

Kraus, Daniel

Working Paper

On the existence of credit rationing and screening with loan size in competitive markets with imperfect information

Thünen-Series of Applied Economic Theory - Working Paper, No. 131

Provided in Cooperation with:

University of Rostock, Institute of Economics

Suggested Citation: Kraus, Daniel (2013) : On the existence of credit rationing and screening with loan size in competitive markets with imperfect information, Thünen-Series of Applied Economic Theory - Working Paper, No. 131, Universität Rostock, Institut für Volkswirtschaftslehre, Rostock

This Version is available at:

<https://hdl.handle.net/10419/81142>

Standard-Nutzungsbedingungen:

Die Dokumente auf EconStor dürfen zu eigenen wissenschaftlichen Zwecken und zum Privatgebrauch gespeichert und kopiert werden.

Sie dürfen die Dokumente nicht für öffentliche oder kommerzielle Zwecke vervielfältigen, öffentlich ausstellen, öffentlich zugänglich machen, vertreiben oder anderweitig nutzen.

Sofern die Verfasser die Dokumente unter Open-Content-Lizenzen (insbesondere CC-Lizenzen) zur Verfügung gestellt haben sollten, gelten abweichend von diesen Nutzungsbedingungen die in der dort genannten Lizenz gewährten Nutzungsrechte.

Terms of use:

Documents in EconStor may be saved and copied for your personal and scholarly purposes.

You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.

If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.

Thünen-Series of Applied Economic Theory
Thünen-Reihe Angewandter Volkswirtschaftstheorie

Working Paper No. 131

**On the Existence of Credit Rationing and Screening with
Loan Size in Competitive Markets with Imperfect Information**

by

Daniel Kraus

Universität Rostock

Wirtschafts- und Sozialwissenschaftliche Fakultät
Institut für Volkswirtschaftslehre
2013

On the Existence of Credit Rationing and Screening with Loan Size in Competitive Markets with Imperfect Information

Daniel Kraus[†]

Department of Economics, University of Rostock

This version: September 10, 2013

Abstract Although credit rationing has been a stylized fact since the groundbreaking papers by Stiglitz and Weiss (1981, hereinafter S-W) and Besanko and Thakor (1987a, hereinafter B-T), Arnold and Riley (2009) note that credit rationing is unlikely in the S-W model, and Clemenz (1993) shows that it does not exist in the B-T model. In this chapter, I explain why credit rationing, more specifically rationing of loan applicants, does exist in a competitive market with imperfect information, and occurs only for low-risk loan applicants. In cases of indivisible investment technologies, low-risk applicants are rationed. In cases of divisible investment technologies, rationing of loan size is restricted to rationing of loan applicants. In the event that the difference in the marginal return between the investment technologies is sufficiently small relative to the difference in their riskiness, rationing of loan size alone yields high opportunity costs; in addition, low-risk loan applicants are rationed in this case.

Keywords: Asymmetric Information, Financial Intermediation, Credit Rationing
JEL code: G21, D82

*Acknowledgements: I am grateful to Anjan Thakor, Hellmuth Milde, Radha Gopalan, Ron Masulis, Doris Neuberger, Philipp Wichardt, Guisepepe Coco and the seminar participants of the Olin Business School, Washington University in St. Louis, the DIW in Berlin and the University of Rostock for helpful comments and discussions. Financial support by University of Rostock is gratefully acknowledged.

[†]Postal address for correspondence: Department of Economics, University of Rostock, Ulmenstr. 69, D-18057 Rostock, Germany; e-mail: daniel.kraus@uni-rostock.de.

1 Introduction

In their pioneering paper, Stiglitz and Weiss (1981, hereinafter S-W) state that credit rationing exists in markets with borrowers who privately know the riskiness of their investment technology. In fact, by credit rationing, S-W mean rationing of loan applicants; that is, in the equilibrium, some loan applicants are rejected while other observationally identical loan applicants are accepted. While S-W established the foundation for the literature on adverse selection and credit rationing in financial markets, Arnold and Riley (2009, hereinafter A-R) cast some doubt on S-W's globally hump-shaped expected bank profit as a function of loan rate. They show that this expected profit function cannot have the hump shape suggested by S-W.

A-R recall from S-W that, although all investment technologies have the same mean return, they differ with respect to their riskiness. This implies that riskier investment projects have a higher return than less risky projects if they are successful. From these investment technologies follows that a bank that merely offers a flat rate of interest to observationally identical borrowers effectively pools borrowers with differing levels of riskiness. Raising this flat loan rate simply drives the less risky applicants out of the loan; there is no 'hump'. Based on this observation, A-R show that any equilibrium with rationing must have at least two loan rates, with credit rationing for the lower loan rate and no rationing for the higher loan rate. However, running a numerical analysis, A-R conclude that credit rationing is unlikely.

Besanko and Thakor (1987a, hereinafter B-T) provide another explanation for credit rationing in a market under perfect competition. In their model, the loan applicants' privately known investment technology differs only in riskiness, but not with respect to return. In the B-T equilibrium, low-risk applicants are rationed if they cannot provide sufficiently high collateral. However, Clemenz (1993) points out that the situation described by B-T does not constitute a Nash-equilibrium. Clemenz shows that another profitable loan contract exists for low-risk loan applicants without credit rationing, but at a higher loan rate.

Credit rationing implies opportunity costs for the rejected low-risk loan applicants. To reduce these informational costs, banks use loan size to screen borrowers' riskiness. However, Milde and Riley (1988), Schmidt-Mohr (1997, hereinafter S-M) and Bester (1985) show that loan size must be rationed, thus incurring opportunity costs for the borrowers, as loan size rationing means that, for the given loan rate, they obtain a smaller loan size than desired.

Empirical evidence confirms that rationing exists for both loan applicants (Cole

1998, Blackwell and Winters 1997) and loan size (Petersen and Rajan 1994, 1995). Besanko and Thakor (1987b) consider these two types of rationing to be co-existing screening devices but cannot explain why rationing occurs. This raises the research question of why credit rationing exists, specifically rationing of observationally identical loan applicants. Further, why does loan size rationing exist, and why does it co-exist with rationing of observationally identical loan applicants?

In this chapter, I show that credit rationing is more costly for high-risk than for low-risk loan applicants. Thus, rationing works as a screening instrument that makes low-risk loan applicants better off although it imposes opportunity costs on them. Divisibility of investment technologies enables banks to use loan size as a further screening device. When possible, banks prefer to ration loan size rather than loan applicants. Only if the difference in the marginal return between the investment technologies is sufficiently small relative to the difference in their riskiness, rationing of loan size alone becomes too expensive and, as a result, banks ration loan applicants as well.

The remainder of this chapter is organized as follows: Section 2 analyzes the competitive loan market Nash-equilibrium for indivisible and divisible investment technologies, and Section 3 concludes the chapter.

2 The Analysis

2.1 The General Set-Up

The risk-neutral entrepreneur E_i considers raising a loan in a competitive market to invest it in a project. Before $t = 1$, E_i privately observes the technology of his investment project, which has either a low (L) or a high risk (H), in the sense of its success probability, such that $p^L > p^H$. The risk-neutral bank knows only that L occurs with probability α and that H occurs with probability $1 - \alpha$. At $t = 1$, there are three stages. In the first stage, the bank offers a menu of loan contracts. In the second stage, E_i either applies for one of these loan contracts or chooses his outside option, with payoff 0. In the third stage, it is realized whether or not the bank will grant E_i the loan. If E_i does not obtain a loan, he chooses his outside option.¹ If E_i

¹Stiglitz and Weiss (1981), Arnold and Riley (2009) and Clemenz (1993) note that rejected loan applicants may choose the contract with the higher loan rate. However, including this possibility in my model does not have any effect on the design of L 's contract. Thus, for simplicity, I assume that a rejected loan applicant chooses his outside option.

obtains the loan, he invests it in his project. In this case, at $t = 2$, the project return of his investment is realized. The return depends on investment technology. With success probability p^i , its return is R^i ; with probability $1 - p^i$, it is zero. The timing of events is summarized in Figure 1.

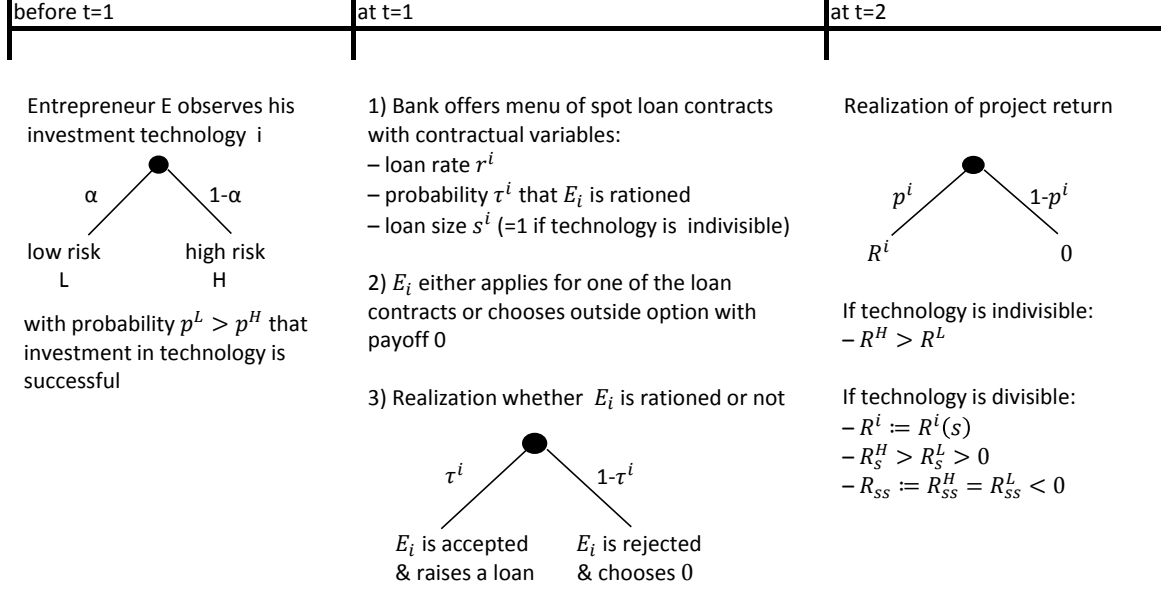


Figure 1: Timeline of events.

2.2 Indivisible Investment Technology

Consider an entrepreneur E_i who has the opportunity to invest in a project with an indivisible technology. The riskier technology has a higher return (i.e., $R^H > R^L$). To finance that project, E_i needs to borrow a loan of size one. The bank offers E_i a loan contract comprising loan rate r , the probability τ that he is rationed, and a loan size equal to one. At $t = 1$, the bank's expected profit from a loan with contractual variables (τ, r) to E_i is given by

$$\Pi^{Bank}(\tau, r; p^i) := \tau(p^i r - \rho) \quad (1)$$

where ρ denotes the gross deposit rate, which represents the bank's costs of funds. If E_i applies for loan contract (τ, r) , his expected profit function is given by

$$\Pi^i(\tau, r) := \tau p^i (R^i - r). \quad (2)$$

In a Nash-equilibrium, every bank takes offers of competing banks as given and

independent of its own actions. A loan contract does not exist outside the Nash-equilibrium with which the bank, if offered, will make a positive profit. Thus, a competitive bank makes zero expected profits with borrower E_i :

$$\Pi^{Bank}(\tau^i, r^i; p^i) = 0. \quad (3)$$

As the bank cannot observe E_i 's type, it can only separate L and H by an incentive compatible loan policy $(\tau, r) := (\tau^i, r^i)^i$, with $i = L, H$, which satisfies

$$\Pi^L(\tau^L, r^L) \geq \Pi^L(\tau^H, r^H) \quad (4)$$

$$\Pi^H(\tau^H, r^H) \geq \Pi^H(\tau^L, r^L) \quad (5)$$

Naturally, E_i prefers the contract that maximizes his expected profit function. He chooses to apply for his optimal loan contract if

$$\Pi^i(\tau, r) := \tau p^i (R^i - r) \geq 0. \quad (6)$$

From this follow Propositions 1 and 2.

Proposition 1 (Equilibrium) *In a competitive loan market with asymmetric information about divisible investment technologies, a Nash-equilibrium exists only if the share of L is sufficiently small; that is,*

$$\alpha < \hat{\alpha}_{Indiv} := 1 - \rho^{-1} p^H (p^L R^L - p^H R^H) (p^L - p^H)^{-1}.$$

Then, in the Nash-equilibrium, the bank's optimal loan policy (r^, τ^*) , with loan rate r and probability τ that E_i is rationed, is separating and given by*

$$\begin{aligned} r^{H*} &= \rho/p^i, \quad \tau^{H*} = 1 \\ r^{L*} &= \rho/p^i, \quad \tau^{L*} = (R^H - \rho/p^H) (R^L - \rho/p^L)^{-1} < 1. \end{aligned}$$

Proposition 2 (Welfare) *In a competitive loan market with asymmetric information about indivisible investment technologies, H always obtains his first-best loan contract, while L does not and is rationed; that is, $\tau^{L*} < 1$.*

The intuition of Proposition 2 is as follows. To evaluate welfare, consider the Nash-equilibrium under full information as a benchmark. It is straightforward to show

that

$$r_{FB}^{i*} = \rho/p^i \quad (7)$$

$$\tau_{FB}^{i*} = 1. \quad (8)$$

Then, interest rate r_{FB}^{i*} covers loan costs, and E_i is not rationed. To understand why the Nash-equilibrium under asymmetric information is not first-best, consider the slope of E_i 's iso-profit curve, which is described by the total differential of (2) with respect to r and τ

$$\frac{dr}{d\tau} \Big|_{\Pi^i = \text{const.}} = \frac{R^i - r}{\tau} > 0. \quad (9)$$

As $R^H > R^L$, H always has a steeper iso-profit curve than L . That means that H is always willing to pay a higher r than L to be marginally less rationed. See Figure 2 for an illustration of credit rationing.²

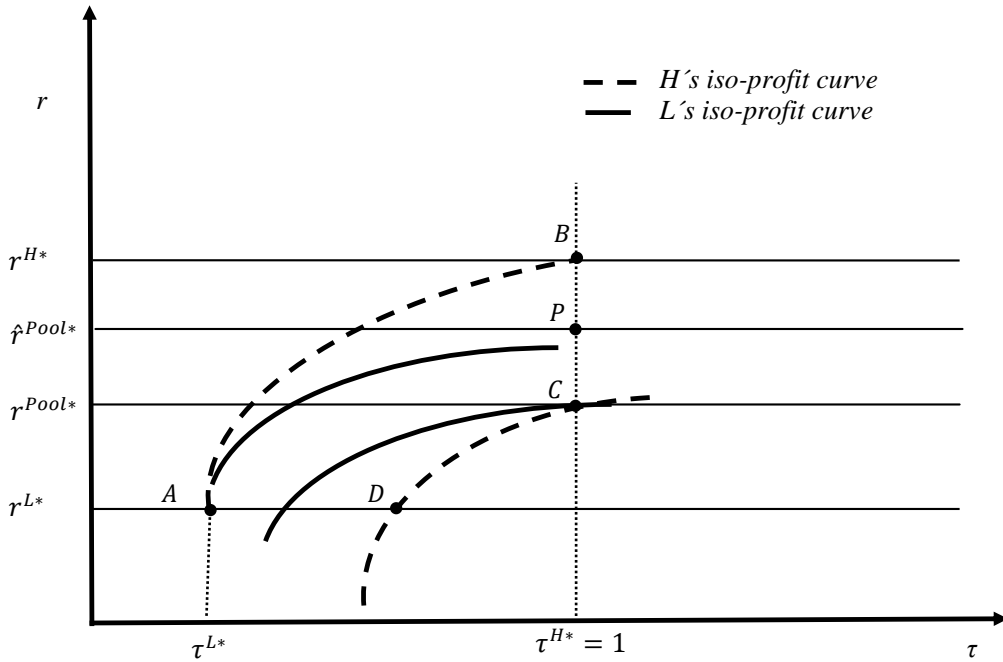


Figure 2: The Nash-equilibrium in a competitive loan market with asymmetric information about indivisible investment technologies. If the pooling loan rate r^{Pool} is sufficiently large, L prefers contract A to pooling contract P . In this case, a Nash-equilibrium does exist and is separating. H chooses contract B , and L chooses contract A , which rations him (i.e., $\tau^{L*} < 1$). If r^{Pool} is small enough that L prefers C to A , a Nash-equilibrium does not exist.

²(9) satisfies the single-crossing property and has a maximum as $d^2r/d\tau^2 \Big|_{\Pi^i = \text{const.}} = -(R^i - r)/\tau^2 < 0$.

Consider a competitive bank that offers contract P with a flat rate of interest \hat{r}^{Pool} such that it makes zero profit. This contract pools L and H . As L has a lower success probability than the average of the pool, he has to pay a higher interest rate, $\hat{r}^{Pool} > r_{FB}^{L*}$. L always prefers a lower iso-profit curve, as, for a given τ , a smaller r increases his expected profit. Thus, in a competitive loan market, another bank can attract L with contract A , which has a lower loan rate of $r^{L*} = r_{FB}^{L*}$, although this means that L is rationed (i.e. $\tau^{L*} < 0$). In contrast to L , H is willing to pay a higher r to avoid being rationed and chooses his first-best contract B .

This separating equilibrium is not a Nash-equilibrium, if $\alpha > \hat{\alpha}_{Indiv}$. In this case, r^{Pool*} is sufficiently low that L prefers pooling contract C to A . C is not a contract in the Nash-equilibrium, as another bank can attract L with contract D . In this case, the bank makes negative profits with contract C and thus no longer offers it. H also chooses D , so the bank also makes negative profits with D . Thus, a Nash-equilibrium exists only if $\alpha < \hat{\alpha}_{Indiv}$.

I will now review A-R and Clemenz to show why credit rationing is unlikely in the S-W model and impossible in the B-T model but does exist in the Nash-equilibrium of my model.

S-W and A-R consider a loan market which is characterized by asymmetric information about investment technologies, which all have the same mean return μ but differ with respect to their riskiness in the sense of mean preserving spreads. Their argument for credit rationing relies on a bank whose expected profit as a function of loan demand is hump-shaped. A-R show that this function can never be globally (cf. S-W's Figure 4, p. 397), but only locally (cf. A-R's Figure 1, p. 2015) hump-shaped. Thus, the bank offers at least two equilibrium loan rates (cf. A-R's Figure 2, p. 2016 and S-W's Figure 5, p. 398). The lower loan rate entails an excess demand for loans (i.e., rationing of loan applicants). To satisfy the rejected applicants, the bank offers a second contract at higher than the Walrasian loan rate. Analyzing this credit rationing equilibrium numerically, A-R conclude that such a rationing equilibrium is unlikely. However, A-R and S-W only consider pooling contracts; in so doing, they rule out loan policies that reveal the borrowers' riskiness.

To examine whether, in the Nash-equilibrium, there is a loan policy that reveals the borrowers' riskiness truthfully, I simplify S-W and A-R's model and consider a high-risk (H) and a low-risk (L) investment technology. Following A-R's notation, E_i 's random gross return is $\tilde{R}^i = \mu + \tilde{z}^i$. The random \tilde{z}^i has a zero mean, but a distribution with support $[-\lambda^i, \lambda^i]$, where $z^H > z^L$. Moreover, A-R assume $\tilde{R}^i > 0$,

which implies $\tilde{z}^i \leq \mu$. The assumption of a second-order stochastic dominance means that L 's and H 's distribution of returns $F_i(z)$ differ such that

$$\int_l^u F_H(z) dz \geq \int_l^u F_L(z) dz, \quad (10)$$

where $u > l$. In contrast to A-R, I also include the probability τ that E_i is rationed in his expected profit function, which is given by³

$$\Pi^i(r, \tau) = \tau \left[\mu + \int_{-\lambda^i}^{-(\mu-r)} F_i(z) dz - r \right]. \quad (11)$$

From above results E_i 's isoprofit curve, which is given by the total differentiation of (11) with respect to r and τ :

$$\frac{dr}{d\tau} \Big|_{\Pi^i = \text{const.}} = \frac{\mu + \int_{-\lambda^i}^{-(\mu-r)} F_i(z) dz - r}{\tau (1 - F_i(-(\mu-r)))} > 0. \quad (12)$$

$$\frac{d^2r}{d\tau^2} \Big|_{\Pi^i = \text{const.}} = -\frac{\mu + \int_{-\lambda^i}^{-(\mu-r)} F_i(z) dz - r}{\tau^2 (1 - F_i(-(\mu-r)))} < 0. \quad (13)$$

As (11) and $F_H(z) \geq F_L(z)$, $\forall z$, H always has a steeper iso-profit curve than L . This corresponds to the properties of E_i 's iso-profit curve (9) in my model. Thus, an equilibrium that constitutes a Nash-equilibrium can never be pooling. Instead, the Nash-equilibrium, if it exists, is always separating. Thus, credit rationing of L is likely in the S-W model.

Clemenz (1993) shows that B-T's credit rationing equilibrium does not constitute a Nash-equilibrium. B-T's investment technologies differ with respect to their riskiness only, not with respect to their return in the event of a successful project such that, in my notation, $R := R^H = R^L$. Then, including collateral as a screening instrument, L has a steeper iso-profit curve in the (r, τ) -space than H . Clemenz shows that a bank can deviate from B-T's rationing equilibrium and make positive profits by offering a loan contract with no rationing at a higher loan rate. Without collateral, there is also no credit rationing in the B-T model. To understand this, consider E_i 's iso-profit curve

$$\frac{dr}{d\tau} \Big|_{\Pi^i = \text{const.}} = \frac{R - r}{\tau} > 0. \quad (14)$$

³This payoff function corresponds to equation (1) in Arnold and Riley (2009, p. 2013). Their notation is adapted to the notation used in the present chapter.

Thus, L 's and H 's iso-profit curve does not differ, which makes screening with credit rationing impossible.

2.3 Divisible Investment Technology

Now, consider an entrepreneur E_i who has the opportunity to invest a loan in a divisible technology. The riskier technology entails an invertible higher marginal return for all s , $R_s^H > R_s^L > 0$, but at a decreasing rate; that is, $R_{ss} := R_{ss}^H = R_{ss}^L < 0$.⁴ At $t = 1$, the bank's expected profit from a loan to E_i - with the contractual variables loan rate r , size s and probability τ that E_i is rationed - is given by

$$\Pi^{Bank}(r, \tau, s) = \tau (p^i r - \rho) s. \quad (15)$$

If E_i chooses this loan contract, his expected profit function is given by

$$\Pi^i(r, \tau, s) = \tau p^i (R^i(s) - rs). \quad (16)$$

In a Nash-equilibrium, a competitive bank does not make negative expected profits with any contract. Outside the equilibrium, there is no contract with which the bank, if offered, will make a positive profit. Thus, the bank's loan policy $(r, \tau, s) := (r^i, \tau^i, s^i)^i$ maximizes $\Pi^i(r, \tau, s)$, subject to

$$\Pi^L(r^L, \tau^L, s^L) \geq \Pi^L(r^H, \tau^H, s^H) \quad (17)$$

$$\Pi^H(r^H, \tau^H, s^H) \geq \Pi^H(r^L, \tau^L, s^L) \quad (18)$$

$$\Pi^i(r^i, \tau^i, s^i) \geq 0. \quad (19)$$

$$\Pi^{Bank}(r^i, \tau^i, s^i) = 0. \quad (20)$$

$$0 \leq \tau^i \leq 1 \quad (21)$$

where $i = L, H$. Note that (17) and (18) are the incentive-compatible constraints, (19) is E_i 's participation constraint that guarantees that E_i raises a loan, (20) is the bank's zero expected profit condition, and (21) is the feasibility constraint for the rationing of E_i .

⁴S-M and Bester also include loan size as a contractual variable, but they do not consider the possibility of the rationing of loan applicants. While Bester's assumption B corresponds to my investment technology, S-M's technological characteristics are a special case of my model. S-M's investment projects always have the same mean return; i.e., $p^L R^L(s) = p^H R^H(s)$ for all s .

From this follow Propositions 3 and 4.

Proposition 3 (Equilibrium) *In a competitive loan market with asymmetric information about divisible investment technologies, a Nash-equilibrium exists only if the share of L is sufficiently small; that is,*

$$\alpha < \hat{\alpha}_{Div} := \Pi^L (r^{L*}, \tau^{L*}, s^{L*}) = \Pi^{L, Pool} (r^{Pool*}, s^{Pool*})$$

with $r^{Pool*} = \rho (\hat{\alpha}_{Div} p^L + (1 - \hat{\alpha}_{Div}) p^H)^{-1}$. This Nash-equilibrium is not first-best if the difference between H 's and L 's marginal return (i.e., $\Delta R_s := R_s^H - R_s^L$) is sufficiently small relative to the difference in their riskiness ($\Delta p := p^L - p^H$). In this case, the bank's optimal loan policy (r^*, τ^*, s^*) , which comprises gross loan rate r , loan size s and probability τ that E_i is rationed, is separating such that H 's respectively L 's loan contract is

$$\begin{aligned} r^{H*} &= \rho/p^H, \quad \tau^{H*} = 1, \quad s^{H*} := p^H (R_s^H (s^{H*}) - \rho/p^H) = 0 \\ r^{L*} &= \rho/p^L \\ \tau^{L*} &= \begin{cases} 1 & \text{if } \frac{\epsilon_{\Pi_{L^*}^L(\bar{\tau}^L=1), s^L}}{\epsilon_{\Pi_{L^*}^H(\bar{\tau}^L=1), s^L}} \geq 1 \\ \frac{R_s^H(s^{H*}) - r^{H*} s^{H*}}{R_s^H(s^{L*}) - r^{L*} s^{L*}} < 1 & \text{otherwise} \end{cases} \\ s^{L*} &:= \begin{cases} R_s^L(s^{L*}) - (R_s^H(s^{L*}) - R_s^L(s^{L*})) \frac{p^H}{p^L - p^H} = r^{L*} & \text{if } \frac{\epsilon_{\Pi_{L^*}^L(\bar{\tau}^L=1), s^L}}{\epsilon_{\Pi_{L^*}^H(\bar{\tau}^L=1), s^L}} \geq 1 \\ \frac{\partial \Pi_{L^*}^L / \Pi_{L^*}^L}{\partial s^L / s^{L*}} = \frac{\partial \Pi_{L^*}^H / \Pi_{L^*}^H}{\partial s^L / s^{L*}} & \text{otherwise.} \end{cases} \end{aligned}$$

where $\Pi_{L^*}^i(\bar{\tau}^L = 1) := \Pi^i(r^{L*}, \bar{\tau}^L = 1, s^{L*})$ denotes E_i 's expected profit if he chooses the loan contract designed for L , assuming that the bank does not ration him (i.e., $\bar{\tau}^L = 1$) and $\epsilon_{\Pi_{L^*}^i(\bar{\tau}^L=1), s^L} := \frac{\partial \Pi_{L^*}^i(\bar{\tau}^L=1) / \Pi_{L^*}^i(\bar{\tau}^L=1)}{\partial s^L / s^{L*}}$. There is a unique interior solution for s^{i*} .

Proposition 4 (Welfare) *In a competitive loan market with asymmetric information about divisible investment technologies, H always obtains his first-best contract, whereas L does not if the difference between H 's and L 's marginal return (i.e., $\Delta R_s := R_s^H - R_s^L$) is sufficiently small relative to the difference in their probability of success ($\Delta p := p^L - p^H$).*

To evaluate welfare, consider the equilibrium under full information as a benchmark. In this scenario, the bank knows E_i 's type and, in the Nash-equilibrium, the optimal

loan policy maximizes E_i 's expected profit subject to (20). It is a straightforward process to verify that the Nash-equilibrium under symmetric information is

$$r_{FB}^{i*} = \rho/p^i \quad (22)$$

$$\tau_{FB}^{i*} = 1 \quad (23)$$

$$s_{FB}^{i*} := R_s^i (s_{FB}^{i*}) = \rho/p^i \quad (24)$$

Under full information, interest rate r_{FB}^{i*} covers loan costs, E_i is not rationed (i.e., $\tau_{FB}^{i*} = 1$), and size s_{FB}^{i*} equates E_i 's marginal return in the event of a project success and the bank's marginal lending costs.

The intuition of Proposition 3 is as follows. Consider the slope of E_i 's iso-profit curve which is the total differential of his expected profit function (16) with respect to r and s :

$$\frac{dr}{ds} \Big|_{\Pi^i = \text{const.}} = \frac{R_s^i - r}{s} \quad (25)$$

As $R_s^H > R_s^L$, $\forall s > 0$, H is always willing to pay a higher r for a marginal increase of s ; that is, H always has a steeper iso-profit curve than L .⁵ See Figure 3 for an illustration.

Again, a competitive bank offers a menu of contracts A and B , which satisfy the incentive compatibility constraints (17) and (18) as well as the bank's zero expected profit condition (20). B has a higher loan rate and a larger loan size than A . L has no incentive to choose B , as $r^{H*} > r^{L*}$, although s^{H*} is too large for him. Thus, L 's incentive compatibility constraint (17) is not binding, and H always obtains his first-best contract.

However, H 's incentive compatibility constraint (18) is binding if H is willing to accept A 's smaller-than-optimal loan size $s^{L*} < s^{H*}$ in order to pay A 's lower $r^{L*} < r^{H*}$. This applies if the difference between H 's and L 's first-best loan size (i.e., $\Delta s_{FB}^* := s_{FB}^{H*} - s_{FB}^{L*}$) is sufficiently small relative to the difference in their first-best loan rate ($\Delta r_{FB}^* := r_{FB}^{H*} - r_{FB}^{L*}$). Δr_{FB}^* increases with the difference in the probability of success $\Delta p := p^L - p^H$, while Δs_{FB}^* increases with the difference in the marginal return of H and L (i.e., $\Delta R_s := R_s^H - R_s^L$). Thus, H chooses L 's first-best contract if ΔR_s is sufficiently small relative to Δr_{FB}^* . To prevent this, under asymmetric information, contract A rations L 's loan size in order to deter H . Loan size $s^{L*} < s_{FB}^{L*}$ is rationed because, in the Nash-equilibrium, for a given r^{L*} , L would like to raise a larger loan.

⁵(25) satisfies the single-crossing property and has a maximum as $d^2r/ds^2|_{\Pi^i = \text{const.}} = (R_{ss}^i s - (R_s^i - r))/s^2 < 0$ as $R_s^i > 0$ and $R_{ss} < 0$, $\forall s > 0$.

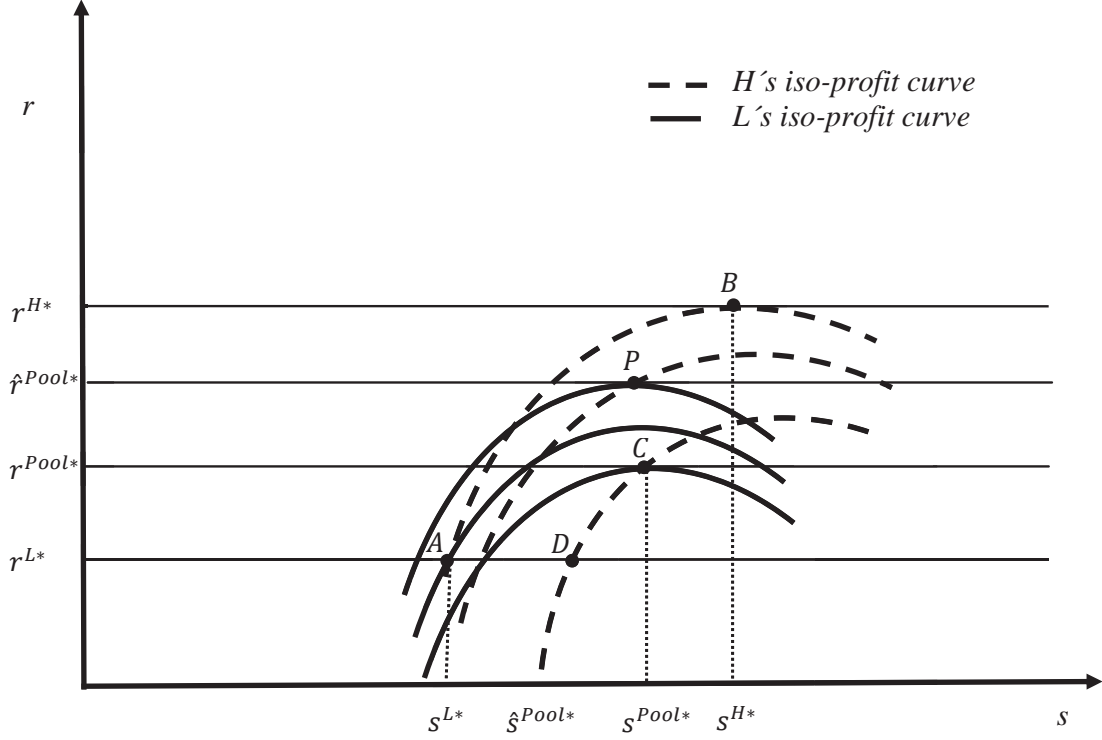


Figure 3: Nash-equilibrium in a competitive loan market with asymmetric information about indivisible investment technologies. The equilibrium is separating if H chooses contract B and L chooses contract A . If pooling interest rate r^{Pool} is sufficiently small, L and H will prefer pooling contract C in order to get on a lower iso-profit curve. In this case, deviation contract D attracts L . However, as this makes C unprofitable, H also chooses D , which, in turn, also becomes unprofitable. Thus, the bank offers no contracts, and a Nash-equilibrium does not exist.

This imposes opportunity costs on L , which increase with rationing of loan size. As soon as these costs are sufficiently high, specifically when

$$\epsilon_{\Pi_{L^*}^L(\bar{\tau}^L=1),s^L} > \epsilon_{\Pi_{L^*}^H(\bar{\tau}^L=1),s^L}, \quad (26)$$

in addition to loan size rationing, the bank rations L .

Such a Nash-equilibrium only exists if the pooling interest rate r^{Pool^*} is sufficiently high. In this case, pooling contract P does not constitute a contract in the Nash-equilibrium. The reason is that another bank can attract exclusively L with contract A which has a lower interest rate $r^{L^*} = r_{FB}^{L^*}$ and a smaller loan size s^{L^*} than P .

Otherwise, a Nash-equilibrium does not exist. In this case, r^{Pool^*} is sufficiently low, such that L prefers pooling contract C to A . However, C does not constitute a

Nash-equilibrium, as another bank can attract exclusively L with deviation contract D . Now, only H chooses C . Because the bank then makes negative profits with C , it chooses to stop offering it. Thus, H also chooses D . As a result, the bank also makes negative profits with D and then offers no loan contracts in a Nash-equilibrium. As r^{Pool*} decreases with α , a Nash-equilibrium does not exist for a sufficiently high $\alpha \geq \hat{\alpha}_{Div}$.

3 Concluding Remarks

Although A-R note that credit rationing is unlikely in the S-W model and Clemenz shows that it does not exist in the B-T model, this chapter shows that there is an appropriate framework for credit rationing. I analyze investment projects with technological characteristics similar to those in A-R's and S-W's model, but different from those in B-T's respectively Clemenz' model. In contrast to A-R and S-W, I consider loan policies that are incentive-compatible in the sense that they truthfully reveal the borrowers' privately known riskiness.

In the case of indivisible projects, rationing occurs if a riskier investment technology yields a higher return. In the case of divisible projects, rationing occurs only iff the marginal return on investment is sufficiently similar for both technologies relative to the difference in their riskiness. While high-risk borrowers always obtain their first-best contract, rationing occurs for low-risk borrowers. If the investment technology is divisible, the bank rations loan size. Only in the event that the rationing of loan size alone becomes too expensive does the bank also ration loan applicants.

These results suggest that future empirical research should pay more attention to borrowers' privately known investment technologies. Knowing the characteristics of these technologies would facilitate a better understanding of why rationing occurs in loan markets with imperfect information. Furthermore, it is left to future research to test empirically whether observationally identical but unobservably less risky borrowers are rationed more than their riskier counterparts.

Appendix

Proof of Proposition 1. Consider a competitive loan market with asymmetric information about indivisible investment technologies. In a competitive loan market, the bank makes zero profits in the Nash equilibrium. Thus, equation (1) can be solved

for

$$r^{i*} = \rho/p^i. \quad (27)$$

Recall from (7) that E_i then pays his first best loan rate, i.e. $r^{i*} = r_{FB}^{i*}$. As $p^L > p^H$, L has a lower first best loan rate than H , i.e. $r_{FB}^{L*} < r_{FB}^{H*}$ and L has no incentive to choose H 's first best loan contract. Thus, I can drop L 's incentive compatible constraint (4) and set $\tau^{H*} = 1$. However, H has an incentive to choose L 's first best loan contract. An incentive compatible loan policy must satisfy H 's incentive compatible constraint (5), which is written out

$$p^H (R^H - \rho/p^H) \geq \tau^L p^H (R^H - \rho/p^L). \quad (28)$$

Solve equation (28) for τ^L :

$$\tau^{L*} \leq \frac{R^H - \rho/p^H}{R^H - \rho/p^L}. \quad (29)$$

It is straightforward to see that τ^{L*} is always smaller than one as $p^L > p^H$.

Figure 2 illustrates that a Nash-equilibrium does only exist if L prefers separating loan contract A to pooling contract P

$$\Pi^L (\tau^{L*}, r^{L*}) > \Pi^L (\tau^{Pool*}, r^{Pool*}). \quad (30)$$

Recall that the bank does only ration to deter H . As P pools H and L rationing does not occur, i.e. $\tau^{Pool*} = 1$. Written out, inequality (30) is

$$\tau^{L*} p^L (R^L - r^{L*}) \geq p^L (R^L - r^{Pool*}). \quad (31)$$

Set (29), (27) and $r^{Pool*} = \alpha \rho/p^L + (1 - \alpha) \rho/p^H$ in (31) to get

$$(p^H R^H - \rho) (p^L R^L - \rho)^{-1} p^L (R^L - \rho/p^L) \geq p^L (R^L - (\alpha \rho/p^L + (1 - \alpha) \rho/p^H)). \quad (32)$$

Then, solve (32) for α

$$\alpha < \hat{\alpha}_{Indiv} := 1 - \rho^{-1} p^H (p^L R^L - p^H R^H) (p^L - p^H)^{-1}. \quad (33)$$

■

Proof of Proposition 2. To evaluate welfare of E_i under asymmetric information, compare it to the full information Nash-equilibrium. H does always obtain

the first best contract. While L is not rationed under full information, he is rationed under asymmetric information. To understand the effect of rationing on L 's welfare, differentiate L 's expected profit with respect to τ^L :

$$\partial \Pi^L / \partial \tau^L = p^L (R^L(s) - rs) > 0. \quad (34)$$

Thus, rationing of L decreases his expected profit, which is why he does not obtain his first best contract under asymmetric information. ■

Proof of Proposition 3. Consider a competitive loan market with asymmetric information about divisible investment technologies. In this market, a bank with expected profit function $\pi^{Bank}(r, \tau, s; p^i)$ offers a menu of loan contracts $(r, \tau, s) := (r^i, \tau^i, s^i)^i$ to E_i with contractual variables loan rate r , probability τ that E_i is rationed and size s . E_i 's expected profit function from this loan contract is $\pi^i(r, \tau, s)$. He chooses his optimal loan contract or outside option 0.

This proof is organized as follows. First, I show that L 's incentive compatible constraint (17) is never binding. Second, I show for which divisible investment technologies H 's incentive compatibility constraint (18) is not binding. Third, given that investment technologies are such that (18) is binding, I formulate the Lagrangian. I differentiate this Lagrangian with respect to contractual variables r, τ, s and Lagrange multipliers to determine the menu of loan contracts in the Nash-equilibrium. Finally, I show under which conditions a Nash-equilibrium does exist.

Proof that L 's incentive compatibility constraint is never binding. To proof this, analyze L 's incentive compatibility constraint (17) under full information. Written out, and reduced by p^L on both sides of the inequality, (17) is

$$R^L(s_{FB}^{L*}) - r_{FB}^{L*} s_{FB}^{L*} > R^L(s_{FB}^{H*}) - r_{FB}^{H*} s_{FB}^{H*}. \quad (35)$$

As $p^L > p^H$, L pays a lower loan rate than H , i.e. $r_{FB}^{L*} > r_{FB}^{H*}$. As s_{FB}^{L*} is a unique interior solution, L does not profit from another loan size. Concluding, L has no incentive to choose H 's first best loan contract.

Proof for which investment technologies H 's incentive compatibility constraint is not binding. To proof this, analyze H 's incentive compatibility constraint (18) under full information. Writing out (18) and reducing it by p^H on both

sides of the inequality results in

$$R^H (s_{FB}^{L*}) - r_{FB}^{L*} s_{FB}^{L*} > R^H (s_{FB}^{H*}) - r_{FB}^{H*} s_{FB}^{H*}. \quad (36)$$

Transform (36) to

$$R^H (s_{FB}^{H*}) - R^H (s_{FB}^{L*}) < (r_{FB}^{H*} s_{FB}^{H*} - r_{FB}^{L*} s_{FB}^{L*}). \quad (37)$$

As R_s^i is invertible, $R_s^i (s_{FB}^{i*}) = \rho/p^i$ can be transformed to $s_{FB}^{i*} = R_s^{i-1} (\rho/p^i)$. Thus, the first best s_{FB}^{i*} depends on R_s^i and p^i , i.e. $s_{FB}^{i*} (R_s^i, p^i)$ and I can rewrite (37) as

$$\begin{aligned} R^H (s_{FB}^{H*} (R_s^H, p^H)) - R^H (s_{FB}^{L*} (R_s^L, p^L)) < \\ (p^L s_{FB}^{H*} (R_s^H, p^H) - p^H s_{FB}^{L*} (R_s^L, p^L)) \frac{\rho}{p^H p^L}. \end{aligned} \quad (38)$$

In equation (37) I can see that H incurs opportunity costs from L 's smaller loan as $s_{FB}^{H*} > s_{FB}^{L*}$, but benefits from L 's lower loan rate, $r_{FB}^{H*} > r_{FB}^{L*}$. Now, regard equation (38). As for a higher $R_s^{i-1} (x)$, $\forall x$, s^{H*} becomes larger, $\Delta s_{FB}^* := s_{FB}^{H*} - s_{FB}^{L*}$ increases with $\Delta R_s := R_s^H - R_s^L$. As $r_{FB}^{i*} = \rho/p^i$, $\Delta r := r_{FB}^{H*} - r_{FB}^{L*}$ increases with $\Delta p := p^L - p^H$. Concluding, H 's incentive compatibility constraint (18) is only binding if ΔR_s is sufficiently small relative to Δp .

Lagrangian and FOC. As L 's incentive compatibility constraint (17) is never binding, it can be dropped. Then, H does always obtain his first best contract $(r^{H*}, \tau^{H*}, s^{H*})$, so that $\Pi_{H^*}^H := \Pi^H (r^{H*}, \tau^{H*}, s^{H*})$. In the following, suppose that ΔR_s is sufficiently small relative to Δp such that H 's incentive compatibility constraint (18) is binding. Then, L 's loan contract is chosen such that it maximizes his expected profit subject to (18). In brief, the Lagrangian is

$$\begin{aligned} L (r^L, \tau^L, s^L; \lambda_I, \lambda_B) = \\ \Pi^L (r^L, \tau^L, s^L) + \lambda_I [\Pi_{H^*}^H - \Pi^H (r^L, \tau^L, s^L)] + \lambda_B \Pi^{Bank} (r^L, \tau^L, s^L; p^L) \end{aligned} \quad (39)$$

and written out

$$\begin{aligned} L (r^L, \tau^L, s^L; \lambda_I, \lambda_B) = & \tau^L p^L (R^L (s^L) - r^L s^L) \\ & + \lambda_I [\Pi_{H^*}^H - \tau^L p^H (R^H (s^L) - r^L s^L)] \\ & + \lambda_B \tau^L (p^L r^L - \rho) s^L. \end{aligned} \quad (40)$$

To determine the bank's optimal contract, at first, calculate the first order conditions of (40) with respect to contractual variables (r^L, τ^L, s^L) and Lagrange multipliers λ_I and λ_B ⁶

$$\partial L / \partial r^L = \tau^L s^L (-p^L + \lambda_I p^H + \lambda_B p^L) = 0 \quad (41)$$

$$\partial L / \partial s^L = \tau^L [p^L (R_s^L - r^L) - \lambda_I p^H (R_s^H - r^L) + \lambda_B (p^L r^L - \rho)] = 0 \quad (42)$$

$$\partial L / \partial \tau^L = p^L (R^L - r^L s^L) - \lambda_I p^H (R^H - r^L s^L) + \lambda_B (p^L r^L - \rho) s^L = 0 \quad (43)$$

$$\partial L / \partial \lambda_I = \Pi_{H^*}^H - \tau^L p^H (R^H - r^L s^L) = 0 \quad (44)$$

$$\partial L / \partial \lambda_B = \tau^L (p^L r^L - \rho) s^L = 0. \quad (45)$$

Then, solve first order condition

- (41) for λ_I

$$\lambda_I = \frac{p^L}{p^H} (1 - \lambda_B); \quad (46)$$

- (42) for λ_B

$$\lambda_B = 1 - \frac{p^H}{p^L} \lambda_I. \quad (47)$$

As (46) and (47) only contain λ_I and λ_B , I can determine the Lagrange multipliers by setting

- (46) into (47) and solve for λ_B^*

$$\lambda_B^* = \frac{R_s^H - R_s^L}{R_s^H - r^L} = 1 - \frac{R_s^L - r^L}{R_s^H - r^L}; \quad (48)$$

- λ_B^* into (46) and solve for λ_I^*

$$\lambda_I^* = \frac{p^L (R_s^L - r^L)}{p^H (R_s^H - r^L)}. \quad (49)$$

Menu of loan contracts in the equilibrium. In the equilibrium, the bank offers the menu of loan contracts (r^{L*}, τ^{L*}, s^*) . Because of perfect competition, the bank solves (41) for r^{L*} and makes zero profits

$$r^{L*} = \rho / p^L. \quad (50)$$

⁶For brevity, I denote $R^L := R^L(s)$ for $i = L, H$.

Then, to determine the optimal contract, the bank first checks whether to ration L or not. Based on the initial situation that there is no rationing, i.e. $\hat{\tau}^L = 1$, the bank does not ration L if rationing of loan size, i.e. a decrease of s^L , yields lower opportunity costs for L than rationing of loan applicants, i.e. a decrease of τ^L . Then, L benefits more from a marginal increase of τ^L than of s^L :

$$\begin{aligned} p^L (R_s^L - r^L) - \lambda_I^* p^H (R_s^H - r^L) &\geq \\ p^L (R^L - r^L s^L) - \lambda_I^* p^H (R^H - r^L s^L). &\end{aligned} \quad (51)$$

Set (49) and (50) into (51)

$$\begin{aligned} p^L (R_s^L - \rho/p^L) - \frac{p^L (R_s^L - \rho/p^L)}{p^H (R_s^H - \rho/p^L)} p^H (R_s^H - \rho/p^L) &\geq \\ p^L (R^L - s^L \rho/p^L) - \frac{p^L (R_s^L - \rho/p^L)}{p^H (R_s^H - \rho/p^L)} p^H (R^H - s^L \rho/p^L) &\end{aligned} \quad (52)$$

It is straightforward to see that left side of equation (52) is zero. Transform (52) to get

$$\frac{p^H (R_s^H - \rho/p^L)}{p^H (R^H - s^L \rho/p^L)} \leq \frac{p^L (R_s^L - \rho/p^L)}{p^L (R^L - s^L \rho/p^L)} \quad (53)$$

If condition (53) holds, in the equilibrium, L is not rationed, i.e. $\tau^{L*} = 1$. Then, the bank sets λ_B^* and λ_I^* into (42) and chooses s^* such that

$$R_s^L (s^{L*}) - \frac{\rho}{p^L} = (R_s^H (s^{L*}) - R_s^L (s^{L*})) \frac{p^H}{p^L - p^H}. \quad (54)$$

There is an interior solution for L 's optimal loan size s^* . First, regard the left side of equation (54). L 's marginal return R_s^L decreases with s as $R_{ss}^L < 0$. Thus, the left side of equation (54) decreases with s . Second, regard the right side of equation (54). As $R_{B,ss} \geq R_{G,ss}$, term $R_{B,s} - R_{G,s}$ is non-decreasing with investment size. Thus, s^* can be chosen to equate the left and the right side of (54).

If condition (53) does not hold, the bank sets

- (49) and (48) into (43) to get

$$\frac{p^H (R_s^H (s^{L*}) - \rho/p^L)}{p^H (R^H (s^{L*}) - s^{L*} \rho/p^L)} \leq \frac{p^L (R_s^L (s^{L*}) - \rho/p^L)}{p^L (R^L (s^{L*}) - s^{L*} \rho/p^L)} \quad (55)$$

- (49) and (48) into (44) to get

$$\tau^{L*} = \frac{\Pi_{H^*}^H}{p^H (R^H - s^{L*} \rho / p^L)} \quad (56)$$

Conditions for existence of Nash-equilibrium. From Figure 3, I know that a Nash-equilibrium does only exist if L prefers the separating to the pooling contract, i.e. $\Pi^L (r^{L*}, s^{L*}) > \Pi^{L,Pool} (r^{Pool*}, s^{Pool*})$. The bank's expected profit function is

$$\Pi^{Bank,Pool} (r, s) := (p^{Pool} r - \rho) s. \quad (57)$$

Under a perfect competition, the bank makes zero profits. It chooses loan rate r^{Pool*} such that $\Pi^{Bank,Pool} (r, s) = 0$. Trivial transformations result in $r^{Pool*} = \rho (\alpha p^L + (1 - \alpha) p^H)^{-1}$. Thus, a higher fraction of L , α , decreases r^{Pool*} . A lower r^{Pool*} again increases L 's profit from the pooling loan contract $\Pi^{L,Pool} (r^{Pool*}, s^{Pool*})$. Thus, L chooses the separating loan contract if $\alpha < \hat{\alpha}_{Div}$ where

$$\hat{\alpha}_{Div} := \Pi^L (r^{L*}, s^{L*}) = \Pi^{L,Pool} (r^{Pool*}, s^{Pool*}) \quad (58)$$

with

$$r^{Pool*} = \rho (\hat{\alpha}_{Div} p^L + (1 - \hat{\alpha}_{Div}) p^H)^{-1}. \quad (59)$$

■

Proof of Proposition 4. To analyze welfare of E_i in a competitive loan market Nash-equilibrium, I compare his loan contract $(r^{i*}, \tau^{i*}, s^{i*})$ to his first best contract $(r_{FB}^{i*}, \tau_{FB}^{i*}, s_{FB}^{i*})$. While it is straightforward to see that $r^{H*} = r_{FB}^{H*}$ and that $\tau^{i*} < 1$ decreases welfare, I need to prove that $s^{L*} < s_{FB}^{L*}$. For that compare (54) and (24). They only differ in their right side. While the right side of (24) is zero, the right side of (54) is bigger than zero. Thus, $s^{L*} < s_{FB}^{L*}$. ■

References

- Arnold, L.G., Riley, J.G., 2009. On the Possibility of Credit Rationing in the Stiglitz-Weiss Model. *The American Economic Review* 99:5, 2012-2021.
- Besanko, D., Thakor, A., 1987a. Collateral and Rationing: Sorting Equilibria in Monopolistic and Competitive Loan Markets. *International Economic Review*

28:3, 671-689.

- Besanko, D., Thakor, A., 1987b. Competitive Equilibrium in the Loan Market under Asymmetric Information. *Journal of Economic Theory* 42, 167-182.
- Bester, H., 1985. The Level of Investment in Credit Markets with Imperfect Information. *Journal of Institutional and Theoretical Economics* 141, 503-515.
- Blackwell, D.W., Winters, D.B., 1997. Banking Relationships and the Effect of Monitoring on Loan Pricing. *The Journal of Financial Research* 20:2, 275-289.
- Clemenz, G., 1993. A Note on the Nonexistence of a Rationing Equilibrium in the Besanko-Thakor Model. *International Economic Review* 34, 727-737.
- Cole, R., 1998. The Importance of Relationships to the Availability of Loan. *Journal of Banking and Finance* 22, 959-977.
- Milde, H., Riley, J.G., 1988. Signaling in Credit Markets. *The Quarterly Journal of Economics* 103:1, 101-129.
- Petersen, M.A., Rajan, R.G., 1994. The Benefits of Lending Relationships: Evidence for Small Business Data. *The Journal of Finance* 49:1, 3-37.
- Petersen, M.A., Rajan, R.G., 1995. The Effect of Credit Market Competition on Lending Relationships. *The Quarterly Journal of Economics* 110, 407-443.
- Schmidt-Mohr, U., 1997. Rationing versus Collateralization in Competitive And monopolistic Loan Markets with Asymmetric Information. *The European Economic Review* 41, 1321-1342.
- Stiglitz, J., Weiss, A., 1981. Credit Rationing in Markets with Hidden Information. *The American Economic Review* 71:3, 393-410.