

Schneider, Maik; Winkler, Ralph

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## Growth and Welfare under Endogenous Lifetime

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# Growth and Welfare under Endogenous Lifetime\*

Maik T. Schneider<sup>†</sup> and Ralph Winkler<sup>‡</sup>

<sup>†</sup> CER-ETH – Center of Economic Research at ETH Zurich  
ZUE D15, CH-8092 Zurich, Switzerland; schneider@mip.mtec.ethz.ch

<sup>‡</sup> Department of Economics and  
Oeschger Centre for Climate Change Research, University of Bern  
Schanzeneckstrasse 1, CH-3001 Bern, Switzerland; mail@ralph-winkler.de  
phone: +41 31 631 4508, fax: +41 31 631 3783 (corresponding author)

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## Abstract:

Welfare aspects of longevity increases are often discussed neglecting the complex relationship between longevity and economic growth, which is the focal point of another literature. Combining both perspectives, we develop an endogenous growth OLG-framework to investigate how longevity affects economic growth and welfare. In our model, life expectancy is determined endogenously by individuals' investments in healthcare. In our benchmark specification growth effects are small, but direct welfare gains from longer lifetimes can be substantial. In the generalized model specification the growth effect dominates and may even lead to overall welfare losses from longevity increases. We interpret our results with respect to real world data and discuss the importance of assumptions about the engine of growth as well as the externalities associated with healthcare investments.

**Keywords:** economic growth, endogenous longevity, healthcare expenditures, healthcare technology, welfare analysis

**JEL-Classification:** O40, I10, J10

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## 1 Introduction

Most countries in the world have witnessed a substantial increase in human longevity over the last decades. Higher expected lifetime has been accompanied, at least in the developed world, by a significant rise of healthcare expenditures. Between 1960 and 2000, for example, life expectancy in the U.S. rose from 69.8 to 77.1 years, while health expenditures, as a share of GDP, increased from 5.2% to 13.4% (according to OECD data).

How does this increased longevity translate into welfare gains? A recent empirical literature argues that higher expected lifetimes have increased welfare substantially. For example, according to ? welfare improvements induced by mortality reductions between 1960 and 2000 in North America amount to about 12% of overall welfare gains over this period. Even when accounting for the corresponding additional health expenditures, ? provide evidence that these welfare gains remain considerable. While arguing that welfare increases of longer expected lifetimes are not well captured by standard welfare measures such as GDP per capita, these studies are silent about the possible repercussions of increasing longevity on the economy's growth rate, an issue that is at the heart of a recent theoretical literature finding that the growth effect of increased longevity can be either positive or negative depending on the particular set-up. Even though the dynamic relationship between longevity and economic growth should affect aggregate welfare, there has been little connection between the two literatures.

In this paper, we study the relationship between endogenous investments in longevity, economic growth and welfare. For this purpose, we develop an endogenous growth model in which longevity is endogenously determined by the households' demand for healthcare services,<sup>1</sup> and analyze how economic growth and welfare reacts to endogenous changes in life expectancy induced by improvements in the healthcare technology. We introduce a theoretical model that is, on the one hand, rich enough to capture endogenous growth, endogenous lifetime and the costs of healthcare, and, on the other hand, is parsimonious enough to allow for analytical solutions both of the individual household's problem and the aggregate economy. In particular, we employ the simplest conceivable model bringing together all essential ingredients by combining the household side of perpetual youth models in the tradition of ? with the production side of an endogenous growth model in the style of ? amended by a healthcare sector. To capture the welfare effects of increased longevity, we consider two different types of healthcare improvements. The first type decreases the baseline mortality, which is independent of individual investments in healthcare. One could think of improvements in the sanitary infrastructure or behavioral changes such as reduced smoking. The second type increases the marginal productivity of

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<sup>1</sup> Even though empirical evidence on the relationship between health expenditures and life expectancy is ambiguous, there is no doubt that expected lifetime is not given per se but can be influenced by investments in healthcare, such as improving sanitation, buying medication and inoculation, consulting a physician, etc. (????).

healthcare expenditures. Examples include better medication or therapeutic breakthroughs, such as new diagnostic tools or surgeries.<sup>2</sup>

In our benchmark model specification, we find that improvements in the healthcare technology always lead to higher steady state growth rates. Intuitively, longevity influences the age structure of the economy. Higher longevity increases the relative share of old (“rich”) to young (“poor”) households and leads *ceteris paribus* to higher capital per capita accumulation. Our numerical calculations suggest that the effect on the growth rate is rather small. This lends support to the empirical literature on the welfare effects of longevity which abstracts from growth effects of increased longevity. However, for the welfare gains of increased longevity the type of healthcare improvements matters. For a given increase in longevity, welfare improvements are substantially higher if increased longevity is induced by improvements of the first type (baseline mortality decreases) compared to improvements of the second type (higher efficiency of healthcare treatment). Using the data set of ? amended by data on healthcare expenditures, we obtain higher welfare improvements than ? in less developed regions, where healthcare improvements were predominantly of the first type, and lower welfare improvements in highly developed regions, where longevity increases were mainly triggered by healthcare improvements of the second type.

Our results for the growth effects of healthcare investments are very sensitive to the spillover specification in the production sector, which drives endogenous growth in our model. Generalizing the specification of the spillover effect, we show that longevity increases may lead to negative growth effects when technological improvements in the health sector trigger higher healthcare expenditures. Even small changes of the spillover specification yield negative growth effects which are consistent with real world data but involve welfare losses outweighing the welfare increase induced by higher longevity, and thus entail a negative overall welfare effect. By contrast, increases in expected lifetime enjoyed without healthcare expenditures always induce positive growth and welfare effects independent of the spillover specification. This further highlights the importance of the source of longevity increases. Of course, which spillover specification best describes reality is an empirical question that is beyond the scope of this paper. However, our results emphasize the complex relationship between longevity and aggregate welfare and the need for more empirical work in this field.

It is obvious that equilibrium health expenditures cannot be efficient when endogenously determined increases in expected lifetimes result in long-run welfare losses. In fact, we identify two externalities associated with healthcare investments in our model. The first, which is well

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<sup>2</sup> Our model emphasizes that increases in healthcare expenditures and longevity are mainly driven by the availability of better healthcare technologies, a view supported, for example, by ?, ?, ? and ?. In particular, we neglect wealth effects as driving factors for increased healthcare expenditures as discussed by ?. Abstracting from wealth effects, we focus on homothetic preferences allowing for a balanced growth path. Such preferences are a common feature in the literature on growth and longevity.

known, stems from the price taking behavior of households with respect to annuities and leads to over-investment in healthcare. The second externality is a consequence of our growth model. In our benchmark model specification, households under-invest in healthcare, as they do not take into account that increased longevity on the aggregate level induces a positive effect on the economy's growth rate. We show that healthcare investments in the market equilibrium cannot be inefficiently high and give a condition under which they are inefficiently low. However, when the growth effect of increased healthcare investments turns negative in the generalized spillover specification, households over-invest rather than under-invest in healthcare services. Consequently, we derive conditions for which healthcare investments in the market equilibrium are inefficiently high. Thus, the nature of the growth effect of longevity also has direct implications for healthcare policy.

The paper is organized as follows. The next section relates our paper to the literature. In Section 3, we introduce the model and provide a detailed discussion of the household's maximization problem with respect to healthcare. Section 4 determines the market equilibrium and derives the dynamics of the aggregate economy. We investigate the effects of improvements in the healthcare technology on the economy's steady state dynamics and provide numerical examples that illustrate the growth and welfare effects in Section 5. In Section 6, we identify the externalities of the market equilibrium. Section 7 extends the analysis to different spillover specifications. Finally, Section 8 concludes.

## 2 Related Literature

As outlined above, our paper combines two strands of the literature. First, our model relates to the literature on the welfare consequences of increased longevity, for example, ? and ?. These papers argue that longevity is an important component of aggregate welfare, and neglecting it, *ceteris paribus*, underestimates changes in welfare if longevity has increased. In contrast to our model, these papers abstract from the relationship between longevity and (consumption) growth, and also do not distinguish between different channels of longevity increases. We show that (i) the welfare effects of longevity depend on the channel of improvement in the healthcare technology, and that (ii) growth effects of longevity hinge on both the spillover specification of the endogenous growth engine and the type of the healthcare technology improvements. In our benchmark model, growth effects of increased longevity are positive but very small. This provides a justification for neglecting growth effects of increasing lifetime, at least for studies capturing relatively short time horizons of a few decades. However, this may not hold anymore if longevity results in negative growth effects, the corresponding welfare effects of which may even outweigh the direct welfare increase of longer lifetimes, as suggested by our generalized spillover set-up.

Second, our paper is related to the literature exploring the relationship between growth and life expectancy.<sup>3</sup> By now, there exists a considerable body of literature that examines *exogenous* variations in expected lifetime on economic development. A longer lifetime increases the returns from (human) capital accumulation, thereby stimulating savings and economic growth (??). However, if one considers a vintage (human) capital structure, in which later vintages are more productive, there is an additional effect working in the opposite direction. An increase in longevity increases the average age of agents and, consequently, the average age of (human) capital. As a consequence, the relationship between longevity and growth takes an inverted U-shape, i.e. economic growth increases with longevity for small values of expected lifetime and decreases for large ones (??). The second channel is strengthened if the retirement age is endogenous (?). As all these models consider longevity to be exogenous, they do not account for the potential costs associated with longer expected lifetimes. ?, ?, ? and ? analyze a neoclassical growth model with endogenous longevity, which is determined by either households' or government's investments in health. Although savings and healthcare expenditures compete for the same resources, they are complements in equilibrium. Thus, higher economic development is accompanied by longer average lifetime. Combining endogenous growth with endogenous longevity ? and ? find non-monotonic relationships between longevity and growth. In these papers longevity is endogenous but determined by an externality of aggregate variables. In contrast, we set up an endogenous growth model, in which each household's average life expectancy directly depends on the household's investments in healthcare.

### 3 The Model

The model comprises a continuum of households. Like in ?, households born at time  $s \in (-\infty, \infty)$  face a hazard rate  $p(s)$  of dying that is constant throughout the lifetime of each household. In our model, however, the hazard rate may vary among households of different cohorts, as it is determined by the level of medical treatment the household gets throughout its lifetime. At time of birth, households choose a level of medical treatment  $h(s)$ , which is fixed over the entire lifetime and determines the hazard rate via a healthcare technology  $H(h(s))$ .

As the hazard rate is constant over the entire lifetime, all households born at time  $s$  and still

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<sup>3</sup> More remotely our paper is also related to the literature on demographic transitions and the literature on the growth effects of epidemics such as AIDS. The former analyzes the relationship between fertility, mortality and growth. Longevity is either exogenous (????), endogenously determined via an externality of aggregate variables such as average income or human capital (?????) or endogenously determined by the healthcare investments of the parents (?). Within the latter, ? concludes that the AIDS epidemic in South Africa, despite being a humanitarian disaster, has rather positive effects on long-run growth. ? and ? are less optimistic and emphasize that epidemics may lead to poverty traps.

alive face an expected remaining lifetime  $T(s)$  at any time  $t \geq s$  given by

$$T(s) = \int_t^\infty (t' - t)p(s) \exp[-p(s)(t' - t)]dt' = \frac{1}{p(s)} . \quad (1)$$

Although the lifetime of each household is stochastic, we assume that the size of each cohort is large enough for cohort sizes to decline deterministically over time. At all times a new cohort is born. We abstract from fertility choices of households and assume that cohort size grows at the constant and exogenously given rate  $\nu$ .<sup>4</sup> Normalizing the cohort size at time  $t = 0$  to unity, we obtain for the size of the population at time  $t$

$$N(t) = \int_{-\infty}^t \exp[\nu s] \exp[-p(s)(t - s)]ds . \quad (2)$$

Households exhibit identical ex ante preferences and face equal hazard rates for the same levels of medical treatment. Households born at time  $s$  maximize expected discounted lifetime utility derived from consumption  $U$

$$U(s) \equiv \int_s^\infty V(c(t, s)) \exp [-(\rho + p(s))(t - s)] dt , \quad (3)$$

where  $V(c(t, s))$  denotes the instantaneous utility derived from consumption  $c(t, s)$  at time  $t$  of the household born at time  $s$ , and  $\rho$  is the constant rate of time preference. We impose standard curvature properties on the instantaneous utility function ( $V' > 0$  and  $V'' < 0$ ). ? showed that optimal investments in healthcare crucially depend on two characteristics of the instantaneous utility function: (i) the intertemporal elasticity of substitution and (ii) the difference in instantaneous utility between being alive and dead. As our definition of lifetime utility (3) normalizes instantaneous utility of being dead to zero, a utility representation with  $V(c) > 0$  for all  $c > 0$  avoids that households may wish to be dead rather than alive. As a consequence, we employ an instantaneous utility function with intertemporal substitution elasticity  $\sigma$  larger than one<sup>5</sup>

$$V(c(t, s)) \equiv \frac{c(t, s)^{1-\frac{1}{\sigma}}}{1 - \frac{1}{\sigma}} , \quad \sigma > 1 . \quad (4)$$

At any time alive, each household is endowed with one unit of labor that is supplied inelastically to the labor market at wage  $w(t)$ . In addition, households may save and borrow assets  $b(t, s)$

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<sup>4</sup> The parameter  $\nu$  can be mapped into the economy's fertility rate, which specifies how many children are born on average by each woman (or by our abstract genderless individual). The fertility rate is independent of the size of the actual population.

<sup>5</sup> ?, ? and ? use  $V(c(t, s)) = c(t, s)^{1-\frac{1}{\sigma}}/(1 - 1/\sigma) + \lambda$  with some positive constant  $\lambda$ . This allows either to employ intertemporal substitution elasticities of  $\sigma < 1$  (?) or to calibrate the model to different values of a statistical life without changing the intertemporal elasticity of substitution (??). We use the functional form (4) representing homothetic preferences which allow for a balanced growth path.

at the interest rate  $r(t)$ . Households are born without assets and may contract against the risk of leaving unanticipated bequests on a perfectly competitive life insurance market. Each unit of assets buys a life annuity paying the return  $a$ , as long as the household is alive. If insurance companies learn the probability of dying  $p(s)$  of each cohort at no costs, and if cohort sizes are large enough, so that insurance companies can offer risk free annuities, perfect competition among insurance companies leads to fair annuity payments  $a(t, s) = r(t) + p(s)$ . In line with ? and ?, among others, we assume that households take  $a(t, s)$  as given. Given that negative bequests are prohibited, households hold their entire wealth in fair annuities. Denoting the costs of healthcare by  $M(h(s))$ , the households' budget constraint reads

$$\dot{b}(t, s) = a(t, s)b(t, s) + w(t) - c(t, s) - M(h(s)) , \quad t \geq s , \quad (5)$$

with  $b(s, s) = 0$ .

The economy comprises two production sectors: the consumption-good and the healthcare sector. We assume that both sectors operate at perfectly competitive conditions. As all firms have access to the same constant returns to scale production technologies, we restrict the analysis to one representative firm in each sector.

### 3.1 Consumption-good production

The representative firm in the consumption-good sector produces a homogeneous consumption good via a Cobb-Douglas production technology  $Y(t) \equiv K(t)^\alpha (A(t)L^F(t))^{1-\alpha}$ , where  $\alpha \in (0, 1)$  and  $K(t)$  and  $L^F(t)$  denote the aggregate amount of capital and labor employed in consumption-good production, respectively.  $A(t)$  characterizes the technological level of the economy that is exogenous to the representative firm. We assume a “learning-by-investing” externality similar to ? but corrected for scale effects:  $A(t) \equiv K(t)/L^F(t)$ .<sup>6</sup> Capital depreciates at the constant rate  $\delta$ . Profit maximization of the representative firm yields factor prices equal to the marginal productivities

$$r(t) = \alpha - \delta , \quad (6a)$$

$$w(t) = (1 - \alpha) \frac{K(t)}{L^F(t)} . \quad (6b)$$

As the interest rate is constant due to the “learning-by-investing” externality, we introduce the notation  $r \equiv r(t) = \alpha - \delta$  and  $a(s) \equiv a(s, t) = r + p(s)$ .

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<sup>6</sup> ? assumes that  $A(t) \equiv K(t)$ . By specifying the learning externality to be proportional to capital per worker, we correct for scale effects. This specification was introduced by ?.



### 3.2 Healthcare sector

The representative firm in the healthcare sector provides medical treatment by employing labor. Without loss of generality, we assume that one unit of labor produces one unit of medical treatment. Given the level of medical treatment  $h(s)$  of all cohorts alive, the total amount of labor employed in the healthcare sector equals

$$L^H(t) = \int_{-\infty}^t h(s) \exp[\nu s] \exp[-p(s)(t-s)] ds . \quad (7)$$

In the labor market equilibrium, labor employed in the healthcare sector has to earn the same wage as labor employed in the consumption-good sector. As a consequence, the healthcare sector offers medical treatment at marginal costs  $w(t)$  and we obtain  $M(h(s)) = h(s)w(t)$  for the costs of medical treatment of a household born at time  $s$ .

The level of medical treatment  $h(s)$  determines the hazard rate  $p(s)$  via the healthcare technology  $H(h(s))$

$$p(s) = H(h(s)) \equiv p_{max} - \psi h(s) . \quad (8)$$

Without medical treatment ( $h = 0$ ) households face the hazard rate  $p(s) = p_{max}$  of dying. The hazard rate  $p(s)$  decreases with constant returns  $\psi$  in the level of medical treatment  $h(s)$ . The parameter  $\psi < p_{max}$  reflects the productivity of healthcare investments and may be interpreted as the quality level of the health system or the state of the art in medical treatment. It denotes the maximum amount by which one may reduce the hazard rate against  $p_{max}$  by spending all wage income on healthcare. While  $p_{max}$  reflects, for example, the sanitary infrastructure of the economy,  $\psi$  increases with the human capital of physicians, the efficiency of hospitals and the like.

The specification of the healthcare technology (8) implies that improvements in the healthcare technology may come in two qualitatively different ways. First, the maximal hazard rate  $p_{max}$  may decrease implying that all households, independently of their levels of healthcare spending, experience a lower hazard rate of dying. In fact, a decrease in  $p_{max}$  offers higher life expectancy for free (at least for the individual household). Historic examples in this respect could be new knowledge about germ theory leading to better hygienic standards and a change in personal behavior. We also interpret the introduction of most vaccines and drugs as a decrease in  $p_{max}$ , because these drugs are usually not very expensive. As an example, think of penicillin which led to substantial declines in mortality in the last century. Second, the state of the art in medical treatment  $\psi$  may increase implying that the same amount of healthcare spending reduces the hazard rate more than before. However, only households with positive healthcare spending benefit from the improved healthcare technology. One may think of improvements such as

magnetic resonance imaging, coronary heart bypass grafting, transplantation, and the like.<sup>7</sup> In Sections 5 and 7 we shall see that these two channels of healthcare technology improvements differently affect the economy's growth rate and the households' lifetime utility.

### 3.3 The individual household's problem

Inserting  $M(h(s)) = h(s)w(t)$  into the households' budget constraint (5) yields

$$\dot{b}(t, s) = a(s)b(t, s) + (1 - h(s))w(t) - c(t, s) , \quad t \geq s . \quad (9)$$

Thus, we can interpret the level of medical treatment  $h(s)$  as the fraction of labor income a household spends throughout its entire life for healthcare services. This implies  $h(s) \in [0, 1]$ , as households are born without assets and must not be indebted when dying.

Households maximize expected intertemporal utility (3) subject to conditions (9) and  $b(s, s) = 0$  by choosing an optimal level of medical treatment  $h(s)$  and an optimal consumption path  $c(t, s)$ . We solve this maximization problem in two steps. First, we determine the optimal consumption path for some given level of healthcare expenditures  $c^*(t, s, h)$ . Second, we obtain the optimal healthcare expenditures  $h^*(s)$  by inserting  $c^*(t, s, h)$  back into the expected intertemporal utility function (3).

For a given level of medical treatment  $h(s)$ , which implies a given hazard rate  $p(s)$  via the healthcare technology (8), the household's maximization problem yields the Euler equation

$$\frac{\dot{c}(t, s)}{c(t, s)} = \sigma[a(s) - \rho - p(s)] , \quad t \geq s . \quad (10)$$

For given  $h(s)$  the behavior of a household born at time  $s$  is characterized by the system of differential equations (9) and (10), the initial condition  $b(s, s) = 0$  and the transversality condition for the stock of assets  $\lim_{t \rightarrow \infty} b(t, s) \exp[-a(s)(t - s)] = 0$ . Under the assumptions that  $(1 - \sigma)a(s) + \sigma(\rho + p(s)) > 0$  and the long-run growth rate of wages  $w(t)$  is smaller than  $a(s)$ ,<sup>8</sup> we obtain for the optimal paths of consumption  $c^*(t, s, h)$  and assets  $b^*(t, s, h)$

$$c^*(t, s, h) = c^*(s, s, h) \exp[\sigma(a(s) - \rho - p(s))(t - s)] , \quad (11a)$$

$$b^*(t, s, h) = \frac{c^*(t, s, h)}{(1 - \sigma)a(s) + \sigma(\rho + p(s))} - (1 - h(s))W(t, s) , \quad (11b)$$

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<sup>7</sup> Although it makes perfect sense to conceptionally distinguish the two different channels of improvements in the healthcare technology, we want to emphasize that most real world improvements impact simultaneously on  $p_{max}$  and  $\psi$ . For example, knowledge about germ theory led to better hygienic standards not only in every day life, thereby decreasing  $p_{max}$ , but also in medical treatment, which increased  $\psi$ .

<sup>8</sup> Without these assumptions, the household's problem is not well defined. We shall see in Section 4 that the condition that the long-run growth rate of wages  $w(t)$  is smaller than  $a(s)$  always holds in the market equilibrium.

$$c^*(s, s, h) = [(1 - \sigma)a(s) + \sigma(\rho + p(s))] (1 - h(s))W(s, s) . \quad (11c)$$

where  $W(t, s) \equiv \int_t^\infty w(t') \exp[-a(s)(t' - t)] dt'$  denotes the expected net present value of the household's future labor income at time  $t$ .

Inserting the optimal consumption path  $c^*(t, s, h)$  into the household's lifetime utility (3) and recalling that  $p(s) = H(h(s))$ , the necessary condition for the optimal level of healthcare expenditures  $h^*(s)$  reads

$$\begin{aligned} & \int_s^\infty \frac{\partial V(c^*(t, s, h))}{\partial h(s)} \exp[-(\rho + p(s))(t - s)] dt \\ & \leq \int_s^\infty (t - s)H'(h(s))V(c^*(t, s, h)) \exp[-(\rho + p(s))(t - s)] dt . \end{aligned} \quad (12)$$

For an interior solution the equality sign holds, while the inequality sign applies in case of the corner solution  $h(s) = 0$ . The intuition for this condition is straightforward. In the optimum the decrease in expected lifetime utility from a marginal increase of healthcare expenditures due to decreasing lifetime consumption has to equal the increase in expected lifetime utility due to an increasing expected lifetime.<sup>9</sup> If the marginal benefits of healthcare investments do not outweigh the corresponding costs for any feasible level of healthcare expenditures, the optimal level of healthcare is given by the corner solution  $h(s) = 0$ .<sup>10</sup>

Inserting equations (4), (11a), and (11c) into equation (12), we obtain for the first-order condition

$$F(h(s)) \equiv \frac{\sigma\psi}{(\sigma - 1)[(1 - \sigma)a(s) + \sigma(\rho + p(s))]} - \frac{1}{1 - h(s)} \leq 0 . \quad (13)$$

This determines a unique optimal level of healthcare expenditures  $h^*(s)$ .

**Proposition 1 (Existence and uniqueness of household optimum)**

*There exists a unique optimal level of healthcare expenditures  $h^*(s)$  which is given by*

$$h^*(s) = \begin{cases} \max \left[ 0, \frac{\sigma\psi - (\sigma - 1)[(1 - \sigma)a(s) + \sigma(\rho + p_{max})]}{\sigma\psi(2 - \sigma)} \right] & , \quad \text{if } \sigma < 2 , \\ 0 & , \quad \text{if } \sigma \geq 2 . \end{cases}$$

In the proof of Proposition 1, provided in the appendix, we show that there is no interior

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<sup>9</sup> The dependence of this trade-off between the quality and quantity of life on the households' intertemporal elasticity of substitution  $\sigma$  is discussed in detail in the working paper version (?). There we emphasize that in our growth model the equilibrium healthcare expenditures follow a U-shaped curve in  $\sigma$  rather than monotonically declining one as typically found in models that neglect long-run growth (see e.g., ?????).

<sup>10</sup> In the first-order condition (12), the trade-off between investments in healthcare and average lifetime involves not only marginal but also absolute welfare comparisons. This illustrates the importance of the difference in instantaneous utility between being alive and dead, as mentioned earlier.

solution for  $\sigma \geq 2$ . The corner solution  $h(s) = 1$  cannot be optimal, as this implies no consumption and zero lifetime utility, while both are positive for any other value  $h(s) \in [0, 1)$ . As a consequence, the corner solution  $h(s) = 0$ , i.e. no healthcare expenditures, is the optimal solution. For  $\sigma < 2$ , there exists a local maximum, which is the optimal solution if it is in the feasible range  $[0, 1)$ . Otherwise again the corner solution  $h(s) = 0$  is optimal.

#### 4 Market Equilibrium and Aggregate Dynamics

To investigate the aggregate economy, we introduce aggregate household variables per capita derived by integrating over all living individuals and dividing by the population size of the economy

$$z(t) \equiv \frac{\int_{-\infty}^t z(t, s) \exp[\nu s] \exp[-p(t-s)] ds}{N(t)}, \quad (14)$$

where  $z(t)$  and  $z(t, s)$  denote aggregate per capita respectively individual household variables.<sup>11</sup> The economy consists of five markets: the labor market, the capital market, the consumption good market, the market for annuities and the market for healthcare. We assume the economy to be in market equilibrium at all times  $t$ . In particular, this implies that labor demand equals the population size,  $L^F(t) + L^H(t) = N(t)$  and capital per capita equals aggregate assets per capita,  $k(t) = b(t)$ .

As the interest rate  $r$  is constant, the equilibrium on the market for annuities,  $a(s) = r + p(s)$ , implies that the first-order condition (13) is identical for all households irrespective of their date of birth. Consequently, all households spend the same fraction  $h(s) = h$  of income for medical treatment implying that the hazard rate  $p(s) = p$  and the population growth rate  $\dot{N}(t)/N(t) = \nu$  are also constant. By setting  $a \equiv r + p$ , we obtain for the optimal healthcare expenditures in the market equilibrium:

$$h^* = \begin{cases} \max \left[ 0, \sigma - \frac{(\sigma-1)[(1-\sigma)r + \sigma\rho + p_{max}]}{\psi} \right] & , \quad \text{if } \sigma < 2, \\ 0 & , \quad \text{if } \sigma \geq 2. \end{cases} \quad (15)$$

Via the healthcare technology (8), the optimal level of healthcare expenditures  $h^*$  in the market equilibrium, which is completely determined by the set of exogenous parameters, maps into the optimal hazard rate  $p^*$ . Introducing the abbreviation<sup>12</sup>

$$x(p) \equiv (1-\sigma)a(s) + \sigma(\rho + p) = r + p - \sigma(r - \rho) > 0, \quad (16)$$

<sup>11</sup> We are aware of the slight abuse of notation, which we consider to be justified to keep notation at a minimum.

<sup>12</sup> Note that  $x(p) > 0$  is necessary for the household's maximization problem to be well defined.

which reflects the difference between the return on annuities  $r + p$  and the growth rate of the households' consumption  $\sigma(r - \rho)$ , we can characterize the aggregate dynamics dependent on the hazard rate  $p$ .

**Proposition 2 (Aggregate system dynamics)**

(i) *The dynamics of the aggregate economy is characterized by:*

$$\dot{c}(t) = \sigma(r - \rho)c(t) - (p + \nu)x(p)k(t) , \quad (17a)$$

$$\dot{k}(t) = (1 - \delta - \nu)k(t) - c(t) . \quad (17b)$$

(ii) *The dynamics of the aggregate economy is governed by a balanced growth path, i.e. aggregate consumption  $c(t)$  and aggregate capital  $k(t)$  are growing at the same constant rate  $g$ . The sign of the growth rate  $g$  is determined by*

$$g \begin{matrix} \geq \\ \leq \end{matrix} 0 \quad \Leftrightarrow \quad x(p)(p + \nu) \begin{matrix} \leq \\ \geq \end{matrix} \sigma(r - \rho)(1 - \delta - \nu) .$$

The proof is given in the appendix. Note that we neglect trivial steady states where  $c(t) = k(t) = 0$ , for all  $t$ . The proposition establishes that there is a unique balanced growth path for any given hazard rate  $p$ . As shown in the proof, the growth rate  $g$  equals

$$g(p) = \frac{1}{2} \left[ 1 + \sigma(r - \rho) - \delta - \nu - \sqrt{[1 - \delta - \nu - \sigma(r - \rho)]^2 + 4x(p)(p + \nu)} \right] . \quad (18)$$

Inserting the optimal hazard rate in the market equilibrium yields the growth rate in the market equilibrium  $g^* = g(p^*)$ . Thus, the aggregate system dynamics is fully characterized by the set  $(h^*, p^*, g^*)$ .<sup>13</sup>

## 5 Improvements in the Healthcare Technology

We are particularly interested in how the aggregate economy is affected by changes in the healthcare technology. As discussed earlier, the healthcare technology (8) exhibits two parameters influencing the hazard rate  $p$  of the households. A decline in the parameter  $p_{max}$  reduces the hazard rate that households face without investments in healthcare. A rise in the parameter  $\psi$  increases the reduction of the hazard rate that is purchased for any given healthcare investment  $h$ . As stated in the following proposition, an improvement of the healthcare technology either via a decrease in  $p_{max}$  or an increase in  $\psi$  leads to a higher rate of growth.

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<sup>13</sup> As shown in the proof of Proposition 2, the economy does not exhibit transitional dynamics for a given value of  $p$ . This reflects the typical dynamics in  $AK$ -models. By neglecting transitional dynamics, we implicitly assume that all individuals alive at an initial date have chosen health expenditures  $h^*$ . This is a realistic assumption as choosing  $h^*$  would have been optimal, given the exogenous parameters have not changed.

**Proposition 3 (Improvements in the healthcare technology)**

For interior levels of healthcare in the market equilibrium, the following conditions hold:

$$\begin{aligned}
 (i) \quad & \frac{dh^*}{dp_{max}} < 0, & \frac{dp^*}{dp_{max}} > 0, & \frac{dg^*}{dp_{max}} < 0, \\
 & \frac{dh^*}{d\psi} > 0, & \frac{dp^*}{d\psi} < 0, & \frac{dg^*}{d\psi} > 0, \\
 (ii) \quad & -\frac{dp^*}{dp_{max}} = \frac{dp^*}{d\psi} = -\sigma, & & -\frac{dh^*}{dp_{max}} < \frac{dh^*}{d\psi}.
 \end{aligned}$$

The proof of Proposition 3, given in the appendix, shows that better healthcare technology affects the equilibrium growth rate  $g^*$  in two ways. First, there is a direct effect. Ceteris paribus, a decrease in  $p_{max}$  or an increase in  $\psi$  lowers the hazard rate  $p^*$ , which implies an increase in the equilibrium growth rate  $g^*$ .<sup>14</sup> Second, there is an indirect effect. When the healthcare technology is more productive, households invest a higher share of income in healthcare implying an increase in  $h^*$ . This additional reduction in the hazard rate  $p^*$  further increases the equilibrium growth rate  $g^*$ .

An important insight is conveyed by the conditions in Proposition 3 (ii). In an interior equilibrium ( $h^* > 0$ ), the magnitude by which a marginal improvement in the healthcare technology increases expected lifetimes is determined by the households' intertemporal elasticity of substitution. The higher is the intertemporal elasticity of substitution, the larger is the effect of a marginal improvement in the healthcare technology on life expectancy. The magnitude of the effect is independent of whether the improvement results from a decrease in  $p_{max}$  or an increase in  $\psi$ . However, the channel by which the healthcare technology improves is crucial for the effect on equilibrium healthcare expenditures  $h^*$ : An increase in longevity via a marginal increase in  $\psi$  incurs higher costs in equilibrium relative to a marginal decrease in  $p_{max}$ .

**5.1 Magnitude of effects**

We provide a numerical example to get an idea of the magnitude of the comparative static effects induced by an improvement in the healthcare technology. Table 1 illustrates the case where  $p_{max} = 1/60$  (implying 60 years of expected lifetime without healthcare investments) and improvements in the productivity of healthcare expenditures  $\psi$  are such that – in equilibrium – life expectancy is increased in steps of 5 years. For each state of technology characterized by  $p_{max}$  and  $\psi$ , the table gives the corresponding equilibrium levels of healthcare spending  $h^*$ ,

<sup>14</sup> A detailed discussion on the effect of the economy's age structure, characterized by  $p$  and  $\nu$ , on the economy's growth rate can be found in the working paper version (?).

$p_{max}$	1/60						
$\psi$	0.97%	1.06%	1.13%	1.19%	1.25%	1.30%	1.34%
$h^*$	0	12.12%	21.05%	27.91%	33.33%	37.74%	41.38%
$T = 1/p^*$	60	65	70	75	80	85	90
$g^*$	2.179%	2.187%	2.194%	2.200%	2.204%	2.208%	2.212%
$\Delta g^*/g^*$	0%	0.383%	0.310%	0.255%	0.212%	0.179	0.153%
$\Delta U_T$	0%	0.750%	1.513%	1.933%	2.154%	2.255%	2.282%
$\Delta U_g$	0%	0.293%	0.248%	0.212%	0.184%	0.160%	0.141%
$\Delta U_p$	0%	0.457%	1.265%	1.721%	1.971%	2.094%	2.140%

**Table 1:** Equilibrium values for healthcare expenditures  $h^*$ , life expectancy  $T$ , growth rate  $g^*$  and relative welfare increase  $\Delta U_T$  (equivalent variation) for different parameters of  $\psi$ .

the expected lifetime  $T = 1/p^*$  and the corresponding growth rate of the economy  $g^*$ . For all parameters we choose plausible real world values.<sup>15</sup>

We observe that the growth rate  $g^*$  increases very little in response to a higher productivity of healthcare investments  $\psi$  accompanied by a higher expected lifetime  $T$ . The relative increase of the growth rate induced by an additional five years of expected lifetime is shown in the row labeled  $\Delta g^*/g^*$  and ranges between 0.383% for an increase of expected lifetime from 60 to 65 and 0.153% for an increase in expected lifetime from 85 to 90 years.

To elicit the welfare gains from increased lifetime, we compare the expected lifetime utilities of two persons: one person faces a healthcare technology leading to hazard rate  $p_1^*$  and an according growth rate  $g(p_1^*)$ , while the other person has access to a healthcare technology leading to a hazard rate  $p_2^*$  and a growth rate  $g(p_2^*)$ . The hazard rates are such that the latter household's life expectancy exceeds the one of the former household by five years. We employ a consumption-equivalent welfare measure and consider an equivalent variation. That is, we derive the relative change in consumption at any instant of time which the person living in the environment with hazard rate  $p_1^*$  must realize such that this person experiences the same expected lifetime utility as the person living in the environment with hazard rate  $p_2^*$ .<sup>16</sup>

The results are shown in the row labeled  $\Delta U_T$  of Table 1. The relative consumption increase ranges between 0.750% for an increase of expected lifetime from 60 to 65 and 2.282% for an increase in expected lifetime from 85 to 90 years. Our previous analysis showed that these utility gains of longevity originate from two different sources: (i) the direct utility of a longer lifetime

<sup>15</sup> For the intertemporal elasticity of substitution  $\sigma$  we follow ? who suggest a value of  $\varepsilon = (u'(c)c)/u(c) = 0.346$  which is also used by ?. For our instantaneous utility function (4) this translates into  $\sigma = 1.529$ , which we round to  $\sigma = 1.5$ . The remaining parameters are set to  $\alpha = 0.33$ ,  $r = 3.5\%$ ,  $\rho = 2\%$ ,  $\nu = 0$ . A sensitivity analysis (available upon request) shows that our results are qualitatively very robust to reasonable changes in the parameter values.

<sup>16</sup> Details can be found in Appendix A.8.

and (ii) the utility gain associated with a higher growth rate. Decomposing the relative utility gain of five year increases in longevity into these two sources, the row labeled  $\Delta U_g$  shows the relative utility increase stemming from the second source.<sup>17</sup> The residuum to the total relative utility gain from increased life expectancy is presented in the last row labeled  $\Delta U_p$ . It reflects the utility gain due to the increase in longevity compared to the economy in which households live five years less in expectation but face the same growth rate.<sup>18</sup> We observe that most of the utility gain is due to the direct effect of a longer expected lifetime and only a small fraction is attributable to the increase of the growth rate.

Overall, the utility gains from longevity due to a higher productivity of healthcare investments seem rather limited. This may change when we consider five-year increases of expected lifetime resulting from decreases in  $p_{max}$ , as – according to Proposition 3 – a marginal decrease in  $p_{max}$  induces a smaller rise in the healthcare expenditures compared to a marginal increase in  $\psi$ . In Table 2 we demonstrate the case where  $\psi$  is so low that no investments in healthcare are always optimal and, thus, increases of expected lifetime solely stem from the reduction of  $p_{max}$ . As a consequence, improvements in longevity come without direct costs to households. We concentrate on this polar case for two reasons. First, it highlights the difference between increases in longevity via the two different channels, as utility gains are highest if longevity increases come without healthcare costs. Second, this case is methodologically identical to models with exogenous changes in longevity and, hence, allows us to compare our results with this literature. Apart from  $\psi$  and  $p_{max}$  all parameter values are identical to the example shown in Table 1 (see footnote 15).

We observe the same growth rates as in the previous example, as in our model the growth rate only depends on the equilibrium life expectancy  $p^*$  and the other exogenous parameters. As a consequence, also the relative utility gains attributable to an increase in the growth rate are identical. However, the relative utility gains stemming from an increase in longevity are now substantially higher (more than fourfold), leading to an according increase of the total relative welfare gains.

Thus, our model indicates that improvements in healthcare technology may have a large impact on overall welfare. However, this impact is rather driven by increasing lifetime utility due to an increasing life expectancy than by the effects of longevity on economic growth. Therefore, our model supports the assumption (implicitly) made by several papers on the welfare aspects of longevity (see, for example, ? and ?) that the welfare gains from an increase in longevity are

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<sup>17</sup> More precisely,  $\Delta U_g$  measures the relative increase of consumption at each instant of time for which a hypothetical household with expected lifetime  $1/p_2^*$  and growth rate  $g(p_1^*)$  would be equally well off in terms of expected lifetime utility compared to a household facing life expectancy  $1/p_2^*$  and the growth rate  $g(p_2^*)$ . Again  $1/p_2^* - 1/p_1^* = 5$  years.

<sup>18</sup> The results change minimally when  $\Delta U_g$  is derived as the residuum and  $\Delta U_p$  is calculated as follows. Household 1 exhibits the lifetime  $1/p_1^*$  which is five years less than the life expectancy  $1/p_2^*$  of household 2. Both households experience the growth rate  $g(p_2^*)$ .



$p_{max}$	1/60	1/65	1/70	1/75	1/80	1/85	1/90
$\psi$	0.5%						
$h^*$	0	0	0	0	0	0	0
$T = 1/p^*$	60	65	70	75	80	85	90
$g^*$	2.179%	2.187%	2.194%	2.200%	2.204%	2.208%	2.212%
$\Delta g^*/g^*$	0%	0.383%	0.310%	0.255%	0.212%	0.179%	0.153%
$\Delta U_T$	0%	14.647%	12.997%	11.625%	10.469%	9.485%	8.639%
$\Delta U_g$	0%	0.293%	0.248%	0.212%	0.184%	0.160%	0.141%
$\Delta U_p$	0%	14.354%	12.749%	11.413%	10.286%	9.325%	8.498%

**Table 2:** Equilibrium values for healthcare expenditures  $h^*$ , life expectancy  $T$ , growth rate  $g^*$  and relative welfare increase  $\Delta U_T$  (equivalent variation) for different parameters of  $p_{max}$ .

not well reflected in the GDP-growth rate. However, our numerical example also shows that the magnitude of the welfare gains due to a higher expected lifetime strongly depends on the channel by which this increase in longevity is reached, and, in particular, by the accompanied rise in health expenditures. Welfare gains are considerably higher if increases in expected lifetime come as windfall gains from a decrease in the maximal hazard rate  $p_{max}$  together with no healthcare expenditures compared to improvements in the productivity of healthcare treatment  $\psi$ .

## 5.2 Welfare gains between 1960 and 2000

The previous discussion indicates that the welfare consequences of increased longevity depend substantially on the healthcare costs associated with it. Consequently, it is interesting to ask by how much welfare increased due to the the longevity increases over the last decades. For this purpose, we apply our model to the development of healthcare expenditures and average lifetime between the years 1960 and 2000 for seven world regions.<sup>19</sup> The results for all seven regions and details on the data for the numerical exercise are given in the appendix. In the following discussion, we concentrate on a developed (North America) and a developing (South Asia) region.

The levels of  $h$  given in Table 3 are the observed health expenditures per GDP multiplied by 3/4. This factor has been chosen for the following reason: On the one hand,  $h$  in our model is the share of labor income spent on healthcare rather than the share of total GDP. Assuming a labor share of 2/3, we divide data on health expenditures per GDP by this number. On the other hand, not all health expenditures are effective in prolonging life. Assuming that half of the expenditures affect the individuals' life expectancy leads to the factor of 3/4 given above.

<sup>19</sup> To be able to compare our results with previous studies, we use the original data set of ?, which we amend by data on healthcare expenditures.

Region	N. America		N. Am. (exo. p)		S. Asia	
Year	1960	2000	1960	2000	1960	2000
$T = 1/p^*$	69.9	77.3	69.9	77.3	44.0	62.7
$h^*$	3.9%	9.8%	0%	0%	0%	3.1%
$g^*$	2.44%	2.45%	2.44%	2.45%	2.38%	2.45%
$r$	3.69%		3.69%		3.74%	
$\nu$	1.14%		1.14%		2.22%	
$g_\emptyset$	2.44%		2.44%		2.42%	
$\hat{g}$	2.57%		2.65%		3.30%	
$\Delta U_T / \Delta U$	5.63%		8.50%		23.26%	
$\Delta U_g / \Delta U$	0.31%		0.28%		1.02%	
$\Delta U_p / \Delta U$	5.31%		8.22%		22.24%	

**Table 3:** Welfare gains for North America and South Asia from 1960 to 2000.

We have no data on health expenditures for South Asia in 1960, which we estimate to be (close to) zero.<sup>20</sup> In line with our theoretical model, we assume that increases in average lifetime stem from improvements of the healthcare technology. This implies that the growth rate increases accordingly. We assume that between 1960 and 1980 the respective world region experienced growth of income per capita consistent with the healthcare expenditure and average lifetime data of 1960, and between 1980 and 2000 income per capita grew consistently with 2000 data. Using the parameters  $\alpha = 0.33$ ,  $\sigma = 1.5$  and  $\rho = 0.02$  of the previous numerical example, the depreciation rate has been adjusted such that the simulated average growth rate in each region is identical to the observed average growth rate between 1960 and 2000.

Table 3 shows the results. In North America average life expectancy increased from 69.9 in 1960 to 77.3 years in 2000. At the same time the healthcare expenditures (in percentage of labor income) increased from 3.9% to 9.8%. In South Asia, life expectancy rose from 44.0 in 1960 to 62.7 years in 2000. Healthcare expenditures equalled 3.1% in 2000 and are estimated (close to) zero for 1960. Over this 40 year period the average annual growth rate of income per capita equalled 2.44% for North America and 2.42% for South Asia. Population grew by an average annual rate of 1.14% in North America and 2.22% in South Asia.

Now we compare the expected lifetime utility of a person born under the conditions of the year 2000 with that of a person born in 1960 in the same region. Let us denote the overall consumption equivalent welfare increase between 1960 and 2000 by  $\Delta U$ .<sup>21</sup> Of course, due to

<sup>20</sup> As further discussed in the appendix, our qualitative results are very robust with respect to reasonable variations of the level of health expenditures.

<sup>21</sup> According to our earlier definition in the previous section,  $\Delta U$  represents the factor by which consumption of a person born in 1960 must be adapted to be as well off as a person born under the conditions of the year 2000.

economic growth a person born in 2000 was better off than a person born in 1960 even without increases in life expectancy. The share of the consumption equivalent welfare gain that is attributable to the increase in average lifetime is given in the row  $\Delta U_T/\Delta U$ .<sup>22</sup> We find that a share of 5.63% of the total welfare gains in North America between 1960 and 2000 originated from the higher expected lifetimes. The last two rows recall our previous finding that almost all the utility gains originate directly from a longer expected lifetime ( $\Delta U_p/\Delta U$ ) rather than indirectly via an increased growth rate of GDP per capita ( $\Delta U_g/\Delta U$ ). The row labelled  $\hat{g}$  reports the average annual “full-income” growth rate of GDP per capita that would have been necessary between 1960 and 2000 to give a person born in 2000 the same utility without the increase in longevity. The table indicates that the growth rate had to be 2.57% instead of 2.44% to compensate for the utility gain of increased expected lifetime.

To get an idea of the role played by healthcare expenditures, we contrast our results with a thought experiment in which longevity increases come without costs. The results are reported in the second column headed “North America (exogenous p)”. Without healthcare expenditures the total relative welfare gains from higher longevity would amount to 8.50%, an increase of more than 50% relative to the real world scenario. This is also reflected in the higher full-income growth rate of 2.65% compared to 2.57% when healthcare costs are considered.

These results might indicate that relative welfare gains due to increased longevity are considerably higher in developing countries where relatively cheap measures (such as better sanitation, better access to standard vaccines, etc.) involve relatively high increases in  $p_{max}$  compared to developed countries where further increases in average lifetime are mainly due to improvements of expensive cutting-edge medical treatment. As an example for a developing region, we report in the third column of Table 3 the relative welfare gain of higher longevity between 1960 and 2000 in South Asia amounting to 23.26%. We obtain a full-income growth rate of 3.3% while on average GDP per capita only grew by 2.42% per annum in this region.

For the period between 1960 and 2000, ? report full-income growth rates for North America of 2.7% and for South Asia of 3.1%. Hence, the re-examination of their data set in light of our theoretical model supports the conclusion that when additionally considering longevity improvements the world’s welfare inequality has become smaller than solely GDP-based measures suggest. In fact, the full-income growth rates given in Table 3 suggest an even stronger convergence of welfare which is mainly due to the fact that we explicitly consider costs of improvements in life expectancy.<sup>23</sup> As a consequence, welfare gains for a given increase in longevity are higher if this increase is accompanied by no or small changes in healthcare expenditures.

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<sup>22</sup> Details on the numerical calculations can be found in the appendix.

<sup>23</sup> While ? did not include healthcare expenditures in their analysis, they conjectured that this would make a difference in the relative welfare gains from longevity increases enjoyed by developed and developing countries.

## 6 Inefficient Market Equilibria

An important policy question is whether the market equilibrium, as analyzed in the previous sections, is efficient. In fact, the model comprises three externalities that are not accounted for in the market equilibrium.

First, there is a “learning-by-investing” externality (?). At any time, firms take the technological level  $A(t)$  of the economy as given, neglecting the positive spillovers the employment of capital exerts on the economy’s production output  $Y(t)$  via an increase in the technological level. As is well known, this leads to an inefficiently low level of asset holdings that could be corrected, for example, by subsidizing household savings.

Second, there are two additional externalities associated with healthcare expenditures. To identify these, we take the total derivative of an individual’s lifetime utility with respect to healthcare expenditures and use (11a), (13), and  $w(t) = \exp[gt]$  to obtain

$$\frac{dU(s)}{dh} = \frac{c^*(s, s, h)^{1-\frac{1}{\sigma}}}{x(p)} \left[ F(h) + \frac{da}{dh} \left( \frac{1}{x(p)} - \frac{1}{y(p)} \right) + \frac{dg(p)}{dh} \left( \frac{1}{y(p)} + s \right) \right]. \quad (20)$$

The term  $y(p) \equiv r + p - g$  represents the difference between the return of annuities  $a$  and the economy’s growth rate  $g$ . The function  $F(h)$  constitutes the individual’s first-order condition with respect to healthcare expenditures (12). In the decentralized market equilibrium with an interior solution the first-order condition equals zero. The remaining two summands in brackets denote the impact of healthcare investments on the equilibrium return for annuities,  $a$ , and the economy’s growth rate,  $g$ , which the households do not take into account. The existence of these two externalities is independent of the positive spillovers from capital accumulation. The reason is that internalizing the “learning-by-investing” externality increases the effective interest rate. The additional two externalities, however, do not disappear for any level of the interest rate. They occur because households do not take into account that increased longevity reduces the equilibrium return of annuities (for a given interest rate) and increases the economy’s growth rate via the age structure of the economy. The externality of healthcare spending on the rate of return for annuities  $a$ , reflected by the expression  $da/dh (1/x(p) - 1/y(p))$ , is negative because  $da/dh = -\psi$  and  $y(p) - x(p) = \sigma(r - \rho) - g > 0$ .<sup>24</sup> Hence, households tend to over-invest in healthcare (?). However, the last expression in brackets in (20) representing the externality on the economy’s equilibrium growth rate  $g$  is positive implying under-investment in healthcare. Thus, a decrease in healthcare expenditures has a positive effect on the rate of return from annuities  $a$ , but a negative effect on  $g$  and vice versa. Whether the equilibrium level of healthcare expenditures is inefficient in the sense that there exists a balanced growth path

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<sup>24</sup> The term  $y(p) - x(p)$  represents the difference between the growth rate of individual household consumption and the growth rate of per capita consumption, which is positive due to equation (17a).

on which all households are better off by investing either more or less in healthcare depends on the magnitude of the welfare losses associated with the two opposed externalities.

All terms in brackets in equation (20) are independent of time except for the last term that reflects the increase of the wage-level of a generation born at time  $s$  due to a marginal change in the growth rate. This term increases with  $s$ ,<sup>25</sup> implying that the welfare loss due to a decrease in the steady-state growth rate is larger the later a household is born. Even small changes in the economy's growth rate  $g$  have huge welfare effects for generations living in the far distant future. As the term is linear in  $s$ , there exists some  $\bar{s}$  for any decrease in healthcare expenditures such that all generations born at  $s > \bar{s}$  are worse off, although early generations may benefit. Thus, it cannot occur that healthcare expenditures in the market equilibrium are inefficiently high in the sense that there exists a lower level of healthcare expenditures for which all households born at  $s \geq 0$  would be better off. Whether healthcare expenditures are inefficiently low in the market equilibrium depends on the relative strengths of the two externalities for the generation born at  $s = 0$ . Proposition 4 gives a condition for which under-investment in healthcare occurs.

**Proposition 4 (Inefficient levels of healthcare expenditures)**

*For interior levels of healthcare expenditures, households invest inefficiently low amounts in healthcare in the market equilibrium if  $dg(p)/dp < 1 - y(p)/x(p)$ .*

The proof is given in the appendix.

## 7 Spillover Effects

In our model we have specified the “learning-by-investing” spillovers as a function of the capital-stock per worker. Although this is a reasonable assumption, one may ask how our results would change with a different specification. To answer this question, we consider the most general spillover definition correcting for scale effects:

$$A(t) \equiv \frac{K(t)}{L^F(t) + (1 - \eta)L^H(t)}, \quad \eta \in [0, 1], \quad (21)$$

which captures all spillover magnitudes between the two polar cases  $\eta = 1$  representing our previous model and  $\eta = 0$  reflecting spillovers depending on capital per capita  $A(t) = k(t)$ . While condition (13) for the individual household's optimal choice of healthcare expenditures remains unchanged, as households take prices as given, the new specification of  $A(t)$  affects

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<sup>25</sup> Note that by setting  $w(t) = \exp[gt]$ , we normalize the wage rate at  $t = 0$  to unity. This implies that we compare the lifetime utility of all generations born at  $s \geq 0$ .

the interest and the wage rate

$$r(t) = \alpha \left[ \frac{L^F(t)}{L^F(t) + (1 - \eta)L^H(t)} \right]^{1-\alpha} - \delta, \quad (22a)$$

$$w(t) = (1 - \alpha) \frac{K(t)}{L^F(t)} \left[ \frac{L^F(t)}{L^F(t) + (1 - \eta)L^H(t)} \right]^{1-\alpha}. \quad (22b)$$

Given the equilibrium levels of  $h(s)$ ,  $L^H(t)$  can still be expressed by (7). For  $\eta < 1$  the interest rate declines with the level of healthcare expenditures, as  $L^H(t)$  increases and  $L^F(t) = N(t) - L^H(t)$  declines. On the one hand, a decline in labor employed in consumption-good production reduces the marginal productivity of capital. On the other hand, for  $\eta > 0$  the technological level  $A(t)$  increases, as the capital intensity in consumption-good production increases which amplifies the learning externality.<sup>26</sup> The first effect dominates the second implying a negative influence of health-care expenditures on the interest rate. Note that in the basic version of our model, which corresponds to  $\eta = 1$ , the two effects cancel out leading to the constant interest rate  $r(t) = \alpha - \delta$ . This interest rate also results in the general set-up if no healthcare expenditures are optimal in the market equilibrium for all generations, i.e.  $h(s) = 0$  for all  $s$ , implying  $L^F(t) = N(t)$  and  $L^H(t) = 0$  at all times  $t$ .

In the following, we restrict attention to the steady state market equilibrium in which the interest rate  $r(t) = r$  is constant. For this to hold, optimal healthcare expenditures  $h$  have to be constant implying that also the hazard rate  $p$  and the population growth rate  $\dot{N}(t)/N(t) = \nu$  are constant. We obtain for the interest rate and the wage rate in steady state<sup>27</sup>

$$\bar{r}(h) = \alpha \left[ \frac{1 - h}{1 - \eta h} \right]^{1-\alpha} - \delta, \quad (23a)$$

$$\bar{w}(h, t) = \frac{1 - \alpha}{1 - h} k(t) \left[ \frac{1 - h}{1 - \eta h} \right]^{1-\alpha}. \quad (23b)$$

Inserting  $\bar{a}(h, p) = \bar{r}(h) + p$  into (13) determines the healthcare expenditures in the steady state market equilibrium.

### Proposition 5 (Existence of steady state healthcare level)

*Given equation (21) for the technological level of the economy, there exists a steady state equilibrium level of healthcare expenditures,  $\bar{h}^*$ .*

The proof is given in the appendix. In contrast to the basic model, the aggregate dynamics in the steady state depends not only on the hazard rate  $p$  but also on the level of healthcare

<sup>26</sup> An alternative interpretation of the effect of healthcare expenditures on  $A(t)$  would be that better health increases the workers' productivity.

<sup>27</sup> We indicate steady state values in the general spillover setting by a bar and, where applicable, use  $h$  as an additional argument to highlight the difference to the basic version of the model.

expenditures  $h$ .

**Proposition 6 (Steady state aggregate dynamics)**

Given equation (21) for the technological level of the economy, the steady state dynamics of the aggregate economy

(i) is characterized by:

$$\begin{aligned}\dot{c}(t) &= \sigma [\bar{r}(h) - \rho] c(t) - \bar{x}(h, p)(p + \nu)k(t) , \\ \dot{k}(t) &= \left[ \frac{\bar{r}(h)}{\alpha} + \frac{1 - \alpha}{\alpha} \delta - \nu \right] k(t) - c(t) ,\end{aligned}$$

(ii) is governed by a balanced growth path given by

$$\begin{aligned}\bar{g}(h, p) &= \frac{1}{2} \left\{ \frac{\bar{r}(h)}{\alpha} + \frac{1 - \alpha}{\alpha} \delta - \nu + \sigma [\bar{r}(h) - \rho] \right\} \\ &\quad - \frac{1}{2} \sqrt{\left\{ \frac{\bar{r}(h)}{\alpha} + \frac{1 - \alpha}{\alpha} \delta - \nu - \sigma [\bar{r}(h) - \rho] \right\}^2 + 4\bar{x}(h, p)(p + \nu)} ,\end{aligned}\tag{25}$$

where  $\bar{x}(h, p) \equiv \bar{r}(h) + p - \sigma(\bar{r}(h) - \rho)$ .

The proof is given in the appendix. Equation (25) shows that not only the hazard rate  $p$  matters for the steady state growth rate, as it is the case in the basic model, but also the level of healthcare expenditures by which it is achieved. If there are no healthcare investments in steady state,  $\bar{h}^* = 0$ , we are back to the basic model implying  $\bar{g}(0, p) = \bar{g}(0, p_{max}) = g(p_{max})$ . By virtue of Proposition 3, for  $\bar{h}^* = 0$  a decreasing  $p = p_{max}$  leads to an increasing equilibrium growth rate  $\partial \bar{g}(0, p) / \partial p < 0$ . This does not necessarily hold if the reduction in  $p$  is accompanied by an increase in  $h$ . We obtain for the total derivative of  $\bar{g}(h, p)$  with respect to  $h$

$$\frac{d\bar{g}(h, p)}{dh} = \frac{\partial \bar{g}(h, p)}{\partial p} \frac{dp}{dh} + \frac{\partial \bar{g}(h, p)}{\partial \bar{r}(h)} \frac{d\bar{r}(h)}{dh} .\tag{26}$$

The first term is positive, as  $\partial \bar{g}(h, p) / \partial p < 0$  (see Proof of Proposition 3) and  $dp/dh < 0$ . The sign of the second term is ambiguous, as the sign of  $\partial \bar{g}(h, p) / \partial \bar{r}(h)$  is ambiguous and  $d\bar{r}(h)/dh < 0$ . In contrast to our basic model, it is now possible that the steady state growth rate declines in response to an increase in longevity. This happens, if  $\partial \bar{g}(h, p) / \partial \bar{r} > 0$  and sufficiently large so that the second term outweighs the first. Thus, the difference between improvements in longevity originating from increases in  $p_{max}$  accompanied by zero healthcare expenditures and those from an increase of  $\psi$  is even further pronounced compared to the basic model. In the basic model, both types of longevity increases had (small) positive growth effects but substantial welfare differences. With the alternative spillover mechanism and  $\eta < 1$  we may even experience negative growth effects in response to longevity increases.

Region	North America			
$T$	$T(1960) = 69.9 \quad T(2000) = 77.3$			
$h$	0	$h(1960) = 3.9\% \quad h(2000) = 9.8\%$		
$\eta$	$\in [0, 1]$	1	0.95	0.9
$r(1960)$	3.69%	3.69%	3.73%	3.76%
$r(2000)$	3.69%	3.69%	3.65%	3.62%
$g^*(1960)$	2.44%	2.44%	2.49%	2.55%
$g^*(2000)$	2.45%	2.45%	2.39%	2.34%
$\hat{g}$	2.65%	2.57%	2.54%	2.51%
$\Delta U_T / \Delta U$	8.50%	5.63%	2.07%	-1.70%
$\Delta U_g / \Delta U$	0.28%	0.31%	-3.29%	-7.17%
$\Delta U_p / \Delta U$	8.22%	5.31%	5.35%	5.47%

**Table 4:** Utility gains for North America from 1960 to 2000 for different values of  $\eta$ .

This also affects the inefficiency result given in Proposition 4. It is now possible that the steady state investments in healthcare are inefficiently high in the sense that all households born at  $s \geq 0$  were better off in a steady state where all households invest less in healthcare.

**Proposition 7 (Inefficient levels of healthcare expenditures)**

*Given equation (21) for the technological level of the economy, in an interior steady state, healthcare expenditures  $\bar{h}^*$  are inefficiently high if  $d\bar{g}(h, p)/dh|_{h=\bar{h}^*} < 0$ .*

The proof of Proposition 7, provided in the appendix, shows that under the given condition both externalities connected with healthcare investments – on the return of annuities and on the steady state growth rate – have a negative impact on expected lifetime utility.

Finally, we illustrate how the results of our numerical example given in Section 5.2 change under the alternative spillover specification. More precisely, we show results for  $\eta$  ranging from 1 (which corresponds to our previous setup) to 0.9. In addition, we also show the results for a hypothetical North America in which no healthcare expenditures are undertaken and all improvements in longevity solely stem from an increase in  $p_{max}$ . In this case the new specification of the spillover effect collapses to the basic model for any value of  $\eta$ . Thus, the results are identical to the second row in Table 3.

For both values of  $\eta < 1$  given in Table 4, we observe that the steady state growth rate declines in response to the observed increase of the healthcare expenditures. The effect is more pronounced the smaller is  $\eta$ . As in Section 5.2, we adjust the depreciation rate such that the average annual growth rate of GDP per capita matches the observed value of 2.44%. This implies that the growth rate drops from 2.49% in 1960 to 2.39% in 2000 for  $\eta = 0.95$  and from 2.55% in 1960 to 2.34% in 2000 for  $\eta = 0.9$ . In contrast to the results in Section 5.2,



the welfare effects of these changes in the growth rate may be substantial, as can be seen in the row labeled  $\Delta U_g/\Delta U$ . In fact, while for  $\eta = 0.95$  the total welfare gain attributable to increases in longevity is still positive, for  $\eta = 0.9$  the utility loss from the decreasing growth rate outweighs the direct utility increase due to increased average lifetime so that the welfare gain between 1960 and 2000 had been higher by 1.70% without the increase in life expectancy.

These results emphasize that the growth effects of increases in longevity may drastically depend on the associated healthcare costs. In any case, welfare and growth effects are the highest if the increases in longevity stem from a decrease of the maximal hazard rate  $p_{max}$  together with zero healthcare expenditures. If increases in longevity are accompanied by increases in healthcare expenditures, as is always the case for increases in the efficiency of healthcare  $\psi$ , the growth effects of increased longevity are either small (in case of our basic model) or may even be negative (in case of our alternative spillover specification). In fact, the negative growth effects may even outweigh the direct utility gains from increased longevity. At least for the richest regions like North America or Western Europe, this might challenge the conclusions of ? and ? who argue that recent increases in longevity induced non-negligible positive welfare gains.

It is important to recognize that the negative welfare effects in the case of  $\eta = 0.9$  result from the negative externality of healthcare expenditures on the economy's growth rate. In general, these externalities call for governmental action. In most countries, the healthcare system is heavily regulated. Usually health insurance systems result in inefficiently high demand for healthcare implying higher healthcare spending relative to the pure market equilibrium considered in our model (see, for example, ?, ?, ?, ?). As a consequence, when government subsidies for healthcare are taken into account welfare losses may be even higher than those shown for the case  $\eta = 0.9$  in Table 4.

Which model specification applies is an important empirical question. To answer this question, one could test whether the return on capital is affected by healthcare expenditures. One of the challenges of such an exercise would be to isolate healthcare expenditures that prolong life from those that do not. For the existing empirical literature, our model is consistent with recent results suggesting that the long-run effects of longevity on GDP per capita are either moderately positive (?), insignificant or even negative (?). By contrast, many (earlier) contributions to this literature usually found substantial positive effects of longevity on economic growth.<sup>28</sup> In an attempt to reconcile these different findings, ? argue that both the level of life expectancy and the increase in life expectancy have to be considered and find that both have significantly positive effects on per-capita GDP growth. However, they also indicate that when restricting attention to OECD-countries in the post-1960 period, the effects weaken. Our model allows for two different interpretations of this result. First, in our basic model the growth rate

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<sup>28</sup> See, for example, ?, ?, ?, ?, ?. Only very few studies find small negative or no effects of longevity of growth such as ?. An overview can be found in ?.

is increasing and concave with respect to longevity. Hence, the model predicts that the effect of an increase in life expectancy on the growth rate becomes smaller the higher is the level of longevity. Second, in the alternative spillover specification the growth rate of the economy may decline in response to higher healthcare costs. The longevity increases in the developed countries in the recent past have mainly originated from “big medicine” involving expensive and intensive personal interventions rather than the eradication of infectious diseases with relatively cheap hygienic measures (see, for example, ?, ?). As a consequence, our theory would predict smaller growth and welfare gains derived from increased life expectancy.<sup>29</sup>

## 8 Conclusion

We developed an overlapping generations endogenous growth model in continuous time to investigate the link between life expectancy, which is the result of endogenous investments in healthcare, economic growth and welfare. We have shown how the direct and indirect welfare effects (via the growth rate) of longer expected lifetimes induced by improvements in the healthcare technology depend on (i) the type of the healthcare technology improvement and (ii) the spillover specification in the economy’s consumption good sector. While the direct positive welfare effect dominates a very small growth effect in our benchmark model specification, the indirect welfare effect can outweigh the direct welfare effect in the generalized setting and even lead to overall welfare losses induced by longevity increases.

Our results have the following implications. First, while our benchmark specification lends some support to studies on welfare aspects of increased lifetimes neglecting growth effects, this disregard may lead to substantially wrong conclusions if reality is better reflected by the generalized version of our model. Which specification better suits reality constitutes an interesting question for further empirical work on the relationship between longevity and growth.

Second, many countries have recently made or are currently making efforts to develop welfare measures that include several components in addition to GDP (see, for example, ?). With respect to longevity, which is one of these complementing indicators, we have seen that neglecting the relationship between longevity and GDP is not innocuous. Similar complex relationships with respect to growth are to be expected for other components, for example, inequality. Thus, our paper points to a conceptional issue associated with such generalized indices of economic welfare. While they better reflect the average welfare levels of individual households, identifying the contribution of a single component to aggregate welfare is a complex task, as neglecting the repercussions on economic growth may substantially bias the results. This is of particular importance, when such analyses serve as the basis for policy advice.

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<sup>29</sup> ? hypothesize that gains in life expectancy at young age mattered more than gains in life expectancy at old age supposedly for reasons of labor market participation and education. We do not refute these reasons but emphasize the importance of healthcare expenditures associated with the increase in expected lifetime.

Third, we have shown that the magnitude of the welfare gains strongly depends on the channel by which the healthcare technology improves. A reduction of the baseline mortality yields higher welfare gains than an increase in the productivity of healthcare expenditures. This finding tends to be reinforced when considering more general specifications of the spillover effects in the production sector. Several authors have argued that the recent increases in longevity in the developed countries are mainly the result of “big medicine” rendering healthcare expenditures more productive in treating life-threatening diseases rather than a decrease in the baseline mortality level via cheap measures, such as improved sanitation. Extrapolating this development, our model suggests that the prospects for future welfare gains from increased longevity are rather modest.

This paper analyzes the complex interplay between endogenous longevity, endogenous economic growth and welfare in a model that abstracts from various issues which deserve further scrutiny. In order to be able to analytically investigate the aggregate economy, we employ a rather simplistic household model. Interesting extension in this direction include age-dependent mortality, retirement decisions or endogenous fertility. On the level of the aggregate economy, we have shown that the decentralized market solution exhibits several externalities that call for government action. Augmenting the model with realistic features of national health systems would allow to examine their effects on growth and welfare and to evaluate potential policy interventions. Finally, we only considered exogenous improvements in the healthcare technology. Endogenizing these improvements is a further challenge for future research.

## Appendix

### A.1 Proof of Proposition 1

First, the corner solution  $h(s) = 1$  cannot be an optimal solution, as consumption and lifetime utility would drop to zero, while both are positive for any value  $h(s) \in [0, 1)$ .

Second, there exists at most one  $h^*(s)$  with  $F(h^*(s)) = 0$ . To see this, set  $F(h(s)) = 0$  and re-arrange terms to yield<sup>30</sup>

$$1 - h(s) = \frac{\sigma(\sigma - 1)(\rho + p_{max}) - (\sigma - 1)^2 a(s)}{\sigma\psi} - (\sigma - 1)h(s) . \quad (\text{A.1})$$

Both, the left-hand and the right-hand side are linear equations in  $h(s)$ , which intersect at most once and are identical in the special case that  $\sigma = 2$  and  $\sigma(\sigma - 1)(\rho + p_{max}) - (\sigma - 1)^2 a(s) = \sigma\psi$  hold simultaneously. This special case is precluded, however, as the latter condition contradicts the necessary condition  $(1 - \sigma)a(s) + \sigma(\rho + p(s)) > 0$  for all  $p(s) \in [p_{max}, p_{max} - \psi]$  for the household's problem to be well defined.

Third, the local extremum given by  $F(h(s)) = 0$  is a local maximum only if  $\sigma < 2$ . Differentiating  $F(h(s))$  with respect to  $h(s)$  and evaluating at the local extremum yields:

$$\left. \frac{\partial F(h(s))}{\partial h(s)} \right|_{F(h(s))=0} = \frac{\sigma^2 \psi^2}{(\sigma - 1)^2 \tilde{x}^2(h(s))} (\sigma - 2) \begin{matrix} \geq \\ \leq \end{matrix} 0 \quad \Leftrightarrow \quad \sigma \begin{matrix} \geq \\ \leq \end{matrix} 2 , \quad (\text{A.2})$$

where  $\tilde{x}(h(s)) = [(1 - \sigma)a(s) + \sigma(\rho + p_{max} - \psi h(s))]$ . As a consequence, an interior optimal solution can only exist for  $\sigma \in (1, 2)$  and thus  $h^*(s) = 0$  if  $\sigma \geq 2$ . Even for  $\sigma \in (1, 2)$ , the optimal solution may be the corner solution  $h(s) = 0$ . This holds if  $F(h(s)) < 0$  for all  $h(s) \in [0, 1)$ .  $\square$

### A.2 Proof of Proposition 2

(i) Aggregate dynamics: To derive the aggregate system dynamics, we evaluate equation (11b) in the market equilibrium, aggregate according to equation (14) and differentiate with respect to  $t$ :

$$\dot{c}(t) = x(p) \left[ \dot{k}(t) + (1 - h)\dot{W}(t) \right] , \quad (\text{A.3})$$

where  $W(t) \equiv \int_t^\infty w(t') \exp[-(r + p)(t' - t)] dt'$  denotes the net present value of the household's lifetime labor income. Evaluating the budget constraint in the market equilibrium and

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<sup>30</sup> Obviously, this re-arrangement is only identical to  $F(h(s)) = 0$  if  $h(s) \neq 1$ . However, we have already seen that  $h(s) = 1$  cannot be an optimal solution.

aggregating according to equation (14), we obtain

$$\dot{b}(t) = (r - \nu)b(t) + (1 - h)w(t) - c(t) . \quad (\text{A.4})$$

Inserting  $\dot{W}(t)$  and equation (A.4) into equation (A.3) yields equation (17a). We derive (17b) by observing that in the market equilibrium  $w(t) = k(t)(1 - \alpha)/(1 - h)$  and inserting it into equation (A.4).

(ii) Balanced growth path: By contradiction, we prove that the dynamics of the economy is governed by a unique balanced growth path (BGP) given a fixed hazard rate  $p$ .

We start by asserting two facts: First, there is a unique economically feasible ratio  $c(t)/k(t)$  such that  $\dot{c}(t)/c(t) \equiv g_c(t) = g_k(t) \equiv \dot{k}(t)/k(t)$ . This follows from solving the equations of motion for  $c(t)/k(t)$  given that  $g_c(t) = g_k(t)$ . As  $x(p)(p + \nu) > 0$  for all  $p > 0$ , there is only one economically feasible solution (with  $c(t)/k(t) > 0$ )

$$\frac{c(t)}{k(t)} = \zeta \equiv \frac{1}{2} \left[ (1 - \delta - \nu) - \sigma(r - \rho) + \sqrt{[(1 - \delta - \nu) - \sigma(r - \rho)]^2 + 4x(p)(p + \nu)} \right] . \quad (\text{A.5})$$

Second, we observe in equations (17a) and (17b) that  $g_c(t)$  is increasing with  $c(t)/k(t)$  while  $g_k(t)$  is decreasing with  $c(t)/k(t)$ .

Now suppose that  $g_c(t) > g_k(t)$ . According to the two facts above, this can only be the case if  $c(t)/k(t) > \zeta$ . The condition  $g_c(t) > g_k(t)$  then implies that  $c(t)/k(t)$  further increases which in turn will increase the future gap between  $g_c$  and  $g_k$ , leading to  $\lim_{t \rightarrow \infty} g_k(t) = -\infty$ . By the same line of argument, the economy's dynamics imply for  $g_k(t) > g_c(t)$  that  $\lim_{t \rightarrow \infty} g_c(t) = -\infty$ . As both cases yield economically infeasible solutions the only remaining possibility is  $g_c(t) = g_k(t)$  implying  $c(t)/k(t) = \zeta$ . Since the latter ratio does not depend on time  $t$  and is unique, the economy must be on a unique BGP  $g_c(t) = g_k(t) = g$  at all times. The BGP-growth rate  $g$  can be calculated by inserting (A.5) into  $g = (1 - \delta - \nu) - c(t)/k(t)$

$$g = \frac{1}{2} \left[ (1 - \delta - \nu) + \sigma(r - \rho) - \sqrt{[(1 - \delta - \nu) - \sigma(r - \rho)]^2 + 4x(p)(p + \nu)} \right] . \quad (\text{A.6})$$

After some minor manipulations, we observe that the growth rate on the BGP is positive if and only if  $x(p)(p + \nu) < \sigma(r - \rho)(1 - \delta - \nu)$ . Consequently,  $g < 0$  if  $x(p)(p + \nu) > \sigma(r - \rho)(1 - \delta - \nu)$  and  $g = 0$  if  $x(p)(p + \nu) = \sigma(r - \rho)(1 - \delta - \nu)$ .  $\square$

### A.3 Proof of Proposition 3

(i) Differentiating the equilibrium level of healthcare expenditures, as given in (15), with respect to  $\psi$  and  $p_{max}$  yields:

$$\frac{dh^*}{d\psi} = \frac{(\sigma - 1)[(1 - \sigma)r + \sigma\rho + p_{max}]}{\psi^2} > 0, \quad \frac{dh^*}{dp_{max}} = -\frac{\sigma - 1}{\psi} < 0. \quad (\text{A.7})$$

From the healthcare technology (8), we obtain

$$\frac{dp^*}{d\psi} = \frac{\partial p^*}{\partial \psi} + \frac{\partial p^*}{\partial h^*} \frac{dh^*}{d\psi} = -\left(h^* + \psi \frac{dh^*}{d\psi}\right) < 0, \quad (\text{A.8a})$$

$$\frac{dp^*}{dp_{max}} = \frac{\partial p^*}{\partial p_{max}} + \frac{\partial p^*}{\partial h^*} \frac{dh^*}{dp_{max}} = 1 - \psi \frac{dh^*}{dp_{max}} > 0. \quad (\text{A.8b})$$

For the growth rate  $g^*$  we obtain

$$\frac{dg^*}{d\psi} = \frac{\partial g(p)}{\partial p} \Big|_{p=p^*} \frac{dp^*}{d\psi} > 0, \quad \frac{dg^*}{dp_{max}} = \frac{\partial g(p)}{\partial p} \Big|_{p=p^*} \frac{dp^*}{dp_{max}} < 0. \quad (\text{A.9})$$

The inequalities follow from  $\partial g(p)/\partial p < 0$ , which we obtain by differentiating equation (18) with respect to the hazard rate  $p$

$$\frac{\partial g(p)}{\partial p} = -\frac{x(p) + p + \nu}{\sqrt{[1 - \delta - \nu - \sigma(r - \rho)]^2 + 4(p + \nu)x(p)}} < 0. \quad (\text{A.10})$$

(ii) Inserting the derivatives in (A.7) and the interior solution for  $h^*$  from (15) into equations (A.8) gives  $dp^*/d\psi = -\sigma$  and  $dp^*/dp_{max} = \sigma$ . Using (A.7), the condition  $-dh^*/dp_{max} < dh^*/d\psi$  translates to

$$\frac{\sigma - 1}{\psi} < \frac{(\sigma - 1)[(1 - \sigma)r + \sigma\rho + p_{max}]}{\psi^2} \Leftrightarrow (1 - \sigma)r + \sigma\rho + p_{max} - \psi > 0. \quad (\text{A.11})$$

The latter condition is satisfied by assumption, as otherwise the households' maximization problem is not well defined.  $\square$

### A.4 Proof of Proposition 4

Healthcare investment in the market equilibrium is inefficiently low if  $dU(s)/dh|_{h=h^*} > 0$  for all  $s \geq 0$ , as this would imply that a marginal increase in healthcare expenditures would increase the lifetime utility of all generations born at  $s \geq 0$ . For an interior solution  $F(h^*) = 0$ . As a

consequence, the condition is satisfied for all  $s \geq 0$  if

$$\frac{da}{dh} \left( \frac{1}{x(p)} - \frac{1}{y(p)} \right) + \frac{dg(p)}{dh} \frac{1}{y(p)} > 0. \quad (\text{A.12})$$

As  $da/dh = dp/dh = -\psi$  and  $dg(p)/dh = dg(p)/dp \cdot dp/dh$ , (A.12) holds if the inequality given in the proposition is satisfied.  $\square$

## A.5 Proof of Proposition 5

The individual household's choice of optimal healthcare expenditures, as given in Proposition 1, remains unchanged by the new definition of the technological level of the economy. As a consequence, the optimal level of healthcare in the steady state market equilibrium equals  $\bar{h}^* = 0$  for  $\sigma \geq 2$ . For  $\sigma < 2$ , we insert  $\bar{a}(h, p) = \bar{r}(h) + p$  into the first-order condition (13) and obtain

$$\bar{F}(h) \equiv \frac{\sigma\psi}{(\sigma-1)[(1-\sigma)\bar{r}(h) + \sigma\rho + p]} - \frac{1}{1-h}. \quad (\text{A.13})$$

Note that  $\lim_{h \rightarrow 1} \bar{F}(h) = -\infty$ , as the first term remains finite<sup>31</sup> and the second term diverges to  $-\infty$  for  $h \rightarrow 1$ . Thus, there exists an  $\bar{h}^*$  with  $\bar{F}(\bar{h}^*) = 0$  and  $d\bar{F}(h)/dh|_{h=\bar{h}^*} < 0$  if  $\bar{F}(h) > 0$  for some  $h \in [0, 1)$ . Otherwise the optimal level of healthcare equals  $\bar{h}^* = 0$ .  $\square$

## A.6 Proof of Proposition 6

(i) Aggregate dynamics: Using equations (23) instead of (6), we derive the aggregate steady state dynamics analogously to part (i) of the proof of Proposition 2. To derive the equation of motion for the capital stock  $k(t)$  insert  $b(t) = k(t)$  and  $\bar{w}(h, t) = (1-\alpha)/[\alpha(1-h)] \cdot \bar{r}(h)k(t)$ .

(ii) Balanced growth path: The existence and uniqueness of a balanced growth path can be shown as in part (ii) of the proof of Proposition 2. Replacing  $(1-\delta-\nu)$  in equation (A.6) by  $\bar{r}(h)/\alpha + \delta(1-\alpha)/\alpha - \nu$  yields  $\bar{g}(h, p)$  as given in the proposition.  $\square$

## A.7 Proof of Proposition 7

Analogously to the proof of Proposition 4 the steady state level of healthcare expenditures  $\bar{h}^*$  is inefficiently high if  $\partial U(s)/\partial h|_{h=\bar{h}^*} < 0$  for all  $s \geq 0$ . The equation corresponding to equation

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<sup>31</sup> Note that  $(1-\sigma)\bar{r}(h) + \sigma\rho + p > 0$  for all  $p \in [p_{max}, p_{max} - \psi]$  is necessary for the household's maximization problem to be well defined.

(20) in the case of the general spillover specification reads

$$\frac{dU(s)}{dh} = \frac{c^*(s, s, h)^{1-\frac{1}{\sigma}}}{\bar{x}(h, p)} \left[ \bar{F}(h) + \frac{d\bar{a}(h, p)}{dh} \left( \frac{1}{\bar{x}(h, p)} - \frac{1}{\bar{y}(h, p)} \right) + \frac{d\bar{g}(h, p)}{dh} \left( \frac{1}{\bar{y}(h, p)} + s \right) \right], \quad (\text{A.14})$$

where  $\bar{y}(h, p) = \bar{r}(h) + p - \bar{g}(h, p)$ . For an interior level of healthcare expenditures  $\bar{F}(h) = 0$ . Moreover,  $d\bar{a}(h, p)/dh = d\bar{r}(h)/dh + dp/dh < 0$  and  $\bar{y}(h, p) - \bar{x}(h, p) = \sigma[\bar{r}(h) - \rho] - \bar{g}(h, p) > 0$ . Thus,  $\partial U(s)/\partial h|_{h=\bar{h}^*} < 0$  if  $d\bar{g}(h, p)/dh|_{h=\bar{h}^*} < 0$ .  $\square$

## A.8 Details on the numerical exercises

We use the original data set of ?<sup>32</sup> and amend it by data on health expenditures from 1960 and 2000. For the year 2000 the WHO<sup>33</sup> provides data of healthcare expenditures per GDP for all countries. Data for healthcare expenditures in 1960 is limited. The OECD<sup>34</sup> provides data on healthcare expenditures per capita in 1960 for some of their members. In fact, complete data on healthcare expenditures is only available for North America. Assuming the missing values to be zero, the data listed in Table 5 for healthcare expenditures in 1960 is a lower bound for real healthcare expenditures. According to ? income per capita is GDP per capita in 1996 international prices adjusted for terms of trade (Penn World Tables 6.1). Data on life expectancy at birth is taken from the World Bank Development Indicators. The average growth rate  $g_\emptyset$  has been calculated from income levels in 1960 and 2000. Accordingly, the average population growth rate  $\nu$  has been calculated from population data in 1960 and 2000. All regional aggregates are population weighted sums of country data.

To derive our consumption-based welfare measure, we insert (11a), (11c),  $a^* = r + p^*$  and  $w(t) = w_0(s) \exp[g^*(t - s)]$  into the household's expected utility function (3)

$$U(w_0(s), p^*, g^*, h^*) = \frac{\sigma}{\sigma - 1} \left[ \frac{w_0(s)(1 - h^*)}{r + p^* - g^*} \right]^{1-\frac{1}{\sigma}} [r + p^* - \sigma(r - \rho)]^{-\frac{1}{\sigma}}. \quad (\text{A.15})$$

Comparing expected utilities of two households, we seek for the relative change in consumption  $\theta$  at all times alive for which household 1's expected utility coincides with household 2's

<sup>32</sup> Available at [http://www.aeaweb.org/aer/data/mar05\\_data\\_becker.zip](http://www.aeaweb.org/aer/data/mar05_data_becker.zip).

<sup>33</sup> Available at <http://www.who.int/gho/en/>.

<sup>34</sup> Available at <http://puck.sourceoecd.org>.



expected utility<sup>35</sup>

$$\begin{aligned} \frac{\sigma}{\sigma-1} \left[ (1+\theta) \frac{w_0^1(s_1)(1-h_1^*)}{r+p_1^*-g_1^*} \right]^{1-\frac{1}{\sigma}} [r+p_1^*-\sigma(r-\rho)]^{-\frac{1}{\sigma}} = \\ \frac{\sigma}{\sigma-1} \left[ \frac{w_0^2(s_2)(1-h^*)}{r+p_2^*-g_2^*} \right]^{1-\frac{1}{\sigma}} [r+p_2^*-\sigma(r-\rho)]^{-\frac{1}{\sigma}} \end{aligned} \quad (\text{A.16})$$

Solving for  $\theta$  yields

$$\theta = \frac{w_0^2(s_2)(1-h_2^*)(r+p_1^*-g_1^*)}{w_0^1(s_1)(1-h_1^*)(r+p_2^*-g_2^*)} \left[ \frac{r+p_1^*-\sigma(r-\rho)}{r+p_2^*-\sigma(r-\rho)} \right]^{\frac{1}{\sigma-1}} - 1 \quad (\text{A.17})$$

In the welfare comparison between 1960 and 2000,  $\Delta U$  reflects the value of  $\theta$  when inserting 1960 levels for  $w_0^1(s_1)$ ,  $p_1^*$ ,  $g_1^*$  and  $h_1^*$ , and 2000 levels for  $w_0^2(s_2)$ ,  $p_2^*$ ,  $g_2^*$  and  $h_2^*$ . The total consumption-equivalent welfare difference  $\Delta U_T$  represents the value of  $\theta$  when we compare a household born in 2000, given there had not been an increase in longevity, with a household born in 2000 under real world conditions. Thus,  $p_1^*$ ,  $g_1^*$  and  $h_1^*$  are given by the observed 1960 levels, and  $p_2^*$ ,  $g_2^*$  and  $h_2^*$  by the corresponding 2000 levels, but  $w_0^1(s_1)/w_0^2(s_2) = \exp[g_1^* \cdot 40] / \exp[g_\emptyset \cdot 40]$  accounting for the difference in wages if wages would not have grown at the observed average growth rate  $g_\emptyset$  but at the growth rate  $g_1^*$  consistent with 1960 levels of healthcare and longevity. Hence, the share of the overall welfare gain originating from the longevity increase  $\Delta U_T / \Delta U$  is given by the ratio of the corresponding  $\theta$ s. The decomposition into direct utility from a longer expected lifetime and the growth effect is accomplished along the same lines.

Table 5 shows the calculation results for seven world regions (E&CA: Europe and Central Asia, EA&P: East Asia and Pacific, LA&C: Latin America and the Caribbean, ME&NA: Middle East and North Africa, NAM: North America, SASIA: South Asia and SSA: Sub-Saharan Africa). NAM (exo) denotes a hypothetical North America in which no healthcare expenditures are undertaken and increases in longevity solely stem from a decrease in  $p_{max}$ .

To estimate the sensitivity of our results with respect to the missing data on healthcare expenditures in 1960, Table 6 shows the same calculations for the seven world regions but with rather high estimates on the healthcare expenditures in 1960. We observe that our results are robust to reasonable changes in the healthcare expenditures.

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<sup>35</sup> See also ?, who use a similar approach.

Region	E & CA	EA & P	LA & C	ME & NA	NAM	NAM (exo)	SASIA	SSA
inc.(1960)	6810.37	1316.87	3459.36	1935.13	12379.8	12379.8	892.08	1470.48
inc.(2000)	18280.6	5866.24	7161.46	5524.89	32880.2	32880.2	2345.84	1573.02
$T(1960)$	67.99	42.05	56.26	47.89	69.89	69.89	44.04	40.55
$T(2000)$	76.22	70.71	70.46	68.94	77.25	77.25	62.73	46.02
$h(1960)$	1.64%	1.03%	0%	0%	3.91%	0%	0%	0%
$h(2000)$	6.02%	4.05%	4.97%	4.03%	9.77%	0%	3.12%	4.84%
$r$	3.70%	4.58%	3.33%	3.87%	3.69%	3.69%	3.74%	2.32%
$\nu$	0.71%	1.64%	2.24%	2.54%	1.14%	1.14%	2.22%	2.73%
$g_{\emptyset}$	2.47%	3.73%	1.82%	2.62%	2.44%	2.44%	2.42%	0.17%
$g^*(1960)$	2.46%	3.69%	1.80%	2.59%	2.44%	2.44%	2.38%	0.15%
$g^*(2000)$	2.47%	3.78%	1.84%	2.66%	2.45%	2.45%	2.45%	0.19%
$\hat{g}$	2.65%	5.10%	2.25%	3.47%	2.57%	2.65%	3.30%	0.4%
$\Delta U_T/\Delta U$	7.50%	16.66%	20.95%	21.01%	5.63%	8.50%	23.26%	72.33%
$\Delta U_g/\Delta U$	0.31%	0.47%	1.08%	0.90%	0.31%	0.28%	1.02%	65.64%
$\Delta U_p/\Delta U$	7.20%	16.19%	19.87%	20.12%	5.31%	8.22%	22.24%	6.69%

**Table 5:** Numerical results for all seven world regions.

Region	E & CA	EA & P	LA & C	ME & NA	NAM	SASIA	SSA
$h(1960)$	3.0%	1.5%	2.0%	1.5%	3.91%	1.0%	2.0%
$h(2000)$	6.02%	4.05%	4.97%	4.03%	9.77%	3.12%	4.84%
$r$	3.70%	4.58%	3.33%	3.87%	3.69%	3.74%	2.32%
$\nu$	0.71%	1.64%	2.24%	2.54%	1.14%	2.22%	2.73%
$g_{\emptyset}$	2.47%	3.73%	1.82%	2.62%	2.44%	2.42%	0.17%
$g^*(1960)$	2.46%	3.69%	1.80%	2.59%	2.44%	2.38%	0.15%
$g^*(2000)$	2.47%	3.78%	1.84%	2.66%	2.45%	2.45%	0.19%
$\hat{g}$	2.67%	5.10%	2.28%	3.49%	2.57%	3.31%	0.44%
$\Delta U_T/\Delta U$	8.11%	16.69%	21.79%	20.27%	5.63%	23.44%	73.35%
$\Delta U_g/\Delta U$	0.30%	0.47%	1.05%	0.89%	0.31%	1.01%	6.38%
$\Delta U_p/\Delta U$	7.81%	16.22%	20.74%	20.39%	5.31%	22.44%	66.97%

**Table 6:** Sensitivity analysis with respect to healthcare expenditures in 1960.