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# Endogenously Excludable Goods

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## Abstract

In this paper we analyze the optimal interplay between the public and private enforcement of property rights. In doing so we endogenize the distinction between public and club goods on the one and private and common-pool goods on the other hand. The private enforcement of property rights is seen as a substitute for public enforcement that results in a contest between the private defender of its property and the potential appropriator. Public enforcement changes the opportunity costs in this contest. We characterize how optimality conditions for the provision of private and public goods change, how an optimal enforcement policy looks like, and compare the solutions with other institutional alternatives.

*Keywords:* Private Goods, Public Goods, Club Goods, Common-Pool Goods, Private Enforcement, Public Enforcement, Contest

*JEL classification:* D74, H41, H42, K42

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# 1 Introduction

One of the most common distinctions in the literature on public economics is the one between private and common-pool and between public and club goods. The distinction rests on the criteria of rivalry in consumption on the one hand and costs of exclusion on the other hand. A public good, for example, is one that is non-rival in consumption and for which the costs of exclusion are prohibitive. A common-pool good is rival in consumption with equally high costs of exclusion. But what does it mean to have prohibitive costs of exclusion? The most standard use of the concept of exclusion costs refers to the public enforcement of property rights. While for the case of common-pool goods property rights can be assigned in principle, their enforcement by the state cannot be guaranteed because the costs of enforcement are too high. This classification of allocation problems defines a useful starting point for economic analysis, at the same time it contains two conceptual weaknesses.

(1) Even in a situation without any public enforcement of property rights agents will start up defending their property against infringements by other agents. The lack of public enforcement creates a situation similar to anarchy. However, as has been observed by several other scholars,<sup>1</sup> anarchy is not equal to amorphous implying that some regularities will finally emerge even in a situation without public enforcement. Grossman (2001), for example, defines effective property rights as those rights that can be defended by the agents. Hence, even in the absence of public enforcement it is not clear that we will, for example, necessarily observe the tragedy of the commons (Hardin 1968) for the case of private goods. The aspect of private enforcement activities, however, has only rarely been studied.<sup>2</sup>

(2) The costs of public enforcement are neither zero nor prohibitive but rather a finite function of the level of public enforcement to be implemented. Whether they turn out to be prohibitive depends as well on the nature of the allocation problem as on the technology of private enforcement available as an alternative. From this perspective it becomes a question of the optimal allocation of scarce resources that ultimately determines the optimal degree of public enforcement. It is common to build in locks in doors and alarm systems in cars while at the same time police is

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<sup>1</sup>See for example Hirshleifer (1995) and Grossman (2001).

<sup>2</sup>Exceptions are Clotfelter (1977,78), de Meza and Gould (1992), Helsley and Strange (1994), Polinsky (1980) and Shavell (1991).

patrolling the streets. This observation has attracted few attention by the profession.

There is a large body of literature studying aspects of public enforcement of law.<sup>3</sup> The basic tradeoff that is modeled in this literature is between the costs of enforcement faced by a government authority and the benefits of enforcement, which depend on the gains from a reduced probability of crime and the type of punishment and the associated disutility of the potential criminal. This approach is particularly useful for the determination of the optimal degree of enforcement and the design of optimal punishment strategies. However, it abstracts from the possibility of individuals to engage in private counter-actions against crime.

It is the purpose of this paper to study the tradeoff between private and public enforcement in greater detail. If one allows for private enforcement the question about the right modeling strategy arises. Existing but non-enforced property rights create a situation formally similar to anarchy. Anarchy has become a valuable area of research during the last couple of years by the application of conflict and contest models which by now constitute the paradigmatic tool for studying various aspects of anarchy.<sup>4</sup> Hence, we merge the literature on public enforcement with the literature on conflict and appropriation to build a general-equilibrium model of private and public enforcement for the case of private and public goods in order to answer the following questions:

- What are the conditions for the optimal allocation of private and public goods? In particular, how do the marginal conditions known from standard theory have to be modified?
- What is the optimal mix between private and public enforcement? What are the conditions under which a society should optimally dispense with public enforcement?
- Under what conditions is the public supply of goods better than the private supply?

The answer to the first question is important for the empirical evaluation of the allocative properties of institutions. Are observed deviations from the marginal con-

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<sup>3</sup>For an excellent survey see Polinsky and Shavell (2000).

<sup>4</sup>See Bush and Mayer (1974), Skaperdas (1992), Hirshleifer (1995), Grossman and Kim (1995), Grossman(2001) among others.

ditions the result of ill-designed institutional structures (and therefore call for reforms) or are they manifestations of transaction costs implied by the costs of the legal system? The answer to the second question allows to build an endogenous theory of common-pool goods for the case of private and non-excludable public goods for the case of public goods. The answer to the third question adds a new argument to the literature about the advantages and disadvantages of centralization<sup>5</sup> based on the transaction costs of decentralized versus centralized mechanisms.

We define both types of goods only with respect to the specific interdependence between the agents' utility functions, not with respect to exclusion.<sup>6</sup> A private good is perfectly rival in consumption; if one agent profits from its use all other agents cannot profit from it. A public good is perfectly non-rival in consumption; the fact that one agent profits from its use does *in itself* not exclude all other agents from profiting from the good. However, exclusion can be used as a mechanism to achieve efficiency. Neglecting crowding, concerts in a concert hall are a good example for a public good. The private enforcement of property rights could, for example, be achieved by the employment of one or several door-keepers. Public enforcement would imply the control of the payment of the entrance fee by, for example, the police. In reality we observe both types of enforcement interacting. Concert halls are protected by private door-keepers, however, the actual punishment of a spectator without valid ticket is usually carried over to the justice.

This example raises the question about the relationship between private and public enforcement. Private and public enforcement is composed out of a number of different activities. Depending on the activity their relationship can differ with respect to the degree of substitution. Private guards and police are close substitutes from the point of a single agent: the purpose of both activities is to prevent and detect appropriation. Private guards and courts have a larger degree of complementarity: as before the purpose of the private guard is to prevent and detect appropriation, whereas the purpose of the court is to decide about the punishment.

In addition to the characterization of the optimal mix of private and public

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<sup>5</sup>See for example the paper by Hart, Shleifer and Vishny (1997). They use a multi-task model with incomplete contracts to explain the difference between public and private supply of goods. In their model private and public ownership differs with respect to the incentives to carry out the different tasks.

<sup>6</sup>For the case of public goods see Samuelson (1954).

enforcement we discuss institutional alternative institutional settings. The costs of private and public enforcement of property rights are a major factor of the transaction costs of decentralized or market mechanisms. Centralization is an institutional alternative that implies different types of costs. A comparison of both types of mechanisms reveals an important difference between rival and nonrival goods: *whereas for the case of private goods the centralization of supply does not eliminate the incentive for appropriation in general, this incentive is eliminated if the supply of non-rival goods is centralized.* Our analysis suggests that these resource costs are an important factor for the determination of optimal institutional structures: even if public enforcement is prohibitively costly but private enforcement is cheap, decentralization may dominate centralization. On the other hand, even with high costs of centralization, decentralization may be worse if the good induces large incentives for appropriation and defense.

A second institutional alternative is the banning of private defense activities. In a number of cases it might be easier for the state to enforce the banning of legal activities like the defense of property than to enforce the banning of illegal activities like stealing. It turns out that banning of private enforcement can never be optimal for the case of rival goods but that it can be optimal for the case of nonrival goods. The reason for this result is closely related to the reason why centralization may dominate decentralization: banning private enforcement reduces the resource costs of the conflict. These resources are then free for productive activities. This increase in production has to be compared to the dilution of incentives to produce the contested good. For the case of rival goods these incentives are reduced to zero, whereas this is not the case for nonrival goods.

It is worthwhile to relate our approach more closely to the very interesting papers by de Meza and Gould (1992) and Helsley and Strange (1994). De Meza and Gould analyze a situation where property rights exist but are not enforced by the government. Private enforcement incurs a fixed cost. The authors show that there can be too much or too little enforcement in equilibrium, depending on the costs of enforcement. The reason for this property is that private enforcement activities have an external effect on the other individuals. This externality can be either positive or negative. Our paper differs from de Meza and Gould in three major respects. First, they analyze the private enforcement of property rights on a resource, while the property rights on the resulting goods are assumed to be perfectly enforced. We

analyze the opposite situation with non-appropriable resources (time) but costs of enforcement of property rights for the resulting goods. Second, de Meza and Gould abstract from the possibility that unstable property rights might give rise to appropriative and defensive activities, which is done in this paper. Third, they restrict attention to the private enforcement of property rights for a rival resource, whereas we analyze the relationship between private and public enforcement for both, private and public goods. Helsley and Strange analyze optimal private enforcement of property rights as a mechanism to discriminate prices between different types of agents. In their model, consumers can either use the good provided by the agency – and pay the according user fee – or try to use the good without paying, which results in a payment ‘lottery,’ where they are either not detected and pay nothing or they are detected and pay a fine. This modeling strategy differs from the one used in this paper in two major respects. It is a general property of the classical law-enforcement literature that fines are maximal in equilibrium. Hence, we do not explicitly model the determination of fines in our model. However, contrary to the model by Helsley and Strange we assume that appropriative activities require the investment of some resources by the appropriator. In addition we focus of the interplay between private and public enforcement.

The paper proceeds as follows. In Section 2 we analyze the case of private goods. In Section 3 we turn attention to the case of public goods. In Section 4 we extend the analysis the questions of public supply of goods and the prohibition of private enforcement. We conclude in Section 5.

## 2 Private Goods

Consider an economy populated by two (small) economic agents  $i = 1, 2$ .<sup>7</sup> Both agents have set up a state and government whose concern is to enforce property rights.<sup>8</sup> We assume that every agent has an initial claim to the goods that are

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<sup>7</sup>The assumption that both agents are small means that they treat prices as parameters.

<sup>8</sup>The restriction to two economic agents is made for analytical convenience. In an economy with more than two agents some matching process must be modeled that specifies the agents contesting each other. See Alesina and Spolaore (1997) for such a matching mechanism. While indispensable in the Alesina and Spolaore model an extension of our model to more than two agents would greatly complicate the analysis without generating further qualitative insights.

produced with his own resources (formal property right).

We assume that each agent is endowed with one unit of time that he can use for four different activities. Agent  $i$  uses a fraction  $l_i$  to produce a private good  $y_i$ . The production function  $y_i = g(l_i)$  is increasing and concave in  $l_i$ ,  $g' > 0$ ,  $g'' \leq 0$ . Every agent has the property right to all the goods produced. He can, however, try to appropriate some of the goods produced by the other agent. By the same token, he has to defend against theft. Hence, he can use a fraction  $d_i$  for defensive measures and a fraction  $a_i$  to appropriate some of the property of agent  $j$ . Finally a fraction  $f_i$  of the time resources can be used to produce an unalienable good (leisure). Denote by  $\delta_i = \{l_i, d_i, a_i, f_i\}$ ,  $i = 1, 2$ , and  $\delta = \{\delta_1, \delta_2\}$  the vector of decision variables. Finally, the agents derive utility out of the consumption of both private goods and leisure,  $u_i = u(c_i^1, c_j^2, f_i)$ ,  $i = 1, 2, j \neq i$ , where superscripts denote the different goods and  $u$  is strictly quasi concave.

The state has the right and power to use  $T$  units of time from each agent. The total amount of resources ‘taxed’ is used to enforce property rights.<sup>9</sup>

We denote by  $\theta \in [0, 1]$  a parameter measuring the degree of public enforcement, where we use the convention that  $\theta = 0$  implies perfect public enforcement (successful appropriative activities are impossible) and  $\theta = 1$  implies no public enforcement at all. The amount of time resources needed from every agent for  $\theta$  units of public enforcement is measured by  $T = z(\theta)$  with  $\partial z/\partial \theta < 0$ .

With imperfect or absent public enforcement we assume that agents engage in appropriative as well as defensive activities. We denote by  $\pi_i \in [0, 1]$  the fraction of good  $i$  that can be successfully defended against the appropriation of agent  $j$  by agent  $i$ . It is a function of appropriative as well as defensive activities,  $a_j, d_i$ , and of the degree of public enforcement,  $\theta$ ,  $\pi_i = \pi_i(d_i, a_j, \theta)$ . It is natural to assume the following properties of  $\pi_i$ :

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<sup>9</sup> $T$  can therefore be seen as a measure for the total per-capita time resources used for the enforcement of property rights. It is therefore a measure of the size of the government and bureaucracy. If, for example,  $T = 0.05$ , 5% of the total time of the economy is devoted to the implementation and enforcement of rules. This interpretation may sound a little bit schizophrenic in a two-person economy because it would imply that part of their time the agents have to enforce the rules that during the rest of their day they try to break. If, however, the restriction to only two persons is seen as a simplifying assumption for the more general case of a  $n$ -person economy  $T$  can be seen as a measure of the number of agents working for the state.

1.  $\partial\pi_i/\partial d_i > 0$ ,  $\partial\pi_i/\partial a_j < 0$ ,  $\partial\pi_i/\partial\theta < 0$ ,
2.  $\partial^2\pi_i/\partial d_i^2 < 0$ ,  $\partial^2\pi_i/\partial a_j^2 > 0$ ,
3.  $\pi(d_i, a_j, 0) = 1$ ,  $\pi(x, x, \theta) = 1/(1 + \theta)$ .

The first assumption states that both agents can influence the outcome in their favor by increasing  $d_i$  or  $a_j$  respectively, and that an increase in  $\theta$  favors the aggressor. The second set of assumptions guarantee the existence of an interior solution that is a maximum. The third assumption states that with perfect public enforcement appropriation cannot be successful and gives a normalization of the defended fraction if both agents invest the same amount of time for appropriation and defense. For example, without public enforcement ( $\theta = 1$ ) the good is equally divided if both agents invest the same amount of time. This assumption precludes cases where one agent is weaker or stronger than the other agent in anarchy.

We assume the following sequential structure of the game:

- Stage 0: At stage 0 the government chooses a level of public enforcement  $\theta$  and a corresponding tax rate  $T$ ,  $t = \{\theta, T\}$ . Its objective is to maximize the sum of utilities of both agents.
- Stage 1: At stage 1 the agents allocate their remaining time budget to productive, defensive, and appropriative activities as well as to leisure. This defines a primary distribution of the private goods,  $E = \{E_1^1(\delta, t), E_1^2(\delta, t), E_2^1(\delta, t), E_2^2(\delta, t)\}$  with  $E_1^1 = \pi_1 y_1$ ,  $E_2^1 = (1 - \pi_1)y_1$ ,  $E_2^2 = \pi_2 y_2$ ,  $E_1^2 = (1 - \pi_2)y_2$ , where subscripts on variables denote the agent and superscripts denote the good.
- Stage 2: At stage 2 the agents can trade their endowment under conditions of perfect competition (i.e. both agents take prices as parameters). The goods are immediately consumed.

## 2.1 General Theory

The game is solved by backward induction.

**Stage 2:** At stage 2 both agents take as given the public enforcement level fixed at stage 0 and the outcome of the contest at stage 1 that defines endowments  $E$ . Denote by  $c = \{c_1^1, c_1^2, c_2^1, c_2^2\}$  the final consumption levels and by  $p_2$  the relative

price of good 2 in terms of good 1. The agents maximize their utility by the choice of  $c_i^1, c_i^2$ :

$$\max_{c_i^1, c_i^2} u(c_i^1, c_i^2, f_i) \quad s.t. \quad c_i^1 + p_2 c_i^2 = E_i^1 + p_2 E_i^2. \quad (1)$$

We denote by  $\lambda_i$  the Lagrangian multiplier associated with the budget constraint. The first order conditions,

$$\frac{\partial u / \partial c_i^2}{\partial u / \partial c_i^1} = p_2, \quad i = 1, 2, \quad (2)$$

reveal that there is trade efficiency: for a given amount of goods agents trade until their marginal rates of substitution coincide. This result follows from the sequencing of the game we have chosen. It allows us to concentrate on productive inefficiencies caused by imperfect enforcement.

The solution to this problem gives rise to Marshallian demand functions  $c_i^j(\delta, t, p_2)$ ,  $i, j = 1, 2$ . A market equilibrium at stage 2 is a price  $p_2(\delta, t)$  such that  $c_1^j(\delta, t, p_2) + c_2^j(\delta, t, p_2) = E_1^j(\delta, t) + E_2^j(\delta, t)$ ,  $j = 1, 2$ . The resulting equilibrium demand functions are denoted by  $c_i^j(\delta, t)$ ,  $i, j = 1, 2$ . Inserting these functions into the utility functions yields indirect utility functions  $v_i(\delta, t)$ .

**Stage 1:** At stage 1 the agents maximize their indirect utility function  $v_i(\delta, t)$  by the choice of  $\delta_i$ ,  $i = 1, 2$ . A Nash equilibrium of the game at stage 1 is a vector  $\delta^N(t)$  such that  $\delta_i^N(t) \in \arg \max_{\delta_i} v_i(\delta_i, \delta_j^N, t)$  s.t.  $l_1 + d_i + a_i + f_i = 1 - T$ ,  $i = 1, 2$ ,  $j \neq i$ . Due to the symmetry of the problem we know that a symmetric equilibrium with  $\delta_1^N = \delta_2^N$  exists.

Substituting for  $f_i$  in  $v_i$  by inserting the constraint, the first-order condition with respect to  $l_i$  can be calculated by the use of the envelope theorem:

$$\frac{\partial v_i}{\partial l_i} = -\frac{\partial u}{\partial f_i} + \lambda_i \pi_i \frac{\partial g}{\partial l_i} + \Delta_{l_i} = 0, \quad i = 1, 2, \quad (3)$$

where

$$\Delta_{l_i} = \lambda_i (E_i^2 - c_i^2) \frac{dp_2}{dl_i}$$

is the marginal utility of the change in the net-income position of agent  $i$  resulting from a change in the equilibrium price. This effect is similar to the terms-of-trade effect in the trade literature: an increase in  $l_i$  will have an impact on the equilibrium price. If  $(E_i^2 - c_i^2)$  is positive, agent  $i$  is a net-seller of good 2, which implies that he profits from an increase in the equilibrium price. The existence of this price effect depends on the interpretation of the model. If the two agents are representative

for a large number of agents,  $dp_2/dl_i = 0$ , because the effect of the labor-supply decision of a single agent on the equilibrium price is equal to zero as a result of large number of buyers and sellers on this market. If however, we are in a situation of a bilateral monopoly it is rational for the agents to internalize the strategic effect of their decisions at stage 1 on the equilibrium price at stage 2. We will use the first convention in the discussion to follow.<sup>10</sup>

(3) reveals that as long as  $\pi_i < 1$  there is production inefficiency compared to a situation of complete public enforcement. In a standard Arrow-Debreu type general-equilibrium model (3) would be

$$\frac{\partial v_i}{\partial l_i} = -\frac{\partial u}{\partial f_i} + \lambda_i \frac{\partial g}{\partial l_i} = 0, \quad i = 1, 2,$$

implying that the marginal rate of substitution,  $(\partial v_i/\partial l_i)/\lambda_i$ , equals the marginal rate of transformation,  $\partial g/\partial l_i$  (the marginal productivity of leisure is equal to one). Incompletely enforced property rights reduce the incentive to produce because part of the marginal return to labor is appropriated by the other agent. One should, however, bear in mind that this dilution of incentives is only inefficient with respect to a standard of reference that disregards the costs of perfect enforcement. Qualifying the allocation as inefficient would therefore be a result of a ‘Nirvana approach’ (Demsetz 1977) as long as no mechanism is found that implements a Pareto-superior allocation.

The first-order conditions with respect to  $d_i$  and  $a_i$  are:

$$\frac{\partial v_i}{\partial d_i} = -\frac{\partial u}{\partial f_i} + \lambda_i \frac{\partial \pi_i}{\partial d_i} g(l_i) + \Delta_{d_i} = 0, \quad i = 1, 2, \quad (4)$$

$$\frac{\partial v_i}{\partial a_i} = -\frac{\partial u}{\partial f_i} - \lambda_i \frac{\partial \pi_j}{\partial a_i} g(l_j) + \Delta_{a_i} = 0, \quad i = 1, 2, \quad (5)$$

where

$$\Delta_{d_i} = \lambda_i (E_i^2 - c_i^2) \frac{dp_2}{dd_i}, \quad \Delta_{a_i} = \lambda_i (E_i^2 - c_i^2) \frac{dp_2}{da_i},$$

are the strategic price effects, which we assume to be equal to zero.

The interpretation of both conditions is straightforward, The first terms on the right-hand side of the equations measure the marginal costs of an increase in defensive or appropriative activities, whereas the second terms measure their marginal

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<sup>10</sup>Asset specificity can be a reason why equilibrium prices can be influenced by strategic investment decisions even in the case of private goods. See Alchian, Crawford, and Klein (1975) and Williamson (1975).

return. The marginal costs are equal to the decrease in utility due to a reallocation of time away from leisure. The marginal return is equal to the change in the fraction of the defended or appropriated good times the marginal utility of this good. At the agent optimum both coincide. Rearranging (4) yields

$$\frac{\partial \pi_i}{\partial d_i} = \frac{1}{g(l_i)} \frac{\partial u / \partial f_i}{\lambda_i}.$$

The left-hand side of this equation shows the effect of a change in private enforcement on the fraction of the good owned that can be secured against appropriation. The right-hand side shows the marginal costs of such an increase in security. It is equal to the weighted marginal rate of substitution between leisure and consumption. Equations (4) and (5) combined show the optimal relationship between defensive and appropriative activities:

$$\frac{\partial \pi_i}{\partial d_i} g(l_i) = \frac{\partial \pi_j}{\partial a_i} g(l_j), \quad (6)$$

appropriative and defensive activities are chosen in a way that balances their marginal impacts on utility.

Inserting the Nash-equilibrium values of  $a$  into the indirect utility functions  $v_i$  gives rise to indirect utility functions  $w_i(t)$ . Using the state's budget constraint  $T = z(\theta)$  this can be further reduced to  $W_i(\theta)$ .

**Stage 0:** At stage 0 the government maximizes the utilitarian sum of utilities by its choice of a public-enforcement policy,

$$\max_{\theta} W_1(\theta) + W_2(\theta). \quad (7)$$

The derivative of (7) with respect to  $\theta$  is

$$- \frac{\partial u}{\partial f_1} z^t + \lambda_1 \left( \pi_1^{d_2} \frac{\partial d_2}{\partial \theta} g(l_1) - p_2 \pi_2^{a_2} \frac{\partial a_2}{\partial \theta} g(l_2) + p_2 (1 - \pi_2) \frac{\partial g}{\partial l_2} \frac{\partial l_2}{\partial \theta} \right) \quad (8)$$

$$- \frac{\partial u}{\partial f_2} z^t + \lambda_2 \left( -\pi_1^{a_1} \frac{\partial a_1}{\partial \theta} g(l_1) + (1 - \pi_1) \frac{\partial g}{\partial l_1} \frac{\partial l_1}{\partial \theta} + p_2 \pi_2^{d_1} \frac{\partial d_1}{\partial \theta} g(l_2) \right). \quad (9)$$

The first terms in each line measure the welfare gain due to the direct resource gain of a decrease in public enforcement (remember that larger values of  $\theta$  imply less public enforcement). The terms within large brackets measure the welfare loss due to a change in aggressive and defensive activities. Using the symmetry of the model (8) simplifies to

$$2 \left( -\frac{\partial u}{\partial f_i} \lambda_i z^t + p_j (1 - \pi_j) \frac{\partial g}{\partial l_j} \frac{\partial l_j}{\partial \theta} \right).$$

The following (Kuhn-Tucker) first-order condition is sufficient for the optimal level of public enforcement:

$$\frac{\partial u / \partial f_i}{\lambda_i} z^t - p_j(1 - \pi_j) \frac{\partial g}{\partial l_j} \frac{\partial l_j}{\partial \theta} \geq 0 \quad \Leftrightarrow \quad \theta \begin{cases} = 1 \\ \in [0, 1] \\ = 0 \end{cases} . \quad (10)$$

The above first-order condition is only necessary for the optimal level of public enforcement because multiple values of  $\theta$  can exist that fulfill (10). As we will see in the following example, it may be the case that some  $\theta \in (0, 1)$  constitutes a local maximum, whereas the global maximum is reached at  $\theta = 1$ .

What is the economic intuition for this condition? An interior optimal level of public enforcement is reached if the marginal resource costs of an increase in enforcement (larger  $T$ ) is equal to the marginal increase in the total production of private goods due to a reduction in appropriative and defensive activities, all weighted by the marginal rate of substitution between leisure and consumption.

We are now in a position to distinguish between four types of private goods depending on the costs of public enforcement:

1. ‘pure’ private goods if public enforcement is perfect,  $\theta = 0$ , and there is no private enforcement at all,
2. ‘impure’ private goods if public and private enforcement interact,  $\theta \in (0, 1)$ ,
3. ‘impure’ common-pool goods if there is no public but only private enforcement of property rights,  $\theta = 1$ .
4. ‘pure’ common-pool goods if there is neither private nor public enforcement.

In the latter two cases it could be argued that the formal concept of property rights makes no sense because there exists no public protection. However, such a conclusion would be premature because the public non-enforcement of property rights is a rational strategy in this case given the costs of enforcement. An empirical example for an impure common-pool good is the early stage of the ‘California Land Run’ where the state granted the legal property of land to the person who first occupied it without having any means to protect these rights in the beginning. A lot of business transactions with either small value or severe verification problems are also

not protected by any public enforcement of property rights. Indeed it can be argued that in such cases formal property becomes a meaningful concept only if it can be defended by the agents (Grossman 2001).

Please also note that the tragedy of the commons (Hardin 1968) associated with common-pool goods gets a different perception in this model. First of all there is never a need for the government not to assign formal property rights to private goods because this formal assignment is without costs. In the absence of public enforcement there will in general be a positive degree of appropriation in equilibrium. In this respect the tragedy of the commons enters the picture, which is reflected in the diluted incentives to produce. However, this tragedy is not necessarily a tragedy from a normative point of view: given the technology for enforcement it is optimal. There exists an *optimal* externality associated with the enforcement technology.<sup>11</sup>

## 2.2 Functional specification

In this section we analyze the properties of optimal enforcement using a functional specification. We assume that households have a Cobb-Douglas utility function

$$u_i(c_i^1, c_i^2, f_i) = c_i^1 \cdot c_i^2 \cdot f_i. \quad (11)$$

The production of the private good is linear in labor input,  $g(l_i) = l_i$ , and the conflict function is of the Tullock type

$$\pi_i(d_i, a_j, \theta) = \frac{1}{1 + \theta \frac{a_j}{d_i}}, \quad (12)$$

where  $\theta \in [0, 1]$  is a parameter that measures the effectiveness of appropriative and defensive activities. Its interpretation as public-enforcement variable implies a specific assumption about the relationship between private and public enforcement, namely that they have an elasticity of substitution equal to 1. In particular private and public enforcement are no perfect substitutes and therefore different intermediate goods. Larger values of  $\theta$  ceteris paribus increase the fraction of the good that

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<sup>11</sup>Even if resources are completely exploited or a species exterminated due to non-enforced property rights it may be an optimal strategy given the opportunity costs of conservation. However, the efficiency of such a policy depends on alternative institutional structures.

is appropriated,

$$\frac{\partial \pi_i}{\partial \theta} = -\frac{\frac{a_j}{d_i}}{\left(1 + \theta \frac{a_j}{d_i}\right)^2} < 0.$$

At the same time, a larger value of  $\theta$  may decrease or increase the marginal effectiveness of appropriative as well as defensive activities, depending on the sign of  $\theta a_j - d_i$ ,

$$\begin{aligned}\frac{\partial^2 \pi_i}{\partial d_i \partial \theta} &= \frac{a_j(\theta a_j - d_i)}{(d_i + \theta a_j)^3}, \\ \frac{\partial^2 \pi_i}{\partial a_j \partial \theta} &= \frac{d_i(d_i - \theta a_j)}{(d_i + \theta a_j)^3}.\end{aligned}$$

We do not yet specify the functional form of  $T = z(\theta)$  because we want to see how the results depend on the public-enforcement technology.

**Stage 2:** At stage 2 the agents maximize their utility by trading good one for good two taking as given the outcome of the appropriative contest at stage 1,  $E = \{E_1^1, E_1^2, E_2^1, E_2^2\}$ . Solving both agents' maximization problems gives the following Marshallian demand functions:

$$c_i^1 = \frac{1}{2} \frac{p^1 E_i^1 + p^2 E_i^2}{p^1}, \quad (13)$$

$$c_i^2 = \frac{1}{2} \frac{p^1 E_i^1 + p^2 E_i^2}{p^2}. \quad (14)$$

$$(15)$$

In equilibrium it must be that  $c_1^1 + c_2^1 = E_1^1 + E_2^1$ . The second market can be neglected by Walras' law. Normalizing the price of good 1 to be equal to 1, the equilibrium price is equal to

$$p^2 = \frac{E_1^1 + E_2^1}{E_1^2 + E_2^2}. \quad (16)$$

Given this information and  $E_1^1 = p_1 g(l_1)$  etc. we can calculate the equilibrium values of the Marshallian demand functions:

$$c_1^1 = \frac{1}{2} l_1 \left( 1 - \frac{d_2}{d_2 + \theta a_1} + \frac{d_1}{d_1 + \theta a_2} \right), \quad (17)$$

$$c_1^2 = \frac{1}{2} l_2 \left( 1 - \frac{d_2}{d_2 + \theta a_1} + \frac{d_1}{d_1 + \theta a_2} \right), \quad (18)$$

$$c_2^1 = \frac{1}{2} l_1 \left( \frac{d_1 d_2 + \theta a_2 (2d_2 + \theta a_1)}{(d_2 + \theta a_1)(d_1 + \theta a_2)} \right), \quad (19)$$

$$c_2^2 = \frac{1}{2} l_2 \left( \frac{d_1 d_2 + \theta a_2 (2d_2 + \theta a_1)}{(d_2 + \theta a_1)(d_1 + \theta a_2)} \right). \quad (20)$$

Inserting these expressions in the utility function yields the indirect utility functions  $v_1(d_1, d_2, a_1, a_2, l_1, l_2, \theta, T)$ ,  $v_2(d_1, d_2, a_1, a_2, l_1, l_2, \theta, T)$ .

**Stage 1:** Calculating the first-order conditions of both agents' optimization problems and solving them simultaneously yields the following Nash equilibrium:

$$l_i = \frac{1}{2} \left( 1 + \frac{2\theta(T-1)}{1+4\theta+\theta^2} - T \right), \quad (21)$$

$$d_i = \frac{\theta(1-T)}{1+4\theta+\theta^2}, \quad (22)$$

$$a_i = \frac{\theta(1-T)}{1+4\theta+\theta^2}. \quad (23)$$

Inserting these values into the indirect utility function  $v_i$  yields the indirect utility function  $w_i(\theta, T)$ .

**Stage 0:** At stage 0 the government maximizes the utilitarian sum of both agents' indirect utility functions  $w_i$  under the constraint that spending on public enforcement has to be 'financed' out of the time budget of the agents,  $T = z(\theta)$ . Inserting this constraint into  $w_i(\theta, T)$  yields a function

$$W_i(\theta) = -\frac{(z(\theta)-1)^3(1+\theta)^6}{32(1+\theta(4+\theta))^3}, \quad (24)$$

which measures the agents' utility given budget-balancing by the government.

We start the discussion by an analysis of the degree of crime, which is defined as  $(1 - \pi_i(\theta))$ , the equilibrium fraction of goods that is appropriated by the non-owner of the good. Inserting the optimal values into  $p_i$  yields  $(1 - \pi_i) = 1 - \frac{1}{1+\theta}$  and is displayed in Figure 1. Intuitively, for  $\theta = 0$  property rights are perfectly publicly enforced, which implies that there is no crime at all. At the other extreme, at  $\theta = 1$ , there is no public enforcement at all, and both agents are equally strong, which implies that 50% of all goods are illegally appropriated. We can calculate the fraction of resources that is wasted at that point as  $d_i(\theta = 1) = 1/6$  and  $a_j(\theta = 1) = 1/6$ , which implies that 1/3 of all time resources are devoted to criminal activities and the private defense against them.

In order to understand the structure of optimal enforcement of property rights we will distinguish between two different cases in the following. In case (a) perfect public enforcement would require 100% of the time resources of the agents (high-cost case). In case (b) a perfect public enforcement would require 10% of the time resources of the agents (low-cost case).

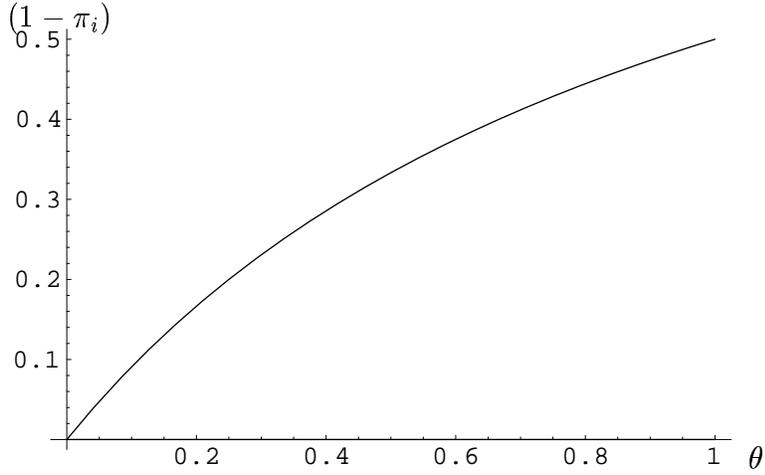


Figure 1: Equilibrium fraction of crime.

Case (a) is plotted in Figure 2. In Figure 2 you find the welfare levels for  $z(\theta) = 1 - \theta^q$  where  $q \in \{0, 1/18, 1/10, 1/8, 2/10\}$ . The uppermost graph is the one for  $q = 1$ , whereas the lowmost graph is the one for  $q = 2/10$ . It is intuitively clear that for  $q = 0$ , which is equivalent to saying that public enforcement of property rights is without cost, it is optimal to perfectly and publicly enforce the property rights. This is the benchmark case from Arrow-Debreu general-equilibrium theory. More interestingly, if the costs of public enforcement increase, it becomes optimal to less than perfectly enforce the property rights publicly. Interestingly there exist situations (in our example at  $q = 1/8$ ) when a certain degree of public enforcement constitutes a local maximum but not a global one; it is globally optimal not to publicly enforce property rights at all (impure common-pool good). To summarize, it is optimal to have

- a pure private good if  $q = 0$ ,
- an impure private good if  $q = 1/18$  or  $1/10$ , and
- an impure common-pool good if  $q = 1/8$  or  $2/10$ .

Case (b) is plotted in Figure 3. The graphs in Figure 3 are plotted for  $z(\theta) = 0.1 - 0.1\theta^q$  where  $q \in \{0, 1/18, 1/10, 1/8, 2/10\}$ . The uppermost graph is the one for  $q = 1$ , whereas the lowermost graph is the one for  $q = 2/10$ , hence, this example

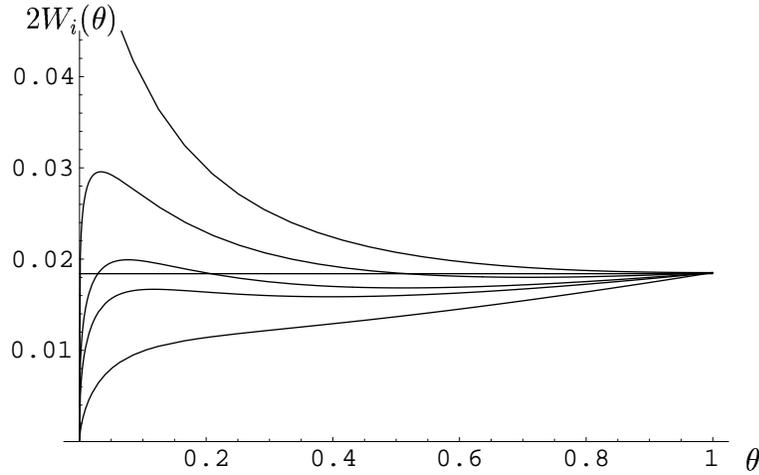


Figure 2: Welfare as a function of public enforcement (high-cost case).

has the same structure as the one underlying Figure 1. It can be seen that if the potential total resource costs of public enforcement are relatively low, the common-pool phenomenon disappears. However, it can also be seen that perfect private goods exist only for low fixed costs of enforcement. If enforcement costs are correlated with the level of conflict it is always optimal not to perfectly enforce property rights publicly.

### 3 Public Goods

We now turn to the analysis of public goods. In order to do so we have to modify and extend the above model. We assume that each agent can produce a public good and a private good as well as leisure. As before, leisure can neither be traded nor can it be appropriated. Both agents can produce the same private good, which is not due to appropriation but which can be traded. The public good produced differs for both agents and is due to appropriation by the other agent. Appropriation of the public good by agent  $j$  means that agent  $j$  illegally uses some fraction of the public good, but that agent  $i$  is still in a position to use it completely. Hence, appropriation is restricted in the sense that the appropriating agent is not able to steal the public

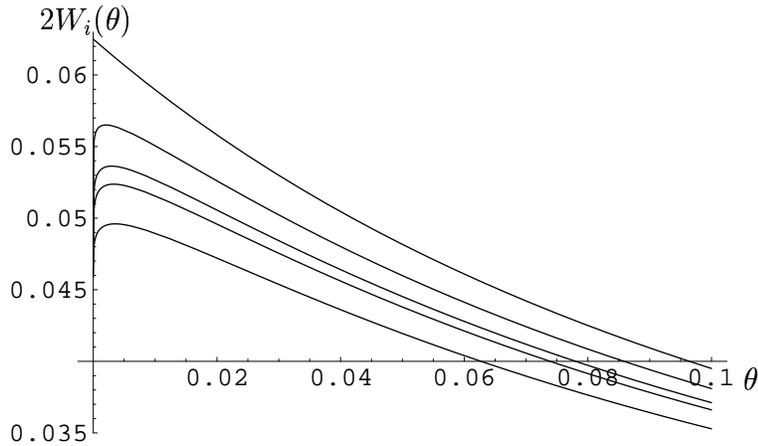


Figure 3: Welfare as a function of public enforcement (low-cost case).

good entirely and to exclude the producer.<sup>12</sup>

There is one fundamental difference between private and public goods with respect to the interpretation of the public-enforcement parameter  $\theta$ . It is reasonable to assume that the degree of appropriation is restricted in the absence of public enforcement (the other assumption would imply that private defense is impossible), and it is convenient to assume that both agents are equally strong in this case, which boils down to  $\theta = 1$ . This need not be the case for public goods. A restriction of  $\theta$  to be in  $[0, 1]$  implies that in the absence of public enforcement and with equal investments in appropriation and defense the appropriator is restricted to the use of half of the public good. This is not convincing because the good is non-rival and we therefore allow  $\theta$  to be in  $[0, \bar{\theta}]$  in the following, where  $\bar{\theta}$  can be any positive real number.

We denote by  $c_i, y_i$  the consumption and production levels of the private good by agent  $i$  and by  $c_i^{Pi}$  the production level of the public good produced by agent  $i$  as well as his consumption level, and by  $c_i^{Pij}$  the consumption level of this good by agent  $j$ . The utility functions are  $u(c_i, c_i^{Pi}, c_j^{Pji}, f_i)$ ,  $i = 1, 2$ .

The sequential structure of the game parallels the one from the last section:

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<sup>12</sup>Such a situation would be formally similar to the appropriation of private goods.

Stage 0: At stage 0 the government chooses a level of public enforcement and a corresponding tax rate  $t = \{\theta, T\}$ . Its objective is to maximize the sum of utilities of both agents.

Stage 1: At stage 1 the agents allocate their remaining time budget to productive (private and public good), defensive, and appropriative activities, as well as to leisure. This defines a primary distribution  $E = \{E_1(\delta, t), E_2(\delta, t), E_1^{P1}(\delta, t), E_1^{P2}(\delta, t), E_2^{P1}(\delta, t), E_2^{P2}(\delta, t)\}$  with  $E_1 = y_1$ ,  $E_2 = y_2$ ,  $E_1^{P1} = c_1^{P1}$ ,  $E_2^{P1} = (1 - \pi_1)c_1^{P1}$ ,  $E_2^{P2} = c_2^{P2}$ ,  $E_1^{P2} = (1 - \pi_2)c_2^{P2}$  of the private and public goods, where again subscripts denote the agent and superscripts denote the good.

Stage 2: At stage 2 the agents can trade their endowment taking prices as given.

### 3.1 General Theory

**Stage 2:** At stage 2 both agents take as given the public enforcement level determined at stage 0 and the outcome of the contest at stage 1. We normalize the price of the private good to be equal to 1 and denote by  $p_1^P$  and  $p_2^P$  the prices of the public goods. Denote by  $c_i^{Pd}$  and  $c_i^{Ps}$  the demand and the supply by agent  $i$  of public good  $i$ . The agents maximize their utility by the choice of  $c_i$ ,  $c_i^{Ps}$ , and  $c_j^{Pd}$  :

$$\max_{c_i, c_j^{Pd}, c_i^{Ps}} u(c_i, c_i^{Pi}, E_i^{Pj} + c_i^{Pd}, f_i) \quad s.t. \quad c_i + p_j^P c_i^{Pd} = p_i^P c_i^{Ps} + y_1. \quad (25)$$

The budget constraint has a straightforward interpretation: total expenditures for the consumption of the private good and the public good of the other agent are denoted on the left-hand side, whereas total revenues are denoted on the right-hand side. They are composed out of the revenues from the selling of the private good plus the revenues from the selling of the access to the public good.

We denote by  $\lambda_i$  the Lagrange multiplier associated with the budget constraint. The first order condition with respect to  $c_i^{Ps}$  is

$$\lambda_i p_i^P \geq 0, \quad i = 1, 2. \quad (26)$$

The Lagrange multiplier measures the maximum increase in utility for an increase in income and is therefore positive. Hence, for every positive price of the public good the agents are willing to sell as much of the public good as possible. This directly

reflects that a public good is non-rival in consumption. Increasing the access to agent  $j$  does not reduce the access by agent  $i$ . Hence, in equilibrium it must be that  $c_i^{Ps} = \pi_i c_i^{Pi}$ . This finding shows that a market for the public good is efficient in the following sense: there is no waste in the use of the good due to a limitation of access. This shows that exclusion is a means or a *mechanism* to guarantee efficiency but that nobody will actually be excluded in equilibrium.<sup>13</sup> Whether or not the exclusion mechanism can guarantee an efficient supply of public goods depends on the associated costs as will be shown in the following.

The first order conditions with respect to  $c_i$  and  $c_i^{Pd}$  show that

$$\frac{\partial u / \partial c_j^P}{\partial u / \partial c_i} = p_2^P, \quad i = 1, 2, \quad (27)$$

the marginal rate of substitution between the public and the private good is equal to the user price of agent  $i$ . We will use this finding in the analysis of stage 1 to establish a modified Samuelson condition for the optimal supply of public goods.

The solution to the problem gives rise to Marshallian demand and supply functions  $c_i(\delta, t, p_1^P, p_2^P)$ ,  $c_i^{Pd}(\delta, t, p_1^P, p_2^P)$ ,  $c_i^{Ps}(\delta, t, p_1^P, p_2^P)$ ,  $i, j = 1, 2$ . A market equilibrium at stage 2 is a price vector  $p_1^P(\delta, t), p_2^P(\delta, t)$  such that  $c_1(\delta, t, p_1^P, p_2^P) + c_2(\delta, t, p_1^P, p_2^P) = y_1 + y_2$ , for the private and  $c_1^{Pd}(\delta, t) = c_1^{Ps}(\delta, t)$ ,  $c_2^{Pd}(\delta, t) = c_2^{Ps}(\delta, t)$  for the public goods. The resulting equilibrium demand functions are denoted by  $c_i(\delta, t)$ ,  $c_i^{pk}(\delta, t)$ ,  $i = 1, 2$ ,  $k = s, d$ . Inserting these functions into the utility functions yields indirect utility functions  $v_i(\delta, t)$ .

**Stage 1:** At stage 1 the agents maximize their indirect utility  $v_i(\delta, t)$  by the choice of  $\delta_i$ ,  $i = 1, 2$ . As before, a Nash equilibrium of the game at stage 1 is a vector  $\delta^N(t)$  such that  $\delta_i^N(t) \in \arg \max_{\delta_i} v_i(\delta_i, \delta_j^N, t)$  s.t.  $l_i + d_i + a_i + c_i^P + f_i = 1 - T$ ,  $i = 1, 2$ ,

<sup>13</sup>This result differs from Fraser (1996) and Janeba and Swope (2001) who get positive exclusion in equilibrium. Their result stems from the assumption that user fees cannot be type specific. Because of the similarity of increasing-returns to scale and public goods the plausibility of the assumptions can be evaluated using a monopoly model. There, exclusion in equilibrium corresponds to a situation of Cournot-price setting, whereas no exclusion in equilibrium corresponds to perfect price discrimination. Hence, the discussion if Lindahl prices can be used boils down to the question if a monopoly can effectively discriminate prices. In a world with symmetric information it is obvious that a monopoly will discriminate prices. However, even with asymmetric information it is a standard result that type-specific contracts will be used in equilibrium (Stiglitz 1977). We therefore believe that the result of non-exclusion in equilibrium is a good approximation, both empirically as well as theoretically.

$j \neq i$ , where for convenience we have assumed that the production of the public good is linear in labor with a marginal productivity equal to 1. Due to the symmetry of the problem we know that a symmetric equilibrium with  $\delta_1^N = \delta_2^N$  exists.

Different to the case of private goods, markets for public goods are characterized by a monopoly if they are efficient.<sup>14</sup> This difference may have an influence on the modeling strategy. When faced with the decision to produce the supplier of a public good might internalize the effect of his supply decision on the equilibrium prices on the personalized markets for his goods. We will analyze both situations, a parametric treatment of prices and an internalization of price effects, in the following.

Substituting for  $f_i$  in  $v_i$  by inserting the constraint, the first-order conditions can be calculated by the use of the envelope theorem:

$$\frac{\partial v_i}{\partial l_i} = -\frac{\partial u}{\partial f_i} + \lambda_i \frac{\partial g}{\partial l_i} + \lambda_i \Delta_{l_i} = 0, \quad i = 1, 2, \quad (28)$$

$$\frac{\partial v_i}{\partial c_i^P} = -\frac{\partial u}{\partial f_i} + \frac{\partial u}{\partial c_i^P} + \lambda_i p_i^P \pi_i + \lambda_i \Delta_{c_i^P} = 0, \quad i = 1, 2, \quad (29)$$

$$\frac{\partial v_i}{\partial d_i} = -\frac{\partial u}{\partial f_i} + \lambda_i p_i^P \frac{\partial \pi_i}{\partial d_i} c_i^P + \lambda_i \Delta_{d_i} = 0, \quad i = 1, 2, \quad (30)$$

$$\frac{\partial v_i}{\partial a_i} = -\frac{\partial u}{\partial f_i} - \frac{\partial u}{\partial c_j^P} \frac{\partial \pi_j}{\partial a_i} c_j^P + \lambda_i \Delta_{a_i} = 0, \quad i = 1, 2, \quad (31)$$

where

$$\Delta_{\sigma_i} = \left( \frac{dp_i^P}{d\sigma_i} \pi_i c_i - \frac{dp_j^P}{d\sigma_i} (1 - \pi_j) c_j \right) \quad \sigma_i \in \{l_i, c_i^P, a_i, d_i\}, i = 1, 2, j \neq i \quad (32)$$

measures the net effect of a change in equilibrium prices. As before this effect can be interpreted as a net-income effect of price changes weighted by the marginal utility of income: the first term in brackets is equal to the increase of the value of the own public good, whereas the second term is equal to the increase in the value of the public good of the other agent. The difference is equal to the net-income effect of the agent.

Rearranging and combining (28) and (29) using (27) gives a modified Samuelson condition for the decentralized supply of public goods:

$$\frac{\partial u_1 / \partial c_i^P}{\partial u_1 / \partial c_1} + \frac{\partial u_2 / \partial c_i^P}{\partial u_2 / \partial c_2} \pi_i = \frac{\partial g}{\partial l_i} + \left( \Delta_{l_i} - \Delta_{c_i^P} \right), \quad i = 1, 2. \quad (33)$$

<sup>14</sup>The average costs are increasing in the number of suppliers.

The decentralized provision of public goods induces two types of distortions: the right-hand side of (33) is equal to the marginal rate of transformation between the public good (whose marginal productivity is equal to 1) and the private good plus the net-income effect due to anticipated price changes. The left-hand side is equal to the sum of marginal rates of substitution between the public and the private good, weighted by the appropriative activities. If  $\pi_i = 1$  and without price effects this condition boils down to the Samuelson condition with perfect public enforcement (Samuelson 1954). However, the fact that public enforcement is in general imperfect implies that an externality exists that dilutes the incentives to produce the public good: because a fraction  $(1 - \pi_i)$  of the public good is used by agent  $j$  without any charge, agent  $i$  internalizes only a fraction  $\pi_i$  in his optimization problem. This deviation from the Samuelson condition does not imply that the resulting equilibrium is inefficient. Under the condition that public enforcement is optimally less than perfect it is the optimal deviation of the rule taking into consideration enforcement costs.

The second distortion is a consequence of the anticipated change in equilibrium prices. Because markets for public goods are more likely to be thin in the sense that the number of suppliers is small, each supplier may anticipate that his supply decision has an influence on the equilibrium prices he can charge. This effect may exist even with perfect enforcement,  $\pi_i = 1$ . What is the effect of this anticipation? Assume that all goods are normal goods. In this case, an increase in leisure of agent  $i$  will tend to increase the price of public good  $i$  and decrease the price of public good  $j$ , which implies  $\Delta_{l_i} > 0$ . An increase in public good  $i$  has the opposite effect of equilibrium prices, which implies  $\Delta_{c_i^P} < 0$ . Hence,  $\Delta_{l_i} - \Delta_{c_i^P} > 0$ , which increases the incentive for underprovision of the public good: the incentives to provide the public good are diluted because each agent anticipates that an increase in supply of the public good tends to reduce its relative price.

Simple manipulations of (30) and (31) show the optimal relationship between aggressive and defensive investments:

$$p_1^P c_1^P \frac{\partial \pi_1}{\partial d_1} = -p_2^P c_2^P \frac{\partial \pi_2}{\partial a_1} + (\Delta_{a_i} - \Delta_{d_i}). \quad (34)$$

As for the case of private goods, the optimal balance of appropriative and defensive investments is given when their marginal return is equal, corrected by a term measuring the net effect of a change in investment on equilibrium prices.

Inserting the Nash-equilibrium values of  $a$  into the indirect utility functions  $v_i$  gives rise to indirect utility functions  $w_i(t)$ . Using the state's budget requirement  $T = z(\theta)$  this can be further reduced to  $W_i(\theta)$ .

**Stage 0:** At stage 0 the government maximizes the utilitarian sum of utilities by its choice of a public-enforcement policy,

$$\max_{\theta} W_1(\theta) + W_2(\theta). \quad (35)$$

The derivative of (35) with respect to  $\theta$  is

$$\begin{aligned} & - \frac{\partial u}{\partial f_1} z^t + \frac{\partial u}{\partial c_2^P} \left( - \frac{\partial \pi_2}{\partial d_2} \frac{\partial d_2}{\partial \theta} + (1 - \pi_2) \frac{\partial c_2^P}{\partial \theta} \right) + \lambda_1 p_1^P \frac{\partial \pi_1}{\partial a_2} \frac{\partial a_2}{\partial \theta} \\ & - \frac{\partial u}{\partial f_2} z^t + \frac{\partial u}{\partial c_1^P} \left( - \frac{\partial \pi_1}{\partial d_1} \frac{\partial d_1}{\partial \theta} + (1 - \pi_1) \frac{\partial c_1^P}{\partial \theta} \right) + \lambda_2 p_2^P \frac{\partial \pi_2}{\partial a_1} \frac{\partial a_1}{\partial \theta} \\ & + \frac{\partial u}{\partial f_1} \left( \frac{dp_1^P}{d\theta} \pi_1 c_1 - \frac{dp_2^P}{d\theta} (1 - \pi_2) c_2 \right) + \frac{\partial u}{\partial f_2} \left( \frac{dp_2^P}{d\theta} \pi_2 c_2 - \frac{dp_1^P}{d\theta} (1 - \pi_1) c_1 \right) \end{aligned} \quad (36)$$

The first terms in each line measure the welfare gain due to the direct resource gain of a decrease in public enforcement (remember that larger  $\theta$  imply less public enforcement). The terms within large brackets measure the welfare loss due to a change in aggressive and defensive activities. Using the symmetry of the model (36) simplifies to

$$2 \left( - \frac{\partial u}{\partial f_i} z^t + p_i^P \lambda_i (1 - \pi_i) \frac{\partial c_i^P}{\partial \theta} + \lambda_i \frac{dp_i^P}{d\theta} (2\pi_i - 1) c_i \right).$$

The following (Kuhn-Tucker) first-order condition is sufficient for the optimal level of public enforcement:

$$\frac{\partial u / \partial f_i}{p_i^P \lambda_i} z^t - (1 - \pi_i) \frac{\partial c_i^P}{\partial \theta} - \frac{dp_i^P}{d\theta} (2\pi_i - 1) c_i \begin{cases} \geq 0 \\ \leq 0 \end{cases} \Leftrightarrow \theta \begin{cases} = 1 \\ \in [0, 1] \\ = 0 \end{cases}. \quad (37)$$

Again, the above first-order condition is only necessary for the optimal level of public enforcement because multiple values of  $\theta$  can exist that fulfill (37).

What is the economic intuition for this condition? An interior optimal level of public enforcement is reached if the marginal resource costs of an increase in enforcement (larger  $T$ ) are equal to the marginal increase in the production of the public good due to a reduction in appropriative and defensive activities plus the incentive effect of a change in equilibrium prices, all weighted by the marginal rate of substitution between leisure and consumption.

Accordingly, four types of public goods can be distinguished depending on the costs of public enforcement:

1. ‘pure’ public good if there is neither private not public enforcement of property rights,
2. ‘impure’ public goods if no public enforcement is optimal,
3. ‘impure’ club goods if public and private enforcement interact, and
4. ‘pure’ club goods if there is only public enforcement of property rights.

### 3.2 Numerical specification

As for the case of private goods, we will further analyze the properties of optimal enforcement using a numerical specification. Again, we assume that households have a Cobb-Douglas utility function

$$u_1(c_1, c_1^{P1}, c_1^{P2}, f_1) = c_1 \cdot c_1^{P1} \cdot c_1^{P2} \cdot f_1, \quad (38)$$

$$u_2(c_2, c_2^{P1}, c_2^{P2}, f_2) = c_2 \cdot c_2^{P1} \cdot c_2^{P2} \cdot f_2, \quad (39)$$

and the conflict function is of the Tullock type where this time,  $\pi_1$  ( $\pi_2$ ) denotes the fraction of public good 1 (2) that is successfully excluded by agent 1 (2).

$$\pi_i(d_i, a_j, \theta) = \frac{1}{1 + \theta \frac{a_j}{d_i}}. \quad (40)$$

As before we do not yet specify the functional form of the public enforcement technology  $T = z(t)$ .

**Stage 2:** At stage 2 the agents maximize their utility by trading the private good for the public good of the other agent. Solving both agents’ maximization problems gives the following Marshallian demand functions:

$$c_1 = \frac{1}{2} (y_1 + p_1^P \pi_1 c_1^P + p_2 (1 - \pi_2) c_2^P), \quad (41)$$

$$c_1^{Pd} = \frac{1}{2} \left( \frac{y_1}{p_1^P} - (1 - \pi_2) c_2^P + \frac{p_1^P}{p_2^P} \pi_1 c_1^P \right), \quad (42)$$

$$c_1^{Ps} = \pi_1 c_1^P, \quad (43)$$

$$c_2 = \frac{1}{2} (y_2 + p_2^P \pi_2 c_2^P + p_1 (1 - \pi_1) c_1^P), \quad (44)$$

$$c_2^{Pd} = \frac{1}{2} \left( \frac{y_2}{p_2^P} - (1 - \pi_1) c_1^P + \frac{p_2^P}{p_1^P} \pi_2 c_2^P \right), \quad (45)$$

$$c_2^{Ps} = \pi_2 c_2^P. \quad (46)$$

In equilibrium it must be that  $c_1 + c_2 = y_1 + y_2$  for the private good and  $c_1^{Pd} = \pi_2 c_2^P$ ,  $c_2^{Pd} = \pi_1 c_1^P$  for the public goods. The equilibrium prices are

$$p_1^P = \frac{(d_2 + \theta a_1)(d_1 + \theta a_2)(l_1 c_2^P d_2(d_1 + \theta a_2) + l_2(2c_1^P d_1(d_2 + \theta a_1) + a_1 \theta c_2^P(d_1 + \theta a_2)))}{2a_2(c_1^P)^2 d_1 \theta (d_2 + \theta a_1)^2 + 2a_1(c_2^P)^2 d_2 \theta (d_1 + \theta a_2)^2 + c_1^P c_2^P (d_2 + \theta a_1)(d_1 + \theta a_2)(3d_1 d_2 + a_1 a_2 \theta^2)}, \quad (47)$$

$$p_2^P = \frac{(d_2 + \theta a_1)(d_1 + \theta a_2)(l_2 c_1^P d_1(d_2 + \theta a_1) + l_1(a_2 c_1^P \theta(d_2 + \theta a_1) + 2c_2^P d_2(d_1 + \theta a_2)))}{2a_2(c_1^P)^2 d_1 \theta (d_2 + \theta a_1)^2 + 2a_1(c_2^P)^2 d_2 \theta (d_1 + \theta a_2)^2 + c_1^P c_2^P (d_2 + \theta a_1)(d_1 + \theta a_2)(3d_1 d_2 + a_1 a_2 \theta^2)}. \quad (48)$$

Inserting these prices into the Marshallian demand functions and these in turn into the utility function yields the indirect utility functions  $v_i(l_1, d_1, a_1, c_1^P, f_1, l_2, d_2, a_2, c_2^P, f_2, \theta, T)$ . They are equal to

$$v_1 = \frac{l_1 \cdot c_1^P \cdot (c_1^P d_1(d_2 + \theta a_1) + a_1 c_2^P \theta(d_1 + \theta a_2))(1 - a_1 - l_1 - c_1^P - d_1 - T)}{(d_2 + \theta a_1)(d_1 + \theta a_2)} \quad (49)$$

$$v_2 = \frac{l_2 \cdot c_2^P \cdot (a_2 c_1^P \theta(d_2 + \theta a_1) + d_2 c_2^P(d_1 + \theta a_2))(1 - a_2 - l_2 - c_2^P - d_2 - T)}{(d_2 + \theta a_1)(d_1 + \theta a_2)} \quad (50)$$

**Stage 1:** At stage 1 both agents simultaneously choose their production, defense, and appropriation levels. The Nash equilibrium can be derived as in the last section. The setup of the model is symmetric again, which implies that there exists a symmetric equilibrium. The equilibrium values are:

$$l_i = \frac{1 + 2\theta + \theta^2 - T - 2\theta T - \theta^2 T}{4 + 9\theta + 3\theta^2}, \quad (51)$$

$$c_1^P = c_2^P = \frac{2 + 3\theta + \theta^2 - 2T - 3\theta T - \theta^2 T}{4 + 9\theta + 3\theta^2}, \quad (52)$$

$$d_i = \frac{\theta(1 - T)}{4 + 9\theta + 3\theta^2}, \quad (53)$$

$$a_i = \frac{\theta(1 - T)}{4 + 9\theta + 3\theta^2}. \quad (54)$$

Inserting these functions into the indirect utility functions  $v_i$  gives rise to indirect utility functions  $W_i(\theta, T)$ . Using the condition for budget balance,  $T = z(\theta)$ , yields indirect utility functions

$$W_i(\theta) = \frac{(1 + \theta)^6 (2 + \theta)^2 (z(\theta) - 1)^4}{(4 + 3\theta(3 + \theta))^4}. \quad (55)$$

**Stage 0:** We can now look at the optimal enforcement policy of the government. It turns out that the results are qualitatively similar to the ones obtained for the case of private goods if we restrict  $\theta \in [0, 1]$  both times. However, quantitatively they differ. There exists an intermediate range of enforcement costs for which perfect public enforcement is welfare maximizing for the case of private goods whereas

perfect private enforcement is welfare maximizing for the case of public goods. This result is intuitive: in the case of private goods an increase in the consumption of, for example, agent 2 directly harms agent 1 because the good is rival in consumption and indirectly because of a loss of total income. This is not the case for public goods. The only adverse effect created by the illegal use of the public good by agent 1 is its reduced income. Hence, a lack of public enforcement incurs a higher cost for the agents for the case of private goods than for the case of public goods.

In Figure 4 we analyze a situation when public enforcement generates no costs. For the purpose of a better exposition we have rescaled the utility function by the factor 10. The figure shows the graphs of the aggregate welfare level (first decreasing

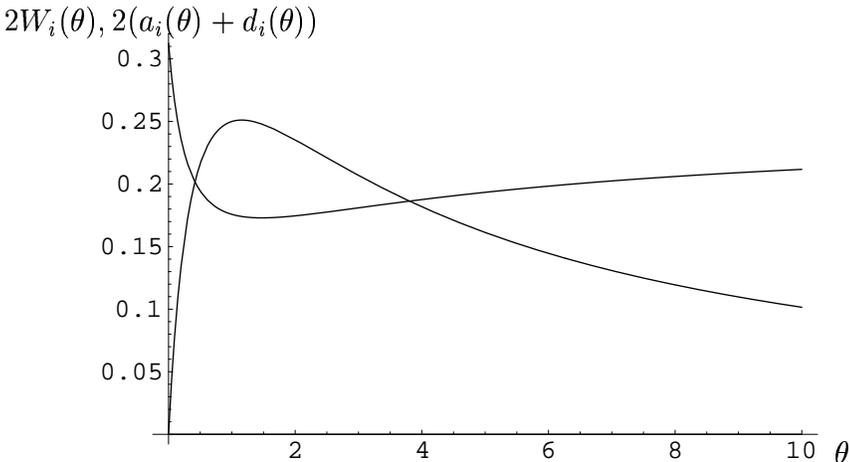


Figure 4: Welfare and investments in appropriation and defense as a function of public enforcement (public enforcement generates no costs).

and then increasing) and the aggregate investments in the contest (first increasing and then decreasing). The graphs is displayed for  $\theta \in [0, 10]$ . The following property can be easily established:  $\lim_{\theta \rightarrow \infty} 2W_i(\theta) = 2/81$  and  $2W_i(0) = 2/64$ , which shows that in fact public enforcement dominates private enforcement if it generates no costs. The more interesting property of this case is that welfare is not monotonous in  $\theta$ . Beginning at  $\theta = 0$  it is decreasing first but if  $\theta$  is getting large enough it starts to increase: a decrease in the level of public enforcement leads to an increase in welfare. The intuition for this result is as follows: an increase in  $\theta$  starting at  $\theta = 0$  increases both, appropriative and defensive activities. Hence, part of the resources

of the economy are wasted in the contest. This effect increases for a while because the opportunity costs of investing in the contest are decreasing. If  $\theta$  is getting very large, the marginal effectiveness of appropriation is getting very large, which implies that the opportunity costs of a marginal investment in the contest start increasing again: the agents start to reduce their appropriative and defensive activities. This phenomenon is similar to the paradox of power first mentioned by Hirshleifer (1991).

Figure 5 displays an example for the case of linear public-enforcement technologies for different total resource uses for perfect public enforcement for the assumption that  $\theta \in [0, 10]$ .<sup>15</sup> To be more specific, we assume that perfect public enforcement uses either 5% or 10% of total time resources, and the technologies are  $z(\theta) = 0.05 - 0.005\theta$ ,  $z(\theta) = 0.1 - 0.01\theta$ . The upper graph represents the 5%-case,

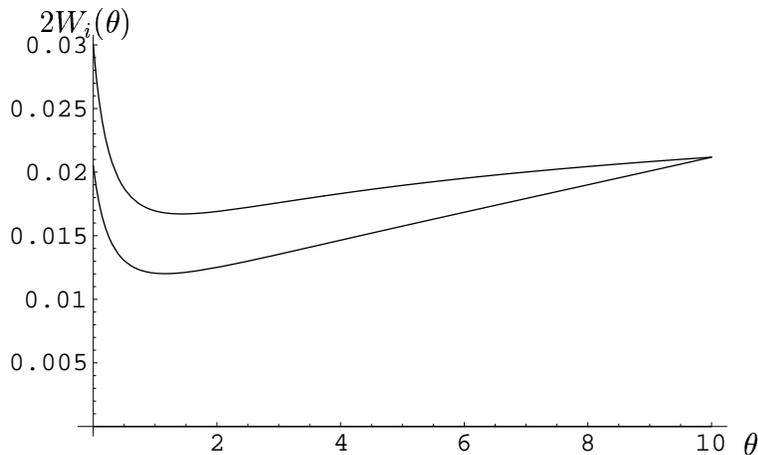


Figure 5: Welfare as a function of public enforcement (linear cost functions).

whereas the lower graph represents the 10% case. In both cases it is optimal to have either perfect public or perfect private enforcement depending on the amount of total resources needed.<sup>16</sup> However, the policy conclusion differs drastically: in the

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<sup>15</sup>In all cases with positive public-enforcement costs such a restriction is necessary in order to specify the cost function in a meaningful way. It has to be excluded that the total costs of enforcement become negative.

<sup>16</sup>With linear resource costs of public enforcement the indirect utility functions rotate around the welfare level at  $\theta = 10$ . If public enforcement would create only fixed costs,  $z$ , the welfare levels would shift parallel. Interior solutions with both, private and public enforcement, would result with

low-cost case it is optimal to create a pure public good, whereas the the high-cost case it is optimal to create a club good.

## 4 Institutional alternatives

### 4.1 Private or Public Supply?

A straightforward question that arises from the above analysis is whether the public supply of goods can be an alternative to their private supply. The idea is that public supply might economize on enforcement costs. We will discuss this issue for both types of goods in the following.

What do we mean by public provision? The goods are publicly supplied if the state buys the goods from the agents and distributes them according to a (normative) principle (in this case according to the maximization of the utilitarian sum of utilities). In order to finance the expenditures the government has to levy taxes. Please note that in our model of symmetric information the setup of the model provides maximum incentives for public provision as long as we stick to the assumption that they are financed by the use of the same tax base that is used for the financing of public enforcement. The reason for this is that a tax on labor creates no distortion in this economy.

**Private goods.** Assume that in addition to the enforcement tax  $T$  the state levies a tax  $T'$  in order to finance the provision of private goods produced by the individuals. The state can easily solve the maximization problems of the individuals, and hence distribute the goods according to the optimal rule. However, such a policy does not imply that the agents stop or even reduce their appropriative and defensive activities: given the distribution of private goods by the state both agents face the following tradeoff (assume that  $g(l_i) = l_i$  for convenience): for the appropriative activity an agent receives a marginal return that is equal to the marginal utility of the good,  $\lambda_i$  times the change in the fraction of the good he receives,  $\partial\pi_i/\partial a_i$ . The marginal costs are equal to the marginal utility of leisure,  $\partial u/\partial f_i$ . At the optimum the marginal utility of consumption is equal to the marginal utility of leisure. Hence, starting from a situation of optimal public provision an agent will start ap-  

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non-linear resource costs.

appropriating if  $\partial\pi_i/\partial a_i(0, 0, \theta) > 1$ : the public provision of private goods does not necessarily eliminate appropriative behavior by the agents, and therefore private enforcement activities. The reason for this lies in the fact that private goods are rival in consumption: it may pay for a single agent to appropriate even though the socially optimal allocation is already reached. For the case of the Tullock function,  $\partial\pi_i/\partial a_j(0, 0, \theta) \rightarrow \infty$ , which implies that the public provision does not eliminate the incentive to appropriate and defend.

**Public goods.** The conclusion changes if we analyze the case of public goods. Assume as before that in addition to the enforcement tax  $T$  the state levies a tax  $T'$  in order to finance the provision of public goods produced by the individuals. If the state pays each individual a subsidy  $p_i^P$  as derived in (27) for each unit of public good produced, every individual has an incentive to produce the efficient quantity of this good if he takes this price exogenous. This would eliminate the strategic effect of production resulting from the influence on equilibrium prices. However, if it is realistic to assume that the producer of the public good anticipates price effects in a decentralized solution, there is no reason to assume that he would not anticipate them in a centralized solution that is also characterized by a bilateral monopoly. Deviations from the the first-best solution resulting from strategic quantity choices therefore are not an inherent phenomenon of decentralization.

The incentives to engage in appropriative and therefore defensive activities vanish because each agent has access to the total amount of the good. Hence, the state can implement the efficient allocation of the public good. For the example of the last section this allocation can be easily calculated as  $c_i = f_i = 1/4$  and  $c_i^P = 1/2$ . The associated utility is equal to  $u(1/4, 1/2, 1/2, 1/4) = 1/64$ . The sum of utilities,  $1/32$ , is equal to the case where public enforcement incurs no cost,  $z(\theta) = 0$ . This observation demonstrates the general insight by Coase (1960): *in the absence of transaction costs, the private provision of goods is equivalent to the public provision of goods. The example shows that this is also true if the goods are non-rival.*

The result that the state can implement the first-best efficient allocation of public goods despite the fact that individuals can in principle engage in appropriative and defensive activities is important and should further be elaborated. The reason for this property lies exactly in the non-rival nature of public goods. If both individuals have access to the total amount of the public goods there is no longer a reason to engage in appropriative activities. As we have seen above, this property is different

from the case of private goods where the incentive to appropriate may still exist. Hence, *an argument for the public supply of goods is that public supply eliminates the incentive to engage in appropriative and defensive activities.* This holds even true if the costs of public enforcement are arbitrarily low but positive.

To illustrate how a theory of private and public supply can be operationalized in this model we analyze the special case where the resource costs of public enforcement are fixed,  $z(\theta) = z$ , and the resource costs of creating public supply are fixed,  $C$ .<sup>17</sup>

It is straightforward to show that the indirect utility with private supply is

$$W_i = \frac{(1 - z)^4(1 + \theta)^6(2 + \theta)^2}{(4 + 3\theta(3 + \theta))^4},$$

and with public supply is

$$W_i = \frac{1}{64}(1 - C)^4.$$

A comparison of these functions yields the following results:

- If  $z < 0.61434$  public enforcement dominates private enforcement and vice versa.
- If  $C < 0.61434$  public supply dominates private enforcement and vice versa.
- If  $z < C$  public enforcement dominates public supply and vice versa.

All three institutional structures are optimal for different intervals of the parameter values. Figure 6 summarizes the relevant parameter constellations. It is optimal to decentralize and to rely on public enforcement in area  $H$ , it is optimal to decentralize and to rely on private enforcement in area  $I$ , and it is optimal to centralize in area  $J$ . It is easy to see that it is not only the cost of exclusion that is decisive for the optimal institutional structure. The restriction to fixed-costs implies that the u-shaped structure of  $W_i$  is preserved, which generates the clear-cut results. A more complex optimal institutional structure can be deduced for more complex cost functions where both, private and public enforcement can coincide for intervals of parameter values. The important observation is that our model allows to operationalize the critical resource costs which define the borderline between private and public supply and the optimal mix of private and public enforcement. In addition it

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<sup>17</sup>Remember that  $C$  are not the resource costs of the production of goods but the additional resource costs resulting from a centralized agency that supplies them

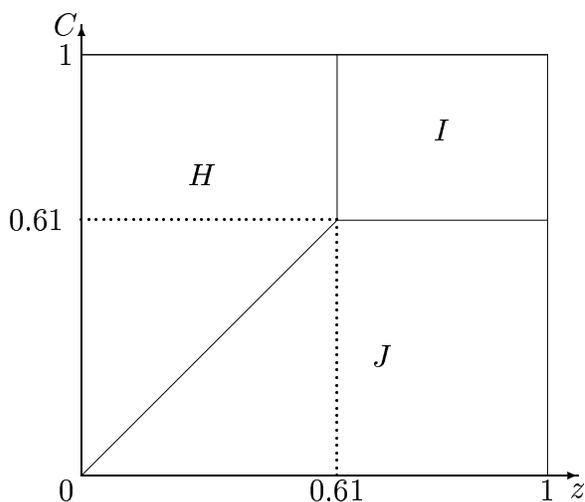


Figure 6: Optimal institutional structure in the fixed-cost model.

allows to evaluate the impact of technical innovations that influence the enforcement technologies on the optimal institutional structure. Perhaps most importantly it can be concluded that *even with prohibitive costs of public enforcement a decentralized supply of public goods may dominate the public supply if the costs of centralization are large but still not prohibitive*. This result is in stark contrast to the conventional wisdom on public goods, and we believe that it can be empirically supported. The music industry is a good example for such a good and we will come back to this example in the conclusions.

As we have already said, the setup of the model provides a maximum bias in favor of public supply. Hence we will conclude the section with a short discussion about the types of distortions can we expect to occur if public goods are supplied by the state. In our model there is no inefficiency due to asymmetric information and taxes induce no distortion in the economy. In how far must our argument be qualified if we take into consideration these facts? Asymmetric information is also relevant if the public goods are privately provided because the supplier has to know the utility function of the buyer in order to calculate the right price. However, the state has to know both individuals' utility functions. Does this make a difference? It is a straightforward application of the result by Crampton, Gibbons, and Klemperer (1987) that public property might *reduce* the incentive problem due to asymmetric information because of a change in the participation constraints (compared to a

situation of private property. This result can directly be applied to the case of public goods in a two-agent economy. However, as Rob (1982) has shown for economies with more than two agents the allocation problem for private and public goods differ. Hence, the evidence is mixed. The introduction of distortionary taxes does not lead to any clear-cut results as well, because it is a-priori unclear whether the total amount of taxes needed for the optimal degree of public enforcement exceeds the total amount of taxes needed for the public provision of goods.

## 4.2 Regulating the market: banning private enforcement?

A lot of the production possibilities of the economy are wasted because of the contest nature of private enforcement. One alternative policy measure could therefore be to forbid private enforcement. Forbidding private enforcement is different from forbidding crime because in a number of cases the measures undertaken to defend are easier to observe than the appropriative measures. The state can for example levy a prohibitive tax on locks, alarm systems, and other means to protect against appropriation and do not accept law suits by property owners. Hence, for the sake of the argument assume that government monitoring of compliance with this policy incurs no costs. As a result,  $d_i = 0$  and  $a_i = \epsilon$ ,  $\epsilon > 0$ ,  $\epsilon \rightarrow 0$ , implying  $\pi_i = 0$  for all  $\theta > 0$ . Hence, an immediate consequence of this policy is to make resources available for productive use.

**Private goods:** For the case of private goods the available time would exclusively be used for leisure as can be seen from (3):

$$\frac{\partial v_i}{\partial l_i} = -\frac{\partial u}{\partial f_i} + \lambda_i \pi_i \frac{\partial g}{\partial l_i} = -\frac{\partial u}{\partial f_i} < 0.$$

With strictly quasi-concave utility functions such an allocation can never dominate the one resulting with private enforcement. A policy of prohibiting private enforcement has the effect that everything that is produced will be stolen, hence, nothing will be produced.

**Public goods:** For the case of public goods the available time would be used for leisure, the production of the private, and the production of the public good. The condition for the individually optimal supply of the public good, (33), is

$$\frac{\partial u_1 / \partial c_i^P}{\partial u_1 / \partial c_1} = \frac{\partial g}{\partial l_i}, \quad i = 1, 2,$$

each agent produces public goods as to equalize his marginal rate of substitution with his marginal rate of transformation. Interestingly this rule parallels the one resulting from the classical analysis of Cournot-Nash behavior for the case of voluntary joint contributions to a single public good (Cornes and Sandler 1996). In the standard Cournot-Nash model every agent takes as given the amount of the public good provided by the other agent when deciding how much to contribute. Hence, voluntary contribution implicitly assumes that no other contractual arrangements concerning the supply of public goods can be made. This implies that each agent can only take into consideration the effect of an increase in the public good on his utility. In this sense voluntary contribution is the consequence of non-enforced (neither publicly nor privately) property rights, as our analysis shows.

Can such an allocation dominate the one with optimal private and public enforcement? No general answer can be given to this question. For the case of Cobb-Douglas utility functions analyzed in the example, the equilibrium allocation with banned private enforcement can easily be calculated. First we note that  $d_i = 0$  and  $a_i > 0$ ,  $a_i \rightarrow 0$ ,  $T = 0$  in equilibrium. If there is complete free riding, no trade can take place at stage 2, hence,  $c_i = y_i$ . Every agent treats the public good as if it were a private good. This implies that he divides his time budget equally among private and public goods and leisure,  $c_i = c_i^P = f_i = 1/3$ . The resulting level of utility is therefore equal to  $v_i(1/3, 1/3, 1/3, 1/3) = 1/81$ , which implies that the sum of utilities is equal to  $2/81$ . Assume on the contrary that for the case of optimal enforcement policies no public enforcement turns out to be optimal,  $\theta = 1$ . In this case, (55) is equal to  $2W(1) = 9/512$ , hence  $2v_i(1/3, 1/3, 1/3, 1/3) > 2W(1)$ : in all cases where the enforcement technology is too expensive for any degree of public enforcement it is better to ban private enforcement. This result, however, is strongly driven by our restriction to two agents. With prohibited private enforcement the deviation from the optimal allocation will increase in the number of agents.

Figure 7 shows examples for enforcement technologies where either ‘enforcement’ or ‘banning’ turns out to be optimal. The uppermost graph represents the case of costless public enforcement. The straight horizontal line represents the policy of banning private enforcement, whereas the other graph represent the situation  $z(\theta) = 0.1 - 0.01\theta$ .

The rivalry in consumption is responsible for the sharp difference between private and public goods. The total banning of enforcement destroys incentives for

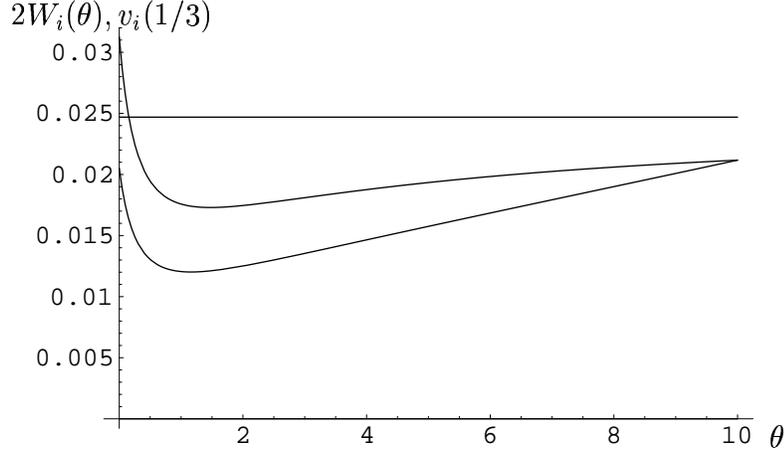


Figure 7: Banning enforcement?

production for the case of private goods, whereas these incentives are only diluted for the case of public goods. Hence, the implied reduction in costs can never over-compensate the loss in production for the case of private goods. For the case of public goods, this cost-reduction effect can over-compensate the negative effect of diluted incentives depending on the costs of enforcement.

## 5 Conclusions

The analysis has shown that the introduction of enforcement costs for property rights implies a number of changes compared to the standard results of the literature on private as well as public goods. First, depending on the costs of public enforcement the marginal conditions that characterize the relevant optimum may change from the first-best conditions. Second, depending on the costs of public enforcement there exist cases where it is optimal to rely solely on private enforcement, and there are cases where a mix between private and public enforcement is optimal.

The fact that public goods are non-rival in consumption implies an a-priori advantage of public supply for these goods even if the enforcement costs are arbitrarily low. The reason for this property is that public supply eliminates incentives to engage in appropriative activities. This feature distinguishes public goods from private

goods where even with public supply incentives for appropriation may exist.

By a similar line of reasoning it may be welfare improving to ban private enforcement activities for public goods, whereas such a strategy is never optimal for private goods. The implied dilution of incentives to produce may be less severe than the costs necessary to prevent appropriation.

The approach also allows us to close a gap in the literature between the concepts of public goods and increasing returns to scale. It has long been recognized (see for example Hillman, 2002) that public goods can be seen as a special case of increasing returns to scale with marginal costs of an additional user are equal to zero. This similarity allows to derive a number of consequences for the regulation of markets for goods with this type of increasing returns to scale. An example that can be analyzed using this model is the music industry. Due to the internet the supply of music has basically the character of a public good because the marginal costs of an additional individual listening to some piece of music are more or less equal to zero. However, this public good is privately supplied. The public enforcement of property rights for music is extremely costly because violations of property rights are almost impossible to detect in the internet. Hence, producers rely almost exclusively on private enforcement.

Given these facts, what is the best way to regulate this market? First, the exclusive public supply of music will most probably be inefficient, albeit for reasons beyond our model. Our model assumes that the identity of the good to be supplied is common knowledge. This is only true for music that already exists. However, music is a good whose half-life period is extremely short and it is almost impossible to imagine that a public bureaucracy would be able to correctly anticipate the future taste of the consumers. However, the public provision of the public good can play a major role in the form of publicly financed radio stations. In addition to this, is there an argument in favor of private enforcement? If music were a private good the answer would be a clear 'yes.' However, banning can have a positive welfare effect if the share of resources invested in the private-enforcement contest is relatively large. For the case of Cobb-Douglas utility and Tullock contest functions a ban of private enforcement would most probably improve welfare, especially if public enforcement is costly.

An interesting extension of the model would be to allow for more individuals and more goods that may differ with respect to the exclusion technology. The qual-

itative properties with respect to optimal enforcement policies and optimal degree of centralization remain the same for every good, however, goods with different exclusion technologies can add ‘overlapping’ supply patterns to the model that may help to better understand the spacial structure of private and – especially – public goods supply. Musgrave, for example, makes the conjecture that for such an overlapping pattern of supply “... [the] detailed mapping would thus call for a maze of service units, creating excessive costs of administration...” (1998). In reality we observe such a maze of service units, some individuals are members of a TV provider located somewhere, others are members of the opera house in city  $A$  in country  $B$ , whereas they join the military protection of country  $C$  and the public infrastructure of city  $D$ . Our model can explain why such a maze can in fact exist and be efficient. The explanatory variable of the structure of such a maze are the costs of the exclusion mechanism in comparison to the costs of centralization.

## 6 References

- Alchian, A.A., Crawford, R., Klein, B., 1978. Vertical Integration, Appropriable Rents, and the Competitive Contracting Process. *Journal of Law and Economics* 21. 297-326.
- Alesina, A., Spolaore, E., 1997. International Conflict, Defense Spending and the Size of Countries. *Quarterly Journal of Economics*
- Bush, W.C., Mayer, L.S., 1974. Some Implications of Anarchy for the Distribution of Property. *Journal of Economic Theory* 8 (4), 401-412.
- Clotfelter, C.T., 1977. Public Services, Private Substitutes, and the Demand for Protection against Crime. *American Economic Review* 67, 867-877.
- Clotfelter, C.T., 1978. Private Security and the Public Safety. *Journal of Urban Economics* 5, 388-402.
- Coase, R., 1960. The Problem of Social Cost. *Journal of Law and Economics* 3, 1-44.
- Cornes, R., Sandler, T., 1996. *The Theory of Externalities, Public Goods and Club Goods*. Cambridge University Press,

- Crampton, P., Gibbons, R., Klemperer, P.D., 1987. Dissolving a Partnership Efficiently. *Econometrica* 55, 612-632.
- de Meta, D., Gould, J.R., 1992. The Social Efficiency of Private Decisions to Enforce Property Rights. *Journal of Political Economy* 100 (3), 561-580.
- Demsetz, H., 1977. Information and Efficiency. Another Viewpoint. *Journal of Law and Economics* 12, 1-22.
- Fraser, C.D., 1996. On the Provision of Excludable Public Goods. *Journal of Public Economics* 60, 111-130.
- Grossman, H., 2001. The Creation of Effective Property Rights. *American Economic Review* 91, 347-352.
- Grossman, H., Kim, M., 1995. Swords or Plowshares? A Theory of the Security of Claims to Property. *Journal of Political Economy* 103 (6), 1275-1288.
- Hardin, G., 1968. The Tragedy of the Commons. *Science* 162, 1243-1248.
- Hart, O.D., Shleifer, A., Vishny, R.W. 1997. The Proper Scope of Government: Theory and Application to Prisons, *Quarterly Journal of Economics* 112, 1127-1161.
- Helsley, R.W., Strange, W.C., 1994. Exclusion and the Private Enforcement of Property Rights. *Journal of Public Economics* 53, 291-308.
- Hirshleifer, J., 1991. The Paradox of Power. *Economics and Politics* 3, 177-200.
- Hirshleifer, J., 1995. Anarchy and its Breakdown. *Journal of Political Economy* 103 (1), 26-52.
- Janeba, E., Swope, K.J., 2001. Taxes or Fees? The Political Economy of Providing Excludable Public Goods. Mimeo, University of Colorado.
- Musgrave, R.G., 1998. Approaches to a Fiscal Theory of Political Federalism. In: W. Oates (Ed.): *The Economics of Fiscal Federalism and Local Finance*, Elgar: Cheltenham and Northampton, 187-212.
- Polinsky, A.M., 1980. Private versus Public Enforcement of Fines. *Journal of Legal Studies* 9, 105-127.

- Polinsky, A.M., Shavell, S., 2000. The Economic theory of Public Enforcement of Law. *Journal of Economic Literature* 38, 45-76.
- Rob, R., 1982. Asymptotic Efficiency of the Demand-Revealing Mechanism. *Journal of Economic Theory* 28, 207-220.
- Samuelson, P.A., 1954. The Pure Theory of Public Expenditure. *Review of Economics and Statistics* 36, 387-389.
- Shavell, S., 1991. Individual Precautions to Prevent Theft: Private versus Socially Optimal Behavior. *International Review of Law and Economics* 11, 123-132.
- Skaperdas, S., 1992. Cooperation, Conflict, and Power in the Absence of Property Rights. *American Economic Review* 82 (4), 720-739.
- Stiglitz, J.E., 1977. Monopoly, Nonlinear Pricing, and Imperfect Information: The Insurance Market. *Review of Economic Studies* 44, 407-430.
- Williamson, O., 1975. *Markets and Hierarchies. Analysis of Antitrust Implications.* New York.