Björnerstedt, Jonas; Stennek, Johan

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Jonas Björnerstedt and Johan Stennek

Research Institute of Industrial Economics
P.O. Box 55665
SE-102 15 Stockholm, Sweden
info@ifn.se
www.ifn.se

Immediate Agreement in Interdependent Bilateral Bargaining*

Jonas Björnerstedt
Swedish Competition Authority
Jonas.Bjornerstedt@kkv.se

Johan Stennek
RIIE and CEPR
Johan.Stennek@riie.se

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Abstract

This note provides sufficient conditions for immediate agreement in an extensive form model of interdependent bilateral bargaining. The model is suggested by Björnerstedt and Stennek (2006) as a work horse for studying bilateral oligopoly. The key feature of this model is that the firms are represented by separate agents in all negotiations in which they are involved. There is immediate agreement in equilibrium, essentially if production is strictly convex or if the agents use Markov strategies.

Key Words: bilateral oligopoly, intermediate goods, bargaining, market network, trade links

JEL classification: C70 L10 L40 D20 D40

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1 Introduction

In Björnerstedt & Stennek (2006), we propose a model of bilateral oligopoly based on interdependent bilateral bargaining. We provide conditions guaranteeing existence of equilibria with immediate agreement, and also discuss the properties of such equilibria in terms of market efficiency, how the surplus is divided between buyers and sellers, and the kinds of buyer-seller networks that will be formed. All these properties of the outcome are closely linked to the negotiations being successfully terminated immediately. In the current note, we therefore investigate under what conditions all equilibria specify immediate agreement.

Delay equilibria can arise for example when the intermediate goods are complements in producing the final goods. Downstream firms will not want to bring forward an agreement in a single negotiation, as that will increase the price it will have to pay for other inputs. If the firm expects delay in one negotiation, it may not want to conclude an agreement in another negotiation, and vice versa.

Any immediate agreement has to be on bilaterally efficient quantities. All pairs of up- and downstream firms has to agree on the quantity that maximizes the sum of the two firms' profits, taking all other quantities as given. Reversely, any bilaterally efficient quantity vector, is the outcome of an equilibrium with immediate agreement. And since there may exist many bilaterally efficient quantity vectors, there may also exist multiple equilibria with immediate agreements. This multiplicity of immediate agreement equilibria can give rise to additional "non Markov" equilibria with delay, which are not eliminated by a standard Markov assumption. To illustrate the insufficiency of the standard Markov assumption, consider a simple meeting game where four players are to meet either at Times square or Grand Central Sta-

tion at noon, or with delay at quarter past. A player's payoff is the number of people he meets, discounted by δ if he waits. All players can observe all actions at noon. An equilibrium we want to rule out is that where all players meet at quarter past at (say) Times square. Such a delay can be enforced if, following a deviation with one player at Times square already at noon, all the others' expectations change to make Grand Central Station their best choice at quarter past.

When there is no strategic interaction in the final goods market, immediate agreement can be ensured by imposing a Markov restriction which is slightly stronger than the standard one. The formal definition can be found in the proof of Proposition 1, and consists of two parts. The first requires the continuation strategies to be the same for payoff-equivalent histories, as in Maskin and Tirole (2001), the second strengthens the Markov assumption, by requiring measurability over an even coarser partitioning of histories. Given a certain history, all representatives know that (u, d) have agreed on a contract structure c_{ud} at some time $s < t$ and hence, will implement c_{ud} in period t and in all future periods. Given a certain other history, all representatives believe with probability one that (u, d) will agree on c_{ud} at t, and hence, will implement c_{ud} in period t and all future periods. The two histories are payoff equivalent for all players, except the representatives in negotiation (u, d).[1] Consequently, we require that the players do not make a distinction at t on whether they know or believe that c_{ud} will be implemented from t onwards. The only reason why one representative might want to make such a distinction is if some other representatives do. By excluding such coordinated switching, early agreements cannot be punished.[2]

[1] All representatives in the ongoing negotiations have the same strategy sets and their continuation payoffs are identical in the two subgames.

[2] This restriction reduces the information on which firms condition their behavior or

Another case when there can be no delay is when *bilateral efficiency is unique*, i.e. when there exists a unique bilaterally efficient quantity vector regardless of what contracts have been agreed upon. Bilateral efficiency is unique for example when goods are physically homogenous but transportation costs differ between different buyer-seller pairs, or when downstream firms consider all intermediate goods differentiated (variable revenues are strictly concave). With unique bilateral efficiency, it is not possible to switch between different equilibria to punish early agreements. Intuitively, for equilibria with delay to exist, firms that deviate and make an early agreement must be punished. To construct an equilibrium with punishments and rewards, the punishers must deviate from their unique bilaterally efficient quantity. As it is not in their interest to do so, deviations from the punishment strategy must be punished. Every punishment or reward reduces the number of open negotiations. As the last pair of firms would not have an incentive to deviate from bilateral efficiency, the sequence of punishments is not credible.

With competition in the final goods market and an infinite horizon, immediate agreement on bilaterally efficient quantities cannot be guaranteed. Delay may then, for example, be used as a form of collusion in the final goods market. Some negotiations are kept open as a threat of flooding the market, following deviations from the collusive agreement.[3] In case the horizon is finite, however, there is immediate agreement. Delay equilibria unravel, as agreement on bilaterally efficient quantities will always be optimal in the last period.

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beliefs, without violating their rationality; that is, it is optimal for a firm to use the prescribed strategy, if all other firms do so. We have defined the Markov condition in terms of strategy profiles. It could instead be defined in terms of individual belief profiles.

[3]The result is somewhat akin to the literature on repeated games. The particular collusion mechanism here is due to the fact that firms cannot renegotiate contracts to punish defectors. The details of the example are discussed below.

Proposition 1 *Assume that (i) goods are substitutes, and that (ii) either firms are price takers in the final goods market or time is finite, and that (iii) the equilibrium is Markov or bilateral efficiency is unique. Then, the equilibrium quantity vector q is bilaterally efficient and there is immediate agreement in all negotiations where $q_{ud} > 0$.*

If there is strategic interaction in the final goods market, the Markov or uniqueness assumptions do not ensure immediate agreement. Consider a homogeneous goods market with two symmetric upstream firms u' and u'' with constant marginal cost selling to two identical firms d' and d'' competing a la Cournot in the final goods market. Let Q_M and Q_C denote the monopoly and symmetric Cournot quantities. If d' and d'' buy $Q_M/2$ from u' and u'' respectively, there will be an equilibrium with no agreement in the two remaining negotiations (u', d'') and (u'', d'). Although the unique bilaterally efficient q has $q_{u''d'} = q_{u'd''} = Q_C - Q_M/2$, non-agreement can be sustained. If either negotiation is concluded, it is optimal for the other pair to come to agreement as well. As this reduces the final goods price, the deviation would be punished. Similarly, the open negotiations sustain agreement on $Q_M/2$. Given a deviation, equilibrium prescribes immediate agreement in these negotiations.

2 The Model

2.1 Preliminaries

In an intermediate goods market, there are $U < \infty$ upstream and $D < \infty$ downstream firms. We assume that there are no vertically integrated firms. Let Ω be the set of all UD pairs (u, d) where u is an upstream firm and d is a

downstream firm. Time is discrete, indexed by t from zero to infinity. Goods are delivered from upstream to downstream firms based on dated contracts. A *contract* at t is a pair $c_{ud}(t) = (q_{ud}(t), p_{ud}(t))$ specifying a quantity $q_{ud}(t)$ and a price $p_{ud}(t)$. A *contract structure* $c(t)$ is a UD-tuple of contracts $c_{ud}(t)$, one for all $(u,d) \in \Omega$. Likewise q and p are the vectors of all q_{ud} and p_{ud}. We write (c_{-ud}, c'_{ud}) to indicate the contract structure given by c for all $(i,j) \neq (u,d)$ and c'_{ud} for (u,d). The corresponding convention is used for vector q.

The short-run cost function for an upstream firm u is denoted by $C^u(q)$, and the short-run revenue function for a downstream firm d is denoted $R_d(q)$. The marginal costs of production are positive, that is $\partial C^u/\partial q_{ud} > 0$ and $\partial R_d/\partial q_{ud} > 0$. A firm's cost is only affected by its own production, that is $\partial C^u/\partial q_{ij} = 0$ if $i \neq u$ Similarly, we say that there is *no interaction in the final goods market* when $\partial R_d/\partial q_{ij} = 0$ if $j \neq d$. We will say that production is (strictly) convex if $C^u(q)$ is (strictly) convex in $\{q_{uj}\}_{j=1}^{D}$ and $R_d(q)$ is (strictly) concave in $\{q_{id}\}_{i=1}^{U}$ for all u and d. To ensure finite production we assume convexity and that $\partial^2 C^u/\partial (q_{ud})^2 > 0$ and $\partial^2 R_d/\partial (q_{ud})^2 < 0$. It is assumed that goods are delivered free on board, and that the downstream firms' cost functions include all transportation costs.

The per-period profit of an upstream firm u and a downstream firm d is a function of the contract structure $c(t)$ and given by

$$\pi^u(c(t)) = \sum_{j=1}^{D} p_{uj}(t)\, q_{uj}(t) - C^u(q(t)), \tag{1a}$$

$$\pi_d(c(t)) = R_d(q(t)) - \sum_{i=1}^{U} p_{id}(t)\, q_{id}(t). \tag{1b}$$

Total profits are a function of the sequence of contract structures $\{c(t)\}_{t=0}^{\infty}$

given by $\sum_{t=0}^{\infty}\delta^{t}\pi^{u}\left(c\left(t\right)\right)$ for upstream firm u and $\sum_{t=0}^{\infty}\delta^{t}\pi_{d}\left(c\left(t\right)\right)$ for downstream firm d, where δ is the common discount factor.

Note that

$$\pi_{d}\left(c\right)-\pi_{d}\left(c_{ud},\left(0,0\right)\right)=R_{d}\left(q\right)-R_{d}\left(q_{-ud},0\right)-p_{ud}q_{ud} \tag{2}$$

is the additional per-period profit that buyer d can obtain from an agreement with seller u, taking the all other agreements as given. Similarly,

$$\pi^{u}\left(c\right)-\pi^{u}\left(c_{ud},\left(0,0\right)\right)=p_{ud}q_{ud}-C^{u}\left(q\right)+C^{u}\left(q_{-ud},0\right) \tag{3}$$

is the additional per-period profit that buyer u can obtain from an agreement with buyer d, taking the all other agreements as given.

Now, we define three concepts that are central to the subsequent analysis. First, the bilateral surplus of a seller-buyer pair (u,d) is defined as the additional aggregate profit of the two firms, generated by their contract, taking all other contracts as given, that is

$$\begin{aligned}&\left[\pi^{u}\left(c\right)+\pi_{d}\left(c\right)\right]-\left[\pi^{u}\left(c_{-ud},\left(0,0\right)\right)+\pi_{d}\left(c_{-ud},\left(0,0\right)\right)\right] &(4)\\ =\;& R_{d}\left(q\right)-C^{u}\left(q\right)-R_{d}\left(q_{-ud},0\right)+C^{u}\left(q_{-ud},0\right) &(5)\end{aligned}$$

Definition 1 *The quantity q_{ud} is bilaterally efficient if it maximizes the bilateral surplus. Consider a set of seller-buyer pairs $\Omega'\subseteq\Omega$, and a fixed contract structure c. Let $N\left(c,\Omega'\right)\subset\mathbb{R}_{+}^{U\times D}$ be the set of bilaterally efficient quantity vectors, where q_{ud} is bilaterally efficient for all $(u,d)\in\Omega'$, and q_{ud} is given by c for $(u,d)\in\Omega\backslash\Omega'$.*

A bilaterally efficient quantity vector is a quantity, one for each pair, such that no pair can increase their aggregate profit, if all other pairs agree upon their

bilaterally efficient quantity. Lemma 1 proves the existence of bilaterally efficient quantity vectors. If production is strictly convex, the bilaterally efficient quantity vector is unique.

The third central concept is equal split of the bilateral surplus.

Definition 2 *A price p_{ud} yields an equal split of the bilateral surplus if*

$$\pi^u(c) - \pi^u(c_{-ud}, (0,0)) = \pi_d(c) - \pi_d(c_{-ud}, (0,0)). \tag{6}$$

Disregarding the distribution of profits between firms, we define social welfare W as the sum of profits of all upstream and downstream firms:

$$W(\{c(t)\}_{t=0}^{\infty}) = \sum_{i=1}^{U}\sum_{t=0}^{\infty}\delta^t\pi^i(c(t)) + \sum_{j=1}^{D}\sum_{t=0}^{\infty}\delta^t\pi_j(c(t)). \tag{7}$$

Note that prices affect the distribution of wealth but not social welfare and are consequently not included as an argument in the welfare function.

Definition 3 *We say that a* bilaterally efficiency is unique *if $N(c,\Omega)$ is a singleton for all c and Ω.*

Lemma 1 *For any $\Omega' \subseteq \Omega$, and c, if production is convex, the set of bilaterally efficient quantities $N(c,\Omega')$ is a non-empty compact, convex set. If production is strictly convex and there is no interaction in the final goods market, there is a bilaterally efficient q in all subgames.*

Proof. The choice set is convex since it is defined by non-negativity and equality (for $(u,d) \in \Omega\backslash\Omega'$) constraints. The choice set is bounded in the case of strictly convex production. By the maximum theorem we see that as the welfare function W is concave there exists a non-empty compact convex

set of quantities $N(c, \Omega') \subset \mathbb{R}_+^{U \times D}$ maximizing W. Moreover, if the welfare function is strictly concave $N(c, \Omega')$ is a singleton.

The welfare function is maximized when the sum (over firms) of per-period profits is maximized. Moreover, $\partial W / \partial q_{ud} = \partial \pi_d / \partial q_{ud} + \partial \pi^u / \partial q_{ud}$ as $\partial \pi^u / \partial q_{ij} = 0$ for $i \neq u$ and $\partial \pi_d / \partial q_{ij} = 0$ for $j \neq d$. If production is strictly convex, there is a unique bilaterally efficient q coinciding with the socially optimal quantities. The selection is consistent, as fixing a q_{ud} at an optimal level does not affect the optimality of other quantities. ■

2.2 The Extensive Form

Each firm is represented by a separate agent in every negotiation in which the firm is involved. All agents, or representatives, maximize their respective firm's profit.

At every date t there is a stage-game. For all stages t, and for all negotiations $(u, d) \in \Omega$, $\rho_{ud}(t) \in \{u, d\}$ indicates which of the two firms that is allowed to make a bid to the other at t. In particular, we assume that offers are alternating. (Let $\rho(t)$ be the UD-tuple that specifies the *order of moves* for all negotiations at time t.) A bid $b_{ud}(t)$ is a pair $(q_{ud}(t), p_{ud}(t)) \in \mathbb{R}_+^2$ where $q_{ud}(t)$ is a quantity and $p_{ud}(t)$ is the price. The other firm is allowed to respond $r_{ud}(t) \in \{y, n\}$ where n indicates reject and y indicates accept. While negotiating, there is assumed to be an implicit contract specifying $q_{ud} = 0$. Once a bid is accepted, the negotiation is ended. Contracts are binding, and there is no renegotiation. Hence, if $r_{ud}(T) = y$, then $c_{ud}(t) = b_{ud}(T)$ for all $t \geq T$. Production occurs in every stage, immediately after the round of negotiation, according to the (possibly implicit) contract $c_{ud}(t)$.

The *link structure* is defined as the set of buyer-seller pairs that negotiate. It is denoted by $\Omega^L \subseteq \Omega$. We say that the link structure is complete if $\Omega^L = \Omega$,

and incomplete otherwise. In the case the link structure is incomplete, we simply impose the restriction that for all t, and for all pairs $(u,d) \in \Omega \backslash \Omega^L$, $b_{ud}(t) = (0,0)$ and $r_{ud}(t) = y$.

The *action* in bargain $(u,d) \in \Omega$ at time t, denoted $a_{ud}(t)$, is the ordered triple $(\rho_{ud}(t), b_{ud}(t), r_{ud}(t))$. The action at time t, is the UD-tuple $(a_{11}(t), ..., a_{ud}(t))$. A *history* at time t, denoted h_t, is a t-tuple of actions $(a_0, ..., a_{t-1})$, with h_0 denoting the "empty" history at $t = 0$. Let H_t be the set of possible h_t. Let $c_{ud}(h_t) = b_{ud}(T)$ if $r_{ud}(T) = y$ for some $T < t$. Let $\Gamma(h_T)$ denote the *subgame* that is induced by the history h_T at time T.

At time t, both the bidder and the respondent know h_t. The respondent also knows the bid to which he must respond. The respondent does not know, however, other bids in the same stage game, not even those given to or by other representatives of his own firm.

For the representative of upstream firm u in negotiation (u,d), the *strategy* $\overline{\alpha}_{ud}$ is a function that for each history h_t specifies a bid $\overline{b}_{ud}$ if $\rho_{ud}(t) = u$, or a response $\overline{r}_{ud}$ conditional on the downstream firm's bid if $\rho_{ud}(t) = d$:

$$\overline{b}_{ud}(h_t) : H_t \to \mathbb{R}^2_+, \text{ and}$$
$$\overline{r}_{ud}(h_t, b) : H_t \times \mathbb{R}^2_+ \to \{y, n\}.$$

For the representative of the downstream firm, $\underline{\alpha}_{ud}$, $\underline{b}_{ud}$ and $\underline{r}_{ud}$ are defined similarly. A strategy profile α specifies a strategy for all representatives of all firms. We restrict attention to pure strategies.

Consider a strategy profile α. Let $h_t(h_T, \alpha)$ with $t \geq 0$ be the history such that (i) for $t \leq T$, it is on the path to h_T, and (ii) for $t > T$, it is induced by α contingent on h_T having been reached. Let the *continuation payoffs* of

strategy profile α at time T with history h_{T+1} be defined as

$$\Pi^u\left(h_{T+1},\alpha\right)=\sum_{t=T}^{\infty}\delta^{t-T}\pi^u\left(c\left(h_t\left(h_T,\alpha\right)\right)\right), \tag{8}$$

and

$$\Pi_d\left(h_{T+1},\alpha\right)=\sum_{t=T}^{\infty}\delta^{t-T}\pi_d\left(c\left(h_t\left(h_T,\alpha\right)\right)\right). \tag{9}$$

Note that h_{T+1} includes the actions at time T.

Notice that this is a game of imperfect information. In a stage game, the representatives are not informed about events taking place simultaneously in other negotiations. The concept of subgame perfect equilibrium is sufficient to ensure the optimality of the bidders' actions in all negotiations, but it does not ensure the optimality of the responses to out of equilibrium bids. Hence, we need to employ the concept of sequential equilibrium (cf. Rubinstein & Wolinsky, 1990). The potential problem with subgame perfect equilibria is that, off the equilibrium path, subgame perfection does not restrict the beliefs of the respondent about other bids. However, in a sequential equilibrium the beliefs have to satisfy a consistency requirement which implies that, after unexpected offers, the beliefs agree with the equilibrium strategies of the other representatives.[4]

[4]To see this, consider the negotiation between u and d. Let $\Pr\left\{b_{ij}\mid b_{ud}\right\}$ represent the respondents belief about the bid given in negotiation (i,j) after having recieved the bid b_{ud}. First, note that if the respondent receives the equilibrium bid (denoted β_{ud}), he must believe that all other simultaneous bids are equilibrium bids. Hence

$$\Pr\left\{b_{ij}=k\mid\beta_{ud}\right\}=\begin{cases}1 & \text{if } k=\beta_{ij}\\ 0 & \text{if } k\neq\beta_{ij}\end{cases}.$$

This follows from the definition of Nash equilibrium. Second, note that all bids given by different representatives are statistically independent (even if the representatives belong to the same firm). Hence, in order for the beliefs at different information sets to satisfy

3 Equilibrium

We begin by proving most difficult part of Proposition 1, that immediate agreement follows if bilateral efficiency is unique.

Proposition 2 *For any link structure* $\Omega \subseteq \overline{\Omega}$*, if bilateral efficiency is unique, there exists a unique sequential equilibrium, implying immediate agreement on* $q \in N(c, \Omega)$

As the proof of Proposition 2, is somewhat involved, we provide an outline of the argument. In a subgame in which there is only one ongoing negotiation a simple application of standard Rubinstein-Ståhl bargaining, shows that firms agree immediately on the bilaterally efficient quantity and that (as $\delta \to 1$) they split the bilateral surplus equally.

In Lemma 2 it is assumed that there exists some date T where all have agreed upon contracts. It is shown that all contracts agreed upon at T must conform to bilateral efficiency and equal split of the bilateral surplus. The reason is the following. A unilateral deviation in a single negotiation (u, d) does not affect other negotiations. Hence, negotiation (u, d) can be analyzed as if it is the only ongoing negotiation already in period T (although this can not be strictly true until period $T + 1$). Thus, there must be immediate

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the consistency requirement, it is necessary that

$$\Pr\{b_{ij} = k \mid b_{ud}\} = \Pr\left\{b_{ij} = k \mid \widetilde{b}_{ud}\right\} \text{ for all } k,\, b_{ud},\, \widetilde{b}_{ud}.$$

Together, the Nash requirement and the consistency requirement implies

$$\Pr\{b_{ij} = k \mid b_{ud}\} = \begin{cases} 1 & \text{if } k = \beta_{ij} \\ 0 & \text{if } k \neq \beta_{ij} \end{cases} \text{ for all } b_{ud}.$$

Hence, also after unexpected offers, the beliefs agree with the equilibrium strategies of the other representatives.

agreement on the prescribed contract.[5]

The final four lemmas prove that there cannot exist delay in equilibrium. Given that players prefer agreement on zero quantities to not agreeing, Lemma 3 shows that an equilibrium cannot prescribe delay in a subgame if only negotiations without gains from trade remain.

Lemma 4 is concerned with subgames $\Gamma(h_T)$ in which the equilibrium α prescribes that some negotiations will never be concluded. Thus, α induces delay in $\Gamma(h_T)$. It is shown that there must exist some subgame $\Gamma(h_S)$ of $\Gamma(h_T)$ in which α induces delay with strictly fewer ongoing negotiations. The logic of the proof is as follows. Consider a subgame in which no further agreements should be concluded according to α. Firms have incentives to conclude their negotiations since there are gains from trade. Actually by concluding an agreement in negotiation (u, d) and by refusing agreement in all other negotiations, u and d can guarantee themselves a positive additional payoff. To uphold the equilibrium α (with zero additional payoff), any firm that rejects a profitable bid must be rewarded (thus receiving positive payoff), while not rewarding the bidder (otherwise, bidding with subsequent rejection would be profitable). To reward the respondent, it is necessary to conclude some negotiations (exploiting some gains from trade). However, all negotiations cannot be concluded at the same time, since then the agreements need to conform to Lemma 2 also giving the bidder positive additional payoff. Once some, but not all, negotiations are concluded, there is a subgame with delay and strictly fewer ongoing negotiations.

Lemma 5 is concerned with subgames $\Gamma(h_T)$ in which the equilibrium α prescribes that some negotiations will be concluded at a date $t > T$.

[5]It is in Lemma 2 where the assumption that respondents do not know the bids in other negotiations is crucial.

Thus, α induces delay in $\Gamma(h_T)$. It is shown that there must exist some subgame $\Gamma(h_S)$ of $\Gamma(h_T)$ in which α induces delay with strictly fewer ongoing negotiations. The logic of the proof is as follows. Note that firms have incentives to conclude their negotiations without delay since then the gains from trade can be exploited immediately. Consider the possibility that u proposes an agreement in (u, d) one period earlier than prescribed, that is already at $t-1$. If there is delay conditional on agreement, the lemma is proved. If not, there must be delay in a subset of negotiations in the subgame after d has rejected. If all ongoing negotiations are concluded at the same time, they will conform to Lemma 2 both in case of acceptance and in case of rejection. Both u and d will gain by this deviation, contradicting the assumption that α was an equilibrium.

To prove the proposition, we show that if a strategy profile α induces delay in a subgame, then it is not an equilibrium. The logic of the proof is as follows. Assume that α is an equilibrium that induces delay in a subgame. Then, the conditions of Lemma 4 or 5 hold. In both cases, the lemmas imply that there exist subgames with delay with fewer ongoing negotiations. Hence, the conditions of Lemma 4 or 5 hold also for that subgame. Repeated application of the lemmas generates an infinite sequence of subgames with delay, in which a smaller and smaller but non-empty set of negotiations remain ongoing. Since the number of initial negotiations is finite, we obtain a contradiction.

Thus, there is immediate agreement. Moreover, by Lemma 2 all contracts conform to bilateral efficiency and equal split of the bilateral surplus (as $\delta \to 1$).

After showing that there is immediate agreement when bilateral efficiency is unique, Proposition 1 is shown. In order to do so, the Markov assumption

is first formalized.

4 Proofs

Given that all negotiations except one have reached agreement, we will now show that there exists a unique bargaining solution with 1) immediate agreement, 2) bilateral efficiency, 3) equal split of bilateral surplus.

Consider a subset of negotiations $\Omega' \subseteq \Omega$, and a fixed contract structure c, with the associated vector of quantities q. The contract structure $\widehat{c}(c, \Omega', t)$ is defined as follows. For $(u,d) \notin \Omega'$, $\widehat{c}_{ud}(c, \Omega', t) = c_{ud}$ (where c_{ud} is the relevant entry in c). Let $\widehat{q}(c, \Omega') \in N(c, \Omega')$ as defined in Lemma 1 and $\widehat{p}_{ud}(c, \Omega')$ be the set of unique Rubinstein-Ståhl prices, taking all quantities $\widehat{q}(c, \Omega')$ as given.

To make notation less cumbersome, we introduce the following notation. Let $\Omega(h_T) \subseteq \Omega$ denote the set of ongoing negotiations (u,d) at the beginning of subgame $\Gamma(h_T)$, that is $\Omega(h_T) = \{(u,d) \in \Omega : r_{ud}(T-1) = n\}$.

Consider a history h_t. Let $\widehat{c}_{ud}(h_t)$ be the set of bilaterally efficient quantities and prices implying equal split of the bilateral surplus, conditional on the contracts agreed upon according to h_t:

$$\widehat{c}_{ud}(h_t) \begin{cases} \widehat{c}_{ud}(c(h_t), \Omega(h_t), t) & (u,d) \in \Omega(h_t) \\ c_{ud}(h_t) & (u,d) \notin \Omega(h_t) \end{cases}.$$

The difference between $c(h_t)$ and $\widehat{c}(h_t)$ is that in the former quantity, those that have not made an agreement are supposed to have the implicit contract, while in the latter contract structure they have their simultaneous FOC contracts.

Consider a history h_T and the subgame $\Gamma(h_T)$ induced by h_T.

$\Omega_t(h_T,\alpha) = \{(u,d) \in \Omega(h_t(h_T,\alpha)) : (u,d) \notin \Omega(h_{t+1}(h_T,\alpha))\}$. This is the set of negotiations that (according to α, and conditional on h_T being reached) come to an agreement at time $t \geq 0$.

$\Omega_\infty(h_T,\alpha) = \{(u,d) : \forall t,\ (u,d) \in \Omega(h_t(h_T,\alpha))\}$. This is the set of negotiations that (according to α, and conditional on h_T being reached) will not reach an agreement in finite time.

$\Omega_\infty^+(h_T,\alpha) = \{(u,d) \in \Omega_\infty(h_T,\alpha) : \forall t,\ \widehat{q}_{ud}(h_t(h_T,\alpha)) \neq 0\}$. This is the set of negotiations that (according to α, and conditional on h_T being reached) will not reach an agreement in finite time, even though there exists gains from trade in every period, i.e. $\widehat{q}_{ud}(h_t(h_T,\alpha)) \neq 0$ for all t.

$\Omega_\infty^0(h_T,\alpha) = \Omega_\infty(h_T,\alpha) \setminus \Omega_\infty^+(h_T,\alpha)$. This is the set of negotiations that (according to α, and conditional on h_T being reached) will not reach an agreement in finite time, and there are eventually no gains from trade, i.e. $\widehat{q}_{ud}(h_t) = 0$ for some $h_t(h_T,\alpha)$ (note that $\widehat{q}_{ud}(h_t(h_T,\alpha))$ is monotonically decreasing).

For $t > T$, the subgame $\Gamma(h_t(h_T,\alpha))$ is the subgame of $\Gamma(h_T)$ at t induced by α, contingent on h_T being reached.

Lemma 2 *Assume that with the equilibrium strategy profile α at h_T, no negotiations remain ongoing after T, i.e. $\Omega_T(h_T,\alpha) = \Omega(h_T)$. Then for $(u,d) \in \Omega(h_T)$, $b_{ud}(T) = \widehat{c}_{ud}(h_T)$.*

Proof. Consider $(u,d) \in \Omega(h_T)$. Deviations from prescribed equilibrium at T by u or d will not affect h_T by the informational assumptions. Hence, for $(i,j) \neq (u,d)$ and $t \geq T$, the contracts $c_{ij}(h_t) = c_{ij}(h_T)$ are not affected

by the actions $b_{ud}(t)$ and $r_{ud}(t)$. The existence of a unique subgame perfect equilibrium follows from **?**. There is a unique q such that for each $(u,d) \in \Omega(h_T)$, q_{ud} is bilaterally efficient. As the price p_{ud} is a function only of the quantities agreed upon, the set of prices for all $(u,d) \in \Omega'$ is uniquely determined. ■

Assumption 1 *Firms u and d strictly prefer agreeing upon a contract specifying $q_{ud}(T) = 0$ over agreeing upon 0 at $T+1$ or not agreeing.*

This is a weak assumption that there exists bargaining costs. However, we assume those costs to be small, in fact of a lexicographic lower order than other costs or revenues. Using this assumption, the next lemma shows that if *none* of the ongoing negotiations have gains from trade, then there cannot be delay.

In the next three lemmas, we will show that an equilibrium α can have delay in a subgame $\Gamma(h_T)$ only if it prescribes delay in a subgame $\Gamma(h_S)$ with fewer firms bargaining. Note that h_S does not have to be on the equilibrium path of α given h_T.

Definition 4 *Strategy profile α **induces delay** in $\Gamma(h_T)$ if $\Omega(h_{T+1}(h_T,\alpha)) \neq \emptyset$.*

Lemma 3 *If $\{(u,d) \in \Omega(h_t(h_T,\alpha)) : \widehat{q}_{ud}(c(h_T),\Omega(h_T)) = 0\} = \Omega(h_t(h_T,\alpha)) \neq \emptyset$ for strategy profile α in subgame $\Gamma(h_T)$ for $t > T$, then α induces delay in h_T.*[6]

Proof. Consider a deviation at time T prescribing agreement on $q_{ud}(T) = 0$. Agreeing assures u and d at least the gain of agreeing in their bargain. In

[6] *Note that $\{(u,d) \in \Omega(h_t(h_T,\alpha)) : \widehat{q}_{ud}(c(h_T),\Omega(h_T)) = 0\}$ is the set of bargains that have no gains from trade.*

order for the deviation not to be accepted, a strict subset of negotiations have to be agreed upon at $T' > T$. If none ever agree, d will gain by accepting. If all agree at T', u will gain by making the deviating offer. the respondent is rewarded only by delayed agreement, this agreement can not entail agreement for (u, d). Otherwise u would gain by giving an offer that is rejected over equilibrium play. If not all agree at T', use the same reasoning for the strictly smaller subset of players in this subgame. Thus in order for deviation not to be profitable $q_{ij}(T') > 0$ for some $(i, j) \in \Omega(h_T)$. As both i and j would prefer $q_{ij} = 0$, at least one of the firms obtains a lower payoff by playing according to equilibrium α off the equilibrium path. In order for α to be a sequential equilibrium, the loss for this firm has to be compensated by a gain in some other bargain (i, k). As k then has a loss that has to be compensated, iteratively applying the argument implies that α is not an equilibrium. ■

The profile induces delay if not all negotiations are concluded immediately. Note that α induces delay in $\Gamma(h_T)$ if, and only if, $\Omega_\infty(h_T, \alpha) \neq \emptyset$ or $\exists t > T : \Omega_t(h_T, \alpha) \neq \emptyset$. An equilibrium strategy profile α that induces delay in $\Gamma(h_T)$ will satisfy the conditions of one of the two following Lemmas.

Lemma 4 *Consider an equilibrium α and a subgame $\Gamma(h_T)$. If $\Omega_\infty(h_T, \alpha) \neq \emptyset$, then there exists a subgame $\Gamma(h_S)$ of $\Gamma(h_T)$ (with $S > T$) such that α induces delay in $\Gamma(h_S)$ and $\emptyset \neq \Omega(h_S) \subset \Omega_T(h_T)$.*

Proof.

Case 1: If any agreements are made in equilibrium along the equilibrium path (i.e. $\Omega_t(h_T, \alpha) \neq \emptyset$ for some $t \geq T$), the result follows immediately by considering the subgame $\Gamma(h_S) = \Gamma(h_t(h_T, \alpha))$. Thus $\Omega(h_T) = \Omega_\infty(h_T, \alpha)$.

Case 2: Consider a bid for $(u, d) \in \Omega^+_\infty(h_T, \alpha)$ specifying the contract

$\widehat{c}_{ud}(h_T)$ with $\widehat{q}_{ud}(h_T) > 0$ in period T. Note that one can chose some negotiation with $\widehat{q}_{ud}(h_T) > 0$ since we have $\Omega^+_\infty(h_T, \alpha) \neq \emptyset$, as Lemma 3 would otherwise imply that α is not a SPE. Without loss of generality, assume that u makes the bid. Let h^+_{T+1} denote the history where d accepts the bid and h^-_{T+1} where d rejects.

Note that since h_T is the same for both h^+_{T+1} and $h_{T+1}(h_T, \alpha)$, $c_{ij}(h_T)$ is the same for all $(i,j) \neq (u,d)$. (This formalizes that others are unaffected.)

Conditional on acceptance, two outcomes are logically possible. (Note that at least one negotiation is ongoing, since $\Omega(h_T)$ cannot be a singleton, which in turn follows from the fact that otherwise the equilibrium strategy profile α could not prescribe delay in $\Gamma(h_T)$.) If there is delay in the ongoing negotiations the lemma is proved. Hence, in the rest of the proof we assume the opposite. If there is no delay in the ongoing negotiations, so that all agree immediately, contracts are given by Lemma 2. In this case, both u and d strictly gain from u's deviation.

Conditional on rejection, three outcomes are logically possible. We show that the first two possibilities are not consistent with equilibrium. Thus, the third possibility is the crucial one.

1. Assume that α prescribes that no players ever agree, that is $\Omega^+_\infty\left(h^-_{T+1}, \alpha\right) = \Omega(h_T)$. As this gives the equilibrium continuation payoffs $\Pi_d(h_T, \alpha)$, acceptance is strictly better for d (and thus for u). Hence, the deviation is profitable. Thus, for equilibrium it is required that $\Omega^+_\infty\left(h^-_{T+1}, \alpha\right) \neq \Omega(h_T)$, which is assumed in the rest of the proof.

2. Assume that α prescribes that all players agree at $t > T$, that is $\Omega_t\left(h^-_{T+1}, \alpha\right) = \Omega(h_T)$. If all players agree, Lemma 2 implies agreements at $\widehat{c}(h_T)$. Hence, both u and d obtain strictly higher payoffs than $\Pi_d(h_T, \alpha)$. Firm u will thus strictly gain by proposing $\hat{c}_{ud}(h_T)$,

since that is profitable both in case of acceptance and in case of rejection. Thus, for equilibrium $\Omega_t\left(h_{T+1}^-, \alpha\right) \neq \Omega\left(h_T\right)$, for all $t > T$. This is assumed in the rest of the proof.

3. Thus the assumptions of the lemma imply that α prescribes that there exists a largest $t > T$ such that $\emptyset \neq \Omega_t\left(h_{T+1}^-, \alpha\right) \neq \Omega\left(h_T\right)$.

   (a) If $\Omega_\infty^+\left(h_{T+1}^-, \alpha\right) \neq \emptyset$, the subgame $\Gamma\left(h_S\right) = \Gamma\left(h_{t+1}\left(h_{T+1}^-, \alpha\right)\right)$ satisfies the claim of the lemma.

   (b) If $\Omega_\infty^+\left(h_{T+1}^-, \alpha\right) = \emptyset$, lemma 3 implies that $\Omega_\infty^0\left(h_{T+1}^-, \alpha\right) = \emptyset$. Moreover, the largest date of agreement t must be larger than $T+1$, as otherwise everyone has to agree at $T+1$. In this case the subgame $\Gamma\left(h_S\right) = \Gamma\left(h_{t-1}\left(h_{T+1}^-, \alpha\right)\right)$ satisfies the claim of the lemma. ■

Lemma 5 *Consider an equilibrium α and a subgame $\Gamma\left(h_T\right)$. If $\Omega_t\left(h_T, \alpha\right) \neq \emptyset$ for some $t > T$, then there exists a subgame $\Gamma\left(h_S\right)$ of $\Gamma\left(h_T\right)$ (with $S > T$) such that α induces delay in $\Gamma\left(h_S\right)$ and $\emptyset \neq \Omega\left(h_S\right) \subset \Omega_T\left(h_T\right)$.*

Proof. Let $t > T$ be last period of agreement in subgame. Let $h_t^\alpha = h_t\left(h_T, \alpha\right)$. Note that $\Omega\left(h_T\right)$ cannot be a singleton. Otherwise the equilibrium strategy profile α could not prescribe delay in $\Gamma\left(h_T\right)$.

Case 1: Assume that there are nonzero contracts that will never conclude according to equilibrium, that is $\Omega_\infty^+\left(h_T, \alpha\right) \neq \emptyset$. Then, the lemma follows immediately from considering the subgame $\Gamma\left(h_S\right) = \Gamma\left(h_t\left(h_T, \alpha\right)\right)$. In that subgame, $\Omega\left(h_S\right) \subset \Omega\left(h_T\right)$ since $\Omega_t\left(h_T, \alpha\right) \neq \emptyset$. Moreover, $\Omega\left(h_S\right) \neq \emptyset$ since $\Omega_\infty^+\left(h_T, \alpha\right) \neq \emptyset$.

Case 2: Assume that $\Omega_\infty^+\left(h_T, \alpha\right) = \emptyset$. Then, by Lemma 3, $\Omega_\infty^0\left(h_T, \alpha\right) = \emptyset$. Otherwise, there would exist subgames with only $(\widehat{q}_{ud} = 0)$-negotiations

left. Note that if $\nexists (u,d) \in \Omega_t (h_T, \alpha) : \hat{q}_{ud} > 0$, then α is not an equilibrium according to Lemma 3. A contradiction. Hence, there exists a pair (u,d) such that $\hat{q}_{ud}(h_t^\alpha) > 0$. Consider a deviation for $(u,d) \in \Omega_t (h_T, \alpha)$ specifying the contract $\widehat{c}_{ud}(h_t^\alpha)$, with $\hat{q}_{ud}(h_t^\alpha) > 0$, already in period $t-1$. Without loss of generality we assume that it is $\rho_{ud}(t-1) = u$ who makes the bid, and that d is the respondent. Let h_t^+ denote the history where d accepts the bid and h_t^- where d rejects.

Conditional on acceptance, two outcomes are logically possible for the subgame $\Gamma\left(h_t^+\right)$. First, if there is delay in $\Gamma\left(h_t^+\right)$ then the lemma follows immediately from considering the subgame $\Gamma(h_S) = \Gamma\left(h_t^+\right)$. Note that $\Omega(h_S) \subset \Omega(h_T)$ since $(u,d) \notin \Omega(h_S)$. Moreover, $\Omega(h_S) \neq \emptyset$, since $\Omega(h_T)$ includes at least two elements. The second logical possibility is that there is no delay in $\Gamma\left(h_t^+\right)$. This is assumed in the rest of the proof.

Conditional on rejection, three outcomes are logically possible for the subgame $\Gamma\left(h_t^-\right)$.

1. Assume that α prescribes that no players ever agree, that is $\Omega_\infty\left(h_t^-, \alpha\right) = \Omega(h_t^\alpha)$. The result then directly follows from Lemma 4.

2. Assume that α prescribes that all players agree at $t' \geq t$, that is $\Omega_{t'}\left(h_t^-, \alpha\right) = \Omega(h_t^\alpha)$. We will show that this assumption cannot hold in equilibrium. If all players agree at $t' \geq t$, contracts are given by $\hat{c}(h_t^\alpha)$ by Lemma 2. Since there is no delay conditional on agreement, d strictly gains by accepting. To see this we compare the payoffs in the two cases. At time $t-1$ the continuation payoffs of rejecting the offer

is:

$$\Pi_d\left(h_t^-,\alpha\right)=\pi_d\left(c\left(h_t^\alpha\right)\right)+\delta\sum_{s=t}^{t'-1}\delta^{s-t}\pi_d\left(c\left(h_t^\alpha\right)\right)+\sum_{s=t'}^{\infty}\delta^{s-t}\pi_d\left(\widehat{c}\left(h_t^\alpha\right)\right) \tag{10}$$

At time $t-1$ the continuation payoffs of rejecting the offer is:

$$\Pi_d\left(h_t^+,\alpha\right)=\pi_d\left(c_{-ud}\left(h_t^\alpha\right),\widehat{c}_{ud}\left(h_t^\alpha\right)\right)+\delta\sum_{s=t}^{\infty}\delta^{s-t}\pi_d\left(\widehat{c}\left(h_t\right)\right). \tag{11}$$

This implies that

$$\Pi_d\left(h_t^+,\alpha\right)-\Pi_d\left(h_t^-,\alpha\right)=\pi_d\left(c_{-ud}\left(h_t^\alpha\right),\widehat{c}_{ud}\left(h_t^\alpha\right)\right)-\pi_d\left(c\left(h_t^\alpha\right)\right) \tag{12}$$

$$+\delta\sum_{s=t}^{t'-1}\delta^{s-t}\left[\pi_d\left(\widehat{c}\left(h_t^\alpha\right)\right)-\pi_d\left(c\left(h_t^\alpha\right)\right)\right]>0. \tag{13}$$

Similarly the deviating proposer u gains from obtaining bilateral payoff one period earlier, given that d accepts:

$$\Pi^u\left(h_t^+,\alpha\right)-\Pi^u\left(h_t^\alpha,\alpha\right)=\pi^u\left(c_{-ud}\left(h_t^\alpha\right),\widehat{c}_{ud}\left(h_t^\alpha\right)\right)-\pi^u\left(c\left(h_t^\alpha\right)\right), \tag{14}$$

where the equilibrium continuation payoffs at time $t-1$ is given by

$$\Pi^u\left(h_t^\alpha,\alpha\right)=\pi^u\left(c\left(h_t^\alpha\right)\right)+\delta\sum_{s=t}^{\infty}\delta^{s-t}\pi^u\left(\widehat{c}\left(h_t^\alpha\right)\right). \tag{15}$$

This contradicts the assumption that α is an equilibrium. More precisely, if α is an equilibrium then it cannot be the case that $\Omega_{t'}\left(h_t^-,\alpha\right)=\Omega\left(h_t^\alpha\right)$ for some $t'\geq t$.

3. Assume that α prescribes that a subset of players agree at $t'>t$.

Formally, there exists a largest $t' > t$ such that $\emptyset \neq \Omega_{t'}\left(h_t^-, \alpha\right) \neq \Omega\left(h_t^\alpha\right) \subseteq \Omega\left(h_T\right)$.

(a) If $\Omega_\infty\left(h_t^-, \alpha\right) \neq \emptyset$, then the subgame $\Gamma\left(h_S\right) = \Gamma\left(h_{t'+1}\left(h_t^-, \alpha\right)\right)$ satisfies the claim of the lemma.[7]

(b) If $\Omega_\infty\left(h_t^-, \alpha\right) = \emptyset$, we must have $t' > t$, as otherwise everyone would have to agree at t. In this case the subgame $\Gamma\left(h_S\right) = \Gamma\left(h_{t'-1}\left(h_t^-, \alpha\right)\right)$ satisfies the claim of the lemma. ■

Proof of Proposition 2, p 12 First, we show that there exists sequential equilibrium, with agreement in the first period. Second, we show uniqueness. This is done by showing that if strategy profile α induces delay in a subgame $\Gamma\left(h_T\right)$, then α is not an equilibrium.

Consider the equilibrium strategy: In all periods, bid $\bar{c}_{ud}$ as given by Lemma 1 and accept all bids $\underline{c}_{ud}$ such that $\pi^u\left(c_{-ud}, \underline{c}_{ud}\right) \geq \delta\pi^u\left(c_{-ud}, \overline{c}_{ud}\right)$. In any period t all players will thus give bids that are accepted. Given that all others play this strategy, a deviating bid in any period t will give player i a lower payoff. Lemma 1 shows that the equilibrium strategy gives the maximal payoff for player i contingent on all others playing according to equilibrium strategy. If player i's bid results in some bids being rejected, payoffs to player i will also be strictly lower. All players will in period $t+1$ play according to the equilibrium strategy, agreeing on the same contracts as would have been agreed upon in period t without deviation. The payoff is thus the same, but discounted one period. As payoffs are a discounted sum of uniformly bounded per-period payoffs, the game is continuous at infinity. Our bargaining game satisfies the conditions for the one-stage deviation principle[8].

[7] If $\Omega_\infty\left(h_t^-, \alpha\right) \neq \emptyset$ then $\Omega_\infty^+\left(h_t^-, \alpha\right) \neq \emptyset$, by Lemma 3.

[8] For a discussion of the one-stage deviation principle in the context of sequential equi-

Assume that α is an equilibrium strategy profile that induces delay in $\Gamma(h_T)$. As α induces delay, by definition either the conditions of Lemmas 4 and 5 hold. In both cases, the lemmas imply that there exist subgames with delay with fewer negotiations. Repeated application of the lemmas give an infinite sequence of subgames $\Gamma(h_{T_1})$, $\Gamma(h_{T_2})$,... where $\Omega\left(h_{T_{k+1}}\right) \subset \Omega(h_{T_k})$ and $\Omega(h_{T_k}) \neq \emptyset$ for all T_k. As $\Omega(h_T)$ is a finite set, we obtain a contradiction. ■

4.1 Proof of Proposition 1

Before proving the proposition, we begin by defining the Markov condition described above.

Definition 5 *A strategy profile σ is Markov if:*

1. *The strategy profile, contingent on the observed history, is a function of (c, Ω) only, i.e. $\sigma(h_t) = \sigma(c(h_t), \Omega(h_t))$.*
2. *Consider two states $\left(\widetilde{c}, \widetilde{\Omega}\right)$ and (c, Ω) with $\widetilde{\Omega} = \Omega \backslash \{(u, d)\}$ and $\widetilde{c} = (c_{-ud}, \widetilde{c}_{ud})$. If $\sigma(c, \Omega)$ prescribes immediate agreement on $\widetilde{c}_{ud}$, then $\sigma(c, \Omega) = \sigma\left(\widetilde{c}, \widetilde{\Omega}\right)$.*

Two histories giving rise to (c, Ω) and (c', Ω) may, in fact, be payoff equivalent. As such equivalence is irrelevant for our purposes, this distinction is ignored for the sake of notational simplicity.

We say that strategy profile σ induces delay in $\Gamma(h_t)$ if not all negotiations are immediately concluded, i.e. if $\Omega(h_{t+1}(h_t, \sigma)) \neq \emptyset$. We say that a subgame $\Gamma(h_t)$ is nontrivial if there exist some profitable agreements in

librium, see Osborne & Rubinstein (1994). For a discussion of the one-stage deviation principle in the context of infinite-horizon games, see Fudenberg & Tirole (1991).

ongoing negotiations, i.e. for some $(u,d) \in \Omega(h_t)$, we have that $q_{ud} > 0$, where $q \in N(c(h_t), \Omega(h_t))$.

Lemma 6 *Assume that there is no downstream market power. Consider a Markov equilibrium σ and a nontrivial subgame $\Gamma(h_T)$. If σ induces delay at T, then there exists a subgame $\Gamma(h_S)$ of $\Gamma(h_T)$ (with $S > T$), such that σ induces delay in $\Gamma(h_S)$ and $\Omega(h_S) \subset \Omega(h_T)$.*

Proof: Assume that there exists some date $t > T$ such that some, but not all, negotiations in $\Omega(h_T)$ are concluded. The Lemma then immediately follows, since there is delay in subgame $\Gamma(h_{t+1})$. Two cases remain to be considered.

Case 1: Assume that σ prescribes that everybody agrees at $t > T$. A deviation specifying the same $\hat{q}_{ud}$ at $t-1$ will increase the payoff for (u,d), as by the Markov assumption, the actions of everybody else will be the same. This contradicts that σ prescribes equilibrium play.

Case 2: Assume that nobody ever agrees in subgame $\Gamma(h_T)$. As the subgame is nontrivial, consider a negotiation $(u,d) \in \Omega(h_{S-1}(h_T,\sigma))$ where $q_{ud} > 0$ in some period $S-1 \geq T$ for $q \in N(c(h_{S-1}(h_T,\sigma)), \Omega(h_{S-1}(h_T,\sigma)))$. Let h_S^+ be the history where u suggests q_{ud} in period $S-1$, and d accepts (all others play according to σ). Let h_S^- be the history where d rejects. Conditional on h_S^-, the Markov assumption ensures that no agreement is reached in the subgame. Conditional on h_S^+, three outcomes are logically possible: immediate agreement, no agreement, or agreement at different times in the other negotiations. In the last case, the Lemma is immediately proved.

It cannot be the case that σ prescribes immediate agreement or no agreement ever in $\Gamma(h_S)$, as this would imply that d's acceptance of the bid $\hat{q}_{ud}$ is a profitable deviation. If d accepts and all negotiations end at S, Proposition 1 in Björnerstedt & Stennek (2006) implies that the agreement is on

$q \in N\left(c\left(h_S\right), \Omega\left(h_S\right)\right)$ and the corresponding prices $\widehat{p}$. As goods are substitutes and with no downstream market power, an agreement on a bilaterally efficient q cannot reduce the profits of u and d relative no agreement at all. Finally, d accepting the bid implies that it is better for u to suggest q_{ud} than equilibrium play in period $S-1$ (no agreements). Thus, σ induces delay in $\Gamma\left(h_S\right)$ with fewer active negotiations.□

Proof of Proposition 1, p 4 First, we show that with no downstream market power, the Markov condition guarantees immediate agreement. Assume, to the contrary, that σ is an equilibrium strategy profile inducing delay in $\Gamma\left(h_T\right)$. According to Lemma 6, there exists a subgame with delay with fewer negotiations. Repeated application of the lemmas gives an infinite sequence of subgames $\Gamma\left(h_{T_1}\right)$, $\Gamma\left(h_{T_2}\right)$,... where $\Omega\left(h_{T_{k+1}}\right) \subset \Omega\left(h_{T_k}\right)$ and $\Omega\left(h_{T_k}\right) \neq \emptyset$ for all T_k. As $\Omega\left(h_T\right)$ is a finite set, we obtain a contradiction. When bilateral efficiency is unique, the proof follows from Proposition 2.

Assume now that there is downstream market power, and that there is a last period T for agreements. In the last period, everyone will agree on bilaterally efficient quantities. A deviation in one negotiation in $T-1$ is then profitable if either play is Markov or if the bilaterally efficiency is unique. Thus, agreement will be in the first period.□

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