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**Working Paper**

## Optimal monetary policy in an operational medium-sized DSGE model

Sveriges Riksbank Working Paper Series, No. 225

**Provided in Cooperation with:**

Central Bank of Sweden, Stockholm

*Suggested Citation:* Adolfson, Malin; Laséen, Stefan; Lindé, Jesper; Svensson, Lars E.O. (2008) : Optimal monetary policy in an operational medium-sized DSGE model, Sveriges Riksbank Working Paper Series, No. 225, Sveriges Riksbank, Stockholm

This Version is available at:

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# Optimal Monetary Policy in an Operational Medium-Sized DSGE Model: Technical Appendix\*

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First draft: October 2006  
This version: August 2008

## Abstract

Technical appendix to “Optimal Monetary Policy in an Operational Medium-Sized DSGE Model”

JEL Classification: E52, E58

Keywords: Optimal monetary policy, forecasts, judgment

This technical appendix of Adolfson, Laséen, Lindé, and Svensson [1] contains the detailed specification of the matrices  $A$ ,  $B$ ,  $C$ , and  $H$  in the model, as well as the detailed specification of the measurement equation and the related matrices  $\bar{D}_0$ ,  $\bar{D}$ , and  $\bar{D}_s$ .

## 1. Definition of the matrices $A$ , $B$ , $C$ , $D$ , and $H$ and the instrument rule $[f_X \ f_x]$

We want to identify the submatrices in the model,

$$X_{t+1} = A_{11}X_t + A_{12}x_t + B_1i_t + C\varepsilon_{t+1},$$

$$Hx_{t+1|t} = A_{22}x_t + A_{21}X_t + B_2i_t.$$

We specify the predetermined variables  $X_t = (X_t^{\text{ex}}, X_t^{\text{pd}})'$ , the forward-looking variables  $x_t$ , the policy instrument  $i_t$ , and the shock vector  $\varepsilon_t$  as follows (the number of the elements in each vector

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\*All remaining errors are ours. The views expressed in this paper are solely the responsibility of the authors and should not be interpreted as reflecting the views of Sveriges Riksbank.

is also listed):

$$\begin{aligned}
X_t^{\text{ex}} \equiv & \left[ \begin{array}{cccc} \hat{\epsilon}_t & 1 & \hat{\tau}_t^y & 21 \\ \hat{\epsilon}_{t-1} & 2 & \hat{\tau}_t^c & 22 \\ \hat{\mu}_{zt} & 3 & \hat{\tau}_t^w & 23 \\ \hat{\mu}_{z,t-1} & 4 & \hat{g}_t & 24 \\ \hat{\nu}_t & 5 & \hat{\tau}_{t-1}^k & 25 \\ \hat{\zeta}_t^c & 6 & \hat{\tau}_{t-1}^y & 26 \\ \hat{\zeta}_t^h & 7 & \hat{\tau}_{t-1}^c & 27 \\ \hat{\zeta}_t^q & 8 & \hat{\tau}_{t-1}^w & 28 \\ \hat{\lambda}_t^f & 9 & \hat{g}_{t-1} & 29 \\ \hat{\lambda}_t^{mc} & 10 & \hat{\pi}_t^* & 30 \\ \hat{\lambda}_t^{mi} & 11 & \hat{y}_t^* & 31 \\ \hat{\lambda}_t^{\hat{s}} & 12 & \hat{R}_t^* & 32 \\ \hat{\phi}_t & 13 & \hat{\pi}_{t-1}^* & 33 \\ \hat{\Upsilon}_t & 14 & \hat{g}_{t-1}^* & 34 \\ \hat{\tilde{z}}_t^* & 15 & \hat{R}_{t-1}^* & 35 \\ \hat{\tilde{z}}_{t-1}^* & 16 & \hat{\pi}_{t-2}^* & 36 \\ \hat{\lambda}_{xt} & 17 & \hat{y}_{t-2}^* & 37 \\ \hat{\varepsilon}_{Rt} & 18 & \hat{R}_{t-2}^* & 38 \\ \hat{\pi}_t^c & 19 & \hat{\pi}_{t-3}^* & 39 \\ \hat{\pi}_{t-1}^c & 20 & \hat{R}_{t-3}^* & 41 \end{array} \right], \quad X_t^{\text{pd}} \equiv \left[ \begin{array}{cccccc} \hat{k}_t & 1 & 42 & \hat{k}_{t-1} & 16 & 57 \\ \hat{m}_t & 2 & 43 & \hat{q}_{t-1} & 17 & 58 \\ \hat{R}_{t-1} & 3 & 44 & \hat{\mu}_{t-1} & 18 & 59 \\ \hat{\pi}_{t-1}^d & 4 & 45 & \hat{a}_{t-1} & 19 & 60 \\ \hat{\pi}_{t-1}^{mc} & 5 & 46 & \hat{\gamma}_{t-1}^{mod} & 20 & 61 \\ \hat{\pi}_{t-1}^{mi} & 6 & 47 & \hat{\gamma}_{t-1}^{mid} & 21 & 62 \\ \hat{y}_{t-1} & 7 & 48 & \hat{\gamma}_{t-1}^{x*} & 22 & 63 \\ \hat{\pi}_{t-1}^x & 8 & 49 & \hat{x}_{t-1} & 23 & 64 \\ \hat{\bar{w}}_{t-1} & 9 & 50 & \hat{\text{mc}}_{t-1}^x & 24 & 65 \\ \hat{c}_{t-1} & 10 & 51 & \hat{k}_{t-1} & 25 & 66 \\ \hat{i}_{t-1} & 11 & 52 & \hat{u}_{t-1} & 26 & 67 \\ \hat{\psi}_{z,t-1} & 12 & 53 & \hat{\pi}_{t-2}^d & 27 & 68 \\ \hat{P}_{k't-1} & 13 & 54 & \hat{\pi}_{t-2}^{mc} & 28 & 69 \\ \Delta \hat{S}_{t-1} & 14 & 55 & \hat{\pi}_{t-3}^d & 29 & 70 \\ \hat{H}_{t-1} & 15 & 56 & \hat{\pi}_{t-3}^{mc} & 30 & 71 \end{array} \right], \\
x_t \equiv & \left[ \begin{array}{cccc} \hat{\pi}_t^d & 1 & \hat{k}_t & 13 \\ \hat{\pi}_t^{mc} & 2 & \hat{q}_t & 14 \\ \hat{\pi}_t^{mi} & 3 & \hat{\mu}_t & 15 \\ \hat{y}_t & 4 & \hat{a}_t & 16 \\ \hat{\pi}_t^x & 5 & \hat{\gamma}_t^{mcd} & 17 \\ \hat{\bar{w}}_t & 6 & \hat{\gamma}_t^{mid} & 18 \\ \hat{c}_t & 7 & \hat{\gamma}_t^{x*} & 19 \\ \hat{i}_t & 8 & \hat{\bar{x}}_t & 20 \\ \hat{\psi}_{zt} & 9 & \hat{\text{mc}}_t^x & 21 \\ \hat{P}_{k't} & 10 & \hat{k}_{t+1} & 22 \\ \Delta \hat{S}_t & 11 & \hat{m}_{t+1} & 23 \\ \hat{H}_t & 12 & & \end{array} \right], \quad i_t \equiv \hat{R}_t, \\
\varepsilon_t \equiv & \left[ \begin{array}{cccccccccccc} \varepsilon_{\epsilon t} & \varepsilon_{zt} & \varepsilon_{\nu t} & \varepsilon_{\zeta^c t} & \varepsilon_{\zeta^h t} & \varepsilon_{\zeta^q t} & \varepsilon_{\lambda t} & \varepsilon_{\lambda^{mc} t} & \varepsilon_{\lambda^{mi} t} & \dots \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & \dots \\ \varepsilon_{\tilde{\phi} t} & \varepsilon_{\Upsilon t} & \varepsilon_{\tilde{z}^* t} & \varepsilon_{\lambda_x t} & \varepsilon_{\varepsilon_R t} & \varepsilon_{\hat{\pi} t} & \varepsilon'_{\tau t} & \varepsilon'_{x^* t} & & \\ 10 & 11 & 12 & 13 & 14 & 15 & 16 : 20 & 21 : 23 & & \end{array} \right]'.
\end{aligned}$$

### 1.1. Predetermined variables

First we specify the exogenous variables and the shocks, then the endogenous predetermined variables.

#### Row 1 - Stationary technology shock

$$\hat{\epsilon}_{t+1} = \rho_\epsilon \hat{\epsilon}_t + \sigma_{\epsilon_\epsilon} \varepsilon_{\epsilon,t+1}.$$

$$\begin{aligned}\hat{\epsilon}_t &: A_{11}(1,1) = \rho_\epsilon, \\ \varepsilon_{\epsilon,t+1} &: C(1,1) = \sigma_{\epsilon_\epsilon}.\end{aligned}$$

The notation means that the value of the persistence of the technology shock,  $\rho_\epsilon$ , is placed in the first row and first column of the matrix  $A_{11}$ . The notation " $\hat{\epsilon}_t :$ " means that the parameter value pertains to that specific variable.

#### Row 2 - Lagged stationary technology shock

$$\hat{\epsilon}_t = \hat{\epsilon}_t.$$

$$\hat{\epsilon}_t : A_{11}(2,1) = 1.$$

#### Row 3 - Permanent technology shock

$$\hat{\mu}_{z,t+1} = \rho_{\mu_z} \hat{\mu}_{zt} + \sigma_{\varepsilon_z} \varepsilon_{z,t+1}.$$

$$\begin{aligned}\hat{\mu}_{zt} &: A_{11}(3,3) = \rho_\epsilon, \\ \varepsilon_{z,t+1} &: C(3,2) = \sigma_{\epsilon_\epsilon}.\end{aligned}$$

#### Row 4 - Lagged permanent technology shock

$$\hat{\mu}_{zt} = \hat{\mu}_{zt}.$$

$$\hat{\mu}_{zt} : A_{11}(4,3) = 1.$$

#### Row 5 - Firm money demand shock

$$\hat{\nu}_{t+1} = \rho_\nu \hat{\nu}_t + \sigma_{\varepsilon_\nu} \varepsilon_{\nu,t+1}.$$

$$\begin{aligned}\hat{\nu}_t &: A_{11}(5,5) = \rho_\nu, \\ \varepsilon_{\nu,t+1} &: C(5,3) = \sigma_{\epsilon_\nu}.\end{aligned}$$

#### Row 6 - Consumption preference shock

$$\hat{\zeta}_{t+1}^c = \rho_{\zeta^c} \hat{\zeta}_t^c + \sigma_{\varepsilon_{\zeta^c}} \varepsilon_{\zeta^c,t+1}.$$

$$\begin{aligned}\hat{\zeta}_t^c &: A_{11}(6,6) = \rho_{\zeta^c}, \\ \varepsilon_{\zeta^c,t+1} &: C(6,4) = \sigma_{\epsilon_{\zeta^c}}.\end{aligned}$$

**Row 7 - Labor supply shock**

$$\hat{\zeta}_{t+1}^h = \rho_{\zeta^h} \hat{\zeta}_t^h + \sigma_{\varepsilon_{\zeta^h}} \varepsilon_{\zeta^h, t+1}.$$

$$\begin{aligned}\hat{\zeta}_t^h &: A_{11}(7, 7) = \rho_{\zeta^h}, \\ \varepsilon_{\zeta^h, t+1} &: C(7, 5) = \sigma_{\varepsilon_{\zeta^h}}.\end{aligned}$$

**Row 8 - Household money demand shock**

$$\hat{\zeta}_{t+1}^q = \rho_{\zeta^q} \hat{\zeta}_t^q + \sigma_{\varepsilon_{\zeta^q}} \varepsilon_{\zeta^q, t+1}.$$

$$\begin{aligned}\hat{\zeta}_t^q &: A_{11}(8, 8) = \rho_{\zeta^q}, \\ \varepsilon_{\zeta^q, t+1} &: C(8, 6) = \sigma_{\varepsilon_{\zeta^q}}.\end{aligned}$$

**Row 9 - Markup shock - domestic firms**

$$\hat{\lambda}_{t+1}^d = \rho_\lambda \hat{\lambda}_t^d + \sigma_{\varepsilon_\lambda} \varepsilon_{\lambda, t+1}.$$

$$\begin{aligned}\hat{\lambda}_t^d &: A_{11}(9, 9) = \rho_\lambda, \\ \varepsilon_{\lambda, t+1} &: C(9, 7) = \sigma_{\varepsilon_\lambda}.\end{aligned}$$

**Row 10 - Shock to substitution elasticity between domestic and foreign consumption goods**

$$\hat{\lambda}_{t+1}^{mc} = \rho_{\lambda^{mc}} \hat{\lambda}_t^{mc} + \sigma_{\varepsilon_{\lambda^{mc}}} \varepsilon_{\lambda^{mc}, t+1}.$$

$$\begin{aligned}\hat{\lambda}_t^{mc} &: A_{11}(10, 10) = \rho_{\eta^{mc}}, \\ \varepsilon_{\lambda^{mc}, t+1} &: C(10, 8) = \sigma_{\varepsilon_{\lambda^{mc}}}.\end{aligned}$$

**Row 11 - Shock to substitution elasticity between domestic and foreign investment goods**

$$\hat{\lambda}_{t+1}^{mi} = \rho_{\lambda^{mi}} \hat{\lambda}_t^{mi} + \sigma_{\varepsilon_{\lambda^{mi}}} \varepsilon_{\lambda^{mi}, t+1}.$$

$$\begin{aligned}\hat{\lambda}_t^{mi} &: A_{11}(11, 11) = \rho_{\eta^{mi}}, \\ \varepsilon_{\lambda^{mi}, t+1} &: C(11, 9) = \sigma_{\varepsilon_{\eta^{mi}}}.\end{aligned}$$

**Row 12 - Risk premium shock**

$$\hat{\tilde{\phi}}_{t+1} = \rho_{\tilde{\phi}} \hat{\tilde{\phi}}_t + \sigma_{\varepsilon_{\tilde{\phi}}} \varepsilon_{\tilde{\phi}, t+1}.$$

$$\begin{aligned}\hat{\tilde{\phi}}_t &: A_{11}(12, 12) = \rho_{\tilde{\phi}}, \\ \varepsilon_{\tilde{\phi}, t+1} &: C(12, 10) = \sigma_{\varepsilon_{\tilde{\phi}}}.\end{aligned}$$

**Row 13 - Investment specific technology shock**

$$\hat{\Upsilon}_{t+1} = \rho_{\Upsilon} \hat{\Upsilon}_t + \sigma_{\varepsilon_{\Upsilon}} \varepsilon_{\Upsilon,t+1}.$$

$$\begin{aligned}\hat{\Upsilon}_t &: A_{11}(13, 13) = \rho_{\Upsilon}, \\ \varepsilon_{\Upsilon,t+1} &: C(13, 11) = \sigma_{\varepsilon_{\Upsilon}}.\end{aligned}$$

**Row 14 - Relative (asymmetric) technology shock (between foreign and domestic)**

$$\hat{\tilde{z}}_{t+1}^* = \rho_{\tilde{z}^*} \hat{\tilde{z}}_t^* + \sigma_{\varepsilon_{\tilde{z}^*}} \varepsilon_{\tilde{z}^*,t+1}.$$

$$\begin{aligned}\hat{\tilde{z}}_t^* &: A_{11}(14, 14) = \rho_{\tilde{z}^*}, \\ \varepsilon_{\tilde{z}^*,t+1} &: C(14, 12) = \sigma_{\varepsilon_{\tilde{z}^*}}.\end{aligned}$$

**Row 15 - Lagged relative (asymmetric) technology shock (between foreign and domestic)**

$$\hat{\tilde{z}}_t^* = \hat{\tilde{z}}_t^*.$$

$$\hat{\tilde{z}}_t^* : A_{11}(15, 14) = 1.$$

**Row 16 - Markup shock - exporting firms**

$$\hat{\lambda}_{x,t+1} = \rho_{\lambda_x} \hat{\lambda}_{xt} + \sigma_{\varepsilon_{\lambda_x}} \varepsilon_{\lambda_x,t+1}.$$

$$\begin{aligned}\hat{\lambda}_{xt} &: A_{11}(16, 16) = \rho_{\lambda_x}, \\ \varepsilon_{\lambda_x,t+1} &: C(16, 13) = \sigma_{\varepsilon_{\lambda_x}}.\end{aligned}$$

**Row 17 - Monetary policy shock**

$$\epsilon_{R,t+1} = \rho_{\varepsilon_R} \epsilon_{Rt} + \sigma_{\varepsilon_{\varepsilon_R}} \varepsilon_{\varepsilon_R,t+1}.$$

$$\begin{aligned}\epsilon_{Rt} &: A_{11}(17, 17) = \rho_{\varepsilon_R}, \\ \varepsilon_{\varepsilon_R,t+1} &: C(17, 14) = \sigma_{\varepsilon_{\varepsilon_R}}.\end{aligned}$$

**Row 18 - Inflation target**

$$\hat{\pi}_{t+1}^c = \rho_{\hat{\pi}^c} \hat{\pi}_t^c + \sigma_{\varepsilon_{\hat{\pi}^c}} \varepsilon_{\hat{\pi}^c,t+1}.$$

$$\begin{aligned}\hat{\pi}_t^c &: A_{11}(18, 18) = \rho_{\hat{\pi}^c}, \\ \varepsilon_{\hat{\pi}^c,t+1} &: C(18, 15) = \sigma_{\varepsilon_{\hat{\pi}^c}}.\end{aligned}$$

**Row 19 - Lagged inflation target**

$$\hat{\pi}_t^c = \hat{\pi}_t^c.$$

$$\hat{\pi}_t^c : A_{11}(19, 18) = 1.$$

**Row 20–29 - Fiscal VAR(2)**

$$\tau_{t+1} = \tilde{\Theta}_1 \tau_t + \tilde{\Theta}_2 \tau_{t-1} + e_{\tau,t+1}, \quad (1.1)$$

where  $\tilde{\Theta}_1 = \Theta_0^{-1} \Theta_1$ ,  $\tilde{\Theta}_2 = \Theta_0^{-1} \Theta_2$  and  $e_{\tau t} = \Theta_0^{-1} S_\tau \varepsilon_{\tau t} \sim N(0, \Sigma_\tau)$ . Rewrite (1.1) as VAR(1):

$$\begin{bmatrix} \hat{\tau}_{t+1} \\ \hat{\tau}_t \end{bmatrix} = \begin{bmatrix} \tilde{\Theta}_1 & \tilde{\Theta}_2 \\ I & 0 \end{bmatrix} \begin{bmatrix} \hat{\tau}_t \\ \hat{\tau}_{t-1} \end{bmatrix} + \begin{bmatrix} e_{\tau,t+1} \\ 0 \end{bmatrix}$$

$$\begin{aligned} \hat{\tau}_t & : A_{11}(20:24, 20:24) = \tilde{\Theta}_1, \\ \hat{\tau}_{t-1} & : A_{11}(20:24, 25:29) = \tilde{\Theta}_2, \\ \hat{\tau}_t & : A_{11}(25:29, 20:24) = I, \\ \varepsilon_{\tau,t+1} & : C(20:24, 16:20) = \text{chol}\left(\Theta_0^{-1} S_\tau (\Theta_0^{-1} S_\tau)'\right)' . \end{aligned}$$

**Row 30–41 - Foreign VAR(4)**

$$X_{t+1}^* = \tilde{\Phi}_1 X_t^* + \tilde{\Phi}_2 X_{t-1}^* + \tilde{\Phi}_3 X_{t-2}^* + \tilde{\Phi}_4 X_{t-3}^* + e_{X^*,t+1}, \quad (1.2)$$

where  $\tilde{\Phi}_1 = \Phi_0^{-1} \Phi_1$ ,  $\tilde{\Phi}_2 = \Phi_0^{-1} \Phi_2$ , etc., and  $e_{X^*t} = \Phi_0^{-1} S_{x^*} \varepsilon_{x^*t} \sim N(0, \Sigma_{x^*})$ . Rewrite (1.2) as VAR(1):

$$\begin{bmatrix} X_{t+1}^* \\ X_t^* \\ X_{t-1}^* \\ X_{t-2}^* \end{bmatrix} = \begin{bmatrix} \tilde{\Phi}_1 & \tilde{\Phi}_2 & \tilde{\Phi}_3 & \tilde{\Phi}_4 \\ I & 0 & 0 & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & I & 0 \end{bmatrix} \begin{bmatrix} X_t^* \\ X_{t-1}^* \\ X_{t-2}^* \\ X_{t-3}^* \end{bmatrix} + \begin{bmatrix} e_{X^*,t+1} \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{aligned} X_t^* & : A_{11}(30:32, 30:32) = \tilde{\Phi}_1, \\ X_{t-1}^* & : A_{11}(30:32, 33:35) = \tilde{\Phi}_2, \\ X_{t-2}^* & : A_{11}(30:32, 36:38) = \tilde{\Phi}_3, \\ X_{t-3}^* & : A_{11}(30:32, 39:41) = \tilde{\Phi}_4, \\ X_t^* & : A_{11}(33:35, 30:32) = I, \\ X_{t-1}^* & : A_{11}(36:38, 33:35) = I, \\ X_{t-2}^* & : A_{11}(39:41, 36:38) = I, \\ \varepsilon_{x^*,t+1} & : C(30:32, 21:23) = \text{chol}\left(\Phi_0^{-1} S_{x^*} (\Phi_0^{-1} S_{x^*})'\right)' . \end{aligned}$$

**Row 42 - Physical capital stock** (note that  $\hat{k}_{t+1}$  is predetermined variable no. 42 in period  $t+1$  and forward-looking variable no. 22 in period  $t$ ):

$$\begin{aligned} \hat{k}_{t+1} & = \hat{k}_{t+1}, \\ \hat{k}_t & : A_{12}(42, 22) = 1. \end{aligned}$$

**Row 43 - Financial assets** (note that  $\hat{m}_{t+1}$  is predetermined variable no. 43 in period  $t+1$  and forward-looking variable no. 23 in period  $t$ ):

$$\begin{aligned} \hat{m}_{t+1} & = \hat{m}_{t+1}, \\ \hat{m}_{t+1} & : A_{12}(43, 23) = 1. \end{aligned}$$

**Row 44–66 - Lagged forward-looking variables**

$$\hat{R}_t : B_1(44, 1) = 1$$

$$\begin{aligned}\hat{\pi}_t^d &: A_{12}(45, 1) = 1 \\ \hat{\pi}_t^{mc} &: A_{12}(46, 2) = 1 \\ \hat{\pi}_t^{mi} &: A_{12}(47, 3) = 1 \\ \hat{y}_t &: A_{12}(48, 4) = 1 \\ \hat{\pi}_t^x &: A_{12}(49, 5) = 1 \\ \hat{w}_t &: A_{12}(50, 6) = 1 \\ \hat{c}_t &: A_{12}(51, 7) = 1 \\ \hat{i}_t &: A_{12}(52, 8) = 1\end{aligned}$$

$$\begin{aligned}\hat{\psi}_{zt} &: A_{12}(53, 9) = 1 \\ \hat{P}_{k't} &: A_{12}(54, 10) = 1 \\ \Delta \hat{S}_t &: A_{12}(55, 11) = 1 \\ \hat{H}_t &: A_{12}(56, 12) = 1 \\ \hat{u}_t &: A_{12}(57, 13) = 1 \\ \hat{q}_t &: A_{12}(58, 14) = 1 \\ \hat{\mu}_t &: A_{12}(59, 15) = 1 \\ \hat{a}_t &: A_{12}(60, 16) = 1 \\ \hat{\gamma}_t^{mcd} &: A_{12}(61, 17) = 1 \\ \hat{\gamma}_t^{mid} &: A_{12}(62, 18) = 1 \\ \hat{\gamma}_t^{x*} &: A_{12}(63, 19) = 1 \\ \hat{\bar{x}}_t &: A_{12}(64, 20) = 1 \\ \widehat{\text{mc}}_t^x &: A_{12}(65, 21) = 1\end{aligned}$$

**Row 66 - Lagged physical capital stock**

$$\hat{\bar{k}}_{t-1} : A_{11}(66, 42) = 1$$

**Row 67 - Lagged capacity utilization**

$$\hat{u}_{t-1} \equiv \hat{k}_{t-1} - \hat{\bar{k}}_{t-1}$$

$$\begin{aligned}\hat{k}_{t-1} &: A_{11}(67, 57) = 1 \\ \hat{\bar{k}}_{t-1} &: A_{11}(67, 66) = -1\end{aligned}$$

## Rows 68-71 - Lagged inflation

$$\begin{aligned}\hat{\pi}_{t-1}^d &: A_{11}(68, 45) = 1 \\ \hat{\pi}_{t-1}^{mc} &: A_{11}(69, 46) = 1 \\ \hat{\pi}_{t-2}^d &: A_{11}(70, 68) = 1 \\ \hat{\pi}_{t-2}^{mc} &: A_{11}(71, 69) = 1\end{aligned}$$

### 1.2. Forward-looking variables

#### Row 1 - Domestic inflation

$$\begin{aligned}\hat{\pi}_t^d - \hat{\pi}_t^c &= \frac{\beta}{1 + \kappa_d \beta} \left( \hat{\pi}_{t+1|t}^d - \rho_{\hat{\pi}^c} \hat{\pi}_t^c \right) + \frac{\kappa_d}{1 + \kappa_d \beta} \left( \hat{\pi}_{t-1}^d - \hat{\pi}_t^c \right) \\ &\quad - \frac{\kappa_d \beta (1 - \rho_{\hat{\pi}^c})}{1 + \kappa_d \beta} \hat{\pi}_t^c + \frac{(1 - \xi_d)(1 - \beta \xi_d)}{\xi_d (1 + \kappa_d \beta)} \left( \widehat{mc}_t^d + \hat{\lambda}_t^d \right), \\ \widehat{mc}_t^d &\equiv \alpha \left( \hat{\mu}_{zt} + \hat{H}_t - \hat{k}_t \right) + \hat{w}_t + \hat{R}_t^f - \hat{\varepsilon}_t, \\ \hat{R}_t^f &= \frac{\nu R}{v(R-1)+1} \hat{R}_{t-1} + \frac{\nu(R-1)}{v(R-1)+1} \hat{\nu}_t \\ \widehat{mc}_t^d &\equiv \alpha \hat{H}_t - \alpha \hat{k}_t + \hat{w}_t + \frac{\nu R}{v(R-1)+1} \hat{R}_{t-1} + \frac{\nu(R-1)}{v(R-1)+1} \hat{\nu}_t - \hat{\varepsilon}_t + \alpha \hat{\mu}_{zt}\end{aligned}$$

Rewrite the above equations as:

$$\begin{aligned}-\frac{\beta}{1 + \kappa_d \beta} \hat{\pi}_{t+1|t}^d &= -\hat{\pi}_t^d + \frac{\kappa_d}{1 + \kappa_d \beta} \hat{\pi}_{t-1}^d + \\ &\quad + \frac{(1 - \xi_d)(1 - \beta \xi_d)}{\xi_d (1 + \kappa_d \beta)} \alpha \hat{H}_t - \frac{(1 - \xi_d)(1 - \beta \xi_d)}{\xi_d (1 + \kappa_d \beta)} \alpha \hat{k}_t \\ &\quad + \frac{(1 - \xi_d)(1 - \beta \xi_d)}{\xi_d (1 + \kappa_d \beta)} \hat{w}_t + \frac{(1 - \xi_d)(1 - \beta \xi_d)}{\xi_d (1 + \kappa_d \beta)} \frac{\nu R}{v(R-1)+1} \hat{R}_{t-1} + \\ &\quad + \frac{(1 - \xi_d)(1 - \beta \xi_d)}{\xi_d (1 + \kappa_d \beta)} \frac{\nu(R-1)}{v(R-1)+1} \hat{\nu}_t + \frac{(1 - \xi_d)(1 - \beta \xi_d)}{\xi_d (1 + \kappa_d \beta)} \alpha \hat{\mu}_{zt} \\ &\quad - \frac{(1 - \xi_d)(1 - \beta \xi_d)}{\xi_d (1 + \kappa_d \beta)} \hat{\varepsilon}_t + \frac{(1 - \xi_d)(1 - \beta \xi_d)}{\xi_d (1 + \kappa_d \beta)} \hat{\lambda}_t^d \\ &\quad - \frac{(1 - \kappa_d)(\beta \rho_{\hat{\pi}^c} - 1)}{1 + \kappa_d \beta} \hat{\pi}_t^c.\end{aligned}$$

$$\begin{aligned}
\hat{\pi}_{t+1|t}^d & : H(1,1) = -\frac{\beta}{1 + \kappa_d \beta} \\
\hat{\pi}_t^d & : A_{22}(1,1) = -1 \\
\\
\hat{\pi}_{t-1}^d & : A_{21}(1,45) = \frac{\kappa_d}{1 + \kappa_d \beta} \\
\hat{\pi}_t^c & : A_{21}(1,18) = 1 - \frac{\beta}{1 + \kappa_d \beta} \rho_{\bar{\pi}^c} - \frac{\kappa_d}{1 + \kappa_d \beta} - \frac{\kappa_d \beta (1 - \rho_{\bar{\pi}^c})}{1 + \kappa_d \beta} \\
\hat{\lambda}_t^d & : \left[ A_{21}(1,9) = \frac{(1 - \xi_d)(1 - \beta \xi_d)}{\xi_d (1 + \kappa_d \beta)} \right] \\
\hat{\lambda}_t^d & : A_{21}(1,9) = 1 \text{ (rescaled)} \\
\text{Marginal cost} & : \\
\hat{w}_t & : A_{22}(1,6) = \frac{(1 - \xi_d)(1 - \beta \xi_d)}{\xi_d (1 + \kappa_d \beta)} \\
\hat{H}_t & : A_{22}(1,12) = \frac{(1 - \xi_d)(1 - \beta \xi_d)}{\xi_d (1 + \kappa_d \beta)} \alpha \\
\hat{k}_t & : A_{22}(1,13) = -\frac{(1 - \xi_d)(1 - \beta \xi_d)}{\xi_d (1 + \kappa_d \beta)} \alpha \\
\hat{\varepsilon}_t & : A_{21}(1,1) = -\frac{(1 - \xi_d)(1 - \beta \xi_d)}{\xi_d (1 + \kappa_d \beta)} \\
\hat{\mu}_{zt} & : A_{21}(1,3) = \frac{(1 - \xi_d)(1 - \beta \xi_d)}{\xi_d (1 + \kappa_d \beta)} \alpha \\
\hat{\nu}_t & : A_{21}(1,5) = \frac{(1 - \xi_d)(1 - \beta \xi_d)}{\xi_d (1 + \kappa_d \beta)} \frac{\nu(R-1)}{v(R-1)+1} \\
\hat{R}_{t-1} & : A_{21}(1,44) = \frac{(1 - \xi_d)(1 - \beta \xi_d)}{\xi_d (1 + \kappa_d \beta)} \frac{\nu R}{v(R-1)+1}
\end{aligned}$$

## Row 2 - Imported consumption inflation

$$\begin{aligned}
\hat{\pi}_t^{mc} - \hat{\pi}_t^c & = \frac{\beta}{1 + \kappa_{mc} \beta} \left( \hat{\pi}_{t+1|t}^{mc} - \rho_{\bar{\pi}^c} \hat{\pi}_t^c \right) + \frac{\kappa_{mc}}{1 + \kappa_{mc} \beta} \left( \hat{\pi}_{t-1}^{mc} - \hat{\pi}_t^c \right) \\
& \quad - \frac{\kappa_{mc} \beta (1 - \rho_{\bar{\pi}^c})}{1 + \kappa_{mc} \beta} \hat{\pi}_t^c + \frac{(1 - \xi_{mc})(1 - \beta \xi_{mc})}{\xi_{mc} (1 + \kappa_{mc} \beta)} \left( \widehat{mc}_t^{mc} + \hat{\lambda}_t^{mc} \right), \\
\widehat{mc}_t^{mc} & \equiv -\widehat{mc}_t^x - \hat{\gamma}_t^{x*} - \hat{\gamma}_t^{mcd}
\end{aligned}$$

$$\begin{aligned}
\hat{\pi}_{t+1|t}^{mc} & : H(2, 2) = -\frac{\beta}{1 + \kappa_{mc}\beta} \\
\hat{\pi}_t^{mc} & : A_{22}(2, 2) = -1 \\
\hat{\pi}_{t-1}^{mc} & : A_{21}(2, 46) = \frac{\kappa_{mc}}{1 + \kappa_{mc}\beta} \\
\hat{\pi}_t^c & : A_{21}(2, 18) = 1 - \frac{\beta}{1 + \kappa_{mc}\beta}\rho_{\bar{\pi}^c} - \frac{\kappa_{mc}}{1 + \kappa_{mc}\beta} - \frac{\kappa_{mc}\beta(1 - \rho_{\bar{\pi}^c})}{1 + \kappa_{mc}\beta} \\
& \quad \left[ \hat{\lambda}_t^{mc} : A_{21}(2, 10) = \frac{(1 - \xi_{mc})(1 - \beta\xi_{mc})}{\xi_{mc}(1 + \kappa_{mc}\beta)} \right] \\
\hat{\lambda}_t^{mc} & : A_{21}(2, 10) = 1 \text{ (rescaled)} \\
\text{Marginal cost} & : \\
\widehat{\text{mc}}_t^x & : A_{22}(2, 21) = -\frac{(1 - \xi_{mc})(1 - \beta\xi_{mc})}{\xi_{mc}(1 + \kappa_{mc}\beta)} \\
\hat{\gamma}_t^{x*} & : A_{22}(2, 19) = -\frac{(1 - \xi_{mc})(1 - \beta\xi_{mc})}{\xi_{mc}(1 + \kappa_{mc}\beta)} \\
\hat{\gamma}_t^{mc} & : A_{22}(2, 17) = -\frac{(1 - \xi_{mc})(1 - \beta\xi_{mc})}{\xi_{mc}(1 + \kappa_{mc}\beta)}
\end{aligned}$$

### Row 3 - Imported investment inflation

$$\begin{aligned}
\hat{\pi}_t^{mi} - \hat{\pi}_t^c & = \frac{\beta}{1 + \kappa_{mi}\beta} \left( \hat{\pi}_{t+1|t}^{mi} - \rho_{\bar{\pi}^c} \hat{\pi}_t^c \right) + \frac{\kappa_{mi}}{1 + \kappa_{mi}\beta} \left( \hat{\pi}_{t-1}^{mi} - \hat{\pi}_t^c \right) \\
& \quad - \frac{\kappa_{mi}\beta(1 - \rho_{\bar{\pi}^c})}{1 + \kappa_{mi}\beta} \hat{\pi}_t^c + \frac{(1 - \xi_{mi})(1 - \beta\xi_{mi})}{\xi_{mi}(1 + \kappa_{mi}\beta)} \left( \widehat{\text{mc}}_t^{mi} + \hat{\lambda}_t^{mi} \right), \\
\widehat{\text{mc}}_t^{mi} & \equiv -\widehat{\text{mc}}_t^x - \hat{\gamma}_t^{x*} - \hat{\gamma}_t^{mi, d}
\end{aligned}$$

$$\begin{aligned}
\hat{\pi}_{t+1|t}^{mi} & : H(3, 3) = -\frac{\beta}{1 + \kappa_{mi}\beta} \\
\hat{\pi}_t^{mi} & : A_{22}(3, 3) = -1 \\
\hat{\pi}_{t-1}^{mi} & : A_{21}(3, 47) = \frac{\kappa_{mi}}{1 + \kappa_{mi}\beta} \\
\text{Marginal cost} & : \\
\hat{\gamma}_t^{mid} & : A_{22}(3, 18) = -\frac{(1 - \xi_{mi})(1 - \beta\xi_{mi})}{\xi_{mi}(1 + \kappa_{mi}\beta)} \\
\hat{\gamma}_t^{x*} & : A_{22}(3, 19) = -\frac{(1 - \xi_{mi})(1 - \beta\xi_{mi})}{\xi_{mi}(1 + \kappa_{mi}\beta)} \\
\widehat{\text{mc}}_t^x & : A_{22}(3, 21) = -\frac{(1 - \xi_{mi})(1 - \beta\xi_{mi})}{\xi_{mi}(1 + \kappa_{mi}\beta)} \\
& \quad \left[ \hat{\lambda}_t^{mi} : A_{21}(3, 11) = \frac{(1 - \xi_{mi})(1 - \beta\xi_{mi})}{\xi_{mi}(1 + \kappa_{mi}\beta)} \right] \\
\hat{\lambda}_t^{mi} & : A_{21}(3, 11) = 1 \text{ (rescaled)} \\
\hat{\pi}_t^c & : A_{21}(3, 18) = 1 - \frac{\beta}{1 + \kappa_{mi}\beta}\rho_{\bar{\pi}^c} - \frac{\kappa_{mi}}{1 + \kappa_{mi}\beta} - \frac{\kappa_{mi}\beta(1 - \rho_{\bar{\pi}^c})}{1 + \kappa_{mi}\beta}
\end{aligned}$$

#### Row 4 - Output gap

$$0 = -\hat{y}_t + \lambda_f \hat{\epsilon}_t + \lambda_f \alpha \hat{k}_t - \lambda_f \alpha \hat{\mu}_{zt} + \lambda_f (1 - \alpha) \hat{H}_t$$

$$\begin{aligned}\hat{y}_t &: A_{22}(4, 4) = -1 \\ \hat{H}_t &: A_{22}(4, 12) = \lambda_f (1 - \alpha) \\ \hat{k}_t &: A_{22}(4, 13) = \lambda_f \alpha\end{aligned}$$

$$\begin{aligned}\hat{\epsilon}_t &: A_{21}(4, 1) = \lambda_f \\ \hat{\mu}_{zt} &: A_{21}(4, 3) = -\lambda_f \alpha\end{aligned}$$

#### Row 5 - Export price inflation

$$\begin{aligned}\hat{\pi}_t^x - \hat{\pi}_t^c &= \frac{\beta}{1 + \kappa_x \beta} \left( \hat{\pi}_{t+1|t}^x - \rho_{\bar{\pi}^c} \hat{\pi}_t^c \right) + \frac{\kappa_x}{1 + \kappa_x \beta} \left( \hat{\pi}_{t-1}^x - \hat{\pi}_t^c \right) \\ &\quad - \frac{\kappa_x \beta (1 - \rho_{\bar{\pi}^c})}{1 + \kappa_x \beta} \hat{\pi}_t^c + \frac{(1 - \xi_x)(1 - \beta \xi_x)}{\xi_x (1 + \kappa_x \beta)} \left( \widehat{mc}_t^x + \hat{\lambda}_t^x \right),\end{aligned}$$

$$\begin{aligned}\hat{\pi}_{t+1|t}^x &: H(5, 5) = -\frac{\beta}{1 + \kappa_x \beta} \\ \hat{\pi}_t^x &: A_{22}(5, 5) = -1 \\ \hat{\pi}_{t-1}^x &: A_{21}(5, 49) = \frac{\kappa_x}{1 + \kappa_x \beta}\end{aligned}$$

Marginal cost :

$$\begin{aligned}\widehat{mc}_t^x &: A_{22}(5, 21) = \frac{(1 - \xi_x)(1 - \beta \xi_x)}{\xi_x (1 + \kappa_x \beta)} \\ &\quad \left[ \hat{\lambda}_t^x : A_{21}(5, 16) = \frac{(1 - \xi_x)(1 - \beta \xi_x)}{\xi_x (1 + \kappa_x \beta)} \right] \\ \hat{\lambda}_t^x &: A_{21}(5, 16) = 1 \text{ (rescaled)} \\ \hat{\pi}_t^c &: A_{21}(5, 18) = 1 - \frac{\beta}{1 + \kappa_x \beta} \rho_{\bar{\pi}^c} - \frac{\kappa_x}{1 + \kappa_x \beta} - \frac{\kappa_x \beta (1 - \rho_{\bar{\pi}^c})}{1 + \kappa_x \beta}\end{aligned}$$

#### Row 6 - Real wage

$$\begin{aligned}0 &= E_t \left\{ \begin{array}{l} \eta_0 \hat{w}_{t-1} + \eta_1 \hat{w}_t + \eta_2 \hat{w}_{t+1} + \eta_3 (\hat{\pi}_t^d - \hat{\pi}_t^c) + \eta_4 (\hat{\pi}_{t+1}^d - \rho_{\bar{\pi}^c} \hat{\pi}_t^c) \\ \quad + \eta_5 (\hat{\pi}_{t-1}^c - \hat{\pi}_t^c) + \eta_6 (\hat{\pi}_t^c - \rho_{\bar{\pi}^c} \hat{\pi}_t^c) \\ \quad + \eta_7 \hat{\psi}_{zt}^\tau + \eta_8 \hat{H}_t + \eta_9 \hat{\tau}_t^y + \eta_{10} \hat{\tau}_t^w + \eta_{11} \hat{\zeta}_t^h \end{array} \right\} \\ \hat{\pi}_t^c &= (1 - \omega_c) \left( \gamma^{cd} \right)^{-(1-\eta_c)} \hat{\pi}_t^d + \omega_c (\gamma^{cmc})^{-(1-\eta_c)} \hat{\pi}_t^{mc}\end{aligned}$$

where

$$\eta \equiv \begin{bmatrix} b_w \xi_w \\ \sigma_L \lambda_w - b_w (1 + \beta \xi_w^2) \\ b_w \beta \xi_w \\ -b_w \xi_w \\ b_w \beta \xi_w \\ b_w \xi_w \kappa_w \\ -b_w \beta \xi_w \kappa_w \\ 1 - \lambda_w \\ -(1 - \lambda_w) \sigma_L \\ -(1 - \lambda_w) \frac{\tau^y}{(1 - \tau^y)} \\ -(1 - \lambda_w) \frac{\tau^w}{(1 + \tau^w)} \\ -(1 - \lambda_w) \end{bmatrix} \equiv \begin{bmatrix} \eta_0 \\ \eta_1 \\ \eta_2 \\ \eta_3 \\ \eta_4 \\ \eta_5 \\ \eta_6 \\ \eta_7 \\ \eta_8 \\ \eta_9 \\ \eta_{10} \\ \eta_{11} \end{bmatrix}. \quad (1.3)$$

Collect terms and use the definition of  $\hat{\pi}_t^c$ :

$$\begin{aligned} \eta_2 \hat{w}_{t+1|t} + \eta_4 \hat{\pi}_{t+1|t}^d &= -\eta_0 \hat{w}_{t-1} - \eta_1 \hat{w}_t - \left[ \eta_3 + \eta_6 \left( (1 - \omega_c) (\gamma^{cd})^{-(1-\eta_c)} \right) \right] \hat{\pi}_t^d \\ &\quad - \eta_6 \omega_c (\gamma^{cmc})^{-(1-\eta_c)} \hat{\pi}_t^{mc} \\ &\quad - \eta_5 \left( (1 - \omega_c) (\gamma^{cd})^{-(1-\eta_c)} \right) \hat{\pi}_{t-1}^d - \eta_5 \left( (\omega_c) (\gamma^{cmc})^{-(1-\eta_c)} \right) \hat{\pi}_{t-1}^{mc} \\ &\quad - \eta_7 \hat{\psi}_{zt}^\tau - \eta_8 \hat{H}_t - \eta_9 \hat{\tau}_t^y - \eta_{10} \hat{\tau}_t^w - \eta_{11} \hat{\zeta}_t^h + [\eta_3 + \rho_{\hat{\pi}^c} \eta_4 + \eta_5 + \rho_{\hat{\pi}^c} \eta_6] \hat{\pi}_t^c \end{aligned}$$

$$\hat{\pi}_{t+1|t}^d : H(6, 1) = \eta_4$$

$$\hat{w}_{t+1|t} : H(6, 6) = \eta_2$$

$$\begin{aligned}\hat{\pi}_t^d &: A_{22}(6, 1) = -\left[\eta_3 + \eta_6 \left((1 - \omega_c) (\gamma^{cd})^{-(1-\eta_c)}\right)\right] \\ \hat{\pi}_t^{mc} &: A_{22}(6, 2) = -\eta_6 \omega_c (\gamma^{cmc})^{-(1-\eta_c)} \\ \hat{w}_t &: A_{22}(6, 6) = -\eta_1 \\ \hat{\psi}_{zt}^\tau &: A_{22}(6, 9) = -\eta_7 \\ \hat{H}_t &: A_{22}(6, 12) = -\eta_8\end{aligned}$$

Rescaling the labor supply shock so its coefficient is of the same magnitude as  $\hat{w}_t$

$$\begin{aligned}\left[ \hat{\zeta}_t^h : A_{21}(6, 7) = -\eta_{11} \right] \\ \hat{\zeta}_t^h &: \text{if } \eta_{11} \begin{cases} > 0 \Rightarrow A_{21}(6, 7) = \eta_1 \\ < 0 \Rightarrow A_{21}(6, 7) = -\eta_1 \\ = 0 \Rightarrow A_{21}(6, 7) = \eta_{11} \end{cases} \\ \hat{\pi}_t^c &: A_{21}(6, 18) = (\eta_3 + \rho_{\hat{\pi}} \eta_4 + \eta_5 + \rho_{\hat{\pi}} \eta_6) \\ \hat{\tau}_t^y &: A_{21}(6, 21) = -\eta_9 \\ \hat{\tau}_t^w &: A_{21}(6, 23) = -\eta_{10} \\ \hat{\pi}_{t-1}^d &: A_{21}(6, 45) = -\eta_5 (1 - \omega_c) (\gamma^{cd})^{-(1-\eta_c)} \\ \hat{\pi}_{t-1}^{mc} &: A_{21}(6, 46) = -\eta_5 \omega_c (\gamma^{cmc})^{-(1-\eta_c)} \\ \hat{w}_{t-1} &: A_{21}(6, 50) = -\eta_0\end{aligned}$$

## Row 7 - Consumption

$$E_t \left[ \begin{array}{l} -b\beta\mu_z \hat{c}_{t+1} + (\mu_z^2 + b^2\beta) \hat{c}_t - b\mu_z \hat{c}_{t-1} + b\mu_z (\hat{\mu}_{zt} - \beta \hat{\mu}_{z,t+1}) + \\ + (\mu_z - b\beta) (\mu_z - b) \hat{\psi}_{zt} + \frac{\tau^c}{1+\tau^c} (\mu_z - b\beta) (\mu_z - b) \hat{\tau}_t^c + (\mu_z - b\beta) (\mu_z - b) \hat{\gamma}_t^{cd} \\ - (\mu_z - b) (\mu_z \hat{\zeta}_t^c - b\beta \hat{\zeta}_{t+1}^c) \end{array} \right] = 0$$

$$\begin{aligned}\hat{\gamma}_t^{cd} &= \omega_c (\gamma^{cmc})^{-(1-\eta_c)} \hat{\gamma}_t^{mc} \\ \hat{\mu}_{z,t+1|t} &= \rho_{\mu_z} \hat{\mu}_{zt} \\ \hat{\zeta}_{t+1|t}^c &= \rho_{\zeta^c} \hat{\zeta}_t^c\end{aligned}$$

$$\begin{aligned}b\beta\mu_z \hat{c}_{t+1|t} &= (\mu_z^2 + b^2\beta) \hat{c}_t - b\mu_z \hat{c}_{t-1} + b\mu_z (1 - \beta \rho_{\mu_z}) \hat{\mu}_{zt} \\ &+ (\mu_z - b\beta) (\mu_z - b) \hat{\psi}_{zt} + \frac{\tau^c}{1+\tau^c} (\mu_z - b\beta) (\mu_z - b) \hat{\tau}_t^c \\ &+ (\mu_z - b\beta) (\mu_z - b) \omega_c (\gamma^{cmc})^{-(1-\eta_c)} \hat{\gamma}_t^{mc} - (\mu_z - b) (\mu_z - b\beta \rho_{\zeta^c}) \hat{\zeta}_t^c\end{aligned}$$

$$\begin{aligned}
\hat{c}_{t+1|t} & : H(7, 7) = b\beta\mu_z \\
\hat{c}_t & : A_{22}(7, 7) = (\mu_z^2 + b^2\beta) \\
\hat{c}_{t-1} & : A_{21}(7, 51) = -b\mu_z \\
\\
\hat{\psi}_{zt} & : A_{22}(7, 9) = (\mu_z - b\beta)(\mu_z - b) \\
\hat{\gamma}_t^{mcd} & : A_{22}(7, 17) = (\mu_z - b\beta)(\mu_z - b)\omega_c(\gamma^{cmc})^{-(1-\eta_c)} \\
\\
\hat{\mu}_{zt} & : A_{21}(7, 3) = b\mu_z \left(1 - \beta\rho_{\mu_z}\right) \\
& \text{Rescaling of } \hat{\zeta}_t^c \text{ to give it the same coefficient as } \hat{c}_t \\
& \left[ \hat{\zeta}_t^c : A_{21}(7, 6) = -(\mu_z - b)(\mu_z - b\beta\rho_{\zeta^c}) \right] \\
\hat{\zeta}_t^c & : A_{21}(7, 6) = -(\mu_z^2 + b^2\beta) \\
\hat{\tau}_t^c & : A_{21}(7, 22) = \frac{\tau^c}{1 + \tau^c}(\mu_z - b\beta)(\mu_z - b)
\end{aligned}$$

### Row 8 - Investment

$$E_t \left\{ \hat{P}_{kt} + \hat{\Upsilon}_t - \hat{\gamma}_t^{i,d} - \mu_z^2 S''(\mu_z) \left[ (\hat{i}_t - \hat{i}_{t-1}) - \beta (\hat{i}_{t+1} - \hat{i}_t) + \hat{\mu}_{zt} - \beta \hat{\mu}_{z,t+1} \right] \right\} = 0$$

$$\begin{aligned}
\hat{\gamma}_t^{i,d} & = \omega_i (\gamma^{i,mi})^{-(1-\eta_i)} \hat{\gamma}_t^{mi,d} \\
\hat{\mu}_{z,t+1|t} & = \rho_{\mu_z} \hat{\mu}_{zt}
\end{aligned}$$

$$\begin{aligned}
\mu_z^2 S''(\mu_z) \beta \hat{i}_{t+1|t} & = \mu_z^2 S''(\mu_z) (1 + \beta) \hat{i}_t - \mu_z^2 S''(\mu_z) \hat{i}_{t-1} - \hat{P}_{kt} \\
& + \omega_i (\gamma^{i,mi})^{-(1-\eta_i)} \hat{\gamma}_t^{mi,d} + \mu_z^2 S''(\mu_z) (1 - \beta \rho_{\mu_z}) \hat{\mu}_{zt} - \hat{\Upsilon}_t
\end{aligned}$$

$$\begin{aligned}
\hat{i}_{t+1|t} & : H(8, 8) = \mu_z^2 S''(\mu_z) \beta \\
\hat{i}_t & : A_{22}(8, 8) = \mu_z^2 S''(\mu_z) (1 + \beta) \\
\hat{i}_{t-1} & : A_{21}(8, 52) = -\mu_z^2 S''(\mu_z) \\
\\
\hat{P}_{kt} & : A_{22}(8, 10) = -1 \\
\hat{\gamma}_t^{mi,d} & : A_{22}(8, 18) = \omega_i (\gamma^{i,mi})^{-(1-\eta_i)} \\
\\
\hat{\mu}_{zt} & : A_{21}(8, 3) = \mu_z^2 S''(\mu_z) \left(1 - \beta \rho_{\mu_z}\right) \\
& \text{Rescaling of } \hat{\Upsilon}_t \text{ to give it the same coefficient as } \hat{i}_t \\
\hat{\Upsilon}_t & : A_{21}(8, 13) = -\mu_z^2 S''(\mu_z) (1 + \beta)
\end{aligned}$$

**Row 9 - Lagrange multiplier (first order condition with respect to  $m_{t+1}$ )**

$$\begin{aligned} \text{E}_t \left[ -\mu \hat{\psi}_{zt} + \mu \hat{\psi}_{z,t+1} - \mu \hat{\pi}_{t+1}^d - \mu \hat{\mu}_{z,t+1} + (\mu - \beta \tau^k) \hat{R}_t + \frac{\tau^k}{1 - \tau^k} (\beta - \mu) \hat{\tau}_{t+1}^k \right] &= 0 \\ \hat{\mu}_{z,t+1|t} &= \rho_{\mu_z} \hat{\mu}_{zt} \end{aligned}$$

$$\begin{aligned} \mu \hat{\psi}_{z,t+1|t} - \mu \hat{\pi}_{t+1|t}^d &= \mu \hat{\psi}_{zt} - (\mu - \beta \tau^k) \hat{R}_t + \mu \hat{\mu}_{z,t+1|t} - \frac{\tau^k}{1 - \tau^k} (\beta - \mu) \hat{\tau}_{t+1|t}^k \\ \hat{\mu}_{z,t+1|t} &= \rho_{\mu_z} \hat{\mu}_{zt} \end{aligned}$$

$$\begin{aligned} 0 &= -\mu \hat{\psi}_{zt} + \mu \hat{\psi}_{z,t+1|t} - \mu \hat{\mu}_{z,t+1|t} + (\mu - \beta \tau^k) \hat{R}_t - \mu \hat{\pi}_{t+1|t}^d + \frac{\tau^k}{1 - \tau^k} (\beta - \mu) \hat{\tau}_{t+1|t}^k \\ \hat{\mu}_{z,t+1|t} &= \rho_{\mu_z} \hat{\mu}_{zt} \\ \hat{\tau}_{t+1|t}^k &= \tilde{\Theta}_1(1, 1) \hat{\tau}_t^k + \tilde{\Theta}_1(1, 2) \hat{\tau}_t^y + \tilde{\Theta}_1(1, 3) \hat{\tau}_t^c + \tilde{\Theta}_1(1, 4) \hat{\tau}_t^w + \tilde{\Theta}_1(1, 5) \hat{g}_t \\ &\quad + \tilde{\Theta}_2(1, 1) \hat{\tau}_{t-1}^k + \tilde{\Theta}_2(1, 2) \hat{\tau}_{t-1}^y + \tilde{\Theta}_2(1, 3) \hat{\tau}_{t-1}^c + \tilde{\Theta}_2(1, 4) \hat{\tau}_{t-1}^w + \tilde{\Theta}_2(1, 5) \hat{g}_{t-1} \end{aligned}$$

$$\begin{aligned} \mu \hat{\psi}_{z,t+1|t} - \mu \hat{\pi}_{t+1|t}^d &= \mu \hat{\psi}_{zt} + \mu \rho_{\mu_z} \hat{\mu}_{zt} - (\mu - \beta \tau^k) \hat{R}_t \\ &\quad - \frac{\tau^k}{1 - \tau^k} (\beta - \mu) \tilde{\Theta}_1(1, 1) \hat{\tau}_t^k - \frac{\tau^k}{1 - \tau^k} (\beta - \mu) \tilde{\Theta}_1(1, 2) \hat{\tau}_t^y \\ &\quad - \frac{\tau^k}{1 - \tau^k} (\beta - \mu) \tilde{\Theta}_1(1, 3) \hat{\tau}_t^c - \frac{\tau^k}{1 - \tau^k} (\beta - \mu) \tilde{\Theta}_1(1, 4) \hat{\tau}_t^w \\ &\quad - \frac{\tau^k}{1 - \tau^k} (\beta - \mu) \tilde{\Theta}_1(1, 5) \hat{g}_t - \frac{\tau^k}{1 - \tau^k} (\beta - \mu) \tilde{\Theta}_2(1, 1) \hat{\tau}_{t-1}^k \\ &\quad - \frac{\tau^k}{1 - \tau^k} (\beta - \mu) \tilde{\Theta}_2(1, 2) \hat{\tau}_{t-1}^y - \frac{\tau^k}{1 - \tau^k} (\beta - \mu) \tilde{\Theta}_2(1, 3) \hat{\tau}_{t-1}^c \\ &\quad - \frac{\tau^k}{1 - \tau^k} (\beta - \mu) \tilde{\Theta}_2(1, 4) \hat{\tau}_{t-1}^w - \frac{\tau^k}{1 - \tau^k} (\beta - \mu) \tilde{\Theta}_2(1, 5) \hat{g}_{t-1} \end{aligned}$$

$$\begin{aligned} \hat{\pi}_{t+1|t}^d &: H(9, 1) = -\mu \\ \hat{\psi}_{z,t+1|t} &: H(9, 9) = \mu \end{aligned}$$

$$\begin{aligned} \hat{\psi}_{zt} &: A_{22}(9, 9) = \mu \\ \hat{R}_t &: B_2(9, 1) = -(\mu - \beta \tau^k) \end{aligned}$$

$$\begin{aligned}
\hat{\mu}_{zt} & : A_{21}(9, 3) = \mu \rho_{\mu_z} \\
\hat{\tau}_t^k & : A_{21}(9, 20) = -\frac{\tau^k}{1 - \tau^k} (\beta - \mu) \tilde{\Theta}_1(1, 1) \\
\hat{\tau}_t^y & : A_{21}(9, 21) = -\frac{\tau^k}{1 - \tau^k} (\beta - \mu) \tilde{\Theta}_1(1, 2) \\
\hat{\tau}_t^c & : A_{21}(9, 22) = -\frac{\tau^k}{1 - \tau^k} (\beta - \mu) \tilde{\Theta}_1(1, 3) \\
\hat{\tau}_t^w & : A_{21}(9, 23) = -\frac{\tau^k}{1 - \tau^k} (\beta - \mu) \tilde{\Theta}_1(1, 4) \\
\hat{g}_t & : A_{21}(9, 24) = -\frac{\tau^k}{1 - \tau^k} (\beta - \mu) \tilde{\Theta}_1(1, 5) \\
\\
\hat{\tau}_{t-1}^k & : A_{21}(9, 25) = -\frac{\tau^k}{1 - \tau^k} (\beta - \mu) \tilde{\Theta}_2(1, 1) \\
\hat{\tau}_{t-1}^y & : A_{21}(9, 26) = -\frac{\tau^k}{1 - \tau^k} (\beta - \mu) \tilde{\Theta}_2(1, 2) \\
\hat{\tau}_{t-1}^c & : A_{21}(9, 27) = -\frac{\tau^k}{1 - \tau^k} (\beta - \mu) \tilde{\Theta}_2(1, 3) \\
\hat{\tau}_{t-1}^w & : A_{21}(9, 28) = -\frac{\tau^k}{1 - \tau^k} (\beta - \mu) \tilde{\Theta}_2(1, 4) \\
\hat{g}_{t-1} & : A_{21}(9, 29) = -\frac{\tau^k}{1 - \tau^k} (\beta - \mu) \tilde{\Theta}_2(1, 5)
\end{aligned}$$

#### Row 10 - Price of capital

$$\begin{aligned}
0 &= \hat{\psi}_{zt} + \hat{\mu}_{z,t+1|t} - \hat{\psi}_{z,t+1|t} - \frac{\beta(1 - \tilde{\delta})}{\mu_z} \hat{P}_{k,t+1|t} + \hat{P}_{kt} - \frac{\mu_z - \beta(1 - \tilde{\delta})}{\mu_z} \hat{r}_{t+1|t}^k \\
&\quad + \frac{\tau^k}{1 - \tau^k} \frac{\mu_z - \beta(1 - \tilde{\delta})}{\mu_z} \hat{\tau}_{t+1|t}^k, \\
\hat{r}_{t+1|t}^k &= \rho_{\mu_z} \hat{\mu}_{zt} + \hat{w}_{t+1|t} + \frac{\nu R}{v(R-1)+1} \hat{R}_t + \frac{\nu(R-1)}{v(R-1)+1} \rho_{\nu} \hat{\nu}_t + \hat{H}_{t+1|t} - \hat{k}_{t+1|t} \\
\hat{\tau}_{t+1|t}^k &= \tilde{\Theta}_1(1, 1) \hat{\tau}_t^k + \tilde{\Theta}_1(1, 2) \hat{\tau}_t^y + \tilde{\Theta}_1(1, 3) \hat{\tau}_t^c + \tilde{\Theta}_1(1, 4) \hat{\tau}_t^w + \tilde{\Theta}_1(1, 5) \hat{g}_t \\
&\quad + \tilde{\Theta}_2(1, 1) \hat{\tau}_{t-1}^k + \tilde{\Theta}_2(1, 2) \hat{\tau}_{t-1}^y + \tilde{\Theta}_2(1, 3) \hat{\tau}_{t-1}^c + \tilde{\Theta}_2(1, 4) \hat{\tau}_{t-1}^w + \tilde{\Theta}_2(1, 5) \hat{g}_{t-1}
\end{aligned}$$

$$\begin{aligned}
& \hat{\psi}_{z,t+1|t} + \frac{\beta(1-\tilde{\delta})}{\mu_z} \hat{P}_{k,t+1|t} + \frac{\mu_z - \beta(1-\tilde{\delta})}{\mu_z} \hat{w}_{t+1|t} + \frac{\mu_z - \beta(1-\tilde{\delta})}{\mu_z} \hat{H}_{t+1|t} \\
& - \frac{\mu_z - \beta(1-\tilde{\delta})}{\mu_z} \hat{k}_{t+1|t} \\
= & \hat{\psi}_{zt} + \left[ \rho_{\mu_z} - \frac{\mu_z - \beta(1-\tilde{\delta})}{\mu_z} \rho_{\mu_z} \right] \hat{\mu}_{zt} + \hat{P}_{kt} - \frac{\mu_z - \beta(1-\tilde{\delta})}{\mu_z} \frac{\nu R}{v(R-1)+1} \hat{R}_t \\
& - \frac{\mu_z - \beta(1-\tilde{\delta})}{\mu_z} \frac{\nu(R-1)}{v(R-1)+1} \rho_\nu \hat{\nu}_t \\
& \frac{\tau^k}{1-\tau^k} \frac{\mu_z - \beta(1-\tilde{\delta})}{\mu_z} \tilde{\Theta}_1(1,1) \hat{\tau}_t^k + \frac{\tau^k}{1-\tau^k} \frac{\mu_z - \beta(1-\tilde{\delta})}{\mu_z} \tilde{\Theta}_1(1,2) \hat{\tau}_t^y \\
& + \frac{\tau^k}{1-\tau^k} \frac{\mu_z - \beta(1-\tilde{\delta})}{\mu_z} \tilde{\Theta}_1(1,3) \hat{\tau}_t^c + \frac{\tau^k}{1-\tau^k} \frac{\mu_z - \beta(1-\tilde{\delta})}{\mu_z} \tilde{\Theta}_1(1,4) \hat{\tau}_t^w \\
& + \frac{\tau^k}{1-\tau^k} \frac{\mu_z - \beta(1-\tilde{\delta})}{\mu_z} \tilde{\Theta}_1(1,5) \hat{g}_t + \frac{\tau^k}{1-\tau^k} \frac{\mu_z - \beta(1-\tilde{\delta})}{\mu_z} \tilde{\Theta}_2(1,1) \hat{\tau}_{t-1}^k \\
& + \frac{\tau^k}{1-\tau^k} \frac{\mu_z - \beta(1-\tilde{\delta})}{\mu_z} \tilde{\Theta}_2(1,2) \hat{\tau}_{t-1}^y + \frac{\tau^k}{1-\tau^k} \frac{\mu_z - \beta(1-\tilde{\delta})}{\mu_z} \tilde{\Theta}_2(1,3) \hat{\tau}_{t-1}^c \\
& + \frac{\tau^k}{1-\tau^k} \frac{\mu_z - \beta(1-\tilde{\delta})}{\mu_z} \tilde{\Theta}_2(1,4) \hat{\tau}_{t-1}^w + \frac{\tau^k}{1-\tau^k} \frac{\mu_z - \beta(1-\tilde{\delta})}{\mu_z} \tilde{\Theta}_2(1,5) \hat{g}_{t-1}
\end{aligned}$$

$$\begin{aligned}
\hat{w}_{t+1|t} & : H(10,6) = \frac{\mu_z - \beta(1-\tilde{\delta})}{\mu_z} \\
\hat{\psi}_{z,t+1|t} & : H(10,9) = 1 \\
\hat{P}_{k,t+1|t} & : H(10,10) = \frac{\beta(1-\tilde{\delta})}{\mu_z} \\
\hat{H}_{t+1|t} & : H(10,12) = \frac{\mu_z - \beta(1-\tilde{\delta})}{\mu_z} \\
\hat{u}_{t+1|t} & : H(10,13) = -\frac{\mu_z - \beta(1-\tilde{\delta})}{\mu_z} \\
\hat{\psi}_{zt} & : A_{22}(10,9) = 1 \\
\hat{\psi}_{zt} & : A_{22}(10,9) = 1 \\
\hat{P}_{kt} & : A_{22}(10,10) = 1
\end{aligned}$$

$$\begin{aligned}
\hat{R}_t & : B_2(10, 1) = -\frac{\mu_z - \beta(1 - \tilde{\delta})}{\mu_z} \frac{\nu R}{v(R-1)+1} \\
\hat{\mu}_{zt} & : A_{21}(10, 3) = \rho_{\mu_z} - \frac{\mu_z - \beta(1 - \tilde{\delta})}{\mu_z} \rho_{\mu_z} \\
\hat{\nu}_t & : A_{21}(10, 5) = -\frac{\mu_z - \beta(1 - \tilde{\delta})}{\mu_z} \frac{\nu(R-1)}{v(R-1)+1} \rho_\nu \\
\hat{\tau}_t^k & : A_{21}(10, 20) = \frac{\tau^k}{1 - \tau^k} \frac{\mu_z - \beta(1 - \tilde{\delta})}{\mu_z} \tilde{\Theta}_1(1, 1) \\
\hat{\tau}_t^y & : A_{21}(10, 21) = \frac{\tau^k}{1 - \tau^k} \frac{\mu_z - \beta(1 - \tilde{\delta})}{\mu_z} \tilde{\Theta}_1(1, 2) \\
\hat{\tau}_t^c & : A_{21}(10, 22) = \frac{\tau^k}{1 - \tau^k} \frac{\mu_z - \beta(1 - \tilde{\delta})}{\mu_z} \tilde{\Theta}_1(1, 3) \\
\hat{\tau}_t^w & : A_{21}(10, 23) = \frac{\tau^k}{1 - \tau^k} \frac{\mu_z - \beta(1 - \tilde{\delta})}{\mu_z} \tilde{\Theta}_1(1, 4) \\
\hat{g}_t & : A_{21}(10, 24) = \frac{\tau^k}{1 - \tau^k} \frac{\mu_z - \beta(1 - \tilde{\delta})}{\mu_z} \tilde{\Theta}_1(1, 5) \\
\hat{\tau}_{t-1}^k & : A_{21}(10, 25) = \frac{\tau^k}{1 - \tau^k} \frac{\mu_z - \beta(1 - \tilde{\delta})}{\mu_z} \tilde{\Theta}_2(1, 1) \\
\hat{\tau}_{t-1}^y & : A_{21}(10, 26) = \frac{\tau^k}{1 - \tau^k} \frac{\mu_z - \beta(1 - \tilde{\delta})}{\mu_z} \tilde{\Theta}_2(1, 2) \\
\hat{\tau}_{t-1}^c & : A_{21}(10, 27) = \frac{\tau^k}{1 - \tau^k} \frac{\mu_z - \beta(1 - \tilde{\delta})}{\mu_z} \tilde{\Theta}_2(1, 3) \\
\hat{\tau}_{t-1}^w & : A_{21}(10, 28) = \frac{\tau^k}{1 - \tau^k} \frac{\mu_z - \beta(1 - \tilde{\delta})}{\mu_z} \tilde{\Theta}_2(1, 4) \\
\hat{g}_{t-1} & : A_{21}(10, 29) = \frac{\tau^k}{1 - \tau^k} \frac{\mu_z - \beta(1 - \tilde{\delta})}{\mu_z} \tilde{\Theta}_2(1, 5)
\end{aligned}$$

### Row 11 - Change in nominal exchange rate (UIP)

$$(1 - \tilde{\phi}_s) E_t \Delta \hat{S}_{t+1} = \tilde{\phi}_s \Delta \hat{S}_t + (\hat{R}_t - \hat{R}_t^*) + \tilde{\phi}_a \hat{a}_t - \hat{\phi}_t$$

$$\begin{aligned}
\Delta \hat{S}_{t+1} & : H(11, 11) = (1 - \tilde{\phi}_s) \\
\Delta \hat{S}_t & : A_{22}(11, 11) = \tilde{\phi}_s \\
\hat{a}_t & : A_{22}(11, 16) = \tilde{\phi}_a
\end{aligned}$$

$$\hat{R}_t : B_2(11, 1) = 1$$

$$\begin{aligned}
\hat{\phi}_t & : A_{21}(11, 12) = -1 \\
\hat{R}_t^* & : A_{21}(11, 32) = -1
\end{aligned}$$

**Row 12 - Hours worked (aggregate resource constraint)**

$$\begin{aligned}
& (1 - \omega_c) (\gamma^{cd})^{\eta_c} \frac{c}{y} (\hat{c}_t + \eta_c \hat{\gamma}_t^{cd}) + (1 - \omega_i) (\gamma^{id})^{\eta_i} \frac{\tilde{i}}{y} (\hat{i}_t + \eta_i \hat{\gamma}_t^{id}) + \frac{g}{y} \hat{g}_t + \frac{y^*}{y} (\hat{y}_t^* - \eta_f \hat{\gamma}_t^{x*} + \hat{z}_t^*) \\
& = \lambda_d [\hat{\varepsilon}_t + \alpha (\hat{k}_t - \hat{\mu}_{zt}) + (1 - \alpha) \hat{H}_t] - (1 - \tau^k) r^k \frac{\bar{k}}{y} \frac{1}{\mu_z} (\hat{k}_t - \hat{\bar{k}}_t) \\
0 & = (1 - \omega_c) (\gamma^{cd})^{\eta_c} \frac{c}{y} (\hat{c}_t + \eta_c \hat{\gamma}_t^{cd}) + (1 - \omega_i) (\gamma^{id})^{\eta_i} \frac{\tilde{i}}{y} (\hat{i}_t + \eta_i \hat{\gamma}_t^{id}) + \frac{g}{y} \hat{g}_t + \frac{y^*}{y} (\hat{y}_t^* - \eta_f \hat{\gamma}_t^{x*} + \hat{z}_t^*) \\
& - \lambda_d [\hat{\varepsilon}_t + \alpha (\hat{k}_t - \hat{\mu}_{zt}) + (1 - \alpha) \hat{H}_t] + (1 - \tau^k) r^k \frac{\bar{k}}{y} \frac{1}{\mu_z} (\hat{k}_t - \hat{\bar{k}}_t) \\
& \hat{\gamma}_t^{cd} = \omega_c (\gamma^{cmc})^{-(1-\eta_c)} \hat{\gamma}_t^{mc} \\
& \hat{\gamma}_t^{i,d} = \omega_i (\gamma^{i,mi})^{-(1-\eta_i)} \hat{\gamma}_t^{mi,d} \\
0 & = (1 - \omega_c) (\gamma^{cd})^{\eta_c} \frac{c}{y} \hat{c}_t + (1 - \omega_c) (\gamma^{cd})^{\eta_c} \frac{c}{y} \eta_c \omega_c (\gamma^{cmc})^{-(1-\eta_c)} \hat{\gamma}_t^{mc} + (1 - \omega_i) (\gamma^{id})^{\eta_i} \frac{\tilde{i}}{y} \hat{i}_t \\
& + (1 - \omega_i) (\gamma^{id})^{\eta_i} \frac{\tilde{i}}{y} \eta_i \omega_i (\gamma^{i,mi})^{-(1-\eta_i)} \hat{\gamma}_t^{mi,d} - \frac{y^*}{y} \eta_f \hat{\gamma}_t^{x*} + \frac{g}{y} \hat{g}_t + \frac{y^*}{y} \hat{y}_t^* + \frac{y^*}{y} \hat{z}_t^* \\
& - \lambda_d \hat{\varepsilon}_t + \lambda_d \alpha \hat{\mu}_{zt} - \lambda_d (1 - \alpha) \hat{H}_t - \left[ \lambda_d \alpha - (1 - \tau^k) r^k \frac{\bar{k}}{y} \frac{1}{\mu_z} \right] \hat{k}_t - (1 - \tau^k) r^k \frac{\bar{k}}{y} \frac{1}{\mu_z} \hat{\bar{k}}_t \\
& \hat{c}_t : A_{22}(12, 7) = (1 - \omega_c) (\gamma^{cd})^{\eta_c} \frac{c}{y} \\
& \hat{i}_t : A_{22}(12, 8) = (1 - \omega_i) (\gamma^{id})^{\eta_i} \frac{\tilde{i}}{y} \\
& \hat{H}_t : A_{22}(12, 12) = -\lambda_d (1 - \alpha) \\
& \hat{k}_t : A_{22}(12, 13) = - \left[ \lambda_d \alpha - (1 - \tau^k) r^k \frac{\bar{k}}{y} \frac{1}{\mu_z} \right] \\
& \hat{\gamma}_t^{mc} : A_{22}(12, 17) = (1 - \omega_c) (\gamma^{cd})^{\eta_c} \frac{c}{y} \eta_c \omega_c (\gamma^{cmc})^{-(1-\eta_c)} \\
& \hat{\gamma}_t^{mi,d} : A_{22}(12, 18) = (1 - \omega_i) (\gamma^{id})^{\eta_i} \frac{\tilde{i}}{y} \eta_i \omega_i (\gamma^{i,mi})^{-(1-\eta_i)} \\
& \hat{\gamma}_t^{x*} : A_{22}(12, 19) = -\frac{y^*}{y} \eta_f \\
& \hat{\varepsilon}_t : A_{21}(12, 1) = -\lambda_d \\
& \hat{\mu}_{zt} : A_{21}(12, 3) = \lambda_d \alpha \\
& \hat{z}_t^* : A_{21}(12, 14) = \frac{y^*}{y} \\
& \hat{g}_t : A_{21}(12, 24) = \frac{g}{y} \\
& \hat{y}_t^* : A_{21}(12, 31) = \frac{y^*}{y} \\
& \hat{\bar{k}}_t : A_{21}(12, 42) = - (1 - \tau^k) r^k \frac{\bar{k}}{y} \frac{1}{\mu_z}
\end{aligned}$$

**Row 13 - Capital-services flow**

$$\begin{aligned}
\hat{u}_t &= \frac{1}{\sigma_a} \hat{r}_t^k - \frac{1}{\sigma_a} \frac{\tau^k}{(1-\tau^k)} \hat{\tau}_t^k, \\
\hat{u}_t &\equiv \hat{k}_t - \hat{\bar{k}}_t, \\
\hat{r}_t^k &= \hat{\mu}_{zt} + \hat{w}_t + \hat{R}_t^f + \hat{H}_t - \hat{k}_t. \\
0 &= -(1+\sigma_a) \hat{k}_t + \sigma_a \hat{\bar{k}}_t + \hat{\mu}_{zt} + \hat{w}_t + \hat{R}_t^f + \hat{H}_t - \frac{\tau^k}{(1-\tau^k)} \hat{\tau}_t^k \\
0 &= -(1+\sigma_a) \hat{k}_t + \sigma_a \hat{\bar{k}}_t + \hat{w}_t + \hat{H}_t + \frac{\nu R}{v(R-1)+1} \hat{R}_{t-1} + \hat{\mu}_{zt} + \frac{\nu(R-1)}{v(R-1)+1} \hat{\nu}_t - \frac{\tau^k}{(1-\tau^k)} \hat{\tau}_t^k \\
\hat{w}_t &: A_{22}(13, 6) = 1 \\
\hat{H}_t &: A_{22}(13, 12) = 1 \\
\hat{k}_t &: A_{22}(13, 13) = -(1+\sigma_a) \\
\hat{\mu}_{zt} &: A_{21}(13, 3) = 1 \\
\hat{\nu}_t &: A_{21}(13, 5) = \frac{\nu(R-1)}{v(R-1)+1} \\
\hat{\tau}_t^k &: A_{21}(13, 20) = -\frac{\tau^k}{(1-\tau^k)} \\
\hat{\bar{k}}_t &: A_{21}(13, 42) = \sigma_a \\
\hat{R}_{t-1} &: A_{21}(13, 44) = \frac{\nu R}{v(R-1)+1}
\end{aligned}$$

**Row 14 - Real money balances**

$$\begin{aligned}
\hat{q}_t &= \frac{1}{\sigma_q} \left[ \hat{\zeta}_t^q + \frac{\tau^k}{1-\tau^k} \hat{\tau}_t^k - \hat{\psi}_{zt} - \frac{R}{R-1} \hat{R}_{t-1} \right], \\
0 &= -\sigma_q \hat{q}_t - \hat{\psi}_{zt} - \frac{R}{R-1} \hat{R}_{t-1} + \hat{\zeta}_t^q + \frac{\tau^k}{1-\tau^k} \hat{\tau}_t^k, \\
\hat{\psi}_{zt} &: A_{22}(14, 9) = -1 \\
\hat{q}_t &: A_{22}(14, 14) = -\sigma_q \\
\hat{\zeta}_t^q &: A_{21}(14, 8) = 1 \\
\hat{\tau}_t^k &: A_{21}(14, 20) = \frac{\tau^k}{1-\tau^k} \\
\hat{R}_{t-1} &: A_{21}(14, 44) = -\frac{R}{R-1}
\end{aligned}$$

**Row 15 - Loan market clearing**

$$\begin{aligned}
\nu \bar{w} H \left( \hat{\nu}_t + \hat{w}_t + \hat{H}_t \right) &= \frac{\mu \bar{m}}{\pi^d \mu_z} \left( \hat{\mu}_t + \hat{m}_t - \hat{\pi}_t^d - \hat{\mu}_{zt} \right) - q \hat{q}_t, \\
0 &= -\nu \bar{w} H \hat{\nu}_t - \nu \bar{w} H \hat{w}_t - \nu \bar{w} H \hat{H}_t + \frac{\mu \bar{m}}{\pi^d \mu_z} \hat{\mu}_t + \frac{\mu \bar{m}}{\pi^d \mu_z} \hat{m}_t - \frac{\mu \bar{m}}{\pi^d \mu_z} \hat{\pi}_t^d - \frac{\mu \bar{m}}{\pi^d \mu_z} \hat{\mu}_{zt} - q \hat{q}_t, \\
\hat{\pi}_t^d &: A_{22}(15, 1) = -\frac{\mu \bar{m}}{\pi^d \mu_z} \\
\hat{w}_t &: A_{22}(15, 6) = -\nu \bar{w} H \\
\hat{H}_t &: A_{22}(15, 12) = -\nu \bar{w} H \\
\hat{q}_t &: A_{22}(15, 14) = -q \\
\hat{\mu}_t &: A_{22}(15, 15) = \frac{\mu \bar{m}}{\pi^d \mu_z} \\
& \\
\hat{\nu}_t &: A_{21}(15, 5) = -\nu \bar{w} H \\
\hat{\mu}_{zt} &: A_{21}(15, 3) = -\frac{\mu \bar{m}}{\pi^d \mu_z} \\
\hat{m}_t &: A_{21}(15, 43) = \frac{\mu \bar{m}}{\pi^d \mu_z}
\end{aligned}$$

**Row 16 - Net foreign assets**

$$\begin{aligned}
\hat{a}_t &= -y^* \widehat{\text{mc}}_t^x - \eta_f y_t^* \hat{\gamma}_t^{x*} + y^* \hat{y}_t^* + y^* \hat{z}_t^* + (c^m + \tilde{i}^m) \hat{\gamma}_t^f \\
&\quad - c^m \hat{c}_t + c^m \eta_c (1 - \omega_c) \left( \gamma^{cd} \right)^{-(1-\eta_c)} \hat{\gamma}_t^{mcd} \\
&\quad - \tilde{i}^m \hat{u}_t + \tilde{i}^m \eta_i (1 - \omega_i) \left( \gamma^{id} \right)^{-(1-\eta_i)} \hat{\gamma}_t^{mid} + \frac{R}{\pi \mu_z} \hat{a}_{t-1}, \\
\hat{\gamma}_t^f &= \widehat{\text{mc}}_t^x + \hat{\gamma}_t^{x*}
\end{aligned}$$

$$\begin{aligned}
0 &= -\hat{a}_t - [y^* - (c^m + \tilde{i}^m)] \widehat{\text{mc}}_t^x - [\eta_f y_t^* - (c^m + \tilde{i}^m)] \hat{\gamma}_t^{x,*} + y^* \hat{y}_t^* + y^* \hat{z}_t^* \\
&\quad - c^m \hat{c}_t + c^m \eta_c (1 - \omega_c) \left( \gamma^{cd} \right)^{-(1-\eta_c)} \hat{\gamma}_t^{mcd} \\
&\quad - \tilde{i}^m \hat{u}_t + \tilde{i}^m \eta_i (1 - \omega_i) \left( \gamma^{id} \right)^{-(1-\eta_i)} \hat{\gamma}_t^{mid} + \frac{R}{\pi \mu_z} \hat{a}_{t-1},
\end{aligned}$$

$$\begin{aligned}
\hat{c}_t & : A_{22}(16, 7) = -c^m \\
\hat{i}_t & : A_{22}(16, 8) = -i^m \\
\hat{a}_t & : A_{22}(16, 16) = -1 \\
\hat{\gamma}_t^{mcd} & : A_{22}(16, 17) = c^m \eta_c (1 - \omega_c) \left( \gamma^{cd} \right)^{-(1-\eta_c)} \\
\hat{\gamma}_t^{mid} & : A_{22}(16, 18) = i^m \eta_i (1 - \omega_i) \left( \gamma^{id} \right)^{-(1-\eta_i)} \\
\hat{\gamma}_t^{x*} & : A_{22}(16, 19) = -[\eta_f y_t^* + (c^m + i^m)] \\
\widehat{\text{mc}}_t^x & : A_{22}(16, 21) = -[y^* + (c^m + i^m)]
\end{aligned}$$

$$\begin{aligned}
\hat{\tilde{z}}_t^* & : A_{21}(16, 14) = y^* \\
\hat{y}_t^* & : A_{21}(16, 31) = y^* \\
\hat{a}_{t-1} & : A_{21}(16, 60) = \frac{R}{\pi \mu_z}
\end{aligned}$$

**Row 17 - Relative price - imported cons vs. domestic**

$$0 = -\hat{\gamma}_t^{mcd} + \hat{\gamma}_{t-1}^{mcd} + \hat{\pi}_t^{mc} - \hat{\pi}_t^d$$

$$\begin{aligned}
\hat{\pi}_t^d & : A_{22}(17, 1) = -1 \\
\hat{\pi}_t^{mc} & : A_{22}(17, 2) = 1 \\
\hat{\gamma}_t^{mcd} & : A_{22}(17, 17) = -1
\end{aligned}$$

$$\hat{\gamma}_{t-1}^{mcd} : A_{21}(17, 61) = 1$$

**Row 18 - Relative price - imported investment vs. domestic**

$$0 = -\hat{\gamma}_t^{mid} + \hat{\gamma}_{t-1}^{mid} + \hat{\pi}_t^{mi} - \hat{\pi}_t^d$$

$$\begin{aligned}
\hat{\pi}_t^d & : A_{22}(18, 1) = -1 \\
\hat{\pi}_t^{mi} & : A_{22}(18, 3) = 1 \\
\hat{\gamma}_t^{mid} & : A_{22}(18, 18) = -1
\end{aligned}$$

$$\hat{\gamma}_{t-1}^{mid} : A_{21}(18, 62) = 1$$

**Row 19 - Relative price - export vs. foreign**

$$0 = -\hat{\gamma}_t^{x*} + \hat{\gamma}_{t-1}^{x*} + \hat{\pi}_t^x - \hat{\pi}_t^*$$

$$\begin{aligned}
\hat{\pi}_t^x & : A_{22}(19, 5) = 1 \\
\hat{\gamma}_t^{x*} & : A_{22}(19, 19) = -1 \\
\hat{\pi}_t^* & : A_{21}(19, 30) = -1 \\
\hat{\gamma}_{t-1}^{x*} & : A_{21}(19, 63) = 1
\end{aligned}$$

**Row 20 - Real exchange rate**

$$\begin{aligned}\hat{\tilde{x}}_t &= -\omega_c(\gamma^{cmc})^{-(1-\eta_c)}\hat{\gamma}_t^{mcd} - \hat{\gamma}_t^{x,*} - \widehat{\text{mc}}_t^x \\ 0 &= -\hat{\tilde{x}}_t - \omega_c(\gamma^{cmc})^{-(1-\eta_c)}\hat{\gamma}_t^{mcd} - \hat{\gamma}_t^{x,*} - \widehat{\text{mc}}_t^x\end{aligned}$$

$$\begin{aligned}\widehat{\text{mc}}_t^x &= \widehat{\text{mc}}_{t-1}^x + \hat{\pi}_t^d - \hat{\pi}_t^x - \Delta \hat{S}_t \\ \hat{\gamma}_t^{x,*} &= \hat{\gamma}_{t-1}^{x,*} + \hat{\pi}_t^x - \hat{\pi}_t^* \\ \hat{\gamma}_t^{mcd} &= \hat{\gamma}_{t-1}^{mcd} + \hat{\pi}_t^{mc} - \hat{\pi}_t^d\end{aligned}$$

$$\begin{aligned}\hat{\gamma}_t^{mcd} &: A_{22}(20, 17) = -\omega_c(\gamma^{cmc})^{-(1-\eta_c)} \\ \hat{\gamma}_t^{x,*} &: A_{22}(20, 19) = -1 \\ \hat{\tilde{x}}_t &: A_{22}(20, 20) = -1 \\ \widehat{\text{mc}}_t^x &: A_{22}(20, 21) = -1\end{aligned}$$

**Row 21 - Marginal cost export**

$$0 = -\widehat{\text{mc}}_t^x + \widehat{\text{mc}}_{t-1}^x + \hat{\pi}_t^d - \hat{\pi}_t^x - \Delta \hat{S}_t$$

$$\begin{aligned}\hat{\pi}_t^d &: A_{22}(21, 1) = 1 \\ \hat{\pi}_t^x &: A_{22}(21, 5) = -1 \\ \Delta \hat{S}_t &: A_{22}(21, 11) = -1 \\ \widehat{\text{mc}}_t^x &: A_{22}(21, 21) = -1\end{aligned}$$

$$\widehat{\text{mc}}_{t-1}^x : A_{21}(21, 65) = 1$$

**Row 22 - Physical capital stock** (note that  $\hat{\bar{k}}_{t+1}$  is predetermined variable no. 42 in period  $t+1$  and forward-looking variable no. 22 in period  $t$ ):

$$0 = -\hat{\bar{k}}_{t+1} + (1 - \tilde{\delta})\frac{1}{\mu_z}\hat{\bar{k}}_t - (1 - \tilde{\delta})\frac{1}{\mu_z}\hat{\mu}_{zt} + \left[1 - (1 - \tilde{\delta})\frac{1}{\mu_z}\right]\hat{\Upsilon}_t + \left[1 - (1 - \tilde{\delta})\frac{1}{\mu_z}\right]\hat{i}_t,$$

$$\begin{aligned}\hat{\bar{k}}_{t+1} &: A_{22}(22, 22) = -1, \\ \hat{\bar{k}}_t &: A_{21}(22, 42) = (1 - \tilde{\delta})\frac{1}{\mu_z}, \\ \hat{\mu}_{zt} &: A_{21}(22, 3) = -(1 - \tilde{\delta})\frac{1}{\mu_z}, \\ \hat{\Upsilon}_t &: \left[A_{21}(22, 13) = 1 - (1 - \tilde{\delta})\frac{1}{\mu_z}\right], \\ \hat{\Upsilon}_t &: A_{21}(22, 13) = \tilde{S}''\mu_z^2(1 + \beta)\left(1 - (1 - \tilde{\delta})\frac{1}{\mu_z}\right) \quad (\text{rescaled shock!}) \\ \hat{\bar{i}}_t &: A_{22}(22, 8) = 1 - (1 - \tilde{\delta})\frac{1}{\mu_z}.\end{aligned}$$

**Row 23 - Financial assets** (note that  $\hat{m}_{t+1}$  is predetermined variable no. 43 in period  $t+1$  and forward-looking variable no. 23 in period  $t$ )

$$0 = -\hat{m}_{t+1} + \hat{\mu}_t - \hat{\mu}_{zt} - \hat{\pi}_t^d + \hat{m}_t$$

$$\begin{aligned}\hat{m}_{t+1} &: A_{22}(23, 23) = -1 \\ \hat{\mu}_{zt} &: A_{21}(23, 3) = -1 \\ \hat{m}_t &: A_{21}(23, 43) = 1 \\ \hat{\pi}_t^d &: A_{22}(23, 1) = -1 \\ \hat{\mu}_t &: A_{22}(23, 15) = 1\end{aligned}$$

### 1.3. Specifying D

We also specify the target variables, for convenience including some variables of interest which may have zero weight in the loss function. They are the quarterly and four-quarterly CPI inflation gaps relative to steady-state inflation, which equals the official inflation target; the output gap relative to steady-state output (we may also use the gap relative to potential output, meaning the flexprice and flexwage output level); the quarterly interest rate; the quarterly interest-rate differential; and the real exchange rate. The inflation and interest rates are all measured at an annual rate. Then the vector of target variables,  $Y_t$ , and the corresponding matrix  $D$  satisfy

$$Y_t \equiv [4\hat{\pi}_t^{cpi}, \bar{\hat{\pi}}_t^{cpi}, \hat{y}_t, 4\hat{R}_t, 4(\hat{R}_t - \hat{R}_{t-1}), \hat{x}_t]' \equiv D \begin{bmatrix} X_t \\ x_t \\ i_t \end{bmatrix},$$

where

$$\bar{\hat{\pi}}_t^{cpi} = \hat{\pi}_t^{cpi} + \hat{\pi}_{t-1}^{cpi} + \hat{\pi}_{t-2}^{cpi} + \hat{\pi}_{t-3}^{cpi}.$$

Quarterly CPI inflation gap:

$$4\hat{\pi}_t^{cpi} = 4[(1 - \omega_c)(\gamma^{cd})^{-(1-\eta_c)}\hat{\pi}_t^d + \omega_c(\gamma^{cmc})^{-(1-\eta_c)}\hat{\pi}_t^{mc}], \quad (1.4)$$

$$\begin{aligned}\hat{\pi}_t^d &: D(1, n_X + 1) = 4(1 - \omega_c)(\gamma^{cd})^{-(1-\eta_c)}, \\ \hat{\pi}_t^{mc} &: D(1, n_X + 2) = 4\omega_c(\gamma^{cmc})^{-(1-\eta_c)}.\end{aligned}$$

Four-quarter CPI inflation gap:

$$\bar{\hat{\pi}}_t^{cpi} = \hat{\pi}_t^{cpi} + \hat{\pi}_{t-1}^{cpi} + \hat{\pi}_{t-2}^{cpi} + \hat{\pi}_{t-3}^{cpi}. \quad (1.5)$$

$$\begin{aligned}\hat{\pi}_t^d &: D(2, n_X + 1) = (1 - \omega_c)(\gamma^{cd})^{-(1-\eta_c)}, \\ \hat{\pi}_{t-1}^d &: D(2, 45) = (1 - \omega_c)(\gamma^{cd})^{-(1-\eta_c)}, \\ \hat{\pi}_{t-2}^d &: D(2, 68) = (1 - \omega_c)(\gamma^{cd})^{-(1-\eta_c)}, \\ \hat{\pi}_{t-3}^d &: D(2, 70) = (1 - \omega_c)(\gamma^{cd})^{-(1-\eta_c)}, \\ \hat{\pi}_t^{mc} &: D(2, n_X + 2) = \omega_c(\gamma^{cmc})^{-(1-\eta_c)}, \\ \hat{\pi}_{t-1}^{mc} &: D(2, 46) = \omega_c(\gamma^{cmc})^{-(1-\eta_c)}, \\ \hat{\pi}_{t-2}^{mc} &: D(2, 69) = \omega_c(\gamma^{cmc})^{-(1-\eta_c)}, \\ \hat{\pi}_{t-3}^{mc} &: D(2, 71) = \omega_c(\gamma^{cmc})^{-(1-\eta_c)}.\end{aligned}$$

Alternatively, we could incorporate the exogenous time-varying inflation target in the inflation gap,<sup>1</sup>

$$Y_t \equiv [4(\hat{\pi}_t^{cpi} - \hat{\pi}_t^c), \bar{\pi}_t^{cpi} - 4\hat{\pi}_t^c, \hat{y}_t, 4\hat{R}_t, 4(\hat{R}_t - \hat{R}_{t-1}), \hat{x}_t]' \equiv D \begin{bmatrix} X_t \\ x_t \\ i_t \end{bmatrix},$$

in which case

$$\begin{aligned}\hat{\pi}_t^c & : D(1, 18) = -4, \\ \hat{\pi}_t^c & : D(2, 18) = -4.\end{aligned}$$

Output gap,

$$\hat{y}_t : D(3, n_X + 4) = 1.$$

Interest rate,

$$\hat{R}_t : D(4, n_X + n_x + n_i) = 4.$$

Interest-rate differential,

$$\begin{aligned}\hat{R}_t & : D(5, n_X + n_x + n_i) = 4, \\ \hat{R}_{t-1} & : D(5, 44) = -4.\end{aligned}$$

Real exchange rate,

$$\hat{x}_t : D(6, n_X + 20) = 1.$$

#### 1.4. Defining $f_X$ and $f_x$ - Interest rate rule

$$i_t = f_X \check{X}_t + f_x \check{x}_t$$

$$\begin{aligned}\hat{R}_t & = \rho_R \hat{R}_{t-1} + (1 - \rho_R) \left[ \hat{\pi}_t^c + r_\pi (\hat{\pi}_{t-1}^c - \hat{\pi}_t^c) + r_y \hat{y}_{t-1} + r_x \hat{x}_{t-1} \right] \\ & \quad + r_{\Delta\pi} (\hat{\pi}_t^c - \hat{\pi}_{t-1}^c) + r_{\Delta y} (\hat{y}_t - \hat{y}_{t-1}) + \varepsilon_{Rt}\end{aligned}$$

$$\begin{aligned}\hat{R}_t & = \rho_R \hat{R}_{t-1} + (1 - \rho_R) \hat{\pi}_t^c + (1 - \rho_R) r_\pi \hat{\pi}_{t-1}^c - (1 - \rho_R) r_\pi \hat{\pi}_t^c \\ & \quad + (1 - \rho_R) r_y \hat{y}_{t-1} + (1 - \rho_R) r_x \hat{x}_{t-1} + r_{\Delta\pi} \hat{\pi}_t^c - r_{\Delta\pi} \hat{\pi}_{t-1}^c + r_{\Delta y} \hat{y}_t - r_{\Delta y} \hat{y}_{t-1} + \varepsilon_{Rt} \\ & = \rho_R \hat{R}_{t-1} + (1 - \rho_R) [1 - r_\pi] \hat{\pi}_t^c + r_{\Delta\pi} (1 - \omega_c) (\gamma^{cd})^{-(1-\eta_c)} \hat{\pi}_t^d \\ & \quad + r_{\Delta\pi} (\omega_c) (\gamma^{cmc})^{-(1-\eta_c)} \hat{\pi}_t^{mc} \\ & \quad + [(1 - \rho_R) r_\pi - r_{\Delta\pi}] (1 - \omega_c) (\gamma^{cd})^{-(1-\eta_c)} \hat{\pi}_{t-1}^d \\ & \quad + [(1 - \rho_R) r_\pi - r_{\Delta\pi}] \omega_c (\gamma^{cmc})^{-(1-\eta_c)} \hat{\pi}_{t-1}^{mc} \\ & \quad + r_{\Delta y} \hat{y}_t + [(1 - \rho_R) r_y - r_{\Delta y}] \hat{y}_{t-1} + (1 - \rho_R) r_x \hat{x}_{t-1} + \varepsilon_{Rt}\end{aligned}$$

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<sup>1</sup> Note that one could alternative replace  $4\hat{\pi}_t^c$  by  $\hat{\pi}_t^c + \hat{\pi}_{t-1}^c + \hat{\pi}_{t-2}^c + \hat{\pi}_{t-3}^c$  and consider the latter the time-varying four-quarter inflation target.

$$\begin{aligned}
\hat{R}_{t-1} & : f_X(1, 44) = \rho_R \\
\hat{\pi}_t^c & : f_X(1, 18) = (1 - \rho_R)[1 - r_\pi] \\
\hat{\pi}_t^d & : f_x(1, 1) = r_{\Delta\pi}(1 - \omega_c)(\gamma^{cd})^{-(1-\eta_c)} \\
\hat{\pi}_t^{mc} & : f_x(1, 2) = r_{\Delta\pi}\omega_c(\gamma^{cmc})^{-(1-\eta_c)} \\
\hat{\pi}_{t-1}^d & : f_X(1, 45) = [(1 - \rho_R)r_\pi - r_{\Delta\pi}](1 - \omega_c)(\gamma^{cd})^{-(1-\eta_c)} \\
\hat{\pi}_{t-1}^{mc} & : f_X(1, 46) = [(1 - \rho_R)r_\pi - r_{\Delta\pi}](\omega_c)(\gamma^{cmc})^{-(1-\eta_c)} \\
\hat{y}_t & : f_x(1, 4) = r_{\Delta y} \\
\hat{y}_t^{flex} & : f_x(1, n_x + 4) = -r_{\Delta y} \\
\hat{y}_{t-1} & : f_X(1, 48) = (1 - \rho_R)r_y - r_{\Delta y} \\
\hat{y}_{t-1}^{flex} & : f_X(1, n_X + 48) = -((1 - \rho_R)r_y - r_{\Delta y}) \\
\hat{x}_{t-1} & : f_X(1, 64) = (1 - \rho_R)r_x \\
\varepsilon_{Rt} & : f_X(1, 17) = 1
\end{aligned}$$

## 2. Potential output under flexible prices and wages - expanding the state

The flexible price and wage model is parameterized under i)  $\xi^d = \xi^{mc} = \xi^{mi} = \xi^x = \xi^w = 0$ , ii) setting the four markup as well as the fiscal shocks to zero ( $\sigma_{\varepsilon_\lambda} = \sigma_{\varepsilon_{\lambda}mc} = \sigma_{\varepsilon_{\lambda}mi} = \sigma_{\varepsilon_{\lambda}x} = 0$ , and  $\Theta_0^{-1}S_\tau = 0_{5x5}$ , respectively), and iii) we solve the system under the assumption that  $\hat{\pi}^{cpi} = 0$ . The flexprice model can be written

$$\begin{bmatrix} X_{t+1} \\ H^f x_{t+1|t} \end{bmatrix} = A^f \begin{bmatrix} X_t \\ x_t \\ i_t \end{bmatrix} + \begin{bmatrix} C \\ 0_{n_x \times n_\varepsilon} \\ 0_{n_i \times n_\varepsilon} \end{bmatrix} \varepsilon_{t+1}, \quad (2.1)$$

where,

$$H^f \equiv \begin{bmatrix} H \\ 0_{1 \times n_x} \end{bmatrix}, \quad A^f \equiv \begin{bmatrix} A_{11} & A_{12} & B_1 \\ A_{21} & A_{22} & B_2 \\ 0_{1 \times n_X} & e_1 & 0 \end{bmatrix},$$

and

$$\begin{aligned}
\hat{\pi}_t^d & : e_1(1, 1) = (1 - \omega_c)(\gamma^{cd})^{-(1-\eta_c)}, \\
\hat{\pi}_t^{mc} & : e_1(1, 2) = \omega_c(\gamma^{cmc})^{-(1-\eta_c)}.
\end{aligned}$$

This implies the following solution

$$\begin{aligned}
X_t^{flex} & = M^{flex} X_{t-1}^{flex} + C^{flex} \varepsilon_t, \\
\tilde{x}_t^{flex} & = \tilde{F}^{flex} \tilde{X}_t^{flex}, \\
& = \tilde{F}^{flex} M^{flex} X_{t-1}^{flex} + \tilde{F}^{flex} C^{flex} \varepsilon_t,
\end{aligned}$$

where  $\tilde{x}_t^{flex} = \begin{bmatrix} x_t^{flex} \\ i_t^{flex} \end{bmatrix}$  and  $\tilde{F}^{flex} = [ F_x^{flex} \ F_i^{flex} ]'$ .

We expand the state of the distorted economy with the predetermined state variables in the flexprice model so that  $\dot{X}_t = (X', X_t^{flex'})'$ , which implies

$$\dot{X}_{t+1} = \dot{A}_{11} \dot{X}_t + \dot{A}_{12} x_t + \dot{B}_1 i_t + \dot{C} \varepsilon_{t+1},$$

$$\dot{H}x_{t+1|t} = \dot{A}_{21}\dot{X}_t + \dot{A}_{22}x_t + \dot{B}_2i_t,$$

where  $\dot{A}_{11} = \begin{bmatrix} A_{11} & 0_{nX*nX} \\ 0_{nX*nX} & M^{flex} \end{bmatrix}$ ,  $\dot{A}_{12} = \begin{bmatrix} A_{12} \\ 0_{nX*nx} \end{bmatrix}$ ,  
 $\dot{A}_{21} = [A_{21} \ 0_{nx*nX}]$ ,  $\dot{A}_{22} = A_{22}$ ,  $\dot{B}_1 = \begin{bmatrix} B_1 \\ 0_{nX*ni} \end{bmatrix}$ ,  $\dot{B}_2 = B_2$ ,  
 $\dot{C} = \begin{bmatrix} C \\ C^{flex} \end{bmatrix}$ , and  $\dot{H} = H$ .

We also need to change the target variable matrix to include the flexprice output gap

$$Y_t \equiv [4\hat{\pi}_t^{cpi}, \bar{\pi}_t^{cpi}, \bar{\pi}_t^d, (y_t - y_t^{flex}), 4\hat{R}_t, 4(\hat{R}_t - \hat{R}_{t-1}), \hat{x}_t]' \equiv \dot{D} \begin{bmatrix} X_t \\ X_t^{flex} \\ x_t \\ i_t \end{bmatrix}$$

$$\dot{D} = [D_X \ D_{X^{flex}} \ D_x \ D_i]',$$

where  $D$  above has been partitioned such that  $D = [D_X \ D_x \ D_i]'$  and  $D_{X^{flex}}$  is defined so that

$$\hat{y}_t^{flex} : D_{X^{flex}}(4, n_X + 4) = -F_{x,4}^{flex}.$$

### 3. The measurement equation and the definition of the matrices $\bar{D}_0$ , $\bar{D}$ , and $\bar{D}_s$

Let  $s_t \equiv (X'_t, \Xi'_{t-1}, x'_t, i'_t)'$  and call  $s_t$  the state of the economy (although it includes nonpredetermined variables). The solution to the model under optimal policy under commitment can be written in Klein form as

$$s_{t+1} \equiv \begin{bmatrix} \tilde{X}_{t+1} \\ x_{t+1} \\ i_{t+1} \end{bmatrix} = \begin{bmatrix} M & 0 & 0 \\ f_x M & 0 & 0 \\ f_i M & 0 & 0 \end{bmatrix} s_t + \begin{bmatrix} I \\ f_x \\ f_i \end{bmatrix} \tilde{C} \varepsilon_{t+1}.$$

As noted in Adolfson et al. [1, Appendix C], it can also be written in AIM form as

$$s_{t+1} = \bar{B}s_t + \bar{C}\varepsilon_{t+1}, \quad (3.1)$$

where

$$\bar{C} \equiv \Phi\Psi$$

and the innovation  $\bar{C}\varepsilon_t$  has the distribution  $N(0, Q)$  with  $Q \equiv E[\bar{C}\varepsilon_{t+1}(\bar{C}\varepsilon_{t+1})'] = \bar{C}\bar{C}'$ . Consider the state unobservable, let  $Z_t$  denote the  $n_Z$ -vector of observable variables, and let the measurement equation be

$$Z_t = \bar{D}_0 + \bar{D}_s s_t + \eta_t, \quad (3.2)$$

where  $\eta_t$  is an  $n_Z$ -vector of measurement errors with distribution  $N(0, \Sigma_\eta)$  (the vector  $\eta_t$  here should not be confused with vector  $\eta$  of coefficients specified in (1.3)). We assume that the matrix  $R$  is diagonal with very small elements.<sup>2</sup> Equations (3.1) and (3.2) correspond to the state-space form for the derivation of the Kalman filter in Hamilton [3].

Note that the matrix  $\bar{D}_s$  is related to the matrix  $\bar{D} \equiv [\bar{D}_X \ \bar{D}_x \ \bar{D}_i]$  (partitioned conformably with  $X_t$ ,  $x_t$ , and  $i_t$ ) as

$$\bar{D}_s = [\bar{D}_X \ 0 \ \bar{D}_x \ \bar{D}_i],$$

since  $s_t$  includes the vector of predetermined variables  $\Xi_{t-1}$ .

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<sup>2</sup> The role of the measurement errors in the estimation of the model and the state of the economy is further discussed in Adolfson et al. [2].

### 3.1. Difference specification

The vector of observable variables is

$$Z_t = \begin{bmatrix} \pi_t^d & \Delta \ln(W_t/P_t) & \Delta \ln C_t & \Delta \ln I_t & \hat{x}_t & R_t & \hat{H}_t & \dots \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 & \dots \\ \Delta y_t & \Delta \ln \tilde{X}_t & \Delta \ln \tilde{M}_t & \pi_t^{cpi} & \pi_t^{def,i} & \Delta \ln Y_t^* & \pi_t^* & R_t^* \\ 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 \end{bmatrix}' \quad (3.3)$$

and corresponds to the data used in estimating Ramses.

Next, we need specify how the data corresponds to the model variables:

$$\begin{aligned} \pi_t^d &= 400(\pi - 1)\pi + 4\pi\hat{\pi}_t^d, \\ \Delta \ln(W_t/P_t) &= 100 \ln \mu_z + \Delta \hat{w}_t + \hat{\mu}_{zt}, \\ \Delta \ln C_t &= 100 \ln \mu_z + \Delta \hat{c}_t + \frac{c}{c^d + c^m} \eta_c \frac{c^d}{c} \frac{c^m}{c} \left[ (\gamma^{cmc})^{-1} - (\gamma^{cd})^{-1} \right] \Delta \hat{\gamma}_t^{mcd} + \hat{\mu}_{zt}, \\ \Delta \ln I_t &= 100 \ln \mu_z + \Delta \hat{i}_t + \frac{\tilde{i}}{\tilde{i}^d + \tilde{i}^m} \eta_i \frac{\tilde{i}^d}{\tilde{i}} \frac{\tilde{i}^m}{\tilde{i}} \left[ (\gamma^{i,mi})^{-1} - (\gamma^{i,d})^{-1} \right] \Delta \hat{\gamma}_t^{mi,d} \\ &\quad + (1 - \tau^k) r^k \bar{k} \frac{1}{\mu_z} \Delta \hat{u}_t + \hat{\mu}_{zt}, \\ \hat{x}_t &= \hat{\tilde{x}}_t, \\ R_t &= 400(R - 1)R + 4R\hat{R}_t, \\ \hat{H}_t &= \hat{H}_t, \\ \Delta y_t &= 100 \ln \mu_z + \Delta \hat{y}_t + \hat{\mu}_{zt}, \\ \Delta \ln \tilde{X}_t &= 100 \ln \mu_z + \Delta \left[ \hat{y}_t^* - \eta_f \hat{\gamma}_t^{x,*} + \hat{\tilde{z}}_t^* \right] + \hat{\mu}_{zt}, \\ \Delta \ln \tilde{M}_t &= 100 \ln \mu_z + \frac{c^m}{c^m + \tilde{i}^m} \Delta \hat{c}_t - \frac{c^m}{c^m + \tilde{i}^m} \eta_c (1 - \omega_c) (\gamma^{cd})^{-(1-\eta_c)} \Delta \hat{\gamma}_t^{mcd} \\ &\quad + \frac{\tilde{i}^m}{c^m + \tilde{i}^m} \Delta \hat{i}_t - \frac{\tilde{i}^m}{c^m + \tilde{i}^m} \eta_i (1 - \omega_i) (\gamma^{i,d})^{-(1-\eta_i)} \Delta \hat{\gamma}_t^{mi,d} + \hat{\mu}_{zt}, \\ \pi_t^{cpi} &\equiv 400(\pi - 1)\pi + 4\pi\hat{\pi}_t^{cpi} + 4\pi \frac{\tau^c}{1 + \tau^c} \Delta \hat{\tau}_t^c \\ &= 400(\pi - 1)\pi + 4\pi(1 - \omega_c) (\gamma^{cd})^{-(1-\eta_c)} \hat{\pi}_t^d \\ &\quad + 4\pi\omega_c (\gamma^{cmc})^{-(1-\eta_c)} \hat{\pi}_t^{mc} + 4\pi \frac{\tau^c}{1 + \tau^c} \Delta \hat{\tau}_t^c, \\ \pi_t^{def,i} &= 400(\pi - 1)\pi + 4\pi \tilde{i}_{\pi^d} \hat{\pi}_t^d + 4\pi \tilde{i}_{\pi^m} \hat{\pi}_t^{mi} \\ &\quad + 4\pi \left[ \left( \tilde{i}_{\pi^d} - \frac{I^d}{I^d + I^m} \right) \eta_i \omega_i (\gamma^{i,mi})^{-(1-\eta_i)} \right. \\ &\quad \left. - \left( \tilde{i}_{\pi^m} - \frac{I^m}{I^d + I^m} \right) \eta_i (1 - \omega_i) (\gamma^{i,d})^{-(1-\eta_i)} \right] \Delta \hat{\gamma}_t^{mi,d}, \\ \Delta \ln Y_t^* &= 100 \ln \mu_z + \Delta \hat{y}_t^* + \Delta \hat{\tilde{z}}_t^* + \hat{\mu}_{zt}, \\ \pi_t^* &= 400(\pi - 1)\pi + 4\pi\hat{\pi}_t^*, \\ R_t^* &= 400(R - 1)R + 4R\hat{R}_t^*. \end{aligned} \quad (3.4)$$

#### Domestic inflation

$$\pi_t^d = 400(\pi - 1)\pi + 4\pi\hat{\pi}_t^d,$$

$$\begin{aligned}\bar{D}_0(1,1) &= 400(\pi - 1)\pi \\ \bar{D}_s(1, n_{\tilde{X}} + 1) &= 4\pi\end{aligned}$$

### Change in real wage

$$\Delta \ln(W_t/P_t) = 100 \ln \mu_z + \hat{\bar{w}}_t - \hat{\bar{w}}_{t-1} + \hat{\mu}_{zt},$$

$$\begin{aligned}\bar{D}_0(2,1) &= 100 \ln \mu_z \\ \hat{\bar{w}}_t &: \bar{D}_s(2, n_{\tilde{X}} + 6) = 1 \\ \hat{\bar{w}}_{t-1} &: \bar{D}_s(2, 50) = -1 \\ \hat{\mu}_{zt} &: \bar{D}_s(2, 3) = 1\end{aligned}$$

### Change in real consumption

$$\begin{aligned}\Delta \ln C_t &= 100 \ln \mu_z + \hat{c}_t - \hat{c}_{t-1} + \frac{c}{c^d + c^m} \eta_c \frac{c^d}{c} \frac{c^m}{c} \left[ (\gamma^{cmc})^{-1} - (\gamma^{cd})^{-1} \right] \hat{\gamma}_t^{mcd} \\ &\quad - \frac{c}{c^d + c^m} \eta_c \frac{c^d}{c} \frac{c^m}{c} \left[ (\gamma^{cmc})^{-1} - (\gamma^{cd})^{-1} \right] \hat{\gamma}_{t-1}^{mcd} + \hat{\mu}_{zt},\end{aligned}$$

$$\begin{aligned}\bar{D}_0(3,1) &= 100 \ln \mu_z \\ \hat{c}_t &: \bar{D}_s(3, n_{\tilde{X}} + 7) = 1 \\ \hat{c}_{t-1} &: \bar{D}_s(3, 51) = -1 \\ \hat{\gamma}_t^{mcd} &: \bar{D}_s(3, n_{\tilde{X}} + 17) = \frac{c}{c^d + c^m} \eta_c \frac{c^d}{c} \frac{c^m}{c} \left[ (\gamma^{cmc})^{-1} - (\gamma^{cd})^{-1} \right] \\ \hat{\gamma}_{t-1}^{mcd} &: \bar{D}_s(3, 61) = - \frac{c}{c^d + c^m} \eta_c \frac{c^d}{c} \frac{c^m}{c} \left[ (\gamma^{cmc})^{-1} - (\gamma^{cd})^{-1} \right] \\ \hat{\mu}_{zt} &: \bar{D}_s(3, 3) = 1\end{aligned}$$

### Change in real investment

$$\begin{aligned}\Delta \ln I_t &= 100 \ln \mu_z + \hat{\tilde{i}}_t - \hat{\tilde{i}}_{t-1} + \frac{\tilde{i}}{\tilde{i}^d + \tilde{i}^m} \eta_i \frac{\tilde{i}^d}{\tilde{i}} \frac{\tilde{i}^m}{\tilde{i}} \left[ (\gamma^{i,mi})^{-1} - (\gamma^{i,d})^{-1} \right] \hat{\gamma}_t^{mi,d} \\ &\quad - \frac{\tilde{i}}{\tilde{i}^d + \tilde{i}^m} \eta_i \frac{\tilde{i}^d}{\tilde{i}} \frac{\tilde{i}^m}{\tilde{i}} \left[ (\gamma^{i,mi})^{-1} - (\gamma^{i,d})^{-1} \right] \hat{\gamma}_{t-1}^{mi,d} \\ &\quad + (1 - \tau^k) r^k \bar{k} \frac{1}{\mu_z} \hat{k}_t - (1 - \tau^k) r^k \bar{k} \frac{1}{\mu_z} \hat{\bar{k}}_t - (1 - \tau^k) r^k \bar{k} \frac{1}{\mu_z} \hat{u}_{t-1} + \hat{\mu}_{zt}, \\ \hat{u}_t &= \hat{k}_t - \hat{\bar{k}}_t\end{aligned}$$

$$\begin{aligned}
\bar{D}_0(4, 1) &= 100 \ln \mu_z \\
\hat{\tilde{i}}_t &: \bar{D}_s(4, n_{\tilde{X}} + 8) = 1 \\
\hat{\tilde{i}}_{t-1} &: \bar{D}_s(4, 52) = -1 \\
\hat{\gamma}_t^{mi,d} &: \bar{D}_s(4, n_{\tilde{X}} + 18) = \frac{\tilde{i}}{\tilde{i}^d + \tilde{i}^m} \eta_i \frac{\tilde{i}^d}{\tilde{i}} \frac{\tilde{i}^m}{\tilde{i}} \left[ (\gamma^{i,mi})^{-1} - (\gamma^{i,d})^{-1} \right] \\
\hat{\gamma}_{t-1}^{mi,d} &: \bar{D}_s(4, 62) = -\frac{\tilde{i}}{\tilde{i}^d + \tilde{i}^m} \eta_i \frac{\tilde{i}^d}{\tilde{i}} \frac{\tilde{i}^m}{\tilde{i}} \left[ (\gamma^{i,mi})^{-1} - (\gamma^{i,d})^{-1} \right] \\
\hat{k}_t &: \bar{D}_s(4, n_{\tilde{X}} + 13) = (1 - \tau^k) r^k \bar{k} \frac{1}{\mu_z} \\
\hat{\bar{k}}_t &: \bar{D}_s(4, 42) = -(1 - \tau^k) r^k \bar{k} \frac{1}{\mu_z} \\
\hat{u}_{t-1} &: \bar{D}_s(4, 67) = -(1 - \tau^k) r^k \bar{k} \frac{1}{\mu_z} \\
\hat{\mu}_{zt} &: \bar{D}_s(4, 3) = 1
\end{aligned}$$

**Real exchange rate**

$$\begin{aligned}
\hat{\tilde{x}}_t &= \hat{\tilde{x}}_t \\
\hat{\tilde{x}}_t : \bar{D}_s(5, n_{\tilde{X}} + 20) &= 1
\end{aligned}$$

**Interest rate**

$$R_t = 400(R - 1)R + 4R\hat{R}_t,$$

$$\begin{aligned}
\bar{D}_0(6, 1) &= 400(R - 1)R \\
\hat{R}_t &: \bar{D}_s(6, n_{\tilde{X}} + n_x + n_i) = 4R
\end{aligned}$$

**Hours worked**

$$\begin{aligned}
\hat{H}_t &= \hat{H}_t, \\
\hat{H}_t : \bar{D}_s(7, n_{\tilde{X}} + 12) &= 1
\end{aligned}$$

**Change in output (GDP)**

$$\Delta y_t = 100 \ln \mu_z + \hat{y}_t - \hat{y}_{t-1} + \hat{\mu}_{zt},$$

$$\begin{aligned}
\bar{D}_0(8, 1) &= 100 \ln \mu_z \\
\hat{y}_t &: \bar{D}_s(8, n_{\tilde{X}} + 4) = 1 \\
\hat{y}_{t-1} &: \bar{D}_s(8, 48) = -1 \\
\hat{\mu}_{zt} &: \bar{D}_s(8, 3) = 1
\end{aligned}$$

### Change in real exports

$$\Delta \ln \tilde{X}_t = 100 \ln \mu_z + \hat{y}_t^* - \hat{y}_{t-1}^* - \eta_f \hat{\gamma}_t^{x,*} + \eta_f \hat{\gamma}_{t-1}^{x,*} + \hat{\tilde{z}}_t^* - \hat{\tilde{z}}_{t-1}^* + \hat{\mu}_{zt},$$

$$\begin{aligned}\bar{D}_0(9,1) &= 100 \ln \mu_z \\ \hat{y}_t^* &: \bar{D}_s(9,31) = 1 \\ \hat{y}_{t-1}^* &: \bar{D}_s(9,34) = -1 \\ \hat{\gamma}_t^{x,*} &: \bar{D}_s(9,n_{\tilde{X}}+19) = -\eta_f \\ \hat{\gamma}_{t-1}^{x,*} &: \bar{D}_s(9,63) = \eta_f \\ \hat{\tilde{z}}_t^* &: \bar{D}_s(9,14) = 1 \\ \hat{\tilde{z}}_{t-1}^* &: \bar{D}_s(9,15) = -1 \\ \hat{\mu}_{zt} &: \bar{D}_s(9,3) = 1\end{aligned}$$

### Change in real imports

$$\begin{aligned}\Delta \ln \tilde{M}_t &= 100 \ln \mu_z + \frac{c^m}{c^m + \tilde{i}^m} \hat{c}_t - \frac{c^m}{c^m + \tilde{i}^m} \hat{c}_{t-1} - \frac{c^m}{c^m + \tilde{i}^m} \eta_c (1 - \omega_c) (\gamma^{cd})^{-(1-\eta_c)} \hat{\gamma}_t^{mcd} \\ &\quad + \frac{c^m}{c^m + \tilde{i}^m} \eta_c (1 - \omega_c) (\gamma^{cd})^{-(1-\eta_c)} \hat{\gamma}_{t-1}^{mcd} + \frac{\tilde{i}^m}{c^m + \tilde{i}^m} \hat{i}_t - \frac{\tilde{i}^m}{c^m + \tilde{i}^m} \hat{i}_{t-1} \\ &\quad - \frac{\tilde{i}^m}{c^m + \tilde{i}^m} \eta_i (1 - \omega_i) (\gamma^{i,d})^{-(1-\eta_i)} \hat{\gamma}_t^{mi,d} + \frac{\tilde{i}^m}{c^m + \tilde{i}^m} \eta_i (1 - \omega_i) (\gamma^{i,d})^{-(1-\eta_i)} \hat{\gamma}_{t-1}^{mi,d} + \hat{\mu}_{zt},\end{aligned}$$

$$\begin{aligned}\bar{D}_0(10,1) &= 100 \ln \mu_z \\ \hat{c}_t &: \bar{D}_s(10,n_{\tilde{X}}+7) = \frac{c^m}{c^m + \tilde{i}^m} \\ \hat{c}_{t-1} &: \bar{D}_s(10,51) = -\frac{c^m}{c^m + \tilde{i}^m} \\ \hat{\gamma}_t^{mcd} &: \bar{D}_s(10,n_{\tilde{X}}+17) = -\frac{c^m}{c^m + \tilde{i}^m} \eta_c (1 - \omega_c) (\gamma^{cd})^{-(1-\eta_c)} \\ \hat{\gamma}_{t-1}^{mcd} &: \bar{D}_s(10,61) = \frac{c^m}{c^m + \tilde{i}^m} \eta_c (1 - \omega_c) (\gamma^{cd})^{-(1-\eta_c)} \\ \hat{i}_t &: \bar{D}_s(10,n_{\tilde{X}}+8) = \frac{\tilde{i}^m}{c^m + \tilde{i}^m} \\ \hat{i}_{t-1} &: \bar{D}_s(10,52) = -\frac{\tilde{i}^m}{c^m + \tilde{i}^m} \\ \hat{\gamma}_t^{mi,d} &: \bar{D}_s(10,n_{\tilde{X}}+18) = -\frac{\tilde{i}^m}{c^m + \tilde{i}^m} \eta_i (1 - \omega_i) (\gamma^{i,d})^{-(1-\eta_i)} \\ \hat{\gamma}_{t-1}^{mi,d} &: \bar{D}_s(10,62) = \frac{\tilde{i}^m}{c^m + \tilde{i}^m} \eta_i (1 - \omega_i) (\gamma^{i,d})^{-(1-\eta_i)} \\ \hat{\mu}_{zt} &: \bar{D}_s(10,3) = 1\end{aligned}$$

### CPI inflation

$$\begin{aligned}\pi_t^{cpd} &= 400(\pi - 1)\pi + 4\pi(1 - \omega_c)\hat{\pi}_t^d \\ &\quad + 4\pi\omega_c\hat{\pi}_t^{mc} + 4\pi\frac{\tau^c}{1+\tau^c}\hat{\tau}_t^c - 4\pi\frac{\tau^c}{1+\tau^c}\hat{\tau}_{t-1}^c,\end{aligned}$$

$$\begin{aligned}
\bar{D}_0(11,1) &= 400(\pi - 1)\pi \\
\hat{\pi}_t^d &: \bar{D}_s(11, n_{\tilde{X}} + 1) = 4\pi(1 - \omega_c) \\
\hat{\pi}_t^{mc} &: \bar{D}_s(11, n_{\tilde{X}} + 2) = 4\pi\omega_c \\
\hat{\tau}_t^c &: \bar{D}_s(11, 22) = 4\pi \frac{\tau^c}{1 + \tau^c} \\
\hat{\tau}_{t-1}^c &: \bar{D}_s(11, 27) = -4\pi \frac{\tau^c}{1 + \tau^c}
\end{aligned}$$

### Inflation investment deflator

$$\begin{aligned}
\hat{\pi}_t^{def,i} &= \frac{\tilde{i}^d}{\tilde{i}^d + \frac{\eta^{mi}}{\eta^{mi}-1}\tilde{i}^m}\hat{\pi}_t^d + \frac{c^m}{\left(c^d + \frac{\eta^{mc}}{\eta^{mc}-1}c^m\right)}\frac{\eta^{mc}}{\eta^{mc}-1}\hat{\pi}_t^{mc} \\
&+ \left[ \begin{array}{l} \left( \frac{\tilde{i}^d}{\tilde{i}^d + \frac{\eta^{mi}}{\eta^{mi}-1}\tilde{i}^m} - \frac{\tilde{i}^d}{\tilde{i}^d + \tilde{i}^m} \right) \eta_i \omega_i (\gamma^{i,mi})^{-(1-\eta_i)} \\ - \left( \frac{\tilde{i}^m}{\left(\tilde{i}^d + \frac{\eta^{mi}}{\eta^{mi}-1}\tilde{i}^m\right)} \frac{\eta^{mi}}{\eta^{mi}-1} - \frac{\tilde{i}^m}{\tilde{i}^d + \tilde{i}^m} \right) \eta_i (1 - \omega_i) (\gamma^{i,d})^{-(1-\eta_i)} \end{array} \right] \Delta \hat{\gamma}_t^{mcd}. \\
\Delta \hat{\gamma}_t^{mi,d} &= \hat{\pi}_t^{mi} - \hat{\pi}_t^d \\
\hat{\pi}_t^{def,i} &= 400(\pi - 1)\pi \\
&+ 4\pi \left\{ \begin{array}{l} \frac{\tilde{i}^d}{\tilde{i}^d + \frac{\eta^{mi}}{\eta^{mi}-1}\tilde{i}^m} - \left( \frac{\tilde{i}^d}{\left(\tilde{i}^d + \frac{\eta^{mi}}{\eta^{mi}-1}\tilde{i}^m\right)} - \frac{\tilde{i}^d}{\tilde{i}^d + \tilde{i}^m} \right) \eta_i \omega_i (\gamma^{i,mi})^{-(1-\eta_i)} \\ + \left( \frac{\tilde{i}^m}{\left(\tilde{i}^d + \frac{\eta^{mi}}{\eta^{mi}-1}\tilde{i}^m\right)} \frac{\eta^{mi}}{\eta^{mi}-1} - \frac{\tilde{i}^m}{\tilde{i}^d + \tilde{i}^m} \right) \eta_i (1 - \omega_i) (\gamma^{i,d})^{-(1-\eta_i)} \end{array} \right\} \hat{\pi}_t^d \\
&+ 4\pi \left\{ \begin{array}{l} \frac{\tilde{i}^m}{\left(\tilde{i}^d + \frac{\eta^{mi}}{\eta^{mi}-1}\tilde{i}^m\right)} \frac{\eta^{mi}}{\eta^{mi}-1} + \left( \frac{\tilde{i}^d}{\tilde{i}^d + \frac{\eta^{mi}}{\eta^{mi}-1}\tilde{i}^m} - \frac{\tilde{i}^d}{\tilde{i}^d + \tilde{i}^m} \right) \eta_i \omega_i (\gamma^{i,mi})^{-(1-\eta_i)} \\ - \left( \frac{\tilde{i}^m}{\left(\tilde{i}^d + \frac{\eta^{mi}}{\eta^{mi}-1}\tilde{i}^m\right)} \frac{\eta^{mi}}{\eta^{mi}-1} - \frac{\tilde{i}^m}{\tilde{i}^d + \tilde{i}^m} \right) \eta_i (1 - \omega_i) (\gamma^{i,d})^{-(1-\eta_i)} \end{array} \right\} \hat{\pi}_t^{mi}.
\end{aligned}$$

$$\bar{D}_0(12,1) = 400(\pi - 1)\pi$$

$$\begin{aligned}
\hat{\pi}_t^d &: \bar{D}_s(12, n_{\tilde{X}} + 1) = 4\pi \left\{ \begin{array}{l} \frac{\tilde{i}^d}{\left(\tilde{i}^d + \frac{\eta^{mi}}{\eta^{mi}-1}\tilde{i}^m\right)} \\ - \left( \frac{\tilde{i}^d}{\left(\tilde{i}^d + \frac{\eta^{mi}}{\eta^{mi}-1}\tilde{i}^m\right)} - \frac{\tilde{i}^d}{\tilde{i}^d + \tilde{i}^m} \right) \eta_i \omega_i (\gamma^{i,mi})^{-(1-\eta_i)} \\ + \left( \frac{\tilde{i}^m}{\left(\tilde{i}^d + \frac{\eta^{mi}}{\eta^{mi}-1}\tilde{i}^m\right)} \frac{\eta^{mi}}{\eta^{mi}-1} - \frac{\tilde{i}^m}{\tilde{i}^d + \tilde{i}^m} \right) \eta_i (1 - \omega_i) (\gamma^{i,d})^{-(1-\eta_i)} \end{array} \right\} \\
\hat{\pi}_t^{mi} &: \bar{D}_s(12, n_{\tilde{X}} + 3) = 4\pi \left\{ \begin{array}{l} \frac{\tilde{i}^m}{\left(\tilde{i}^d + \frac{\eta^{mi}}{\eta^{mi}-1}\tilde{i}^m\right)} \frac{\eta^{mi}}{\eta^{mi}-1} \\ + \left( \frac{\tilde{i}^d}{\tilde{i}^d + \frac{\eta^{mi}}{\eta^{mi}-1}\tilde{i}^m} - \frac{\tilde{i}^d}{\tilde{i}^d + \tilde{i}^m} \right) \eta_i \omega_i (\gamma^{i,mi})^{-(1-\eta_i)} \\ - \left( \frac{\tilde{i}^m}{\left(\tilde{i}^d + \frac{\eta^{mi}}{\eta^{mi}-1}\tilde{i}^m\right)} \frac{\eta^{mi}}{\eta^{mi}-1} - \frac{\tilde{i}^m}{\tilde{i}^d + \tilde{i}^m} \right) \eta_i (1 - \omega_i) (\gamma^{i,d})^{-(1-\eta_i)} \end{array} \right\}
\end{aligned}$$

### Foreign output

$$\Delta \ln Y_t^* = 100 \ln \mu_z + \hat{y}_t^* - \hat{y}_{t-1}^* + \hat{\tilde{z}}_t^* - \hat{\tilde{z}}_{t-1}^* + \hat{\mu}_{zt},$$

$$\begin{aligned}\bar{D}_0(13, 1) &= 100 \ln \mu_z \\ \hat{\mu}_{zt} &: \bar{D}_s(13, 3) = 1 \\ \hat{\tilde{z}}_t^* &: \bar{D}_s(13, 14) = 1 \\ \hat{\tilde{z}}_{t-1}^* &: \bar{D}_s(13, 15) = -1 \\ \hat{y}_t^* &: \bar{D}_s(13, 31) = 1 \\ \hat{y}_{t-1}^* &: \bar{D}_s(13, 34) = -1\end{aligned}$$

### Foreign inflation

$$\pi_t^* = 400(\pi - 1)\pi + 4\pi\hat{\pi}_t^*,$$

$$\begin{aligned}\bar{D}_0(14, 1) &= 400(\pi - 1)\pi \\ \hat{\pi}_t^* &: \bar{D}_s(14, 30) = 4\pi\end{aligned}$$

### Foreign interest rate

$$R_t^* = 400(R - 1)R + 4R\hat{R}_t^*.$$

$$\begin{aligned}\bar{D}_0(15, 1) &= 400(R - 1)R \\ \hat{\pi}_t^* &: \bar{D}_s(15, 32) = 4R\end{aligned}$$

## 3.2. Cointegration specification

The vector of observable variables is

$$Z_t = \left[ \begin{array}{ccccccccccccc} \pi_t^d & \ln(W_t/P_t) - \ln Y_t & \ln C_t - \ln Y_t & \ln I_t - \ln Y_t & \hat{x}_t & R_t & \hat{H}_t & \dots \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 & \dots \\ \Delta \ln Y_t & \ln \tilde{X}_t - \ln Y_t & \ln \tilde{M}_t - \ln Y_t & \pi_t^{cpi} & \pi_t^{def,i} & \ln Y_t^* - \ln Y_t & \pi_t^* & R_t^* \\ 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 \end{array} \right]' \quad (3.5)$$

and corresponds to the data used in estimating Ramses. However, it should be noted that in the current version of [1] we are only using the difference specification in equation (3.3).

Next, we need to specify how the cointegrated data corresponds to the model variables (see the section above for the specification of the non-cointegrated variables):

$$\begin{aligned}\ln(W_t/P_t) - \ln Y_t &= \hat{w}_t - \hat{y}_t, \\ \ln C_t - \ln Y_t &= \hat{c}_t + \frac{c}{c^d + c^m} \eta_c \frac{c^d}{c} \frac{c^m}{c} \left[ (\gamma^{cmc})^{-1} - (\gamma^{cd})^{-1} \right] \hat{\gamma}_t^{mcd} - \hat{y}_t, \\ \ln I_t - \ln Y_t &= \hat{i}_t + \frac{\tilde{i}}{\tilde{i}^d + \tilde{i}^m} \eta_i \frac{\tilde{i}^d}{\tilde{i}} \frac{\tilde{i}^m}{\tilde{i}} \left[ (\gamma^{i,mi})^{-1} - (\gamma^{i,d})^{-1} \right] \hat{\gamma}_t^{mi,d} + (1 - \tau^k) r^k \bar{k} \frac{1}{\mu_z} \hat{u}_t - \hat{y}_t, \\ \ln \tilde{X}_t - \ln Y_t &= \hat{y}_t^* - \eta_f \hat{\gamma}_t^{x,*} + \hat{\tilde{z}}_t^* - \hat{y}_t, \\ \ln \tilde{M}_t - \ln Y_t &= \frac{c^m}{c^m + \tilde{i}^m} \hat{c}_t - \frac{c^m}{c^m + \tilde{i}^m} \eta_c (1 - \omega_c) (\gamma^{cd})^{-(1-\eta_c)} \hat{\gamma}_t^{mcd} \\ &\quad + \frac{\tilde{i}^m}{c^m + \tilde{i}^m} \hat{i}_t - \frac{\tilde{i}^m}{c^m + \tilde{i}^m} \eta_i (1 - \omega_i) (\gamma^{i,d})^{-(1-\eta_i)} \hat{\gamma}_t^{mi,d} - \hat{y}_t, \\ \ln Y_t^* - \ln Y_t &= \hat{y}_t^* + \hat{\tilde{z}}_t^* - \hat{y}_t.\end{aligned} \quad (3.6)$$

### Real wage

$$\ln(W_t/P_t) = \widehat{w}_t - \hat{y}_t,$$

$$\begin{aligned}\widehat{w}_t & : \bar{D}_s(2, n_{\tilde{X}} + 6) = 1, \\ y_t & : \bar{D}_s(2, n_{\tilde{X}} + 4) = -1.\end{aligned}$$

### Real consumption

$$\ln C_t - y_t = \hat{c}_t + \frac{c}{c^d + c^m} \eta_c \frac{c^d}{c} \frac{c^m}{c} \left[ (\gamma^{cmc})^{-1} - (\gamma^{cd})^{-1} \right] \widehat{\gamma}_t^{mcd} - y_t,$$

$$\begin{aligned}\hat{c}_t & : \bar{D}_s(3, n_{\tilde{X}} + 7) = 1, \\ \widehat{\gamma}_t^{mcd} & : \bar{D}_s(3, n_{\tilde{X}} + 17) = \frac{c}{c^d + c^m} \eta_c \frac{c^d}{c} \frac{c^m}{c} \left[ (\gamma^{cmc})^{-1} - (\gamma^{cd})^{-1} \right], \\ \hat{y}_t & : \bar{D}_s(3, n_{\tilde{X}} + 4) = -1.\end{aligned}$$

### Real investment

$$\begin{aligned}\ln I_t - \ln Y_t & = \widehat{i}_t + \frac{\tilde{i}}{\tilde{i}^d + \tilde{i}^m} \eta_i \frac{\tilde{i}^d}{\tilde{i}} \frac{\tilde{i}^m}{\tilde{i}} \left[ (\gamma^{i,mi})^{-1} - (\gamma^{i,d})^{-1} \right] \widehat{\gamma}_t^{mi,d} + (1 - \tau^k) r^k \bar{k} \frac{1}{\mu_z} \hat{k}_t - (1 - \tau^k) r^k \bar{k} \frac{1}{\mu_z} \widehat{k}_t - \hat{y}_t \\ \hat{u}_t & = \hat{k}_t - \widehat{\bar{k}}_t,\end{aligned}$$

$$\begin{aligned}\widehat{\bar{k}}_t & : \bar{D}_s(4, n_{\tilde{X}} + 8) = 1, \\ \widehat{\gamma}_t^{mi,d} & : \bar{D}_s(4, n_{\tilde{X}} + 18) = \frac{\tilde{i}}{\tilde{i}^d + \tilde{i}^m} \eta_i \frac{\tilde{i}^d}{\tilde{i}} \frac{\tilde{i}^m}{\tilde{i}} \left[ (\gamma^{i,mi})^{-1} - (\gamma^{i,d})^{-1} \right], \\ \hat{k}_t & : \bar{D}_s(4, n_{\tilde{X}} + 13) = (1 - \tau^k) r^k \bar{k} \frac{1}{\mu_z}, \\ \widehat{\bar{k}}_t & : \bar{D}_s(4, 42) = - (1 - \tau^k) r^k \bar{k} \frac{1}{\mu_z}, \\ \hat{y}_t & : \bar{D}_s(4, n_{\tilde{X}} + 4) = -1.\end{aligned}$$

### Real exports

$$\ln \tilde{X}_t - \ln Y_t = \hat{y}_t^* - \eta_f \widehat{\gamma}_t^{x,*} + \widehat{\tilde{z}}_t^* - \hat{y}_t,$$

$$\begin{aligned}\hat{y}_t^* & : \bar{D}_s(9, 31) = 1, \\ \widehat{\gamma}_t^{x,*} & : \bar{D}_s(9, n_{\tilde{X}} + 19) = -\eta_f, \\ \widehat{\tilde{z}}_t^* & : \bar{D}_s(9, 14) = 1, \\ \hat{y}_t & : \bar{D}_s(9, n_{\tilde{X}} + 4) = -1.\end{aligned}$$

## Real imports

$$\begin{aligned}
\ln \tilde{M}_t - \ln Y_t &= \frac{c^m}{c^m + \tilde{i}^m} \hat{c}_t - \frac{c^m}{c^m + \tilde{i}^m} \eta_c (1 - \omega_c) (\gamma^{cd})^{-(1-\eta_c)} \hat{\gamma}_t^{mc} \\
&\quad + \frac{\tilde{i}^m}{c^m + \tilde{i}^m} \hat{i}_t - \frac{\tilde{i}^m}{c^m + \tilde{i}^m} \eta_i (1 - \omega_i) (\gamma^{i,d})^{-(1-\eta_i)} \hat{\gamma}_t^{mi,d} - \hat{y}_t, \\
\hat{c}_t &: \bar{D}_s(10, n_{\tilde{X}} + 7) = \frac{c^m}{c^m + \tilde{i}^m}, \\
\hat{\gamma}_t^{mc} &: \bar{D}_s(10, n_{\tilde{X}} + 17) = -\frac{c^m}{c^m + \tilde{i}^m} \eta_c (1 - \omega_c) (\gamma^{cd})^{-(1-\eta_c)}, \\
\hat{i}_t &: \bar{D}_s(10, n_{\tilde{X}} + 8) = \frac{\tilde{i}^m}{c^m + \tilde{i}^m}, \\
\hat{\gamma}_t^{mi,d} &: \bar{D}_s(10, n_{\tilde{X}} + 18) = -\frac{\tilde{i}^m}{c^m + \tilde{i}^m} \eta_i (1 - \omega_i) (\gamma^{i,d})^{-(1-\eta_i)}, \\
\hat{y}_t &: \bar{D}_s(10, n_{\tilde{X}} + 4) = -1.
\end{aligned}$$

## Foreign output

$$\ln Y_t^* - \ln Y_t = \hat{y}_t^* + \hat{z}_t^* - \hat{y}_t,$$

$$\begin{aligned}
\hat{z}_t^* &: \bar{D}_s(13, 14) = 1, \\
\hat{y}_t^* &: \bar{D}_s(13, 31) = 1, \\
\hat{y}_t &: \bar{D}_s(13, n_{\tilde{X}} + 4) = -1.
\end{aligned}$$

### 3.3. Smoothed estimates

The Kalman filter calculates a forecast of the state vector  $s_t$ ,  $s_{t|t-1}$ , as a linear function of previous observations,

$$s_{t|t-1} \equiv \text{E}[s_t | \mathcal{Z}_{t-1}], \quad (3.7)$$

where  $\mathcal{Z}_{t-1} \equiv (Z_{t-1}, Z_{t-2}, \dots, Z_1)$ . The matrix  $P_{t|t-1}$  represents the MSE of this forecast

$$P_{t|t-1} \equiv \text{E} \left[ (s_t - s_{t|t-1}) (s_t - s_{t|t-1})' | \mathcal{Z}_{t-1} \right].$$

The key equations of the Kalman filter are

$$s_{t|t} = s_{t|t-1} + P_{t|t-1} \bar{D}'_s (\bar{D}_s P_{t|t-1} \bar{D}'_s + \Sigma_\eta)^{-1} (Z_t - \bar{D}_0 - \bar{D}_s s_{t|t-1}), \quad (3.8)$$

$$s_{t+1|t} = \bar{B} s_{t|t}, \quad (3.9)$$

$$P_{t|t} = P_{t|t-1} - P_{t|t-1} \bar{D}'_s (\bar{D}_s P_{t|t-1} \bar{D}'_s + \Sigma_\eta)^{-1} \bar{D}_s P_{t|t-1}, \quad (3.10)$$

$$P_{t+1|t} = \bar{B} P_{t|t} \bar{B}' + Q. \quad (3.11)$$

The smoothed estimates of  $s_t$ ,  $s_{t|T}$  for  $t \leq T$ , are denoted

$$s_{t|T} \equiv \text{E}[s_t | \mathcal{Z}_T]. \quad (3.12)$$

The sequence of smoothed estimates  $\{s_{t|T}\}_{t=1}^T$  can be calculated by first calculating the sequences  $\{s_{t|t}\}_{t=1}^T$ ,  $\{s_{t+1|t}\}_{t=1}^{T-1}$ ,  $\{P_{t|t}\}_{t=1}^T$  and  $\{P_{t+1|t}\}_{t=1}^{T-1}$  and then calculate

$$s_{t|T} = s_{t|t} + P_{t|t} \bar{B}' P_{t+1|t}^{-1} (s_{t+1|T} - s_{t+1|t}) \quad (3.13)$$

by iterating backward through the sample for  $t = T-1, T-2, \dots, 1$  (Hamilton [3, chapt. 13.6]).

## References

- [1] Adolfson, Malin, Stefan Laséen, Jesper Lindé, and Lars E.O. Svensson (2008), “Optimal Monetary Policy in an Operational Medium-Sized DSGE Model,” Sveriges Riksbank Working Paper Series No. 225.
- [2] Adolfson, Malin, Stefan Laséen, Jesper Lindé, and Mattias Villani (2006), “Evaluating an Estimated New Keynesian Small Open Economy Model”, working paper, [www.riksbank.se](http://www.riksbank.se), *Journal of Economic Dynamics and Control*, forthcoming .
- [3] Hamilton, James D. (1994), *Time Series Analysis*, Princeton University Press.