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Constitution or Conflict?

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Abstract
Constitutional resolution of disputes between constituent groups of a polity avoids the incremental costs of civil conflict. But, the political process prescribed by a constitution provides a viable alternative to civil conflict if and only if the constitution is self enforcing. This paper presents theoretical and historical analyses of the possibility of designing a self-enforcing constitution. The theoretical analysis reveals that the following factors make it possible to design a self-enforcing constitution that a polity can use to resolve a given dispute:

- No party to the dispute has a big advantage in civil conflict.
- The parties to the dispute expect the incremental costs of civil conflict to be large relative to the importance of the dispute.
- The parties to the dispute have almost as much concern for the future consequences of the choices that they make as for the current consequences.

The theoretical analysis also reveals that a self-enforcing constitution can require limitations on the prerogatives of winners of constitutional contests such that the outcomes of constitutional contests do not matter too much. The historical analysis illustrates the relevance of the theoretical analysis by applying these theoretical results to two dramatic examples of constitutional failure: the secession of eleven Southern states from the Union in 1861 and the National Socialist revolution in Germany in 1933.

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A constitution attempts to create a political process for resolving disputes among the constituent groups of a polity. Constitutional resolution of disputes, although not costless, is desirable because it avoids the incremental costs of civil conflict. These incremental costs, which distinguish civil conflicts from constitutional political processes, involve the allocation of scarce resources to arming and to other conflictual activities and, if conflict escalates beyond the threat of the use of force, also can include the traumatic effects of violence and possibilities of collateral damage and even the loss of life.

Sometimes constitutions succeed, but other times constitutions fail to prevent civil conflict. The problem is that the political process prescribed by a constitution provides a viable alternative to civil conflict if and only if the parties to the dispute to be resolved voluntarily choose to abide by the constitution — that is, if and only if the constitution is self enforcing. If any party to a dispute does not voluntarily accept the outcome of the constitutional political process, even if this party is eventually coerced into complying, then the polity has been left to resolve the dispute by civil conflict.

This paper presents theoretical and historical analyses of the possibility of designing a self-enforcing constitution. The theoretical analysis is concerned with discovering the configurations of exogenous parameters under which in the face of given dispute the set of self-enforcing constitutions is not empty. The relevant exogenous parameters include measures of the importance of the dispute to be resolved, the probabilities associated with the outcome of a civil conflict, the expected incremental costs of a civil conflict, and the concern for the future consequences of choices relative to current consequences. The theoretical analysis is also concerned with discovering some properties of a self-enforcing constitution.

The question of the possibility of designing a self-enforcing constitution is logically prior to two related questions. First, if the set of self-enforcing constitutions is not empty, is the possibility of designing a self-enforcing constitution realized? Second, which constitution is chosen from the set of self-enforcing constitutions, if that set includes more than one element? The analysis in this paper determines the preconditions for addressing these
important questions.$^1$

The most important implication of the theoretical analysis is that the following characteristics of the configuration of exogenous parameters make it possible to design a self-enforcing constitution that a polity can use to resolve a given dispute:

- No party to the dispute has a big advantage in civil conflict.

- The parties to the dispute expect the incremental costs of civil conflict to be large relative to the importance of the dispute.

- The parties to the dispute have almost as much concern for the future consequences of the choices that they make as for the current consequences.

Alternatively, either a big advantage in civil conflict for one of the parties to the dispute, or expectations that the incremental costs of civil conflict are small relative to the importance of the dispute, or little concern for future consequences can preclude the design of a self-enforcing constitution. The theoretical analysis also reveals that a self-enforcing constitution

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$^1$The answers to both of these questions depend on, among other things, whether a consensus of the constituent groups of a polity is necessary to establish a constitution, or whether one constituent group can impose a constitution on others, or an outsider can impose a constitution. A literature on “democratic consolidation” focuses on the possibility of a consensus in favor of a democratic rather than an autocratic constitution. See Gerard Alexander (2002) for an overview and recent contribution to this literature. In work that is more closely related to the present paper, Daron Acemoglu and James Robinson (2001) and Adam Przeworski (1991, 2001) ask whether a constitution can enable a polity to avoid civil conflict. But, these authors take civil conflict to be only a mechanism for switching between democratic and nondemocratic constitutions. In other related work, Avinash Dixit, Gene Grossman, and Faruk Gul (2000), who generalize the seminal work of Alberto Alesina (1988), pose as alternatives a constitution that specifies limits on the prerogatives of the party in power and a constitution without such limits. But, these authors implicitly assume that both of these constitutions would be viable. None of these contributions view a constitution and civil conflict to be alternative ongoing methods for resolving disputes, as in the present paper.
can require limitations on the prerogatives of winners of constitutional contests such that the outcomes of constitutional contests do not matter too much.

The historical analysis illustrates the relevance of the theoretical analysis by using the theoretical results to explicate two dramatic examples of constitutional failure. One example is the secession of eleven Southern states from the Union in 1861 and the ensuing American Civil War. In this example the theory helps us to identify changes in exogenous factors that, after seventy years in which artful compromises had allowed disputes over slavery to be resolved constitutionally, prevented another successful renegotiation of the constitution. The other example is the National Socialist revolution in Germany in 1933. In this example the theory suggests why, despite repeated attempts to deal with divisions among economic and social interests constitutionally, in the end there was no viable alternative to civil conflict for resolving the dispute over the demand for a Nazi dictatorship.

I. THEORY

A Generic Dispute

Assume that a polity faces a recurring dispute between two constituent groups over a specific economic or social issue. These constituent groups can be either narrow, like rival ruling elites or rival political cliques, or broad, like ethnic groups or social classes.

Possible examples of disputes include the following:

- A political squabble: the share of political patronage that goes to the politicians associated with one clique rather than another.

- A kleptocratic rivalry: the share of kleptocratic rent that goes to one ruling elite rather than another.

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2A more general model would consider a set of disputes involving many issues and many groups. In practice some disputes might be resolvable constitutionally, while other disputes lead to civil conflict. In this event a constitution might continue to function during a civil conflict.
• A divisive distributional issue: the share of national income that goes to one social
class rather than another.

• A divisive ideological issue: the degree of tolerance for the religious practices of one
ethnic group, these practices being offensive to another ethnic group.

Let Party A and Party B denote the groups involved in the dispute. The term “Party”
is used here in the generic sense of a group with a common purpose. Party A and Party B
do not necessarily correspond to political parties.

Let $X, X \in [0,1]$, denote the realization of the disputed issue. Party A prefers $X$
to be larger, whereas Party B prefers $X$ to be smaller. To implement this difference in
preferences, assume that in each iteration of the dispute the realization $X$ adds the amount
$\alpha X, \alpha \in (0, \infty)$, to the utility of Party A and the amount $\beta(1 - X), \beta \in (0, \infty)$, to the
utility of Party B. The preference parameters, $\alpha$ and $\beta$, are weights that calibrate the
importance of the dispute about $X$ for each party. This formulation implicitly assumes that
the choice of $X$ in each iteration of the dispute affects only the current utility of the Parties
and that the choice of $X$ is costlessly reversible in the next iteration. Also assume that each
party knows how important the dispute is for the other party as well as for itself.

A Simple Constitution

A constitution includes two essential components: the specification of the form and timing
of constitutional contests and the specification of the prerogatives of winners of constitutional
contests. These components can be themselves the subject of the constitution, or they can be
derived from general principles expressed in the constitution. In addition, these components,
or the general principles from which they are derived, can be embodied either in explicit
provisions of the constitution or in implicit understandings.

Both of the essential components of actual constitutions vary widely. In electoral democ-
racies constitutional contests involve periodic competition for the votes of an electorate. In
contrast, in aristocracies constitutional contests involve competition for the favor of wise
men, elders, or hereditary rulers. Many constitutions combine democratic and aristocratic features. For example, under the Constitution of the United States a recent inconclusive electoral competition led to a second competition in which the candidates for President competed for the favor of a court of appointed judges. The theoretical analysis in this paper does not distinguish either between democratic and nondemocratic constitutions, a distinction that is often ambiguous, or between explicit constitutional provisions and implicit understandings.

Consider initially a simple constitution that prescribes a periodical constitutional contest to determine which Party has the prerogative to choose \( X \) until the next constitutional contest, but that places no limits on the prerogatives of the winning Party in choosing \( X \). Let Party A have probability \( P \), and let Party B have probability \( 1 - P \), of winning each constitutional contest. Assume that these probabilities, which depend on, among other things, the design of the constitution, are known to both Parties.

Because this simple constitution places no limits on the prerogatives of the winning Party, if Party A were to win a constitutional contest, then it would set \( X \) equal to one, its most preferred value, whereas, if Party B were to win a constitutional contest, then it would set \( X \) equal to zero, its most preferred value. Thus, under this simple constitution the expected value of \( X \), denoted by \( \bar{X} \), equals \( P \).

To contrast this simple constitution with a civil conflict, let Party A have probability \( Q \), and let Party B have probability \( 1 - Q \), of winning a civil conflict. Assume that these probabilities are exogenous and known to both Parties.\(^3\) Assume also that the winner of a civil conflict, like the winner of a constitutional contest under this simple constitution, would set \( X \) equal the winner’s most preferred value.

Finally, normalize the costs of a constitutional contest to zero, and let the positive numbers, \( C_A \) and \( C_B \), which are calibrated in units of utility, denote the expected incremental costs of a civil conflict to the respective Parties. As mentioned above, these incremental costs

\(^3\)An interesting extension of the model would be to allow \( Q \) to be a state variable whose current value depends on past realizations of constitutional contests and civil conflicts.
can include the use and the destruction of scarce resources and the loss of life. Assume that the expectations, $C_A$ and $C_B$, are exogenous and known to both Parties.

With this simple constitution constitutional resolution of a dispute differs from a civil conflict in two ways. First, the probabilities associated with winning a constitutional contest are socially constructed. Second, a constitutional contest is less costly than a civil conflict.

**Hypothetical Commitments**

Assume, for the moment, that both Parties were committed to abide by the constitution. Under this assumption it would be possible to design a simple constitution that is attractive to both Parties if and only if there exists one or more values of $P$ and, equivalently, $\bar{X}$, such that for both Parties in any iteration of the dispute over $X$ the expected contribution to utility from using a constitutional contest to resolve the dispute would be larger than the expected contribution to utility from a civil conflict. Given the properties of a civil conflict, such values of $\bar{X}$ must satisfy the following two conditions:

\begin{align*}
(1) & \quad \alpha \bar{X} > \alpha Q - C_A. \\
(2) & \quad \beta (1 - \bar{X}) > \beta (1 - Q) - C_B.
\end{align*}

The LHS of condition (1) is the expected contribution to the utility of Party A from a constitutional contest, whereas the RHS of condition (1) is the expected contribution to the utility of Party A from a civil conflict. Condition (2) applies analogously to Party B.

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4 An alternative way to make the expected cost of civil conflict larger than the expected cost of a constitutional contest would be to assume that the Parties are risk averse and that under a constitution, but not with civil conflict, the Parties can implement supplementary understandings that decrease the variance of $X$. Alesina (1988) and Dixit, Grossman, and Gul (2000) explore the possibility of collusion by political parties to decrease risk.

Both condition (1) and condition (2) can be satisfied if and only if there exist one or more values of $X$ that satisfy

$$Q + C_B / \beta > X > Q - C_A / \alpha.$$  

Condition (3) implies that, if both Parties were committed to abide by the constitution, then an incremental cost of civil conflict for either party would be sufficient to make it possible to design a simple constitution that is attractive to both parties. Condition (3) also implies that in a constitution that is attractive to both Parties, the expectation of the constitutional outcome is not too favorable to either one Party or the other.

Abide or Abrogate?

In reality Parties cannot commit themselves to abide by a constitution.\(^6\) Moreover, without constitutional commitments the existence of one or more values of $X$ that satisfy condition (3) is not sufficient for a constitution to provide a viable alternative to civil conflict. As explained above, to be viable a constitution must be self enforcing.

Suppose that a polity establishes a simple constitution with a value of $X$ that satisfies condition (3). Suppose also that an initial constitutional contest takes place, and that one of the Parties is the winner. Now the other Party, the loser of the constitutional contest, has to make a choice. It can decide to abide by the constitution, thereby allowing the winning Party to exercise its constitutional prerogative to set $X$. Or, it can abrogate the constitution by attempting to prevent Party A from exercising this constitutional prerogative, thereby initiating a civil conflict. (If the constitutional contest was an election, then we commonly say that the losing party can either accept or not accept the results of the election.)

Assume that the losing Party will abrogate the constitution only if the present value of the contribution to its expected utility from abiding by the constitution would be smaller

\(^6\)This assertion abstracts from the possibility that a Party could bond itself to abide by the constitution by offering collateral or other hostages. A large literature deals with the question of whether or not political actors can bond themselves. See, for example, Jeremy Bulow and Kenneth Rogoff (1989).
than the present value of the contribution to its expected utility from abrogating the constitution. To determine these present values, assume that, as long as both Parties abide by the constitution, the Parties can resolve future iterations of the dispute about $X$ constitutionally, but that, if either Party were to abrogate the constitution, then the Parties would be left to resolve future iterations of the dispute by civil conflict.\footnote{If the consequences of abrogation were less drastic — for example, if refusal to abide by the constitution in the current iteration of the dispute did not preclude with certainty the possibility of resolving future iterations of the dispute constitutionally — then the configurations of exogenous parameters under which it is possible to design a self-enforcing constitution would be more restricted.}

Given these assumptions, the Party that has lost a constitutional contest, whether that is Party A or Party B, abides by the constitution only if $\bar{X}$ satisfies both of the following conditions:

\begin{align}
\rho \frac{1}{1-\rho} \alpha \bar{X} &\geq \frac{1}{1-\rho} (\alpha Q - C_A), \\
\rho \frac{1}{1-\rho} \beta (1 - \bar{X}) &\geq \frac{1}{1-\rho} [\beta (1 - Q) - C_B].
\end{align}

The parameter $\rho$, $\rho \in [0,1)$, in conditions (4) and (5) is an exogenous discount factor, that, in addition to reflecting pure time preference, can depend on the hazard rate for termination of the dispute about $X$. The larger is $\rho$ the more concern that the Parties have for the future consequences of the choices that they make relative to the current consequences.

The LHS of condition (4) is the present value of the expected contribution to the utility of Party A, if it has lost a constitutional contest, from abiding by the constitution in the current iteration of the dispute, given that both Parties will abide by the constitution in future iterations of the dispute. The RHS of condition (4) is the present value of the expected contribution to the utility of Party A from abrogating the constitution. Condition (5) applies analogously to Party B. The specifications of conditions (4) and (5) assume that the Parties expect the parameters, $\alpha$, $\beta$, $Q$, $C_A$, $C_B$, and $\rho$, as well as the expectation, $\bar{X}$, associated with the current constitution, to remain unchanged in the future.
Taken together conditions (4) and (5) imply the following proposition, which is derived in the mathematical appendix:

If and only if the configuration of exogenous parameters satisfies

\[
\max \{ Q - \frac{C_A}{\alpha}, \ 1 - Q - \frac{C_B}{\beta}, \ 1 - \frac{C_A}{\alpha} - \frac{C_B}{\beta} \} \leq \rho, 
\]

then the set of values of \( \bar{X} \) that satisfy both condition (4) and condition (5) is not empty.

This proposition means that, if and only if the configuration of exogenous parameters satisfies condition (6), then it is possible to design a simple self-enforcing constitution that, regardless of which Party wins a constitutional contest, can resolve a recurring dispute about \( X \) without civil conflict.\(^8\)\(^9\)

Condition (6) implies that the following characteristics of the configuration of exogenous parameters make constitutional resolution of a dispute a viable alternative to civil conflict:

- Neither the probability \( Q \) nor the probability \( 1 - Q \) is too large. This property means that neither Party has a big advantage in civil conflict.

- The ratios, \( \frac{C_A}{\alpha} \) and \( \frac{C_B}{\beta} \), which calibrate for each party the expected incremental costs of civil conflict relative to the importance of the dispute about \( X \), are large.

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\(^8\)With some effort we could derive the conditions under which constitutional resolution of a dispute provides a viable alternative to civil conflict for as long as one Party wins constitutional contests, but under which that Party would abrogate the constitution if the other Party were to win a constitutional contest. Among possible examples of such a situation might be the Chilean constitution, which apparently was viable as long as the Right held political power, but which in 1973 the Right abrogated, when the Left gained political power.

\(^9\)If conditions (4) and (5) are satisfied, then condition (3) is satisfied. This property implies that a simple constitution that is self-enforcing also would be attractive to both Parties if they were committed by abide by it.
• The discount factor, $\rho$, is large.

Alternatively, either a large value of either $Q$ or $1 - Q$, or small values of $C_A/\alpha$ and $C_B/\beta$, or a small value of $\rho$ can preclude the design of a simple self-enforcing constitution.

The observation that large values of $C_A/\alpha$ and $C_B/\beta$ help to make a self-enforcing constitution possible is especially interesting because it conveys both good news and bad news. The good news is that polities have a good chance of avoiding civil conflicts that would have large incremental costs, if the Parties to disputes anticipate these large incremental costs — that is, if $C_A$ and $C_B$ are large. The bad news is that polities do not have a good chance of avoiding civil conflicts over important disputes — that is, over disputes for which $\alpha$ and $\beta$ are large. For example, unless the Parties have almost as much concern for the future consequences as for the current consequences of the choices that they make, if for both Parties the expected incremental cost of civil conflict is small relative to the importance of the dispute, then it is not possible to design a simple self-enforcing constitution.\(^{10}\)

To illustrate the relevance of this theoretical analysis we use these results to explicate two dramatic historical examples of constitutional failure. Before that, however, let us consider the possibility that, even if the configuration of exogenous parameters is such that a simple constitution is not viable, a more complex constitution might be viable.

**Limits on the Prerogatives of the Winning Party**

Consider a more complex constitution that, in addition to prescribing a periodical constitutional contest, also includes limits on the prerogatives of the winning Party in choosing $X$. These limits, which, as suggested above, can be either explicit or implicit, create another difference between constitutional resolution of a dispute and civil conflict.

Let the limits on the prerogatives of the winning Party be that the winning Party will not choose $X$ to be either larger than $X_A$ or smaller than $X_B$. If $X_A$ is smaller than one, then

\(^{10}\)It is worth noting that condition (6) can be satisfied only if either $C_A/\alpha$ or $C_B/\beta$ is positive, but that, if $C_A/\alpha$ and $C_B/\beta$ are sufficiently large, then condition (6) can be satisfied even if $\rho$ equals zero.
this limit is a binding constraint on Party A. If $X_B$ is positive, then this limit is a binding constraint on Party B. Under this complex constitution $\bar{X}$ equals $PX_A + (1 - P)X_B$. The triple, $S$, where $S = \{\bar{X}, X_A, X_B\}$, describes this complex constitution.

It is easy to show that, if both Parties were committed to abide by the constitution, then, as with a simple constitution, it would be possible to design a complex constitution that is attractive to both Parties if and only if there exists one or more values of $\bar{X}$ that satisfy condition (3). Thus, under the assumption of constitutional commitments, an incremental cost of civil conflict for either party would be sufficient to make it possible to design either a simple constitution or a complex constitution that is attractive to both parties.

Now we ask under what configuration of exogenous parameters is it possible to design a complex constitution that is self enforcing. Suppose that a polity establishes a complex constitution described by a triple, $S$, and that a constitutional contest takes place. Given that this complex constitution imposes a binding constraint on the prerogatives of the winning Party, both the losing Party and the winning Party have to decide whether to abide by the constitution or to abrogate the constitution.

Suppose that Party A is the winner of the constitutional contest, and consider the choice that Party B faces. Party B, having lost the constitutional contest, abides by the constitution only if $X_A$ and $\bar{X}$ are small enough to satisfy the following condition:

$$ (7) \quad \beta \left[ (1 - X_A) + \frac{\rho}{1 - \rho} (1 - \bar{X}) \right] \geq \frac{1}{1 - \rho} [\beta (1 - Q) - C_B]. $$

The LHS of condition (7) is the present value of the expected contribution to the utility of Party B from both Parties abiding by the constitution in the current iteration of the dispute and in future iterations of the dispute, conditional on Party A having won the current constitutional contest. The RHS of condition (7) is the present value of the expected contribution to the utility of Party B from abrogating the constitution. The specification of condition (7) assumes that Party B expects the parameters, $\beta$, $Q$, $C_B$, and $\rho$, as well as the triple, $S$, to remain unchanged in the future. A comparison of the LHS of condition
(7) with the LHS of condition (5) shows how a constraint on Party A, in the form of $X_A$ smaller than one, makes it more attractive for Party B to abide by the constitution.\(^{11}\)

Now consider Party A, the winner of the constitutional contest. If Party B does not abrogate the constitution, then Party A can exercise its constitutional prerogative to choose $X$ for the current iteration of the dispute. In choosing $X$, either Party A can abide by the understanding that it will not set $X$ larger than $X_A$ or Party A can behave opportunistically, disregard this limit, set $X$ larger than $X_A$, and thereby abrogate the constitution.

Assume that Party A will abide by the limit on its constitutional prerogative only if the present value of the expected contribution to its utility from setting $X$ equal to $X_A$ is at least as large as the present value of the expected contribution to its utility from setting $X$ equal to one, which is its best opportunistic choice. Accordingly, Party A abides by the constitution only if $X_A$ and $\bar{X}$ are large enough to satisfy the following condition:

\[
\alpha (X_A + \frac{\rho}{1-\rho} \bar{X}) \geq \alpha + \frac{\rho}{1-\rho} (\alpha Q - C_A). \tag{8}
\]

The LHS of condition (8) is the present value of the expected contribution to the utility of Party A from both Parties abiding by the constitution in the current iteration of the dispute and in future iterations of the dispute, conditional on Party A having won the current constitutional contest. The RHS of condition (8) is the present value of the expected contribution to the utility of Party A from being opportunistic in current iteration of the dispute and, thereby, causing civil conflict in future iterations of the dispute. The specification of condition (8) assumes that Party A expects the parameters, $\alpha$, $\rho$, $Q$, and $C_A$, as well as the triple, $S$, to remain unchanged in the future. The LHS of condition (8) shows how a binding constraint on Party A, in the form of $X_A$ smaller than one, causes Party A to consider abrogating the constitution.

\(^{11}\)We might suppose that, because Party A prefers $X$ to be larger, if condition (7) is satisfied, then it is satisfied as an equality. The conclusions derived below do not depend on whether condition (7) is satisfied as an inequality or as an equality.
Taken together conditions (7) and (8) and the analogous conditions that apply if Agent B has won the current constitutional contest imply the following proposition, which is derived in the mathematical appendix:

If and only if the configuration of exogenous parameters satisfies

\[
\max \frac{\sqrt{Q - C_A/\alpha}}{Q + C_B/\beta}, \quad \frac{1 - Q - C_B/\beta}{1 - Q + C_A/\alpha}, \quad \frac{1 - C_A/\alpha - C_B/\beta}{1 + C_A/\alpha + C_B/\beta} \leq \rho,
\]

then the set of triples, \( S \), that satisfy condition (7), condition (8), and the analogous conditions that apply if Agent B has won the current constitutional contest is not empty.

This proposition means that, if and only if the configuration of exogenous parameters satisfies condition (9), then it is possible to design a complex self-enforcing constitution that, regardless of which Party wins a constitutional contest, can resolve a recurring dispute about \( X \) without civil conflict.

Condition (9) has the same implications as condition (6) regarding characteristics of the configuration of exogenous parameters that make constitutional resolution of a dispute a viable alternative to civil conflict — namely, neither \( Q \) nor \( 1 - Q \) is too large, \( C_A/\alpha \) and \( C_B/\beta \) are large, and \( \rho \) is large. More interestingly, comparing condition (9) with condition (6), we obtain the following result, which is derived in the mathematical appendix:

The set of configurations of exogenous parameters that satisfy condition (6) is a proper subset of set of configurations of exogenous parameters that satisfy condition (9).

This result means that, even if it is not possible to design a simple self-confirming constitution, it can be possible to design a self-confirming constitution that includes binding limits of the prerogatives of the Party that wins a constitutional contest. To understand this result, recall that, if the loser of a constitutional contest abrogates the constitution, then the Parties
begin to incur the incremental costs of civil conflict in the current iteration of the dispute, whereas if the winner of a constitutional contest abrogates the constitution, then the Parties do not begin to incur the incremental costs of civil conflict until the next iteration of the dispute. Consequently, in determining the possibility of designing a self-enforcing constitution, the effect of a binding constraint on the Party that wins a constitutional contest in making it more attractive for the loser to abide by the constitution outweighs the effect of a binding constraint in making it attractive for the winner to consider abrogating the constitution.

We also observe that, conditional on Party A winning the current constitutional contest, condition (7) requires that the discounted present value of current and future realizations of $X$ is not too large, whereas condition (8) requires that the discounted present value of current and future realizations of $X$ is not too small. These properties mean that under a complex constitution that provides a viable alternative to civil conflict which Party wins the current constitutional contest cannot matter too much to either one Party or the other.

II. HISTORY

Why Secession? The Received Answer

In the course of American history the most consequential disputes between constituent groups of the polity involved slavery, the issue that divided Northern and Southern interests in ante-bellum United States. The disputes over slavery are especially interesting because for seventy years artful constitutional compromises, which involved the evolution of a complex constitution, enabled Northern interests and Southern interests to resolve these disputes by constitutional means. A critical element of this complex constitution was that throughout

12Postulating a polarity between the North and the South over the issue of slavery is a crude, but useful, simplification. In fact, neither Northern nor Southern interests were monolithic. Nevertheless, according to James McPherson (2001), “Since the 1950s most professional historians have come to agree with Lincoln’s assertion that slavery ‘was, somehow, the cause of the war’.” McPherson convincingly debunks the claim that the main Southern interest was not in defending slavery, but in “a noble cause, the cause of state rights, constitutional liberty, and consent of the governed.”
the ante-bellum period the understood prerogatives of winners of electoral contests under the Constitution largely shielded national politics from the issue of slavery.\textsuperscript{13}

Nevertheless, as Barry Weingast (1998, pages 167-168) points out, “Because the country was growing, each new generation had to renew the arrangements that began when the founding fathers created a system with strong constitutional protection for slavery.” In 1861, however, both the existing constitution and new attempts to renegotiate the constitution dramatically failed, and events culminated in the secession of eleven Southern states from the Union and the ensuing Civil War.

Why did the issue of slavery eventually result in civil conflict? I take the received answer, my account of which is largely based on Robert Fogel (1989), McPherson (1988, 2001), David Potter (1976), and Weingast (1998), to involve three main elements:

\textbf{First}, by the middle of the nineteenth century, as Potter (1976, page 93) explains, “The longstanding sectional equilibrium within the Union was disappearing and the South was declining into a minority status, outnumbered in population, long since outnumbered and outvoted in the House, and protected only by balance in the Senate.” But, neither the Compromise of 1850, which admitted California to the Union as a free state, while allowing settlers in New Mexico and Utah to decide, under the principle of “squatter sovereignty”, whether these territories should become free or slave states, nor the Kansas-Nebraska Act of 1854, which organized the Kansas and Nebraska Territories under the principle of squatter sovereignty, resulted in the admission of additional slave states, as maintaining balance in the Senate would have required. In addition, as Potter (1976, page 93) stresses, “There was not one slave territory waiting to be converted into another slave state, while all of the upper part of the Louisiana Purchase, all of the Oregon territory, and now all of the Mexican Cession stood ready to spawn free states in profusion.” With their failure to gain

\textsuperscript{13}Prior to the establishment in 1854 of the Republican Party, the main political parties, Whigs and Democrats, had national constituencies, and the sectionally divisive issue of slavery was not central in the competition between the parties.
admittance of Kansas as a slave state it was clear that Southern interests had permanently lost the protection of balance in the Senate.

**Second**, prior to the election of 1860 every President has been either a Southerner or a Northerner who had significant Southern support. But, by 1860 more rapid population growth in the North than in the South allowed Abraham Lincoln, the candidate of the newly formed Republican Party, to be elected without carrying a single Southern state. This unprecedented development confirmed that under the existing rules Southern interests now had lost the protection of the Presidential veto.

**Third**, the free-soil platform of the Republican Party, which called for the prohibition of slavery in the territories, implied a new understanding about the prerogatives of winners of electoral contests under the Constitution and, in effect, rescinded the understanding that the Constitution excluded slavery policy from national politics.\(^\text{14}\) Although the Republican platform did not call for emancipation, the new president, Lincoln, as quoted by Potter (1976, page 427) and McPherson (1988, page 179), had denounced slavery as “morally wrong”, had stated that “this government cannot endure, permanently half slave and half free”, and had expressed his hope for the “ultimate extinction” of slavery. According to Fogel (1989, page 381), the Republicans were “determined to restrict slavery’s political and economic domination to guarantee that the federal government promoted northern interests and principles.” On the Southern side, according to McPherson (2001), “Jefferson Davis...justified secession as an act of self-defense against the incoming Lincoln administration, whose policy of excluding slavery from the territories would make ‘property in slaves so insecure as to be comparatively worthless,...thereby annihilating in effect property worth thousands of millions of dollars’.” According to this account, in abrogating the Constitution the Southern secessionists were reacting both to demographic developments and to the proactive stance of

\(^{14}\)Weingast (1998) argues that this understanding depended on balance in the Senate and, hence, that the rescinding of this understanding was not an independent development, but rather a result of the increasing dominance of Northern interests in constitutional contests.
the Republican Party in rescinding an implicitly understood limitation on the prerogatives of the winner of a constitutional contest.

We can easily adapt this analysis to the language of our model. Let Party A denote Northern interests, the winner of both the battle over statehood for Kansas and the presidential election of 1860, and let Party B denote Southern interests. (Recall that the term “Party” as used here refers to a group with a common purpose and not necessarily to a political party.) Also, let $X$ equal to one represent the free-soil policy that Northern interests favored, and that Southerners saw as leading to destruction of the wealth of slave owners, and let $X$ equal to zero represent a policy of unrestricted property rights for slave owners, without geographical limitations, that Southern interests favored. Intermediate values of $X$ could represent a more moderate set of policies, which might include some geographical limitations on the property rights of slave owners and/or emancipation with compensation to the slave owners.

Using the language of our model, the first two elements in the received answer imply an increase in $P$, and the third element implies an increase in $X_A$. Apparently, these increases in $P$ and $X_A$ were so large that condition (7) was no longer satisfied. Accordingly, the existing constitution was no longer self enforcing, as Party B (Southern interests) found that the present value of its expected utility from abiding by the Constitution would be smaller than the present value of its expected utility from abrogating the Constitution.

**Why Secession? A Deeper Analysis**

The problem with this received answer is that it does not go far enough. Specifically, although the received answer implies that the existing constitution no longer satisfied condition (7) and, hence, can account for the failure of the existing constitution, the received answer does not imply that condition (9), which applies to the configuration of exogenous parameters, was no longer satisfied. But, if condition (9) were still satisfied, then the set of self-confirming constitutions would not have been empty. In that case it would have been
possible to avoid civil conflict by revising the constitution either explicitly or implicitly. The required revisions either would have changed the form of constitutional contests to reverse the increasing political dominance of Northern interests or would have constructed a new understanding limiting the prerogatives of Northern interests, as the likely winner of future constitutional contests. Moreover, given their experience in devising compromises, Northern and Southern interests should have been capable of realizing such a renegotiation of the constitution, if it were feasible. Certainly, there were many ideas for another compromise in the air.

One idea was to reconstitute the Union as a federation of the set of Northern states and the set of Southern states. In his proposal for a “concurrent majority”, Calhoun envisaged a dual presidency, with one president representing the North and one representing in the South, and each with the power to veto legislation. Of course, such a reform proposal had no chance, as Northern interests, having worked hard to destroy sectional balance in the Senate, would hardly be willing to accept a sectionally balanced presidency.

Another possibility would have been to limit the prerogatives of Northern interests to a policy choice no more extreme than the British example of emancipation with compensation. But, Fogel (1989, page 412) tells us that “whatever the opportunity for a peaceful abolition of slavery before 1845, it surely was nonexistent after that date. To Southern slaveholders, West Indian emancipation was a complete failure...They could see plainly that the economy of the West Indies was in shambles, that the personal fortunes of the West Indian planters had collapsed, and that assurances made to these planters in 1833 to obtain their acquiescence to compensated emancipation were violated as soon as the planters were reduced to political impotency.” In terms of our model, this account implies that, even if a limitation on $X_A$, the prerogatives of Northern interests, to a policy choice no more extreme than compensated emancipation would have satisfied condition (7), Southern interests did not view such a limitation to be credible.

The proposed Crittenden Compromise, perhaps the most serious of several futile at-
tempts to amend the Constitution in order to prevent civil conflict, embodied another set of possibilities for limiting the prerogatives of Northern interests. The Crittenden Compromise, formally introduced in Congress in December 1860, would have given explicit constitutional protection to slavery in those states, and in the District of Columbia, where slavery already existed and in those remaining territories in which slavery was to be allowed according to the Missouri Compromise of 1820. Both Northern and Southern interests rejected this compromise. The Republicans, led by President-elect Lincoln, would not accept any scheme that infringed on the free-soil plank of their platform. And, according to Fogel (1989, page 413), the Southerners by then “were convinced that northern hostility to slavery precluded a union that would promoted [Southern] economic, political, and international objectives.”

Finally, Northern interests might have accepted the establishment of an independent Southern Confederacy. Assuming that the Confederacy would have no territorial ambitions beyond the borders of the eleven secessionist states, such a peaceful dissolution of the Union would have allowed Northern interests to implement their free-soil policy in the territories. But, the fervent opposition of Southern interests to the exclusion of slavery from the territories belies this assumption. As Roger Ransom (1989, page 167) emphasizes, “The South of the mid-nineteenth century was an expansionist system that coveted land to the west and to the south...If they gained status as an independent nation, slave owners would be free to pursue a ‘foreign policy’ just as inimical to the North’s interests as that pursued by the ‘slave power’ when it had control of the federal government within the union.” And, an independent Confederacy, unconstrained by the interstate commerce clause of the Constitution, would have had enhanced strategic advantages, including, for example, the ability to control access to the sea via the Mississippi River. Fogel (1989, page 416) argues that acceptance of an independent Southern Confederacy would only have postponed a war over slavery and its expansion and “that the delay would have created circumstances far more favorable to a southern victory.”

Massimo Bordignon and Sandro Brusco (2001) analyze the optimality of including secession rules in the
Why was civil conflict unavoidable?

This analysis suggests that in 1861 Northern and Southern interests failed to avoid civil conflict not only because the existing constitution was no longer self enforcing, but also, and more importantly, because constitutional resolution of the dispute over slavery no longer provided a viable alternative to civil conflict. In other words, the fatal fact was not that the existing constitution no longer satisfied condition (7), but that the configuration of exogenous parameters no longer satisfied condition (9).

What made civil conflict, which was avoided before the election of 1860, unavoidable in 1861? Historical scholarship, when combined with our theoretical model, suggests that the critical change was that, as a result of developments in the years leading up to 1861, the dispute between Northern and Southern interests became too important to be resolved through a constitutional political process. In terms of our model historical scholarship suggests that an increase in either $\alpha$ or $\beta$ or both upset condition (9).

Fogel’s account of northern ante-bellum politics suggests a plausible story that is consistent with an increase in $\alpha$ in the years leading up to 1861. From the late 1840s, mainly because of increased immigration, incomes and living conditions of native, northern, non-farm workers became increasingly depressed. Fogel (1989, page 356) tells us that this depression of living conditions was “one of the most severe and protracted economic and social catastrophes of American history.”

As a consequence of this working-class depression land policy became increasingly important. Free homesteads, opening western lands for settlement by the working poor, became a paramount demand of northern labor. But, the objective of Southern interests that new territories be opened to slavery stood in the way of free homesteads. Thus, as Fogel (1989, page 350) explains, land policy “drew into direct conflict with Slave Power the northern constitution of a federal union. They consider a potential dispute over the value of the federal union. In their analysis, in contrast to present analysis of the dispute over the expansion of slavery, secession resolves the dispute.
working-class leaders who had previously remained aloof from the anti-slavery movement.”

The result was the coalescing of free soil and nativist factions into the new Republican Party.

In addition, the evidence about the economics of slavery, as summarized by Fogel (1989) and Ransom (1989), suggests a plausible reason for an increase in $\beta$ in the years leading up to 1861. According to Fogel (1989, page 412), “From the mid-1840s on...the slave economy of the South was vigorous and growing rapidly. Whatever the pessimism of [slave owners] during the economic crises of 1826-1831 and 1840-1845, during the last half of the 1840s and most of the 1850s they foresaw a continuation of their prosperity and, save for the political threat from the North, numerous opportunities for its expansion. The main thrust of cliometric research has demonstrated that this economic optimism was well founded...”

As Ransom (1989, page 47) puts it, “On the eve of the Civil War, American slaveholders were coming off a decade and a half of exuberant growth and expansion.”

As it turned out, the actual incremental costs to both Northern interests and Southern interests of the ensuing civil conflict, including six hundred thousand men killed and thousands more maimed, certainly were larger than the expected incremental costs, $C_A$ and $C_B$. We can speculate whether, if both Northern interests and Southern interests had not underestimated the incremental costs of the ensuing civil conflict, the perceived configuration of exogenous parameters still would have failed to satisfy condition (9), even with the increased importance of the dispute, as reflected in increased values of $\alpha$ and $\beta$. But, it is only hindsight that suggests that a constitutional resolution, facilitated by a renegotiation of the constitution, would have been better for both Northern interests and Southern interests than the actual consequences of the civil conflict. In the event, the Civil War resolved the dispute. In the poignant words of Potter (1976, page 583), “Slavery was dead; secession was dead; and six hundred thousand men were dead.”

**The National Socialist Revolution**

In 1919 the Weimar Constitution was adopted to replace the abolished monarchy in
the wake of Germany’s defeat in the Great War. The Constitution established a hybrid combination of parliamentary and presidential democracy. This hybrid provided two different political processes for resolving disputes among constituent groups of the polity. The usual process was to be a parliamentary election and the creation of a parliamentary government, whose acts were subject to presidential veto. But, in an “emergency” the President could appoint a presidential government and authorize it to govern by decree, in effect suspending the parliament (Reichstag) and assuming dictatorial powers for himself.

As seems clear from historical accounts — see, for example, William Shirer (1960) — the Weimar Constitution, despite carefully crafted limitations on the prerogatives of winners of constitutional contests, faced an impossible task in resolving disputes that reflected deep divisions among economic and social interests. In German society and politics workers, themselves split between Communists and Social Democrats, were aligned against employers, landless peasants were aligned against Junker landlords, monarchists were aligned against republicans, and nationalists and revanchists were aligned with the military against real or imagined subversives, whom, as Shirer (1960, page 157) tells us, the Deutsche Zeitung declaimed as “internationalist traitors and pacifist swine”.

These divisions resulted in political turmoil and other symptoms of social disintegration: widespread strikes, extremist political movements, and the formation of private paramilitary forces, which fought pitched battles in the streets. One consequence of the turmoil was indecisive economic policy, with a resulting hyperinflation, and, later, an inability to mount an effective response to the world-wide depression. These economic problems caused a further deepening of economic and social divisions.

The National Socialist German Workers’ (Nazi) Party, with Adolf Hitler as its leader, was founded in the midst of this chaos. The Nazi Party was a populist, nationalist, revanchist, and anti-Semitic movement that purported to bridge the divide between the left and the right of the political spectrum. The Nazis promised that, once they were in power, they would reestablish social order and take decisive action to solve economic problems. The
Nazis emerged as an important political force for the first time in the parliamentary election of 1930.

Did the Nazis seek political power mainly to further a set of economic and social goals? Or, were they mainly opportunists who sought political power as a means to wealth and personal aggrandizement? In either case their behavior reveals that the Nazi leaders were exceptionally ambitious men who put an unusually high value on achieving their objectives.\(^\text{16}\)

Thus, if, using the language of our model, we let Party B denote the Nazis, and let \(X\) equal to zero, the outcome most favored by the Nazis, represent a Nazi dictatorship that would be unconstrained in carrying out the Nazi program, where \(X\) equal to one would represent the exclusion of the Nazis from political power, then it seems clear that the parameter \(\beta\) was large. In this context, \(\beta\) calibrates the value that the Nazis attached to their objective of a monopoly of political power.

Four times between March 1932 and March 1933 the Nazis attempted, but failed, to win a constitutional contest for political power. In the first attempt Hitler ran for president. He campaigned vigorously, promising, as Shirer (1960, page 159) tells us, “jobs for the workers, higher prices for the farmers, more business for the businessmen, and a big Army for the militarists.” But, Hitler finished a distant second to President Hindenberg in both the initial vote on March 13th, 1932, and in the runoff election of April 10th.

The second attempt was in the parliamentary election of July 31st. The Nazis had maneuvered to bring about the dissolution of the Reichstag with the hope that a new election would sweep them into power. According to Shirer (1960, page 166), “the Nazis threw themselves into the campaign with more fanaticism and force than ever before.” Nevertheless, the results left the Nazis, although with 37 per cent of the vote the largest party in the Reichstag, far short of a even a simple majority.

\(^{16}\)Regarding the rank-and-file Nazis, according to Shirer (1960, page 206), “Most of them had belonged to the ragged army of the dispossessed and the unsatisfied. They...believed that the revolution that they had fought by brawling in the streets would bring them loot and good jobs.”
The third attempt was in the parliamentary elections of November 6th. Again the Nazis had maneuvered to bring about the dissolution of the Reichstag. But, this time an increasingly widespread view that the Nazis were socially disruptive turned off both financial supporters and voters and, in turn, demoralized party comrades. The Nazis lost votes and seats in the Reichstag.

The final attempt was in the parliamentary election of March 5th, 1933. Social disintegration had led to what Shirer (1960, page 185) describes as “the inexplicable weakness, that now bordered on paralysis, of existing institutions — the Army, the churches, the trade unions, the political parties — [and] of the vast non-Nazi middle class and the highly organized proletariat.” In this setting, on January 30th, 1933, President Hindenberg had allowed the Nazis, in coalition with the Nationalists, to form a parliamentary government with Hitler as Chancellor. According to Shirer (1960, page 5), Goebbels, one of Hitler’s lieutenants, wrote that night in his diary, “The new Reich has been born. Fourteen years of work have been crowned with victory. The German revolution has begun.” But, Hitler’s new government had the support of only a minority in the Reichstag. Also, the Nazis held only three of eleven posts in the cabinet.

Yet again the Nazis, confident of overwhelming victory in a new election, maneuvered to bring about the dissolution of the Reichstag. Again, as Shirer (1960, page 189) tells us, “Goebbels was jubilant. ‘Now it will be easy,’ he wrote in his diary on February 3, ‘to carry on the fight, for we can call on all of the resources of the State. Radio and press are at our disposal. We shall stage a masterpiece of propaganda. And this time, naturally, there is no lack of money.’”

In addition to mounting a forceful election campaign, Hitler, on the pretext of a threat of a Communist revolution, prevailed on President Hindenberg to allow him to suspend civil liberties. Hitler also got Goering, another of his lieutenants, into the post of Prussian Interior Minister, whereby the Nazis gained control of the powerful Prussian state police. The police, supplemented by Nazi storm troopers, arrested political opponents and banned
their publications. Despite these tactics, the election of March 5th gave the Nazis and their allies only a slim majority in the Reichstag, far less than the Nazis needed to amend the Constitution and to establish a dictatorship by constitutional processes.

In this election, although the Nazis used tactics that violated the spirit of democracy, they still had adhered to the letter of the Weimar Constitution. But, the results of the election confirmed that the Nazis had little or no chance of achieving their goal of a Nazi dictatorship through electoral competition. Faced with the unpalatable prospect of their opponents exercising constitutional prerogatives to block the Nazi program, the Nazis, although still operating under a veneer of legality, abrogated the Weimar Constitution.

The Nazis won the ensuing civil conflict so quickly and easily that observers have blamed the opposition, perhaps unfairly, for giving up without a fight. On March 23rd, 1933, with a large number of opposition deputies detained by the police, in violation of the constitutional provision of legislative immunity from arrest, and with Nazi storm troopers lining the aisles, the Nazis rammed through the Reichstag an “Enabling Act” that gave dictatorial powers to Chancellor Hitler. Hitler then moved quickly to destroy any potential opposition. Within a few months, state legislatures were dissolved, and Nazi governors were appointed in each state; all political parties, other than the Nazi party, were banned; labor unions were dissolved; freedom of speech and press were abolished. In late June 1934 Hitler consolidated his personal power, and appeased the Army and his supporters on the Right, by carrying out a bloody purge of leftist elements among the Nazis. Finally, a month later, on the death of President Hindenberg, Hitler abolished the office of President, and extracted a personal oath of loyalty from the members of the armed forces.

In terms of our model why were the Nazis and their opponents unable to resolve their dispute over political power according to a constitutional political process? It is easy to see why the Weimar Constitution failed. The historical account suggests that $1 - Q$, the probability of the Nazis winning a civil conflict, was large and that $C_B$, the incremental cost for the Nazis of a civil conflict was small. Given these parameters, $1 - P$, the probability
of the Nazis winning a political contest under the Weimar Constitution, apparently was too small to satisfy condition (7).

But, again we want to ask a deeper question. Why was condition (9), according to which constitutional resolution of a dispute can provide a viable alternative to civil conflict, not satisfied? As we have seen, if the configuration of exogenous parameters had satisfied condition (9), then it would have been possible to avoid civil conflict either by revising the form of constitutional contests or by constructing, either explicitly or implicitly, a new understanding limiting the prerogatives of winners of constitutional contests. The historical account suggests that in Germany in the early 1930s the configuration of exogenous parameters failed to satisfy equation (7) because, in addition to $1 - Q$ being large, $\beta$ also was large relative to $C_B$. Like the dispute over the expansion of slavery in ante-bellum United States, the disputed demand for a Nazi dictatorship was too important relative to the expected incremental cost of civil conflict to be resolved by a constitutional political process.

Summary

Constitutional resolution of disputes between constituent groups of a polity avoids the incremental costs of civil conflict. But, the political process prescribed by a constitution provides a viable alternative to civil conflict if and only if the constitution is self-enforcing. The theoretical analysis in this paper revealed that the following characteristics of the configuration of exogenous parameters make it possible to design a self-enforcing constitution that a polity can use to resolve a given dispute:

- No party to the dispute has a big advantage in civil conflict.

- The parties to the dispute expect the incremental costs of civil conflict to be large relative to the importance of the dispute.

- The parties to the dispute have almost as much concern for the future consequences of the choices that they make as for the current consequences.
Alternatively, either a big advantage in civil conflict for one of the parties to the dispute, or expectations that the incremental costs of civil conflict are small relative to the importance of the dispute, or little concern for future consequences can preclude constitutional resolution of the dispute. The theoretical analysis also revealed that a viable constitution can require limitations on the prerogatives of winners of constitutional contests such that the outcomes of constitutional contests do not matter too much.

The paper illustrated the relevance of the theoretical analysis by applying these theoretical results to two dramatic historical examples of constitutional failure: the secession of eleven Southern states from the Union in 1861 and the National Socialist revolution in Germany in 1933. Historical accounts suggest that civil conflict was unavoidable in the American case because the dispute over the expansion of slavery had become too important, and perhaps also because the parties to the dispute underestimated the incremental costs of civil conflict. Historical accounts suggest that civil conflict was unavoidable in the German case because the disputed demand for a Nazi dictatorship was intrinsically too important and because, at least for the Nazis, the incremental cost of civil conflict was small.

Both of these examples illustrate an unfortunate reality. As long as the expected incremental costs of civil conflict are not too large, constitutional political processes cannot resolve important disputes. If the constituent groups of a polity are deeply divided and, hence, are unwilling to accept meaningful restrictions on the prerogatives of winners of constitutional contests, then civil conflict is unavoidable.
Mathematical Appendix

Derivation of Condition (6):

Condition (4) is satisfied if and only if $\bar{X}$ is sufficiently large that

$$\bar{X} \geq \frac{Q - C_A/\alpha}{\rho}.$$  

Condition (5) is satisfied if and only if $\bar{X}$ is sufficiently small that

$$\bar{X} \leq 1 - \frac{1 - Q - C_B/\beta}{\rho}.$$ 

Conditions (i) and (ii) have the following implications:

(6.1) If and only if the RHS of condition (i) is not larger than one, then the set of values of $\bar{X}$, $\bar{X} \in [0, 1]$, that satisfy condition (i) is not empty.

(6.2) If and only if the RHS of condition (ii) is not negative, then the set of values of $\bar{X}$, $\bar{X} \in [0, 1]$, that satisfy condition (ii) is not empty.

(6.3) If and only if the RHS of condition (ii) is not smaller than the RHS of condition (i), then the union of the set of values of $\bar{X}$ that satisfy condition (i) and the set of values of $\bar{X}$ that satisfy condition (ii) is not empty.

If and only if condition (6) obtains, where

$$\max \{Q - \frac{C_A}{\alpha}, \; 1 - Q - \frac{C_B}{\beta}, \; 1 - \frac{C_A}{\alpha} - \frac{C_B}{\beta} \} \leq \rho,$$

then conditions (6.1), (6.2), and (6.3) obtain.

Derivation of Condition (9):

Both condition (7) and condition (8) are satisfied if and only if the triple, $S$, satisfies

$$(iii) \quad \frac{1}{1 - \rho} \cdot (Q + C_B/\beta) \geq X_A + \frac{\rho}{1 - \rho} \cdot \bar{X} \geq 1 + \frac{\rho}{1 - \rho} \cdot (Q - C_A/\alpha).$$
The analogous condition that applies if Agent B has won the current constitutional contest is satisfied if and only if $S$ satisfies

$$(iv) \quad \frac{\rho}{1-\rho} (Q + C_B / \beta) \geq X_B + \frac{\rho}{1-\rho} \bar{X} \geq \frac{1}{1-\rho} (Q - C_A / \alpha).$$

Conditions (iii) and (iv) have the following implications:

(9.1) If and only if the LHS of condition (iii) is not smaller than the RHS of condition (iii), then the set of values of $X_A$ that satisfy condition (iii) is not empty.

(9.2) If and only if the LHS of condition (iv) is not smaller than the RHS of condition (iv), then the set of values of $X_B$ that satisfy condition (iv) is not empty.

(9.3) If and only if the difference between the LHS of condition (iii) and the RHS of condition (iv) is not smaller than the difference between the RHS of condition (iii) and the LHS of condition (iv), then the set of values of $\bar{X}$ that satisfy both condition (iii) and condition (iv) is not empty.

If and only if condition (9) obtains, where

$$(9) \quad \max \left\{ \frac{\rho}{1-\rho} (Q - C_A / \alpha), \frac{1-Q-C_B / \beta}{Q + C_B / \beta}, \frac{1-C_A / \alpha - C_B / \beta}{1+C_A / \alpha + C_B / \beta} \right\} \leq \rho,$$

then conditions (9.1), (9.2), and (9.3) obtain.

Derivation of the result that the set of configurations of exogenous parameters that satisfy condition (6) is a proper subset of the set of configurations of exogenous parameters that satisfy condition (9):

Define $Y \equiv Q - C_A / \alpha$, and define $Z \equiv 1 - Q - C_B / \beta$.

Using these definitions, we can rewrite condition (6) as

$$(6) \quad \max\{Y, Z, Y + Z\} \leq \rho,$$
and we can rewrite condition (9) as

$$\max \frac{\sqrt{2}}{Y} \frac{Z}{1-Z}, \frac{Z}{1-Y}, \frac{Y+Z}{2-Y-Z} \leq \rho.$$  

Also, note that \( \max\{Y, Z, Y+Z\} \leq 1.\)

To compare condition (6) and condition (9) we have to consider four cases:

1. If both \( Y \leq 0 \) and \( Z \leq 0 \), then the configuration of exogenous parameters satisfies both condition (6) and condition (9).

2. If both \( Y > 0 \) and \( Z > 0 \), then \( \max\{Y, Z, Y+Z\} = Y + Z \). In addition, with \( Y > 0 \) and \( Z > 0 \), and with \( \max\{Y, Z, Y+Z\} \leq 1 \), we have

$$Y + Z > \max \frac{\sqrt{2}}{Y} \frac{Z}{1-Z}, \frac{Z}{1-Y}, \frac{Y+Z}{2-Y-Z}.$$  

3. If \( Y \leq 0 \) and \( Z > 0 \), then \( \max\{Y, Z, Y+Z\} = Z \). In addition, with \( Y \leq 0 \) and \( Z > 0 \), and with \( \max\{Y, Z, Y+Z\} \leq 1 \), we have

$$Z > \max \frac{\sqrt{2}}{Y} \frac{Z}{1-Z}, \frac{Z}{1-Y}, \frac{Y+Z}{2-Y-Z}.$$  

4. If \( Y > 0 \) and \( Z \leq 0 \), then \( \max\{Y, Z, Y+Z\} = Y \). In addition, with \( Y > 0 \) and \( Z \leq 0 \), and with \( \max\{Y, Z, Y+Z\} \leq 1 \), we have

$$Y > \max \frac{\sqrt{2}}{Y} \frac{Z}{1-Z}, \frac{Z}{1-Y}, \frac{Y+Z}{2-Y-Z}.$$  

In the latter three cases, because \( \max\{Y/(1-Z), Z/(1-Y), (Y+Z)/(2-Y-Z)\} \) is smaller than \( \max\{Y, Z, Y+Z\} \), the configuration of exogenous parameters can satisfy condition (9) even if it does not satisfy condition (6).
References


