

Tangerås, Thomas

Working Paper

Network Competition with Income Effects

IFN Working Paper, No. 903

Provided in Cooperation with:

Research Institute of Industrial Economics (IFN), Stockholm

Suggested Citation: Tangerås, Thomas (2012) : Network Competition with Income Effects, IFN Working Paper, No. 903, Research Institute of Industrial Economics (IFN), Stockholm

This Version is available at:

<https://hdl.handle.net/10419/81509>

Standard-Nutzungsbedingungen:

Die Dokumente auf EconStor dürfen zu eigenen wissenschaftlichen Zwecken und zum Privatgebrauch gespeichert und kopiert werden.

Sie dürfen die Dokumente nicht für öffentliche oder kommerzielle Zwecke vervielfältigen, öffentlich ausstellen, öffentlich zugänglich machen, vertreiben oder anderweitig nutzen.

Sofern die Verfasser die Dokumente unter Open-Content-Lizenzen (insbesondere CC-Lizenzen) zur Verfügung gestellt haben sollten, gelten abweichend von diesen Nutzungsbedingungen die in der dort genannten Lizenz gewährten Nutzungsrechte.

Terms of use:

Documents in EconStor may be saved and copied for your personal and scholarly purposes.

You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.

If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.

IFN Working Paper No. 903, 2012

Network Competition with Income Effects

Thomas P. Tangerås

Network competition with income effects*

Thomas P. Tangerås

Research Institute of Industrial Economics (IFN)

P.O. Box 55665, SE-102 15 Stockholm, Sweden

E-mail: thomas.tangeras@ifn.se

Homepage: www.ifn.se/thomast

March 21, 2013

Abstract

I generalize the workhorse model of network competition to include income effects in call demand. Empirical work has shown call demand to increase with income. Income effects deliver theoretical results consistent with regulatory concern about excessive termination rates: Unregulated network operators competing in two-part tariffs with non-discriminatory call prices negotiate termination rates above cost for any positive income effect. This holds even under termination-based price discrimination if networks are differentiated. Under second-degree price discrimination, network operators profit from increasing termination rates above cost if the share of consumers on discriminatory contracts is high, and consumer informational rent is positive.

Keywords: Income effects, network competition, profit neutrality, second-degree price discrimination, termination-based price discrimination.

JEL classification: L51, L96

*Many thanks to Mark Armstrong, Sven-Olof Fridolfsson, Pär Holmberg, Johannes Mauritzen, Lars Persson, Yossi Spiegel, Johan Stennek, Joacim Tåg, seminar participants at IFN and conference participants at Telecom ParisTech for valuable comments, Max Brimberg for research assistance, and Dina Neiman for proof-reading the manuscript. Financial support from Almega and the NET Institute (www.netinst.org) is gratefully acknowledged.

1 Introduction

The workhorse model of network competition generates a number of puzzling predictions concerning the termination rates network operators charge for connecting calls from one another and regarding call prices. First, theory predicts network profit to be independent of the reciprocal termination rate when operators compete in two-part call tariffs and charge non-discriminatory call prices (Laffont, Rey and Tirole, 1998a). In reality, profit margins tend to fall whenever network operators are forced by regulators to lower their termination rates (Genakos and Valletti, 2011). This discrepancy between predicted and observed effects on profit constitutes a *profit neutrality puzzle*.¹

Profit is no longer independent of the termination rate when network operators engage in price discrimination between calls inside the network (on-net) and calls to other networks (off-net); see Laffont, Rey and Tirole (1998b). Yet, termination-based price discrimination addresses the profit neutrality puzzle only to introduce another. Operators now have an incentive to agree on termination rates below cost (Gans and King, 2001). As off-net calls have a lower perceived marginal cost than on-net calls, the workhorse model predicts off-net call prices below on-net call prices. In reality, off-net calls are nearly always more expensive than on-net calls under price discrimination. This discrepancy between predicted and observed call prices constitutes an *off-net price puzzle*.

In this paper I show that income effects in call demand can solve both the profit neutrality puzzle and the off-net price puzzle. I consider network competition in two-part tariffs under non-discriminatory call prices, under termination-based price discrimination, and when consumers are heterogeneous so that networks engage in second-degree price discrimination. I establish conditions under which even weak income effects are enough to induce unregulated network operators to raise termination rates above cost. A positive markup on termination in turn implies off-net prices above on-net prices under price discrimination.

Extending the model to include income effects is empirically relevant. With income effects, call demand generally depends on the price of all types of calls, the subscription fee and on income. In a study of residential telephony in France, Aldebert et al. (2004) find consumers in higher income classes to display significantly higher demand for local and national calls than consumers in lower income classes, and there are significant cross-price elasticities between local, national and international calls.² For example, the estimated income elasticity of demand for

¹Under profit neutrality, network operators should not oppose to lowering termination rates at the regulator's request. This is *not* how operators usually respond to tighter regulation. Sweden constitutes an illustrative case in point. In 2004, the Swedish regulatory agency for telecommunications, PTS, deemed all four mobile operators, TeliaSonera, Tele2, Vodafone (later Telenor) and Hi3G to have significant market power. PTS instructed the companies to lower their termination rates and has continued to do so every consecutive summer since then. The operators consistently refused to comply during the first three years of the new regulatory regime. The only exception was TeliaSonera who voluntarily lowered the rate on one occasion, in 2007. The termination rate disputes have been settled in court from 2008 and onwards, with final rulings being in favour of PTS on every account. Apparently, the operators have given up the fight: Since 2008, they have only sporadically refused to lower their termination rates in accordance with PTS' demands.

²I have not found any studies of mobile call demand using household data which report income elasticities. Danaher (2002) estimates a positive and significant income effect in a field experiment introducing a new subscription service similar to a cellular telephone service. Income is a dummy variable, taking on the value 1 if annual

local calls was approximately 20. With a monthly subscription fee of 10 Euros and a monthly after-tax income of 2000 Euros, this corresponds to an elasticity of call demand with respect to the subscription fee of approximately 0.1.³ Thus, a 20 percent reduction in the subscription fee would increase call demand by 2 percent. The average residential mobile subscriber in Sweden makes approximately 140 call minutes per month (Table 18 in PTS, 2012), a 2 percent increase of which amounts to 2.8 minutes. The average length of a mobile call is 2.6 minutes (Table 19 in PTS, 2012). Hence, a reduction in the monthly subscription fee from 10 to 8 Euros would cause the average Swedish subscriber to make one additional mobile call per month, with the estimated subscription elasticity of 0.1. This sounds plausible. Observe, however, that most of my results rest upon the assumption of a positive income effect, not that it is particularly strong.

Considering income effects is policy relevant. Based upon the results of the workhorse model one would conclude that the conditions for efficient regulation are favorable. Policy makers could implement the first-best welfare optimum by means of a simple cost-based regulation: Disallow termination-based price discrimination and demand termination rates equal to reported marginal termination cost. As the networks do not care about the termination rate under non-discriminatory pricing, they have no incentive to lie about marginal cost, either. This paper shows that regulated networks instead have an incentive to strongly exaggerate marginal costs under cost-based regulation, even for weak income effects. A well-designed regulatory policy needs to take these incentives into account instead of just accepting reported costs at face value.

Non-discriminatory call prices Each network operator maximizes profit by charging call prices equal to perceived marginal cost and using the subscription fee to compete for customers. Thus, network profit stems entirely from the subscription fee and the termination profit on incoming calls. For call prices below the monopoly level, a higher termination rate means a higher termination profit, but has an adverse effect on the subscription fee. To understand why the subscription fee falls as the termination rate goes up, observe that the operator can save on termination payments by completing a larger share of its calls in its own network. The higher is the termination rate, the stronger is this cost-saving incentive. The way to achieve more on-net termination is through a lower subscription fee and thereby a higher market share. In the workhorse model, the additional termination profit and the lower subscription fee cancel out under non-discriminatory call prices, leaving network profit unaffected by changes in the termination rate.

Income effects open a channel through which higher termination rates soften competition for subscribers. A higher termination rate means lower total call expenditures because the reduction in the subscription fee dominates the cost of higher call prices. This is the documented "wa-

household income exceeds \$60,000 and 0 otherwise. This specification renders it difficult to estimate income elasticities. Grajek and Kretschmer (2009) estimate a positive and significant relationship between mobile usage intensity and GDP per capita in a panel data analysis of 41 countries. Karacuka et al. (2011) find the same result in a study of the Turkish market for mobile telephony.

³Let η_I be the income elasticity of call demand, and denote by $\eta_t = \eta_I t / I$ (the absolute value of) the elasticity of call demand with respect to the subscription fee, t . If $\eta_I = 20$, $t = 10$, and $I = 2000$, then $\eta_t = 0.1$.

terbed" effect (Genakos and Valletti, 2011). A higher residual income (net of call expenditures) implies that marginal utility of income falls. The lower is marginal utility of income, the less important is the subscription fee for the consumers' choice of network, hence competition for consumers is softer. This represents a marginal income effect on the intensity of competition. Because profit neutrality is a knife-edge result, even an infinitesimal income effect is enough to tilt networks to favour very high termination rates under non-discriminatory call prices. Regulation is called for because unregulated networks would negotiate a reciprocal termination rate in excess of the socially optimal termination rate.

Termination-based price discrimination When all calls cost the same, consumers do not care about the size of the network they belong to. Size becomes important for the choice of network whenever network operators charge different prices for calls inside or outside the network. If on-net calls are cheaper than off-net calls, as is usually the case, consumers are drawn to the largest network to save on call expenditures. Cutting the subscription fee becomes extra profitable to the individual operator in this case of tariff-mediated network externalities (Laffont, Rey and Tirole, 1998b) because increased network size attracts additional customers through a *network multiplier*. With constant market size, the multiplier is double: A negative externality arises in the competing network as that network becomes smaller and adds to the positive externality in the own network. In the end, tariff-mediated network externalities simply reinforce competition for a fixed number of subscribers and drives down equilibrium subscription fees at the loss of the industry.

Competition for subscribers is more intense the higher is the termination rate because the price difference between off-net and on-net calls then is larger and the network multiplier stronger. Hence, tariff-mediated network externalities present a motive for network operators to jointly reduce termination rates. Unregulated networks trade the benefit of the marginal income effect and marginal termination profit off against the cost of a stronger marginal network externality in their choice of termination rate. For any positive income effect and if networks are sufficiently differentiated, then network externalities matter relatively less for consumers' choice of network. In this case, unregulated networks negotiate a termination rate above cost, and as a consequence charge off-net prices above on-net prices. Conversely, the marginal network externality dominates the trade-off if the income effect is weak, and causes unregulated networks to agree on a termination rate below cost (Gans and King, 2001).

Second-degree price discrimination Network operators often sell subscriptions featuring on-net/off-net discrimination alongside subscriptions with non-discriminatory call prices. Presumably, these menus of contracts exist to account for consumer heterogeneity. Furthermore, incomplete information about consumer preferences requires retail contracts to be incentive compatible. In this model, the contract with on-net/off-net discrimination is designed for informed consumers. An informed consumer is someone who knows whether the call recipient belongs to the own network or not. The contract with non-discriminatory call prices is intended for uninformed consumers, i.e. those who have no information about which network the call recipi-

ent belongs to.⁴ The incentive problem facing network operators is how to design contracts to prevent informed consumers from taking the non-discriminatory contract.

Consumer net surplus is entirely determined by the value of the non-discriminatory contract if the incentive compatibility (IC) constraint of the on-net/off-net contract is strictly binding. The value of the non-discriminatory contract is independent of network size. But then, the individual operator cannot lower its subscription fee and take advantage of the negative network externality in the competing network present under termination-based price discrimination. Hence, the network multiplier is smaller and retail competition weaker under second-degree price discrimination. This is true even if the share of consumers on on-net/off-net contracts is large, because incentive compatibility constraints are binding at individual consumer level.

The reduction in the network multiplier implies that the marginal impact of a higher termination rate on the intensity of competition is much smaller than under termination-based price discrimination, and tends to drive up operators' preferred termination rate. An additional rent extraction motive comes into play under second-degree price discrimination. Asymmetric information may force networks to leave informational rent to consumers. A higher termination rate makes the on-net/off-net contract more valuable to consumers because a stronger network externality intensifies competition. The value of the non-discriminatory contract falls because a higher termination rate softens competition for uninformed consumers through the marginal income effect. A higher termination rate thus relaxes the IC constraint of the on-net/off-net contract, thereby allowing networks to extract informational rent. Weak network externalities and marginal rent extraction are enough to render a termination rate above cost profitable under second-degree price competition, even for weak income effects.

2 Related literature

The workhorse, *A-LRT*, model of network competition was developed by Armstrong (1998), who considered linear non-discriminatory call prices, and Laffont, Rey and Tirole (1998a and b), who allowed also two-part call tariffs and price discrimination. The network competition literature now is extensive, see Armstrong (2002) and Vogelsang (2003) for surveys of the earlier literature, but until now only two papers have provided possible solutions to both the profit neutrality puzzle and the off-net price puzzle.

Armstrong and Wright (2009) consider network arbitrage as their main mechanism. Two mobile operators prefer mobile-to-mobile (M2M) termination rates below cost to soften network competition. However, they would both like to exercise vertical market power against a third, fixed-line network by setting high fixed-to-mobile (F2M) termination rates. If the fixed-line operator can bypass termination by transiting calls via the competitor's mobile network, then it is impossible to uphold different termination rates. All termination is priced above cost if

⁴Segmentation of consumers according to their degree of information about call recipients has real-world applications. For example, the Swedish mobile operator Tele2 recommends the contract "Kompis" to consumers with many friends within the Tele2 network. "Kompis" features on-net/off-net price discrimination. Instead, Tele2 proposes the contract "Snackis" to consumers who have no idea which mobile operator their friends belong to. "Snackis" has the same call price to all domestic networks (www.tele2.se, March 8, 2013)

fixed-line termination profit is sufficiently important to mobile operators. This result extends trivially to non-discriminatory call prices because then mobile operators do not care about M2M rates, but still care about F2M rates. Even the slightest arbitrage possibility would therefore cause M2M rates to jump to the level of the F2M rate.⁵

Jullien et al. (2013) consider heterogenous call intensities and elastic total subscription demand as their main mechanism. "Light" users hold subscriptions mainly because they value receiving calls and have price elastic total demand. Aggregate demand of "heavy" users (the ones who initiate calls) is constant. The presence of light users softens competition for heavy users because a heavy user in the competitor's network generates a larger termination profit than a heavy user in the own network. The authors show that the networks then agree on termination rates above cost under non-discriminatory prices.⁶ The same result holds when networks apply termination-based price discrimination to heavy users, under some additional conditions.

The present paper differs from Armstrong and Wright (2009) and Jullien et al. (2013) in examining an entirely different mechanism whereby income effects in call demand play an important role for the choice of termination rates.

Dessein (2003) conducts the first analysis of network competition with second-degree price discrimination. He assumes consumers to differ in terms of volume demand and that networks offer menus of contracts with non-discriminatory call prices. Profit is found to be independent of the termination rate even under second-degree price discrimination. The present analysis complements Dessein (2003) by allowing even subscriptions with on-net/off-net discrimination. I show that unregulated networks prefer a termination rate at or below marginal termination cost if consumers' marginal utility of income is constant and equal to unity (as in A-LRT). Hence, the profit neutrality puzzle and the off-net price puzzle both are robust to incomplete information about consumer preferences and second-degree price discrimination alone. An extra ingredient is needed; this paper emphasizes income effects.

Hurkens and López (2013) assume that consumers do not take the effect of a price increase on network size into account when they select which network to subscribe to. Network externalities disappear under passive expectations, so the termination rate is set to maximize termination profit.⁷ Network externalities are weaker than in A-LRT even in the present model, but in my

⁵Armstrong and Wright's (2009) model relies on the assumption that F2M rates are set non-cooperatively, simultaneously with retail prices and subsequent to the (reciprocal) M2M rate. Finding the equilibrium when F2M and M2M rates are set simultaneously and prior to retail prices is still open for research.

⁶The assumption that total subscription demand is elastic and negatively correlated with usage intensity is crucial. Dessein (2003) and Hahn (2004) assume constant aggregate demand of heavy and light users and show that profit neutrality still holds. Dessein (2003) and Armstrong and Wright (2009) allow elastic total subscription demand and find that it is more profitable to lower the termination rate slightly below marginal cost than to increase it marginally. In a model with constant market size, Dessein (2004) finds the profit maximizing termination rate to be below (above) marginal cost if light user demand is less (more) elastic than heavy user demand.

⁷An intermediate stance is to assume that only a share of consumers have passive beliefs. Hoernig (2012) finds the profit-maximizing termination rate to be above cost if and only if at least half of the consumers have passive expectations. Another way to soften network externalities is to assume that every subscriber only takes the actions of some other customers into account - they belong to so called "calling clubs" (Calzada and Valletti, 2008; Gabrielsen and Vagstad, 2008; Hoernig et al., 2011). The smaller is the calling club, the weaker is the network externality and the higher is the termination rate. This result rests on the assumption that the members of the calling club do *not* coordinate the choice of network. In case of coordination, calling clubs have no effect on

case this due to binding incentive constraints which render consumer net surplus independent of network size.⁸

3 Non-discriminatory prices: The profit neutrality puzzle

The model I generalize A-LRT to allow for income effects in call demand. A continuum of consumers with unit measure are uniformly distributed on the unit interval. Each consumer subscribes to one of two networks located at each end of the interval. I assume in this section that all calls originating from a network have the same price, whereas the next sections allow networks to discriminate between calls inside (on-net) and outside (off-net) one's own network. A consumer with income I pays a subscription fee t , places $x \geq 0$ calls at price $r \geq 0$ per call to a fraction $\lambda \in (0, 1]$ of the other subscribers and consumes a numeraire good in quantity $y \geq 0$ to maximize utility $\lambda u(x) + z(y)$, subject to the budget constraint $\lambda rx + y + t \leq I$. Call utility is three times continuously differentiable, increasing and strictly concave: $u' > 0$ and $u'' < 0$. The utility of consuming the numeraire good is three times continuously differentiable, increasing and concave: $z' > 0$ and $z'' \leq 0$.⁹ A-LRT features quasi-linear utility: $z(y) = y$, where I is large. Setting $\lambda \leq 1$ is a simple way of capturing the fact that consumers have a personal network which is (much) smaller than the total network. As utility is strictly increasing in consumption, the budget constraint is binding. Therefore, consumer net surplus equals

$$v(r, t) = \max_x \{ \lambda u(x) + z(I - \lambda rx - t) \}. \quad (1)$$

A difference between this model and A-LRT is that call demand $x(r, t)$ now decreases in the subscription fee t and not only in the call price r ¹⁰:

$$\frac{\partial x}{\partial r} = \frac{z' - z''\lambda rx}{u'' + \lambda z''r^2} < 0, \quad \frac{\partial x}{\partial t} = \frac{-z''r}{u'' + \lambda z''r^2} \leq 0. \quad (2)$$

A consumer located at $k \in [0, 1]$ derives utility $v_0 + v(r_1, t_1) - k/2\sigma$ from subscribing to network 1 and utility $v_0 + v(r_2, t_2) - (1 - k)/2\sigma$ from subscribing to network 2, where $1/2\sigma$ is the

the optimal termination rate. The regulator can easily implement the first-best welfare optimum under passive beliefs or in the presence of calling clubs by prohibiting termination-based price discrimination and enforcing cost-based regulation of termination rates. Expectations about network size and calling clubs do not matter under non-discriminatory prices, and network operators have no incentive to distort the termination rate.

⁸Other extensions of the model have been considered, too: Call externalities neither solve the profit neutrality puzzle, nor the off-net price puzzle (Armstrong, 2002; Jeon et al., 2004; Berger, 2005; Hurkens and López, 2010). Calzada and Valletti (2008) and López and Rey (2012) find that termination rates above cost sometimes can be used to deter entry under termination-based price competition, but not under non-discriminatory call prices; for this later point see also Carter and Wright (2003). Valletti and Cambini (2005) argue that networks might prefer termination rates above cost to curb investments in quality improvement under non-discriminatory prices.

⁹Additively separable utility in x and y is a simplifying, but not crucial, assumption. Tangerås (2013) shows that the main results of this section go through even with non-separable utility $V(x, y; \lambda)$, provided V satisfies appropriate concavity assumptions, and $x < \infty$ even as $\lambda \rightarrow 0$.

¹⁰Call demand depends also on λ , but I have subsumed this in most of the analysis. I am mainly interested in the properties of the model as $\lambda \rightarrow 0$. Call demand is bounded even as λ becomes small: $x \rightarrow u'^{-1}(z'(I - t)r)$ as $\lambda \rightarrow 0$.

virtual transportation cost and a measure of horizontal differentiation. The lower is σ , the more differentiated are the networks. I assume throughout that the utility v_0 of holding a subscription is sufficiently high that all consumers subscribe to one of the two networks, i.e. $s_1 + s_2 = 1$. In the situation where the two networks share the market, the market share of network $i = 1, 2$ equals:

$$s_i = \frac{1}{2} + \sigma(v(r_i, t_i) - v(r_j, t_j)), i \neq j = 1, 2. \quad (3)$$

The profit of network i equals

$$\pi_i = \underbrace{s_i \lambda (r_i - s_i c - (1 - s_i)(c_O + a)) x(r_i, t_i)}_{\text{Call profit}} + \underbrace{s_i (t_i - f)}_{\text{Subscription profit}} + \underbrace{s_i (1 - s_i) \lambda (a - c_T) x(r_j, t_j)}_{\text{Termination profit}}. \quad (4)$$

Each network derives its profits from three sources. First, the network makes a profit on initiated calls if the call price r_i exceeds the perceived marginal call cost $s_i c + (1 - s_i)(c_O + a)$. The marginal cost of an on-net call equals $c = c_O + c_T$, where c_O (c_T) is the marginal cost of call origination (termination). The marginal cost of terminating a call off-net equals the marginal cost of call origination c_O plus the termination rate a . The perceived marginal call cost is the average of the two, weighted by the relative call volumes inside and outside the network. Second, subscription profit is positive if the subscription fee t_i is higher than the marginal subscription cost f . The final term constitutes the termination profit on received off-net calls, which is positive if the (reciprocal) termination rate $a \geq -c_O$ is higher than the marginal cost c_T of call termination.¹¹

Retail equilibrium Raising the call price t_i lowers call demand since the network has fewer subscribers making less calls. This is marginal call profit below:

$$\begin{aligned} \frac{\partial \pi_i}{\partial t_i} = & \underbrace{\lambda (r_i - s_i c - (1 - s_i)(c_O + a)) \left(\frac{\partial s_i}{\partial t_i} x_i + s_i \frac{\partial x_i}{\partial t_i} \right)}_{\text{Marginal call profit}} + \underbrace{\lambda \frac{\partial s_i}{\partial t_i} s_i (a - c_T) x_i}_{\text{Composition effect}} \\ & + \underbrace{s_i + \frac{\partial s_i}{\partial t_i} (t_i - f)}_{\text{Marginal subscription profit}} + \underbrace{\lambda \left(\frac{\partial s_i}{\partial t_i} s_j + s_i \frac{\partial s_j}{\partial t_i} \right) (a - c_T) x(r_j, t_j)}_{\text{Marginal termination profit}}, \end{aligned} \quad (5)$$

where $x_i = x(r_i, t_i)$ and $\partial x_i / \partial t_i = \partial x(r_i, t_i) / \partial t$. The final term in the first row is a cost composition effect. As the number of subscribers falls, more calls are terminated outside than inside the network. The composition effect is negative whenever it is more costly to terminate off-net than on-net calls. The two terms in the second row reflect the marginal effect of a higher subscription fee on subscription profit and termination profit, respectively. Marginal termination demand is ambiguous. On the one hand, termination demand tends to fall because there are fewer subscribers to reach in network i . On the other hand, termination demand tends to increase because there are more subscribers calling from the other network. With full market

¹¹The restriction $a \geq -c_O$ on the termination rate is to avoid arbitrage. For $a < -c_O$, network i could make infinite profits by initiating an unbounded amount of off-net calls. Likewise, a non-negative call price, $r \geq 0$, is necessary to prevent subscribers from making unbounded profits on calls. However, $t \geq 0$ is not necessary because of the assumption that consumers have at most one subscription. Also, I assume (realistically) that income I is so high that $t < I$ at equilibrium.

coverage and a balanced call pattern, marginal termination demand is positive if and only if network i initially has more than 50 percent of the subscribers: $\frac{\partial s_i}{\partial t_i} s_j + s_i \frac{\partial s_j}{\partial t_i} = \frac{\partial s_i}{\partial t_i} (1 - 2s_i)$.

Lemma 1 generalizes the existence and uniqueness results (Proposition 7) in Laffont, Rey and Tirole (1998a) to the case of income effects:

Lemma 1 *Assume that network operators charge two-part tariffs and non-discriminatory call prices. If either (i) networks are differentiated (σ is low); (ii) the termination rate is close to marginal termination cost ($|a - c_T|$ is low) or (iii) each subscriber calls a small fraction of the total network (λ is low), then there exists a unique and symmetric retail equilibrium $(r^*(a), t^*(a))$ characterized by $r^*(a) = c + \frac{1}{2}(a - c_T)$ and*

$$t^* - (f - \frac{1}{2}\lambda(a - c_T)x(r^*(a), t^*)) = \frac{1}{-\frac{\partial s_i}{\partial t_i} \frac{1}{s_i} \Big|_{r_1=r_2=r^*(a), t_1=t_2=t^*}}. \quad (6)$$

Proof: The proof is standard; see Tangerås (2013) for the details.

A small reduction in the call price r has marginal benefit $-\partial v/\partial r = \lambda x z'$ to every consumer in the network. This allows the operator to raise the subscription fee by λx and keep all consumers equally well off as before: $\lambda x \partial v/\partial t = -\lambda x z'$. Hence, market shares remain unchanged. To the operator, the direct loss $\lambda x/2$ in call revenue is exactly offset by a corresponding increase in the subscription revenue of $\lambda x/2$. But, as total call demand increases,¹² the price reduction is strictly profitable if the markup on call prices is positive ($r > c + \frac{1}{2}(a - c_T)$). In the opposite case of a negative markup on call prices, the network operator strictly profits from increasing the call price, thereby contracting call demand. At optimum, therefore, the network operator sets the call price equal to perceived marginal cost, and uses instead the subscription fee to compete for consumers.

The network balances a higher subscription markup against lower subscription demand and takes into account the effect of a higher subscription fee on the marginal cost of initiated calls - the composition effect. The other effects in (5) are zero: Marginal call profit is zero because calls are priced at perceived marginal cost; marginal termination profit is zero because marginal termination demand is zero at symmetric market shares.

Subscription profit, $s_i(t_i - f)$, is strictly quasi-concave in the own subscription fee, and marginal subscription profit is increasing in the subscription fee of the competitor. Quasi-concavity and strategic complementarity carry over to marginal network profit for λ sufficiently low because then marginal call profit, the composition effect and marginal termination profit are small in magnitude relative to marginal subscription profit; see (5). Consequently, (r^*, t^*) constitutes a unique equilibrium when λ is low.

¹²The total effect on call demand is $-\partial x/\partial r + \lambda x(\partial x/\partial t) = -z'/(u'' + \lambda z''r^2) > 0$, where I have substituted in $\partial x/\partial r$ and $\partial x/\partial t$ from (2) and simplified.

The semi-elasticity of subscription demand

$$-\frac{\partial s_i}{\partial t_i} \frac{1}{s_i} \Big|_{r_1=r_2=r^*(a), t_1=t_2=t^*(a)} = 2\sigma z' (I - \underbrace{[\lambda r^*(a)x^*(a) + t^*(a)]}_{T(a)}) \quad (7)$$

is a measure of the intensity of competition for subscribers at equilibrium. The lower is the subscription semi-elasticity, the higher is the markup of the subscription fee $t^*(a)$ over perceived marginal subscription cost $f - \frac{1}{2}\lambda(a - c_T)x^*(a)$, where $x^*(a) = x(r^*(a), t^*(a))$ is equilibrium call demand. Obviously, the subscription semi-elasticity is lower the lower is the degree of network substitutability (σ) because tariffs then matter less for the choice of network. Second, the subscription semi-elasticity is lower the lower is the marginal utility of income (z') because the subscription fee then is less important for consumer net surplus. Notice also that the subscription semi-elasticity is lower the lower are equilibrium consumer expenditures $T(a)$ because marginal utility of income is decreasing ($z'' \leq 0$).

The profit maximizing termination rate Unregulated networks choose the reciprocal termination rate a to maximize industry profit. By symmetry, this is the same as maximizing network profit

$$\pi(a) = \frac{1}{2}(t^*(a) - f) + \frac{1}{4}(a - c_T)\lambda x^*(a).$$

Networks do not profit at all from initiated calls because calls are priced at marginal cost. Instead, network profit stems entirely from the subscription fee and termination profit. Substitute the equilibrium subscription fee (6) into $\pi(a)$ above:

$$\pi(a) = \frac{1}{2} \frac{1}{-\frac{\partial s_i}{\partial t_i} \frac{1}{s_i} \Big|_{r_1=r_2=r^*(a), t_1=t_2=t^*(a)}} = \frac{1}{4\sigma} \underbrace{\frac{1}{z'(I - T(a))}}_{\text{Subscription markup}}. \quad (8)$$

Owing to the balanced call pattern, termination profit is fully offset by the cost composition part of the subscription fee. The only effect of the termination rate on network profit is through its effect on the intensity of competition for subscribers. In the quasi-linear Hotelling framework, network differentiation and marginal utility of income are both constant (σ is constant and $z' = 1$). Thus, the intensity of competition is constant and therefore network profit is independent of the termination rate.¹³ With a positive income effect ($z'' < 0$), marginal utility of income is lower the lower are consumer expenditures $T(a)$. Hence, the networks' incentive to increase or

¹³Profit neutrality holds also in the more general case of constant marginal utility of income different from unity, $z(y) = \beta y$, because then $\pi(a) = 1/4\sigma\beta$, which is still independent of a . More generally, all quasi-linear models in which subscription demand is determined by the difference in consumer net surplus, i.e. $s_i = g(v_i - v)$, feature a constant semi-elasticity, $g'(0)/g(0)$, at symmetric prices and therefore imply profit neutrality. The random utility model first used by Dessein (2003) for the duopoly case and later extended by Calzada and Valletti (2008) to the general $n \geq 2$ network case also belongs to this class of models: $s_i = 1/(1 + (n-1)e^{-\frac{1}{\gamma}\{v_i - v\}})$. However, profit neutrality does not imply that subscription demand is a function of the differences $v_i - v$ in consumer net surplus. For example, $s_i = g((v_i/v)^v - 1)$ has a constant semi-elasticity, $g'(0)/g(0)$, at symmetric prices but is not a function of $v_i - v$.

lower the termination rate depends on the sensitivity of consumer expenditures to changes in the termination rate:

$$\pi'(a) = \underbrace{\frac{1}{4\sigma} \frac{z''(I - T(a))}{z'(I - T(a))^2}}_{\text{Marginal income effect}} T'(a). \quad (9)$$

When call demand constitutes a normal good, consumer expenditures fall as the termination rate goes up ($T'(a) < 0$). This is the documented "waterbed" effect (Genakos and Valletti, 2011). In the more general case of positive income effects, therefore, the networks profit from a high termination rate:

Proposition 1 *Assume that network operators charge two-part tariffs and non-discriminatory call prices. Assume also that at least one of the following holds: (i) networks are differentiated (σ is low); (ii) the termination rate is close to marginal termination cost ($|a - c_T|$ is low); (iii) every subscriber calls only a small fraction of the total network (λ is low).*

1. *Network profit is independent of the termination rate if and only if the income effect is zero ($\pi'(a) = 0$ if and only if $z''(I - T(a)) = 0$).*
2. *Network profit is increasing in the termination rate for any positive income effect ($\pi'(a) > 0$ if $z''(I - T(a)) < 0$).*

Proof: I establish that consumer expenditures $T(a)$ defined in (7) satisfy $T'(a) < 0$. Then, $\pi'(a) = 0$ if and only if $z''(I - T(a)) = 0$, whereas $\pi'(a) > 0$ if $z''(I - T(a)) < 0$; see (9). By implicit differentiation of the equilibrium subscription fee t^* defined in (6):

$$t^{*'}(a) = \frac{\lambda}{2} \frac{(z'' - 2\sigma(z')^2)(x^* + (c + \frac{a-c_T}{2})\frac{\partial x}{\partial r}) + 2\sigma c(z')^2\frac{\partial x}{\partial r}}{(2\sigma(z')^2 - z'')(1 + \lambda(c + \frac{a-c_T}{2})\frac{\partial x}{\partial t}) - 2\sigma\lambda c(z')^2\frac{\partial x}{\partial t}}. \quad (10)$$

Substitute this expression into $T'(a)$ to get

$$\begin{aligned} T'(a) &= \frac{\frac{1}{2}\lambda(x^* + (c + \frac{a-c_T}{2})\frac{\partial x}{\partial r}) + (1 + \lambda(c + \frac{a-c_T}{2})\frac{\partial x}{\partial t})t^{*'}(a)}{\sigma\lambda c(z')^2(\frac{\partial x}{\partial r} - \lambda x^*\frac{\partial x}{\partial t})} \\ &= \frac{(2\sigma(z')^2 - z'')(1 + \lambda(c + \frac{a-c_T}{2})\frac{\partial x}{\partial t}) - 2\sigma\lambda c(z')^2\frac{\partial x}{\partial t}}{-\sigma\lambda c(z')^3} \\ &= \frac{u''z'' - 2\sigma(z')^2u'' - 2\sigma\lambda c(c + \frac{a-c_T}{2})(z')^2z''}{u''z'' - 2\sigma(z')^2u'' - 2\sigma\lambda c(c + \frac{a-c_T}{2})(z')^2z''} < 0 \end{aligned} \quad (11)$$

after simplifications, where in the last line I have used $\partial x/\partial r$ and $\partial x/\partial t$ characterized in (2). ■

Profit neutrality is a knife-edge result and hinges crucially on the semi-elasticity of subscription demand being constant at symmetric prices. In the presence of even the slightest income effect, profit neutrality vanishes because then the semi-elasticity and therefore the intensity of competition depends on consumer expenditures. Even a weak income effect has a strong effect

on the networks' choice of termination rate. Instead of being indifferent, the networks would now like to set the termination rate as high as possible. However, this does not mean that the termination rate will be infinite. The upper bound on the termination rate is the point at which each network would instead deviate and corner the market. For a sufficiently high, Lemma 1 breaks down and network profit is no longer given by (8); see Laffont, Rey and Tirole (1998a) for the formal proof in the quasi-linear case.

Social optimum Proposition 1 establishes that unregulated networks maximize the termination rate. Whether this incentive renders regulatory intervention desirable, depends on the profit maximizing termination rate relative to the social optimum. Under the assumption of unregulated retail competition, the social planner chooses the termination rate a to maximize the sum of consumer net surplus and industry profit

$$w(a) = \lambda(u(x^*(a)) - cx^*(a)) + z(I - T(a)) + T(a) - f.$$

A termination rate different from marginal termination cost ($a \neq c_T$) distorts prices. The first term below identifies the inefficiency associated with distorted consumption. Changing the termination rate also redistributes income between consumers and the networks. The second term below identifies the cost/benefit of redistribution:

$$w'(a) = \underbrace{\frac{\lambda}{2}(a - c_T)x^*(a)}_{\text{Price distortion}} + \underbrace{(1 - z'(I - T(a)))(\frac{\lambda}{2}x^*(a) + t^*(a))}_{\text{Redistribution}}. \quad (12)$$

The socially optimal termination rate a^{opt} trades off efficiency and redistribution. The social planner redistributes income from the networks to consumers by setting a termination rate below marginal termination cost ($a^{opt} \leq c_T$) if consumers value income higher than the networks, i.e. $z'(I - T(c_T)) \geq 1$. Conversely, if consumers' marginal utility of income is low, i.e. $z'(I - T(c_T)) < 1$, then the social planner allows upward distortions in prices ($a^{opt} > c_T$) so as to transfer income to the networks. Even so, there is an upper bound \bar{a} to the termination rate beyond which the social cost of price distortions offsets any welfare benefits of redistribution:

Proposition 2 *Assume that network operators charge two-part tariffs and non-discriminatory call prices. Assume also that at least one of the following holds: (i) networks are differentiated (σ is low); (ii) the termination rate is close to the marginal termination cost ($|a - c_T|$ is low); (iii) every subscriber calls only a small fraction of the total network (λ is low).*

1. If $z'(I - T(c_T)) > 1$, then the socially optimal termination rate a^{opt} is below marginal termination cost c_T .
2. If $z'(I - T(c_T)) \leq 1$, then $a^{opt} \in [c_T, \bar{a}]$, where $\bar{a} \geq c_T$ is defined by $z'(I - T(\bar{a})) = c/(c + \frac{1}{2}(\bar{a} - c_T))$. Termination is priced at marginal termination cost at the social optimum,

$a^{opt} = c_T$, in the special case $z'(I - T(c_T)) = 1$.

Proof: Note that

$$\begin{aligned} 2x^{*'}(a) &= \frac{\partial x}{\partial r} + 2\frac{\partial x}{\partial t}t^{*'}(a) = \frac{(2\sigma(z')^2 - z'')(\frac{\partial x}{\partial r} - \frac{\partial x}{\partial t}\lambda x^*)}{(2\sigma(z')^2 - z'')(1 + \lambda(c + \frac{1}{2}(a - c_T))\frac{\partial x}{\partial t}) - 2\sigma\lambda c(z')^2\frac{\partial x}{\partial t}} \\ &= \frac{(z'' - 2\sigma(z')^2)z'}{u''z'' - 2\sigma(z')^2u'' - 2\sigma\lambda c(c + \frac{1}{2}(a - c_T))(z')^2z''} < 0, \end{aligned} \quad (13)$$

where I have substituted $t^{*'}(a)$ given in (10) into the first row and simplified, and in the second row substituted in $\partial x/\partial r$, $\partial x/\partial t$ from (2) and simplified further. Furthermore,

$$\lambda x^*(a) + 2t^{*'}(a) = \frac{\lambda(\sigma(a - c_T)(z')^2 - (c + \frac{a - c_T}{2})z'')z'}{u''z'' - 2\sigma(z')^2u'' - 2\sigma\lambda c(c + \frac{a - c_T}{2})(z')^2z''}. \quad (14)$$

Substituting (13) and (14) into $w'(a)$ defined in (12) yields

$$\frac{2}{\lambda z'}w'(a) = \frac{\Gamma(a)z'' + (c_T - a)\sigma(z')^3}{u''z'' - 2\sigma(z')^2u'' - 2\sigma\lambda c(c + \frac{a - c_T}{2})(z')^2z''}, \quad (15)$$

where $\Gamma(a) = (c + \frac{a - c_T}{2})z'(I - T(a)) - c$. The denominator on the right-hand side of (15) is positive, so the sign of $w'(a)$ is the same as the sign of the numerator.

1. If $z'(I - T(c_T)) > 1$, then $\Gamma(c_T) = c(z'(I - T(c_T)) - 1) > 0$. In this case, $\Gamma(a) > 0$ for all $a > c_T$ because

$$2\Gamma'(a) = z' - 2(c + \frac{a - c_T}{2})T'(a)z'' = \frac{u''(z'' - 2\sigma(z')^2)z'}{u''z'' - 2\sigma(z')^2u'' - 2\sigma\lambda c(c + \frac{a - c_T}{2})(z')^2z''} > 0,$$

where I have substituted in $T'(a)$ given in (11) and simplified. Thus, $w'(a) < 0$ for all $a > c_T$ because $\Gamma(a)z'' \leq 0$ and $(c_T - a)\sigma(z')^3 < 0$. In this case, the socially optimal termination rate a^{opt} is contained in $[-c_O, c_T]$.

2. If $z'(I - T(c_T)) \leq 1$, then $\Gamma(c_T) \leq 0$. Note also that $\Gamma(a) \geq (c + \frac{a - c_T}{2})z'(I) - c$ because $z'' \leq 0$. Thus, $\Gamma(a) > 0$ for a large enough. Hence, there exists a unique $\bar{a} \geq c_T$ satisfying $\Gamma(\bar{a}) = 0$. Observe that $w'(a) < 0$ for all $a > \bar{a} \geq c_T$ because then $\Gamma(a)z'' \leq 0$ and $(c_T - a)\sigma(z')^3 < 0$. Moreover, $w'(a) > 0$ for all $a < c_T \leq \bar{a}$ because $\Gamma(a)z'' \geq 0$ and $(c_T - a)\sigma(z')^3 > 0$. Therefore, $z'(I - T(c_T)) \leq 1$ implies $a^{opt} \in [c_T, \bar{a}]$. In the special case $z'(I - T(c_T)) = 1$, $\bar{a} = c_T$ and therefore $a^{opt} = c_T$. ■

Propositions 1 and 2 jointly present a case for regulation under non-discriminatory call prices. Unregulated networks would maximize the termination rate, whereas there is an upper bound \bar{a} to the socially optimal termination rate. This holds for any positive income effect. Whether termination should be priced above or below marginal cost at social optimum, depends on the

marginal utility of income, $z'(I - T(c_T))$, which can be difficult to gauge in practice. However, the stronger is the concern with efficiency relative to redistribution, the closer is the social optimum to marginal termination cost.

Network operators have an incentive to exaggerate costs under cost-based regulation of termination rates. Thus, regulatory authorities should view self-reported costs with skepticism. A solution would be to set a rate ceiling which is independent of self-reported costs, but instead targeted to some retail price index adjusted for technological development - the well-known *RPI-X* scheme.

4 Termination-based price discrimination: The off-net price puzzle

The model I now generalize the model in the previous section by allowing the networks to price discriminate between calls within the network (on-net) and calls outside the network (off-net). Assume that all consumers subscribe to one of the two networks. A consumer with income I pays the subscription fee \hat{t} , places $q \geq 0$ calls at call price $p \geq 0$ to a fraction $\lambda \in (0, 1]$ of the \hat{s} consumers who subscribe to the same network, makes $\hat{q} \geq 0$ calls at price $\hat{p} \geq 0$ per call to $\lambda(1 - \hat{s})$ consumers in the other network and consumes the numeraire good in amount $y \geq 0$ to maximize utility $\lambda \hat{s} u(q) + \lambda(1 - \hat{s}) u(\hat{q}) + z(y)$, subject to the budget constraint $\lambda \hat{s} p q + \lambda(1 - \hat{s}) \hat{p} \hat{q} + y + \hat{t} \leq I$. The budget constraint is binding, so consumer net utility equals

$$\hat{v}(p, \hat{p}, \hat{t}, \hat{s}) = \max_{q, \hat{q}} \{ \lambda \hat{s} u(q) + \lambda(1 - \hat{s}) u(\hat{q}) + z(I - \lambda \hat{s} p q - \lambda(1 - \hat{s}) \hat{p} \hat{q} - \hat{t}) \}. \quad (16)$$

Price discrimination creates network externalities in the sense that the value of belonging to a network now also depends on the size of the network and not only on the tariff structure. Because of the income effect, both on-net demand $q(p, \hat{p}, \hat{t}, \hat{s})$ and off-net demand $\hat{q}(p, \hat{p}, \hat{t}, \hat{s})$ depend on the on-net as well as off-net prices, on the subscription fee and on the relative sizes of the two networks. In rational expectations equilibrium where both networks have a positive market share,

$$\hat{s}_i = \frac{1}{2} + \sigma(\hat{v}(p_i, \hat{p}_i, \hat{t}_i, \hat{s}_i) - \hat{v}(p_j, \hat{p}_j, \hat{t}_j, 1 - \hat{s}_i)) \quad (17)$$

defines subscription demand \hat{s}_i as a function of call prices $(p_1, \hat{p}_1, p_2, \hat{p}_2)$ and subscription fees (\hat{t}_1, \hat{t}_2) of the two networks. The profit of network i equals

$$\hat{\pi}_i = \underbrace{\hat{s}_i \lambda [\hat{s}_i (p_i - c) q_i + (1 - \hat{s}_i) (\hat{p}_i - c_O - a) \hat{q}_i]}_{\text{Call profit}} + \underbrace{\hat{s}_i (\hat{t}_i - f)}_{\text{Subscription profit}} + \underbrace{\hat{s}_i (1 - \hat{s}_i) \lambda (a - c_T) \hat{q}_j}_{\text{Termination profit}}.$$

Retail equilibrium By increasing the subscription fee \hat{t}_i , the network operator affects marginal call profit through lower demand for on-net and off-net calls:

$$\begin{aligned}
\frac{\partial \hat{\pi}_i}{\partial \hat{t}_i} = & \underbrace{\lambda \left[\hat{s}_i (p_i - c) \left(\frac{\partial \hat{s}_i}{\partial \hat{t}_i} q_i + \hat{s}_i \frac{\partial q_i}{\partial \hat{t}_i} + \hat{s}_i \frac{\partial q_i}{\partial \hat{s}_i} \frac{\partial \hat{s}_i}{\partial \hat{t}_i} \right) + \hat{s}_j (\hat{p}_i - c_O - a) \left(\frac{\partial \hat{s}_i}{\partial \hat{t}_i} \hat{q}_i + \hat{s}_i \frac{\partial \hat{q}_i}{\partial \hat{t}_i} + \hat{s}_i \frac{\partial \hat{q}_i}{\partial \hat{s}_i} \frac{\partial \hat{s}_i}{\partial \hat{t}_i} \right) \right]}_{\text{Marginal call profit}} \\
& + \underbrace{\lambda \hat{s}_i \frac{\partial \hat{s}_i}{\partial \hat{t}_i} [(p_i - c) q_i - (\hat{p}_i - c_O - a) \hat{q}_i]}_{\text{Composition effect}} + \underbrace{\hat{s}_i + \frac{\partial \hat{s}_i}{\partial \hat{t}_i} (\hat{t}_i - f)}_{\text{Marginal subscription profit}} \\
& + \underbrace{\lambda (a - c_T) \frac{\partial \hat{s}_i}{\partial \hat{t}_i} \left[(1 - 2\hat{s}_i) \hat{q}_j - \hat{s}_i (1 - \hat{s}_i) \frac{\partial \hat{q}_j}{\partial \hat{s}_j} \right]}_{\text{Marginal termination profit}}. \tag{18}
\end{aligned}$$

The first term on the second line is a composition effect, same as under non-discriminatory pricing: Less subscribers means that relatively more calls are terminated off-net. The composition effect could be positive or negative depending on the profitability of on-net calls relative to off-net calls. The second term in the second row captures the trade-off between higher subscription revenue and the marginal loss in subscribers. The final term is the effect on termination demand of charging a higher subscription fee. Because of the income effect in call demand ($\partial \hat{q}/\partial \hat{s} \neq 0$), marginal termination demand could be non-zero even at symmetric market shares.

Lemma 2 generalizes the existence and uniqueness results (Proposition 5) in Laffont, Rey and Tirole (1998b) to the case of income effects:

Lemma 2 *Assume that network operators charge two-part tariffs and price discriminate between on-net and off-net calls. If either (i) networks are differentiated (σ is low); (ii) the termination rate is close to marginal termination cost ($|a - c_T|$ is low); or (iii) every subscriber calls only a small fraction of the total network (λ is low), then there exists a unique and symmetric retail equilibrium $(p^*, \hat{p}^*(a), \hat{t}^*(a))$ characterized by $p^* = c$, $\hat{p}^*(a) = c_O + a$ and*

$$\hat{t}^* - f - \frac{\lambda}{4} (a - c_T) \frac{\partial \hat{q}(c, c_O + a, \hat{t}^*, 1/2)}{\partial \hat{s}} = \frac{1}{-\frac{\partial \hat{s}_i}{\partial \hat{t}_i} \frac{1}{\hat{s}_i} \Big|_{p_1=p_2=c, \hat{p}_1=\hat{p}_2=c_O+a, \hat{t}_1=\hat{t}_2=\hat{t}^*}}. \tag{19}$$

Proof: See Tangerås (2013).

As under non-discriminatory pricing, each operator optimally sets call prices at perceived marginal cost and then uses the subscription fee to compete for subscribers. At optimum, the operator balances a higher subscription markup against lower subscription demand, accounting also for the effect of a higher subscription fee on termination profit ($\partial \hat{q}/\partial \hat{s} \neq 0$). The marginal effect of a higher subscription fee on call profit, the first term in (18), is zero because all calls are priced at marginal cost. The composition effect in (18) is zero because the operator makes zero profit both on off-net and on-net calls. The equilibrium is unique and symmetric if every subscriber calls only a small fraction λ of the total network. The explanation is the same as under

non-discriminatory prices: Subscription profit $\widehat{s}_i(\widehat{t}_i - f)$ constitutes the main part of network profit and is well-behaved whenever λ is small.

Let $q^*(a) = q(c, c_O + a, \widehat{t}^*(a), 1/2)$ be equilibrium on-net demand as a function of the termination rate a , and define equilibrium off-net demand $\widehat{q}^*(a)$ analogously. The subscription fee plus the amount the consumer spends on calls yield consumer expenditures

$$\widehat{T}(a) = \widehat{t}^*(a) + \frac{\lambda}{2} c q^*(a) + \frac{\lambda}{2} (c_O + a) \widehat{q}^*(a).$$

The semi-elasticity of subscription demand

$$-\frac{\partial \widehat{s}_i}{\partial \widehat{t}_i} \frac{1}{\widehat{s}_i} \bigg|_{p_1=p_2=c, \widehat{p}_1=\widehat{p}_2=c_O+a, \widehat{t}_1=\widehat{t}_2=\widehat{t}^*(a)} = \frac{2\sigma z'(I - \widehat{T}(a))}{1 - 2\sigma\lambda(\mu(a) - \widehat{\mu}(a))} \quad (20)$$

measures the intensity of competition for subscribers at equilibrium. As under non-discriminatory pricing, subscription elasticity depends on network differentiation (σ) and marginal utility of income (z'). An additional *network multiplier* influences the subscription elasticity. Under price discrimination, consumer net surplus \widehat{v} depends on the size of the network; see (16). A positive network externality arises if the value

$$\mu(a) = u(q^*(a)) - z'(I - \widehat{T}(a)) c q^*(a)$$

of connecting with a subscriber in the own network is higher than the value

$$\widehat{\mu}(a) = u(\widehat{q}^*(a)) - z'(I - \widehat{T}(a)) (c_O + a) \widehat{q}^*(a)$$

of reaching a subscriber in the other network, i.e. $\partial \widehat{v} / \partial \widehat{s} = \lambda(\mu(a) - \widehat{\mu}(a)) > 0$. A positive network externality implies that it is easier for a network to attract additional consumers by lowering the subscription fee, because a higher market share further accentuates the benefit of belonging to that network. The effect is *double* because of an additional negative network externality of equal magnitude in the competing network. A positive network externality benefits the individual network but is costly to the industry. In the end, the externality only serves to intensify competition for existing subscribers without attracting any new customers. To see this, observe that the subscription semi-elasticity in (20) is higher the stronger is the network externality $\lambda(\mu(a) - \widehat{\mu}(a))$.

The networks influence the strength of the network multiplier by their choice of termination rate because the network externality depends on the price difference $\widehat{p}^* - p^* = a - c_T$ between off-net and on-net calls. For example, the network externality vanishes completely in case termination is priced at marginal cost, because off-net and on-net calls then have the same

price, c .¹⁴ In general, the marginal impact

$$\mu'(a) - \hat{\mu}'(a) = z'(I - \hat{T}(a))\hat{q}^*(a) + z''(I - \hat{T}(a))(cq^*(a) - (a + c_O)\hat{q}^*(a))\hat{T}'(a) \quad (21)$$

on the network externality of increasing the termination rate is ambiguous. A higher off-net price has a direct and negative effect on the value of reaching a subscriber in the other network, which serves to strengthen the externality. This is the first term on the right-hand side above. The price increase also affects consumer expenditures $\hat{T}(a)$ and therefore the marginal utility of income. The second term on the right-hand side represents this income effect, which could be positive or negative. The first effect dominates the second effect whenever the initial price difference between off-net and on-net calls is small enough or the income effect is weak enough. In this case, the networks can soften the externality by lowering the termination rate a .

The profit maximizing termination rate The subscription fee and termination profit are the sole sources of network profit because on-net and off-net calls are priced at marginal cost:

$$\hat{\pi}(a) = \frac{1}{2}(\hat{t}^*(a) - f) + \frac{\lambda}{4}(a - c_T)\hat{q}^*(a).$$

Substitute the equilibrium subscription fee in (19) into $\hat{\pi}(a)$ above and use (20) to rewrite network profit in terms of the subscription markup and adjusted termination profit:

$$\hat{\pi}(a) = \underbrace{\frac{1}{4\sigma} \frac{1 - 2\sigma\lambda(\mu(a) - \hat{\mu}(a))}{z'(I - \hat{T}(a))}}_{\text{Subscription markup}} + \underbrace{\frac{\lambda}{4}(a - c_T)[\hat{q}^*(a) + \frac{1}{2} \frac{\partial \hat{q}(c, a + c_O, \hat{t}^*(a), 1/2)}{\partial \hat{s}}]}_{\text{Adjusted termination profit}}. \quad (22)$$

The marginal effect on network profit of raising the termination rate above marginal termination cost equals:

$$\begin{aligned} \hat{\pi}'(c_T) &= \underbrace{\frac{1}{4\sigma} \frac{z''(I - \hat{T}(c_T))}{z'(I - \hat{T}(c_T))^2} \hat{T}'(c_T)}_{\text{Marginal income effect}} - \underbrace{\frac{\lambda}{2} \frac{\mu'(c_T) - \hat{\mu}'(c_T)}{z'(I - \hat{T}(c_T))}}_{\text{Marginal network externality}} + \underbrace{\frac{\lambda}{4} \hat{q}^*(c_T)}_{\text{Marginal termination profit}} \\ &= \frac{1}{4\sigma} \frac{z''(I - \hat{T}(c_T))}{z'(I - \hat{T}(c_T))^2} \hat{T}'(c_T) - \frac{\lambda}{4} \hat{q}^*(c_T). \end{aligned} \quad (23)$$

The first, positive term is the marginal income effect, which tends to soften competition and raise profit, same as under non-discriminatory prices; see (9). A higher termination rate also implies a stronger network externality, which instead intensifies competition, thereby reducing profit. This is the second, negative term. The third and final term is the positive effect of a marginally higher termination rate on termination profit. The marginal network externality

¹⁴By the first-order conditions for call demand, $u'(\hat{q}^*(a))/u'(q^*(a)) = (c_O + a)/c$, which implies $u'(\hat{q}^*(c_T)) = u'(q^*(c_T))$. By strict monotonicity of u' , $\hat{q}^*(c_T) = q^*(c_T)$. Thus, $\mu(c_T) = \hat{\mu}(c_T)$.

dominates marginal termination profit.¹⁵ If the income effect is weak (z'' is close to zero), then operators prefer a termination rate below cost (Gans and King, 2001). On the other hand, if the networks are differentiated (σ is low), then network externalities matter relatively less for the choice of network. Under these circumstances, operators soften competition and raise profits by setting a termination rate above cost:

Proposition 3 *Assume that network operators charge two-part tariffs and price discriminate between on-net and off-net calls.*

1. *For any positive income effect ($z''(y) < 0$ for all $y \in [0, I]$) and if networks are sufficiently differentiated (σ is low), then unregulated operators set a termination rate above the marginal cost of termination ($\hat{\pi}(\hat{a}^*) > \hat{\pi}(a)$ for some $\hat{a}^* > c_T$ and for all $a \leq c_T$). The off-net price is higher than the on-net price ($\hat{p}^*(\hat{a}^*) - p^* = \hat{a}^* - c_T > 0$).*
2. *The profit maximizing termination rate is below marginal termination cost for any degree of network differentiation σ , if the income effect is sufficiently weak ($z''(y)$ is close to zero for all $y \in [0, I]$) and either (i) the termination rate is close to the marginal termination cost ($|a - c_T|$ is low); or (ii) every subscriber calls only a small fraction of the total network (λ is low).*

Proof: See the Appendix.

Social optimum Welfare equals the sum of consumer net surplus and industry profit:

$$\hat{w}(a) = \frac{\lambda}{2}(u(q^*(a)) - cq^*(a) + u(\hat{q}^*(a)) - c\hat{q}^*(a)) + z(I - \hat{T}(a)) + \hat{T}(a) - f.$$

A termination rate different from marginal termination cost ($a \neq c_T$) distorts prices. The first term below identifies the inefficiency associated with distorted consumption. Changing the termination rate also redistributes income between consumers and the networks. The second term below identifies the cost/benefit of redistribution:

$$\hat{w}'(a) = \underbrace{\frac{\lambda}{2}(a - c_T)\hat{q}^{*'}(a)}_{\text{Price distortion}} + \underbrace{(1 - z'(I - \hat{T}(a)))(\hat{t}^{*'}(a) + \frac{\lambda}{2}\hat{q}^*(a))}_{\text{Redistribution}}. \quad (24)$$

A higher off-net price typically is mitigated by a lower subscription fee. This is the source of the waterbed effect. The direct price effect dominates the subscription effect whenever the networks are differentiated ($\hat{t}^{*'} + \frac{\lambda}{2}\hat{q}^* \geq 0$; see the Appendix). In this case, a social planner who optimally redistributes income from the networks to consumers ($z' \geq 1$) does so by setting the termination rate below cost ($\hat{a}^{opt} \leq c_T$). If the income effect is weak, the reduction in the subscription fee

¹⁵ Recall from (21) that $\mu'(c_T) - \hat{\mu}'(c_T) = z'(I - \hat{T}(c_T))\hat{q}^*(c_T)$.

dominates the higher off-net price ($\hat{t}^{*'} + \frac{\lambda}{2}\hat{q}^* \leq 0$), and the social planner optimally redistributes income to the consumers by setting a termination rate above cost ($\hat{a}^{opt} \geq c_T$):

Proposition 4 *Assume that network operators charge two-part tariffs and price discriminate between on-net and off-net calls. Assume also that consumers value income higher than the networks ($z'(I) \geq 1$).*

1. *For any positive income effect ($z''(y) < 0$ for all $y \in [0, I]$) and if networks are sufficiently differentiated (σ is low), the socially optimal termination rate \hat{a}^{opt} lies below marginal termination cost c_T .*
2. *The socially optimal termination rate lies above marginal termination cost for any degree of network differentiation σ , if the income effect is sufficiently weak ($z''(y)$ is close to zero for all $y \in [0, I]$) and either (i) the termination rate is close to the marginal termination cost ($|a - c_T|$ is low); or (ii) every subscriber calls only a small fraction of the total network (λ is low).*

Proof: See the Appendix.

A comparison of Propositions 3 and 4 reveals that profit maximizing networks sometimes set excessive termination rates and under other circumstances choose termination rates that are too low from society's viewpoint under termination-based price discrimination. Whether termination rates should be regulated up or down, depends on marginal utility of income and the strength of the income effect relative to network differentiation. When networks are differentiated, it follows that $\hat{a}^* > c_T \geq \hat{a}^{opt}$, and the termination rates should be capped. On the other hand, $\hat{a}^* < c_T \leq \hat{a}^{opt}$ if the income effect is weak, and the regulator should impose a termination rate floor.

5 Second-degree price discrimination

The analysis has so far built upon the assumption that all consumers are identical - up to a constant measuring horizontal differentiation. Homogeneity implies that network operators only offer one contract each. I now turn to the more compelling case of heterogenous preferences: Some consumers prefer subscriptions with on-net/off-net discrimination, while others prefer contracts in which all calls are equal. Contracts are required to be incentive compatible because operators cannot distinguish ex ante between different types of consumers.

Second-degree price discrimination has interesting implications for network competition and for unregulated termination rates. In particular, unregulated networks may profit from increasing the termination rate above cost even if they would not do so under complete information termination-based price discrimination, i.e. when a large share of consumers demand on-net/off-net contracts and income effects are weak.

The model There are two types of consumers, informed ones in share θ and uninformed ones in share $1 - \theta$. An informed consumer is someone who knows whether the called party belongs to the own network or is located off-net. An uninformed consumer has no information about the recipient's network. These types are uniformly distributed on the unit interval and statistically independent of one another.

Network i offers an on-net/off-net contract $\{q_i, \hat{q}_i, \hat{T}_i\}$ designed for the informed consumers, where q_i (\hat{q}_i) is the number of on-net (off-net) calls the subscriber is allowed to make to each member of its personal network, and \hat{T}_i is the subscription fee. An additional non-discriminatory contract $\{x_i, T_i\}$ is intended for the uninformed consumers, where x_i denotes the number of calls the subscriber is allowed to make to each member of its personal network, on and off-net alike, and T_i is the subscription fee.¹⁶

Under the assumption that consumers only call other consumers of the same type (see, e.g. Dessein, 2003),¹⁷ the on-net/off-net contract is incentive compatible if and only if

$$\hat{v}_i = \hat{v}(q_i, \hat{q}_i, \hat{T}_i, \hat{s}_i) = \hat{s}_i \lambda u(q_i) + (\theta - \hat{s}_i) \lambda u(\hat{q}_i) - \hat{T}_i \geq \theta \lambda u(x_i) - T_i, \quad (25)$$

where \hat{s}_i ($\hat{s}_j = \theta - \hat{s}_i$) is the number of informed consumers in network i (j).¹⁸

An important aspect of second-degree price discrimination in this dimension is that incentive compatibility is affected, not only by the network's own choice of contracts, but also by the contracts offered by the *competitor* through the network externality: $(\partial \hat{v}_i / \partial \hat{s}_i)(\partial \hat{s}_i / \partial \hat{T}_j) \neq 0$.

The Revelation Principle does not require i 's contracts to respect the IC constraints of j 's consumers. Rather, it is necessary to account for the possibility of incentive incompatibility in subscription demand. Thus

$$\hat{s}_i = \frac{\frac{\theta}{2} + \theta \sigma [\lambda \theta u(\hat{q}_i) - \hat{T}_i - J(\lambda \theta u(q_j) - \hat{T}_j) - (1 - J)(\lambda \theta u(x_j) - T_j)]}{1 - \theta \sigma \lambda [u(q_i) - u(\hat{q}_i) + J(u(q_j) - u(\hat{q}_j))]} \quad (26)$$

is the number of consumers subscribing to i 's on-net/off-net contract in rational expectations equilibrium when the market is fully covered. The indicator function J takes the value 1 if j 's on-net/off-net contract is strictly incentive compatible ($\hat{v}_j > \theta \lambda u(x_j) - T_j$) and the value 0 if it is instead strictly incentive incompatible ($\hat{v}_j < \theta \lambda u(x_j) - T_j$). Observe that the intensity of competition as measured by the semi-elasticity $-(\partial \hat{s}_i / \partial \hat{T}_i) / \hat{s}_i$ depends on whether j 's IC constraint is satisfied or not.

Let $v(x_i, T_i) = (1 - \theta) \lambda u(x_i) + z(I - T_i)$ be consumer net surplus the uninformed consumer achieves by selecting $\{x_i, T_i\}$. Switching to the on-net/off-net contract creates a variance in

¹⁶ A maximum call allowance paired with a subscription fee is a common type of non-discriminatory contract. Specifying separate on-net and off-net call allowances is peculiar, but for simplifying purposes only. The equilibrium on-net/off-net contract can equivalently be reformulated as a two-part tariff, $\{c, c_O + a, \hat{t}^n\}$.

¹⁷ The assumption of type dependent call demand hugely simplifies the analysis without affecting the qualitative insights. I discuss robustness to alternative call patterns at the end of the section.

¹⁸ As a simplification, I assume away income effects for informed consumers: The utility of consuming the numeraire good equals $z(y) = y$, same as in A-LRT. If anything, this departure from Section 4 biases networks to favour termination rates below cost by removing the positive marginal income effect of the on-net/off-net contract present in (23).

consumer net surplus because the consumer then has incomplete information about the number of calls available to her (or about total call expenditures under two-part tariffs). If uninformed consumers are sufficiently risk averse, then networks offer these subscribers non-discriminatory contracts.¹⁹ Also, they would not select an on-net/off-net contract over a non-discriminatory contract. I thus neglect the incentive constraint on the non-discriminatory contract in what follows, but verify that it is indeed satisfied in the relevant range of termination rates (Tangerås, 2013). In a fully covered market, demand for network i 's non-discriminatory contract thus equals

$$s_i = \frac{1-\theta}{2} + (1-\theta)\sigma(v(x_i, T_i) - v(x_j, T_j)). \quad (27)$$

The Lagrangian of network i reads

$$\begin{aligned} L_i = & \underbrace{\widehat{s}_i[\widehat{T}_i - f - \widehat{s}_i c \lambda q_i - \widehat{s}_j(c_O + a)\lambda \widehat{q}_i]}_{\text{Subscription profit, on-net/off-net contract}} + \underbrace{s_i[T_i - f - ((1-\theta)c + s_j(a - c_T))\lambda x_i]}_{\text{Subscription profit, non-discriminatory contract}} \\ & + \underbrace{(a - c_T)\lambda[\widehat{s}_i\widehat{s}_j(J\widehat{q}_j + (1-J)x_j) + s_i s_j x_j]}_{\text{Termination profit}} + \underbrace{\xi_i(\widehat{v}(q_i, \widehat{q}_i, \widehat{T}_i, \widehat{s}_i) - \theta \lambda u(x_i) + T_i)}_{\text{IC constraint}}, \end{aligned} \quad (28)$$

where ξ_i is the Lagrangian multiplier associated with the IC constraint (25). Each network derives its profits from two sources, subscription fees and termination profit. Perceived marginal subscription cost now also includes call cost in addition to the direct cost f .

Retail equilibrium By increasing the subscription fee, \widehat{T}_i , of the on-net/off-net contract the network earns a higher markup per subscriber, but at the cost of lower subscription demand. This trade-off represents marginal subscription profit below:

$$\begin{aligned} \frac{\partial L_i}{\partial \widehat{T}_i} = & \underbrace{\widehat{s}_i + \frac{\partial \widehat{s}_i}{\partial \widehat{T}_i}[\widehat{T}_i - f - \widehat{s}_i c \lambda q_i - \widehat{s}_j(c_O + a)\lambda \widehat{q}_i]}_{\text{Marginal subscription profit}} + \underbrace{\lambda \widehat{s}_i \frac{\partial \widehat{s}_i}{\partial \widehat{T}_i}((c_O + a)\widehat{q}_i - c q_i)}_{\text{Composition effect}} \\ & + \underbrace{\lambda(a - c_T) \frac{\partial \widehat{s}_i}{\partial \widehat{T}_i}(\theta - 2\widehat{s}_i)(J\widehat{q}_j + (1-J)x_j)}_{\text{Marginal termination profit}} - \underbrace{\xi_i(1 - \frac{\partial \widehat{s}_i}{\partial \widehat{T}_i} \lambda(u(q_i) - u(\widehat{q}_i)))}_{\text{Marginal incentive effect}}. \end{aligned} \quad (29)$$

The second term in the first row is the cost composition effect stemming from a larger amount of calls being terminated off-net than on-net as the number of subscribers falls. Marginal termination profit is ambiguous as before, but zero at symmetric market shares owing to the balanced call pattern. An increase in the subscription fee of the on-net/off-net contract makes it harder to satisfy the IC constraint (25), hence the negative marginal incentive effect.

An analogous trade-off faces network i when choosing the optimal subscription fee, T_i , of the

¹⁹The most extreme manifestation would be lexicographic preferences towards price stability.

non-discriminatory contract:

$$\begin{aligned} \frac{\partial L_i}{\partial T_i} = & \underbrace{s_i + \frac{\partial s_i}{\partial T_i}[T_i - f - ((1 - \theta)c + s_j(a - c_T))\lambda x_i]}_{\text{Marginal subscription profit}} + \underbrace{\lambda s_i \frac{\partial s_i}{\partial T_i}(a - c_T)x_i}_{\text{Composition effect}} \\ & + \underbrace{\lambda(a - c_T) \frac{\partial s_i}{\partial T_i}(1 - \theta - 2s_i)x_j}_{\text{Marginal termination profit}} + \underbrace{\xi_i}_{\text{Marginal incentive effect}} \end{aligned} \quad (30)$$

Marginal termination profit is zero at symmetric market shares even here. An increase in the subscription fee of the non-discriminatory contract renders the on-net/off-net contract comparatively more attractive to informed consumers, hence the positive marginal incentive effect.

Lemma 3 *The on-net/off-net contract $\{q^*, \hat{q}^*, \hat{T}^m\}$ defined by $u'(q^*) = c$, $u'(\hat{q}^*) = c_O + a$ and*

$$\hat{T}^m - f - \theta c \lambda q^* = \frac{1 - (1 + \Omega)\theta\sigma\lambda(u(q^*) - u(\hat{q}^*))}{2\sigma} - \frac{2\xi^m}{\theta} \frac{[1 - \Omega\theta\sigma\lambda(u(q^*) - u(\hat{q}^*))]}{2\sigma}, \quad (31)$$

and the non-discriminatory contract $\{x^m, T^m\}$ defined by

$$\frac{u'(x^m) - (c + \frac{1}{2}(a - c_T))z'(I - T^m)}{u'(x^m)} = \frac{2\xi^m}{1 - \theta} \left[\frac{\theta}{1 - \theta} z'(I - T^m) - 1 \right] \quad (32)$$

and

$$T^m - f - (1 - \theta)c\lambda x^m = \frac{1}{2\sigma z'(I - T^m)} \left[1 + \frac{2\xi^m}{1 - \theta} \right], \quad (33)$$

plus the associated complementary slackness conditions

$$\xi^m[\hat{v}(q^*, \hat{q}^*, \hat{T}^m, \theta/2) - \lambda\theta u(x^m) + T^m] = 0, \quad \hat{v}(q^*, \hat{q}^*, \hat{T}^m, \theta/2) \geq \lambda\theta u(x^m) - T^m, \quad \xi^m \geq 0 \quad (34)$$

and the multiplicity restriction

$$(1 - \Omega)[\hat{v}(q^*, \hat{q}^*, \hat{T}^m, \theta/2) - \lambda\theta u(x^m) + T^m] = 0, \quad \Omega \in [0, 1] \quad (35)$$

jointly characterize a symmetric retail equilibrium if $(a - c_T)(\hat{q}^ - x^m) \geq 0$ and either: (i) the networks are differentiated (σ is low); (ii) every subscriber calls only a small fraction of the total network (λ is low); (iii) the termination rate is close to marginal termination cost and the share of informed consumers is high ($|a - c_T|$ is low and θ is high). For every termination rate $a \neq c_T$ with multiple equilibria, the unique pay-off dominant equilibrium is given by $\Omega^m(a) = 0$ if $a > c_T$ and $\Omega^m(a) = 1$ if $a < c_T$.*

Proof: See Tangerås (2013).

Call allowances, q^* and \hat{q}^* , are set at their first-best level (i.e. where marginal call utility equals perceived marginal call cost) in the on-net/off-net contract. This is an incidence of the classical "no-distortion-at-the-top" result. Call allowances are distorted in the non-discriminatory contract to extract consumer informational rent. A reduction in the call allowance, x^m , below the first-best level renders the non-discriminatory contract less attractive to all consumers. The network compensates the uninformed consumer by lowering also the subscription fee, T^m . If her marginal utility of income is high or her personal network relatively small ($\theta z'(I - T^m) > 1 - \theta$), only a small reduction in the subscription fee is required to keep the uninformed consumer equally well off as before. But for the informed consumer, whose marginal utility of income is much lower, or whose personal network, θ , is comparatively larger, the lower fee is insufficient to fully compensate the reduced call allowance. The increased leverage allows the network operator to profitably increase the subscription fee, \hat{T}^m , while still maintaining incentive compatibility.

Call allowances are not necessarily distorted below the first-best level at equilibrium. If uninformed consumers have a low marginal utility of income or a relatively large personal network ($\theta z'(I - T^m) < 1 - \theta$), then the network can increase the call allowance, x^m , slightly and the subscription fee, T^m , substantially without hurting the uninformed consumer. The increased call allowance is insufficient to fully compensate the informed consumer for the higher subscription fee. Now the operator can profitably raise even the subscription fee, \hat{T}^m , up to the point at which the IC constraint is once again binding.

In the special case, $\theta z'(I - T^m) = 1 - \theta$, transfers are simply redistributed between the two consumer types without generating any additional revenue to the firm. The equilibrium call allowance, x^m , thus remains at its first-best level even if the IC constraint is binding; see Dessein (2003) and references therein for similar results.

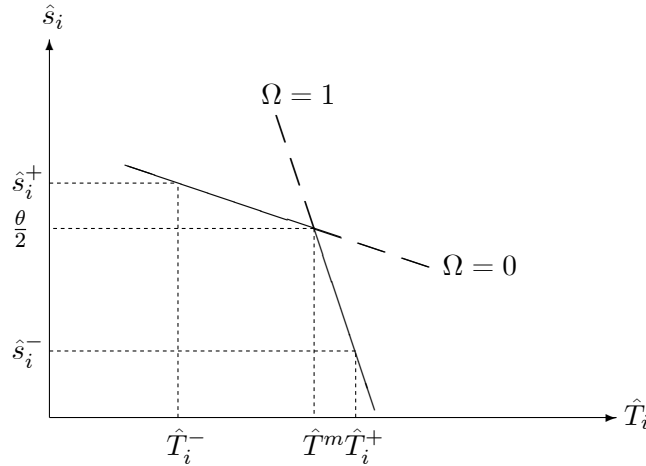


Figure 1: Kinked subscription demand

Binding incentive constraints lead to a kinked subscription demand, as illustrated in Figure 1. The horizontal axis displays the subscription fee, \hat{T}_i , of network i , while subscription demand, \hat{s}_i , is on the vertical axis. Assume that the termination rate exceeds marginal termination cost, so that $q^* > \hat{q}^*$. A small increase in i 's subscription fee from \hat{T}^m to \hat{T}_i^+ satisfying (25) causes

subscribers to move from i to j . The change in relative network size imposes a negative network externality in i and a positive network externality in j ($\partial \hat{v}_i / \partial \hat{s}_j < 0$ and $\partial \hat{v}_j / \partial \hat{s}_i > 0$). This double network externality amplifies the net value of subscribing to network j and leads by the network multiplier to an accentuated drop in network i 's subscriptions from $\theta/2$ to \hat{s}_i^- . Consider now the opposite case of a reduction in \hat{T}^m to \hat{T}_i^- , again constrained by incentive compatibility (25). The lower fee allows i to attract consumers from j , creating a positive network externality in i ($\partial \hat{v}_i / \partial \hat{s}_i > 0$). But j 's on-net/off-net contract is incentive incompatible at i 's lower subscription fee ($\hat{v}_j < \theta u(x^m) - T^m$ for all $\hat{T}_i < \hat{T}^m$ because $\partial \hat{v}_j / \partial \hat{s}_i < 0$), so all consumers in j are on non-discriminatory contracts. The value of a non-discriminatory contract is independent of network size. Therefore, a reduction in i 's subscription fee to \hat{T}_i^- does not create any negative network externality in network j for i to exploit. Because the network multiplier is much smaller, i 's network only increases to \hat{s}_i^+ .

The equilibrium subscription fees, \hat{T}^m and T^m , are determined according to a modified Ramsey rule: The markup of the subscription fee over adjusted marginal subscription cost is set equal to the inverse of the semi-elasticity of subscription demand, modified by an incentive correction. Adjusted marginal subscription cost here is the sum of the direct subscription cost, f , and total call cost, adjusted by the cost composition effects in (29) and (30).

Kinked subscription demand implies that the semi-elasticity of the on-net/off-net contract is undefined at equilibrium. Instead, a continuum of pseudo-elasticities bounded by the directional elasticities are consistent with profit maximization:

$$-\frac{\partial \hat{s}_i}{\partial \hat{T}_i} \frac{1}{\hat{s}_i} \bigg|_{q_1=q_2=q^*, \hat{q}_1=\hat{q}_2=\hat{q}^*(a), \hat{T}_1=\hat{T}_2=\hat{T}^m(a)} = \frac{2\sigma}{1 - (1 + \Omega)\theta\sigma\lambda(u(q^*) - u(\hat{q}^*(a)))}, \Omega \in [0, 1]. \quad (36)$$

Binding IC constraints yield a continuum of symmetric retail equilibria $\Omega \in [0, 1]$, hence the restriction (35). Still, equilibrium selection according to pay-off dominance is straightforward: Competition for subscribers is weaker and network profit higher, the lower is the elasticity of subscription demand. Hence, the pay-off dominant equilibrium is given by $\Omega^m(a) = 0$ for all $a > c_T$ (and by $\Omega^m(a) = 1$ for all $a < c_T$). Under complete information, or if the incentive constraint is non-binding, then the semi-elasticity is well-defined with $\Omega = 1$ in (36), and the symmetric equilibrium is unique.

In the non-discriminatory contract, the equilibrium semi-elasticity of subscription demand

$$-\frac{\partial s_i}{\partial T_i} \frac{1}{s_i} \bigg|_{x_1=x_2=x^m(a), T_1=T_2=T^m(a)} = 2\sigma z'(I - T^m(a)) \quad (37)$$

is identical to (7) in Section 3.

The Lagrangian multiplier $\xi^m(a)$ in Lemma 3 defines the shadow price on the IC constraint. A large $\xi^m(a)$ calls for severe distortions in the allowance, x^m , and a strong downward (upward) adjustment in the subscription fee of the on-net/off-net (non-discriminatory) contract to restore incentive compatibility. On the other hand, the shadow price could be zero at equilibrium. Let $\{q^*, \hat{q}^*, \hat{T}_0^m\}$ and $\{x_0^m, T_0^m\}$ be the complete information, "null" contracts (i.e. assuming that

$\xi^m = 0$ and $\Omega = 1$ in Lemma 3). If termination is priced at marginal cost, then the informed consumer's net benefit of picking her designated null contract equals

$$\begin{aligned} & \theta \lambda(u(q^*) - u(x_0^m(c_T))) + T_0^m(c_T) - \widehat{T}_0^m(c_T) \\ &= \theta \lambda(u(q^*) - u(x_0^m(c_T))) + \frac{1}{2\sigma z'(I - T_0^m(c_T))} - \frac{1}{2\sigma} + (1 - \theta)c\lambda x_0^m(c_T) - \theta c\lambda q^*, \end{aligned} \quad (38)$$

where in the second line I have substituted in the equilibrium subscription fees from Lemma 3. If marginal utility of income does not deviate too much from unity, then call allowances q^* and $x_0^m(c_T)$ and subscription markups are nearly the same in both contracts.²⁰ In this case, the difference between the two contracts mainly is captured by the differences in adjusted marginal costs, the last two terms in the above expression.

The adjusted marginal cost of the on-net/off-net contract is comparatively low if the share θ of informed subscribers is relatively small because the corresponding number of outgoing calls then is low. A low marginal cost implies that the subscription fee $\widehat{T}_0^m(c_T)$ is small compared to $T_0^m(c_T)$, and the null contract $\{q^*, \widehat{q}^*, \widehat{T}_0^m\}$ therefore is incentive compatible. In the opposite case, when the share, θ , of informed consumers is large, then $\{q^*, \widehat{q}^*, \widehat{T}_0^m\}$ is strictly incentive incompatible and therefore $\xi^m(c_T) > 0$. By continuity, $\xi^m(a) > 0$ even for all termination rates a sufficiently close to marginal termination cost c_T .²¹

Finally, network profit not only is kinked, but also discontinuous in general. Each operator makes termination profit $\theta^2(a - c_T)\lambda\widehat{q}^*/4$ on the informed subscribers in the competing network at equilibrium. If the incentive constraint is strictly binding, an infinitesimal deviation is enough to render the on-net/off-net contract of the competing network incentive incompatible, inducing a discontinuous shift in termination profit to approximately $\theta^2(a - c_T)\lambda x^m/4$. The restriction $(a - c_T)(\widehat{q}^* - x^m) \geq 0$ in Lemma 3 ensures that no network can boost its termination profit (or cut termination losses if $a < c_T$) by a slight alteration to its retail contracts.

Termination rates Assume that the two network operators are able to coordinate on the pay-off dominant retail equilibrium. At termination rates above cost, network profit equals

$$\begin{aligned} \pi^m(a) = & \underbrace{\frac{\theta}{4\sigma}[1 - (1 + \Omega^m(a))\theta\sigma\lambda(u(q^*) - u(\widehat{q}^*(a)))]}_{\text{Average subscription markup}} + \frac{1 - \theta}{4\sigma z'(I - T^m(a))} \\ & + \underbrace{\frac{\theta^2\lambda}{4}c(q^* - \widehat{q}^*(a))}_{\text{Net termination profit}} - \underbrace{\frac{\xi^m(a)}{2\sigma}\left(1 - \frac{1}{z'(I - T^m(a))}\right)}_{\text{Consumer informational rent}} \end{aligned} \quad (39)$$

under second-degree price discrimination, after substituting in the subscription fees from Lemma 3 into the Lagrangian (28) and simplifying. Network profit can be divided in three parts: The average subscription markup weighted by the relative sizes of the two consumer groups, net

²⁰Note from eq. (32) that $u'(x_0^m(c_T)) = cz'(I - T_0^m(c_T)) = u'(q^*)z'(I - T_0^m(c_T))$. If $z'(I - T_0^m(c_T)) \rightarrow 1$, then $u'(x_0^m(c_T)) \rightarrow u'(q^*)$.

²¹Tangerås (2013) shows that $\{q^*, \widehat{q}^*, \widehat{T}_0^m\}$ and $\{x^m, T^m\}$ satisfy even the IC constraint of the non-discriminatory contract for a sufficiently close to c_T , θ large enough and $z'(I - T_0^m(c_T)) - 1$ not too small.

termination profit, and consumer informational rent.

Second-degree price discrimination has an ambiguous effect on the average subscription markup. On the one hand, a weaker network externality ($\Omega^m(a) \leq 1$) softens competition for informed consumers and raises the subscription markup of the on-net/off-net contract. On the other hand, incentive compatibility drives up the subscription fee, $T^m(a)$, which lowers the subscription markup of the non-discriminatory contract.

Net termination profit is defined as gross termination profit less the cost composition effect. The composition effect and termination profit cancel out in the non-discriminatory contract, just as in Section 3. A net termination profit remains in the on-net/off-net contract owing to the difference in on-net and off-net call volumes. Net termination profit is independent of second-degree price discrimination because call allowances q^* and $\hat{q}^*(a)$ are at their first-best levels with or without incentive problems.

Consumer informational rent could be positive, zero or negative under second-degree price discrimination. If marginal utility of income of uninformed consumers is relatively high ($z'(I - T^m) > 1$), then competition for uninformed (informed) consumers is relatively intense (weak) as measured by the difference between the two semi-elasticities (37) and (36). This difference in competitive pressure implies that operators tend to balance marginal profit distortions by reducing the subscription fee of the on-net/off-net contract instead of increasing the subscription fee of the non-discriminatory contract. Consequently, the average equilibrium subscription fee, $\theta\hat{T}^m + (1-\theta)T^m$, falls relative to the case with complete information, and consumer informational rent is positive. In the opposite case ($z'(I - T^m) < 1$), network operators restore incentive compatibility mostly by increasing the subscription fee of the non-discriminatory contract. This, in turn, leaves a negative informational rent to consumers at equilibrium.

Despite its effect on network profit, second-degree price discrimination alone does not alter the basic insight of the workhorse model that termination rates above cost are unprofitable under on-net/off-net discrimination. Assume that marginal utility of income is constant and equal to unity for both consumer groups, i.e. $z(y) = y$. Then the subscription markup of the non-discriminatory contract is independent of the termination rate and equal to $(1 - \theta)/4\sigma$. Moreover, consumer informational rent is zero because IC constraints only redistribute rent between informed and informed consumers with no direct effect on the expected subscription fee. What is left for network operators, is to trade the cost of a reinforced network externality off against the benefit of a higher termination profit:

$$\pi^{m'}(a) = \underbrace{\frac{\theta^2\lambda}{4}(1 + \Omega^m)u'(\hat{q}^*)\hat{q}^{*'}(a)}_{\text{Marginal network externality}} - \underbrace{\frac{\theta^2\lambda}{4}c\hat{q}^{*'}(a)}_{\text{Marginal termination profit}} = \frac{\theta^2\lambda}{4}((c_O + a)\Omega^m + a - c_T)\hat{q}^{*'}(a).$$

While second-degree price discrimination may soften the marginal network externality considerably, the externality is still strong enough to dominate marginal termination profit at termination rates above marginal termination cost:

Proposition 5 *If consumers' marginal utility of income is constant and equal to unity (as in*

A -LRT), then unregulated network operators maximize profit under second-degree price discrimination by agreeing on a termination rate at or below marginal termination cost ($z(y) = y$ implies $\pi^m(c_T) > \pi^m(a)$ for all $a > c_T$).

Proof: If $z(y) = y$, then $\pi^m(c_T) = 1/4\sigma$, as is easily verified. Hence,

$$4[\pi^m(c_T) - \pi^m(a)] = \theta^2 \lambda \Omega^m(a)[u(q^*) - u(\hat{q}^*(a))] + \theta^2 \lambda [u(q^*) - cq^* - u(\hat{q}^*(a)) + c\hat{q}^*(a)],$$

which is strictly positive for all $a > c_T$ because $\Omega^m(a) \geq 0$, $q^* > \hat{q}^*(a)$, and q^* is the unique maximizer of $u(q) - cq$. ■

Consider now a more general utility function $z(y) \neq y$. Assume also that the IC (25) is strictly binding.²² In this case, the general trade-off facing network operators is:

$$\begin{aligned} \pi^{m'}(a) = & \underbrace{\frac{\theta^2 \lambda}{4}(a - c_T)\hat{q}^{*'}(a)}_{\text{Marginal network externality plus marginal termination profit}} \\ & + \underbrace{\frac{1 - \theta}{4\sigma} \left(1 + \frac{2\xi^m}{1 - \theta}\right) \frac{z''(I - T^m)}{z'(I - T^m)^2} T^{m'}(a)}_{\text{Marginal income effect}} + \underbrace{\frac{\xi^{m'}(a)}{2\sigma} \left(\frac{1}{z'(I - T^m)} - 1\right)}_{\text{Marginal rent extraction}}. \end{aligned} \quad (40)$$

Marginal termination profit under second-degree price discrimination differs from marginal termination profit $\hat{\pi}'(a)$ under termination-based price discrimination, see (23), in two fundamental aspects. First, second-degree price discrimination weakens the network multiplier to such an extent ($\Omega^m(a) = 0 < 1$) that the marginal network externality now is close in magnitude to marginal termination profit. In fact, the two cancel out at a termination rate equal to marginal cost. Second, a rent extraction motive comes into play under second-degree price discrimination. A higher termination rate makes the on-net/off-net contract more valuable to consumers because a stronger network externality intensifies retail competition. The value of the non-discriminatory contract falls because a higher termination rate softens retail competition for uninformed consumers through the marginal income effect. A higher termination rate thus relaxes the IC constraint of the on-net/off-net contract, thereby causing the shadow price on the IC constraint to fall ($\xi^{m'}(a) < 0$). If consumer informational rent is positive ($z'(I - T^m) > 1$), then network operators extract informational rent by agreeing on a higher termination rate.

The profitability of increasing the termination rate above cost is driven entirely by the strength of the marginal income effect relative to marginal rent extraction. The marginal income effect is positive for any positive income effect ($z'' < 0$) because of the waterbed effect ($T^{m'}(a) < 0$) and dominates the trade-off unless consumer informational rent is too negative ($z'(I - T^m) < 1$ is small). If the income effect is zero, but consumer informational rent positive ($z(y) = \beta y$, $\beta > 1$), marginal rent extraction alone makes it profitable to raise the termination rate above

²²The IC constraint is strictly binding if, for example, the termination rate is sufficiently close to marginal cost ($a - c_T > 0$ is small), the share θ of informed consumers is large and marginal utility of income is not too far from unity, i.e. $|1 - z'(I - T_0^m(c_T))|$ is small; see (38).

cost ($\pi^{m'}(c_T) = \xi^{m'}(c_T)(1 - \beta)/2\sigma\beta > 0$). Proposition 6 summarizes the main result of this Section:

Proposition 6 *Assume that (i) network operators engage in second-degree price discrimination and coordinate on the pay-off dominant retail equilibrium; (ii) a large share of consumers are on on-net/off-net contracts (θ is large enough); (iii) the incentive compatibility constraint is strictly binding ($\xi^m(c_T) > 0$).*

1. *Unregulated network operators profit from increasing the termination rate above marginal termination cost ($\pi^m(a^m) > \pi^m(c_T)$ for some $a^m > c_T$) if either*

- (a) *the income effect in call demand is positive and consumer informational rent is not too negative ($z''(T^m(c_T)) < 0$ and $z'(I - T_0^m(c_T)) - 1$ is not too negative); or*
- (b) *the income effect in call demand effect is zero, but consumer informational rent is positive ($z(y) = \beta y$, $\beta > 1$).*

2. *The unregulated termination rate is excessive from a welfare viewpoint ($a^m > a^{opt}$).*

Proof: See Tangerås (2013).

Under complete information, termination-based price discrimination it is strictly unprofitable to set a termination rate above cost if the income effect is weak; see Proposition 3. But Proposition 6 shows that a minor modification is sufficient to reverse this conclusion and render it strictly profitable to set a termination rate above cost. By introducing a little bit of consumer heterogeneity, in the sense that a small share of consumers demand non-discriminatory contracts ($\theta \lesssim 1$), and assuming network operators to be incompletely informed about preferences, networks find it optimal to engage in second-degree price discrimination.²³

Figure 2 compares network profit, $\hat{\pi}(a)$, under termination-based price discrimination (assuming $z(y) = y$ in (22)) with network profit $\pi^m(a)$ under second-degree price discrimination. Consumer informational rent causes network profit to shift downwards: $\pi^m(c_T) < \hat{\pi}(c_T)$. At the same time, second-degree price discrimination softens competition for informed consumers through a smaller network multiplier, which rotates the profit function counter-clockwise. The network multiplier is so weak under second-degree price discrimination that networks strictly profit from setting a termination rate above cost. While each network suffers a unilateral loss from being incompletely informed about consumer preferences, the industry may nevertheless gain. For high enough termination rates, competition is weak enough to outweigh the informational rent paid to consumers: $\pi^m(a) > \hat{\pi}(a)$ for all $a > b$.

²³In the opposite case, when the share θ of informed consumers is low enough and $|1 - z'(I - T_0^m(c_T))|$ is low, then the null contract $\{q^*, \hat{q}^*, \hat{T}_0^m\}$ is incentive compatible for all termination rates sufficiently low (Tangerås, 2013). If the income effect is positive, then $\pi^{m'}(a) > 0$ for all $a \in [-c_O, c_T]$. Hence, Proposition 1 extends to the case of a small, but positive share of consumers on-net/off-net contracts.

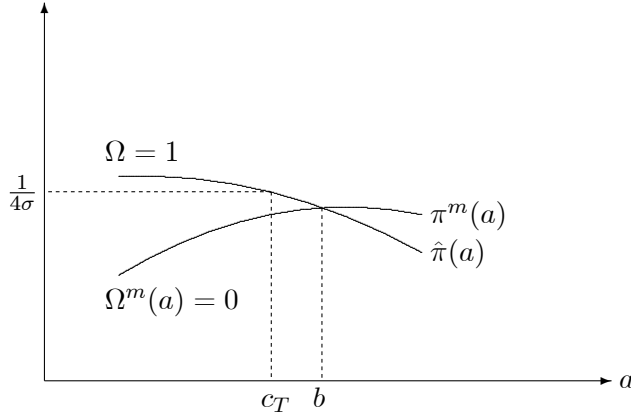


Figure 2: Network profit comparison

Proposition 6 contains statements about the profitability of deviating to a termination rate above cost, but not below cost. There are two reasons why networks would choose not to set termination rates below cost under second-degree price discrimination. First, retail equilibria might fail to exist. If the share, θ , of informed consumers is high and that the incentive constraint is binding ($\xi^m > 0$), then profit maximization calls for a strong downward distortion of the call allowance x^m ; see (32). The downward distortion, combined with the perceived marginal cost of a non-discriminatory call being higher than that of an off-net call implies $x^m < \hat{q}^*$ for all $a < c_T$. But then $(a - c_T)(\hat{q}^* - x^m) < 0$, so each network can achieve a discontinuous cut in its termination loss by an infinitesimal deviation which renders the on-net/off-net contract of the competing network strictly incentive incompatible and moves all the competitor's consumers to the non-discriminatory contract.

Second, termination rates below cost are contained in $[-c_O, c_T)$ to prevent arbitrage. Technological development has pushed marginal costs down towards zero, so the potential for setting termination rates below cost now is limited.²⁴ But the potential for deviating to a termination rate above cost is unconstrained by arbitrage. Hence, $\pi^m(a^m) > \pi^m(c_T) \approx \pi^m(a)$ for all $a \in [-c_O, c_T)$ if marginal call cost, c , is small enough.

Social welfare Expected welfare equals

$$w^m(a) = \frac{\theta^2 \lambda}{2} [u(q^*) - cq^* + u(\hat{q}^*(a)) - c\hat{q}^*(a)] \\ + (1 - \theta)[(1 - \theta)\lambda(u(x^m(a)) - cx^m(a)) + z(I - T^m(a)) + T^m(a)] - f$$

under second-degree price discrimination, assuming that the networks coordinate on the pay-off dominant retail equilibrium. As in the previous sections, the socially optimal termination rate

²⁴Jullien et al. (2013), for example, estimate the sum of call origination and call termination at 2 Eurocents per minute in their calibration of the French market for mobile telephony.

trades off efficiency against redistribution:

$$w^{m'}(a) = \underbrace{\frac{\theta^2 \lambda}{2}(a - c_T)\hat{q}^{*'}(a) + (1 - \theta)^2 \lambda(u'(x^m) - c)x^{m'}(a)}_{\text{Distortion}} + \underbrace{(1 - \theta)(1 - z'(I - T^m(a)))T^{m'}(a)}_{\text{Redistribution}}.$$

If the share, θ , of informed consumers is high enough or marginal utility of income is close enough to unity, then efficiency considerations dominate redistribution, and the social optimum is to set the termination rate close to or below marginal termination cost. The unregulated termination rate is excessive, as stated in Proposition 6.

Discussion The main mechanism which softens competition under second-degree price discrimination is the binding IC constraint on the on-net/off-net contract which eliminates the negative network externality in the competing network and leads to a kinked subscription demand. This mechanism should arise in any model in which IC constraints interact with network externalities and therefore is not particular to this specific model.

The assumption that only consumers of the same type call each other simplifies the analysis, but is not restrictive. Tangerås (2013) considers an alternative setup in which all consumers call each other. Under a type independent call pattern, network externalities depend also on the properties of the non-discriminatory contract because the value of the on-net/off-net contract now is a function of i 's entire network size, $\hat{s}_i + s_i$. Second, marginal termination rates are non-zero even at symmetric market shares.²⁵ These two effects render the characterization of equilibrium retail contracts even more complicated than in Lemma 3 without adding any new qualitative insights regarding the choice of termination rates. Marginal network profit is the same as in (40), except $\lambda(a - c_T)\hat{q}^{*'}(a)/4$ in the first line of (40) is multiplied by θ instead of θ^2 to account for the larger network under type independent call pattern.

6 Conclusion

A main concern with network competition is the termination rates operators charge for connecting calls from each other. The perception is that unregulated networks can use termination rates to distort prices and competition. Therefore, termination rates usually are regulated. Yet, authorities tend to view regulation of telecommunications as temporary, at least in Europe: "As normal market conditions develop, regulation can be rolled back, and competition law, as applied to industry in general, will replace sector-specific intervention" (European Commission, 2007). A relevant question is whether markets now have developed so far that deregulation is viable.

²⁵Demand for termination in network i is given by $\lambda(s_i + \hat{s}_i)(s_j x_j + \hat{s}_j \hat{q}_j)$ under type-independent call demand. The marginal effect of an increase in \hat{T}_i is $\lambda(s_j x_j - s_i \hat{q}_j + (\hat{s}_j - \hat{s}_i)\hat{q}_j)\partial \hat{s}_i / \partial \hat{T}_i$, which is different from zero at symmetric market shares in the general case $x_j \neq \hat{q}_j$.

Most of the literature on network competition concludes that unregulated operators either are indifferent or would negotiate termination rates below marginal termination cost. As technological development has pushed marginal costs down towards zero, the potential for setting termination rates below cost now is limited. Yet, there are circumstances under which operators would instead prefer positive markups; for example, if market coverage is incomplete and total subscription demand elastic (Jullien et al., 2013), or if fixed-to-mobile termination represents a substantial source of income to mobile operators (Armstrong and Wright, 2009). Market coverage, in terms of mobile subscriptions per capita, now exceeds 100 percent in most OECD countries. Mobile telephony is gaining rapidly in importance relative to fixed telephony: In the US, for example, the mobile-only population went from 6.1 percent in 2004 to 22.7 percent in 2009 (all statistics are from OECD, 2011). As the incentive for networks to negotiate excessive termination rates is correspondingly weak in fully covered markets dominated by mobile telephony, these trends point to an increased scope for deregulation. However, there are other reasons why operators would prefer termination rates above cost.

This paper analyses the role of income effects in call demand. I show that weak income effects are enough to drive unregulated termination rates above cost if networks charge non-discriminatory call prices. This holds even under termination-based price discrimination if networks are differentiated. Under second-degree price discrimination, network operators profit from increasing termination rates above cost if the share of consumers on discriminatory contracts is high, and consumer informational rent is positive. Income effects are inherent to consumer preferences and likely to persist independently of market developments on the supply side. Deregulation could therefore lead to a jump in termination rates and corresponding welfare losses even in advanced telecommunications markets.

Appendix

Proof of Proposition 3

If either σ , λ or $|a - c_T|$ is low, then there exists a unique and symmetric equilibrium characterized in Lemma 2. Network profit is then given by $\hat{\pi}(a)$ defined in (22). For future reference, differentiate the two first-order conditions $u'(q) = z'p$ and $u'(\hat{q}) = z'\hat{p}$ to get the following comparative statics:

$$\begin{aligned}
\frac{\partial q}{\partial p} &= ((\hat{u}'' + \lambda(1 - \hat{s})\hat{p}^2 z'')z' - \hat{u}''\lambda\hat{s}pqz'')H & \frac{\partial q}{\partial t} &= -\hat{u}''pz''H \\
\frac{\partial q}{\partial \hat{p}} &= -(\hat{u}''\hat{q} + \hat{p}z')\lambda(1 - \hat{s})pz''H & \frac{\partial q}{\partial s} &= \hat{u}''\lambda p(\hat{p}\hat{q} - pq)z''H \\
\frac{\partial \hat{q}}{\partial p} &= -(u''q + pz')\lambda\hat{s}\hat{p}z''H & \frac{\partial \hat{q}}{\partial t} &= -u''\hat{p}z''H \\
\frac{\partial \hat{q}}{\partial \hat{p}} &= ((u'' + \lambda\hat{s}p^2 z'')z' - u''\lambda(1 - \hat{s})\hat{p}\hat{q}z'')H & \frac{\partial \hat{q}}{\partial s} &= u''\lambda\hat{p}(\hat{p}\hat{q} - pq)z''H,
\end{aligned} \tag{41}$$

where $u'' = u''(q)$, $\hat{u}'' = u''(\hat{q})$ and $H = (u''\hat{u}'' + \lambda(u''(1 - \hat{s})\hat{p}^2 + \hat{u}''\hat{s}p^2)z'')^{-1} > 0$.

Marginal network profit equals:

$$\begin{aligned} 2\hat{\pi}'(a) &= \frac{\lambda}{2}(a - c_T) \left(\frac{\partial \hat{q}}{\partial \hat{p}} + \frac{\partial \hat{q}}{\partial \hat{t}} \hat{t}^{*'} \right) + \hat{t}^{*'} + \frac{\lambda}{2} \hat{q} \\ &= \frac{\lambda}{2}(a - c_T) \left(\frac{\partial \hat{q}}{\partial \hat{p}} - \frac{\lambda}{2} \frac{\partial \hat{q}}{\partial \hat{t}} \hat{q}^* \right) + \left[1 + \frac{\lambda}{2}(a - c_T) \frac{\partial \hat{q}}{\partial \hat{t}} \right] (\hat{t}^{*'} + \frac{\lambda}{2} \hat{q}^*). \end{aligned}$$

The first term of $2\hat{\pi}'(a)$ is positive if $a < c_T$ and negative if $a > c_T$ because

$$\frac{\partial \hat{q}}{\partial \hat{p}} - \frac{\lambda}{2} \frac{\partial \hat{q}}{\partial \hat{t}} \hat{q} = (u'' + \frac{\lambda}{2} c^2 z'') z' H < 0,$$

where I have substituted in the relevant expressions from (41). The term inside the square brackets above is positive if either $a \leq c_T$ or if $a > c_T$ and z'' is close to zero because $\partial \hat{q} / \partial \hat{t} = -u''(a + c_O) z'' H \leq 0$. Hence, the industry optimally sets $\hat{a}^* > c_T$ if $\hat{t}^{*'} + \frac{\lambda}{2} \hat{q}^* \geq 0$ for all $a \in [-c_O, c_T)$, and $\hat{t}^{*'} + \frac{\lambda}{2} \hat{q}^* > 0$ at $a = c_T$. Conversely, $\hat{a}^* < c_T$ if z'' is close to zero, $\hat{t}^{*'} + \frac{\lambda}{2} \hat{q}^* \leq 0$ for all $a > c_T$, and $\hat{t}^{*'} + \frac{\lambda}{2} \hat{q}^* < 0$ for $a = c_T$.

I now evaluate $\hat{t}^{*'}(a) + \frac{\lambda}{2} \hat{q}^*(a)$. Define consumer expenditures

$$E(a, \hat{t}) = \hat{t} + \frac{\lambda}{2} c q(c, c_O + a, \hat{t}, 1/2) + \frac{\lambda}{2} (a + c_O) \hat{q}(c, c_O + a, \hat{t}, 1/2)$$

when both networks price calls at perceived marginal cost, charge the same subscription fee, $\hat{t}_1 = \hat{t}_2 = \hat{t}$, and divide the market equally. The indirect utility of reaching an on-net subscriber under those circumstances equals

$$\psi(a, \hat{t}) = u(q(c, c_O + a, \hat{t}, 1/2)) - z'(I - E(a, \hat{t})) c q(c, c_O + a, \hat{t}, 1/2),$$

with $\hat{\psi}(a, \hat{t})$ analogously defined. With the above definitions, the intensity of competition as measured by semi-elasticity of subscription demand becomes:

$$\zeta(a, \hat{t}) = - \frac{\partial \hat{s}_i}{\partial \hat{t}_i} \frac{1}{\hat{s}_i} \bigg|_{p_1=p_2=c, \hat{p}_1=\hat{p}_2=c_O+a, \hat{t}_1=\hat{t}_2=\hat{t}} = \frac{2\sigma z'(I - E(a, \hat{t}))}{1 - 2\sigma \lambda (\psi(a, \hat{t}) - \hat{\psi}(a, \hat{t}))}, \quad (42)$$

and I can rewrite the equilibrium subscription fee (19) as

$$\hat{t}^* = f + \frac{1}{\zeta(a, \hat{t}^*)} + \frac{\lambda}{4} (a - c_T) \frac{\partial \hat{q}(c, c_O + a, \hat{t}^*, 1/2)}{\partial \hat{s}}.$$

By implicit differentiation, the marginal effect on the subscription fee of increasing the termi-

nation rate equals

$$\widehat{t}^{*'}(a) = \frac{\frac{\lambda}{4}(\frac{\partial \widehat{q}}{\partial s} + (a - c_T) \frac{\partial^2 \widehat{q}}{\partial s \partial p}) - \frac{1}{\zeta^2} \frac{\partial \zeta}{\partial a}}{1 + \frac{1}{\zeta^2} \frac{\partial \zeta}{\partial t} - \frac{\lambda}{4} (a - c_T) \frac{\partial^2 \widehat{q}}{\partial s \partial t}}. \quad (43)$$

and therefore:

$$\widehat{t}^{*'}(a) + \frac{\lambda}{2} \widehat{q}^*(a) = \frac{\frac{\zeta \lambda}{2} \widehat{q}^* - \frac{1}{\zeta} (\frac{\partial \zeta}{\partial a} - \frac{\lambda}{2} \frac{\partial \zeta}{\partial t} \widehat{q}^*) + \frac{\zeta \lambda}{4} [\frac{\partial \widehat{q}}{\partial s} + (a - c_T) (\frac{\partial^2 \widehat{q}}{\partial s \partial p} - \frac{\lambda}{2} \frac{\partial^2 \widehat{q}}{\partial s \partial t} \widehat{q}^*)]}{\frac{1}{\zeta} \frac{\partial \zeta}{\partial t} + \zeta - \frac{\zeta \lambda}{4} (a - c_T) \frac{\partial^2 \widehat{q}}{\partial s \partial t}}, \quad (44)$$

where I have multiplied the numerator and denominator by ζ . Straightforward differentiation of $\zeta(a, \widehat{t})$ in (42) yields

$$\frac{\partial \zeta}{\partial a} = \frac{-2\sigma z'' \frac{\partial E}{\partial a}}{1 - 2\sigma \lambda (\psi - \widehat{\psi})} + \frac{4\sigma^2 \lambda z' (\frac{\partial \psi}{\partial a} - \frac{\partial \widehat{\psi}}{\partial a})}{(1 - 2\sigma \lambda (\psi - \widehat{\psi}))^2} = \zeta (\zeta \lambda \widehat{q} - [1 - \zeta \lambda (cq - (c_O + a) \widehat{q})] \frac{z''}{z'} \frac{\partial E}{\partial a}), \quad (45)$$

$$\frac{\partial \zeta}{\partial \widehat{t}} = \frac{-2\sigma z'' \frac{\partial E}{\partial \widehat{t}}}{1 - 2\sigma \lambda (\psi - \widehat{\psi})} + \frac{4\sigma^2 \lambda z' (\frac{\partial \psi}{\partial \widehat{t}} - \frac{\partial \widehat{\psi}}{\partial \widehat{t}})}{(1 - 2\sigma \lambda (\psi - \widehat{\psi}))^2} = -\zeta [1 - \zeta \lambda (cq - (c_O + a) \widehat{q})] \frac{z''}{z'} \frac{\partial E}{\partial \widehat{t}}, \quad (46)$$

where I have substituted in $\partial \psi / \partial a = cq z'' \partial E / \partial a$, $\partial \psi / \partial \widehat{t} = cq z'' \partial E / \partial \widehat{t}$, $\partial \widehat{\psi} / \partial a = (a + c_O) \widehat{q} z'' \partial E / \partial a - z' \widehat{q}$ and $\partial \widehat{\psi} / \partial \widehat{t} = (a + c_O) \widehat{q} z'' \partial E / \partial \widehat{t}$ and simplified. By utilizing these expressions for $\partial \zeta / \partial a$ and $\partial \zeta / \partial \widehat{t}$, I can write the two first terms in the numerator of (44) as:

$$\begin{aligned} \frac{\zeta \lambda}{2} \widehat{q}^* - \frac{1}{\zeta} (\frac{\partial \zeta}{\partial a} - \frac{\lambda}{2} \frac{\partial \zeta}{\partial t} \widehat{q}^*) &= [1 - \zeta \lambda (cq^* - (c_O + a) \widehat{q}^*)] (\frac{\partial E}{\partial a} - \frac{\lambda}{2} \frac{\partial E}{\partial t} \widehat{q}^*) \frac{z''}{z'} - \frac{\zeta \lambda}{2} \widehat{q}^* \\ &= \frac{\lambda}{2} [1 - \zeta \lambda (cq^* - (c_O + a) \widehat{q}^*)] (a + c_O) H u'' z'' - \frac{\zeta \lambda}{2} \widehat{q}^* \end{aligned} \quad (47)$$

where the second line of the expression arises after substituting in

$$\begin{aligned} \frac{\partial E}{\partial a} &= \frac{\lambda}{2} (c \frac{\partial q}{\partial \widehat{p}} + (c_O + a) \frac{\partial \widehat{q}}{\partial \widehat{p}} + \widehat{q}) = \frac{\lambda}{2} u'' (\widehat{u}'' \widehat{q} + (a + c_O) z') H, \\ \frac{\partial E}{\partial \widehat{t}} &= 1 + \frac{\lambda}{2} (c \frac{\partial q}{\partial \widehat{t}} + (c_O + a) \frac{\partial \widehat{q}}{\partial \widehat{t}}) = u'' \widehat{u}'' H \end{aligned} \quad (48)$$

and simplifying. Note also that the first term in the denominator of (44) can be written as

$$\frac{1}{\zeta} \frac{\partial \zeta}{\partial \widehat{t}} = -[1 - \zeta \lambda (cq^* - (a + c_O) \widehat{q}^*)] \frac{z''}{z'} u'' \widehat{u}'' H. \quad (49)$$

Assume that $z''(y) < 0$ for all $y \in [0, I]$, and evaluate (44) as $\sigma \rightarrow 0$. It is sufficient to study what happens as $\zeta \rightarrow 0$ because $\sigma \rightarrow 0$ implies $\zeta \rightarrow 0$. Take the limits of (44), using (47) and (49):

$$\lim_{\zeta \rightarrow 0} (\widehat{t}^{*'}(a) + \frac{\lambda}{2} \widehat{q}^*(a)) = -\frac{\lambda(a + c_O) z'}{2 \widehat{u}''} > 0 \text{ for all } a > -c_O.$$

Thus, for $z'' < 0$ and σ sufficiently low, there exists an $\hat{a}^* > c_T$ such that $\hat{\pi}(\hat{a}^*) > \hat{\pi}(a)$ for all $a \leq c_T$. By marginal cost pricing, $\hat{p}^*(\hat{a}^*) - p^* = \hat{a}^* - c_T > 0$.

Next, evaluate (44) as $z''(y) \rightarrow 0$ for all $y \in [0, I]$. Note that $\partial \hat{q} / \partial \hat{s} \rightarrow 0$, $\partial^2 \hat{q} / \partial \hat{s} \partial \hat{p} \rightarrow 0$ and $\partial^2 \hat{q} / \partial \hat{s} \partial \hat{t} \rightarrow 0$ as $z'' \rightarrow 0$; see (41). Also, invoking (47) and (49): $\lim_{z'' \rightarrow 0} (t^{*'}(a) + \frac{\lambda}{2} \hat{q}^*(a)) = -\frac{\lambda}{2} \hat{q}^* < 0$. Thus, for z'' sufficiently close to zero, there exists an $\hat{a}^* < c_T$ such that $\hat{\pi}(\hat{a}^*) > \hat{\pi}(a)$ for all $a \geq c_T$. ■

Proof of Proposition 4

If either σ , λ or $|a - c_T|$ is low, then there exists a unique and symmetric equilibrium characterized in Lemma 2. Consider first the case with σ low. Assume throughout that $z'(y) \geq 1$ and $z''(y) < 0$ for all $y \in [0, I]$, Rewrite $\hat{w}'(a)$ in (24) as

$$\hat{w}'(a) = \frac{\lambda}{2} (a - c_T) \left(\frac{\partial \hat{q}}{\partial \hat{p}} - \frac{\partial \hat{q}}{\partial \hat{t}} \frac{\lambda}{2} \hat{q}^* \right) + [1 - z' + \frac{\lambda}{2} (a - c_T) \frac{\partial \hat{q}}{\partial \hat{t}}] (\hat{t}^{*'} + \frac{\lambda}{2} \hat{q}^*).$$

The first term is strictly negative for all $a > c_T$; see the proof of Proposition 3. The term in square brackets is negative for all $a > c_T$ because $\partial \hat{q} / \partial \hat{t} = -u''(a + c_O) z'' H < 0$. From the proof of Proposition 3, $\hat{t}^{*'} + \frac{\lambda}{2} \hat{q}^* > 0$ for all $a > c_T$, provided σ is sufficiently low. Thus, $\hat{w}'(a) < 0$ for all $a > c_T$, and therefore $\hat{a}^{opt} \leq c_T$, provided σ is sufficiently low.

Consider next the case of weak income effects. By invoking (41) and (43), it is straightforward to verify that

$$\lim_{z'' \rightarrow 0} \hat{t}^{*'}(a) = - \frac{\lim_{z'' \rightarrow 0} \frac{\partial \zeta}{\partial a}}{\zeta^2 + \lim_{z'' \rightarrow 0} \frac{\partial \zeta}{\partial \hat{t}}}.$$

From (45) and (46), I obtain $\partial \zeta / \partial a \rightarrow \zeta^2 \lambda \hat{q}^*$ and $\partial \zeta / \partial \hat{t} \rightarrow 0$ as $z'' \rightarrow 0$. Thus, $\hat{t}^{*'}(a) \rightarrow -\lambda \hat{q}^*$ as $z'' \rightarrow 0$, and therefore $\hat{w}'(a)$ satisfies

$$\lim_{z'' \rightarrow 0} \hat{w}'(a) = \frac{\lambda}{2} (a - c_T) \frac{\partial \hat{q}}{\partial \hat{p}} + (z' - 1) \frac{\lambda}{2} \hat{q}^*,$$

which is positive for all $a \in [-c_O, c_T)$. If z'' is sufficiently close to zero, then $\hat{w}'(a) > 0$ for all $a \in [-c_O, c_T)$ and by implication $\hat{a}^{opt} \geq c_T$. ■

References

Aldebert, Marc, Marc Ivaldi and Chantal Roucolle (2004): Telecommunications Demand and Pricing Structure: An Econometric Analysis, *Telecommunication Systems* 25, 89-115.

Armstrong, Mark (1998): Network Interconnection in Telecommunications, *Economic Journal*

108, 545-564.

Armstrong, Mark(2002): The Theory of Access Pricing and Interconnection, in Martin E. Cave, Sumit K. Majumdar and Ingo Vogelsang (eds.) *Handbook of Telecommunications Economics Vol. 1*, Amsterdam: Elsevier, 295-384.

Armstrong, Mark and Julian Wright (2009): Mobile Call Termination, *Economic Journal* 119, F270-F307.

Berger, Ulrich (2005): Bill-and-Keep vs. Cost-Based Access Pricing Revisited, *Economics Letters* 86, 107-112.

Calzada, Joan and Tommaso M. Valletti (2008): Network Competition and Entry Deterrence, *Economic Journal* 118, 1223-1244.

Carter, Michael and Julian Wright (2003): Asymmetric Network Interconnection, *Review of Industrial Organization* 22, 27-46.

Danaher, Peter J. (2002): Optimal Pricing of New Subscription Services: Analysis of a Market Experiment, *Marketing Science* 21, 119-138.

Dessein, Wouter (2003): Network Competition in Nonlinear Pricing, *RAND Journal of Economics* 34, 593-611.

Dessein, Wouter (2004): Network Competition with Heterogeneous Customers and Calling Patterns, *Information Economics and Policy* 16, 323-345.

European Commission (2007): Electronic Communications: New Regulatory Framework Principles, Fact Sheet 13, http://ec.europa.eu/information_society/newsroom/cf/itemlongdetail.cfm?item_id=1398, March 12, 2013.

Gabrielsen, Tommy S. and Steinar Vagstad (2008): Why is On-net Traffic Cheaper than Off-net Traffic? Access Markup as a Collusive Device, *European Economic Review* 52, 99-115.

Gans, Joshua S. and Stephen P. King (2001): Using 'Bill and Keep' Interconnect Arrangements to Soften Network Competition, *Economics Letters* 71, 413-420.

Genakos, Christos and Tommaso Valletti (2011): Testing the "Waterbed" Effect in Mobile Telephony, *Journal of the European Economic Association* 9, 1114-11142.

- Grajek, Michał and Tobias Kretschmer (2009): Usage and Diffusion of Cellular Telephony, 1998–2004, *International Journal of Industrial Organization* 27, 238-249.
- Hahn, Jong-Hee (2004): Network Competition and Interconnection with Heterogenous Subscribers, *International Journal of Industrial Organization* 22, 611-631.
- Hoernig, Steffen, Roman Inderst and Tommaso Valletti (2011): Calling Clubs: Network Competition with Non-Uniform Calling Patterns, CEIS Tor Vergata Research Paper Series #206.
- Hoernig, Steffen (2012): Why Mobile Networks Prefer High Termination Rates: An Exploration of Strategic Delegation, manuscript, Nova School of Business and Economics.
- Hurkens, Sjaak and Ángel L. López (2010): Mobile Termination and Consumer Expectations under the Receiver-Pays Regime, NET Institute WP #10-12.
- Hurkens, Sjaak and Ángel L. López (2013): Mobile Termination, Network Externalities, and Consumer Expectations, manuscript, University of Navarra.
- Jeon, Doh-Shin, Jean-Jacques Laffont and Jean Tirole (2004): On the "Receiver-Pays" Principle, *RAND Journal of Economics* 35, 85-110.
- Jullien, Bruno, Patrick Rey and Wilfried Sand-Zantman (2013): Termination Fees Revisited, manuscript, Toulouse School of Economics.
- Karacuka, Mehmet, Justus Haucap and Ulrich Heimeshoff (2011): Competition in Turkish Mobile Telecommunications Markets: Price Elasticities and Network Substitution, *Telecommunications Policy* 35, 202-210.
- Laffont, Jean-Jacques, Patrick Rey and Jean Tirole (1998a): Network Competition: Overview and Nondiscriminatory Pricing, *RAND Journal of Economics* 29, 1-37.
- Laffont, Jean-Jacques, Patrick Rey and Jean Tirole (1998b): Network Competition: Price Discrimination, *RAND Journal of Economics* 29, 38-56.
- López, Ángel L. and Patrick Rey (2012): Foreclosing Competition through Access Charges and Price Discrimination, manuscript, University of Navarra.
- OECD (2011): OECD Communications Outlook 2011, OECD Publishing.

PTS (2012): Svensk Telemarknad 2011, PTS-ER-2012:17.

Tangerås, Thomas (2013): Network Competition with Income Effects: Mathematical Appendix, available at www.ifn.se/thomast.

Valletti, Tommaso M. and Carlo Cambini (2005): Investments and Network Competition, *RAND Journal of Economics* 36, 446-467.

Vogelsang, Ingo (2003): Price Regulation of Access to Telecommunications Networks, *Journal of Economic Literature* 41, 830-862.