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Focused power: Experiments, the Shapley-Shubik power index, and focal points

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Focused Power: Experiments, the Shapley-Shubik Power Index, and Focal Points

Chris R. Geller, Jamie Mustard, and Ranya Shahwan

Abstract

Experiments evaluate the fit of human behaviour to the Shapley-Shubik power index (SSPI), a formula of voter power. Groups of six subjects with differing votes divide a fixed purse by majority rule in online chat rooms. Earnings proxy for measured power. Chat rooms and processes for selecting subjects reduce or eliminate extraneous forces. Logrolling remains as the primary political force. Subjects' initial proposals for division of the purse allow measurement of effects from focal points and transaction costs. Divisions of purses, net of those effects, closely fit the SSPI, averaging 1.033 of their SSPI values. The SSPI can serve as a control for power imbedded in voting blocs, permitting fuller analysis of other factors that affect political outcomes.

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Keywords Social choice; public choice; elections; bargaining; coalitions; politico economic; voting power; conflict; election; collective action; majority rule

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I. Introduction

The Shapley (1953) value and its normalization the Shapley-Shubik (1954) power index “SSPI” exist in a game-theoretical, mathematical world of axioms and results. Humans exist in a world of space, time and social relationships. Are these two worlds separated by a crack or a chasm? Are the Shapley value and SSPI close enough to our world to inform legal decisions or control for voting power in empirical investigations of social issues? Or, are they beautiful but distant lights in the sky? To investigate the fit of human behaviour to Shapley’s mathematical results, we use bargaining experiments in an environment tailored to approximate the axiomatic assumptions of the value and power indices generally.

Researchers have applied the Shapley value and the SSPI in a broad variety of empirical areas including analysis of power on corporate boards (Leech 2002, Chen 2004), costing of joint projects (Tijs and Branzei 2001), genetic analysis (Lucchetti *et. al.* 2010 and Moretti 2010) and law cases (Felsenthal and Machover 1998).

Others have proposed and used alternate power indices – formulations of voting power. Few empirical studies yield results closely consistent with any power indices. Some scholars hold that such abstract mathematical models cannot relate to human behaviour (section II). With those disputes and absent consistent empirical support, choosing to use a power index and selecting between them may appear arbitrary.

A possible source of discrepancy between empirical results and the Shapley value or SSPI is that the social environments of the various studies do not match to the assumptions of the Shapley value, summarized here and detailed in Section III. 1) The numbers of votes controlled by participants (players) matter, but other characteristics of the players do not matter. 2) The players as a whole gain all available benefits. 3) Voting on one set of issues is not affected by voting on any other sets of issues. Indeed, the matters of interest in scholarly

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studies may be among the empirical conditions that differ from the assumptions, e.g. differences in resources, personality, durable alliances, etc.

This paper cannot and does not attempt to measure the accuracy or validity of the Shapley value, which is a mathematical result. Rather it contributes to discussions of the applicability of the Shapley value and power indices to human behaviour. We are particularly interested in the usefulness of the SSPI as a control for the effects of voting blocks in studies of other sources of political influence.

This article makes three contributions to the current literature. 1) It presents experiments (section IV) involving a relatively large number of subjects with relatively unrestricted communication in conditions approximating Shapley value axioms. 2) It measures the effects of focal points and transaction costs (section V). 3) Netting these effects from gross results, this article provides support that at an underlying level, human behaviour relates closely to the Shapley-Shubik power index and so to its foundation, the Shapley value (section VI).

We use experiments to isolate voting power from other institutional influences, while leaving free reign for negotiation among six voters. We experimentally measure the level of power embodied in large blocks of votes by placing sets of six subjects in computer chat rooms, giving them different numbers of votes, issuing them \$15, and having them divide the money by majority rule. Our measure of power is the percentage of earnings captured by the subjects. Thus, our metric is a direct application of the concept of P-power in particular, the capacity to capture a share of a fixed payoff (Felsenthal and Machover, 1998).

These earnings are averaged over many rounds of play for each of several vote profiles. Effectively, we measure the degree to which power compounds with the size of voting bloc within each profile relative to the degree designated by the SSPI. Results supporting the SSPI would encourage confidence in using it as a control for pure voting power in investigations of other political and socio-economic aspects of group decision making as well as using it in

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commercial and legal contexts. Results divergent from the SSPI, even in this environment tailored for power indices, would undermine confidence in applying it in more natural institutions.

Section II of this article discusses relevant literature. Section III further explains power indices, specifically defining the SSPI. Section IV describes the experiment. Section V gives our analysis of the experiment: evaluating our pursuit of approximating axiomatic conditions, identifying the effects of focal points and transaction costs. Section VI presents results and section VII concludes.

II. Literature

Power indices are abstract mathematical representations of the ability of blocks of votes to affect group decisions. They address votes *per se* in isolation from other political considerations such as resources, traditional alliances, charismatic leadership, or shared agendas. Scholars such as Morriss (2002), Saari and Sieberg (2001), and Holler and Napel (2004) question the usefulness of power indices. Garrett and Tsebelis (1999) note that power indices, being general, do not address particulars of human institutions. Steffen (2002) presents arguments (and counter-arguments) that the abstract mathematics of game theory cannot relate directly to human institutions or human behaviour.

Empirical investigations of power indices have shown weak performance by the SSPI and other power indices. Gelman *et al.* (2004) empirically determine that power in US and European elections increases with the size of voting blocks, but increases at a decreasing rate. Their findings conflict with theoretical results for the SSPI which shows power tending to increase at an increasing rate with the relative size of voting blocks. Navarro and Veszteg (2011) find bargaining power “remarkably different” than the Shapley value.

Felsenthal and Machover (1995, 1998) note the possibility and relevance of empirical justification of power indices. However previous empirical studies involved complex natural

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institutions which may have caused weak results, or used experiments with uncontrolled features that complicate interpretation of the results.

The U.S. Supreme Court cases of *Whitcomb v. Chavis*, 403 U.S. 124 (1971), and *New York City Board of Estimate v Morris*, 489 U.S. 688 (1989) reject the use of a power index on theoretical and intuitive grounds. Empirical support for power indices in general or one in particular may have affected the court's decisions. See Felsenthal and Machover (1998) for a listing of other US court cases that use power indices.

Experiments starting with Kalisch *et al.* (1954) shed some light on bargaining. However as explained by Kahan and Rapoport (1984), for various reasons they do not illuminate the distance between empirical observations and the SSPI. Some experiments lacked sufficient control over personal characteristics so that results were dominated by charisma or even seating position during negotiations. In others, incremental formation of coalitions created ambiguity in the structure of the game in later stages of formation.

Kauppi and Widgrén (2004) found that power as estimated by the SSPI accounted for between 60 and 90 percent of European Union's agricultural spending share per country. However due to the complexity of potential political dealing in the EU, they searched many hypothetical alliances of countries to find vote structures that correspond to the observed subsidies, so the search may have affected their results.

Despite the preceding concerns, power indices and the Shapley value have been applied to predict or account for influence over decisions, establish measures of value or responsibility, and to structure information. Felsenthal and Machover (1998) provide an excellent overview of various political, legal, and business applications in the European Union and its predecessors and in the United States. Applications continue in diverse areas: fuzzy mathematics of strategic investment (Liginlal and Ow 2006), legal review of corporate boards (Casajus *et al.* 2009), financial risk (Tarashev *et al.* 2009), knowledge representation in

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artificial intelligence (Hunter and Konieczny 2010), data envelopment analysis (Li and Liang 2010, and Wu *et. al.* 2009), genetic analysis (Lucchetti *et. al.* 2010, and Moretti 2010), business (Xu and Ruan 2012), pattern recognition in machine learning (Sun *et. al.* 2012), and systems biology (Sajitz-Hermstein and Nikoloski 2012).

III. Shapley-Shubik power index

Shapley (1953) used three assumptions to develop “the value” an abstract measure of the value of playing a game such as buying a lottery ticket or influencing a Member of a Parliament. These games are a subset of bargaining problems. The three axioms were characteristics that a reasonable bargaining model – had one existed – presumably ought to obey. 1) The game is ‘symmetric’; it is the number of votes *per se* controlled by a player that matters, not visual appeal of the number, not the player’s personality or any other characteristic. 2) The game is ‘efficient’ in that all possible gains are captured. 3) If two independent games are merged, then the value for each player in the merged game equals that sum of that player’s value in the two games played separately, ‘additivity’. For example, the value of two lottery tickets from different games, when purchased as a package, equals the sum of the values of the two tickets separately.

Normalizing the Shapley value over all players in a game (usually voters with different numbers of votes) gives the Shapley-Shubik power index (SSPI), the portion of influence for each player in the game (Shapley and Shubik, 1954). However, the players in these games are theoretical constructs, not the flesh and blood voters casting ballots in the institutions that we experience. The SSPI identifies reasonable aggregate outcomes without reference to human behaviour beyond the three assumptions.

Voting power is not directly proportionate to the size of voting blocks. Suppose there are four players (political parties, shareholders, countries, etc) with 52, 45, 2, and 1 vote each while a majority of at least 51 votes decides the outcome: {51; 52, 45, 2, 1}. The first player

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has 100% of the electoral power. With the superficially similar profile {51; 46, 45, 6, 3} any two of the first three players can form a majority of votes, and the fourth cannot help any other players form a majority. The power in this case divides between the four players as: 1/3, 1/3, 1/3, and 0. The first three players have equal power but include very different votes. The latter two have similar numbers of votes and very different power. The fourth player is a ‘dummy,’ unable to turn any losing coalition into a winning coalition. Major power indices – Shapley-Shubik (1954, and Shapley, 1953); as well as Penrose (1946) - Banzhaf (1965), Coleman (1971), Johnston (1977, 1978), Deegan-Packel (1978), Holler-Packel (1983) and Intervals (Taylor and Zwicker, 1997) – agree with these interpretations. When the number of players increases, results become more complex.

The three axioms do not conform to all common institutions to which we might expect power indices to apply and they may be inconsistent with other reasonable assumptions (Felsenthal and Machover, 1998). Since veto power can prevent the use of available resources, it violates Shapley’s axiom of efficiency. Raffles violate additivity because buying a second ticket from the same raffle marginally reduces the probability of the first ticket winning.

However, given the domain to which these three axioms do apply, ‘it is remarkable that no further conditions are required to determine the value uniquely’ (Shapley, 1953). These are sufficient conditions, not necessary ones. As Shapley (1953) notes, other sets of sufficient conditions exist.

By formula, the Shapley value ϕ for a player i is:

$$\phi_i[v] = \sum_{S \subseteq N, i \in S} \left(\frac{(s-1)!(n-s)!}{n!} \right) v(S) - \sum_{S \subseteq N, i \notin S} \left(\frac{s!(n-s-1)!}{n!} \right) v(S),$$

where v is a game expressed as a set function, S is a set of players, N is a carrier of S i.e. S together with any dummy players, s and n are the sizes of S and N .

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Shapley and Shubik (1954) apply the Shapley value to politics. They normalize the Shapley value by dividing each player's value by the sum of all players' values, so that the sum of all players' values equals one. This normalized form of the Shapley value is the SSPI (Shapley-Shubik power index).

Natural environments such as international organizations, legislatures, or boardrooms are excellent environments for *application* of power indices. Power indices should theoretically distinguish influence deriving from votes in these environments and facilitate identifying influences deriving from other sources. However, these environments may be unsuited for direct *evaluations* of power indices because voters in those environments differ in many dimensions and often share interests, violating symmetry. In contrast, this experiment endeavours to approximate Shapley's assumptions, conditions that underlie at least some power indices. Approximating such conditions allows more effective, direct evaluation of the applicability of the Shapley-Shubik power index to human institutional voting power.

IV. Experiment

Experiment outline

The experiments involve groups of six student 'subjects' taking the roles of 'players' and meeting in an online chat room with a proctor and recorder who supervise the play and who assign some number of votes to each subject. Table 1 shows vote profiles. The subjects then negotiate using chat room style English and standard abbreviations to divide a \$15 'purse' between themselves by simple majority-vote rule. This assignment of votes and distribution of money was repeated twelve or twenty-four times in each session of experiments. Each assignment of votes and distribution of money is a 'round'. All six subjects, the proctor and recorder see all communication in the chat room.

Consider an example. One player receives eight votes while the other five receive three votes each, so a majority with at least 12 votes determines division of the money;

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{12;8,3,3,3,3,3}. Identify the players with letters consecutively from largest A (with 8 votes) through to smallest F. Player A may propose to divide the money evenly with B and C. In response, players D could reply with a proposal to accept three dollars to replace C, allowing A and B to have six dollars each. Then E could propose a different division splitting B's and A's money, dividing the purse evenly between C, D, E and F.

If each subject has strictly less than half the votes, no matter how the subjects divide the money, there is always a majority (by votes) that can benefit from a different division of the money. Thus, all distributions of money in which each member of a majority coalition receives a positive payment form a cycling set of outcomes; that is, the core is empty.

Chat rooms permit negotiation based upon logrolling, making deals using one's votes (Tullock, 1976 and Johnston, 1977) in an environment with reduced effects from factors other than votes. Side deals and threats are obvious and avoidable in supervised chat rooms. Personality has less potential for influence when deals are made using brief typed statements directly relevant to voting. In these experiments, subject's identities in the chat rooms consisted of a number shared by all subjects in a particular game and a letter unique to each subject. (For example '528C'; see the screen examples in Appendix 1.) For further protection of anonymity, we changed the player-identities of the subjects every six rounds.

Profiles

The 18 vote profiles in Table 1 meet several criteria. No single player has a majority of votes, so their cores are empty. At least one power index differed widely from the SSPI for each profile so that our results were not driven by some universally agreed concept of power beyond those embodied in the SSPI. The 18 profiles selected for this experiment fall into seven power-identical sets (Table 1) with each set identified by a letter. Within each of these seven sets, each player by rank has the same power as the same ranked player in each of the other profiles.

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We used multiple profiles for several reasons. Although Binmore (2007) found that focal points based on fairness did not affect experimental results in a bargaining game, any of many other focal points and transaction costs may have affected earnings outcomes (Schelling, 1960 and Shubik, 2002). Just as power is embodied in votes, focal points are embodied in perceptions of votes. People cannot use votes without perceiving them, nor can any small set of people confidently find all focal points that others may perceive. Sugden (1995) demonstrated the unconscious and un-anticipatable nature of many focal points. We treat expected transaction costs as a subset of focal points, assuming that experienced subject's expectations of transaction costs approximate actual transaction costs. Inclusion of different sets of power-identical vote profiles as well as visually different profiles within power-identical sets allowed us to isolate effects from focal points.

Some profiles have readily observable focal points such as the three sixes that exactly equal the quota in profile u_I . Others have more subtle focal points such as other sums of votes that exactly equal the quota as in w_I . Profiles differed in appearance with the large player differing from others by wide margins in some cases and small margins in others. In some profiles all smaller players had the same number of votes while their votes differed in other profiles. In some power-identical sets, all smaller players had equal SSPI values and in other sets the values differed.

Subjects

Our experimental design is based on approximating the assumptions of power indices in a human institution. As mentioned earlier, three assumptions underlie the SSPI: efficiency, symmetry, and additivity. Subject homogeneity is the empirical manifestation of symmetry. Subjects' attitudes and perspective can affect their play in cooperative games. We used several methods to limit the effects of subjects' different personalities: various psychological profiles before the experiments and regressions afterwards to identify a set of empirically

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homogeneous subjects. We were not investigating the effects of variation among the subjects, an interesting issue in its own right, but distinct from our topic. So, we could exclude those potential subjects with characteristics incompatible with the SSPI assumptions and to use instructions (Appendix 1) that promoted behaviour consistent with the assumptions. To this end, we collected a variety of data on the subjects: gender, nationality, psychosocial orientation, and risk orientation (Table 2). Regressions using control variables tested the effectiveness of our attempts to eliminate effective variation, and estimate potential distortion of our results from residual effective variation. As shown in the Results section below, these interventions functioned in manners compatible with the SSPI assumptions.

Psychosocial orientation

Psychosocial orientation measures individuals' preferences for receiving payments in comparison with payments to others (Van Lange *et al.*, 1997). This Van Lange *et al* instrument divides subjects into three broad orientations and a residual category. Individualist subjects prefer to receive a higher payment for themselves without regard to payments received by others. Competitive subjects prefer to receive more than others receive, even to the extent of accepting a lower payment for themselves to cause even lower payments to others. Pro-social subjects prefer the highest total for payments to themselves plus those to others; at least they will accept a somewhat decreased payment to themselves in order to gain more for others. The fourth category consists of those who do not answer the instrument consistently.

We excluded subjects with a competitive psychosocial orientation as their willingness to waste money violates the power index assumption of efficiency. We also excluded those who gave inconsistent responses to the psychosocial instrument as their inconsistency may signal confusion, apathy or attempts at manipulation. While these traits may be interesting from a

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behavioural perspective, they would be problematic in our attempt to create an environment tailored to power indices. Thus, all of our subjects were individualist or pro-social.

The magnanimity of pro-social subjects was potentially problematic for the study. We attempted to eliminate the effect of pro-social orientation by structuring the experiments such that all subjects would receive approximately equal voting power over the course of the game (based on averaging the Shapley-Shubik, Banzhaf, and Johnson power indices). Our verbal instructions to subjects assured all subjects that the games were fair and that anyone temporarily in a weak position would be in a strong position later, that all had an equal potential to earn money based upon their votes. The instructions also explicitly encouraged subjects to earn as much as they could. Thus, structural equity replaced the perceived need for subjects to pursue equity. These mechanisms were apparently effective, as subjects with a pro-social orientation received at least as much as did individualistic subjects (Table 3).

Efficiency

We promoted and achieved efficiency through three mechanisms. First, we excluded subjects who were competitive in the Van Lange sense due to their willingness to forego possible gains for the group.

Second, distribution of actual money in the experiments maintained subjects' interest and motivated their active participation Smith (1982). Pilot study debriefings and observations of play showed \$10 per round motivated subjects. However, we used \$15 per round to satisfy university human research ethics requirements, resulting in an hourly compensation comparable to that of a research assistant.

Third, we selected subjects who were more interested in earning high winnings by inviting more subjects than we needed for any particular session, and holding an auction for who would participate. Each potential subject wrote a bid of what they would accept to not

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participate in the experiments. We paid those with the lowest bids and used the others in experiments.

Additivity

The experiments were structurally additive because two games in sequence yielded twice the payment of one game: \$30 for a twelve round session at \$15 per round with six subjects. The participation auction resulted in payoffs that clustered around \$25 for twelve round sessions and \$50 for 24 round sessions. These payoffs, although imprecise and subject to statistical distortion through selectivity, suggest additivity.

Risk

We used a simple test (Appendix 2) for attitude toward risk: aversion, love, or neutrality. We found that gender, risk aversion, psychosocial orientation and national origin did not affect earnings significantly in either the statistical or practical sense. Coefficients on those variables were both small and insignificant (Table 2).

Language proficiency

We also limited the subjects to those with apparent proficiency in English as a pilot study results showed foreign nationality to affect earnings and debriefings suggested English language ability to be the key issue in low foreigner performance. This approach was apparently effective as foreign subjects received at least as much as did domestic subjects (Table 3).

Experience

Experience should matter in performance. Consistent with previous works (for example Kelly and Arrowood, 1960; Komorita and Moore, 1976), in a pilot study six rounds of play imparted enough experience for proficient play. In our analysis we limited our observations

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to those in which all subjects had already participated in a practice round and at least six rounds for money.

We developed an additional control for heterogeneity by selecting homogenous subjects by regressing experiment outcomes on dummies for subjects. Details of these regressions follows description of the experiments.

Experiment process

On twelve days, with one or two sessions per day, we conducted two or three concurrent experiments in a classroom-style computer laboratory, with each experiment involving a group of six subjects playing series of twelve or 24 rounds under the supervision of a proctor and a recorder.

The subjects sat as widely as possible around the laboratory, maintaining at least one computer between every two subjects and seating subjects in the same experimental group more distantly. Each computer used by a subject had chat rooms for multiple player-identities, permitting rapid change of identities between every six rounds. The proctor and recorder for each group of six subjects shared a computer, participating in the chats as a single individual. Each proctor's computer had files giving the listings of votes to be used each round and typical messages used during the rounds. Each recorder had a hardcopy sheet giving the votes for each subject and majorities required for each round as backup and verification against electronic records.

We assigned the subjects to computers without them knowing each other's subject identities. If new subjects were participating, we distributed and read instructions on the game and played a practice round without money. The instructions (Appendix 1) included procedures, rules of the game, suggestions on strategies, and that fifteen dollars would be divided each round. The subjects were students in the university and were familiar with the use of the chat rooms because the platform was used for educational purposes and student

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communication throughout the university. After we provided instructions, each group of six subjects with a proctor and recorder ran independently of other groups.

Subjects within each group had the same information, communicating entirely through chat room windows shared by all group members. At the beginning of each round, the proctor submitted a message to one window, labelled 'Group Chat', on the subjects' monitors saying to wait and do nothing until further notice (Appendix 1 has screen examples). Second, the proctor sent a message to another window, labelled 'Vote Vector', on each subject's monitor giving the votes for each subject and majority required for that round. All subjects in the group saw the same message and each knew the votes of all subjects in their group. This was the only message each round sent to the Vote Vector window. Third, the proctor sent a message to the Group Chat requesting that the subjects confirm their votes. Each subject responded with the number of votes assigned to them that round. The proctor and recorder confirmed each number of votes with the data file and hardcopy sheet, correcting any mistakes until all subjects reported correct votes.

The proctors submitted a message to each Group Chat to begin the games. Subjects submitted messages proposing, rejecting, revising, or accepting various divisions of the money. Subjects could write plain English statements, use conventional chat room abbreviations, or use brief notation provided during the instructions for the game. Subjects proposing a division of money had to identify that proposal uniquely, using their player-identification letter followed by a number. They were not permitted to use threats, statements that would reveal personal information, or deals for anything other than divisions of money that round. Proctors could end a round without any payment to enforce the rules, but they never had to exercise such punishment. Recorders and proctors watched the messages for the emergence of a consensus, a task requiring two people. When subjects appeared to have reached a majority decision, the proctors would wait briefly to permit additional proposals

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then submit a call for votes. Each subject could then submit a message supporting one proposal. Subjects could change their acceptance of a proposal if they wished at any time until they replied to the proctor's call for votes.

The proctor and recorder then counted the votes. If there was no winner, the proctor sent a message saying to continue negotiating. When a proposal received enough votes, the proctor sent a message saying that the round was over, which proposal won, and instructing the subjects to send no further messages until the next round started.

At the end of each round, the recorder wrote how much each subject received on the hardcopy sheet and the proctor confirmed the record with the messages in the Group Chat. After each six rounds, the proctors submitted messages instructing subjects to minimize their Group Chat and Vote Vector and open the alternative version of each to proceed for six more rounds with new identities.

After 12 or 24 rounds as time allowed, we ended the session and tallied each subject's winnings. We paid them precisely to five cents, the smallest local denomination coin, and collected a receipt which included each subject's player-identities during the game.

V. Analysis

Sample

We ran 441 rounds of experiments for money. Many of these rounds included inexperienced subjects or used vote profiles designed as a pre-study for another project, leaving 291 rounds. For several reasons, we took as our observation from each round the earnings of the subject with the most votes 'largest'. The largest subject was uniquely and simply identified. Choosing the subject with an intermediate power index value was available in 56 rounds, but using that subject would have run the risk of missing that the SSPI overestimated the power on either end of the vote range, and underestimating at the other, leaving middle subject's earnings apparently complying with the SSPI. Since subjects divided

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a fixed purse each round, earnings of any one subject contained nearly all earnings information that could be extracted from that round. Consider the six pi profiles (in Table 1) in which all the subjects with fewer votes have identical power index values. The earnings of the five smaller subjects must be one-fifth of the purse not taken by the largest subject.

Thirty subjects participated in at least one experiment round used in this analysis – after they practiced and played six rounds for money as “inexperienced” subjects. Twenty-five of these subjects participated in multiple experiments of either 6 or 24 rounds. Each subject was the ‘largest subject’ (in terms of votes) for between one and 26 rounds. We are missing demographic control data for one subject who was the largest subject for three rounds. Twenty subjects were male, 4 were foreign, 16 were individualist, 14 were pro-social, 4 risk averse, 4 risk loving, 15 risk neutral, and 7 gave inconsistent answers in the risk instrument.

Approximating residual homogeneity

Experience is not sufficient for comparable skill or effectiveness between subjects. We excluded all rounds of experiments that used the subjects whose earnings were outliers by both of two measures. An OLS regression on earnings as the largest player identified a relatively dense cluster of subject dummy coefficients that were sharply bounded from lower coefficients, leaving the subjects with the lower coefficients as outliers. All but one of those subjects with outlying earnings by regression also earned substantially less than other subjects in other rounds. Such doubly confirmed outlying subjects would cause violations of symmetry, and so we excluded all rounds in which any of those subjects participated.

In total there were 20 typical subjects. For 88 rounds, all six subjects were typical and experienced. We consider these 88 rounds to have been played by homogeneous-by-performance subjects, approaching symmetry, and therefore appropriate to use in an environment approximating conditions of the Shapley value and power indices.

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Measuring distortions

Solutions to games that are relatively attractive or conspicuous “focal points” violate the Shapley assumption of symmetry because an aspect of vote matters in addition to their power as votes *per se*. Suppose that two people would each receive \$1000 if they named the same colour, but could not communicate with each other beforehand. What colour should each pick? Blue would be a better choice than ochre. While both are colours, the latter is obscure while the former is a primary colour. The colour the local sports team could be a good choice. Those colours are focal points. Many games have such focal points. Binmore (2007) and others have investigated effects of specific focal points.

Negotiation requires some effort “transaction costs”. Transaction costs might differ between minimal winning coalitions. Coalitions with fewer subjects may be easier than those with more subjects. Coalitions in which all subjects control the same number of votes could be easier or harder to form than coalitions with one subject controlling a high number of votes. Transaction costs and expectations thereof may make some combinations of players more attractive than others, so we treat them jointly with focal points. Being subjective, these differences in costs are difficult to anticipate *a priori*.

The depth of our data allows us to estimate combined effects of focal points and transaction costs with observations that are distinct from the final agreements each round. We use the estimates to net the effects from focal points and transaction costs from gross results.

Initial proposals are structurally different from final agreements. An initial proposal is a statement by a single subject preceding any negotiation over a given vote profile. A final agreement is a consensus of multiple subjects following multiple proposals. Several to dozens of other proposals occur between initial proposals and final votes. Therefore, initial proposals provide an opportunity to estimate effects from focal points and anticipated transactions costs

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for the vote profiles and subjects in this research. To further differentiate data about distribution of the purse from the data to estimate effects from focal points, we exclude experimental rounds used to measure final division of the purse from the rounds used to estimate effects from focal points and transaction costs. Profile t_3 had the same four rounds for measuring focal points and for distribution of the purse, so it is excluded from analysis. Thus we use 84 observations of final division of the purses and 222 observations to measure focal points.

We use over- or under-representation of the largest player, p_l , in initial proposals as a measure of the attractiveness of the larger player resulting in net from focal points and distortions from anticipated transaction costs in vote profile V_j . This motivates the following definitions.

Consider a representative vote profile V_j . Let P denote the set of all players p_i , and define w_i to be the corresponding number of votes held by each player. We only consider vote profiles with exactly one large player, $p_l \in P$, such that $w_l > w_i, \forall i \neq l$. For any set $S \subseteq P$, define $\|S\| = \sum_{p_i \in S} w_i$ as the total number of votes available in a set S and identify $|S|$ as the number of players in a set, S .

We set the quota q as the minimum number of votes necessary to determine an outcome. In this article, we use only simple majority rule, $q = \lfloor (\|P\|/2) + 1 \rfloor$. A minimum winning coalition, $M_X \subset P$, is defined as a set of players with enough votes to determine an outcome of an election, but could not do so in the absence of any player in the coalition. Thus M_X satisfies the following conditions: $\|M_X\| \geq q$, and $\forall p_i \in M_X, \|M_X \setminus p_i\| < q$.

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Let M be the set of all such minimum winning coalitions, $L \subseteq M$ be the set of all such coalitions that contain player p_l , and α be the proportion of all minimum winning coalitions containing the largest player: $\alpha = \frac{|L|}{|M|}$.

Let $M_{x,r} \in M$ be the minimum winning coalition given in the initial proposal for a particular round, r , in the experiment. Then, $\beta_r = \begin{cases} 1, & M_{x,r} \in L \\ 0, & M_{x,r} \notin L \end{cases}$.

The proportion of rounds in which the largest player is included in the initial proposal is given by: $\beta = \frac{\sum_r \beta_r}{\sum_r 1}$.

Thus the over-representation or under-representation of p_l in initial proposals is β/α . If the large player p_l were no more or less attractive than other players in profile V_j , then $\beta/\alpha = 1$. If p_l were more attractive than other players in profile V_j , then $\beta/\alpha > 1$. If p_l were less attractive than other players in profile V_j , then $\beta/\alpha < 1$.

For each completed round r let $e_{r,i}$ be the earnings for player p_i . The largest player's portion of total earnings is then given by: $E = \frac{\sum_l e_{r,l}}{\sum_i \sum_r e_{r,i}}$.

Recall that the subject playing the role of the largest player varies each round. In the absence of any distortions and if the SSPI accurately predicted aggregate earnings, then E would equal $SSPI_l$, where $SSPI_l$ is the SSPI value for the largest player in the profile. If there are distortions from focal points and transaction costs, and initial proposals accurately approximate those distortions, then:

$$\frac{E \cdot \alpha}{\beta} \approx SSPI_l. \tag{1}$$

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The ratio α / β compensates earnings for such distortions. We can compare results across many profiles by normalizing equation 1) to the SSPI for each vote profile, V_j , in the experiment yielding an idealized result of:

$$\left(\frac{E_j \cdot \alpha_j}{\beta_j} \right) / SSPI_{lj} \approx 1 \quad (2)$$

where the subscript j identifies each parameter for a specific profile.

Our test of how closely human institutional results in our experiments approximate the SSPI is how closely the weighted mean across all profiles of equation (2) approximates unity:

$$\sum_j f_j \left(\frac{E_j \cdot \alpha_j}{\beta_j} \right) / SSPI_{lj} \approx 1 \quad (3)$$

where f_j is the portion of profiles in the experiment that are profile V_j .

VI. Results

To implement equation (3) as a test measure, we have chat room records of initial proposals in 222 rounds played by experienced subjects with those rounds not used to measure division of purses. Table 4 presents the average earnings for the largest subject in each profile, a scalar for the effects focal points and transaction costs, scaled average earnings, and the SSPI.

Our measure of voting power is the winnings of the subject with the most votes *averaged* for each profile and scaled for focal points. Observations of results from each round are not appropriate because the Shapley value and SSPI are not predictions of outcomes for individual games, but an expected average. Suppose that the Shapley value correctly specified the average value of participating in a game that included positive outcomes for members of winning coalitions and outcomes of zero for others. Winnings when in the winning coalition would have to average higher than the Shapley value to balance the times

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out of the winning coalition. Whether or not human behaviour conforms to the value, empirical results at the level of iterations of a game will tend to vary widely from the value.

Table 4 also shows the weighted average across all profiles. For ease of comparison across profiles, earnings and the Shapley value are expressed as percentage of the total purse. Thus the Shapley value is expressed as the SSPI.

For 13 of the 17 profiles (bold in Table 4), the scalar increases or decreases earnings in the direction which brings it toward the SSPI. The probability that fourteen or more scalars would be the right direction by chance is 0.072, suggesting that the metric captured important aspects of distortions from focal points and transaction costs.

Table 4 also presents our overall results. Earnings of the largest subject by round tend to have high variation, even for a particular vote profile, due to inherent bimodal distribution. While not all coalition formation yields bimodal results for all players, all vote profiles that conform to the assumptions the Shapley value and most power indices have strongly bimodal results for all players except dummies. Subjects would at least sometimes receive nothing in round of an experiment, when not in a winning coalition. When they do receive a payment it would average greater than “predicted” by a valid power index. Most power indices and the Shapley value do not address variation of results within a game for a specific player, or central tendencies other than totals and means. Therefore, medians, modes and measures of variation do not convey information relevant to evaluating the fit of human behaviour to power indices.

The subject sample population approximates the conditions assumed by the Shapley value and most power indices. An instrument measures the effects of potential focal points and anticipated transactions costs in order to compensate for effects from the inherent need for subjects to perceive voting structures in some form.

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The right-hand column of Table 4 shows the earnings, net of effects from focal points, for the largest player in each vote profile. With a total of 84 observations using subjects selected for relative homogeneity, the overall weighted average earnings scaled for focal points of the subject with the most votes is three and a third percent greater than the SSPI (1.033 of SSPI), closer than in other empirical studies.

This level of precision may well have reassured the courts in *Whitcomb v. Chavis*, 403 U.S. 124 (1971), and *New York City Board of Estimate v Morris*, 489 U.S. 688 (1989) that the Shapley value represent voting power. Likewise, the result may incline empirically minded researchers that the SSPI is a reasonable control for voting power, even the concentration of power in larger players, in studies of broader political influences.

VII. Conclusion

The Shapley value and SSPI are closely related to our human world, although the link is usually masked by potentially interesting social complexities. This experiment reduced many of those complexities and controlled for others in order to focus on the influence of voting blocs themselves. Our aggregated results, net of measured effects from focal points, show that on average subjects earned within only four percent of their SSPI values. Human behaviour in an institution tailored to power indices coincided closely with the Shapley value and SSPI, comparably well to other social science controls. These results support the Shapley value and SSPI as foundations to investigate power in human institutions. Specifically they may control for voting power in investigations of other potential sources of political influence, and make visible the power of voting blocks in legal and business contexts.

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Table 1: Vote profiles with largest player's SSPI

Profile	SSPI ^a
$p_1 = \{31:11,10,10,10,10,10\}$	33.3
$p_2 = \{36:14,12,12,11,11,11\}$	33.3
$p_3 = \{22:8,7,7,7,7,7\}$	33.3
$p_4 = \{16:11,4,4,4,4,4\}$	33.3
$p_5 = \{20:14,5,5,5,5,5\}$	33.3
$p_6 = \{12:8,3,3,3,3,3\}$	33.3
$r_1 = \{13:8,4,3,3,3,3\}$	36.7
$r_2 = \{18:10,7,5,4,4,4\}$	36.7
$r_3 = \{9:5,3,2,2,2,2\}$	36.7
$s_1 = \{10:5,3,3,3,2,2\}$	31.7
$s_2 = \{25:10,8,8,8,7,7\}$	31.7
$s_3 = \{13:6,4,4,4,3,3\}$	31.7
$t_1 = \{24:10,8,8,7,7,6\}$	30.0
$t_2 = \{18:8,6,6,5,5,4\}$	30.0
$t_3 = \{36:14,12,12,11,11,10\}$	30.0
$u_1 = \{18:7,6,6,6,5,5\}$	30.0
$v_1 = \{22:8,7,7,7,7,6\}$	26.7
$w_1 = \{13:8,5,3,3,3,3\}$	40.0

^aSSPI = Power of largest player according to the Shapley-Shubik Index

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Table 2: Data description: Characteristics of vote profiles and subject with the most votes

	Number	Mean %	SD %	Min %	Median %	Max %
Earnings	291	35.4	22.8	0	40.0	73.3
Shapley-Shubik PI	291	32.8	3.0	26.7	33.3	40.0
Male	291	72.5	44.7			
Foreign	291	7.6	26.5			
Individualist	291	50.5	50.1			
Risk Defined	288	86.5	34.3			
Risk Averse	288	16.7	37.3			
Risk Neutral	288	56.9	49.6			
Risk Loving	288	12.8	33.5			
In Winning Coalition	291	74.6	43.6			
Equal	291	38.5	48.7			
Unequal	291	43.0	49.6			

Includes only rounds in which all subjects were experienced.

Foreign (English not first language); Individualist (seeks highest payment to self); Equal (applies to profile used in a round, votes held by each player are relatively equal); Unequal (applies to profile used in a round, votes held by players is relatively unequal).

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Table 2 Continued Data description: Characteristics of vote profiles and the subject with the most votes

	Earn%	SS%	In WC	Male	Foreign	Individ-	Risk	Risk	Risk	Risk	Equal	Unequal
Earnings	1.00	0.17	0.91	-0.02	0.01	-0.06	-0.10	0.09	-0.02	0.00	-0.06	-0.07
Shapley Shubik	0.17	1.00	0.05	0.02	-0.06	-0.04	-0.12	0.07	0.00	-0.03	-0.43	0.51
In Winning Coalition	0.91	0.05	1.00	-0.02	0.02	0.01	-0.06	0.05	-0.01	0.00	0.01	-0.11
Male	-0.02	0.02	-0.02	1.00	-0.46	0.13	-0.31	0.07	0.24	0.00	0.03	-0.03
Foreign	0.01	-0.06	0.02	-0.46	1.00	0.21	0.50	-0.30	-0.11	0.00	-0.09	-0.04
Individualist	-0.06	-0.04	0.01	0.13	0.21	1.00	0.45	-0.04	-0.36	0.07	-0.02	0.04
Risk Averse	-0.10	-0.12	-0.06	-0.31	0.50	0.45	1.00	-0.51	-0.17	0.18	-0.01	-0.06
Risk Neutral	0.09	0.07	0.05	0.07	-0.30	-0.04	-0.51	1.00	-0.44	0.46	0.00	0.11
Risk Loving	-0.02	0.00	-0.01	0.24	-0.11	-0.36	-0.17	-0.44	1.00	0.15	-0.05	0.01
Risk Defined	0.00	-0.03	0.00	0.00	0.00	0.07	0.18	0.46	0.15	1.00	-0.06	0.09
Equal	-0.06	-0.43	0.01	0.03	-0.09	-0.02	-0.01	0.00	-0.05	-0.06	1.00	-0.69
Unequal	-0.07	0.51	-0.11	-0.03	-0.04	0.04	-0.06	0.11	0.01	0.09	-0.69	1.00

Includes only rounds in which all subjects were experienced.

Table 3: Largest subject earnings as portion of SSPI by control variables

		p_i profiles		not p_i profiles		all profiles	
Overall average	Mean	1.010		1.131		1.078	
	SD	0.671		0.698		0.688	
	N	128		163		291	
	Mean in WC	1.390		1.486		1.445	
	SD in WC	0.296		0.330		0.319	
	In WC	93		124		217	
	Conf Int H	1.054		1.176		1.110	
	Conf Int L	0.965		1.086		1.045	
	<hr/>		<hr/>		<hr/>		<hr/>
Male = 1	Female = 0	1	0	1	0	1	0
	Mean	1.015	0.999	1.104	1.212	1.067	1.106
	SD	1.000	0.683	0.706	0.677	0.691	0.684
	N	88	40	123	40	211	80
	Mean in WC	1.395	1.378	1.476	1.515	1.443	1.450
	SD in WC	0.283	0.330	0.335	0.321	0.316	0.330
	In WC	64	29	92	32	156	61
	Conf Int H	1.066	1.088	1.157	1.303	1.104	1.170
	Conf Int L	0.963	0.910	1.052	1.121	1.030	1.042

Includes only rounds in which all subjects were experienced.

Earnings are expressed as a portion of the Shapley-Shubik Power Index value, for ease of comparison.

Mean in WC and SD in WC give the conditional mean and standard deviation of earnings of the largest subject when the largest subject is in the winning coalition.

The mean and standard error for the largest subject's earnings when not part of the winning coalition were, of course, zero.

In WC gives the number of observations with the largest subject in the winning coalition.

Conf_Int_H and Conf_Int_L give the unconditional upper and lower bound of the 95% confidence interval of the earnings of the largest subject whether or not the largest subject was in the winning coalition.

$$\text{Conf_Int_H, Conf_Int_L} = \text{Mean} \pm 2(\text{in WC}/n)[\text{SD in WC}/(\text{in WC})^{1/2}]$$

Table 3 continued: Largest subject earnings as portion of SSPI by control variables

		p_i	profiles	not p_i	profiles	all	profiles
Foreign = 1	Australian = 0	1	0	1	0	1	0
	Mean	0.755	1.031	1.433	1.107	1.125	1.074
	SD	0.723	0.665	0.516	0.706	0.695	0.689
	N	10	118	12	151	22	269
	Mean in WC	1.258	1.399	1.564	1.479	1.456	1.444
	SD in WC	0.425	0.287	0.263	0.336	0.350	0.317
	in WC	6	87	11	113	17	200
	Conf Int H	0.963	1.077	1.579	1.154	1.256	1.107
	Conf Int L	0.547	0.986	1.288	1.059	0.994	1.041
Individualist =1	Mean	0.919	1.106	1.136	1.125	1.039	1.117
Pro-Social = 0	SD	0.683	0.650	0.641	0.755	0.667	0.709
	N	66	62	81	82	147	144
	Mean in WC	1.333	1.429	1.416	1.564	1.388	1.503
	SD in WC	0.312	0.278	0.333	0.313	0.325	0.304
	in WC	45	48	65	59	110	107
	Conf Int H	0.982	1.169	1.203	1.184	1.085	1.161
	Conf Int L	0.855	1.044	1.070	1.067	0.992	1.073
Risk defined =1	Mean	0.997	1.063	1.139	1.081	1.077	1.073
	SD	0.679	0.680	0.697	0.718	0.692	0.694
	N	109	16	140	23	249	39
	Mean in WC	1.393	1.417	1.490	1.462	1.449	1.443
	SD in WC	0.297	0.289	0.330	0.341	0.320	0.316
	in WC	78	12	107	17	185	29
	Conf Int H	1.045	1.188	1.188	1.203	1.112	1.160
	Conf Int L	0.949	0.938	1.090	0.958	1.042	0.986

Table 3 continued: Largest subject earnings as portion of SSPI by control variables

		p_i	profiles	not p_i	profiles	all	profiles
		1	0	1	0	1	0
Risk Averse = 1	Mean	0.844	1.031	1.020	1.157	0.958	1.100
	SD	0.695	0.674	0.718	0.694	0.707	0.686
	N	17	108	31	132	48	240
	Mean in WC	1.305	1.409	1.437	1.497	1.393	1.459
	SD in WC	0.335	0.289	0.328	0.332	0.331	0.316
	in WC	11	79	22	102	33	181
	Conf Int H	0.975	1.078	1.119	1.208	1.037	1.135
	Conf Int L	0.713	0.983	0.921	1.106	0.879	1.065
Risk Neutral =1	Mean	1.055	0.928	1.170	1.084	1.117	1.022
	SD	0.683	0.667	0.683	0.718	0.983	0.700
	N	76	49	88	75	164	124
	Mean in WC	1.432	1.337	1.493	1.478	1.466	1.424
	SD in WC	0.294	0.290	0.330	0.335	0.314	0.324
	in WC	56	34	69	55	125	89
	Conf Int H	1.113	0.997	1.233	1.150	1.160	1.071
	Conf Int L	0.997	0.859	1.108	1.018	1.074	0.973
Risk Loving = 1	Mean	0.881	1.023	1.183	1.123	1.052	1.080
	SD	0.646	0.682	0.741	0.694	0.708	0.690
	N	16	109	21	142	37	251
	Mean in WC	1.282	1.412	1.552	1.477	1.442	1.449
	SD in WC	0.248	0.299	0.345	0.329	0.333	0.317
	in WC	11	79	16	108	27	187
	Conf Int H	0.984	1.072	1.314	1.171	1.146	1.115
	Conf Int L	0.778	0.975	1.051	1.075	0.958	1.045

Table 3 continued: Largest subject earnings as portion of SSPI by control variables

		p_i profiles		not p_i profiles		all profiles	
		1	0	1	0	1	0
Relatively	Mean	1.162	0.895	1.002	1.200	1.081	1.076
equal [†] vote	SD	0.573	0.719	0.765	0.653	0.679	0.695
distribution=1	N	55	73	57	106	112	179
	Mean in WC	1.389	1.390	1.503	1.479	1.441	1.448
	SD in WC	0.268	0.325	0.335	0.330	0.303	0.330
	in WC	46	47	38	86	84	133
	Conf Int H	1.228	0.956	1.074	1.258	1.131	1.119
	Conf Int L	1.096	0.834	0.930	1.142	1.031	1.033
Relatively	Mean			1.054	1.167	0.961	1.165
unequal [†] vote	SD			0.685	0.705	0.707	0.662
distribution=1	N			52	111	125	166
	Mean in WC			1.503	1.479	1.397	1.476
	SD in WC			0.335	0.330	0.334	0.306
	in WC			38	86	86	131
	Conf Int H			1.133	1.222	1.011	1.207
	Conf Int L			0.975	1.111	0.911	1.123

[†]Equal: Largest has less than 1/4 again as many votes as the second largest player. Unequal: Largest has more than 1/2 again as many votes as the second largest player.

Table 4: Earnings net of effects from focal points

Profile j	SSPI $_j$ as percent	Average Earnings as Percent E_j	α / β^*	$E_j \alpha_j / \beta_j$ as Percent	F	Empirical $_j / \text{SSPI}_j$
p_1	33.3	42.2	1.14	48.3	6	1.44
p_2	33.3	53.3	0.85	45.3	3	1.36
p_3	33.3	42.2	0.83	35.2	6	1.05
p_4	33.3	11.7	0.95	11.1	4	0.33
p_5	33.3	29.2	0.97	28.3	8	0.84
p_6	33.3	42.7	1.33	56.9	5	1.71
r_1	36.7	28.3	1.07	30.3	4	0.83
r_2	36.7	49.5	0.80	39.8	7	1.09
r_3	36.7	66.7	0.93	61.9	3	1.69
s_1	31.7	20.0	0.84	16.7	3	0.53
s_2	31.7	50.0	0.90	45.0	4	1.42
s_3	31.7	36.7	0.86	31.4	4	0.99
t_1	30.0	46.7	0.68	31.5	3	1.05
t_2	30.0	52.7	0.88	46.3	5	1.54
t_3	30.0	--	0.62	--	--	--
u_1	30.0	4.8	1.19	5.7	7	0.19
v_1	26.7	39.2	0.69	27.2	8	1.02
w_1	40.0	55.0	0.64	35.0	4	0.88
Weighted overall average					84	1.033 [†]

Includes only rounds played by only experienced subjects who were homogeneous by results

* α / β is the inverse of over-representation of the subject with the most votes in initial proposals.

[†] Weighted overall average for each profile was calculated using nine decimal places.

Bold: In 12 of 17 profiles, using α / β moves results toward SSPI, less than 10% chance of occurring by chance.

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