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How Status Concerns Can Make Us Rich and Happy

Holger Strulik
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Abstract. This paper considers an overlapping generations model of economic growth populated by two types of individuals. Competitive types compare future consumption (i.e. wealth) with the mean. Self-sufficient types derive utility simply from their own consumption and do not compare themselves with others. I derive a condition under which the utility (happiness) of both types increases when the economy is populated by a larger share of competitive types. In the long-run the condition is always fulfilled when the economy is capable of economic growth. The reason for this phenomenon is that competitive types generate higher savings and thus higher aggregate capital stock and income per capita, which raises utility of both types. I show that the result is robust to the consideration of endogenous work effort and that a sufficiently high share of competitive types in a society can be inevitable for long-run economic growth to exist.

Keywords: status preferences, happiness, economic growth.

JEL: D90; E21; O40.

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1. Introduction

In evaluating our well-being we are continuously comparing our achievements with those of others. This fact has been well-known for a long time among psychologists and sociologists and is now increasingly acknowledged and verified by the economists profession (see, for example, Oswald, 1997; Frey and Stutzer, 2002; Layard, 2003; Luttmer, 2005). While it is rarely stated as explicitly as in the opening quote, one can read between the lines that researchers opine that the behavior of building up reference stocks and comparing achievements can impede happiness. In particular, Richard Easterlin argued that the observation that happiness has not increased (by much) over the last 50 years in some developed countries can be explained by the fact that people are evaluating their achievements relative to others (Easterlin, 1974, 1995, 2001). Brickman and Campbell (1971) have created the image of the hedonic treadmill on which we all live, unable to draw perpetual happiness from the status quo. Frank (1985) has added the idea of the positional treadmill for our comparison with friends, neighbors, and colleagues, and the untiring endeavor to compete with others is known in the vernacular as “keeping up with the Joneses”.

From a micro-economic perspective the idea that we could be happier if we were not comparing our consumption achievements with others may be convincing. However, the microeconomic focus neglects that the comparing attitude influences economic behavior, which affects macroeconomic outcomes, and in turn feeds back to individual utility. In this paper, I show that the consideration of general equilibrium effects challenges the view that comparing consumption achievement is bad for happiness. For this purpose, I focus on savings as one particular activity that influences macroeconomic performance. But savings could be replaced or, at the expense of more complexity, augmented by other activities that are conducive to macroeconomic performance and elicited by status concerns such as, for example, education.

Below, I use the term happiness for the utility experienced by the inhabitants of the model economy. Most of the economics literature applies the terms happiness and life satisfaction interchangeably. It has been argued, however, that the term happiness more accurately describes...
the instantaneous, affective component of subjective well-being, while the term life satisfaction is reserved for the evaluative, long-term component of subjective well-being (Deaton, 2008; Stevenson and Wolfers, 2008). In this sense, a more appropriate yet awkward title of the paper may have been “How status concerns make us rich and increases our life satisfaction”. Having made this qualification, I will continue to employ the term ‘happiness’ for its ease of linguistic use.

How does one make inferences about changes in happiness without resorting on interpersonal utility comparisons? Also, we cannot simply evaluate the outcome after a change of the utility function that increased or reduced the propensity for consumption comparisons, because such an experiment would alter the object of investigation.\footnote{For example, suppose that we manipulate the utility function such that individuals derive less utility from leisure. In the new situation, they work harder and experience more utility from consumption and less from leisure. Which utility function should be used to evaluate the outcome? Shall we consider the initial self or the transformed one? Moreover, a modification of status preferences manipulates the curvature of the utility function such that the strict conditions for valid interpersonal utility comparisons are not fulfilled (Harsanyi, 1955; Mas-Collel et al., 1995, Ch. 22).} In this paper I propose the following solution. We consider a society consisting of two types of individuals, self-sufficient types who do not compare consumption with others, and competitive types who compare their consumption (or income) with others. Both types are equipped with an invariant utility function. We then investigate the conditions under which utility of both types increases, given a society consisting of more competitive types.

There is an interesting body of literature arguing that the evolution of status preferences can be attributed to specific purposes, such as the allocation of non-market goods (see e.g. Cole et al., 1992; Samuelson, 2004; Rayo and Becker, 2007). This paper ignores any functional determination of status concerns and follows the mainstream modeling in growth theory by taking status preferences exogenously and without any purpose. This view defines a kind of lower bound for the happiness derived from status concerns because the static micro-effects of such “senseless” comparisons of consumption, income, or wealth are clearly negative. As such, it allows us to focus on the positive macro-economic feedback effects.

The idea for this research was inspired by Carroll, Overland, and Weil (1997, 2000) who introduced comparison utility as formalized by Abel (1990) into a standard Ak growth setup.\footnote{The model has become a kind of benchmark for theoretical and quantitative analyses of the impact of consumer reference stocks on growth and has been developed further in several other papers, for example, Futagami and Shibata (1998), Grossmann (1998), Fisher and Hof (2000), Corneo and Jeanne (2001), Alvarez-Cuadrado et al. (2004), Cozzi (2004), and Alonso-Carrera et al. (2005).}
In this framework, economic growth is increasing in the degree to which individuals compare their own consumption with others (or own past achievements) because more severe comparison elicits higher savings. A similar mechanism is at work in the present model. The present setup, however, is simpler. Instead of an infinitely living representative agent, we consider a two-period overlapping generations model. This simplification facilitates the analysis of heterogenous individuals and the explicit computation and evaluation of utilities.

While the positive feedback effect from status concerns on growth has been stressed by many authors since Carroll et al. (1997), the positive feedback effect on individual utility has remained unnoticed and constitutes the novel finding of this paper. It is perhaps understandable that the existing theoretical literature neglected to establish positive feedback effects from status concerns on happiness because the earlier empirical literature argued that – among the developed countries – citizens of richer countries are not happier than those of poorer countries (see e.g. the survey by Frey and Stutzer, 2002). More recent empirical happiness literature, however, has demonstrated that the income happiness gradient is significantly positive. Richer countries, i.e. those that grew at higher rate in the past, are on average, populated by happier people. Also, within countries, happiness increases with rising GDP per capita (Deaton 2007; Stevenson and Wolfers, 2008). There also seems to be no evidence of satiation (Stevenson and Wolfers, 2013). This paper shows that these observations are compatible with the presence of status concerns. In fact, it may well be that people in richer countries are happier because they live in a society in which most people have strong status concerns and evaluate their consumption, income or wealth relative to that of others.

The paper is organized as follows. The next section presents the basic model, derives economic behavior of individuals depending on their concern for status, and shows how the make up of society effects economic growth. Section 3 computes the utility gradient, i.e. the response of utility when society consists of more competitive types who compare consumption or wealth with others. It derives a condition under which happiness responds positively to increasing status competition in the medium-run, and shows that in the long-run, overall happiness is always higher when there are more competitive types in society. Section 4 extends the model by variable labor supply. In this case, I can no longer prove all results analytically. Instead, I

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3 In particular, the newer literature takes into account that marginal utility from consumption (or income) is decreasing and finds a strong association between the log of income and happiness. See Strulik (2013) for an axiomatic derivation of the curvature of the happiness-income curve under alternative assumptions about relative income comparisons.
show with help of numerical calibrated versions of the model, that the main results from the basic model are preserved. Section 5 concludes.

2. The Model

2.1. Households. Consider a society consisting of two overlapping generations. The population size (measure) of each generation is normalized to one. Each generation consists of a measure $\lambda$ of individuals who compare their income or consumption with others, $0 \leq \lambda < 1$. I call these individuals “competitive”, and index the associated variables with $c$. A measure $1 - \lambda$ of individuals does not compare themselves with others. These individuals are called “self-sufficient”, and the associated variables are indexed by $s$.

Both competitive and self-sufficient individuals maximize utility from consumption in the first and second period of life. For simplicity, the period utility functions have the log form and future achievements of both types of individuals are discounted by factor $\beta$. Consumption in the first and second period are denoted by $c_{t}^{i,1}$ and $c_{t+1}^{i,2}$, $i \in \{c, s\}$. In the first period, both types supply one unit of labor and receive a wage income $w_t$, which is spent on consumption and savings $s_t^i$. Thus, both types face the same budget constraint, $w_t = c_t^i + s_t^i$. Both types also receive the same return on savings $r_{t+1}$, implying $c_{t+1}^{i,2} = (1 + r_{t+1})s_t^i$. The self-sufficient types face the standard utility function $U_s^t = \log c_{t}^{s,1} + \beta \log c_{t+1}^{s,2}$. After substituting first and second period consumption into $U_s^t$ the utility maximization problem reads

$$\max_{s_t^i} U_s^t = \log(w_t - s_t^i) + \beta \log[(1 + r_{t+1})s_t^i],$$

which provides the well-known solution

$$s_t^s = \frac{\beta}{1 + \beta} \cdot w_t.$$ 

Competitive types compare either income or second period consumption (wealth) with the average level of consumption in the economy.\(^4\) In the case of consumption comparisons, utility is assumed to increase in the difference between own and average consumption. Since second period consumption is financed by savings, this is tantamount to assuming that competitive

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\(^4\) Comparing consumption in both periods is pointless as long as labor income is exogenous. Consumption comparisons in both periods are discussed in Section 4, after the introduction of endogenous labor supply. In the Conclusion I briefly discuss the case in which individuals compare consumption only in the first period. Alternatively it has been suggested that individuals compare themselves with the parent generation (de la Croix 1999, 2001).
types compare wealth. The strength of the comparing attitude, capturing the strength of status concerns, is measured by the weight of reference-based consumption, \( \gamma \). A weight \((1 - \gamma)\) is attached to absolute consumption. Formally utility of competitive types is given by

\[
U^c_t = \log c^c_t + \beta \log \left[ \gamma (c^c_{t+1} - \bar{c}^s_{t+1}) + (1 - \gamma) c^c_{t+1} \right], \quad 0 < \gamma \leq 1,
\]

in which \( \bar{c}^2_{t+1} \) denotes average consumption in the second period, \( \bar{c}^2_{t+1} = \lambda c^c_{t+1} + (1 - \lambda) c^s_{t+1} \).

Without the presence of self-sufficient types, \( c^c_{t+1} \) is equal to \( \bar{c}^s_{t+1} \), implying that the utility maximization problem would not be well defined when competitive individuals care only about relative consumption (for \( \gamma = 1 \)). In order to include this case in the analysis we have assumed that there is at least one self-sufficient person in the economy (\( \lambda < 1 \)). After inserting first and second period consumption into \( U^c_t \) the utility maximization problem of competitive types reads

\[
\max_{s_t} U^c_t = \log(w_t - s_t) + \beta \log \left[ (1 + r_{t+1})(s_t - \gamma \bar{s}_t) \right], \quad (3)
\]

in which \( \bar{s}_t \) denotes average savings, \( \bar{s}_t = \lambda s^c_t + (1 - \lambda) s^s_t \).

Alternatively, consider that competitive types experience utility from comparing their life-time income \( y^c = w_t + (1 + r)s_t \) with average income \( \bar{y} \). Suppose second period utility is derived from a weighted sum of consumption and life-time income, \( U^c_t = \log c^c_t + \beta \log \left[ \gamma (y^c_{t+1} - \bar{y}_{t+1}) + (1 - \gamma) c^c_{t+1} \right] \). Because first period income is the same for both types, the income difference is simply \( y^c_{t+1} - \bar{y}_{t+1} = (1 + r_{t+1})(s^c_t - \bar{s}_t) \). Inserting this information and the first period budget constraint into the utility function we arrive at (3). The two problems are observationally equivalent.

The first order condition for maximizing (3) requires that \( \beta(w_t - s^c_t) = s^c_t - \gamma \bar{s}_t \). Inserting the definition of \( \bar{s}_t \) and \( s^s_t \) from (2) we arrive at the solution

\[
s^c_t = \frac{\beta[(1 + \beta) + \gamma(1 - \lambda)]}{(1 + \beta - \gamma\lambda)(1 + \beta)} \cdot w_t. \quad (4)
\]

Computing \( \partial(s^c_t/w_t)/\partial \lambda = \beta \gamma^2/[(1 + \beta)(1 + \beta - \gamma\lambda)^2] > 0 \) verifies that the competitive types’ savings rate is increasing in the number of competitive types in society. Intuitively, the more competitive types there are, the harder it becomes to deviate positively from average consumption and thus the harder the competitive types try to beat the average by saving more. Likewise, we verify that \( \partial s^c_t/\partial \gamma > 0 \). The larger the strength of the comparing attitude, the higher is the competitive types’ savings rate. For \( \lambda \to 0 \) and \( \gamma \to 0 \), competitive types save at the same rate as self-sufficient types. For \( \lambda \to 1 \) and \( \gamma \to 1 \), the savings rate of competitive types converges to
unity. For later use, we plug (2) and (4) into the definition of $\bar{s}_t$ and obtain a simple expression for aggregate savings in the economy.

$$\bar{s}_t = \frac{\beta}{1 + \beta - \gamma \lambda} \cdot w_t.$$  (5)

Inspection proves the following intuitive result.

**Proposition 1.** Aggregate savings are an increasing function of the share of competitive types in society ($\lambda$) and of the strength of the status concerns ($\gamma$).

### 2.2. Firms and General Equilibrium.

The economy is populated by a measure one of identical firms employing labor $\ell_t$ and capital $k_t$ and producing output $y_t$ via a production function $y_t = A_t k_t^{\alpha} \ell_t^{1-\alpha}$, taking total factor productivity $A_t$ as given. Factors are paid according to the marginal product such that when aggregate labor supply equals unity, wages are given by $w_t = (1 - \alpha) A_t k_t^{\alpha}$ and interest rates are given by $r_t = \alpha A_t k_t^{\alpha-1}$. The easiest way to generate economic growth is to follow Romer (1986) and Rebelo (1991) and assume that total factor productivity is a positive function of the aggregate capital stock $A_t = \bar{A} k_t^{1-\alpha}$. Factor returns are then given by

$$w_t = (1 - \alpha) \bar{A} k_t, \quad r_t = \alpha \bar{A}.$$  (6)

A convenient side-effect of the Romer model is that the interest rate is constant (rather than declining in accumulated wealth). Interest rates independent of wealth appear to be a good first approximation of reality (Caselli and Feyrer, 2007).

As usually in the OLG framework we assume that the next young generation works with the capital stock accumulated by the currently young generation. For simplicity we assume that the capital stock depreciates fully within one generation. Since population size has been normalized to unity, the next period’s capital stock is a simple function of aggregate savings, $k_{t+1} = s_t$, that is together with (5) and (6),

$$k_{t+1} = \frac{\beta(1 - \alpha) \bar{A}}{1 + \beta - \gamma \lambda} \cdot k_t.$$  (7)

Solving the difference equation, we obtain

$$k_t = \left( \frac{\beta(1 - \alpha) \bar{A}}{1 + \beta - \gamma \lambda} \right)^t \cdot k_0,$$  (8)

in which $k_0 > 0$ is an arbitrary initial stock of capital. The term in parenthesis in equation (8) is the growth factor. The associated growth rate is $g = \beta(1 - \alpha) \bar{A}/(1 + \beta - \gamma \lambda) - 1$. It is a positive
function of the savings rate, a well-known result from endogenous growth theory. Inspection of (8) verifies the following result.

**Proposition 2.** The growth rate of the economy is an increasing function of the number of competitive types in the economy ($\lambda$) and of the strength of status concerns ($\gamma$).

This completes the description of the economy.

3. Social Composition on Overall Happiness

To evaluate overall happiness, it is crucial to understand how the utility of competitive types responds to an increase of competitive types in society. For this purpose, we begin by computing $c_t^e = w_t - s_t^e$ and $s_t^e - \gamma \bar{s}_t$ from (4) and (5). Inserting this information into (3), we get the maximized indirect utility:

$$U_t^e = \log \left\{ \frac{1 + \beta(1 - \gamma) - \gamma \lambda}{(1 + \beta - \gamma \lambda)(1 + \beta)} \right\} + \log w_t + \beta \log \left\{ \frac{\beta[1 + \beta(1 - \gamma) - \gamma \lambda]}{(1 + \beta - \gamma \lambda)(1 + \beta)} \right\} + \beta \log w_t + (1 + \beta) \log(1 + r_{t+1}).$$

Obviously the expressions in curly brackets depend negatively on $\lambda$. As the number of competitive types in the economy increases, it becomes harder to “beat” average wealth, and there is less utility experienced in the second period. Moreover, the fact that competitive individuals try harder to beat the average and save more reduces utility from consumption in the first period. These are the mechanisms in primary focus within the empirical happiness literature. Yet a higher $\lambda$ through increased savings and higher capital stock has also a positive effect on wages, thereby increasing utility. I call this the general equilibrium channel. Taking the general equilibrium channel into account, it is a priori unclear which effect dominates and whether more competitiveness in society drives down happiness. Taking the derivative with respect to $\lambda$ we obtain

$$\frac{dU_t^e}{d\lambda} = -\frac{\beta(1 + \beta)\gamma^2}{(1 + \beta - \gamma \lambda)(1 + \beta(1 - \gamma) - \gamma \lambda)} + \frac{1 + \beta}{w_t} \cdot \frac{\partial w_t}{\partial \lambda}.$$  

I call $dU_t^e/d\lambda$ the utility gradient. The first negative term on the right-hand side of (9) reflects the well-known effect from the happiness literature. I called it the micro-effect. The second term is the general equilibrium effect, which is usually overlooked in behavioral economics. The effect is positive because a higher share of competitive types leads to a higher capital stock and, through this channel, to higher wages and higher utility. Since interest rates are constant all general equilibrium effects run through higher wages. Lagging (7) by one period, inserting it into (6), and computing the derivative with respect to $\lambda$, the general equilibrium effect is obtained
as a neat expression:

\[ \frac{1 + \beta}{w_l(k_l-1)} \cdot \frac{\partial w_l(k_l-1)}{\partial \lambda} = \frac{(1 + \beta)\gamma}{1 + \beta - \gamma \lambda} > 0. \]  
\[ (10) \]

The general equilibrium effect is positive and increases with increasing \( \lambda \). Plugging it into (9) we arrive at

\[ \frac{dU^c}{d\lambda} = \left\{ \frac{1 + \beta}{1 + \beta - \gamma \lambda} \right\} \left\{ \frac{1 + \beta - 1}{1 + \beta - 2\gamma} - \gamma \lambda \right\} \left\{ \frac{1 + \beta}{1 + \beta - \gamma \lambda} \right\}. \]  
\[ (11) \]

Obviously, all terms in curly brackets in (11) are strictly positive (recall that \( 0 < \gamma \lambda < 1 \)). The sign of the utility gradient hinges on the sign of the term in squared brackets. Inspection of this term proves the following result.

**Proposition 3.** The utility of competitive types is increasing in the share of competitive types in society as long as

\[ \lambda < \lambda^* \equiv \frac{1 + \beta(1 - 2\gamma)}{\gamma}. \]  
\[ (12) \]

The result indicates that there may be an interior solution for the optimal mix of society. The term \( \lambda^* \) is the optimal mix of society from the viewpoint of competitive types. Self-sufficient types would actually like to have more competitive types in society than \( \lambda^* \) because more competitive types generate higher wages. Since self-sufficient types do not suffer from the negative micro-effect their response of utility is simply

\[ \frac{dU^s}{d\lambda} = \frac{1 + \beta}{w_l} \cdot \frac{\partial w_l}{\partial \lambda} > 0. \]

This observation can be stated as follows.

**Corollary 1.** The share of competitive types that maximizes utility of self-sufficient types is larger than \( \lambda^* \).

Returning to Proposition 3, we observe that the right-hand side of (12) is declining in \( \gamma \). This is because strong status concerns reduce the benefit of having many competitors. If competition for consumption achievements (or wealth) is “too severe”, the micro effect dominates the general equilibrium effect (it can be easily verified that (10) is strictly increasing in \( \gamma \)). This leads to the question of whether there is an upper limit of \( \gamma \) for which utility of comparing types increases always when the share of comparing types in society gets larger. Plugging \( \lambda^* = 1 \) into (11), we arrive at the following result.

**Corollary 2.** If status concerns are not too severe, i.e. if \( \gamma < (1 + \beta)/(1 + 2\beta) \), then an
increasing share of competitive types in society always improves utility (happiness) of competitive types and self-sufficient types.

A back-of-the-envelope calculation may be helpful to assess this result. Suppose that the rate of time preference $\rho$ equals 2 percent per year and that a generation takes 25 years. Then $\beta = 1/(1 + 0.02)^{25} = 0.61$ and $\lambda^* = 1$ for $\gamma < 0.72$. This means that the optimal share of competitive types is unity as long as the utility of competitive types is derived by less than 72 percent from status concerns. This value increases to 87 percent when the time preference rate is 5 percent per year with a generation defined as 35 years.

3.1. The Makeup of Society and its Long-Run Development. The general equilibrium effect computed in (10) understates the long-run benefits from a competitive society. It is a medium-run effect, obtained from one period to the next, i.e. for given $k_{t-1}$. The long-run effect is computed by inserting (8) into (6) and taking the derivative with respect to $\lambda$, which provides

$$
\frac{1 + \beta}{w_t(k_0)} \cdot \frac{\partial w_t(k_0)}{\partial \lambda} = t \cdot \frac{(1 + \beta)\gamma}{1 + \beta - \gamma \lambda}.
$$

Plugging the long-run effect into (9), taking the limit $t \to \infty$, and noting that the first term in (9) is finite proves the following result.

**Proposition 4.** In the long-run, for $t \to \infty$, a rising share of competitive types in society always improves utility (happiness) of competitive types and self-sufficient types.

The result is intuitive: the micro effect from consumption comparison is time-invariant while the macro effect is dynamic and increases over time with rising wages. A particularly drastic case is observed when a society stagnates because it consists of “too few” competitive types. From (8) it can be seen that an economy is capable of long-run growth if the growth factor $\beta(1 - \alpha)\bar{A}/(1 + \beta - \gamma \lambda)$ is larger than unity. Solving this condition for $\lambda$, we arrive at the following result.

**Proposition 5.** The economy is capable of long-run growth if $\lambda > \lambda_g \equiv [1 + \beta - \beta(1 - \alpha)\bar{A}]/\gamma$.

If the economy is incapable of growth, it converges towards the origin ($k = 0$). This unrealistic scenario can be avoided by adding income from home production (subsistence farming) to the budget constraint: $w_t + b = c_t + s_t$, in which $b$ denotes income from home production. Then, as
shown in the Appendix, a society for which $\lambda < \lambda_g$ stagnates at

$$k^*_t = \frac{\beta b}{1 + \beta - \gamma \lambda - \beta (1 - \alpha) \bar{A}} > 0. \quad (13)$$

Observe that at the steady state of stagnation, capital stock and thus income of both types is larger when the population share of competitive types $\lambda$ is larger and when the importance of status $\gamma$ is larger. More important, however, is the observation that a society consisting of predominantly self-sufficient types may stagnate while another society populating an economy constrained by the same fundamentals and populated by sufficiently many competitive types grows perpetually. A concern for status may be inevitable for economic growth.

3.2. Income Inequality. Income inequality reaches its peak when society is equally divided into self-sufficient and competitive types, i.e. for $\lambda = 1/2$, and declines with further rising $\lambda$. In order to verify this intuitive proposition it suffices to focus on income inequality in the second period. Type $i$ earns income $y^i_t = (1 + r_{t+1})s^i_t$ and average income is $\bar{y} = (1 + r)(\lambda s^c_t + (1 - \lambda)s^s_t)$. Inequality is measured by the Gini coefficient, defined as half of the relative mean difference:

$$G = \frac{1}{2} \cdot \lambda \cdot \frac{|y^c_t - \bar{y}_t|}{\bar{y}_t} = \frac{1}{2} \cdot \lambda \cdot \frac{\left|\frac{s^c_t}{\bar{s}_t} - 1\right|}{\bar{y}_t} = \frac{\lambda (1 - \lambda) \gamma}{2\beta},$$

where the last equality follows after inserting (4) and (5). Computing the derivative $\partial G / \partial \lambda = \gamma(1 - 2\lambda)/(2\beta)$ verifies that a higher share of competitive types in society reduces income inequality for $\lambda > 1/2$.

Combining the inequality and growth results, we may imagine a medieval or traditional society as characterized by a low value of $\lambda$, $\lambda < \lambda_g \ll 1/2$. Only few people (such as the aristocrats) compare wealth and derive utility from status while society at large lives a self-sufficient life. The average savings rate is low and the economy stagnates. Gradually increasing the share of competitive types initiates a take-off to perpetual growth and leads to first increasing and then decreasing income inequality, i.e. a Kuznets curve (Kuznets, 1955).

4. Endogenous Labor Supply

4.1. Households. In this section, I show that the main results are robust to the introduction of variable labor supply and utility from leisure. This finding may be surprising because it has been argued that another drawback of status concerns is excessive work effort, which leads to higher income but less happiness due to loss of leisure time (Layard, 2005; Clark et al.,
In order to derive most of the results analytically, I assume that utility from leisure is logarithmic. Comparing wealth is no longer observationally equivalent to comparing income because first period income is now endogenous. We begin with a model in which competitive types compare second period consumption or wealth and discuss consumption comparisons in both periods later on. In the first period both types are endowed with one unit of time, which can be spent on leisure or on labor supply \( \ell_i^t \) in order to earn an income \( w_t \ell_i^t, \) \( i = c, s \). The weight of leisure for overall utility is denoted by \( \eta \). Utility of self-sufficient types is thus given by

\[
U_s^t = \log(c_{i}^s, 1_t) + \beta \log(c_{i}^s, 2_t) + 1 + \eta \log(1 - \ell_i^t). \tag{14}
\]

From the first order conditions, we get the solution

\[
s_i^s = \frac{1 + \beta}{1 + \beta + \eta}, \quad \ell_i^s = 1 + \frac{\beta}{1 + \beta + \eta}. \tag{15}
\]

Analogously, utility of competitive types is given by

\[
\max_{s_i^c} U_c^t = \log(w_t \ell_c^t - s_i^c) + \beta \log[(1 + r_{t+1})(s_i^c - \gamma \bar{s}_t)] + \eta \log(1 - \ell_i^c). \tag{16}
\]

Substituting the definition of \( \bar{s}_t \) and \( s_i^s \) from (15) into the first order conditions we get the solution

\[
s_i^c = \frac{\beta \{\beta + (1 + \eta)(1 + \gamma(1 - \lambda))\}}{(1 + \beta + \eta)(\beta + (1 + \eta)(1 - \gamma \lambda))} \cdot w_t \tag{17}
\]

\[
\ell_i^c = \frac{\beta^2 + (1 + \eta)(1 - \gamma \lambda) + \beta [2 - \gamma \lambda + \eta]1 + \gamma - \gamma \lambda}{(1 + \beta + \eta)(\beta + (1 + \eta)(1 - \gamma \lambda))}. \tag{18}
\]

Comparing (18) and (15) we find that competitive types work harder and enjoy less leisure than self-sufficient types, \( \ell_i^c - \ell_i^s = \gamma \eta \beta/(1 + \beta + \eta)[(1 + \beta + \eta)(1 - \gamma \lambda)] > 0 \). We also find that competitive types work harder when there are more competitive types to compete with, \( \partial \ell_i^c/\partial \lambda = \beta \eta (1 + \eta)\gamma^2/(1 + \beta + \eta)[\beta + (1 - \eta)(1 - \gamma \lambda)] > 0 \), and when status concerns are more important, \( \partial \ell_i^c/\partial \gamma = \beta \eta / [\beta + (1 - \eta)(1 - \gamma \lambda)]^2 > 0 \). As for the simple model, we find that competitive types save more when \( \lambda \) or \( \gamma \) increases. Plugging (15), (17) and (18) into the definitions of aggregate savings \( \bar{s}_t = \lambda s_i^c + (1 - \lambda)s_i^s \) and aggregate labor supply \( \bar{\ell}_t = \lambda \ell_i^c + (1 - \lambda)\ell_i^s \) we obtain

\[
\bar{s}_t = \frac{\beta}{\beta + (1 + \eta)(1 - \gamma \lambda)} \cdot w_t, \quad \bar{\ell}_t = \frac{1 + \beta - \gamma \lambda}{\beta + (1 + \eta)(1 - \gamma \lambda)}. \tag{19}
\]
Evaluating $\partial \bar{u}_t / \partial \lambda = \gamma \eta \beta / D$, $\partial \bar{u}_t / \partial \gamma = \eta \beta / D$, $\partial \bar{s}_t / \partial \lambda = \eta (1 + \eta) \gamma w_t / D$, $\partial \bar{s}_t / \partial \gamma = \beta (1 + \eta) w / D$, $D \equiv [\beta + (1 + \eta)(1 - \gamma \lambda)]^2$ confirms the following result.

**Proposition 6.** Aggregate savings $\bar{s}$ and aggregate labor supply are increasing functions of the share of competitive types $\lambda$ in society ($\lambda$) and of the strength of status concerns ($\gamma$).

4.2. **Firms and General Equilibrium.** The production function is the same as for the simple model, $y = A_t k_t^{\alpha \ell_t^{1-\alpha}}$. A simple solution of the model can be derived when there is congestion. Considering this case first, suppose that aggregate productivity depends not only positively on capital stock, but also negatively on aggregate labor supply such that TFP is rising in the capital labor ratio $A_t = \bar{A}(k_t / \bar{\ell}_t)^{1-\alpha}$. As a consequence, the interest rate remains the same as in the simple model, $r_t = \alpha \bar{A}$, and wages are scaled by aggregate labor supply, $w_t = (1 - \alpha)Ak_t / \bar{\ell}_t$. This constitutes a worst case scenario for status concerns to have a positive impact on happiness because two otherwise positive effects are eliminated. First, because of congestion aggregate labor income $w_t \ell_t$ is independent from work effort. Without congestion, increasing work effort would have a positive impact on aggregate labor income and thus on aggregate savings and economic growth. Second, without congestion increasing work effort would lead to a higher equilibrium interest rate, and this way to another positive macro effect on utility.

Inserting $w_t$ and $\ell_t$ into $\bar{s}_t$ we see why the simplification is convenient. The equation of motion, $k_{t+1} = \bar{s}_t$, is exactly the same as for the simple model and given by the linear difference equation (7), which has been solved in (8).

4.3. **Social Composition and Overall Happiness.** Inserting (17)-(19) into (16) and taking the derivative with respect to $\lambda$ provides (after some algebra),

$$\frac{dU_t^c}{d\lambda} = -\frac{\beta (1 + \eta) \beta \gamma^4 \lambda}{a_1 \cdot a_2 \cdot a_3} + \frac{1 + \beta \partial w_t}{w_t} \frac{\partial \bar{u}_t}{\partial \lambda} + \frac{\beta}{1 + r} \frac{\partial r}{\partial \lambda}, \quad a_1 = \beta + (\eta + 1)(1 - \gamma \lambda) > 0,$$

$$a_2 = \beta (\gamma - 1) - (\eta + 1)(1 - \gamma \lambda) < 0, \quad a_3 = \beta (\gamma - 1) - (\eta + 1) [1 - (1 - \gamma \gamma \lambda)] < 0.$$

Again the first term in (20) reflects the micro effect and the second and third term reflect the general equilibrium effect. The micro effect is again unambiguously negative. The third term is zero under congestion because the interest rate does not depend on $\lambda$. The wage effect is obtained by lagging (7) one period, inserting the result and $\ell_t$ from (19) into $w_t = (1 - \alpha)Ak_t / \bar{\ell}_t$.
and taking the derivative with respect to $\lambda$. It provides

$$\frac{\partial w_t(k_{t-1})}{\partial \lambda} \cdot \frac{1 + \beta}{w_t(k_{t-1})} = \frac{(1 + \beta)\gamma \left[\beta(1 - \eta) + (\eta + 1)(1 - \gamma \lambda)\right]}{(1 + \beta - \gamma \lambda) \left[\beta + (\eta + 1)(1 - \gamma \lambda)\right].}$$

(21)

For $\eta = 0$, the general equilibrium effect is reduced to the one obtained for the simple model (10). For $\eta > 1$ the general equilibrium effect falls short of (10). By taking the derivative with respect to $\eta$ it can be easily verified that the general equilibrium effect is deceasing in $\eta$. The reason is that a high preference for leisure mutes the effect of increasing competition $\lambda$ on work effort and thus savings and growth, because competitive individuals are less easily convinced to work harder. In contrast to the basic model, the general equilibrium effect is positive only when $\lambda$ is sufficiently low, namely for $\lambda < [1 + \beta + \eta(1 - \beta)]/[(1 + \eta)\gamma]$.

Figure 1.a: Status Preferences, Happiness, and Economic Performance ($\gamma = 0.3$)

$\lambda$ is the population share of competitive types and $\gamma$ is the strength of status concerns relative to absolute consumption. The utility gradient is the response of utility to an increase of $\lambda$ computed from (20) with (21) and $\partial r/\partial \lambda = 0$. Average labor supply and average savings rate are computed from (19).

The overall effect $dU^c_t/d\lambda$ can no longer be assessed analytically. For a numerical assessment we take up the discussion of the simple model and assume a length of a generation of 25 years and an annual time preference rate of two percent, i.e. $\beta = 0.061$. We set $\eta$ such that self-sufficient types work 20 percent of the time, which is somewhat less than the 25 or 33 percent that is commonly assumed. We need some leeway to arrive at a reasonable value for average labor supply because competitive types work more. This provides the estimate $\eta = 6.4$. The implied savings rate of self-sufficient types is 0.076. Naturally, the average savings rate will be higher because competitive types save at a higher rate. We consider three different values for the strength of status concerns, $\gamma \in \{0.3, 0.6, 0.9\}$.

Figure 1.A. shows the utility gradient $dU^c_t/d\lambda$, the average savings rate per unit of labor
income $\bar{s}_t/w_t$, and average labor supply $\bar{\ell}_t$ for alternative population shares of competitive types ($\lambda$) when the strength of status concerns is relatively low, $\gamma = 0.3$. The utility gradient is positive throughout, indicating that a higher share of competitive types in society leads to more happiness for all values of $\lambda$. The gradient is falling, indicating decreasing marginal utility from a higher share of competitive types in society. Because status concerns are relatively low, the average savings rate and average labor supply respond only minimally to increasing competitiveness of society. The savings rate for example rises to a mere 10 percent for $\lambda \to 1$.

Figure 1.b: Status Preferences, Happiness, and Economic Performance ($\gamma = 0.6$)

Figure 1.b repeats the numerical experiment for an intermediate value of status concerns ($\gamma = 0.6$). The utility gradient becomes negative when $\lambda$ exceeds 0.6, suggesting that happiness of competitive types (but not of self-sufficient types) declines with rising $\lambda$ when there are already more than 60 percent of competitive types in society. Average savings and labor supply converge towards empirically plausible values when $\lambda$ approaches unity.

Finally, Figure 1.c shows the outcome for a high degree of status concerns ($\gamma = 0.9$). The utility gradient gets now negative when $\lambda$ exceeds 0.3 and declines very steeply when $\lambda$ exceeds 0.6. Empirically plausible values for the average savings rate and average labor supply are observed when $\lambda$ is about 0.7, a value at which the utility gradient is negative. Recall, however, that we have computed medium-run effects (for given $k_{t+1}$). As shown in Proposition 4, the long-run effect of increasing $\lambda$ on happiness is always positive, given that the general equilibrium effect is positive.

So far the macro environment has constituted the worst scenario for increasing $\lambda$ to play a positive role. In the next step we abolish the assumption of congestion. Returning to the
original assumption for TFP, $A_t = \bar{A}k_t^{1-\alpha}$, the equilibrium interest rate depends positively on labor supply $r_t = \alpha\bar{A}l_t^{1-\alpha}$ and wages are given by $w_t = (1-\alpha)\bar{A}k_t\bar{l}_t^{\alpha}$. Without congestion we have two previously ignored effects: greater labor supply leads to higher interest rates and higher labor income $w_t\bar{l}_t$. Both channels amplify the positive effects from higher $\lambda$ because greater competition increases labor supply.

The solution without congestion is less elegant and delegated to the Appendix. The general equilibrium effect on wages, however, is again relatively simple:

$$\frac{\partial w_t(k_{t-1})}{\partial \lambda} \cdot \frac{1+\beta}{w_t(k_{t-1})} = \frac{(1+\beta)\gamma \{\beta[1+(1-2\alpha)\eta] + (\eta+1)(1-\gamma\lambda)\}}{(1+\beta-\gamma\lambda) [\beta + (\eta+1)(1-\gamma\lambda)]}.$$  (22)

The expression collapses to (21) for $\alpha \rightarrow 1$. More importantly, $\alpha < 1/2$ is a sufficient, not necessary condition for the general equilibrium effect to be positive. The utility gradient is further modified by a positive effect of $\lambda$ on the interest rate through increasing labor supply. This effect is computed as

$$\frac{\beta}{1+r} \frac{\partial r}{\partial \lambda} = \frac{(1-\alpha)\alpha\bar{A}\beta^2\gamma \left[\alpha\bar{A} + \left(\frac{1+\beta-\gamma\lambda}{\beta + (\eta+1)(1-\gamma\lambda)}\right)^{1-\alpha}\right]}{[\beta + (\eta+1)(1-\gamma\lambda)] (1+\beta-\gamma\lambda)} > 0.$$  (23)

The size of the interest rate effect, however, is negligible. For the present calibration, the interest rate effect is between $1/80$ and $1/20$ of the wage effect, depending on the size of $\lambda$. In order to calibrate the model, we need to specify a value for $\bar{A}$. For this purpose, I use the prediction for long-run growth and set $\bar{A}$ such that the economy grows at an annual rate of 2 percent when the society consists of 90 percent competitive types and the weight of status concerns in utility is 90
percent \((\lambda = \gamma = 0.9)\). Moreover, I set \(\alpha = 1/3\) and keep all the other parameters as specified above.

Figure 2.a: Status Preferences, Happiness, and Economic Performance \((\gamma = 0.6)\)

![Figure 2.a: Status Preferences, Happiness, and Economic Performance \((\gamma = 0.6)\)](image)

The utility gradient is the response of utility to an increase of \(\lambda\) computed from (20) with (22) and (23).

Figure 2.a. shows the utility gradient and average savings and labor supply for alternative \(\lambda\) when \(\gamma = 0.6\) (the case \(\gamma = 0.3\) is similar to Fig 1.a and not replicated here for the sake of brevity). The utility gradient is now positive for all values of \(\lambda\), indicating that happiness of competitive and self-sufficient types increases for all \(\lambda\). Moreover, for relatively small \(\lambda\), below 0.6, the slope of the gradient is increasing, indicating successively larger happiness gains with rising \(\lambda\). The average savings rate and average labor supply assume empirically plausible values when \(\lambda\) converges towards unity.

Figure 2.b shows the outcome when status concerns are very important \((\gamma = 0.9)\). The utility gradient becomes negative when \(\lambda\) exceeds 0.6. For high values of \(\lambda\) (above 0.8), severe status competition leads to excessive behavior among the competitive types and empirically implausible values for average savings and average labor supply.

Empirically, the strength of status concerns is difficult to estimate. The available literature seems to suggest a value of \(\gamma\) somewhere between 0.5 and 0.7. For example, Clark et al. (2008, p. 111) conclude their discussion of the empirical happiness literature: “Together, this suggests a utility function in which two-thirds of aggregate income has no effect because it is status related”. The mean value derived from experiments by Alpizar et al. (2005) is 0.45. In their calibration, Carroll et al. (2000) use 0.5 as the benchmark value. Heaton (1995) estimates a value of 0.71. Boldrin et al. (1997) find that a value of 0.58 best explains the risk premium. If we consider a \(\gamma\) of 0.6 as being close to the “true value”, then Figure 2.a suggests that happiness
of both competitive and self-sufficient types is largest when the share of competitive types in society approaches unity. For the pessimistic alternative with congestion, Figure 1.b suggests that the “optimal” value of $\lambda$ is around 0.6. But again, this holds for the medium run, that is at time $t$ for given $k_{t-1}$. As shown above, in the long-run, that is for given $k_0$, the general equilibrium effect always dominates the micro effect and happiness of both types is maximal when $\lambda$ approaches unity as long as the general equilibrium effect is positive.

Figure 2.b: Status Preferences, Happiness, and Economic Performance ($\gamma = 0.9$)

Finally, we consider consumption comparisons in both periods. The utility function of competitive types is given by

$$U^c_t = \log [(1 - \gamma)c^c_t + \gamma(c_t - \bar{c}_t)] + \beta [(1 - \gamma)s_t + \gamma(s_t - \bar{s}_t)] + \log(1 + r) + \eta \log(1 - \ell_t),$$

in which $\bar{c}_t$ denotes average consumption in the first period, $\bar{c}_t = \lambda c^c_t + (1 - \lambda)s^s_t$. Stating the first order conditions for maximizing the utility function with respect to the budget constraint and inserting the definition of $\bar{c}_t$ and $\bar{s}_t$ into the solution as well as the solution for self-sufficient types $c^s_t$ and $s^s_t$ from above provides, after some algebra,

$$s^c_t = \frac{\beta [1 + \beta + \eta(1 + \gamma - \gamma \lambda)]}{(1 + \beta + \eta) [1 + \beta + \eta(1 - \gamma \lambda)]} \cdot w_t,$$

$$\ell^c_t = \frac{(1 + \beta) [1 + \beta + \eta(1 + \gamma - \gamma \lambda)]}{(1 + \beta + \eta) [1 + \beta + \eta(1 - \gamma \lambda)]}.$$  \hspace{1cm} (24)

As expected, the competitive types save less and work harder when they derive status from consumption in both periods, compared to the case in which they derive status from wealth. This result can best be seen by inserting (24) into the definition of $\bar{s}_t$ and $\bar{\ell}_t$ and compute average
savings and labor supply in the economy. This provides
\[ \bar{s}_t = \frac{\beta}{1 + \beta + \eta(1 - \gamma \lambda)} \cdot w_t, \quad \bar{\ell}_t = \frac{1 + \beta}{1 + \beta + \eta(1 - \gamma \lambda)}. \]  
(25)

Comparing (25) with (19) leads to the following result (proved in the Appendix).

**Proposition 7.** If competitive types derive status from consumption in both periods, aggregate savings are lower and aggregate labor supply is higher compared to the case when competitive types derive status from wealth (second period consumption).

Inspection of (25) shows that the general equilibrium channel is preserved:

**Proposition 8.** If competitive types derive status from consumption in both periods, the aggregate savings rate as well as aggregate labor supply are increasing functions of the share of competitive types in society ($\lambda$).

In conclusion, economic growth increases with increasing population share of competitive types. The size of the medium-run general equilibrium of effect on wages is computed as (see Appendix for details):
\[ \frac{\partial w(k_{t-1})}{\partial \lambda} \cdot \frac{1 + \beta}{w(k_{t-1})} = \frac{(1 + \beta)(1 - 2\alpha)\eta\gamma}{1 + \beta + \eta(1 - \gamma \lambda)}. \]  
(26)

The general equilibrium effect is positive for $\alpha < 0.5$ irrespective of the size of $\lambda$ or $\gamma$. Because of Proposition 7, we expect the medium-run effect to be less powerful and the utility gradient to become negative at a lower value of $\lambda$, compared to Figure 2. In the long-run, however, as in for the case of status from wealth, the general equilibrium effect continues to be the dominating force as long as the capital share is smaller than one-half. This implies that an increasing share of competitive individuals in society improves the happiness levels of self-sufficient and competitive types.

5. Conclusion

This paper has suggested a novel view on the income and happiness nexus. In earlier research, the observation of seemingly insignificant happiness gains from increasing income in developed countries inspired an explanation based on status concerns and the integration of status concerns into economic theory. Recent empirical evidence then showed that happiness (or life satisfaction) does in fact increase in the log of income without an identifiable satiation point. Some
authors conclude from these findings that absolute income matters for happiness and that relative income comparisons are of lesser importance (Stevenson and Wolfers, 2008). This paper has demonstrated that status concerns are not only compatible with perpetual happiness gains from rising income, but that overall happiness can increase when a society consists of a larger share of individuals who compare their consumption achievements, income, or wealth with others. The explanation of this phenomenon is a feedback effect from general equilibrium. A competitive society induces high savings, and thus, a high level of income and productivity, which feeds back positively on utility. In the long-run, the positive general equilibrium effect always dominates the well-known micro effect if an economy is capable of perpetual growth.

The purpose of this paper was to establish one particular channel through which status concerns have a positive impact on overall happiness. Next I briefly discuss some qualifications of the main result with reference to alternative treatments of status concerns in the literature. To begin with, status concerns may not elicit any economically relevant behavior. In the present context this would be the case if consumption comparisons were of multiplicative (rather than subtractive) form and utility remained logarithmic, i.e. if second period utility of competitive types were given by $\beta \log(c_{t+1}^c/c_{t+1}^\bar{c})$. This formulation, however, constitutes a knife-edge assumption. The main mechanism, namely that stronger status concerns are conducive to growth, reappears when the degree of relative risk aversion exceeds one, i.e. when second period utility has the form $\beta(c_{t+1}^c/c_{t+1}^\bar{c})^{1-\sigma}/(1-\sigma)$, $\sigma > 1$, as shown by Carroll et al. (1997). Moreover, the literature in behavioral economics suggests that subjective well-being predicts future behavior (Clark et al., 2008). It thus refutes the result implied by the log-multiplicative form of utility that individual consumption remains unchanged when the consumption level of neighbors and peers rises and individual well-being declines.

The general equilibrium channel remains active when individuals do not only compare future consumption (i.e. wealth) but also current consumption. However, the mechanism breaks down and gets reversed if individuals compare consumption only in the first period (Corneo and Jeanne, 1998). Moav and Neeman (2010, 2012) put this mechanism into a development context and show how conspicuous consumption of the poor prevents investment in education and may create a poverty trap. Interestingly, a crucial assumption for the model to function is that investment in education is unobservable and cannot be used as a status marker. For developed countries, it seems to be more plausible to acknowledge that education is observable. Weiss and
Fersthman (1998) discuss evidence suggesting that in developed countries, education is an even more important marker of status than income or wealth. To capture this channel the present model could be reformulated as an “Ah” growth model, in which individuals invest in education h and in which competitive individuals compare educational outcomes with the mean. It is easy to imagine that the main results will be similar. Fershtman et al. (1996) and Kawamoto (2007) have shown that an increase of the strength of status derived from education promotes economic growth. Fershtman et al. also show that increasing status concerns can reduce growth if production requires low skilled workers and high skilled workers in fixed proportions.

In order to arrive at an explicit solution for maximized utility, a deliberately simple economic framework was chosen, built upon the Ak growth model. Carroll et al. (2000) have argued in a similar context that the linear Ak production function is the ultimate structure of all endogenous growth models, referring to Rebelo (1991). This claim is certainly valid as long as economic growth is driven by factor growth. But even if growth is driven by the growth of ideas, produced by costly R&D, we expect the basic mechanism to be present as long as R&D is positively influenced by the savings rate, as for example in the Romer (1990) model.

An interesting task for future research may be to let the share of competitive types in society evolve endogenously, depending on the socio-economic environment. This would potentially create a unified growth theory (Galor, 2005). In pre-modern times only a few individuals (the aristocrats) had the opportunity to compare their wealth and develop status concerns. The majority of the population lived a self-sufficient life and aggregate savings and/or investment in education were at such low levels that the economy (almost) stagnated. The model would then predict that the economy takes off as more individuals gain access to status competition (emergence of a middle class) and that this take-off to growth is accompanied by first rising and then declining income inequality.

Finally, I would like to stress that this paper does not argue that individuals subjected to status concerns are happier than self-sufficient types. The working hypothesis was that such utility comparisons are impossible. The point was to show that status concerns can create an economic environment which is conducive to the experience of a happy life for both competitive and self-sufficient members of society. From the individual perspective, behind the veil of ignorance, it may well be the best choice to opt for a self-sufficient life in a competitive society. Unfortunately, this option is not available to everyone.
**APPENDIX**

**Derivation of (13).** With home production, the maximization problem of self-sufficient types reads
\[
\max_{s_t^*} U_t^e = \log(w_t + b - s_t^*) + \beta \log[(1 + r_{t+1})s_t^*].
\]
which provides the solution \( s_t^* = \beta(b + w_t)/(1 + \beta) \). Competitive types solve the problem
\[
\max_{s_t^*} U_t^c = \log(w_t + b - s_t^*) + \beta \log[(1 + r_{t+1})(s_t^* - \gamma s_t)].
\]
The first order condition requires that \( \beta(b + w_t) = s_t^* - \gamma s_t \). Inserting the definition of \( \bar{s}_t \) and the solution for \( s_t^* \) we obtain
\[
s_t^* = \frac{\beta[(1 + \beta) + \gamma(1 - \lambda)](b + w_t)}{(1 + \beta - \gamma \lambda)(1 + \beta)}, \quad \bar{s}_t = \frac{\beta(b + w_t)}{1 + \beta - \gamma \lambda}.
\]
The equation of motion \( k_{t+1} \bar{s}_t \) thus becomes
\[
k_{t+1} = \frac{\beta b}{1 + \beta - \gamma \lambda} + \frac{\beta(1 - \alpha)\bar{A}}{1 + \beta - \gamma \lambda} k_t.
\]
If the growth factor in front of \( k_t \) falls short of unity there exists no long-run. Solving the equation for \( k_{t+1} = k_t \) provides the steady state displayed in (13).

**Derivation of (22).** Without congestion wages are given by \( w_t = (1 - \alpha)\bar{A}t^\alpha \). The equation of motion (lagged one period) becomes
\[
k_t = \frac{\beta(1 - \alpha)\bar{A}}{\beta + (1 + \eta)(1 - \gamma \lambda)} \left( \frac{1 + \beta - \gamma \lambda}{\beta + (1 + \eta)(1 - \gamma \lambda)} \right)^{-\alpha} k_{t-1}.
\]
Plugging \( k_t \) back into \( w_t \) we get \( w_t = \beta(1 - \alpha)^2\bar{A}^2(1 + \beta - \gamma \lambda)^{-2\alpha} [\beta + (1 + \eta)(1 - \gamma \lambda)]^{2\alpha - 1} k_{t-1} \).

Taking the derivative with respect to \( \lambda \), dividing by \( w_t \) and multiplying by \( 1 + \beta \) provides (22) in the main text. The interest rate is given by \( r_t = \alpha \bar{A}t^{-\alpha} \), i.e. with (19)
\[
r_t = \alpha \bar{A} \left( \frac{1 + \beta - \gamma \lambda}{\beta + (1 + \eta)(1 - \gamma \lambda)} \right)^{1-\alpha}.
\]
Taking the derivative with respect to \( \lambda \), dividing by \( 1 + r_t \) and multiplying by \( \beta \) provides the expression in the main text.

**5.1. Proof of Proposition 7.** From 0 > \( -\gamma \lambda \) follows \( 1 + \eta(1 - \gamma \lambda) > 1 - \gamma \lambda + \eta(1 - \gamma \lambda) \) and
\[
1 + \beta + \eta(1 - \gamma \lambda) > \beta + (1 + \eta)(1 - \gamma \lambda)
\]
The left hand side in this expression it the denominator of \( \bar{s}_t \) in (24) and the right hand side is the denominator of \( \bar{s}_t \) of (19). The fact that the numerator is the same for both variants of \( \bar{s}_t \) then proves the first part of Proposition 7. The difference between aggregate labor supply in (24) and (19) is
\[
\frac{1 + \beta}{1 + \beta + \eta(1 - \gamma \lambda)} - \frac{1 + \beta - \gamma \lambda}{\beta + (1 + \eta)(1 - \gamma \lambda)} = \frac{\gamma \eta \lambda(1 - \gamma \lambda)}{[\beta + (1 + \eta)(1 - \gamma \lambda)]^2} > 0,
\]
which completes the proof.
Derivation of (26). Without congestion, wages are given by $w_t = (1 - \alpha)\bar{\ell}_t^{-\alpha}$. Using this information and $s_t$ from (25) the law of motion for capital is computed as

$$k_t = \frac{\beta(1 - \alpha)\bar{A}}{1 + \beta + \eta(1 - \gamma \lambda)} \bar{\ell}^{-\alpha} k_{t-1}.$$  

Plugging $k_t$ back into $w_t$ and substituting $\bar{\ell}_t$ from (25) provides

$$w_t = \beta(1 - \alpha)^2 \bar{A}^2 (1 + \beta)^{-2} [1 + \beta + \eta(1 - \gamma \lambda)]^{2\alpha - 1} k_{t-1}.$$  

Taking the derivative with respect to $\lambda$ and dividing by $w_t$ provides (26).
References


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