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Social welfare and wage inequality in search equilibrium with personal contacts

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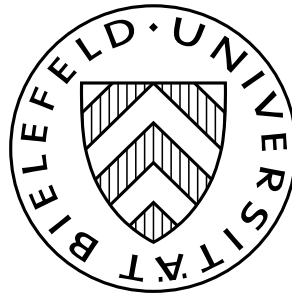
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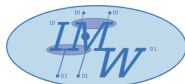
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Social Welfare and Wage Inequality in Search Equilibrium with Personal Contacts

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Abstract

This paper incorporates job search through personal contacts into an equilibrium matching model with a segregated labour market. Job search in the public submarket is competitive which is in contrast with the bargaining nature of wages in the informal job market. Moreover, the social capital of unemployed workers is endogenous depending on the employment status of their contacts. This paper shows that the traditional Hosios (1990) condition continues to hold in an economy with family contacts but it fails to provide efficiency in an economy with weak ties. This inefficiency is explained by a network externality: weak ties yield higher wages in the informal submarket than family contacts. Furthermore, the spillovers between the two submarkets imply that wage premiums associated with personal contacts lead to higher wages paid to unemployed workers with low social capital but the probability to find a job for those workers is below the optimal level.

JEL classification: J23, J31, J64, D10

Keywords:

Personal contacts, family job search, social capital, wages, equilibrium efficiency

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1 Introduction

The aim of this article is to develop a labour market matching model with personal contacts and to investigate the implications of network effects for the equilibrium welfare and wage inequality. The seminal approach to address the question of equilibrium efficiency in modern labour economics is laid down in the contributions by Diamond, Mortensen and Pissarides¹. In a standard search and matching framework Hosios (1990) and later Pissarides (2000) explain the fundamentals of congestion externalities and prove existence of a unique value of the bargaining power parameter delivering efficiency to the decentralized equilibrium. Congestion externalities are internalized at the optimal value of the bargaining power. However an important limitation of this framework is an atomistic structure of the society where the possibility of information exchange in a group of connected workers is largely ignored.

Economic consequences of personal contacts and social networks are analyzed in a different strand of literature dating back to the original papers by Granovetter (1973, 1995) and Montgomery (1991, 1992, 1994). Montgomery (1994) considers a continuum of workers grouped in pairs in a Markov model of employment transitions. He demonstrates that an increase in weak-tie interactions reduces inequality in the employment rates and has a positive effect on the equilibrium welfare if inbreeding by employment status among weak ties is sufficiently low. Nevertheless the model is set in a partial equilibrium framework so that the effects of personal contacts on job creation and recruitment strategies by firms are not taken into account.

This paper combines the two strands of literature in a natural way by embedding the social structure of Montgomery (1994) into the traditional labour market model with search frictions. This allows to consider the implications of personal contacts for wage inequality and social welfare in a unified general equilibrium framework with endogenous wages and job-finding rates. The central issues addressed in this study are then the interaction between a search and a network externality and the channels of spillovers between the public and the informal job market. Wages in the public job market are set competitively, exploiting the fact that a more generous wage offer attracts a larger number of applications. The concept of competitive search employed in this paper is originally introduced in Moen (1997). In contrast, vacancy information in the informal job market is exclusively transmitted through employed personal contacts so that wages are set ex-post via the mechanism of Nash bargaining.

¹Diamond (1982), Mortensen (1982), Pissarides (1984, 1985) and Mortensen and Pissarides (1994).

In order to simplify the model it is assumed that every worker in the labour market has exactly one social link, which can be interpreted as a close relative, a friend or an acquaintance, so the economy is populated by an exogenous number of two-person groups (dyads). In the benchmark model of the paper a pair of connected individuals are fully sharing their labour income and therefore are treated as a single family or a household (strong ties). The model is further extended to relax the assumption of income sharing, which allows to analyze the inherent difference of a personal contact being a strong or a weak tie.

From the perspective of labour demand there is a free-entry of firms both into the public and the informal job market. Upon the decision to enter the labour market firms face a trade-off between a high cost vacancy in the public job market with a large number of searching unemployed workers versus a low cost vacancy only available to workers with an employed personal contact. The closest study to analyze social welfare in an equilibrium search model with a free-entry of firms is Cahuc and Fontaine (2009). The choice of search methods by firms is also endogenous in their model, however there is only one search method prevailing in the equilibrium, whereas in this study both search methods are simultaneously used by workers with employed social contacts.

The model predictions can be summarized in the following way. First of all, the model implies wage dispersion among equally productive risk-neutral workers. This is due to the ex-post differentiation of unemployed workers by social capital, which can be high or low, defined by the employment status of their contact. Only unemployed workers with high social capital have an additional access to the informal job market through their contacts, so the reservation wage of those workers is high. Wage competition between firms opening vacancies in the public job market combined with the endogenous differentiation of unemployed workers results in a segmentation of the public job market. Firms in low wage segment target at unemployed workers with low social capital and a low reservation wage, while the opposite is true for firms in a high wage segment.

Further, this paper considers the question of social welfare in an economy with personal contacts and shows that competitive search equilibrium with strong ties and bargaining in the informal job market is constraint efficient for the Hosios value of the bargaining power. The new contribution of this paper is to prove that wage dispersion between workers with high and low social capital in the public job market is maximized

for the efficient value of the bargaining power. If the bargaining power parameter is low, meaning that wages paid in jobs obtained through personal contacts are low, then a higher value of this parameter has an enlarging effect on wage dispersion in the public job market. The functional relationship between the bargaining power and wage dispersion is reversed if the bargaining power parameter is large. This also means that both wage penalties and wage premiums in the informal job market lead to higher wages paid to unemployed workers with low social capital but the probability to find a job for those workers is below the optimal level.

The model is then extended to relax the assumption of income-sharing within a pair of connected workers. This allows to treat workers as friends or acquaintances helping each other to find a job, so the two economies with strong and weak ties can be compared. In the extended model workers bargaining over wages in the informal job market do not internalize the positive externality imposed on their social contacts inducing firms to pay higher wages. As a consequence competitive search equilibrium with weak ties and bargaining in the informal job market is not efficient at the Hosios value of bargaining power: too few job vacancies are filled in the informal job market. The implications of the described network externality for the public job market are twofold. At low values of the bargaining power the network externality has a neutralizing effect on the externality from search frictions. Workers with low social capital gain from a higher probability to find a job in the low wage segment of the public job market but their wages are lower. On the contrary, workers with high social capital face a lower job-finding rate but are compensated by higher wages. The overall effect on output is positive but these effects are reversed when the bargaining power parameter is above the efficient level.

Finally, theoretical predictions of the model are confronted with the empirical evidence. In general, the role of social networks and personal contacts is strongly emphasized in the empirical literature. The recent contributions are summarized in table 1 and show that between one and two-thirds of the employees in different countries have obtained their current job with a help of a friend or a relative². More specifically, the model predicts a positive correlation in the employment status of connected workers, both relatives and friends. On the empirical level the impact of family ties is closely investigated by Kramarz and Nordstrom Skans (2010). Their results show that a significant proportion of young employees in Sweden work for the same firm as their parents.

²An overview of the early empirical literature before 1990 is presented in Bewley (1999).

The effect of the employment status of friends is analyzed in Capellari and Tatsiramos (2010) who find that in the United Kingdom an additional employed friend increases the probability of finding a job by 3.7%.

Study	Incidence	Wage effects	Country
Staiger (1990)	40%	Positive	United States
Granovetter (1995)	56%	Positive	United States
Pistaferri (1999)	47%	Negative	Italy
Addison, Portugal (2002)	47%	Negative	Portugal
Margolis, Simonnet (2003)	36%	Positive	France
Delattre, Sabatier (2007)	34%	Negative	France
Bentolila, Michelacci, Suarez (2008)	31% 50%	Negative Negative	European Union United States
Ponzo, Scoppa (2010)	31%	Negative	Italy
Pelizzari (2010)	38%*	Positive Negative	Belgium, Netherlands Finland, Portugal, Italy, UK

* – average for the European Union, 14 countries

Table 1: Empirical evidence on job search through personal contacts

The next theoretical prediction of the model concerns the effect of personal contacts on wages. Wages in the informal job market are set ex-post as a result of individual bargaining which is different from competitive wage setting in the public job market. Therefore personal contacts in the model can lead to penalties or premiums in wages depending on the parameter of bargaining power. Nevertheless the model predicts lower wages in the informal job market when personal contacts are strong rather than weak ties. Indeed, when bargaining workers account for the gain of a connected worker only if their labour income is shared, this has a weakening effect on their bargaining position and leads to lower wages. These predictions are similar to the empirical findings (see table 1), in particular, Pelizzari (2010) shows that in the European Union "... premiums and penalties to finding jobs through personal contacts are equally frequent and are of about the same size." (p. 1). However when the distinction between relatives and friends is explicit family ties tend to have a negative effect on wages (see Sylos Labini (2004), Delattre and Sabatier (2007), Kramarz and Nordstrom Skans (2010)).

The plan of the paper is as follows. Section 2 contains an overview of the related literature while section 3 explains notation and the general economic environment of the model. Section 4 contains the labour market model with strong ties which is compared to the economy with weak ties in section 5. Section 6 contains welfare analysis of the decentralized equilibrium, whereas section 7 concludes the paper.

2 Related literature

Early economic studies on social contacts are Montgomery (1991, 1992, 1994) and Mortensen and Vishwanath (1994). The focus of Montgomery (1991) is on the effect of asymmetric information on wage inequality in the presence of the "inbreeding bias", implying clustering of workers with respect to their ability type. As a result the equilibrium is characterized by the positive correlation between ability and wages. Mortensen and Vishwanath (1994) consider the population of workers differing with respect to the probability of receiving job offers through personal contacts, they show that wages paid in jobs obtained through personal contacts are more likely to be higher than wage offers obtained through a direct application. This conclusion is questioned in the recent empirical literature, and moreover, "both the models of Montgomery (1991) and Mortensen and Vishwanath (1994) ignore what may be the most important role for network: to increase the job offer arrival rate." (p. 7, Margolis and Simonnet (2003)).

Recent theoretical literature on personal contacts is represented by the studies of Calvo-Armengol and Jackson (2004, 2007), Fontaine (2004, 2007, 2008) and Bentolila et al. (2010). A larger overview of this literature can be found in Ioannides and Datcher Loury (2004). Calvo-Armengol and Jackson (2004) examine a model of the transmission of job information through a network of social contacts and show that information passing leads to positive correlation between the employment status of agents who are directly or indirectly connected in the network. This effect is also present in the current study but in a richer equilibrium framework with endogenous wages and job-finding rates permitting analysis of the equilibrium welfare.

Calvo-Armengol and Jackson (2007) extend their initial result by showing that networks of agents that start with a worse wage status will have higher drop-out rates and persistently lower wages. The negative effect of social networks on wages is also demonstrated in Bentolila et al. (2010). In particular they show that social contacts can generate a mismatch between the occupational choice and the productive advantage of the worker leading to wage penalties in jobs obtained through personal contacts.

The possibility of wage penalties is also included in the present study if the bargaining power of workers in the informal job market is low, however this wage effect is reversed if the bargaining power parameter is high. This framework is more general and allows to differentiate welfare implications of social networks in both types of labour markets with wage premiums and wage penalties.

Fontaine (2004) considers policies aiming at increasing individuals social capital by enlarging the access to networks and shows that such policies can increase the congestion externalities and induce firms to substitute employee referrals for job advertising. Eventually unemployment can increase and welfare decrease. The theoretical framework of this study is similar to the present work, but the results are different. The interaction between a search and a network externality is not discussed in Fontaine (2004) so the decentralized equilibrium is efficient under the Hosios value of the bargaining power. This is not the case in the present study where weak ties give rise to the inefficiency under the benchmark value of the bargaining power.

The paper is also related to the literature on search externalities and social welfare in an economy with heterogeneous agents. Gautier (2002) shows that mixing two types of workers (high and low skilled) in a single labour market generates additional pooling externalities. Blazquez and Jansen (2008) analyze welfare in this economy and prove that pooling compresses the wage distribution, therefore higher wages of low-ability workers discourage the creation of unskilled jobs. A straightforward extension of the present study to the case of random search and bargaining allows to conclude that the effect of pooling on job creation is reversed. Endogenous heterogeneity of workers with equal productivities leads to lower average wages in the public submarket and encourages firms to open jobs. This finding highlights the importance of the source of worker heterogeneity for the equilibrium efficiency.

Finally, this paper relates to the recent literature on family job search (joint search) represented by Guler et al. (2009) and Ek and Holmlund (2010). This literature shows that joint decisions by spouses give rise to different economic outcomes from the model of single agents. Nevertheless, the major focus of these studies is on income sharing within a family and the possibility to exchange job information between partners is not considered. Therefore this research study is the first to combine the literature on family job search with the literature on social networks. The combined approach shows that risk aversion is not a necessary condition to generate wage dispersion in a model of

family job search. Both this study and Fontaine (2008) show that information sharing between connected workers gives rise to endogenous wage dispersion even if workers are risk-neutral.

3 Labour market modeling framework

The labour market is characterized by the following properties. There is a unit mass of infinitely lived risk neutral workers and an endogenous number of firms, both workers and firms are ex-ante identical and discount the future at rate r . Every worker has exactly one social link, which can be interpreted as a close relative, a friend or an acquaintance. In the baseline model of the paper a pair of connected individuals is treated as a family with a full income-sharing within the household (strong ties). The model extension presented in section 5 considers consequences for the labour market once the income-sharing assumption is relaxed and pairs of connected workers are treated as friends or acquaintances helping each other to find a job (weak ties).

Every worker can be either unemployed, receiving the value of leisure z and searching for a job or employed and producing output $y > z$. Therefore all pairs of workers can be split into three mutually exhaustive groups: employed, mixed or unemployed. The total number of worker-pairs in each group is denoted p_e , p_m and p_u respectively:

$$p_e + p_m + p_u = 0.5$$

Every firm entering the labour market has an option to open a vacancy in the public job market with a high flow cost $c + \rho$ or in the informal job market with a low cost c . Vacancy information in the informal job market is transmitted through employed personal contacts, therefore only unemployed workers in mixed pairs have access to vacancies in the informal job market. In contrast every unemployed worker in the economy has access to vacancy information posted in the public job market. This creates a trade off for the firm: a costly public vacancy with a high number of searching workers $2p_u + p_m$ versus a low cost informal vacancy with a low number of searchers p_m . On-the-job search is prohibited, so that employed workers always forward job information to their unemployed contacts. This model structure implies that unemployed workers searching in the public job market are endogenously differentiated into two groups – with high or low social capital – depending on the employment status of a connected worker.

The concept of competitive search, which was originally introduced in Moen (1997), is used to model search frictions in the public job market. Here firms post vacancies with exact information about the wage, while workers observe vacancy information and direct their search to particular jobs. It is assumed that firms commit to the posted employment contract. This wage-setting mechanism provides foundations for the wage competition between employers: firms offering higher wages are more likely to fill their open vacancies as opposed to the firms with low wage offers.

Endogenous heterogeneity of unemployed workers combined with competitive search implies that the public labour market is segmented into the submarket with low wages w_0 and short waiting queues, targeting at workers with low social capital, and a submarket with high wages w_1 and longer waiting queues, targeting at workers with high social capital. Let v_0 and v_1 denote the total number of vacancies in a low and high wage submarket respectively. Both unemployed workers and firms correctly anticipate the number of job matches m_i and the market tightness θ_i , in each of the submarkets $i = 0, 1$:

$$m_0 = m(2p_u, v_0) \quad \theta_0 = \frac{v_0}{2p_u} \quad \text{and} \quad m_1 = m(p_m, v_1) \quad \theta_1 = \frac{v_1}{p_m}$$

In contrast to the public job market, wages obtained through personal contacts (w_2) are not competitive, but set ex-post via the concept of Nash bargaining. Therefore search through personal contacts is random with a total number of job matches m_2 and the market tightness θ_2 given by:

$$m_2 = m(p_m, v_2) \quad \theta_2 = \frac{v_2}{p_m}$$

The matching function m_i , $i = 0, 1, 2$ is assumed to be increasing in both arguments – unemployment and vacancies, concave, and exhibiting constant returns to scale. Then the job finding rate $\lambda(\theta_i)$ and the vacancy filling rate $q(\theta_i)$ are given by:

$$q(\theta_i) = \frac{m_i}{v_i} = q_0 \theta_i^{-\eta} \quad \lambda(\theta_i) = \theta_i q(\theta_i) = q_0 \theta_i^{1-\eta}, \quad i = 0, 1, 2$$

where $0 < \eta < 1$ is the elasticity of the job filling rate $q(\theta_i)$. Any job can be destroyed for exogenous reasons with a Poisson destruction rate δ . Upon a separation the worker becomes unemployed and the firm may open a new job.

4 Search equilibrium with personal contacts

4.1 Endogenous social capital

Let U and U_e denote asset values of unemployed workers with an unemployed and an employed partner respectively. In the following the concept of social capital is applied in order to distinguish the two types of unemployed workers³. The social capital is called high if the dyad partner of the worker is employed and transmits job information between the worker and the informal job market. The social capital is low if the dyad partner of the worker is unemployed. This means that the social capital is endogenous and is reflected in variables U and U_e .

Further, let W_u^i and W_e^i denote asset values of workers employed at wage w_i with an unemployed and an employed partner. Note that the subindex $\{u, e\}$ shows the employment status of a connected worker. Then, using the continuous time Bellman equations, asset values U , U_e , W_u^i and W_e^i can be written as:

$$rU = z + \lambda(\theta_0)(W_u^0 - U) + \lambda(\theta_0)(U_e - U) \quad (4.1)$$

$$rU_e = z + \lambda(\theta_1)(W_e^1 - U_e) + \lambda(\theta_2)(W_e^2 - U_e) - \delta(U_e - U) \quad (4.2)$$

$$rW_u^i = w_i - \delta(W_u^i - U) + (\lambda(\theta_1) + \lambda(\theta_2))(W_e^i - W_u^i), \quad i = 0, 1, 2 \quad (4.3)$$

$$rW_e^i = w_i - \delta(W_e^i - U_e) - \delta(W_e^i - W_u^i), \quad i = 0, 1, 2 \quad (4.4)$$

Labour market transitions for the special case $w_1 = w_2$ are illustrated in figure 1.

Consider an unemployed pair of workers, both partners are searching in the low wage segment of the public labour market with a job-finding rate $\lambda(\theta_0)$ and a wage w_0 . When either of the workers finds a job, the asset value of this worker is increased to the level W_u^0 with a corresponding job rent $R_u^0 \equiv W_u^0 - U$, while the surplus value of the connected worker is increased to U_e . The gain of the unemployed worker $\Delta U = U_e - U$ is twofold, on the one hand, the worker starts searching in a high wage segment of the public labour market with a high wage w_1 and the job-finding rate $\lambda(\theta_1)$, on the other, the worker obtains access to the informal job market through the employed personal contact. Value gain of the unemployed worker ΔU is then given by:

$$\Delta U = U_e - U = \frac{\lambda(\theta_1)R_e^1 + \lambda(\theta_2)R_e^2 - \lambda(\theta_0)R_u^0}{r + \delta + \lambda(\theta_0)} \quad (4.5)$$

where $R_e^1 = W_e^1 - U_e$, $R_e^2 = W_e^2 - U_e$ are, respectively, worker rents in the case of

³See Coleman (1988) for the definition of social capital.

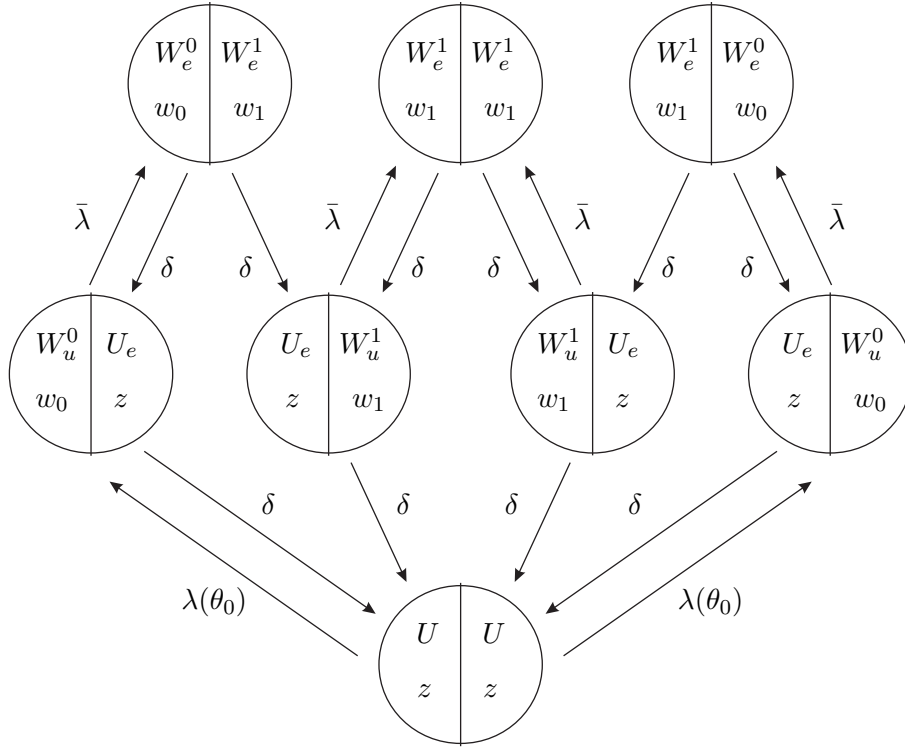


Figure 1: Competitive search with personal contacts, $\bar{\lambda} = \lambda(\theta_1) + \lambda(\theta_2)$, $w_2 = w_1$

accepting a job at wage w_1 in the public job market or a wage w_2 in the informal job market. However, not only unemployed workers gain from a better employment status of their partner. The gain of the employed worker in the event when the unemployed partner finds a job is denoted by $\Delta\Phi = W_e^i - W_u^i$, it results from the fact, that the partner will have a higher surplus value U_e rather than a low value U if the job is destroyed. Therefore the surplus gain $\Delta\Phi$ is given by:

$$\Delta\Phi = W_e^i - W_u^i = W_e^i - W_u^i = \frac{\delta\Delta U}{r + 2\delta + \lambda(\theta_1) + \lambda(\theta_2)} < \Delta U \quad (4.6)$$

Note that value gains of a connected worker ΔU and $\Delta\Phi$ are endogenous in the model.

4.2 Labour market with strong ties

Throughout the rest of section 4 consider an economy where dyad partners are fully sharing their income and therefore are treated as members of the same family and household. This is the case of strong social ties. Let P_u denote asset value of the unemployed household, so that $P_u = 2U$, similarly $P_m^j = U_e + W_u^j$ – asset value of the mixed household where one of the two family members is employed at wage w_j , $j = 0, 1, 2$. Finally let $P_e^{ij} = W_e^i + W_e^j$ denote surplus of the employed household earning wages w_i and w_j , $i, j = 0, 1, 2$. Then Bellman equations for P_u , P_m^j and P_e^{ij}

are written as:

$$rP_u = 2z + 2\lambda(\theta_0)(P_m^0 - P_u) \quad (4.7)$$

$$rP_m^j = z + w_j + \lambda(\theta_1)(P_e^{1j} - P_m^j) + \lambda(\theta_2)(P_e^{2j} - P_m^j) - \delta(P_m^j - P_u) \quad (4.8)$$

$$rP_e^{ij} = w_i + w_j - \delta(P_e^{ij} - P_m^i) - \delta(P_e^{ij} - P_m^j) \quad (4.9)$$

The net job rent of the unemployed household if one of the workers finds a job $P_m^0 - P_u$ can be expressed as follows:

$$(r + \delta)(P_m^0 - P_u) = z + w_0 - rP_u + \lambda(\theta_1)(P_e^{10} - P_m^0) + \lambda(\theta_2)(P_e^{20} - P_m^0) \quad (4.10)$$

For a given vector of variables $\{w_1, \theta_1, w_2, \theta_2\}$ and therefore for fixed surplus values $P_e^{10} - P_m^0$ and $P_e^{20} - P_m^0$ equation (4.7) describes an indifference curve of the unemployed household searching in the low wage segment of the public labour market. The household is indifferent between obtaining a higher wage w_0 yielding a higher job rent $P_m^0 - P_u$ combined with a low job-finding rate $\lambda(\theta_0)$ versus a low wage w_0 combined with a high job-finding rate $\lambda(\theta_0)$. The slope of the indifference curve of the unemployed household ($P_u = cst$) in the variable space $\{\theta_0, w_0\}$ is then obtained from:

$$\lambda'(\theta_0) \frac{d\theta_0}{dw_0} (P_m^0 - P_u) + \lambda(\theta_0) \frac{1}{r + \delta} = 0 \quad (4.11)$$

This indifference curve is decreasing and convex in the space $\{\theta_0, w_0\}$. The total job rent $P_m^0 - P_u$ can be decomposed into the personal gain of the worker R_u^0 and the partner's gain ΔU : $P_m^0 - P_u = R_u^0 + \Delta U$, it can then be expressed as:

$$P_m^0 - P_u = \frac{w_0 - z + \lambda(\theta_1)(P_e^{10} - P_m^0) + \lambda(\theta_2)(P_e^{20} - P_m^0)}{r + \delta + 2\lambda(\theta_0)} \quad (4.12)$$

Further, the net job rent of the mixed household $P_e^{i0} - P_m^0$, when one of the members is employed at wage w_0 and the unemployed member finds a job at wage w_i , can be expressed as:

$$(r + 2\delta)(P_e^{i0} - P_m^0) = w_i + w_0 - rP_m^0 + \delta \frac{w_i - w_0}{r + \delta}, \quad i = 1, 2 \quad (4.13)$$

Note that surplus values $P_e^{10} - P_m^0$ and $P_e^{20} - P_m^0$ are independent of variables $\{w_0, \theta_0\}$ for a given value of P_u , which also means that these surplus values do not depend on the partner's wage:

$$P_e^{10} - P_m^0 = \frac{w_1 - z - \lambda(\theta_2)(P_e^{20} - P_m^0) + \delta(P_m^1 - P_u)}{r + 2\delta + \lambda(\theta_1)} \quad (4.14)$$

$$P_e^{20} - P_m^0 = \frac{w_2 - z - \lambda(\theta_1)(P_e^{10} - P_m^0) + \delta(P_m^2 - P_u)}{r + 2\delta + \lambda(\theta_2)} \quad (4.15)$$

Therefore all of the unemployed workers in mixed households search in the same high wage segment of the public labour market. This simplification of the model is attributed to the assumption of risk neutrality. The total gain of the household $P_e^{i0} - P_m^0$ can be similarly decomposed into the gain of the worker and the gain of the partner: $P_e^{i0} - P_m^0 = R_e^i + \Delta\Phi$.

For a given vector of variables $\{w_0, \theta_0, w_2, \theta_2\}$ the indifference curve of the mixed household where one worker is employed at wage w_0 is given by: $P_m^0 = cst$. Unemployed family members in a mixed household face a similar trade off between a high wage w_1 and therefore a high rent value $P_e^{10} - P_m^0$ combined with a low job arrival rate $\lambda(\theta_1)$ versus a low wage w_1 combined with a high job arrival rate $\lambda(\theta_1)$. The slope of the indifference curve $P_m^0 = cst$ in the space $\{\theta_1, w_1\}$ is then given by:

$$\lambda'(\theta_1) \frac{d\theta_1}{dw_1} (P_e^{10} - P_m^0) + \lambda(\theta_1) \frac{1}{r + \delta} = 0$$

This indifference curve is similarly decreasing and concave in the variable space $\{\theta_1, w_1\}$, however, it will be shown later that $P_e^{10} - P_m^0$ is smaller than $P_m^0 - P_u$ despite the fact that $w_1 > w_0$. Indeed given the equal productivity of workers, it should be the case that the rent gain of a household with a better outside option is lower than the gain of a household with a worse outside opportunity. This means that the indifference curve $P_u = cst$ is flatter than $P_m^j = cst$ in the space $\{\theta, w\}$.

4.3 Firms: wage determination

Firms are free to open a vacancy in the public labour market with a flow cost $c + \rho$ or in the informal market with a lower cost c . In addition, firms can freely choose between the two segments within the public labour market. Let V^0 and V^1 denote asset values of an open vacancy in a low/high wage segment of the public labour market, respectively, and V^2 – vacancy value in the informal job market. Bellman equations for V^0 , V^1 and V^2 are then given by:

$$rV^i = -(c + \rho) + q(\theta_i)(J^i - V^i), \quad i = 0, 1 \quad (4.16)$$

$$rV^2 = -c + q(\theta_2)(J^2 - V^2) \quad (4.17)$$

where J^0 , J^1 and J^2 are the corresponding asset values of a filled job:

$$rJ^i = y - w_i - \delta J^i \quad i = 0, 1, 2 \quad (4.18)$$

Upon the decision to open a vacancy in the public job market firms face a similar trade-off as households. Paying a higher wage w_i , $i = 0, 1$ should be compensated by a higher probability to fill the job $q(\theta_i)$. It can be shown that the firm's indifference curves $V^i = cst$ are downward-sloping and convex in the space $\{\theta_i, w_i\}$. For given values $\{w_1, \theta_1, w_2, \theta_2\}$ denoted as information set I_0 firms in the low wage segment maximize their surplus V^0 , with respect to a combination $\{\theta_0, w_0\}$ and subject to the worker indifference curve $P_u = cst$:

$$V^0(P_u, I_0) = \max_{w_0, \theta_0} V^0(w_0, \theta_0) \quad \text{s.t.} \quad P_u(w_0, \theta_0, I_0) = cst \quad (4.19)$$

Solution of this maximization problem with a free-entry of firms meaning that in the equilibrium $V^0 = 0$ gives rise to the following rent-sharing condition:

$$J^0 = \frac{(1 - \eta)}{\eta} (P_m^0 - P_u), \quad \text{where} \quad P_m^0 - P_u = R_u^0 + \Delta U \quad (4.20)$$

This equation is an extension of the result by Moen (1997) for the case of family job search. The wage w_0 is then given by:

$$w_0 = \eta y + (1 - \eta)[rU - (\lambda(\theta_1) + \lambda(\theta_2))\Delta\Phi - (r + \delta)\Delta U] \quad (4.21)$$

$$= \eta y + (1 - \eta)[rU - (r + 2\delta)(\Delta U - \Delta\Phi)] \quad (4.22)$$

There are two new terms in the reservation wage of the worker. The first of them, $(\lambda(\theta_1) + \lambda(\theta_2))\Delta\Phi$ is a future gain of the worker once the partner finds a job, while the second $(r + \delta)\Delta U$ is an immediate gain of the partner due to the possibility to search in the informal job market. Both gains act to reduce the reservation wage of the worker. Intuitively individuals are ready to work for lower wages if their partners and household members gain from additional job opportunities.

Similarly for given values $\{w_0, \theta_0, w_2, \theta_2\}$ denoted as information set I_1 firms in the high wage segment maximize their surplus V^1 with respect to a combination $\{w_1, \theta_1\}$ and subject to the worker indifference curve $P_m^0 = cst$:

$$V^1(P_m^0, I_1) = \max_{w_1, \theta_1} V^1(w_1, \theta_1), \quad \text{s.t.} \quad P_m^0(w_1, \theta_1, I_1) = cst \quad (4.23)$$

This maximization problem combined with a free-entry requirement $V^1 = 0$ gives rise

to the following rent-sharing condition:

$$J^1 = \frac{(1-\eta)}{\eta}(P_e^{10} - P_m^0), \quad \text{where} \quad P_e^{10} - P_m^0 = R_e^1 + \Delta\Phi \quad (4.24)$$

and the following wage equation:

$$w_1 = \eta y + (1-\eta)[rU_e + \delta\Delta\Phi - (r+\delta)\Delta\Phi] \quad (4.25)$$

$$= \eta y + (1-\eta)[rU + r(\Delta U - \Delta\Phi)] \quad (4.26)$$

The first new term in the reservation wage $\delta\Delta\Phi$ is a future surplus loss of the worker once the partner loses the job, while the second term $(r+\delta)\Delta\Phi$ is an immediate gain of the partner. Here again the immediate surplus gain of the partner $\Delta\Phi$ is reducing the reservation wage of the worker. The partial equilibrium in the public job market is illustrated in figure 2. Firms are indifferent between the two segments since $V^0 = V^1 = 0$.

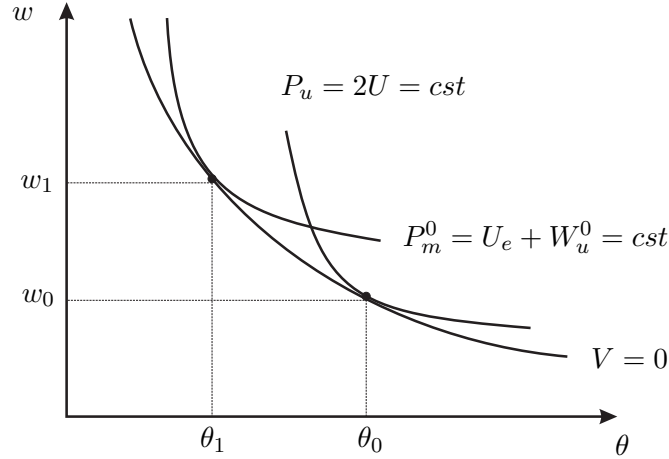


Figure 2: Segmentation in the public labour market

Comparison of equations (4.21) and (4.25) allows to evaluate the wage difference $w_1 - w_0$ showing the extent of wage dispersion in the public job market resulting from the introduction of personal contacts:

$$w_1 - w_0 = (1-\eta)2(r+\delta)(\Delta U - \Delta\Phi) \quad (4.27)$$

$$= (1-\eta)2(r+\delta) \left[\frac{r+\delta+\lambda(\theta_1)+\lambda(\theta_2)}{r+2\delta+\lambda(\theta_1)+\lambda(\theta_2)} \right] \Delta U \quad (4.28)$$

The more valuable is the access to the informal job market ΔU , the higher is the dif-

ference in the reservation wages of unemployed workers with different levels of social capital $2(r + \delta)(\Delta U - \Delta \Phi)$. By implication, a higher difference in the reservation wages leads to a higher wage dispersion in the public job market $w_1 - w_0$.

In the informal job market wages are determined ex-post, after the meeting between the firm and the unemployed worker. I use the concept of Nash bargaining in order to determine wage w_2 , the rent-sharing condition with $V^2 = 0$ and β denoting the worker's bargaining power is then:

$$J^2 = \frac{(1 - \beta)}{\beta}(P_e^{20} - P_m^0), \quad \text{where} \quad P_e^{20} - P_m^0 = R_e^2 + \Delta \Phi \quad (4.29)$$

with the following equation for wage w_2 :

$$w_2 = \beta y + (1 - \beta)[rU + r(\Delta U - \Delta \Phi)] \quad (4.30)$$

Clearly $w_2 = w_1$ if and only if $\beta = \eta$ and $w_2 > (<)w_1$ if and only if $\beta > (<)\eta$. Intuitively, job search through personal contacts is associated with wage premiums if $\beta > \eta$ and it leads to wage penalties otherwise. This completes the analysis of wages.

4.4 The decentralized equilibrium

The free entry of firms into every of the three submarkets implies that in the equilibrium $V^i = 0$, $i = 0, 1, 2$. Inserting these conditions into the asset value equations for V^i produces the following:

$$\frac{c + \rho}{q(\theta_i)} = J^i, \quad i = 0, 1 \quad \frac{c}{q(\theta_2)} = J^2 \quad (4.31)$$

The left hand-side of these equations is the expected cost of opening a vacancy, since $q(\theta_i)$, $i = 0, 1, 2$, describes expected duration of the open vacancy. Expected cost of a vacancy in the equilibrium should be equal to the present value of flow profits from a filled job J^i . The rent-splitting equations (4.20), (4.24) and (4.29) imply that firms in the low wage segment of the public job market obtain fraction $(1 - \eta)$ of the total job surplus $S^0 \equiv J^0 + P_m^0 - P_u$. Firms in the high wage segment obtain a similar fraction of the total job surplus $S^1 \equiv J^1 + P_e^{10} - P_m^0$, while firms operating in the informal job market obtain a fraction $(1 - \beta)$ of the total surplus $S^2 = J^2 + P_e^{20} - P_m^0$, this means:

$$\frac{c + \rho}{q(\theta_i)} = (1 - \eta)S^i, \quad i = 0, 1 \quad \frac{c}{q(\theta_2)} = (1 - \beta)S^2 \quad (4.32)$$

A larger surplus value S^i attracts more entrants into the submarket, which is reflected in a higher value of the market tightness θ_i , $i = 0, 1, 2$. Equations (4.10) and (4.13) allow to rewrite surplus values S^0 , S^1 and S^2 as follows:

$$\begin{aligned}(r + \delta)S^0 &= y + z - rP_u + \lambda(\theta_1)(P_e^{10} - P_m^0) + \lambda(\theta_2)(P_e^{20} - P_m^0) \\ (r + 2\delta)S^1 &= y + w_0 - rP_m^0 + \delta J^0 \quad \text{and} \quad S^2 = S^1\end{aligned}$$

Note that the total surplus S^2 does not directly depend on the exact surplus split between the firm and the worker and therefore does not directly depend on β , which means that $S^1 = S^2$. This equality allows to express the market tightness θ_2 in the equilibrium as a linear function of θ_1 :

$$q(\theta_2) = \frac{c(1 - \eta)}{(c + \rho)(1 - \beta)}q(\theta_1) \quad \Rightarrow \quad \theta_2 = \theta_2(\theta_1), \quad \frac{\partial \theta_2(\theta_1)}{\partial \theta_1} > 0$$

Intuitively a larger surplus $S^1 = S^2$ has a positive effect on both variables θ_1 and θ_2 . Moreover it can be shown that the benchmark case $\beta = \eta$ implies that $\theta_2 > \theta_1$. If $w_1 = w_2$ then more firms exploit the cost advantage of the informal job market. Further, it can be shown that there exists a threshold value $\hat{\beta}$ such that:

$$\begin{cases} \text{if } \beta > \hat{\beta} & \text{then } \theta_2 < \theta_1 \\ \text{if } \beta < \hat{\beta} & \text{then } \theta_2 > \theta_1 \end{cases} \quad \text{where} \quad \hat{\beta} = \eta + \frac{\rho(1 - \eta)}{c + \rho}$$

Using the functional relationship $\theta_2 = \theta_2(\theta_1)$ allows to simplify the characterization of the equilibrium to a vector of variables $\{\theta_0, \theta_1\}$. Using expressions $P_m^0 - P_u = \eta S^0$, $P_e^{20} - P_m^0 = \eta S^1$ and $P_e^{10} - P_m^0 = \beta S^1$ allows the following reformulation of surplus variables S^0 and S^1 :

$$S^0 = \frac{y - z + [\eta\lambda(\theta_1) + \beta\lambda(\theta_2)]S^1}{r + \delta + 2\lambda(\theta_0)\eta} \quad (4.33)$$

$$S^1 = \frac{y - z + \delta S^0}{r + 2\delta + \eta\lambda(\theta_1) + \beta\lambda(\theta_2)} \quad (4.34)$$

The system of equations (4.33)-(4.34) describes spillovers between the submarkets. Consider a worker with an unemployed partner, a larger surplus gain S^1 created in the event when the unemployed partner finds a job (at rate $\lambda(\theta_1)$ or $\lambda(\theta_2)$) has a positive effect on the current surplus value of this worker and therefore on the total surplus value S^0 . Now consider a worker with an employed partner, a larger surplus loss S^0 in the event when the partner loses the job (at rate δ) has a negative effect on the reservation value of the household P_m^0 and therefore a positive effect on the current surplus value S^1 . Lemma 1 describes the effects of variables θ_0 and θ_1 on surplus val-

ues $S^0(\theta_0, \theta_1)$ and $S^1(\theta_0, \theta_1)$.

Lemma 1: Denote $\alpha(\theta_1) = \lambda(\theta_1) + \frac{\beta}{\eta}\lambda(\theta_2(\theta_1))$ – weighted job-finding rate for workers with high social capital, so that $\alpha'(\theta_1) > 0$ then total surplus values $S^0(\theta_0, \theta_1)$ and $S^1(\theta_0, \theta_1)$ can be expressed as follows:

$$S^0 = \frac{(y-z)(r+2\delta+2\eta\alpha(\theta_1))}{(r+2\eta\lambda(\theta_0))(r+2\delta+\eta\alpha(\theta_1))+\delta(r+2\delta)}$$

$$S^1 = \frac{(y-z)(r+2\delta+2\eta\lambda(\theta_0))}{(r+2\eta\lambda(\theta_0))(r+2\delta+\eta\alpha(\theta_1))+\delta(r+2\delta)}$$

Moreover, $S^0(\theta_0, \theta_1)$ is a decreasing function of θ_0 but an increasing function of θ_1 while $S^1(\theta_0, \theta_1)$ is decreasing in both arguments, formally:

$$\frac{\partial S^0(\theta_0, \theta_1)}{\partial \lambda(\theta_0)} < 0 \quad \frac{\partial S^0(\theta_0, \theta_1)}{\partial \alpha(\theta_1)} > 0 \quad \frac{\partial S^1(\theta_0, \theta_1)}{\partial \lambda(\theta_0)} < 0 \quad \frac{\partial S^1(\theta_0, \theta_1)}{\partial \alpha(\theta_1)} < 0$$

Proof: Differentiate surplus variable S^0 with respect to $\alpha(\theta_1)$:

$$\frac{\partial S^0(\theta_0, \theta_1)}{\partial \alpha(\theta_1)} = \frac{(y-z)\eta(r+2\delta)(r+2\delta+2\eta\lambda(\theta_0))}{[(r+2\eta\lambda(\theta_0))(r+2\delta+\eta\alpha(\theta_1))+\delta(r+2\delta)]^2} > 0$$

Differentiate surplus variable S^1 with respect to $\lambda(\theta_0)$:

$$\frac{\partial S^1(\theta_0, \theta_1)}{\partial \lambda(\theta_0)} = -\frac{(y-z)2\eta\delta(r+2\delta+2\eta\alpha(\theta_1))}{[(r+2\eta\lambda(\theta_0))(r+2\delta+\eta\alpha(\theta_1))+\delta(r+2\delta)]^2} < 0$$

Intuitively a larger job finding rate $\lambda(\theta_0)$ has a positive effect on the present value of the unemployed household P^u which has a direct negative effect on surplus $S^0 = J^0 + P_m^0 - P_u$. There is then a spillover into the high wage segment since a lower value of S^0 is reducing the surplus value S^1 . Second, a larger weighted job finding rate $\alpha(\theta_1)$ has a direct positive effect on S^0 but also a negative effect on $S^1 = J^1 + P_e^{10} - P_m^0$ due to a higher present value P_m^0 . Lemma 1 shows that the direct positive effect of $\alpha(\theta_1)$ on S^0 is dominating, however S^1 is lower despite a higher value of S^0 .

The main conclusion following from lemma 1 is that the free-entry condition in the low wage public market segment describes an increasing functional relationship between variables θ_0 and θ_1 . A higher probability to find a job for the partner θ_1 has a positive effect on the total job surplus S^0 , the fraction $1 - \eta$ of this surplus accrues to firms and therefore has a positive effect on the job creation θ_0 :

$$\frac{c+\rho}{q(\theta_0)} = (1-\eta)S^0(\theta_0, \theta_1) \Rightarrow \theta_0 = \theta_0(\theta_1), \quad \frac{\partial \theta_0(\theta_1)}{\partial \theta_1} > 0 \quad (JC_0)$$

In contrast the free-entry condition in the high wage public market segment describes a negative relationship between variables θ_0 and θ_1 – the higher the job finding rate $\lambda(\theta_0)$ which means the easier is it to find an initial job, the lower is the surplus of this job S^0 . This has a negative effect on the surplus S^1 and a lower job creation θ_1 :

$$\frac{c + \rho}{q(\theta_1)} = (1 - \eta)S^1(\theta_0, \theta_1) \Rightarrow \theta_1 = \theta_1(\theta_0), \quad \frac{\partial \theta_1(\theta_0)}{\partial \theta_0} < 0 \quad (JC_1)$$

The unique intersection between the increasing curve $\theta_0(\theta_1)$ and the decreasing curve $\theta_1(\theta_0)$ allows to obtain the equilibrium values of θ_0 and θ_1 , this is illustrated in figure 3. The equilibrium is defined in the following way:

Definition 1 *A competitive search equilibrium with strong ties and bargaining in the informal job market is a vector of variables $\{P_u, P_m^j, P_e^{ij}, V^i, J^i, w_i, \theta_i\}$, $i, j = 0, 1, 2$ satisfying the asset value equations for workers (4.7), (4.8), (4.9), for firms (4.16), (4.17), (4.18), the three rent-sharing equations (4.20), (4.24), (4.29) and the free-entry conditions $V^i = 0$.*

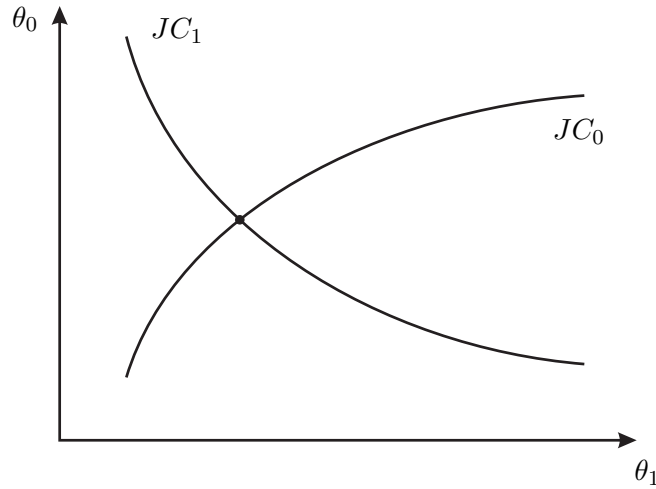


Figure 3: Equilibrium values of θ_0 and θ_1

Proposition 1 shows that there exists a unique search equilibrium with strong ties.

Proposition 1: *There exists a unique competitive search equilibrium with strong ties and bargaining in the informal job market described in definition 1. The equilibrium market tightness variables $\{\theta_0, \theta_1\}$ are obtained from the following system of equations:*

- (a.) *The job creation curve in the public job market (JC_0) describes a positive rela-*

tionship between the market tightness variables θ_0 and θ_1 , specifically:

$$\frac{c + \rho}{q(\theta_0)} = \frac{(1 - \eta)(y - z)(r + 2\delta + 2\eta\alpha(\theta_1))}{(r + 2\eta\lambda(\theta_0))(r + 2\delta + \eta\alpha(\theta_1)) + \delta(r + 2\delta)}$$

(b.) The job creation curve in the public job market (JC_1) describes a negative relationship between the market tightness variables θ_0 and θ_1 , specifically:

$$\frac{c + \rho}{q(\theta_1)} = \frac{(1 - \eta)(y - z)(r + 2\delta + 2\eta\lambda(\theta_0))}{(r + 2\eta\lambda(\theta_0))(r + 2\delta + \eta\alpha(\theta_1)) + \delta(r + 2\delta)}$$

(c.) wage dispersion in the public job market Δw is given by:

$$\Delta w = \frac{2(1 - \eta)\eta(r + \delta)(y - z)(\alpha(\theta_1) - \lambda(\theta_0))}{(r + 2\eta\lambda(\theta_0))(r + 2\delta + \eta\alpha(\theta_1)) + \delta(r + 2\delta)}$$

Wage dispersion $\Delta w = w_1 - w_0$ is increasing in θ_1 and decreasing in θ_0 .

Proof: Parts (a) and (b) follow directly from lemma 1. For part (c) differentiate $\Delta\bar{w} = \Delta w / (2(1 - \eta)\eta(r + \delta)(y - z))$ with respect to θ_1 :

$$\frac{\partial \Delta\bar{w}}{\partial \theta_1} = \frac{(r + 2\eta\lambda(\theta_0))(r + 2\delta + \eta\lambda(\theta_0)) + \delta(r + 2\delta)}{[(r + 2\eta\lambda(\theta_0))(r + 2\delta + \eta\alpha(\theta_1)) + \delta(r + 2\delta)]^2} > 0$$

Corollary 1: *Competitive equilibrium in the family search model with bargaining in the informal job market entails positive wage dispersion among equally productive risk-neutral workers. In particular:*

$$\begin{cases} w_2 < w_0 = w_1 & \text{if } \beta = 0 \\ w_0 < w_1 & w_2 < w_1 & \text{if } 0 < \beta < \eta \\ w_0 < w_1 = w_2 & \text{if } \beta = \eta \\ w_0 < w_1 < w_2 & \text{if } \eta < \beta < 1 \\ w_0 = w_1 < w_2 & \text{if } \beta = 1 \end{cases} \quad (4.35)$$

Corollary 1 shows that interior values of the bargaining power parameter $0 < \beta < 1$ lead to a segmented public labour market, so that $w_0 < w_1$. However, this segmentation disappears at the corner values of the bargaining power $\beta = 0$ and $\beta = 1$. This is due to the fact that the option to search in the informal job market $\lambda(\theta_2)(P_e^{2j} - P_m^j)$ has zero value at $\beta = 0$ and $\beta = 1$. If the bargaining power of workers is zero, then the total household surplus $P_e^{2j} - P_m^j$ is zero, whereas if β is converging to 1, too few firms will use the informal job market to post vacancies, so that θ_2 is converging to zero.

Moreover, Ek and Holmlund (2010) prove that risk aversion is necessary to generate endogenous wage dispersion among equally productive workers in a model with family job search. Corollary 1 extends this result by showing that information sharing between household members can generate wage dispersion even if workers are risk neutral.

4.5 The equilibrium unemployment

Let p_u , p_m and p_e denote the number of unemployed, mixed and employed households respectively. The equilibrium values of these variables can be obtained from the following system of differential equations:

$$\begin{cases} \dot{p}_u = \delta p_m - 2\lambda(\theta_0)p_u \\ \dot{p}_e = (\lambda(\theta_1) + \lambda(\theta_2))p_m - 2\delta p_e \\ 0.5 = p_u + p_m + p_e \end{cases} \quad (4.36)$$

In the stationary equilibrium the inflow of households into a particular state should be equal to the outflow of households from this state, namely $\dot{p}_u = 0$, $\dot{p}_m = 0$, $\dot{p}_e = 0$. The number of households of each type is then:

$$p_u = \frac{0.5\delta^2}{[\delta^2 + \lambda(\theta_0)(2\delta + \lambda(\theta_1) + \lambda(\theta_2))]}$$

$$p_m = \frac{\delta\lambda(\theta_0)}{[\delta^2 + \lambda(\theta_0)(2\delta + \lambda(\theta_1) + \lambda(\theta_2))]}$$

and $p_e = 0.5 - p_u - p_m$. The number of unemployed households is falling in any of the job finding rates $\lambda(\theta_i)$, $i = 0, 1, 2$, in contrast, the number of employed households is increasing. The effects on the number of mixed households are inversely directed: p_m is falling in $\lambda(\theta_1)$ and $\lambda(\theta_2)$ but it is increasing in $\lambda(\theta_0)$.

Further consider the special case $\beta = \eta$, so that the wage distribution in the equilibrium is binary $w_1 = w_2$. Lemma 2 shows the distribution of workers by income categories $\{z, w_0, w_1\}$:

Lemma 2: *Let $\beta = \eta$, then the equilibrium unemployment rate u and the fraction of workers employed at wage w_1 denoted f are given by:*

$$u = \frac{\delta(\delta + \lambda(\theta_0))}{[\delta^2 + \lambda(\theta_0)(2\delta + \lambda(\theta_1) + \lambda(\theta_2))]} \quad f = \frac{\lambda(\theta_1) + \lambda(\theta_2)}{\delta + \lambda(\theta_1) + \lambda(\theta_2)}$$

The fraction $1 - f$ of workers are employed at wage w_0 .

Proof: The equilibrium unemployment is given by $u = 2p_u + p_m$, while the total number of workers e_0, e_1 employed at wages w_0, w_1 are given by:

$$e_0 = \frac{2\delta(p_m + p_e)}{2\delta + \lambda(\theta_1) + \lambda(\theta_2)} \quad e_1 = \frac{(\lambda(\theta_1) + \lambda(\theta_2))(p_m + p_e)}{2\delta + \lambda(\theta_1) + \lambda(\theta_2)} + p_e$$

The fraction f is then $e_1/(e_0 + e_1)$.

Note that the equilibrium unemployment is falling in all of the job finding rates $\lambda(\theta_i)$, $i = 0, 1, 2$, but the wage distribution $\{1 - f, f\}$ is independent of the job finding rate $\lambda(\theta_0)$.

In addition, for the special case $\beta = \eta$, it can be shown that the conditional probability of being unemployed for a worker with an employed contact $P\{u|e\}$ is lower than the conditional probability of being unemployed with an unemployed contact $P\{u|u\}$:

$$P\{u|e\} = \frac{0.5p_m}{p_e + 0.5p_m} = \frac{\delta}{\delta + \lambda(\theta_1) + \lambda(\theta_2)} < P\{u|u\} = \frac{\delta}{\delta + \lambda(\theta_0)}$$

so the labour market exhibits a positive correlation in the employment status of workers within one family. This result follows from $\Delta w > 0$ and so $\lambda(\theta_1) + \lambda(\theta_2) > \lambda(\theta_0)$.

4.6 Comparative statics

Empirical studies presented in section 1 show that wages in jobs obtained through personal contacts can be higher or lower than wages obtained through a direct job application. This section addresses the effect of the bargaining power β , and therefore the effect of wage w_2 , on market tightness variables, wages and wage dispersion in the public job market. Clearly a larger bargaining power parameter β has a positive direct effect on wage w_2 and a negative effect on θ_2 since a larger wage in the informal job market reduces the number of open vacancies. The spillovers of this effect into the public job market are summarized in proposition 2:

Proposition 2: *The economic effects of a larger bargaining power $0 < \beta < 1$ in the informal submarket on variables $\{\theta_0, \theta_1, \theta_2, w_0, w_1, w_2\}$ are summarized in table 2.*

Proof: Appendix I.

The effect of a higher bargaining power β is additionally illustrated on figure 4. For the corner case $\beta = 0$ jobs obtained through personal contacts pay exactly the

$\uparrow \beta$	θ_0	θ_1	θ_2	w_0	w_1	w_2
$\beta < \eta$	+	-	-	-	+	+
$\beta > \eta$	-	+	-	+	-	+

Table 2: Economic effects of a higher bargaining power

reservation wage of the worker and for this reason do not add any additional value, so that $w_0 = w_1$. The situation is similar for $\beta = 1$ meaning that $\theta_2 = 0$. For $0 < \beta < \eta$, the term $\beta\lambda(\theta_2)$ is falling in β due to a lower value of θ_2 that has a negative effect on $\alpha(\theta_1) = \lambda(\theta_1) + \frac{\beta}{\eta}\lambda(\theta_2)$ – the weighted job finding rate. This is raising the total job surplus S^0 and the market tightness θ_0 (see lemma 1). In contrast a lower value of S^1 implies a lower job creation θ_1 . There is then a reverse prediction for wages w_0 and w_1 . The situation is exactly the opposite for $\eta < \beta < 1$ when the effect of a higher bargaining power is dominating so the term $\beta\lambda(\theta_2)$ is increasing. The above analysis shows that wage dispersion in the public job market Δw achieves maximum at $\beta = \eta$ when the economic effect of the informal job market is maximized.

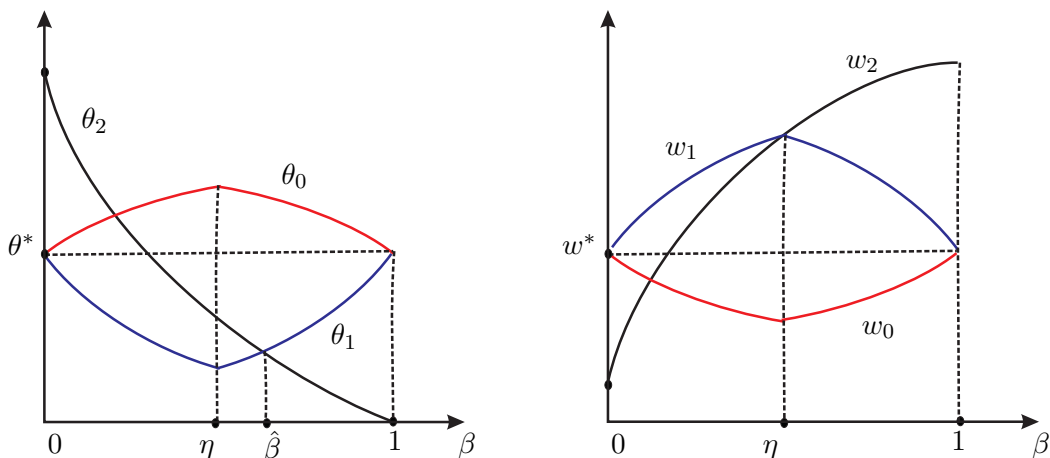


Figure 4: Economic effects of a higher bargaining power parameter β

On the intuitive level figure 4 shows that both wage premiums and wage penalties associated with personal contacts lead to higher wages paid to unemployed workers with low social capital but the probability to find a job for those workers is reduced.

5 Competitive search equilibrium with weak ties

This section investigates the role of income-sharing and the implications of this assumption for the equilibrium outcomes. If the assumption of full income-sharing is relaxed, a pair of connected workers can then be seen as friends or acquaintances (weak ties). Consider the bargaining problem between a firm and a worker in the informal

job market. The total job surplus of the worker is given by:

$$(r + \delta)R_e^2 = w_2 - rU_e - \delta\Delta\Phi$$

Nash bargaining implies that the joint surplus $R_e^2 + J^2$ is shared in the proportion β , so that $\beta J^2 = (1 - \beta)R_e^2$. This gives rise to the following wage equation:

$$w_2 = \beta y + (1 - \beta)[rU + r(\Delta U - \Delta\Phi) + (r + \delta)\Delta\Phi]$$

Compare this to equation (4.30). Wage w_2 is higher *ceteris paribus* in an economy with weak ties due to the additional term $(r + \delta)\Delta\Phi > 0$. When the flow income is shared within a pair of connected workers the total job surplus is $R_e^2 + J^2 + \Delta\Phi$, so the worker obtains a share β of the total surplus net of the gain of a connected worker, specifically $R_e^2 = \beta(R_e^2 + J^2 + \Delta\Phi) - \Delta\Phi = \beta(R_e^2 + J^2) - (1 - \beta)\Delta\Phi$. Clearly this surplus is lower than $\beta(R^2 + J^2)$ in an economy with weak ties, so that workers in the informal job market demand higher wages when their dyad partner's gain is not taken into account.

In a public job market firms in a low wage segment maximize their surplus $V^0(w_0, \theta_0)$ subject to the worker indifference curve $U = cst$. The slope of this indifference curve is identical to the slope of $2U = cst$ which is an indifference curve of a connected pair of unemployed workers. This means that there are no changes in the equation for w_0 , and the total job surplus is split according to $\eta J^0 = (1 - \eta)(R_u^0 + \Delta U)$.

In a high wage segment of the public job market firms maximize their surplus $V^1(w_1, \theta_1)$ subject to the worker indifference curve $U_e = cst$:

$$V^1(U_e, I_1) = \max_{w_1, \theta_1} V^1(w_1, \theta_1) \quad \text{s.t.} \quad U_e(w_1, \theta_1, I_1) = cst$$

$$\text{where} \quad (r + \delta)U_e = z + \lambda(\theta_1)R_e^1 + \lambda(\theta_2)R_e^2 + \delta U$$

The worker indifference curve $U_e = cst$ is steeper in the space $\{\theta, w\}$ than the household indifference curve $U_e + W_u^i = cst$. This gives rise to lemma 3:

Lemma 3: *The slope of the worker indifference curve $U_e(w_1, \theta_1, I_1) = cst$, where I_1 denotes an information set $\{w_0, \theta_0, w_2, \theta_2\}$, is given by:*

$$\lambda'(\theta_1) \frac{d\theta_1}{dw_1} R_e^1 + \frac{\lambda(\theta_1)}{r + \delta} + k\lambda'(\theta_1) \frac{d\theta_1}{dw_1} \Delta\Phi = 0 \quad (5.37)$$

$$\text{where } \frac{1}{k} = 1 + \frac{r}{\delta} \left[\frac{(r + 2\delta)(r + \lambda(\theta_0) + \delta)}{(\lambda(\theta_1) + \lambda(\theta_2))(r + 2\lambda(\theta_0)) + \lambda(\theta_0)(r + 2\delta)} \right] > 1,$$

This gives rise to the following equilibrium equation for w_1 :

$$w_1 = y\eta + (1 - \eta)[rU + r(\Delta U - \Delta\Phi) + (r + \delta)\Delta\Phi(1 - k)]$$

Proof: Appendix II.

Consider an increase in w_1 along the worker indifference curve $U_e = cst$. In the absence of income-sharing, the direct effect of changes in w_1 on the surplus of a connected worker W_u^i is not taken into account, whereas it is accounted for along the household indifference curve $U_e + W_u^i = cst$. The corresponding drop in the surplus of a connected worker is attributed to a lower flow probability for the partner to find a job $\lambda(\theta_1)$:

$$(r + \delta)W_u^i = w_i + \delta U + (\lambda(\theta_1) + \lambda(\theta_2))\Delta\Phi$$

so that

$$(r + \delta)\frac{\partial W_u^i}{\partial w_1} = \lambda'(\theta_1)\frac{\partial\theta_1}{\partial w_1}\Delta\Phi + (\lambda(\theta_1) + \lambda(\theta_2))\frac{\partial\Delta\Phi}{\partial w_1}$$

since U is constant along the household indifference curve $U_e + W_u^i = cst$:

$$U = \frac{\lambda(\theta_0)}{r + 2\lambda(\theta_0)}(U_e + W_u^0)$$

In contrast, the asset value of an unemployed worker U is decreasing along the worker indifference curve $U_e = cst$:

$$\left[r + \frac{(r + \lambda(\theta_0))(r + \delta)}{\lambda(\theta_0)} \right] \frac{\partial U}{\partial w_1} = \lambda'(\theta_1)\frac{\partial\theta_1}{\partial w_1}\Delta\Phi + (\lambda(\theta_1) + \lambda(\theta_2))\frac{\partial\Delta\Phi}{\partial w_1}$$

For $r \rightarrow 0$, expression in the square brackets is converging to δ , so that the indirect effect of a higher w_1 on U and therefore on U_e is converging to the direct effect of w_1 on W_u^i . The difference between the stronger direct and the weaker indirect effect is reflected in the auxiliary variable k , such that $\lim_{r \rightarrow 0} k = 1$. Therefore the wage w_1 is *ceteris paribus* higher than in the economy with household job search due to the additional term $(r + \delta)\Delta\Phi(1 - k)$ but this difference becomes negligibly small for $r \rightarrow 0$. In the following in order to focus on the network externality stemming from the informal job market consider the limiting case $r \rightarrow 0$.

In the absence of income-sharing the surplus splitting rule in the informal job market implies $J^2 = (1 - \beta)(J^2 + R_e^2) = (1 - \beta)(S^2 - \Delta\Phi)$, where $S^2 = J^2 + R_e^2 + \Delta\Phi$ is the total job surplus. Denote $\tilde{\beta}$ – the implied bargaining power, such that $J^2 = (1 - \tilde{\beta})S^2$. The implied bargaining power $\tilde{\beta}$ is a fraction of the firm surplus J^2 in the total job surplus S^2 and can be obtained from:

$$\tilde{\beta} = \beta + \phi(\theta_0, \theta_1, \theta_2)(1 - \beta), \quad \text{where} \quad \phi(\theta_0, \theta_1, \theta_2) = \frac{\Delta\Phi}{S^2} \quad (5.38)$$

Variable $\phi(\theta_0, \theta_1, \theta_2)$ is a fraction of the gain of a connected worker $\Delta\Phi$ in the total job surplus S^2 . The implied bargaining power $\tilde{\beta}$ is larger than β for $0 < \beta < 1$ and captures changes in the informal job market stemming from a relaxed assumption of income sharing. If $\beta = 0$ wage dispersion in the public job market Δw is zero implying $\Delta\Phi = 0$ and $\tilde{\beta} = \beta = 0$. Similarly if $\beta = 1$ it is true that $w_2 = y$, so that differences in the reservation wages of workers are not reflected in wage w_2 . This means that $\tilde{\beta} = \beta = 1$.

Proposition 3 shows that the labour market equilibrium with weak social ties (for $r \rightarrow 0$) is a special case of the equilibrium with strong social ties where the total job surplus S^2 is split in the proportion $\tilde{\beta}$ between the worker and the firm:

Proposition 3: *Consider the case $r \rightarrow 0$, competitive search equilibrium without income sharing is a special case of the search equilibrium described in definition 1 with the implied bargaining power parameter $\tilde{\beta}$:*

$$\tilde{\beta} = \beta + (1 - \beta) \frac{\delta(\eta\lambda(\theta_1) + \beta\lambda(\theta_2) - \eta\lambda(\theta_0))}{2(\delta + \lambda(\theta_0)\eta)(\delta + \lambda(\theta_1) + \lambda(\theta_2)) - (1 - \beta)\delta\lambda(\theta_2)} \quad (5.39)$$

The equilibrium market tightness variables $\{\theta_0, \theta_1, \theta_2\}$ are obtained from the following system of equations:

(a.) *The job creation condition in the low wage segment of the public job market:*

$$\frac{c + \rho}{q(\theta_0)} = \frac{(1 - \eta)(y - z)(\delta + \eta\lambda(\theta_1) + \tilde{\beta}\lambda(\theta_2))}{\eta\lambda(\theta_0)(2\delta + \eta\lambda(\theta_1) + \tilde{\beta}\lambda(\theta_2)) + \delta^2}$$

(b.) *The job creation condition in the high wage segment of the public job market:*

$$\frac{c + \rho}{q(\theta_1)} = \frac{(1 - \eta)(y - z)(\delta + \eta\lambda(\theta_0))}{\eta\lambda(\theta_0)(2\delta + \eta\lambda(\theta_1) + \tilde{\beta}\lambda(\theta_2)) + \delta^2}$$

(c.) *The job creation condition in the informal job market:*

$$q(\theta_2) = \frac{c(1-\eta)}{(c+\rho)(1-\tilde{\beta})}q(\theta_1)$$

Proof: Appendix II.

Proposition 3 proves that relaxing the income-sharing assumption is equivalent to an exogenous shift of the bargaining power in the informal job market from β to $\tilde{\beta}$. Intuitively, the gain of a connected worker $\Delta\Phi$ is ignored in the bargaining process, this strengthens the bargaining position of workers and leads to higher wages w_2 in the informal job market. The implications of the income sharing assumption for the equilibrium efficiency are considered in the next section.

6 Social optimum: welfare maximization

Hosios (1990) and further Pissarides (2000) show, that the Nash wage equation is not likely to internalize search externalities resulting from the dependence of transition probabilities $\lambda(\theta_i)$ and $q(\theta_i)$ on the tightness of the market. Nevertheless Hosios (1990) proves that search externalities may be internalized, if $\beta = \eta$, where η is the elasticity of the job-filling rate $q(\theta_i)$. Further Moen (1997) strengthens this finding by showing that competitive search equilibrium gives rise to a worker surplus share of η , so the equilibrium in his model is constrained efficient.

This section investigates efficiency properties of the competitive search equilibrium with bargaining in the informal job market, and shows that the classical Hosios efficiency condition continues to hold in an economy with strong ties but it fails to deliver efficiency in the economy with weak social ties. To obtain this result, consider the problem of a social planner, whose objective is to maximize the present discounted value of output minus the costs of job creation:

$$\max_{\theta_0, \theta_1, \theta_2} \int_0^{\infty} e^{-rt} \left[\begin{aligned} & p_u 2z + p_m(z+y) + p_e 2y \\ & -(c+\rho)\theta_0 2p_u - (c+\rho)\theta_1 p_m - c\theta_2 p_m \end{aligned} \right] dt \quad (6.40)$$

In addition, the social planner is subject to the same matching constraints as firms and workers, therefore the dynamics of unemployment is described by the system of differential equations (4.36). Proposition 4 presents solution of the planner optimization problem (6.40) subject to the equilibrium conditions \dot{p}_u and \dot{p}_e .

Proposition 4: Consider a social planner choosing the optimal number of vacancies $v_0 = 2p_u\theta_0$, $v_1 = p_m\theta_1$ – in the public job market and $v_2 = p_m\theta_2$ – in the informal job market. Then the optimal job creation is:

$$\frac{c + \rho}{q(\theta_0)} = (1 - \eta)\mu_u \quad \frac{c + \rho}{q(\theta_1)} = (1 - \eta)\mu_e \quad \frac{c}{q(\theta_2)} = (1 - \eta)\mu_e$$

where variables μ_u and μ_e are obtained from the following system of equations:

$$\mu_u = \frac{y - z + (c + \rho)[2\theta_0 - \theta_1] - c\theta_2 + (\lambda(\theta_1) + \lambda(\theta_2))\mu_e}{r + \delta + 2\lambda(\theta_0)} \quad (6.41)$$

$$\mu_e = \frac{y - z + (c + \rho)\theta_1 + c\theta_2 + \delta\mu_u}{r + 2\delta + \lambda(\theta_1) + \lambda(\theta_2)} \quad (6.42)$$

Proof: Appendix III.

Costate variables μ_u and μ_e in the dynamic optimization problem of the social planner correspond to the surplus values S^0 and $S^1 = S^2$ in the decentralized equilibrium. To see this insert the job creation conditions of the social planner $(c + \rho)\theta_0 = \lambda(\theta_0)(1 - \eta)\mu_u$, $(c + \rho)\theta_1 = \lambda(\theta_1)(1 - \eta)\mu_e$ and $c\theta_2 = \lambda(\theta_2)(1 - \eta)\mu_e$ into expressions for μ_u and μ_e :

$$\mu_u = \frac{y - z + (\lambda(\theta_1) + \lambda(\theta_2))\eta\mu_e}{r + \delta + 2\lambda(\theta_0)\eta} \quad (6.43)$$

$$\mu_e = \frac{y - z + \delta\mu_u}{r + 2\delta + (\lambda(\theta_1) + \lambda(\theta_2))\eta} \quad (6.44)$$

Direct comparison of these equations with (4.33) and (4.34) reveals that the decentralized labour market equilibrium with family search, competitive wage setting in the public submarket and bargaining in the informal submarket is constrained efficient if $\beta = \eta$. If the Hosios condition $\beta = \eta$ is not satisfied, welfare in the benchmark equilibrium is reduced due to the presence of congestion externalities in the informal job market: unemployed workers searching for jobs reduce the probability for the remaining unemployed workers to find a job. Similarly new vacancies open in the informal job market reduce the probability for other firms to hire a worker. Overall, economic agents on the same side of the market create negative search externalities, while agents on different sides of the market create positive search externalities for each other.

If the bargaining power parameter is low ($\beta < \eta$) welfare in the benchmark model with strong ties is reduced due to the fact that wages w_2 in the informal job market are too low, while the market tightness θ_2 is too high. In contrast, if the bargaining power is high ($\beta > \eta$) welfare is reduced as a result of wages w_2 being too high while the market tightness θ_2 is too low.

Welfare implications of the network externality in a model without income sharing can be analyzed on the basis of lemma 3, showing that a regime switch from full income sharing within a pair of connected workers to the economy without income sharing and under the simplifying assumption $r \rightarrow 0$, is equivalent to an exogenous shift in the bargaining power parameter from β to $\tilde{\beta}$. This means that at low values of β network externality has a reducing effect on the congestion externality through a positive effect on wages w_2 . Higher wages attract more firms to the informal job market implying a higher value of θ_2 . The overall welfare is then increased as a result of higher output produced in the informal job market. On the contrary, if the bargaining power parameter is high ($\beta > \eta$) network externality has an amplifying effect on the congestion externality. This means an additional upward distortion on wages w_2 corresponding to a lower market tightness θ_2 and a lower equilibrium welfare. These results are summarized in corollary 2:

Corollary 2: *Let $\beta = \eta$, where $\eta = -(\partial q(\theta)/\partial \theta)(\theta/q(\theta))$ – elasticity of the job filling rate $q(\theta)$, then the decentralized competitive search equilibrium with bargaining in the informal job market and*

- *strong social ties described in proposition 1 is constrained efficient;*
- *weak social ties described in proposition 3 is inefficient. Job creation in the informal job market θ_2 and in the low wage segment of the public job market θ_0 are both insufficient, while θ_1 is excessive for $r \rightarrow 0$.*

Corollary 2 shows that welfare in the decentralized search equilibrium with weak ties does not achieve the maximum level at the Hosios value of the bargaining power $\beta = \eta$. The reason for this inefficiency is a positive wage bias in the informal job market, where the implied bargaining power of workers $\tilde{\beta}$ is larger than $\beta = \eta$. This means that the optimal actual bargaining power β^* in an economy with weak ties can be obtained by setting $\tilde{\beta} = \eta$, so that:

$$\frac{\eta - \beta^*}{1 - \beta^*} = \frac{\delta \eta (\lambda(\theta_1) + \lambda(\theta_2) - \lambda(\theta_0))}{2(\delta + \lambda(\theta_0)\eta)(\delta + \lambda(\theta_1) + \lambda(\theta_2))} \quad (6.45)$$

where variables $\theta_0, \theta_1, \theta_2$ constitute an optimal choice of the social planner (for $r \rightarrow 0$) and can be obtained from proposition 3 by setting $\tilde{\beta} = \eta$ (see figure 5).

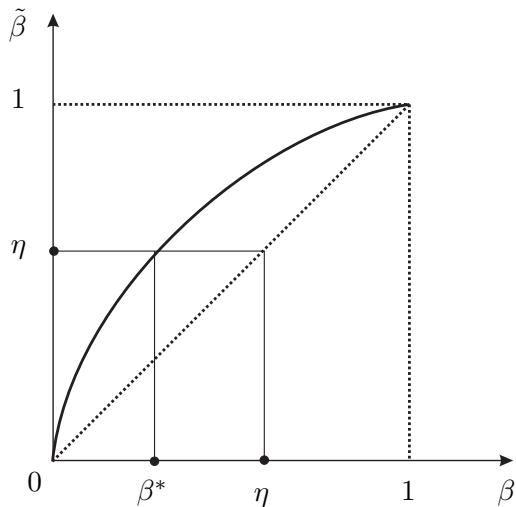


Figure 5: The optimal bargaining power in an economy with weak ties

The optimal bargaining power $\beta^* < \eta$ leads to $\tilde{\beta} = \eta$ and delivers constrained efficiency in a labour market model with weak ties.

7 Conclusions

This paper investigates the implications of job search through personal contacts on social welfare and wage dispersion in an equilibrium model with matching frictions. Upon entry firms have an option to post a high cost vacancy in the public job market or a low cost vacancy in the informal job market. Vacancy information in the informal submarket is only transmitted through employed personal contacts. In the benchmark model of the paper workers are grouped into a continuum of two-person households with a full income and information sharing between the members. Therefore this study combines the literature on joint job search with a focus on income sharing within a family and the literature on social networks with a focus on information sharing.

This paper shows that unemployed workers in mixed households gain from an additional option to screen jobs in the informal labour market, which is a result of information transmission from the employed to the unemployed household members. Ex-post differentiation of workers by social capital reflecting differences in the employment status of a connection gives rise to endogenous wage dispersion among equally productive risk-neutral workers. This is an extension of the result by Ek and Holmlund (2010)

stating that risk-aversion is a necessary condition for wage dispersion in an equilibrium model with family job search. Moreover, the model exhibits a positive correlation in the employment status of household members observed in a number of empirical studies.

Wages in the public job market are set via the mechanism of competitive search utilizing the link between the probability to fill a vacancy and the posted wage offer. Endogenous heterogeneity of workers with respect to their reservation wages induces a segmentation in the public job market. Firms in a low wage segment of the public job market target at workers with low social capital and unemployed personal contacts, in contrast firms in a high wage segment target at workers with high social capital. Wages in the informal job market are set through individual bargaining. This highlights the non-competitive nature of wages paid in jobs obtained through personal contacts and allows for the possibility of wage penalties or wage premiums between the public and the informal job market.

Further, this paper proves that search equilibrium with competition in the public job market and bargaining in the informal market is unique and constrained efficient at the Hosios value of the bargaining power parameter. The new contribution of the paper is then to show that the efficient resource allocation is associated with a maximum wage dispersion in the public job market. This is due to the fact that the total output created in the informal job market is maximized at the efficient allocation, implying the highest value of the employed social contact granting access to the informal market and leading to the maximum heterogeneity in the reservation wages of workers.

The model is further extended to relax the assumption of income sharing, a pair of connected workers can then be interpreted as friends or acquaintances helping each other to find a job (weak ties). From the perspective of positive analysis this paper shows that weak ties yield higher wages in the informal submarket than family contacts, which is supported by the existing empirical evidence. Considered from the normative perspective, higher wages in jobs obtained through personal contacts lead to lower job creation in the informal job market, so the optimal bargaining power in an economy with weak ties is below the traditional Hosios value. If the bargaining power of workers is low, the network inefficiency is neutralizing the classical inefficiency from search frictions. This leads to an unambiguous increase in the total output. The effects on workers with different levels of social capital are however adverse. Unemployed workers with low social capital gain from a higher probability to find jobs in a low wage segment

of the public job market despite a corresponding reduction in wages. On the contrary, unemployed workers with high social capital are confronted with a lower probability to find a job, but are compensated by higher wages. Furthermore welfare and output are reduced and the effects on workers with different level of social capital are reversed if the bargaining power parameter and wages in the informal job market are high.

8 Appendix

Appendix I: Proof of proposition 2:

Differentiate the job creation condition JC_0 with respect to β :

$$-\frac{(c + \rho)q'(\theta_0)}{q^2(\theta_0)} \frac{\partial \theta_0}{\partial \beta} = (1 - \eta) \left[\frac{\partial S^0}{\partial \theta_0} \frac{\partial \theta_0}{\partial \beta} + \frac{\partial S^0}{\partial \alpha(\theta_1)} \frac{\partial \alpha(\theta_1)}{\partial \beta} \right]$$

Let $\eta_s^0 = -(\partial S^0 / \partial \theta_0)(\theta_0 / S^0)$ – elasticity of the total surplus S^0 with respect to θ_0 , $\eta_s^0 > 0$. This yields:

$$\frac{\partial \theta_0}{\partial \beta} (\eta + \eta_s^0) = \frac{\theta_0}{S^0} \frac{\partial S^0}{\partial \alpha(\theta_1)} \frac{\partial \alpha(\theta_1)}{\partial \beta}$$

This means that the sign of $\partial \theta_0 / \partial \beta$ is the same as the sign of $\partial \alpha(\theta_1) / \partial \beta$. Now differentiate the job creation condition JC_2 to obtain:

$$\eta \frac{\partial \theta_1}{\partial \beta} \frac{S^1}{\theta_1} = \frac{\partial S^1}{\partial \theta_0} \frac{\partial \theta_0}{\partial \beta} + \frac{\partial S^1}{\partial \alpha(\theta_1)} \frac{\partial \alpha(\theta_1)}{\partial \beta}$$

Insert expression for $\partial \theta_0 / \partial \beta$ and let $\eta_s^1 = -(\partial S^1 / \partial \theta_0)(\theta_0 / S^1)$ – elasticity of the total surplus S^1 with respect to θ_0 , $\eta_s^1 > 0$. This yields:

$$\eta \frac{\partial \theta_1}{\partial \beta} = \left[\frac{\partial S^1}{\partial \alpha(\theta_1)} \frac{\theta_1}{S^1} - \frac{\eta_s^1}{\eta + \eta_s^0} \frac{\partial S^0}{\partial \alpha(\theta_1)} \frac{\theta_1}{S^0} \right] \frac{\partial \alpha(\theta_1)}{\partial \beta}$$

Denote expression in the square bracket by Ω , so that:

$$\Omega \equiv \frac{\partial S^1}{\partial \alpha(\theta_1)} \frac{\theta_1}{S^1} - \frac{\eta_s^1}{\eta + \eta_s^0} \frac{\partial S^0}{\partial \alpha(\theta_1)} \frac{\theta_1}{S^0} < 0$$

As follows from lemma 1 Ω is negative, so the sign of $\partial \theta_1 / \partial \beta$ is opposite to the sign of $\partial \alpha(\theta_1) / \partial \beta$. Further differentiate equation $q(\theta_2) = c(1 - \eta)q(\theta_1) / [(c + \rho)(1 - \beta)]$ with respect to β to obtain:

$$\frac{\partial \theta_2}{\partial \beta} \frac{\beta}{\theta_2} = \frac{\partial \theta_1}{\partial \beta} \frac{\beta}{\theta_1} - \frac{\beta}{\eta(1 - \beta)}$$

Therefore if $\partial\theta_1/\partial\beta < 0$, it follows that $\partial\theta_2/\partial\beta < 0$. Differentiate variable $\eta\alpha(\theta_1) = \eta\lambda(\theta_1) + \beta\lambda(\theta_2)$ with respect to β :

$$\eta \frac{\partial\alpha(\theta_1)}{\partial\beta} = \eta\lambda'(\theta_1) \frac{\partial\theta_1}{\partial\beta} + \beta\lambda'(\theta_2) \frac{\partial\theta_2}{\partial\beta} + \lambda(\theta_2)$$

This equations implies that if $\partial\theta_1/\partial\beta > 0$ and therefore $\partial\alpha(\theta_1)/\partial\beta < 0$ then it is true that $\partial\theta_2/\partial\beta < 0$. Insert expression for $\partial\theta_2/\partial\beta$ to obtain:

$$\eta \frac{\partial\alpha(\theta_1)}{\partial\beta} = \frac{\partial\theta_1}{\partial\beta} \left[\eta\lambda'(\theta_1) + \beta\lambda'(\theta_2) \frac{\theta_2}{\theta_1} \right] + \lambda(\theta_2) \left[1 - \frac{\beta(1-\eta)}{\eta(1-\beta)} \right]$$

Finally, insert $\partial\alpha(\theta_1)/\partial\beta$ into the equation for $\partial\theta_1/\partial\beta$:

$$\frac{\partial\theta_1}{\partial\beta} \left(\eta^2 - \Omega \left[\eta\lambda'(\theta_1) + \beta\lambda'(\theta_2) \frac{\theta_2}{\theta_1} \right] \right) = \Omega\lambda(\theta_2) \frac{\eta - \beta}{\eta(1-\beta)}$$

Since $\Omega < 0$ expression in the square bracket on the left-hand side is positive, therefore the sign of $\partial\theta_1/\partial\beta$ is solely determined by the sign of $\Omega(\eta - \beta)$ on the right hand-side, which is negative if $\beta < \eta$ and it is positive if $\beta > \eta$. This means:

$$\begin{aligned} \frac{\partial\theta_1}{\partial\beta} < 0 \quad \frac{\partial\theta_2}{\partial\beta} < 0 \quad \frac{\partial\alpha(\theta_1)}{\partial\beta} > 0 \quad \frac{\partial\theta_0}{\partial\beta} > 0 \quad \text{if } \beta < \eta \\ \frac{\partial\theta_1}{\partial\beta} > 0 \quad \frac{\partial\theta_2}{\partial\beta} < 0 \quad \frac{\partial\alpha(\theta_1)}{\partial\beta} < 0 \quad \frac{\partial\theta_0}{\partial\beta} < 0 \quad \text{if } \beta > \eta \end{aligned}$$

Appendix II: Proof of lemma 3. Worker surplus values R_e^1 and R_e^2 are given by:

$$\begin{aligned} (r + 2\delta)R_e^1 &= w_1 - rU_e + \delta(W_u^1 - W_u^0) + \delta(W_u^0 - U_e) \\ (r + 2\delta)R_e^2 &= w_2 - rU_e + \delta(W_u^2 - W_u^0) + \delta(W_u^0 - U_e) \end{aligned}$$

which gives rise to the following derivative functions:

$$\frac{\partial R_e^1}{\partial w_1} = \frac{1}{r + \delta} + \frac{\delta}{r + 2\delta} \frac{\partial W_u^0}{\partial w_1} \quad \text{and} \quad \frac{\partial R_e^1}{\partial w_1} = \frac{\delta}{r + 2\delta} \frac{\partial W_u^0}{\partial w_1}$$

The worker indifference curve $U_e = cst$ is given by:

$$(r + \delta)U_e = z + \lambda(\theta_1)R_e^1 + \lambda(\theta_2)R_e^2 + \delta U$$

with a slope:

$$\lambda'(\theta_1) \frac{\partial\theta_1}{\partial w_1} R_e^1 + \lambda(\theta_1) \frac{\partial R_e^1}{\partial w_1} + \lambda(\theta_2) \frac{\partial R_e^2}{\partial w_1} + \delta \frac{\partial U}{\partial w_1} = 0$$

$$\lambda'(\theta_1) \frac{\partial \theta_1}{\partial w_1} R_e^1 + \frac{\lambda(\theta_1)}{r + \delta} + \delta \frac{\partial W_u^0}{\partial w_1} \left[\frac{(\lambda(\theta_1) + \lambda(\theta_2))}{r + 2\delta} + \frac{\lambda(\theta_0)}{r + 2\lambda(\theta_0)} \right] = 0$$

where the asset value W_u^0 can be obtained from:

$$(r + \delta)W_u^0 = w_0 + \delta U + (\lambda(\theta_1) + \lambda(\theta_2))\Delta\Phi$$

and $\Delta\Phi$ is given by:

$$(r + 2\delta)\Delta\Phi = w_0 - rW_u^0 - \delta(W_u^0 - U_e)$$

The derivative of W_u^0 with respect to w_1 is then:

$$(r + \delta) \frac{\partial W_u^0}{\partial w_1} = \delta \frac{\partial U}{\partial w_1} + \lambda'(\theta_1) \frac{\partial \theta_1}{\partial w_1} \Delta\Phi - \frac{(\lambda(\theta_1) + \lambda(\theta_2))}{r + 2\delta} \frac{\partial W_u^0}{\partial w_1} (r + \delta)$$

$$(r + \delta) \frac{\partial W_u^0}{\partial w_1} \left[1 - \frac{\delta \lambda(\theta_0)}{(r + \delta)(r + 2\lambda(\theta_0))} + \frac{(\lambda(\theta_1) + \lambda(\theta_2))}{r + 2\delta} \right] = \lambda'(\theta_1) \frac{\partial \theta_1}{\partial w_1} \Delta\Phi$$

giving rise to equation (5.37) in lemma 3.

Proof of proposition 3. Consider the case $r \rightarrow 0$, at the implied bargaining power $\tilde{\beta}$ surplus values S^0 and S^1 are given by:

$$S^1 = \frac{(y - z)(\delta + \lambda(\theta_0)\eta)}{\lambda(\theta_0)\eta(2\delta + \eta\lambda(\theta_1) + \tilde{\beta}\lambda(\theta_2)) + \delta^2} \quad S^0 = \frac{(y - z)(\delta + \eta\lambda(\theta_1) + \tilde{\beta}\lambda(\theta_2))}{\lambda(\theta_0)\eta(2\delta + \eta\lambda(\theta_1) + \tilde{\beta}\lambda(\theta_2)) + \delta^2}$$

The surplus difference ΔU is then:

$$\delta\Delta U + (\lambda(\theta_1) + \lambda(\theta_2))\Delta\Phi = (\eta\lambda(\theta_1) + \tilde{\beta}\lambda(\theta_2))S^1 - \lambda(\theta_0)\eta S^0$$

$$2\Delta\Phi(\delta + \lambda(\theta_1) + \lambda(\theta_2)) = \frac{(y - z)\delta(\eta\lambda(\theta_1) + \tilde{\beta}\lambda(\theta_2) - \eta\lambda(\theta_0))}{\lambda(\theta_0)\eta(2\delta + \eta\lambda(\theta_1) + \tilde{\beta}\lambda(\theta_2)) + \delta^2}$$

The implied bargaining power $\tilde{\beta}$ is then given by:

$$\tilde{\beta} = \beta + (1 - \beta) \frac{\Delta\Phi}{S^2} = \beta + (1 - \beta) \frac{\delta(\eta\lambda(\theta_1) + \tilde{\beta}\lambda(\theta_2) - \eta\lambda(\theta_0))}{2(\delta + \eta\lambda(\theta_0))(\delta + \lambda(\theta_1) + \lambda(\theta_2))}$$

Extracting $\tilde{\beta}$ from the above equation produces expression (5.39) in proposition 3.

Appendix III: Proof of proposition 4:

The current value Hamiltonian for the social planner problem is:

$$\begin{aligned} H &= p_u 2z + p_m(z + y) + p_e 2y - (c + \rho)\theta_0 2p_u - (c + \rho)\theta_1 p_m - c\theta_2 p_m \\ &+ \mu_u [2\lambda(\theta_0)p_u - \delta p_m] + \mu_e [(\lambda(\theta_1) + \lambda(\theta_2))p_m - 2\delta p_e] \\ &+ \mu [0.5 - [p_u + p_m + p_e]] \end{aligned}$$

where μ_u , and μ_e are costate variables corresponding to p_u and p_e respectively. The optimal social planner solution must satisfy:

$$\begin{aligned} \frac{\partial H}{\partial p_m} &= z + y - (c + \rho)\theta_1 - c\theta_2 - \mu_u \delta + \mu_e (\lambda(\theta_1) + \lambda(\theta_2)) - \mu = 0 \\ \frac{\partial H}{\partial p_u} &= 2z - 2(c + \rho)\theta_0 + \mu_u 2\lambda(\theta_0) - \mu = -r\mu_u \\ \frac{\partial H}{\partial p_e} &= 2y - \mu_e 2\delta - \mu = r\mu_e \\ \frac{\partial H}{\partial \theta_0} &= -(c + \rho)2p_u + \mu_u 2\lambda'(\theta_0)p_u = 0 \\ \frac{\partial H}{\partial \theta_1} &= -(c + \rho)p_m + \mu_e \lambda'(\theta_1)p_m = 0 \\ \frac{\partial H}{\partial \theta_2} &= -cp_m + \mu_e \lambda'(\theta_2)p_m = 0 \end{aligned}$$

9 References

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