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The responsibility criterion: Consumer sovereignty without the assumption of coherent preferences

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CONSUMER SOVEREIGNTY
WITHOUT THE ASSUMPTION
OF COHERENT PREFERENCES**

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Abstract

This paper proposes a formulation of consumer sovereignty, for use in normative economics, which does not presuppose individuals' preferences to be coherent. The fundamental intuition, that opportunity and responsibility have moral value, is formalised as a *responsibility criterion* for assessing opportunities in an economic system. A model of an exchange economy is presented, in which rational arbitrageurs compete to make profits by trading with non-rational consumers. In equilibrium, this economy satisfies the responsibility criterion. One interpretation of this result is that, in a competitive environment, the overall effects of money pumps are benign, even if individuals' preferences are persistently incoherent.

Key words: Opportunity, responsibility, consumer sovereignty, money pump.

1. Introduction

This paper reconsiders one of the oldest questions in economics: What, if anything, is good about allocating resources through markets? Since I shall be focusing on perfectly competitive exchange economies, it may seem that I am tackling a question for which the answer is already known. What, the reader may ask, is there to add to the first fundamental theorem of welfare economics, that competitive equilibrium is Pareto-efficient?

My starting point is that that theorem tells us what is good about markets, only if we accept two presuppositions. First, since the theorem is framed in terms of Pareto efficiency, we need to accept preference satisfaction as a normative criterion. Second, since it is a postulate of the theorem that each individual has consistent preferences over the set of possible consumption bundles, we need to be confident that individuals really do have such preferences. Among economists, these presuppositions were once accepted almost universally; but current developments in normative and behavioural economics are making each of them seem less secure. In this paper, I propose a form of normative economics that is compatible with these developments, while retaining the principle of consumer sovereignty that has been central to the mainstream tradition of welfare economics.

In normative economics, there is now a growing emphasis on criteria of *opportunity* rather than of preference satisfaction. In opportunity-based theories, normative value is attached to the size and richness of an individual's *opportunity set* – that is, the set of options from which he is free to choose. Some contributors to this literature argue that opportunity has intrinsic value as an element of well-being in its own right (Sen, 1992; Arrow, 1995). Others argue that the extent to which a person satisfies his preferences is not a proper concern of public policy or of a theory of justice: justice requires equality of opportunity, not equality of preference satisfaction (Cohen, 1989; Roemer 1998). Debate continues about how, for any given person, opportunity sets should be ranked; but there is general agreement that such rankings should not be determined solely by the degree to which each opportunity set allows the individual to satisfy given preferences.¹ The fundamental theorems of welfare economics do not tell us whether markets are effective in providing individuals with opportunity.

Over the last twenty years, there has been an accumulation of evidence that the behaviour of real human decision-makers deviates systematically from the

¹ This literature is reviewed by Roemer (1996) and Sugden (1998). Roemer focuses on opportunity-based theories of justice whilst Sugden focuses on the ranking of opportunity sets.

predictions of conventional choice theory.² It now seems clear that these anomalies are not mere artefacts of particular experimental and survey designs. Beyond this point, opinion among researchers in the field is divided. Some maintain that most anomalies are transient effects which tend to disappear with market experience; others propose that anomalies can be explained by theories in which individuals act on preferences which, although deviating in various ways from the standard assumptions, are context-independent; still others argue that decision-making behaviour is so context-dependent that it is better modelled as the product of a suite of mental routines, and that preferences are ‘constructed’ only in response to specific problems. The jury is still out; but it would surely be prudent to begin to think about what kinds of normative economics would be possible if conventional assumptions about preferences had to be given up.

In this paper, I propose a new approach to normative economics, based on the intuition that individual opportunity and responsibility have moral value. In Section 2, I explain this intuition. In Section 3, I formalise it as a normative criterion – the *responsibility criterion* – for assessing the opportunities that individuals possess in an economic system. This criterion does not presuppose that individuals have coherent preferences: indeed, it does not refer to preferences in any way. In Section 4, I define a condition of *market-clearing* which is similar to the Walrasian concept of competitive equilibrium, but which does not refer to preferences. I show that if the market-clearing condition is satisfied, so is the responsibility criterion.

This result parallels the first theorem of welfare economics, but leaves open the question of whether, in the absence of the usual assumptions about individual rationality, there are any forces that tend to induce market-clearing. In Section 5, I address this question by developing a model in which the opportunities that are available to consumers are provided by profit-seeking arbitrageurs. I define a concept of *free-entry equilibrium* which represents the implications of competition among such arbitrageurs, and show that (provided that there is at least some trade in every good) free-entry equilibrium implies market-clearing and hence the satisfaction of the responsibility criterion. In Section 6, I offer some interpretations of these results. I show that, in a certain sense, my approach treats markets as collections of money pumps operated with the intention of extracting value from consumers. The overall effect of these money pumps is benign, not because consumers are induced to form coherent preferences, but because of the effects of competition among arbitrageurs.

² This evidence is reviewed by Camerer (1995). Alternative interpretations of this evidence are offered by Camerer (1996), Plott (1996), Binmore (1999), Loewenstein (1999) and Starmer (1999).

2. Moral Integration and the Virtue of Responsibility

I start from this normative intuition: It is good that each person is free to get what he or she wants, in so far as this is possible within the constraints imposed by other people’s being free to get what *they* want. This, in essence, is the intuition that underlies the familiar concept of *consumer sovereignty*. However, consumer sovereignty is normally formulated in terms of the satisfaction of coherent preferences. Each individual, it is usually claimed, is the best or proper judge of her own well-being, and those judgements of well-being are revealed in her preferences. My aim is to reformulate the idea of consumer sovereignty in a way that does not require assumptions about the coherence of preferences.

If this aim is to be achieved, we need to find some way of saying that it is good that an individual is free to satisfy her preferences, whether or not those preferences reveal any internally consistent set of judgements about well-being – or, indeed, whether or not they show any internal consistency at all. We need to allow the individual to act on different preferences at different times, in different situations, and in response to different ‘framings’ of what, according to conventional economic theory, is the same decision problem. Having allowed all this, we need to be able to say that it is good that, at each moment, she is free to satisfy whatever preferences she then has.

This way of thinking about consumer sovereignty runs counter to ideas about preferences that are deeply embedded in economics. It is a folk saying in the discipline that, as far as theory is concerned, an individual *is* a preference ordering: everything the theorist needs to know about a person is contained in that person’s preferences. Viewed in this perspective, a person who lacks a coherent set of preferences appears as lacking an integrated sense of his own self: there seems to be no firm basis for statements about what, in his own judgement, is good *for him*. So it is perhaps not surprising that, when economists consider inconsistencies in an individual’s preferences across time, they so often use models of *multiple selves*. That is, the individual is modelled as a collection of distinct selves, each with its own preferences; interactions between these selves are represented in much the same way as are interactions between different people. Typically, each self is treated as having its own view about the welfare of the individual as a whole; if these views differ, the selves are playing a strategic game with one another.³ By representing preference inconsistencies in this way, economists have conserved the identity between selfhood and preference, but at a high cost. A person whose preferences are inconsistent across time has to be treated as lacking the self-integration of a coherent moral agent.

³ This way of representing dynamic inconsistency can be traced to Strotz (1956).

I propose a different approach. If we are to have a normative theory which respects each individual's decisions irrespective of whether they reveal consistent preferences, we need to be able to represent a person's continuing sense of selfhood without referring to her preferences. To this end, I propose a concept of *moral integration*.

The intuitive idea is that a person is morally integrated to the extent that, at each moment in her life, she *identifies with* her own actions, past, present and future. A morally integrated person treats her past actions as her own, whether or not they were what she now desires them to have been. Similarly, she treats her future actions as her own, even if she does not yet know what they will be, and whether or not she expects them to be what she now desires them to be. To treat a past action as one's own is to take *ex post* responsibility for it, rather than attributing it to an alien past self. To treat a future action as one's own is to take *ex ante* responsibility for it, rather than conceiving of oneself as the principal in a principal-agent interaction with an alien future self.

The significance of this concept of moral integration is that it can underpin the claim that opportunity has moral value. That is, by viewing moral integration as a normative standard, and by treating the associated concept of responsibility as a virtue, we can recognise opportunity as having value.

Consider the set of opportunities that are open to some individual across time. Is it a good thing that this set is larger rather than smaller? In conventional welfare economics, more opportunity is better than less only to the extent that it allows the individual to achieve a more preferred outcome; if the individual lacks coherent preferences, there seems to be no way of answering the question. In a model of multiple selves, the question is ill-specified: it has to be posed separately in relation to each self, and increases in lifetime opportunity may be judged to be good from the viewpoint of one self and bad from that of another. (For example, in problems involving self-control, an earlier self may approve of restrictions on the opportunities of a later self.) But from the viewpoint of a morally integrated individual, any increase in her own lifetime opportunity is good for her in the following unambiguous sense: the larger her opportunity set is, the more she – construed as an agent with a continuing existence through time – is free to do. This is true whether or not her actions across time are consistent with any one set of coherent preferences.

I do not claim that everyone shares the moral intuitions that I have been sketching out. Not everyone believes that more opportunity (for himself, or for anyone else) is better than less. Not everyone believes that taking responsibility for one's life as a whole is a virtue. These are moral positions that belong to the tradition of classical liberalism. But just the same is true of the principle of

consumer sovereignty, as it is used in conventional welfare economics. My aim is to formulate a coherent approach to normative economics which will appeal to people of a broadly liberal persuasion.

3. The Responsibility Criterion

I now propose a formal normative criterion which encapsulates the intuitions presented in Section 2. I begin by considering the opportunity set of a single individual.

I work with a simple model in which there are m goods, where $m > 1$. Let $x_i = (x_{i1}, \dots, x_{im})$ represent the quantities of these goods held by a given consumer i ; any such vector is a *bundle*. One such bundle, denoted z_i , is i 's *initial endowment*. The model is intended to represent the opportunities that are open to i , over some period of time, to change his holdings of goods through some process of trade. The economic agents with whom i might trade are not modelled explicitly at this stage; they are represented only by the trading opportunities they offer. The aim is to formulate a normative criterion for assessing those opportunities.

Formally, these opportunities are represented by a *trading opportunity function* $\varphi_i(\cdot)$. To every bundle $x_i \in \mathbb{R}_+^m$, this function assigns a set $\varphi_i(x_i)$, such that $x_i \in \varphi_i(x_i)$. The interpretation is that i can expect that if, at any time, he holds the bundle x_i , he can exchange it for any bundle in the set $\varphi_i(x_i)$. The concept of a 'trading opportunity' is a primitive of the model; but in interpreting the model, I shall make the informal assumption that, at all times, i knows $\varphi_i(\cdot)$. Thus, he knows all the opportunities that are open to him across time. I say that i can *expect* to be able to trade on the terms specified by $\varphi_i(\cdot)$, rather than that he *is* able to trade on those terms, so as to allow the possibility that other agents are offering trading 'opportunities' to i which, if i were actually to take up, those agents could not honour. My reason for allowing this possibility will become clear later.

The *opportunity set* for consumer i , denoted O_i , comprises all those bundles that i can reach, starting from his initial endowment z_i , by sequences of trades that are compatible with $\varphi_i(x_i)$. Formally, for all bundles x_i , x_i is an element of O_i if and only if there is some sequence of bundles y^0, \dots, y^t , such that $y^0 = z_i$, $y^t = x_i$, and, for all $s = 1, \dots, t$, $y^s \in \varphi_i(y^{s-1})$. Prior to any trade, and conceiving himself as a morally integrated actor, i knows that each of the bundles in O_i is a feasible option for him. For each such bundle, i has the information necessary to form a trading plan which, if acted out, will take him to that bundle. However, there is no assumption that i *in fact* forms any such plan or that, if he does form one, he carries it out.

Now let us assume the viewpoint of some social planner who has the power to decide, within certain resource constraints, what initial endowment and what

trading opportunities i will be given. Specifically, suppose that the resource constraints can be represented by some non-empty *feasible set* of bundles X_i : the resources at the planner's disposal make it possible for i to have any one of the bundles in this set, but do not allow him to have any bundle from outside the set. What opportunities should i be given?

In this simple case, it is natural to interpret the principle of consumer sovereignty as requiring that i 's opportunity set should contain every feasible bundle – that is, as requiring that $X_i \subseteq O_i$. (Recall that my interpretation of 'opportunity' does not require that all opportunities are feasible. Thus, we do not need to require that $O_i = X_i$.)

It will turn out to be useful to consider an equivalent *ex post* characterisation of this criterion of opportunity. Consider any opportunity set O_i . Suppose that, in response to these opportunities, i engages in a sequence of trades which takes him to some feasible bundle $x_i^* \in O_i$. *Ex post*, in the situation in which i holds x_i^* , i might point to some other bundle $x_i' \neq x_i^*$, and ask the planner to explain or justify the fact that his holding is not x_i' . To this demand, one surely adequate response is to point out that x_i' is not an element of the feasible set. If this is the case, i 's not having x_i' is an unavoidable consequence of resource constraints. A second possible response is to point out that x_i' is an element of O_i . If this is the case, i 's not having x_i' is attributable to decisions that he has taken knowingly of his own free will: as a continuing moral agent, he is responsible for what has come about. These ideas can be expressed through the following criterion. For the case of a single consumer who ends up with some feasible bundle $x_i^* \in O_i$, the *responsibility criterion* is satisfied if and only if, for every conceivable bundle $x_i \neq x_i^*$, either $x_i \notin X_i$ or $x_i \in O_i$.

The responsibility criterion can be satisfied without the consumer's acting on coherent preferences. For example, suppose there are just two goods and that i 's initial endowment is $z_i = (10, 10)$. Suppose that feasibility constraints allow i to increase his holdings of good 2 by 4/5 unit for every unit of good 1 he forgoes, and to increase his holdings of good 1 by 4/5 unit for every unit of good 2 he forgoes. (Perhaps any change in i 's holdings incurs transport costs.) Formally, the feasible set is defined by the condition that, for any bundle $x_i = (x_{i1}, x_{i2})$, $x_i \in X_i$ if and only if $\max[5x_{i1} + 4x_{i2}, 4x_{i1} + 5x_{i2}] \leq 90$. The condition $X_i \subseteq O_i$ is satisfied if i is offered the opportunity to buy whatever quantities of good 2 he wishes at a price of 5/4 units of good 1 per unit of good 2, and to sell whatever quantities of good 2 he wishes at a price of 4/5. Suppose i is given these opportunities, and responds in the following way. First, feeling a desire to have more of good 1, he sells five units of good 2 and takes four units of good 1 in exchange. Then, changing his mind and feeling a desire to have more of good

2, he buys back the five units of good 2, giving up 6.25 units of good 1. He now has the holding $x_i^* = (7.75, 10)$. If we stipulate that all coherent preferences rank larger bundles above smaller ones, this sequence of trades cannot be interpreted as the product of any set of coherent preferences. Still, the responsibility criterion is satisfied. The consumer may acknowledge that, through changing his mind, has made an unambiguous loss; but that loss was the result of decisions that he made freely, and with which, as a morally integrated agent, he still identifies.

I now consider how the responsibility criterion can be generalised to cases involving more than one consumer. Consider an economy with m goods and n consumers. In interpreting this model, I assume that each consumer is concerned only about his own holdings of goods.⁴

An *allocation* $x = (x_1, \dots, x_n)$ is a profile of bundles of goods, one bundle for each consumer; the *initial allocation* $z = (z_1, \dots, z_n)$ is a profile of initial endowment bundles. For each consumer i there is a trading opportunity function $\phi_i(\cdot)$ and a corresponding opportunity set O_i , defined as before. The *feasible set* of allocations X is some set of allocations such that $z \in X$. The interpretation is that X comprises all those allocations that are feasible, given the resources of the economy. I shall consider only exchange economies without transaction costs. In such an economy, X contains all those allocations x which satisfy the constraints

$$\sum_i (x_{ij} - z_{ij}) \leq 0 \quad j = 1, \dots, m. \quad (1)$$

Just as, in the one-person case, I did not require that all opportunities were feasible, so, in the n -person case, I do not require that every logically possible n -tuple of choices made by the n consumers from their respective opportunity sets is feasible. I shall be modelling the opportunities that consumers have by virtue of having access to a market. When a market is in equilibrium, consumers severally have opportunities which each chooses not to exercise and which, if all consumers tried to exercise simultaneously, could not be realised. (For example, each of us takes himself to be free to buy as much of any major consumer good as he can afford at the current price; but if all of us simultaneously tried to buy much more than we customarily do of the same good, some of us would find our plans frustrated.)

⁴ Since preferences do not appear in the model, this condition cannot be stated as a formal property of preferences. It must be treated as a precondition for the applicability of the model to real-world solutions.

The responsibility criterion can now be stated in its n -person form. Given a profile of opportunity sets $O = (O_1, \dots, O_n)$ from which consumers have made choices leading to a feasible allocation x^* , the responsibility criterion is satisfied if and only if, for every allocation $x \neq x^*$, *either* $x \notin X$ *or* there is some consumer j such that $x_j \neq x_j^*$ and $x_j \in O_j$. In other words: every feasible allocation other than the one that has in fact come about assigns to some consumer a bundle that that consumer had, but did not take, the opportunity to achieve.

This criterion can be interpreted as a generalisation of the idea that, in a one-person economy, that person should be able to take responsibility for the outcome he experiences. The n -person criterion requires that, for every feasible alternative to the actual outcome, *someone* can take responsibility for that alternative not having come about. To see what this means, suppose that, after x^* has come about, some consumer i asks for a justification of the fact that his holding is x_i^* and not some other bundle x_i' . In order for this complaint to have substance, i needs to be able to point to some feasible allocation x' in which his own bundle is x_i' . If $x_i' \in O_i$, the fact that x_i' has not come about is the result of i 's own free choices, for which he must take responsibility. But what if, although $x_i' \notin O_i$, there is some other consumer j for whom $x_j' \neq x_j^*$ and $x_j' \in O_j$? If this is the case, then in proposing x' , i is proposing that j ends up with a bundle, different from the one she in fact holds, that *she* has already had the opportunity to achieve. That is, i is proposing to undo the outcome for j of j 's free choices.

One might say that the responsibility criterion is the analogue in the domain of opportunity of the Pareto criterion in the domain of preference satisfaction. If the Pareto principle is satisfied, any feasible alternative to the allocation that has in fact come about involves someone's moving to a less preferred position. If the responsibility criterion is satisfied, any feasible alternative to the allocation that has in fact come about involves undoing an outcome for which some individual is responsible.

4. Market-Clearing Prices and the Responsibility Criterion

I now define a property of initial endowments, opportunity sets and final outcomes which is sufficient to ensure that the responsibility criterion is satisfied.

Consider any exchange economy with n consumers and m goods. Let z be the initial allocation. Let O be the profile of opportunity sets, and let x^* be an allocation which comes about as a result of each consumer i choosing x_i^* from O_i . I shall say that the triple (x^*, z, O) is *market-clearing* if the following conditions are satisfied:

- A1. *Feasibility*. For each good j : $\sum_i (x_{ij}^* - z_{ij}) \leq 0$.
- A2. *No waste*. For each good j : $\sum_i (x_{ij}^* - z_{ij}) \geq 0$.
- A3. *Single price*. There is some vector of finite, strictly positive prices $p = (p_1, \dots, p_m)$ such that, for each individual i : $O_i = \{x_i: \sum_j p_j (x_{ij} - z_{ij}) \leq 0\}$.

The following result can be proved:

Result 1: For all x^, z, O : if (x^*, z, O) is market-clearing, then O satisfies the responsibility criterion.*

The proof (which is given in the Appendix) is a simple adaptation of a familiar revealed-preference proof of the first fundamental theorem of welfare economics.

Result 1 tells us that *if* each good has a single price at which all individuals are free to buy and sell whatever quantities they wish, *and if*, as a result of choices that individuals make in relation to those prices, total final holdings of each good are exactly equal to total initial endowments, *then* the responsibility criterion is satisfied. This result is a first step towards a normative justification of the market in terms of the responsibility criterion; but it is no more than this.

If this result is to justify the market, there has to be an argument which shows that market forces induce market-clearing prices. The concept of market-clearing used in Result 1 is essentially the same as Walrasian equilibrium in conventional general equilibrium theory – except that no assumptions are made about preferences. Walrasian equilibrium is normally interpreted in terms of consumers who respond to market prices by making utility-maximising bids to buy and sell goods: the Walrasian auctioneer (who personifies the forces of the market) interacts with *rational* consumers. But is market-clearing still an appropriate equilibrium concept if individuals are not assumed to have coherent preferences?

5. Arbitrage

The fundamental idea I now develop is that, whether or not consumers have coherent preferences, arbitrage induces market-clearing prices. At this point, I introduce a new class of actors into the theoretical framework: *traders*. So far, I have modelled consumers as facing ‘trading opportunities’, which they use to acquire bundles of goods that they desire to consume. Those trading opportunities have been treated as primitives. I now open that black box: traders are the agents who offer those opportunities.

I begin with an intuitive account of my model of the trading process.

In this model, traders do not consume goods; they merely buy them from, and sell them to, consumers with the objective of making profits. In general, goods are not directly traded against one another. Instead, each of goods 2, ..., m is traded separately against good 1, which will be called *money*. For consumers, money has intrinsic value in consumption as well as serving as a medium of exchange. Traders measure profits in terms of increases in their holdings of money; holdings of other goods have no intrinsic value to them.

In thinking about the model, it might help to imagine the following scenario. Suppose the consumers are the inhabitants of an island, unable to travel to or from the mainland. They hold endowments of the various goods, including money. The traders live on the mainland. They are able to travel to and from the island, but the only good that they can transport in either direction is money. They come to the island equipped with stocks of money to be used as working capital. Through trading with consumers, each trader hopes to return to the mainland with more money than she took out.

The distinction between consumers and traders is intended to represent two different roles that economic actors can play. As consumers, individuals attempt to satisfy their subjective desires by acquiring bundles of goods that they want to consume. As traders, individuals attempt to make profits by buying goods at low prices and reselling them at higher prices. My model will attribute a high degree of rationality to traders, in the sense of their being able to foresee opportunities for profit, and to pursue such opportunities consistently. However, it is *not* assumed that traders have consistent preferences in the m -dimensional space of bundles of goods, any more than this is assumed of consumers. The rationality of traders consists in their ability to make accurate predictions of demand and supply, and to solve *objective* maximisation problems. All that is assumed about their subjective tastes is that they prefer more money to less.

The real-world counterparts of the traders of the model are professional entrepreneurs and arbitrageurs, who have accumulated experience of the particular markets in which they work and who have been subject to a natural selection process. (Traders who consistently make losses do not accumulate the reserves necessary to engage in large-scale arbitrage.) Thus, I suggest, it is reasonable to make the modelling assumption that traders are well-informed and instrumentally rational in the pursuit of profit.

The market of the model works in the following way. Initially, each trader's holding of each non-money good is zero. Consumers in the aggregate have positive holdings of all m goods. Prior to any trade taking place, each trader sets the values of certain parameters which specify the trades she is willing to make. For each non-money good, she sets a buying price (the money price at which she is willing to buy the good from consumers), a selling price (the money price at which she is willing to sell it), a buying constraint (the maximum total quantity she is willing to sell), and a selling constraint (the maximum total quantity she is willing to sell).

Trading takes place over an interval of time, during which the prices and constraints set by each trader are fixed. We may think of consumers milling around a public market place in which traders are displaying their buying and selling prices. Constraints are not displayed, but if a trader has already sold (or bought) as much as her constraint allows, her selling (or buying) price is no longer displayed. At any time, a consumer may approach a trader and ask to trade on the terms that trader is offering. If a consumer's trading request would require a trader to exceed a constraint, trade takes place up to that constraint. A consumer may volunteer to pay more than the selling price quoted by the trader he approaches, or to accept less than the quoted buying price; any such offers (which are effectively gifts of money from the consumer to the trader) are accepted.⁵

During the trading period, traders' promises to deliver goods are treated, both by consumers and by other traders, as equivalent to possession of the corresponding goods; thus, at any given time, a trader's holding of a good may be negative. However, a trader incurs a severe penalty if, *at the end of the trading period*, she has negative holdings of any good. (To have negative holdings of any good at the end of the trading period is to be unable to make deliveries that have been promised.) Consumers are not permitted to have negative holdings at any time. There is some exogenous limit to the length of the trading period, so that the trading process cannot continue indefinitely.

⁵ This option is included in the model to allow consumers to dispose of their holdings if they so choose. The assumptions of the model will imply that this option is never used, but its existence is significant for the specification of consumers' opportunity sets.

I make just one rationality assumption about consumers: *at any given time, in relation to any given good*, consumers buy only at the lowest price currently offered, and sell only at the highest price currently offered. (One implication of this assumption that consumers never use the option of paying more than, or accepting less than, a quoted price.) This assumption is essential for the model, since it defines the terms on which the traders compete. However, there is no assumption that the various trades that a given consumer makes are *jointly* consistent with any concept of preference – not even a preference for more money rather than less. For example, suppose that the lowest selling price quoted by traders for some good j is higher than the highest buying price. A consumer who is endowed with some positive amount of good j might choose to sell this endowment at the highest buying price on offer and then, immediately afterwards, choose to buy the same quantity back again at the lowest selling price on offer. These two trades, taken together, imply a loss of money and no compensating gain. Still, each trade, considered in isolation, was made on the best available terms.

I now present the model itself. Formally, this is distinct from the model of consumers' opportunities that was developed in Section 4. In the present model, the behaviour of consumers over time is not represented explicitly. The model represents only the final outcomes of consumers' decisions, as these impinge on traders, for given profiles of prices and constraints set by traders.

There is a finite number of traders N , where $N > 1$. Each trader k sets, for each non-money good j , a *buying price* $p_{jk}^B \geq 0$, a *selling price* $p_{jk}^S \geq 0$, a *buying constraint* $c_{jk}^B \geq 0$, and a *selling constraint* $c_{jk}^S \geq 0$. Prices are expressed in money units per unit of good j ; constraints are expressed in units of good j . The interpretation is that k undertakes to sell good j to consumers at price p_{jk}^S in any quantities that consumers request, subject to the constraint that her *total* sales do not exceed c_{jk}^S ; and similarly for buying. The profile of prices and constraints set by all traders for all markets is the *offer configuration*.

For each possible offer configuration F , there is a determinate *outcome* $\phi(F)$. The outcome of an offer configuration specifies how much of each non-money good is bought and sold by each trader, and how much money each trader receives in *gifts* from consumers (that is, as a result of consumers choosing to pay more than a trader's selling price or to accept less than a buying price). The quantity of each non-money good j sold by trader k , as an outcome of offer configuration F , is denoted by $q_{jk}^S(F)$; the total sold by all traders is $q_j^S(F)$. Similarly, $q_{jk}^B(F)$ is the quantity of good j bought by trader k , and $q_j^B(F)$ is the total quantity bought by all traders. Trader k 's receipt of gifts in transactions in which she sells good j is denoted by $g_{jk}^S(F)$; her receipt of gifts in transactions in which she buys good j is $g_{jk}^B(F)$; the totals of such receipts for all traders are

$g_j^S(F)$ and $g_j^B(F)$ respectively. It is convenient to use $p_{jk}^S(F)$ to denote the selling price for good j set by trader k in offer configuration F , $c_{jk}^S(F)$ to denote the selling constraint for good j set by trader k in offer configuration F , and so on.

The following definitions will be used. Consider any offer configuration F and any non-money good j . A trader k is an *active seller* of good j in F if $c_{jk}^S(F) > 0$; she is an *active buyer* of good j if $c_{jk}^B(F) > 0$; she is *active* in F if she is an active buyer or seller of at least one non-money good. A price π is an *effective selling price* for good j in F if there is some trader k for whom $p_{jk}^S(F) = \pi$ and $q_{jk}^S(F) > 0$; similarly, it is an *effective buying price* for good j in F if there is some trader k for whom $p_{ik}^B(F) = \pi$ and $q_{ik}^B(F) > 0$. (That is, a price is effective if some trade takes place at that price.) An effective selling price π for good j is *constrained* if (i) $q_{jk}^S(F) = c_{jk}^S(F)$ holds for every trader k for whom $p_{jk}^S(F) = \pi$, and (ii) for every offer configuration F' which differs from F only in respect of the selling constraint for good j of some such trader k , $c_{jk}^S(F') > c_{jk}^S(F)$ implies $q_{jk}^S(F') > c_{jk}^S(F)$. (That is, an effective selling price is constrained if the selling constraint of every trader who sells at that price is strictly binding.) The concept of a constrained effective buying price is defined in a symmetrical way.

I impose six restrictions on the function $\phi(\cdot)$:

B1. *Respect for constraints.* For all offer configurations F , for all traders k , for all non-money goods j : (i) $c_{jk}^S(F) \geq q_{jk}^S(F) \geq 0$; (ii) $c_{jk}^B(F) \geq q_{jk}^B(F) \geq 0$.

B2. *Price sensitivity.* For all offer configurations F , for all traders k, h , for all non-money goods j : (i) if $p_{jk}^S(F) < p_{jh}^S(F)$ then *either* $q_{jh}^S(F) = 0$ *or* $q_{jk}^S(F) = c_{jk}^S(F)$; (ii) if $p_{jk}^B(F) > p_{jh}^B(F)$ then *either* $q_{jh}^B(F) = 0$ *or* $q_{jk}^B(F) = c_{jk}^B(F)$.

B3. *No gifts.* For all offer configurations F , for all traders k , for all non-money goods j : (i) $g_{jk}^S(F) = 0$; (ii) $g_{jk}^B(F) = 0$.

B4. *Symmetry.* For all offer configurations F , for all traders k, h , for all non-money goods j : (i) if $p_{jk}^S(F) = p_{jh}^S(F)$ and $q_{jk}^S(F) > q_{jh}^S(F)$, then $q_{jh}^S(F) = c_{jk}^S(F)$; (ii) if $p_{jk}^B(F) = p_{jh}^B(F)$ and $q_{jk}^B(F) > q_{jh}^B(F)$, then $q_{jh}^B(F) = c_{jk}^B(F)$.

B5. *Independence of inactive prices.* (i) Let F, F' be two offer configurations which differ only in respect of the selling price of some trader k for some non-money good j . If k is not an active seller of good j either in F or in F' , then $\phi(F') = \phi(F)$. (ii) Let F, F' be two offer configurations which differ only in respect of the buying price of some trader k for some non-money good j . If k is not an active buyer of good j either in F or in F' , then $\phi(F') = \phi(F)$.

B6. *Continuity.* For all non-money goods j : $q_j^S(\cdot)$ and $q_j^B(\cdot)$ are continuous functions of all $p_{hi}^S, p_{hi}^B, c_{hi}^S, c_{hi}^B$ ($i = 1, \dots, N$; $h = 2, \dots, m$).

These restrictions are motivated by the informal description of the model, or are modelling simplifications. B1 requires that traders' selling and buying constraints are not exceeded; this is an immediate implication of the interpretation given to those constraints. B2 requires that if some traders offer higher selling prices or lower buying prices than others, those offering the less favourable prices make trades only if the constraints associated with the more favourable prices bind. This is an implication of the informal assumption that, at every point in time, consumers trade only at the most favourable prices currently available. B3, which requires that consumers never pay more to a trader than her quoted selling price, or accept less than her quoted buying price, is another implication of the same assumption. B4 requires that, except as a consequence of binding constraints, traders who quote the same selling (or buying) price for a given good sell (or buy) equal quantities of it. This is a simplifying assumption which abstracts from the effects of random variation in the assignment of consumers to traders. B5 requires that consumers take no account of the selling prices of traders who are not active sellers, nor of the buying prices of traders who are not active buyers. This is an implication of the informal assumption that, at any time, consumers see only the prices offered by traders whose constraints are not then binding. B6 requires that small changes in traders' prices and constraints do not induce discontinuous changes in the aggregate behaviour of consumers.⁶

The next step is to define a concept of equilibrium for the market of the model. The concept I use, *free-entry equilibrium*, is intended to correspond as closely as possible with the concept of perfectly competitive equilibrium that micro-economics has traditionally applied to sets of competing firms. The intuitive idea is that an offer configuration is a free-entry equilibrium if no trader trades at a loss, and if traders who are not active in the market cannot make positive profits by entering (other things remaining constant).

As with conventional concepts of competitive equilibrium for firms, there is an implicit assumption that, in equilibrium, traders can predict the consequences for themselves of the alternative trading strategies that are available to them. In the context of my model, this requires that the behaviour of consumers is predictable in the aggregate. Notice that this form of predictability does not imply that the choices of individual consumers reveal coherent preferences – for at least two reasons. First, the behaviour of an individual may be predictable by virtue of regularities that are consistent with some theory other than that of rational choice. Second, if the number of consumers is large, predictability in

⁶ This condition allows that small changes in traders' prices might induce discontinuous changes in the *distribution* of trade among traders; discontinuous changes in the distribution of trade should be expected, given the price sensitivity condition.

the aggregate is compatible with stochastic variation in the behaviour of individual consumers. Notice also that we do not need to assume that traders understand the *causes* of the regularities they observe in consumers' behaviour.

Formally, I define an offer configuration F to be a *free-entry equilibrium* if it satisfies the following conditions:

C1. *Non-negative profit.* For each trader k : $\sum_{j=2}^m [p_{jk}^S(F) q_{jk}^S(F) + g_{jk}^S(F)] \geq \sum_{j=2}^m [p_{jk}^B(F) q_{jk}^B(F) - g_{jk}^B(F)]$.

C2. *No shortfalls.* For each trader k , for each non-money good: $q_{jk}^B(F) \geq q_{jk}^S(F)$.

C3. *Existence of potential entrant.* At least one trader is not active in F .

C4. *No profitable entry.* Let F' be any offer configuration which differs from F only in respect of the prices and constraints of some trader k who is not active in F . If $q_{jk}^B(F') \geq q_{jk}^S(F')$ for each non-money good j , then $\sum_{j=2}^m [p_{jk}^S(F') q_{jk}^S(F') + g_{jk}^S(F')] \leq \sum_{j=2}^m [p_{jk}^B(F') q_{jk}^B(F') - g_{jk}^B(F')]$.

C1 requires that no trader makes a loss. Since it is possible to avoid losses by becoming non-active, this is a natural equilibrium condition. C2 requires that no trader's sales of any good exceed her purchases; this is an implicit representation of the assumption that traders are penalised for failure to deliver on the promises they make (and that these penalties are sufficient to deter traders from making promises that they do not expect to honour). C3 is a convenient way of representing the idea, familiar in the theory of perfect competition, that, for every market, there are potential entrants. C4 requires that no non-active trader can make a strictly positive profit by becoming active, while satisfying the condition that sales may not exceed purchases. This is the natural counterpart of C1 as an equilibrium condition for a market with free entry and exit.

The relationship between free-entry equilibrium and market-clearing is encapsulated in the following result, which is proved in the Appendix:

Result 2. For all offer configurations F : if F is a free-entry equilibrium, then for each non-money good j , if there exists any effective selling price or effective buying price for that good, then there is a price $p_j^* > 0$ such that (i) p_j^* is both the unique effective selling price and the unique effective buying price for good j , (ii) p_j^* is unconstrained, both as a selling price and a buying price, (iii) $q_j^B(F) = q_j^S(F)$, and (iv) $g_j^S(F) = g_j^B(F) = 0$.

To interpret this result in relation to consumers' opportunity sets, consider the case in which, for each non-money good, there is some effective selling price or effective buying price. Then we know from parts (i) and (ii) of Result 2 that, for each non-money good j , there is a price p_j^* which is displayed to consumers, both as a buying price and as a selling price, throughout the trading process. Throughout this process, any trade that any consumer chooses to make at this price is in fact made. Counterfactually, had any consumer desired to buy or sell slightly more of any good than he in fact bought or sold, that desire would have been accommodated. Had any consumer wished to dispose of any good, that wish would have been accommodated too (through the consumer paying more than p_j^* to buy some good j , or accepting less than p_j^* for selling it). Given all this, we may reasonably⁷ specify the trading opportunity function of each consumer i as

$$\phi_i(x_i) = \{y_i : \sum_{j=2}^m p_j^*(y_{ij} - x_{ij}) \leq 0\} \quad (2)$$

for all bundles x_i , which implies that the opportunity set of each consumer is given by

$$O_i = \{x_i : \sum_{j=2}^m p_j^*(x_{ij} - z_{ij}) \leq 0\}. \quad (3)$$

This specification implies that the triple (x^*, z, O) satisfies the single-price condition A3.

Part (iii) of Result 2 tells us that, for each non-money good j , total sales by traders are equal to total purchases. Part (iv) tells us that traders receive no gifts from consumers. Since, for each non-money good j , all trade takes place at the same price p_j^* , it follows that total receipts of money by traders are equal to total expenditures. Hence, (x^*, z, O) satisfies both the feasibility condition A1 and the no waste condition A2.

Thus, provided that at least some trade takes place in each non-money good, free-entry equilibrium implies that (x^*, z, O) is market-clearing. Result 1 then allows us to conclude that free-entry equilibrium satisfies the responsibility criterion.

The proviso that at least some trade takes place in each non-money good may seem to weaken these results as a justification of the market; but that proviso is unavoidable. To see why, consider the case in which, for some non-money

⁷ Notice that (2) is an *interpretation* of Result 2, and not a strict implication of it. Formally, trading opportunities belong in the model of consumers' opportunities that was developed in Section 4, while offer configurations belong in the model of interactions among traders that was developed in Section 5. These are distinct models.

good j , the lowest price at which any consumer in any circumstances would sell that good is higher than the highest price at which any consumer in any circumstances would buy it. Then, clearly, it is compatible with free-entry equilibrium for there to be no active buyers or sellers of good j . If that is the case, individual consumers do not have any opportunities to exchange good j for other goods, and so the responsibility criterion is not satisfied.

However, the proviso demands very little. So long as some consumer is willing to sell some quantity of good j at some price greater than that at which some consumer is willing to buy some quantity of it, there cannot be a free-entry equilibrium in which there is no trade in good j . Thus, the proviso requires only that there are *some* potential gains from trade in each good.

6. Markets as Money Pumps

What, if anything, is good about allocating resources through markets? That question was the starting point for the analysis in Sections 3, 4 and 5. My analysis supports a particular answer: that competitive markets provide opportunities which allow individuals to take responsibility for the outcomes they experience.

In interpreting the formal results, it is natural to ask what gives markets this (on my account) morally valuable property. Viewed through the lens of my model, what are the essential features of markets?

I suggest that four features of my model are particularly significant as stylised representations of properties of real markets. First, there is the *profit motive*. There is a significant number of actors who are motivated by a desire to buy cheap and sell dear, and who are alert to opportunities which allow them to do so. Second, there is *free entry*. Potential traders who see opportunities for profit are not prevented from exploiting them. This property is crucial in ensuring that, in equilibrium, no profits can be made through arbitrage. Third, there is *publicness of transactions*. All offers to buy and sell are visible to, and open to, all consumers. It is by virtue of this property that each consumer has opportunities which extend beyond the trades that he is in fact willing to make. Finally, there is *price sensitivity*. Consumers are aware of any differences in the prices offered to them by different traders, and buy and sell only at the most favourable prices currently on offer. It is because of this property that the profit motive leads traders to compete to offer favourable prices to consumers. Summing up, the profit motive (on the part of traders) and price sensitivity (on the part of consumers) supply the motive power for a process of competition. Free entry and publicness are the fundamental rules of the game which govern that process.

In comparison with more familiar models of markets, my model is distinctive in how little it assumes about the rationality of consumers: all it assumes is price sensitivity. Most of the work of generating the morally valuable properties of markets is done by profit-seeking traders. From the viewpoint of those traders, consumers are essentially passive, responding to traders' offers in a predictable but not necessarily rational fashion. We might say that the body of consumers appears to traders rather as a population of fish appears to a set of competing trawler-owners, or as an oil field appears to a set of competing oil-prospectors.

The metaphor of the oil field prompts another: the traders are operators of *money pumps*. In the literature of decision theory, a money pump is a sequence of trading opportunities offered to a particular individual such that, if all those

opportunities are accepted, the resulting sequence of trades generates an unambiguous gain for the trader who offers the opportunities and an unambiguous loss for the person who accepts them. Many theorists have claimed that, in order for an individual to be invulnerable to money pumps, he must have preferences which satisfy conventional axioms of rationality, such as transitivity and the independence axiom of expected utility theory. The suggestion has been that the possibility of money pumps forces economic actors to act on preferences which satisfy rationality axioms. It is a presupposition of this literature (shared by those theorists who deny the validity of money pump arguments) that vulnerability to money pumps, if it exists, is a pathology of individual decision-making.⁸

In contrast, my approach does not make any fundamental distinction between money pumps – that is, trading sequences which impose unambiguous losses on ‘irrational’ individual consumers – and what might be called benign arbitrage – that is, trading activities which generate profits by realising gains from trade between ‘rational’ consumers.⁹ In each case, traders make profits by offering to consumers opportunities which those consumers freely choose to take up. In each case, the existence of profit opportunities is a phenomenon of disequilibrium. Competition between traders ensures that neither kind of profit opportunity exists in equilibrium. However, the preference inconsistencies which make consumers *potential* victims of money pumps may persist in equilibrium. What competition erodes is not individuals’ propensities to act contrary to the axioms of rational choice theory, but the profit margins that can be achieved by trading with individuals who act in this way.

On the account I am offering, it is as a result of providing a field of open competition for would-be arbitrageurs and money-pump operators that markets provide the opportunities that allow individuals to take responsibility for outcomes. A market, we might say, is a complex system of money pumps, each of which is operated with the intention of extracting value from us, the consumers. Nevertheless, that system gives us opportunity and responsibility – whether or not our preferences meet the standards of rational choice theory.

⁸ Recent contributions to the theoretical debate about the validity of money pump arguments include Machina (1989), McClennen (1990), Anand (1993), Border and Segal (1994), Kelsey and Milne (1997) and Cubitt and Sugden (2001).

⁹ Nau and McCardle’s (1991) analysis of the relationship between arbitrage and rationality follows a similar approach to that of the present paper, but in relation to choice among lotteries with money consequences.

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Appendix: Proofs of theorems

Proof of Result 1

Let (x^*, z, O) be any market-clearing equilibrium. To prove Result 1, it is sufficient to show that, for every feasible allocation $x \neq x^*$, there is some individual i for whom $x_i \neq x_i^*$ and $x_i \in O_i$. To allow a proof by contradiction, suppose the contrary. Then there exists some feasible allocation $x' \neq x^*$ such that, for all i , either (a) $x_i' = x_i^*$ or (b) $x_i' \notin O_i$. Let p be the price vector in relation to which O_i satisfies the single-price condition. Trivially, (a) implies $\sum_j p_j (x_{ij}' - x_{ij}^*) = 0$. Since $x_i^* \in O_i$, (b) implies $\sum_j p_j (x_{ij}' - x_{ij}^*) > 0$. Since $x' \neq x^*$, (b) must apply for some individual i . Thus, $\sum_i \sum_j p_j (x_{ij}' - x_{ij}^*) > 0$. But, since x' is feasible and x^* is both feasible and non-wasteful, $\sum_i (x_{ij}' - x_{ij}^*) \leq 0$ for all j , which contradicts the inequality stated in the preceding sentence.

Proof of Result 2

Consider any offer configuration F which is a free-entry equilibrium. Suppose (this is *Supposition 1*) that, for some non-money good j , there is an effective selling price p^S and an effective buying price p^B such that $p^S > p^B$. Let k be any trader who is not active in F ; the existence of such a trader follows from C3. Let F^1 be an offer configuration which differs from F only in respect of k 's selling and buying prices and constraints for good j . Set $p_{jk}^S(F^1) = p^S$, $p_{jk}^B(F^1) = p^B$, and $c_{jk}^S(F^1) = c_{jk}^B(F^1) = \varepsilon$, where $\varepsilon > 0$. By B5 and B6, as $\varepsilon \rightarrow 0$, $q_j^S(F^1) - q_j^S(F) \rightarrow 0$ and $q_j^B(F^1) - q_j^B(F) \rightarrow 0$. Thus, by B1, B2 and B4, for sufficiently small values of ε , $q_{jk}^S(F^1) = q_{jk}^B(F^1) = \varepsilon$. At any such ε , k makes a profit in F^1 , contrary to the 'no profitable entry' condition C4. Hence *Supposition 1* is false.

Thus, for each non-money good, the lowest effective buying price is at least as high as the highest effective selling price. In conjunction with the 'no shortfalls' condition C2, and the 'no gifts' condition B3, this implies that no trader makes a strictly positive profit in her transactions in any non-money good. But, by virtue of the non-negative profit condition C1, no trader makes a strictly negative total profit. So each trader makes zero profit in her transactions in each non-money good. Since effective selling prices are never greater than effective buying prices, it follows that, for each non-money good j , *either* (a) no trade takes place (i.e. there is no effective buying price and no effective selling price for good j) *or* (b) some trade takes place in good j , all such trade – both buying and selling – takes place at the same price p_j^* , and, for each trader, sales and purchases of good j are equal. This proves parts (i) and (iii) of Result 1. Part (iv) follows immediately from B3.

Now suppose that (b) holds for some non-money good j , and let p_j^* be the effective buying and selling price for that good. Suppose (this is *Supposition 2*)

that p_j^* is a constrained selling price. Let k be any trader not active in F , and let F^2 be an offer configuration which differs from F only in respect of k 's selling and buying constraints for good j . Set $p_{jk}^S(F^2) = p_{jk}^B(F^2) = p_j^*$, $c_{jk}^S(F^2) = \varepsilon^S$, and $c_{jk}^B(F^2) = \varepsilon^B$, where $\varepsilon^S, \varepsilon^B > 0$. By the same reasoning as used in the first part of the proof, for sufficiently small values of ε^S and ε^B , $q_{jk}^S(F^2) = \varepsilon^S$ and $q_{jk}^B(F^2) = \varepsilon^B$.

The next step is to show that, for sufficiently small values of ε^S and ε^B , p_j^* is also a constrained selling price in F^2 . Since p_j^* is a constrained selling price in F , if any trader h who is an active seller of good j in F relaxes her selling constraint, all other prices and constraints being held constant at their values in F , h 's sales of good j increase. By virtue of B5, for sufficiently small values of ε^S and ε^B , it must also be true that if h relaxes her selling constraint with all other prices and constraints being held constant at their values in F^2 , her sales increase. It has been shown that for sufficiently small values of ε^S and ε^B , $q_{jk}^S(F^2) = \varepsilon^S$ (where k is the trader who is active in F^2 but not in F). This implies that, starting from sufficiently small values of ε^S and ε^B , if k relaxes her selling constraint with all other prices and constraints being held constant at their values in F^2 , k 's sales increase. Thus p_j^* is a constrained selling price in F^2 .

Now let F^3 be any offer configuration which differs from F only in respect of k 's selling and buying constraints for good j . Set $p_{jk}^S(F^3) = p_j^* + \delta$, $p_{jk}^B(F^3) = p_j^*$, $c_{jk}^S(F^3) = \varepsilon^S$, and $c_{jk}^B(F^3) = \varepsilon^B$, where $\delta > 0$, where ε^S and ε^B take the same values as in F^2 , and such that there is no trader h for whom $p_j^* + \delta > p_{jh}^S(F^3) > p_j^*$ and $c_{jh}^S(F^3) > 0$. (Since the number of traders is finite, the condition that there is no such trader is satisfied at all sufficiently small values of δ .) Now compare the trades that k makes in F^2 and F^3 . It follows from B5 that, as $\delta \rightarrow 0$, $q_j^B(F^3) - q_j^B(F^2) \rightarrow 0$. Thus, by B1, B2 and B3, at sufficiently low values of ε^B , $c_{jk}^B(F^3) = \varepsilon^B$. Similarly, as $\delta \rightarrow 0$, $q_j^S(F^3) - q_j^S(F^2) \rightarrow 0$. Since p_j^* is the only effective selling price for good j in F , no trader in F^2 or F^3 sells good j at any price p such that $p \neq p_j^*$ and $p < p_j^* + \delta$. Since p_j^* is a constrained selling price in F^2 , sales of good j by traders other than k at price p_j^* cannot be greater in F^3 than in F^2 . Thus, total sales by traders other than k at prices less than $p_j^* + \delta$ are no greater in F^3 than in F^2 . By B2, in F^3 , total sales at prices greater than $p_j^* + \delta$ are zero unless k 's selling constraint is binding, i.e. unless $q_{jk}^S(F^3) = q_{jk}^S(F^2)$. Thus, as $\delta \rightarrow 0$, $q_{jk}^S(F^3) - q_{jk}^S(F^2) \rightarrow 0$. Using B5, it follows that there exist strictly positive $\varepsilon^S, \varepsilon^B$ and δ , such that $\varepsilon^S = \varepsilon^B$, $q_{jk}^S(F^3) = \varepsilon^S$, and $q_{jk}^B(F^3) = \varepsilon^B$. At such values of $\varepsilon^S, \varepsilon^B$ and δ , k makes a profit in F^3 , contrary to the 'no profitable entry' condition C4. Hence Supposition 2 is false: p_j^* is *not* a constrained selling price. A symmetrical argument shows that p_j^* is not a constrained selling price. This proves part (ii) of Result 1.