.10)$$

In the following, time indices of variables are neglected where unnecessary.

## 2.5. The Market Solution

The representative household chooses its consumption and the allocation of its physical and human capital to final goods production and education, respectively, in order to maximise its life time utility (2.8) subject to the human capital accumulation function (2.3) and the budget constraint (2.9) taking the tax rates  $\tau_c, \tau_h, \tau_k$ , and  $\tau_p$  as given. Since pollution is a public ‘bad’, economic agents ignore it in

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<sup>13</sup>A more realistic utility function would allow for increasing marginal disutility of pollution  $U_{PP} < 0$ , but this only changes the optimal allocation of resources between abatement and final goods production (2.22). All other first order conditions of the central planner are unaffected and so are the obtained results.

<sup>14</sup>For human capital as a market good, see [16] Milesi-Ferretti & Roubini (1995) in a similar model without environment.

<sup>15</sup>In fact, the pollution tax is a tax on output, since pollution is assumed to be complementary to output.



their maximisation problem. The first order conditions with respect to  $C$ ,  $K$ ,  $H$ ,  $v$ , and  $u$  become

$$\frac{1}{C}e^{-\rho t} = \lambda_k(1 + \tau_c) \Rightarrow -\frac{\dot{\lambda}_k}{\lambda_k} = \frac{\dot{C}}{C} + \rho \quad (2.11)$$

$$-\frac{\dot{\lambda}_k}{\lambda_k} = (1 - \tau_k)r \quad (2.12)$$

$$-\frac{\dot{\lambda}_h}{\lambda_h} = (1 - \beta)B \left[ \frac{(1 - v)K}{(1 - u)H} \right]^\beta \quad (2.13)$$

$$\lambda_k(1 - \tau_k)r = \lambda_h\beta B \left[ \frac{(1 - v)K}{(1 - u)H} \right]^{\beta-1} \quad (2.14)$$

$$\lambda_k(1 - \tau_h)w = \lambda_h(1 - \beta)B \left[ \frac{(1 - v)K}{(1 - u)H} \right]^\beta \quad (2.15)$$

where  $\lambda_k$  and  $\lambda_h$  are the shadow prices of physical capital and human capital in the market solution, respectively. Equation (2.11) implies that the marginal utility of consumption in every period should equal the after-tax shadow price of physical capital. The first Euler condition (2.12) implies that the rate of change in the shadow price of physical capital should equal the after-tax marginal product of capital in the final goods sector. The second Euler condition (2.13) says that the rate of change in the shadow price of human capital should equal the marginal product of human capital in the education sector. Finally, equations (2.14) and (2.15) describe the optimal allocation of physical and human capital between the two sectors.

The transversality conditions to the maximisation problem are:<sup>16</sup>

$$\lim_{t \rightarrow \infty} \lambda_k K = 0 \quad \text{and} \quad \lim_{t \rightarrow \infty} \lambda_h H = 0 \quad (2.16)$$

Equations (2.14) and (2.15) yield the sectoral allocation of resources as a function of the tax rates and the parameters of the model

$$\frac{v}{u} = \left( \frac{\alpha}{1 - \alpha} \frac{1 - \beta}{\beta} \right) \left( \frac{1 - \tau_k}{1 - \tau_h} \right) \frac{1 - v}{1 - u}, \quad (2.17)$$

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<sup>16</sup>These conditions rule out explosive paths by requiring that the present discounted value of each capital good equals zero in the long run. This is a reasonable requirement, since optimizing agents do not want valuable assets at the end of their planning horizon. Consequently, the first transversality condition requires that the real interest rate should be positive.

where equations (2.6) and (2.7) have been used. According to equation (2.17), the after-tax rates of technical substitution between capital and labour must be equalised across sectors in order to achieve an optimal intersectoral allocation of capital and hours. The income taxes on capital and labour affect this allocation in different ways. According to equation (2.17), the capital-labour ratio in the final goods sector  $v/u$  increases relative to the capital-labour ratio in the education sector  $(1-v)/(1-u)$ , whenever the capital income tax declines or the labour income tax increases. In addition, an increase (a decline) in the capital share in final goods production (education) leads to an increase in the capital-labour ratio in final goods production relative to the education sector. Note that the sectoral allocation of resources is unaffected by income taxation, when taxation is comprehensive  $\tau_k = \tau_h$ . In addition, the sectoral allocation of factors is unaffected by a change in the pollution tax, since it affects the return to the two input factors in the same way.

Finally, the Keynes-Ramsey rule describing the optimal consumption path for the market economy is derived by use of equations (2.11) and (2.12), where  $r$  is replaced by equation (2.7):<sup>17</sup>

$$\hat{C} = (1 - \tau_k)(1 - \tau_p)\alpha\frac{Y}{vK} - \rho \equiv R - \rho \quad (2.18)$$

For further analysis it seems to be useful to define  $R$  as the return to capital investment net of tax.

It is obvious that the consumption tax  $\tau_c$  is a lump sum tax, since it is absent in the first order conditions of the market solution (2.11) – (2.15) and hence does not distort the economy. However, all other taxes have effects on the economy. The taxes on capital income and pollution affect the intertemporal incentive to invest in physical capital, described by equation (2.18). In addition, both factor income taxes affect the sectoral allocation of factors, described by equation (2.17).

## 2.6. The Planned Solution

In contrast to the representative household, a benevolent central planner takes the negative side effects of production into account. Consumption, the amount of public abatement activities, and the allocation of physical and human capital between the two sectors are chosen, in order to maximise the discounted life time utility of the representative household (2.8) subject to the human capital

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<sup>17</sup>A hat above a variable indicates its rate of growth.

accumulation function (2.3) and the resource constraint of the economy (2.2). The first order conditions with respect to  $C$ ,  $K$ ,  $H$ ,  $Z$ ,  $v$ , and  $u$  become

$$\mu_k = \frac{1}{C}e^{-\rho t} \Rightarrow -\frac{\dot{\mu}_k}{\mu_k} = \frac{\dot{C}}{C} + \rho \quad (2.19)$$

$$-\frac{\dot{\mu}_k}{\mu_k} = \left(1 - \frac{Z}{Y}\right) \alpha \frac{Y}{vK} \quad (2.20)$$

$$-\frac{\dot{\mu}_h}{\mu_h} = (1 - \beta) B \left[ \frac{(1 - v) K}{(1 - u) H} \right]^\beta \quad (2.21)$$

$$\mu_k = \chi \eta \frac{1}{Z} e^{-\rho t} \quad (2.22)$$

$$\mu_k \left(1 - \frac{Z}{Y}\right) \alpha \frac{Y}{vK} = \mu_h \beta B \left[ \frac{(1 - v) K}{(1 - u) H} \right]^{\beta-1} \quad (2.23)$$

$$\mu_k \left(1 - \frac{Z}{Y}\right) (1 - \alpha) \frac{Y}{uH} = \mu_h (1 - \beta) B \left[ \frac{(1 - v) K}{(1 - u) H} \right]^\beta \quad (2.24)$$

where  $\mu_k$  and  $\mu_h$  are the shadow prices of physical capital and human capital in the social planner solution, respectively. Note that equations (2.19) – (2.24) describe the first best solution of the economy. Equation (2.19) implies that the marginal utility of consumption in every period should equal the shadow price of physical. The first Euler condition (2.20) implies that the rate of change of the shadow price of physical capital should equal the marginal product of capital in the final goods sector. The second Euler condition (2.21) is identical to equation (2.13). Equation (2.22) describes the optimal allocation of resources between public abatement and production of physical capital. According to equation (2.22), the marginal utility of public abatement activities should equal the shadow price of physical capital. Finally, equations (2.23) and (2.24) describe the optimal allocation of physical and human capital.

After eliminating the shadow prices, equations (2.23) and (2.24) yield the optimal sectoral allocation of resources in the socially planned economy:

$$\frac{v}{u} = \left( \frac{\alpha}{1 - \alpha} \frac{1 - \beta}{\beta} \right) \frac{1 - v}{1 - u} \quad (2.25)$$

Finally, the Keynes–Ramsey rule for the socially planned economy is obtained by

use of equations (2.19) and (2.20)

$$\hat{C} = \left(1 - \frac{Z}{Y}\right) \alpha \frac{Y}{vK} - \rho \equiv R_S - \rho, \quad (2.26)$$

where  $R_S$  is defined as the social return to capital investment. As can be seen, the social return equals the private return to capital investment  $\alpha (Y/vK)$  corrected by the optimal marginal damage of pollution  $Z/Y$ , which is external to firms in the market economy.

### 3. The Balanced Growth Path

In this section, the optimal tax rates are derived. Furthermore, the reduced forms of the balanced growth rate and all core variables and ratios are derived. Finally, the small open economy version of the model is considered.

Along a balanced growth path, the variables  $C$ ,  $H$ ,  $K$ ,  $Y$ , and  $Z$  grow at the same constant rate, whereas  $u$  and  $v$  are constant over time. Therefore, the balanced growth rate  $g$  can be defined as follows

$$\begin{aligned} g &\equiv \hat{C} = \hat{H} = \hat{K} = \hat{Y} = \hat{Z} = -\hat{\mu}_* - \rho = -\hat{\lambda}_* - \rho \\ 0 &= \dot{u} = \dot{v} \end{aligned} \quad (3.1)$$

where a hat above a variable indicates its rate of growth. According to condition (3.1), the ratios  $C/K$ ,  $C/Y$ ,  $K/H$ ,  $Z/K$ , and  $Z/Y$  are constant along a balanced growth path. This implies that the level of net pollution  $P$  is constant along the balanced growth path. A constant level of  $P$  is in accord with sustainable environmental development if the ecosystem is assumed to be a renewable resource and the level of pollution does not exceed the natural regeneration.<sup>18</sup>

#### 3.1. Optimal Tax Rates

This section derives the optimal tax rates in the model. In order to derive the first best tax rates, the first order conditions of the market solution are compared with the corresponding first order conditions of the central planner solution. Comparison of equations (2.15) with (2.24) and (2.12) with (2.20) using equation (2.6)

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<sup>18</sup>See [24] Smulders (1995) for a survey on environmental sustainability in endogenous growth models.

and (2.7) reveals the following two conditions for a first best solution:

$$(1 - \tau_h)(1 - \tau_p) = (1 - \tau_k)(1 - \tau_p) = 1 - \left(\frac{Z}{Y}\right)^{CPS} \quad (3.2)$$

where the superscript  $^{CPS}$  denotes the central planner solution. Recall that the abatement–output ratio is constant along a balanced growth path. Note that there are more than one possibility to achieve a first best solution in a market economy. Condition (3.2) can be fulfilled in at least three ways, see table 3.1.

	$\tau_h$	$\tau_k$	$\tau_p$
Case 1	0	0	$\left(\frac{Z}{Y}\right)^{CPS}$
Case 2	$\left(\frac{Z}{Y}\right)^{CPS}$	$\left(\frac{Z}{Y}\right)^{CPS}$	0
Case 3	$1 - \sqrt{1 - \left(\frac{Z}{Y}\right)^{CPS}}$	$1 - \sqrt{1 - \left(\frac{Z}{Y}\right)^{CPS}}$	$1 - \sqrt{1 - \left(\frac{Z}{Y}\right)^{CPS}}$

Table 3.1: Optimal tax rates, when  $P = (Y/Z)^x$

*Case 1:* A first best solution can be reached by setting tax rates on labour and capital equal to zero and the pollution tax equal to the optimal marginal damage of pollution. In this case, the pollution tax corresponds to a Pigouvian tax. Note that the first best solution can be reached without the use of a lump sum instrument in all three cases. Hence, in case 1 the pollution tax fulfills two tasks at the same time. It corrects the inefficient input ratio and generates the exact amount of public revenues to provide the optimal level of abatement. But additionally, there are at least two other ways to reach a first best solution without setting the pollution tax at its Pigouvian level. *Case 2:* A first best solution can be reached by setting both factor income taxes equal to the optimal marginal damage of pollution and the pollution tax equal to zero. Hence, a comprehensive income tax ( $\tau_h = \tau_k$ ) works in the same way as a pollution tax. Therefore, setting a comprehensive income tax equal to the optimal abatement–output ratio is equivalent to a Pigouvian tax. *Case 3:* A first best solution can be reached in many other ways by setting a comprehensive income tax and using a pollution tax in addition. One of them is represented in table 3.1 for  $\tau_h = \tau_k = \tau_p$ . In all three cases, the optimal tax structure holds both along and outside the balanced growth path. Outside the balanced growth path, the tax rates change over time since the optimal abatement–output ratio is not constant. However,

along the balanced growth path the tax rates are constant, since the abatement–output ratio is constant. Finally, it can be stated that the non–lump sum income taxes may be efficient, when the tax revenues collected by the government are spent on abatement activities.

### 3.2. The Market Economy

In the following, we analyse the effects of tax and parameter changes on the reduced forms of all core variables and ratios. At the end of the section, the results are summarised in table 3.2. Along a balanced growth path, the first order conditions of the market economy (2.13) and (2.18), the human capital accumulation constraint (2.3), and the resource constraint of the economy (2.2) can be rewritten by use of condition (3.1):

$$g = (1 - \tau_k)(1 - \tau_p) \alpha \left( \frac{vK}{uH} \right)^{\alpha-1} - \rho = R - \rho \quad (3.3)$$

$$g = (1 - \beta) B \left[ \frac{(1 - v)K}{(1 - u)H} \right]^{\beta} - \rho \quad (3.4)$$

$$g = B \left[ \frac{(1 - v)K}{(1 - u)H} \right]^{\beta} (1 - u) \quad (3.5)$$

$$g = A \left( \frac{vK}{uH} \right)^{\alpha-1} v - \frac{C}{K} - \frac{Z}{K} \quad (3.6)$$

In order to derive the reduced form of the physical to human capital ratio in the final good sector  $(vK) / (uH)$ , we substitute the term  $(1 - v) / (1 - u)$  in equation (3.4) by (2.17) and set it equal to (3.3):

$$\frac{vK}{uH} = \left\{ \frac{\alpha A}{(1 - \beta) B} \left[ \frac{\alpha}{1 - \alpha} \frac{1 - \beta}{\beta} \right]^{\beta} \frac{(1 - \tau_p)(1 - \tau_k)^{(1+\beta)}}{(1 - \tau_h)^{\beta}} \right\}^{\frac{1}{1 - \alpha + \beta}} \quad (3.7)$$

According to equation (3.7), the physical to human capital ratio in the final goods sector  $(vK) / (uH)$  depends positively on the tax on labour income and negatively on the taxes on pollution and capital income. It is obvious that a higher labour income tax leads to a more capital intensive final goods production, whereas studying becomes more labour intensive. The opposite is true for a higher pollution

tax and a higher capital income tax. The effect of a change in the comprehensive income tax on the physical to human capital ratio is identical to that of a change in the pollution tax. This is the reason behind the equivalence of a comprehensive tax and a pollution tax for internalising the external effect of pollution. Although, the pollution tax is equivalent to a tax on output it does reduce the ratio of physical to human capital in final goods production. The intuition behind this is that an output tax reduces the return to physical capital net of tax rate directly, whereas the rate of return to investment in human capital is left unaffected, because education is a non-market activity. Since the return to capital investment should equal the return to human capital investment, final goods production must become more human capital intensive in order to leave the return to capital investment net of tax unaffected.

The reduced forms of the return to capital investment net of tax and the growth rate, see equation (2.18), can now be obtained by introduction of equation (3.7) in (3.3):

$$R = \left[ D (1 - \tau_h)^{(1-\alpha)\beta} (1 - \tau_p)^\beta (1 - \tau_k)^{\alpha\beta} \right]^{\frac{1}{1-\alpha+\beta}} \quad (3.8)$$

$$g = \left[ D (1 - \tau_h)^{(1-\alpha)\beta} (1 - \tau_p)^\beta (1 - \tau_k)^{\alpha\beta} \right]^{\frac{1}{1-\alpha+\beta}} - \rho \quad (3.9)$$

$$\text{where } D \equiv (\alpha A)^\beta [(1 - \beta) B]^{(1-\alpha)} \left( \frac{1 - \alpha}{\alpha} \frac{\beta}{1 - \beta} \right)^{(1-\alpha)\beta} \quad (3.10)$$

According to equation (3.9), the balanced growth rate in the market economy depends positively on the levels of the technology in the final goods sector  $A$  and the education sector  $B$ , while it depends negatively on the capital income tax  $\tau_k$ , the labour income tax  $\tau_h$ , the comprehensive income tax  $\tau_h = \tau_k$ , the pollution tax  $\tau_p$ , and the rate of time preference  $\rho$ . It can be seen that a tax on consumption  $\tau_c$  does not affect the long term growth rate. Furthermore, the value added tax has the characteristics of a lump sum tax, since it does not distort the economy.<sup>19</sup> The result that non-environmental taxes reduce growth has been already shown by [16] Milesi-Ferretti & Roubini (1995) in a similar model without

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<sup>19</sup>In case the more general utility function  $U = \frac{(CP^{-\eta})^{1-\theta} - 1}{1-\theta}$  is used, the reduced form of the growth rate (3.9) changes to:  $g = \frac{1}{\theta} \left\{ \left[ D (1 - \tau_h)^{(1-\alpha)\beta} (1 - \tau_p)^\beta (1 - \tau_k)^{\alpha\beta} \right]^{\frac{1}{1-\alpha+\beta}} - \rho \right\}$ . Obviously, an intertemporal elasticity of substitution  $1/\theta$  unequal to unity does not alter the qualitative growth rate effects of parameter and tax rate changes.

pollution. Furthermore they state that growth and welfare are maximized, when factor income is taxed at a zero rate. Hence, in their model the outcome of the unregulated market economy is a first best solution. However, we consider the environment in addition and show that the long-term growth rate and pollution in an unregulated market economy is too high from a welfare perspective, when pollution harms utility.

In the following, the reduced forms of the fractions of human and physical capital that are devoted to final goods production are determined. The fraction of human capital allocated to final goods production is derived by use of equations (3.4) and (3.5) and introduction of (3.9):

$$u = \beta + (1 - \beta) \rho \left[ D (1 - \tau_h)^{(1-\alpha)\beta} (1 - \tau_p)^\beta (1 - \tau_k)^{\alpha\beta} \right]^{-\frac{1}{1-\alpha+\beta}} \quad (3.11)$$

It can be seen immediately that the fraction of human capital allocated to final goods production depends positively on both factor income taxes, the comprehensive income tax, and the pollution tax. It seems contra-intuitive that an increase in the labor income tax induces agents to spend more time at work. However, the capital-labour ratio  $v/u$  and not the absolute value of  $u$  is relevant for the allocation of physical and human capital between the two production sectors. As we will see below, a labour income tax also increases the fraction of physical capital allocated to final goods production. However,  $v$  rises more than  $u$ , since a labour income tax increases the capital-labour ratio in the final goods sector  $v/u$  relative to capital-labour ratio in the education sector  $(1 - v) / (1 - u)$ , see equation (2.17).

Now, the fraction of physical capital allocated to final goods production is derived by use of equations (2.17) and (3.11) and introduction of (3.9)

$$v = \left[ 1 + \frac{1 - \alpha}{\alpha} \frac{1 - \tau_h}{1 - \tau_k} \frac{1}{1 + \frac{\rho}{\beta \left\{ \left[ D (1 - \tau_h)^{(1-\alpha)\beta} (1 - \tau_p)^\beta (1 - \tau_k)^{\alpha\beta} \right]^{-\frac{1}{1-\alpha+\beta}} - \rho \right\}}} \right]^{-1}, \quad (3.12)$$

where the fraction of physical capital allocated to final goods production depends positively on the labour income tax, the comprehensive income tax, and the pollution tax, while the effect of the capital income tax is presumed to be negative, but cannot be signed unambiguously.

In the following, the reduced form of the abatement-output ratio is derived. Therefore, the budget constraint of the government (2.10) is rewritten as:

$$(1 - \tau_p) [\tau_k \alpha + \tau_h (1 - \alpha)] + \tau_p + \tau_c \frac{C}{Y} = \frac{Z}{Y} \quad (3.13)$$



Introduction of the resource constraint of the economy (3.6) in equation (3.13) yields either

$$\frac{Z}{Y} = \frac{(1 - \tau_p) [\tau_k \alpha + \tau_h (1 - \alpha)] + \tau_p + \tau_c \left[ 1 - \frac{g}{Av} \left( \frac{vK}{uH} \right)^{1-\alpha} \right]}{1 + \tau_c} \quad (3.14)$$

or

$$\frac{C}{Y} = \frac{\left[ 1 - \frac{g}{Av} \left( \frac{vK}{uH} \right)^{1-\alpha} \right] - (1 - \tau_p) [\tau_k \alpha + \tau_h (1 - \alpha)] - \tau_p}{1 + \tau_c} \quad (3.15)$$

It can be seen that a consumption tax increases the abatement–output ratio. However, the exclusive use of the consumption tax cannot yield a first best solution, since it does not correct the sectoral allocation of factors. For the derivation of the other effects we have to set the lump sum consumption tax equal to zero, otherwise the effects cannot be signed. Due to tax interaction effects, a higher pollution tax erodes the tax bases of the non–environmental taxes, which finally could lead to a lower abatement–output ratio and thereby to higher pollution. An increase in the pollution tax raises tax revenue directly, but lowers the factor income tax revenue indirectly through a drop in the returns to capital and labour. However, an increase in the pollution tax increases the abatement–output ratio and hence reduces pollution as long as the taxes on capital and labour are between zero and one. The effects of a comprehensive income tax are equivalent to a pollution tax. Isolated increases in either the labour income tax or the capital income tax raise the abatement–output ratio without eroding any other tax bases.

Even though a consumption tax is a lump sum tax, it has a negative effect on the consumption–output ratio, since resources are reallocated towards public abatement activities, see equation (3.15). However, the effects of all other taxes and parameters on the consumption–output ratio cannot be signed unambiguously. The effects of changes in the tax rates and the parameters on the core variables and ratios are summarized in table 3.2.<sup>20</sup>

By means of the Keynes–Ramsey rule (3.3) and the results in table 3.2 (second and fifth row), the channels through which taxes affect long term economic growth can be shown. The direct effect of higher taxes on capital and pollution is to reduce the net interest rate  $R$  for a given physical to human capital ratio. From equation (3.3), we know that this single effect reduces growth. Due to the indirect

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<sup>20</sup>In table 3.2, the results indicated with a ‘\*’ are obtained by setting  $\tau_c = 0$ , whereas ‘\*’ cannot be signed unambiguously, but is expected to be negative.

	$\tau_c$	$\tau_h$	$\tau_k$	$\tau_h = \tau_k$	$\tau_p$	$A$	$B$	$\rho$
$\frac{\partial g}{\partial x}$	0	-	-	-	-	+	+	-
$\frac{\partial u}{\partial x}$	0	+	+	+	+	-	-	+
$\frac{\partial v}{\partial x}$	0	+	-*	+	+	+	+	-
$\frac{\partial[(vK)/(uH)]}{\partial x}$	0	+	-	-	-	+	-	0
$\frac{\partial(Z/Y)}{\partial x}$	+	+*	+*	+*	+*	0*	0*	0*

Table 3.2: The effects of changes in taxes and parameters on core variables

effect of these taxes, final good production becomes more labour-intensive, which ceteris paribus stimulates growth due to a higher marginal product of physical capital. Table 3.2 shows that the direct effect dominates the indirect effect. A higher labour tax leads to a more capital-intensive final good production, which lowers growth indirectly, see equation (3.3).

In the preceding analyses, the case of productive environmental spillovers have not been taken into account. However, an improvement in the quality of the environment may increase the productivity in the final goods production or in the education sector.<sup>21</sup> Formally, these cases could have been analysed in the present setup by adding a multiplicative term  $P^{-\epsilon}$  on the left hand side of the production function (2.1) or the human capital accumulation function (2.3). However, it is possible to discuss productive environmental spillovers without a formal analysis. Along a balanced growth path, a better environmental quality is namely equivalent to higher levels of the technology  $A$  or  $B$ , because the level of pollution is constant along a balanced growth path. From table 3.2, an increase in productivity is seen to increase economic growth. Hence, environmental improvements stimulate economic growth, ceteris paribus. Whether, this positive growth rate effect dominates the above-mentioned negative growth rate effect depends solely on parameter values.

### 3.3. The Small Open Economy

So far we have analysed the effects of environmental policy in a closed economy setting. In this section, we complement the analysis by investigating the possibilities for a small open economy to lead an independent environmental policy. Throughout, the conventional assumptions of international trade models are

<sup>21</sup>Some empirical evidence suggests that pollution may cause productivity losses, see e.g. [14] Margulis (1992), [1] Ballard & Medema (1993), and [4] Brendemoen & Vennemo (1994).

made: Domestic capital goods and foreign assets  $F$  are perfect substitutes, there is international borrowing and lending, and international trade in capital and consumption goods, but international immobility of labour i.e. human capital. The latter assumption ensures that the small open economy will not specialise over time in either final goods production or education. Furthermore, the world interest rate  $r_f$  cannot be influenced by a small open economy and is therefore exogenous. This implies that the after-tax return to capital invested domestically and abroad is the same, if a residence-based (world wide) tax system<sup>22</sup> is assumed. In addition, we assume that the environmental externality is national in scope.<sup>23</sup> Given these assumptions, the flow resource constraint of the small open economy becomes

$$Y + r_f F = C + \dot{K} + \dot{F} + Z, \quad (3.16)$$

where  $r_f F$  is the interest payment earned on foreign assets and  $\dot{F}$  is investment in foreign assets. In the small open market economy, the first order conditions of the representative agent's maximisation problem are still given by equations (2.11) – (2.15), but in addition the after-tax return to domestic capital should equal the after-tax return to investment in foreign assets:<sup>24</sup>

$$R = (1 - \tau_k)r = (1 - \tau_f)r_f \quad (3.17)$$

The small open economy can still determine the domestic interest rate net of tax by use of its tax policy. But at the same time the domestic interest rate net of tax should equal the net of tax world interest rate. As a consequence, the only difference between the closed economy and the small open economy is that the government must accommodate the tax on foreign capital income in order to fulfil equation (3.17). Thus, the tax on foreign capital income  $\tau_f$  is not a real decision variable of the government. In case the after-tax return is higher on foreign assets, there will be a permanent outflow of capital from the domestic economy. In case the after-tax return is lower on foreign assets, there will be permanent inflow of capital. Obviously, both of these cases are unstable. The endogenous

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<sup>22</sup>This is the prevailing taxation system, see e.g. [18] OECD (1991). Agents pay taxes in their home country on capital income from foreign investments, but receive a tax credit for any taxes paid abroad on this income.

<sup>23</sup>In case of global environmental problems like the anthropogenic greenhouse effect or the destruction of the ozone layer, a small open economy has no influence on the pollution level at all.

<sup>24</sup>Note that in a pure residence-based taxation system domestic and foreign capital income would be taxed at the same rate.

determination of the tax on foreign capital income is derived by introduction of the after-tax interest rate (3.8) in equation (3.17):

$$\tau_f = 1 - \frac{R}{r_f} = 1 - \frac{1}{r_f} \left[ D (1 - \tau_h)^{(1-\alpha)\beta} (1 - \tau_p)^\beta (1 - \tau_k)^{\alpha\beta} \right]^{\frac{1}{1-\alpha+\beta}} \quad (3.18)$$

It can be seen that for given taxes on labour income, capital income and pollution, there exists a unique feasible value of the tax on foreign assets that equalises the returns to domestic and foreign investment. Since the equilibrium interest rate net of tax  $R$  (3.8) is a negative function of the taxes on capital, labour and pollution, the tax on foreign assets is a positive function of these taxes. This result is similar to the result obtained by [15] Milesi-Ferretti & Roubini (1994), except for the effect of the pollution tax.

The key insight from the above analysis is that a small open economy under a residence-based tax system can determine the after-tax return earned by domestic residents as long as the tax rate on foreign capital income is set according to condition (3.18). Thus, the after-tax returns on domestic capital equals the world interest rate net of the tax on foreign capital income levied on domestic residents. Hence, a small open economy can decide its own growth rate through taxation of factor income. In fact, the balanced growth rate is the same in the small open economy and in the closed economy (3.9).<sup>25</sup> Furthermore, a small open economy can use efficient instruments to internalize the pollution externality.

As a consequence, the optimal taxation results of section 3.1 are still valid. However, the government in the small open economy obtains an additional revenue from the tax on foreign assets. This tax revenue must be redistributed in a lump sum manner to guarantee a first best solution. The main difference between the closed and the open economy is that the latter will exhibit no transitional dynamics as long as there are no investment costs present. The domestic capital stock can be changed immediately by borrowing or lending in the international capital markets.

If in contrast capital income is taxed according to the source-based tax principle, the government becomes partially restricted in its tax policy. Under such a system the net of tax interest rate in the small open economy must equal the world interest rate

$$R = (1 - \tau_k) r = r_f, \quad (3.19)$$

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<sup>25</sup>The papers by [10] King & Rebelo (1990) and [22] Rebelo (1992) also show that a residence-based taxation system allows for a wedge between both domestic and world interest rates and growth rates.

Thus, the growth rate of the small open economy is determined by the world interest rate. If on the one hand, the technology of the production processes are identical at home and abroad, the government of the small open economy cannot levy positive taxes on both labour, capital, and pollution, since this would violate condition (3.19) which ensures a stable solution. However, an independent environmental policy can still be lead by the small open economy by setting two of the tax rates and tying the last one according to equation (3.19) given that both taxes and subsidies are available.<sup>26</sup> Thus, the revenues of certain combinations of taxes and subsidies, which does not alter the return to capital net of tax  $R$ , and the revenue of the lump sum consumption tax can be used to increase abatement activities and thereby lower pollution. But note that this is a second best environmental policy. Hence, should the EU decide to move towards a source-based income tax system, then a common EU environmental policy might be necessary, since individual member states no longer can lead an effective environmental policy. If on the other hand, the technologies are different the government must use at least one of the above-mentioned tax instruments to fulfil condition (3.19). As long as the interest rate of the small open economy  $r$  is larger than the world interest rate  $r_f$  there is a possibility for environmental policy. If the interest rate is smaller than the world interest rate, the small open economy must subsidize the interest rate and finance it by a lump sum tax.

#### 4. Capital as the Polluting Factor

After having discussed the possibilities of a small open economy to lead an independent environmental policy, we now turn back to a closed economy setting. In the preceding sections, pollution was assumed to be a function of output and public abatement activities according to equation (2.4). In this section, we assume an another plausible pollution specification. Pollution is now assumed to be a function of physical capital used in the final goods production and public

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<sup>26</sup>If all foreign tax rates are zero, then  $r_f = D^{\frac{1}{1-\alpha+\beta}}$  and the restriction on domestic tax rates becomes:

$$(1 - \tau_p) = (1 - \tau_h)^{-(1-\alpha)} (1 - \tau_k)^{-\alpha}$$

see equation (3.8). Thus, pollution can e.g. be taxed at the rate  $\tau_p = s/(1+s)$ , if factor income is subsidized at the comprehensive rate  $\tau_h = \tau_k = -s$ . The revenue from such a policy is negative, which implies that consumption has to be taxed at an appropriate rate in order to balance the government budget and still engage in public abatement activities, see equation (2.10).

abatement activities. The alternative specification of the net pollution function is

$$P = \left( \frac{vK}{Z} \right)^x \quad (4.1)$$

where pollution is seen to be increasing in the use of the dirty factor and decreasing in public abatement. In contrast to the pollution function (2.4), the alternative specification allows for a reduction in pollution without lowering output, through a substitution of physical by human capital in production. This would lead to a cleaner and a more labour-intensive final goods production.

In the following, we derive the growth rate effects of changes in the tax rates and the parameters and compute the optimal tax rates for this pollution specification given that all other things are equal to section 2.1. In contrast to the setup above, the pollution tax is now levied on the use of physical capital in the final good production. Therefore, the after-tax marginal products of labour and capital change as follows:

$$w = (1 - \alpha) \frac{Y}{uH} \quad (4.2)$$

$$r = \alpha \frac{Y}{vK} - \tau_p \quad (4.3)$$

The wage rate is seen to be unaffected by a pollution tax, while the interest rate is a negative function of the pollution tax. The first order conditions of the market solution (2.11) – (2.15) are unchanged. But due to the changes in the wage and interest rate, the sectoral allocation of factors is no longer unaffected by a pollution tax

$$\frac{v}{u} = \frac{\alpha}{1 - \alpha} \frac{1 - \beta}{\beta} \frac{1 - \tau_k}{1 - \tau_h} \frac{\alpha \frac{Y}{vK} - \tau_p}{\alpha \frac{Y}{vK}} \frac{1 - v}{1 - u}, \quad (4.4)$$

where equations (2.14), (2.15), (4.2) and (4.3) have been used.

The alternative specification of the pollution function changes the differential equations, which govern the balanced growth path. Thus, along the balanced growth path equations (3.4) – (3.6) still hold, while the Keynes–Ramsey rule (3.3) of the market economy changes to:

$$g = R - \rho = (1 - \tau_k) \left( \alpha \frac{Y}{vK} - \tau_p \right) - \rho \quad (4.5)$$

In order to determine the growth rate effects of changes in the tax rates and the parameters, we derive the reduced form of the growth rate by using equations (3.4), (4.4) and (4.5). The implicit function of the reduced form is called  $F$ :

$$F \equiv 0 = (1 - \beta) B \left( \frac{1 - \beta}{\beta} \frac{\alpha}{1 - \alpha} \frac{1 - \tau_k}{1 - \tau_h} \right)^{-\beta} \left\{ \frac{1}{\alpha A} \left[ \frac{1}{\alpha A} \left( \frac{g + \rho}{1 - \tau_k} + \tau_P \right) \right]^{\frac{\alpha}{1 - \alpha}} \left( \frac{g + \rho}{1 - \tau_k} \right) \right\}^{-\beta} - \rho - g \quad (4.6)$$

The effects of parameter and tax rate changes can be derived by using the implicit function rule.<sup>27</sup> The growth rate is negatively affected by taxes on labour income, capital income<sup>28</sup>, and pollution, and by the rate of time preference. Also in this specification, a consumption tax is a lump sum tax and therefore has no effect on economic growth. Growth is stimulated by increases in the levels of technology in both sectors,  $A$  and  $B$ . These results are similar to the second row of table 3.2 in section 3.2.

In the following, the optimal tax rates are found by a comparison of the first order conditions of the representative agent problem (2.11) – (2.15) and the social planner problem (A.3) – (A.8).<sup>29</sup> This comparison yields the following two equations that must be fulfilled in order to achieve the first best optimum:

$$(1 - \tau_k) \left( \alpha \frac{Y}{vK} - \tau_p \right) = \alpha \left( \frac{Y}{vK} \right)^{CPS} - \left( \frac{Z}{vK} \right)^{CPS} \quad (4.7)$$

$$(1 - \tau_h) (1 - \alpha) \frac{Y}{uH} = (1 - \alpha) \left( \frac{Y}{uH} \right)^{CPS} \quad (4.8)$$

Equation (4.7) can be fulfilled in at least two ways, see table 4.1. And inspection of equation (4.8) immediately reveals that the optimal labour income tax is zero.

*Case 1:* A first best solution can be reached by setting taxes on labour and capital income equal to zero and the pollution tax equal to the optimal marginal damage of pollution. Such a pollution tax corresponds to a Pigouvian tax.

*Case 2:* Another way to achieve a first best solution is to set the labour and pollution tax equal to zero and the capital tax equal to the optimal marginal damage of

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<sup>27</sup>See section A.1 in the appendix for the partial derivatives of  $F$ .

<sup>28</sup>We cannot sign the growth rate effect of the capital income tax, but we expect it to be negative.

<sup>29</sup>See section A.2 in the appendix for the first order conditions of the central planner solution with this pollution specification.

	$\tau_h$	$\tau_k$	$\tau_p$
Case 1	0	0	$\left(\frac{Z}{vK}\right)^{CPS}$
Case 2	0	$\left(\frac{Z}{vK}\right)^{CPS} \left[\alpha \left(\frac{Y}{vK}\right)^{CPS}\right]^{-1}$	0

Table 4.1: Optimal tax rates, when  $P = (vK/Z)^x$

pollution divided by the optimal marginal product of capital. It can be seen that the effects of a capital income tax are similar to the effects of a tax on pollution. This is not surprising since the use of physical capital in final good production is the dirty input factor. However, there is one difference: the tax base. A tax on capital is levied on the capital income which consists of the marginal product of capital times the use of physical capital in production  $[\alpha Y / (vK)] vK$ , whereas the tax on pollution is levied solely on the use of physical capital in production  $vK$ . Therefore, it is necessary to correct the optimal marginal damage of pollution by  $[\alpha Y / (vK)]^{-1}$  in order to equate the tax base differences of the two taxes, when a capital income tax is used. Note that both the pollution and the capital income tax lead to a more labor intensive and thereby a cleaner final goods production. Furthermore, there is a rationale for capital income taxation, when capital is the dirty input factor and the pollution tax instrument is not available.

Comparison of table 3.1 and table 4.1 reveals that the optimal labour income tax becomes zero, once human capital becomes a clean input factor in production. The discussion of the small open economy version for the alternative pollution specification is similar to the one in section 3.3 and is therefore neglected.

## 5. Conclusions

This paper has examined the effects of fiscal policy in a two sector endogenous growth model with pollution, where the tax revenues are used to finance public abatement activities. We investigated two plausible pollution specifications: Pollution is either generated by final goods production or by the use of physical capital in final good production. In both cases, the decentralised outcome is inefficient without government intervention. From a welfare perspective there is too much pollution and economic growth is too high in an unregulated market economy, since economic agents do not take the environmental externality into account. Both factor income taxes and the pollution tax reduce economic growth



whereas a consumption tax has the characteristics of a lump sum tax.<sup>30</sup> The tax on pollution improves welfare as long as the level of pollution is above the optimal level. If productive environmental spillovers are considered additionally there may be a stimulating growth effect of a tighter environmental policy depending on the strength of these spillovers. At least productive environmental spillovers partially offset the negative growth effect of providing a better environmental quality.

When pollution is complementary to final goods production, the government can reach a first best solution either by setting the pollution tax equal to the optimal marginal damage of pollution or by setting a comprehensive income tax at the same level. Furthermore, a combination of the comprehensive income tax and the pollution tax can be used to reach a first best solution. The optimal tax analysis implies that non-lump sum taxes may be efficient, when the tax revenue is spent on public abatement activities. Concerning the growth rate effects of taxes on capital income and pollution, there are two effects at work: Both taxes reduce the growth rate directly through a drop in the net of tax interest rate, but stimulate growth indirectly through an increase in the interest rate, because final good production becomes more labour intensive. However, the direct effect dominates the indirect effect. A higher labour income tax leads to a reduction of the interest rate solely through the indirect effect. Although the pollution tax is equivalent to a tax on output, it does affect the physical to human capital ratio in final goods production, since education is assumed to be a non-market activity. Furthermore, we have shown that a higher pollution tax erodes the tax bases of both factor income taxes. Nevertheless it increases the government revenue and thereby the abatement-output ratio, which is accompanied by a better environmental quality.

When pollution is caused by the use of physical capital in production, the first best solution once more can be reached by setting the pollution tax equal to the optimal marginal damage of pollution. However, the negative externality can also be internalised by a capital income tax that equals the optimal marginal damage of pollution divided by the optimal marginal product of capital. The latter correction is necessary to equate the tax base differences between the capital income tax and the pollution tax. Thus, capital income taxation is well-founded, when capital is a dirty input factor, because it induces firms to use more of the clean input factor in production.

In addition, we investigated a small open economy version of the model with

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<sup>30</sup>In a model with identical production processes but augmented by leisure, a consumption tax is growth reducing as well, see [16] Milesi-Ferretti & Roubini (1995).

perfect international capital mobility. We found that under a residence-based income tax system that discriminates between domestic source and foreign source income, a small open economy can lead an independent first best environmental policy, where it chooses its own fiscal policy and thereby determine its own growth rate. However, in order to rule out a permanent inflow or outflow of capital and hence the unstable solution, the tax on foreign capital income is no longer a decision variable of the government. Instead, the government must accommodate any domestic tax rate change by changing the foreign income tax such that the domestic after-tax interest rate equals the after-tax world interest rate.

Under a source-based tax system, the government is partially restricted in its tax policy since the latter must be constrained in such a way that the after-tax interest rate equals the world interest rate. However, in order to lead a first best environmental policy the government must be able to determine its interest rate. Nevertheless, there is still room for an independent second best environmental policy. By means of certain tax-subsidy combinations and the use of the lump sum consumption tax the level of the abatement activities and hence the level of pollution can be varied without changing the interest rate.

## A. Appendix

### A.1. Partial Derivatives of $F$

Partial derivatives of the reduced form of the growth rate  $F$

$$\begin{aligned}
\frac{\partial F}{\partial \tau_h} &= -\beta(1-\beta)B\left(\frac{1-\beta}{\beta}\frac{\alpha}{(1-\alpha)}(1-\tau_k)\right)^{-\beta}(1-\tau_h)^{\beta-1}\Omega_2^{-\beta} < 0 \\
\frac{\partial F}{\partial \tau_p} &= -\beta\Omega_1\Omega_2^{-\beta-1}\frac{\alpha}{1-\alpha}\frac{1}{\alpha A}\Omega_3^{\frac{\alpha}{1-\alpha}}\left[1-\frac{1}{\alpha A}\tau_p\Omega_3^{-1}\right] < 0 \\
\frac{\partial F}{\partial A} &= \frac{1}{A}\beta\Omega_1\Omega_2^{-\beta-1}\frac{1}{1-\alpha}\Omega_3^{\frac{\alpha}{1-\alpha}}\left[1-\frac{1}{\alpha A}\tau_p\Omega_3^{-1}\right] > 0 \\
\frac{\partial F}{\partial B} &= \frac{1}{B}\Omega_1\Omega_2^{-\beta} > 0 \\
\frac{\partial F}{\partial \rho} &= -\beta\Omega_1\Omega_2^{-\beta-1}\frac{1}{\alpha A}\frac{1}{1-\tau_k}\frac{1}{1-\alpha}\Omega_3^{\frac{\alpha}{1-\alpha}}\left\{1-\frac{1}{A}\tau_p\Omega_3^{-1}\right\}-1 < 0 \\
\frac{\partial F}{\partial g} &= -\beta\Omega_1\Omega_2^{-\beta-1}\frac{1}{\alpha A}\frac{1}{1-\tau_k}\frac{1}{1-\alpha}\Omega_3^{\frac{\alpha}{1-\alpha}}\left\{1-\frac{1}{A}\tau_p\Omega_3^{-1}\right\}-1 < 0
\end{aligned} \tag{A.1}$$

where the following definitions are used:

$$\begin{aligned}
\Omega_1 &\equiv (1 - \beta) B \left( \frac{1-\beta}{\beta} \frac{\alpha}{(1-\alpha)} \frac{(1-\tau_k)}{(1-\tau_h)} \right)^{-\beta} > 0 \\
\Omega_2 &\equiv \frac{1}{\alpha A} \left[ \frac{1}{\alpha A} \left( \frac{g+\rho}{1-\tau_k} + \tau_p \right) \right]^{\frac{\alpha}{1-\alpha}} \left( \frac{g+\rho}{1-\tau_k} \right) > 0 \\
\Omega_3 &\equiv \frac{1}{\alpha A} \left( \frac{g+\rho}{1-\tau_k} + \tau_p \right) > 0
\end{aligned} \tag{A.2}$$

To show that the term in the square bracket of the second and third line of equation (A.1) is positive we substitute  $\Omega_3$  by  $\left(\frac{vK}{uH}\right)^{\alpha-1}$ , see equation (4.3). Therefore

$$1 - \frac{1}{\alpha A} \tau_p \Omega_3^{-1} = \frac{1}{\alpha A} \left( \frac{vK}{uH} \right)^{1-\alpha} \left[ \alpha A \left( \frac{vK}{uH} \right)^{\alpha-1} - \tau_p \right] = \frac{1}{\alpha A} \left( \frac{vK}{uH} \right)^{1-\alpha} r > 0,$$

where it has been used that a positive interest rate is a necessary condition for the existence of a balanced growth path. Then it is straight forward to see that the curly bracket in line fifth and sixth of equation (A.1) is positive as well:

$$1 - \frac{1}{A} \tau_p \Omega_3^{-1} = \frac{1}{A} \left( \frac{vK}{uH} \right)^{1-\alpha} \left[ A \left( \frac{vK}{uH} \right)^{\alpha-1} - \tau_p \right] > 0$$

## A.2. First Order Conditions of the Social Planner Solution

When pollution is caused by the use of physical capital in final goods production, the first order conditions of the central planner problem becomes:

$$\nu_k = \frac{1}{C} e^{-\rho t} \tag{A.3}$$

$$-\frac{\dot{\nu}_k}{\nu_k} = \left( \alpha \frac{Y}{vK} - \frac{Z}{vK} \right) \tag{A.4}$$

$$-\frac{\dot{\nu}_h}{\nu_h} = (1 - \beta) B \left[ \frac{(1-v)K}{(1-u)H} \right]^\beta \tag{A.5}$$

$$\mu_k = \chi \eta \frac{1}{Z} e^{-\rho t} \tag{A.6}$$

$$\nu_h \beta B \left[ \frac{(1-v)K}{(1-u)H} \right]^{\beta-1} = \nu_k \left( \alpha \frac{Y}{vK} - \frac{Z}{vK} \right) \tag{A.7}$$

$$\nu_h (1 - \beta) B \left[ \frac{(1-v)K}{(1-u)H} \right]^\beta = \nu_k (1 - \alpha) \frac{Y}{uH} \tag{A.8}$$

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