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Competitive Moment Matching of a New-Keynesian and an Old-Keynesian Model: Extended Version, First Draft

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Abstract

The paper considers two rival models referring to the new macroeconomic consensus: a standard three-equations model of the New-Keynesian variety and dynamic adjustments of a business and an inflation climate in an ‘Old-Keynesian’ tradition. Over the two subperiods of the Great Inflation and Great Moderation, both of them are estimated by the method of simulated moments. An innovative feature is here that it does not only include the autocovariances up to eight lags of quarterly output, inflation and the interest rate, but optionally also a measure of the raggedness of the three variables. In short, the performance of the Old-Keynesian model is very satisfactory and similar to, unless better than, the New-Keynesian model. In particular, the Old-Keynesian model is better suited to match the new moments without deteriorating the original second moments too much.

JEL classification: C52; E32; E37.

Keywords: Sentiment dynamics; new macroeconomic consensus; method of simulated moments; Great Inflation, Great Moderation.

1. Introduction

The paper brings together two strands of economic research on small-scale modelling in the context of the so-called new macroeconomic consensus. First, there is the New-Keynesian approach with its extensive estimation literature. While Bayesian likelihood

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methods have become dominant here in the past decade, a smaller number of studies has alternatively used moment matching procedures. They seek for parameter values such that a set of model-generated summary statistics, or ‘moments’, comes as close as possible to their empirical counterparts. The goodness-of-fit of a model, or a trade-off between the merits and demerits in matching specific moments, can thus also be assessed in finer detail than by just referring to the optimization of a single and relatively abstract objective function.

In particular, this method has recently been applied to a hybrid three-equations model with forward- and backward-looking elements, which focusses on the quarterly output gap, the inflation gap and the interest gap (i.e., the deviations of these variables from a constant or possibly time-varying trend). Within mainstream macroeconomics, the model represents a sort of common-sense middle ground that preserves the insights of standard rational expectations with some sort of sluggish behaviour, while allowing for better empirical fit by dealing directly with a well-known empirical deficiency of the purely forward-looking models. As a result, this class of models has been widely used in applied monetary policy analysis, with the policy implications depending importantly on the values of the coefficients on the expected and lagged variables, respectively.

When estimating the New-Keynesian model, the moments to be matched were the auto- and cross-covariances of the three variables with lags up to eight quarters. Admitting sufficiently backward-looking behaviour (in some contrast to the ordinary likelihood literature), the performance of these estimations on US data for the two sub-periods of the so-called Great Inflation and Great Moderation was so good that they were said to constitute a challenging yardstick for any macroeconomic model of a similar complexity (Franke, 2011a; Franke et al., 2011).

The second strand of research that we address are macroeconomic theories that refuse the paradigm of the optimizing representative agents and their rational expectations. To face the task of providing strong alternatives, two new types of models have been advanced within the three-variables framework that put special emphasis on translating the idea of the famous ‘animal spirits’ into a formal canonical framework. They can thus be briefly characterized as models of sentiment dynamics (Franke, 2008 and 2011a; De Grauwe, 2010). Their cycle-generating properties have been demonstrated by numerical simulations with suitably calibrated parameters, but so far these models have not yet been subjected to econometric procedures.

This is where the present paper sets in. It takes up the model by Franke (2011a), which is based on the notion of endogenously determined transition probabilities with which the individual firms switch between an optimistic and a pessimistic investment attitude. In addition, entering the Phillips curve is an inflation climate the adjustments of which are influenced by a parameter that represents the general credibility of the central bank. For a better contrast, this model is called an Old-Keynesian model. Because of its intrinsic nonlinearities, the second moments for the estimation can no longer be analytically

computed as in the linear New-Keynesian case but have to be simulated over a long time horizon (which introduces the problem of sample variability). The obvious question is, of course, whether the general match of the Old-Keynesian model can compete with the New-Keynesian model (NKM).

Three different versions of the Old-Keynesian model are studied. The first one is deterministic and the exact discrete-time analogue of Franke (2011a), except that it slightly extends the Taylor rule in order to have the same specification with interest rate smoothing as in NKM. The persistent cyclical behaviour in this model is mainly brought about by a sufficiently strong herding mechanism; it renders the long-run equilibrium unstable, while the nonlinearities prevent the dynamics from exploding. The goodness-of-fit that can here be achieved deserves already some respect, although it will clearly fall behind the stochastic NKM.

Versions two and three of our model are stochastic, which, incidentally, will deemphasize the role of herding in the estimations. They introduce the analogous quarterly random shocks from NKM, i.e. demand shocks, supply shocks and monetary policy shocks, all of them being serially uncorrelated. In the second version they only take direct effect in an output equation, the Phillips curve and the Taylor rule, respectively. Being inspired by NKM where in the reduced-form solution each shock acts on each of the three variables, our third version additionally allows the demand shock to act on the adjustments of the inflation climate and the cost push shock to act on the adjustments of the firms' investment attitude.

When estimating the models with the second moments mentioned above one will note that the general pattern of the simulated time series exhibits a similar level of noise. This is in contrast to the empirical series where the quarterly inflation rates are much noisier than the other two variables. As an innovative feature we specify a measure of raggedness of the time series and add these statistics to the other moments. By and large it turns out that the Old-Keynesian model is better suited to match the new moments without deteriorating the original second moments too much. A short and succinct characterization of the overall matching quality will then be that the second version is fairly similar to that of NKM, whereas the third version is markedly superior. The paper thus shows that, in the framework indicated, there is indeed an alternative 'Old-Keynesian' model that can well bear comparison with the workhorse model of orthodox macroeconomics.

The remainder of the paper is organized as follows. The next section describes our estimation approach of the method of simulated moments. Section 3 introduces the two rival models, the New- and the Old-Keynesian one. The estimation results are presented in Sections 4 and 5, where Section 4 deals the period of the Great Inflation and Section 5 with the Great Moderation. Section 4 is actually the main part of the paper as it also contains the discussion of many specification details when they are first applied. Section 6 concludes. Several more technical issues are relegated to a number of appendices.

2. The estimation approach

2.1. The minimization problem

The approach to estimate our New- and Old-Keynesian models is concerned with their dynamic properties in general. They are quantitatively described by a number of summary statistics, also called ‘moments’, and the estimation seeks to identify numerical parameter values such that the model-generated moments come as close as possible to their empirical counterparts. In linear models such as the log-linearized New-Keynesian rational expectations models, all or a larger set of the moments can be computed analytically. If this is not possible, which because of its nonlinearity will be the case for the Old-Keynesian model, the moments can still be computed from numerical simulations of the model. Hence, briefly, we are using the method of moments (MM) or the method of simulated moments (MSM), respectively.

The crucial point is, of course, the choice of the moments, which by some critics is branded as arbitrary. Here the approximate nature of modelling should be taken into account. Since any structural model focusses on a specific purpose, it is only natural that it would be able to match, at best, *some* of the ‘stylized facts’ of an actual economy. MM and MSM, then, require the researcher to definitely make up his or her mind about the dimensions along which the model should be most realistic. Correspondingly, one can look at the single moments and find out which of them are more adequately matched than others. This will also provide more detailed diagnostics about the particular merits and demerits of a model than an objective function that summarizes many (and perhaps infinitely many) features in a single value. In our view, it is the explicit choice of the moments and, in practice, their easy interpretation that are strong arguments just in favour of the moment matching approach.¹

Generally, there are n_m moments that are collected in a column vector $m \in \mathbb{R}^{n_m}$. The moments that are obtained from an empirical sample of T observations are designated m_T^{emp} . The model-generated moments depend on the numerical values of a vector θ of n_θ structural parameters, which are confined to a (rectangular) set $\Theta \subset \mathbb{R}^{n_\theta}$. If analytical expressions for these moments are available (which are the asymptotic moments), we could unambiguously write $m = m(\theta)$. If the moments have to be simulated, the simulation size should be made explicit, i.e., the effective number of periods S over which the model is simulated. As an approximation to the infinite simulation size that would be needed for the asymptotic moments, the final estimations of our discrete-time, quarterly models will be based on $S = 10,000$ quarters.²

¹ Which does not rule out that other estimation procedures have their virtues as well. Note, however, that likelihood methods become fairly complicated if, as it will be the case for us, the models contain unobservable state variables and are nonlinear.

² “Effective” simulation size means that, starting from the steady state position, the models are simulated over $200+S$ quarters and the first 200 quarters are discarded to rule out any transient

The long time horizon reduces, but does not completely eliminate, the sample variability from different random number sequences that are underlying the simulations when the models are stochastic. These sequences may be identified by a natural number $c \in \mathbb{N}$, which corresponds to their random seed. In sum, a model-generated vector of moments is thus determined as $m = m(\theta; c, S)$.

The distance between the vectors of the theoretical and empirical moments is measured by a quadratic function J that is characterized by an $(n_m \times n_m)$ weighting matrix W . The value of this loss function is to be minimized, where it goes without saying that across different parameters θ only simulations with the same random seed are comparable.³ Accordingly, the model is estimated by the following set of parameters:

$$\begin{aligned} \hat{\theta} = \hat{\theta}(c, S) &= \arg \min_{\theta \in \Theta} J(\theta; m_T^{emp}, c, S) \\ &:= \arg \min_{\theta \in \Theta} [m(\theta; c, S) - m_T^{emp}]' W [m(\theta; c, S) - m_T^{emp}] \end{aligned} \quad (1)$$

The problem of how to treat the variability arising from different random seeds c is addressed below.⁴

Minimization of (1) is not a straightforward matter. Given the relatively high number of parameters in our applications, there is for functions of type (1), just as it is the case for likelihood functions, a great danger of a larger number of local extrema, possibly also located at a farther distance from each other. Our search therefore proceeds in two steps. First, in order to reduce the risk of being trapped in a wrong region of the parameter space, we use simulated annealing as a globally effective procedure.⁵

Since, with different initial conditions and different random sequences for the stochastic search, the algorithm does not always settle down in the same region, it is necessary to carry out several minimization runs and identify from them a parameter region with the lowest losses. A situation typically encountered is that two or four out of ten runs

effects (which proves to be more than sufficient).

³ For our random variables, which are of the form $\varepsilon_t \sim N(0, \sigma^2)$, this means that in period t always the same random number $\tilde{\varepsilon}_t$ is drawn from the standard normal $N(0, 1)$ and then, depending on the specific value of σ under examination, ε_t is set equal to $\varepsilon_t = \sigma \tilde{\varepsilon}_t$.

⁴ If in the course of the minimization search procedure for (1) some parameter leaves an admissible interval, it is temporarily reset to the boundary value, the loss J of the thus resulting moments is computed, and then a sufficiently strong penalty is added to J that proportionately increases with the extent of the original violation. In this way also corner solutions to (1) can be safely identified.

⁵ In detail, we apply the algorithm put forward by Corona et al. (1987); see also Goffe et al. (1994) and Goffe (1996). The most critical ‘tuning parameters’ are the reduction factor r_T and the initial temperature T_o . We set $r_T = 0.75$, a conservative value “which is suitable for a function one has little experience with” (Goffe, 1996, p. 172). To obtain T_o , first the median loss M of 500 widely dispersed parameter vectors is computed. Requiring that the algorithm’s (desirable) probability of accepting an increase in the loss is about 0.50, at temperature T_o and at the same step sizes that in the initial procedure have scaled the changes in the single parameter values, subsequently Boltzmann’s formula $\exp(-M/T_o) = 0.50$ is solved for T_o .

fail to find sufficiently low values of the loss function, while the remaining trials yield fairly close values in the same region.⁶

In a second stage, we choose the optimal parameter vector from the stage of simulated annealing and take it as a starting vector for another minimization procedure, the Nelder-Mead simplex search algorithm (see Press et al., 1986, pp. 289–293).⁷ Even when repeatedly restarting it upon convergence until no more noteworthy improvement in the minimization occurs, this algorithm is faster than simulated annealing (it takes a few minutes). Combining the two search strategies, we can be rather confident that for all practical purposes the global minimum of (1) has indeed been found.

Turning to the weighting matrix W that sets up the loss function in (1), an obvious since asymptotically optimal choice would be the inverse of an estimated moment covariance matrix $\hat{\Sigma}_m$ (Newey and McFadden, 1994, pp. 2164f). Unfortunately, the moments underlying our estimations will not be independent of each other, so that such a matrix would be nearly singular and its inverse could not be relied on. The usual option, then, is to employ a diagonal weighting matrix the entries of which are given by the reciprocals of the variances of the single moments.

There are several methods to estimate these variances from the empirical data. Here we employ a bootstrap procedure. It is in detail somewhat different from an ordinary block bootstrap but seems more appropriate to us for the present problem (the details are described in Appendix A1). This being understood, we have

$$W_{ii} = 1 / \widehat{\text{Var}}(m_{T,i}^{emp}), \quad i = 1, \dots, n_m \quad (2)$$

(and of course $W_{ij} = 0$ if $i \neq j$). Clearly, the less precisely a moment is estimated from the data, that is, the higher is its variance, the lower is the weight attached to it in the loss function. Since the width of the confidence intervals around the empirical moments $m_{T,i}^{emp}$ is proportional to the square root of these variances, it may be stated that the model-generated moments obtained from the estimated parameters lie “as much as possible inside these confidence intervals” (Christiano et al., 2005, p. 17). Nevertheless, a formulation of this kind, which with almost the same words can also be found in several other applications, should not be interpreted too narrowly. In particular, a minimum of the loss function in (1) need not simultaneously imply a minimal number of moments outside the confidence intervals.⁸

As useful feature of the approach of moment matching is that it does not only provide a measure of the model’s overall goodness-of-fit, it also allows us to locate potential shortcomings in single moments. A convenient since directly available measure to evaluate

⁶ A shorter simulation horizon of $S = 2,000$ proves to be sufficient for these global investigations. The reduction is helpful here since on average one such run takes about 25 minutes on a standard personal computer.

⁷ It has broader scope than gradient methods to escape from small local valleys.

⁸ An example for this can be found in Franke et al. (2011, p. 11, Table 1).

how serious the deviations of a model-generated moment from the empirical moment may be can be obtained by relating them to the standard errors of the latter. With respect to a moment i , this is the t -statistic

$$t_i = [m_i(\theta; c, S) - m_{T,i}^{emp}] / \sqrt{\widehat{\text{Var}}(m_{T,i}^{emp})}, \quad i = 1, \dots, n_m \quad (3)$$

(reference of t_i to c and S is omitted as it will be understood from the context). Although statistically not exactly justified, we would be satisfied with the matching of a moment if its t -statistic will be less than two in absolute value; otherwise we would have to admit that the model has a certain weakness in this respect. It may also be noted in passing that, with the diagonal weighting matrix from (2), estimation means nothing else than minimizing the sum of the squared t -statistics, which is a fairly intuitive criterion.

After thus establishing the econometric framework, we have to turn to the specific moments on which we want to base our estimations. In the course of discussing the results, we will demand more from the models and correspondingly extend the set of these moments. For the time being, we make the matching criterion explicit from which we start out. It is meant to characterize the fundamental dynamic relationships between the observable variables of the model. For this purpose the second moments are well suited, that is, the nine profiles of the unconditional contemporaneous and lagged auto- and cross-covariances of (the gaps of) output (y), inflation (π) and the interest rate (i).⁹

It may be emphasized that we fix these moments in advance and that their number will not be too small, either.¹⁰ In detail, we are concerned with the nine profiles of $\text{Cov}(p_t, q_{t-h})$ for $p, q = i, y, \pi$ and $h = 0, 1, \dots$ up to some maximal lag H (the hat over i or π for the New-Keynesian model below may be omitted here). Given that the length of the business cycles in the US economy varies between (roughly) five and ten years, the estimations should not be based on too long a lag horizon. A reasonable compromise is a length of two years, so that we will work with $H = 8$ quarters. In this way we have a total of 78 moments to match: 9 profiles with (1+8) lags, minus 3 moments to avoid double counting the zero lags in the cross relationships. To distinguish it from the extended versions below, we denote this loss function by $J^{(78)}$. For a definite reference, we repeat

$$\begin{aligned} J^{(78)} : \quad & \text{loss function constituted by the 78 moments } \text{Cov}(p_t, q_{t-h}) \\ & \text{and the corresponding weights } W_{ii} \text{ from (2),} \\ & \text{for } p, q = i, y, \pi \text{ and } h = 0, 1, \dots, 8 \end{aligned} \quad (4)$$

⁹ The second moments provide similar information to the impulse-response functions of the three types of shocks in the models, in which (or just one of them) many New-Keynesian studies take a greater interest.

¹⁰ This commitment is different from an explicit moment selection procedure as it is, for example, put to use by Karamé et al. (2008). They begin with a large set of moments, estimate their model on them, and then step by step discard the moments that the model reproduces most poorly until an over-identification test fails to reject the model any longer.

2.2. Confidence intervals and J test

Under standard regularity conditions, the parameter estimates $\hat{\theta}$ from (1) are consistent and asymptotically follow a normal distribution around the (pseudo-) true parameter vector. There is moreover an explicit formula for estimates of the corresponding covariance matrix, from which the standard errors of $\hat{\theta}$ are obtained as the square roots of its diagonal elements (Lee and Ingram, 1991, p.202). In the present case, however, this approach faces two problems. First, it may turn out that locally the loss function J may react only very weakly to the changes in some of the parameters. Hence these standard errors become extremely large and, beyond this qualitative message, are not very informative. The second point is that one of the regularity conditions will be violated if the minimizing parameter vector is a corner solution of (1) or close to an upper- or lower-bound of some parameter; trivially, for some components i the distributions of the estimated parameters cannot be symmetrically centred around the point estimates $\hat{\theta}_i$ then.

These problems can be avoided by having recourse to a bootstrap procedure. Besides, it is also expedient for coping with the small sample problem. In this respect it is only natural to employ the bootstrap that already served to obtain the variances of the empirical moments in the weighting matrix W above. From there we have a large set of artificial moments on which, rather than on the single empirical moment vector m_T^{emp} , a model can be re-estimated just as many times as we want. In this way we get a frequency distribution for each of the parameters and can easily compute the confidence intervals from them (see below).

The re-estimations are furthermore useful for a general evaluation of the model's goodness-of-fit. It is well-known that, under the null hypothesis that the model is true, the minimized value of the loss function represents a statistic that is asymptotically chi-square distributed—provided, it has to be added, that the weighting matrix entering the loss function is optimal (Lee and Ingram, 1991, p.204). Clearly, this standard J test for overidentifying restrictions is not applicable here since the latter supposition is not satisfied. The test distribution can, however, be simulated. The main idea is to exploit the information that a distribution $\{\tilde{J}\}$ of the minimized losses in the re-estimations provides to us. After all, the bootstrapped moments that the model then tries to match can be viewed as proxies for the moments that hypothetically would be produced by different small-sample realizations of the unknown real-world data generation process (DGP). Hence the distribution of the losses \tilde{J} incurred from them would lend us a test criterion for a possible rejection of the model, which we would have to do if the previously estimated value $J[\hat{\theta}(c, S); m_T^{emp}, c, S]$ exceeds the 95% quantile of these \tilde{J} .

Although this concept is straightforward, the precise specification of the re-estimation problem requires a little care. Denoting the distribution of the bootstrapped moment vectors by $\{\tilde{m}_T\}$ and omitting the reference to c and S for a moment, it has to be

taken into account that, except for special circumstances, there is no set of parameters θ ensuring $E[m(\theta) - \tilde{m}_T] = 0$ (as there are more moments than parameters in the model). This fact becomes important for the first-order validity of the bootstrapped J test, for which a null hypothesis must be imposed.¹¹ To this end we first observe that $E(\tilde{m}_T) = m_T^{emp}$ should prevail. Furthermore, different (hypothetical) realizations of the real-world DGP would give rise to a distribution of the estimated parameters $\hat{\theta}$ and corresponding moment vectors $m(\hat{\theta})$, while the parameters $\tilde{\theta}$ that are re-estimated on the bootstrapped moments give rise to a distribution $m(\tilde{\theta})$. These two moment distributions should have the same mean values, too; that is, $E[m(\tilde{\theta})] = E[m(\hat{\theta})]$ is supposed to hold true. Taken together, we get the equation $E\{[m(\tilde{\theta}) - \tilde{m}_T] - [m(\hat{\theta}) - m_T^{emp}]\} = 0$. Writing it in this form makes it clear that the moment conditions for the bootstrap version of the J test are to be demeaned by the second term in square brackets.

The corresponding MSM re-estimations take two types of variability into account. First, we allow for the variability in the generation of the data. To this end we apply the procedure in Appendix A1 that we already referred to above, which gives us a collection $\{m_T^b\}_{b=1}^B$ of B moment vectors bootstrapped from the empirical time series of length T . Second, we allow for the sample variability in the model simulations by carrying out each estimation with a different seed c . Thus, practically, each bootstrap sample b has associated with it a random seed $c=c(b)$.

In addition, suppose that we have a large number of original estimations (1) on the empirical moment vector m_T^{emp} . Then, let us settle down on the random seed \bar{c} that yields their median loss. That is, $\hat{\theta}(\bar{c}, S)$ is our benchmark point estimate of the model. On this basis, the collection of the parameter re-estimates $\hat{\theta}^b$ is obtained as follows:

$$\begin{aligned} \hat{\theta}^b &= \arg \min_{\theta \in \Theta} J_g(\theta; b) := \arg \min_{\theta \in \Theta} g(\theta; b)' W g(\theta; b) , \\ g(\theta; b) &:= \{m[\theta; c(b), S] - m_T^b\} - \{m[\hat{\theta}(\bar{c}, S)] - m_T^{emp}\} , \quad b = 1, \dots, B \end{aligned} \tag{5}$$

For the bootstrapped J test it remains to consider the distribution of the values $J_g^b = J_g(\hat{\theta}^b; b)$. At the conventional significance level, the model with estimate $\hat{\theta}(\bar{c}, S)$ would have to be rejected as being inconsistent with the data if the corresponding loss $\hat{J} = J[\hat{\theta}(\bar{c}, S); m_T^{emp}, \bar{c}, S]$ from (1) exceeds the 95% quantile of the distribution $\{J_g^b\}$, otherwise the model would have passed the test. In this way we can also readily construct a p -value of the model. It is given by the value of p that equates the $(1-p)$ -quantile of the distribution $\{J_g^b\}$ to \hat{J} , which says that if \hat{J} were employed as a benchmark for model rejection, then p is the error rate of falsely rejecting the null hypothesis that the model is the true DGP. Hence, in short, the higher this p -value the better the fit. Certainly, it

¹¹ See Hall and Horowitz (1996, p. 897) and Allen et al. (2011, p. 112) for the following recentring procedure.

may not be forgotten that, besides sample variability, this evaluation is conditional on the specific choice of the moments that the model is required to match.¹²

Regarding the confidence intervals that, from the re-estimations, can be built for the i -th component of the originally estimated parameter vector $\hat{\theta}(\bar{c}, S)$, two versions can be set up: the standard percentile interval and Hall's percentile confidence interval. Hall's method has the advantage that it is asymptotically correct, but it may violate the admissible range of a parameter. Therefore we use Hall's interval if no such violation occurs and the standard interval otherwise. The details are spelled out in Appendix A2, equations (A1) and (A2).

Having available a reliable estimate $\hat{\theta}(\bar{c}, S)$, the minimizations in (5) can safely do without further global explorations. That is, we can directly use the simplex algorithm for them and let it start from a simplex around $\hat{\theta}(\bar{c}, S)$. Nevertheless, although a single minimization with several re-initializations does not take very long, the computational effort for, say, a battery of 1000 re-estimation accumulates considerably. For this reason we will only make use of this method for our 'showcase' estimations. For the other versions we content ourselves with the point estimations based on the same random seed \bar{c} as the median loss of the best model version. They will not be perfect but good enough to indicate the basic tendencies to us.

3. The two Keynesian approaches

3.1. *The New-Keynesian model*

The elementary New-Keynesian models are constituted by three equations determining output, inflation and the rate of interest. It should be made clear from the beginning that we are concerned with these variables in gap form, that is, with their deviations from some trend values.¹³ The exact specification of the latter depends on the particular microfoundation from which the equations are derived. In the simpler versions at a textbook level, the 'trend' is just a fixed steady state value, which for the inflation rate will furthermore be equal to zero. More generally, the trend may be given by some moving frictionless output equilibrium or by time-varying target rates of inflation and interest set by the central bank. To allow for a wider range of theoretical interpretations, we will treat the trend as a purely exogenous issue.¹⁴ It can therefore remain in

¹² An application of this bootstrap approach to another New-Keynesian model is Fève et al. (2009), although they only estimate a subset of the parameters. Their moments are given by the impulse-response functions to a monetary policy shock, so that they can be analytically computed and the p -value is not plagued with the sample variability from $c=c(b)$.

¹³ The literature is sometimes rather sloppy in this respect. Cogley et al. (2010, p. 43, fn 1), for example, remarked when discussing inflation persistence that it is not always completely plain in the literature whether the focus is on raw inflation or the inflation gap.

¹⁴ Ireland (2007) and, more ambitiously, Cogley and Sbordone (2008) are two proposals of an endogenous determination of the central bank's moving inflation target. Ireland (p. 1864), however,

the background, so that, with respect to period t , the formulation of the model directly refers to the output gap y_t , the inflation gap $\hat{\pi}_t$, and the interest rate gap \hat{i}_t .¹⁵ On this basis the New-Keynesian model, labelled ‘NK’, that we are going to estimate reads:

$$\begin{aligned}
y_t &= \phi_y E_t y_{t+1} + (1-\phi_y) y_{t-1} - \tau (\hat{i}_t - E_t \hat{\pi}_{t+1}) + v_{y,t} & (0 \leq \phi_y \leq 1) \\
\hat{\pi}_t &= \phi_\pi \beta E_t \hat{\pi}_{t+1} + (1-\phi_\pi) \beta \hat{\pi}_{t-1} + \kappa y_t + v_{\pi,t} & (0 \leq \phi_\pi \leq 1) \\
\hat{i}_t &= \mu_r \hat{i}_{t-1} + (1-\mu_r) (\mu_\pi \hat{\pi}_t + \mu_y y_t) + \varepsilon_{r,t} & \text{(NK)} \\
v_{\pi,t} &= \rho_\pi v_{\pi,t-1} + \varepsilon_{\pi,t} \\
v_{y,t} &= \rho_y v_{y,t-1} + \varepsilon_{y,t}
\end{aligned}$$

As already mentioned, the underlying time unit is one quarter. All of the parameters are supposed to be nonnegative, but regarding the order of magnitude of some of them it should be pointed out that the rates of inflation and interest will be annualized. Clearly, (NK) is a hybrid model with forward-looking as well as backward-looking elements in the dynamic IS relationship (the first equation) and the Phillips curve (the second equation, where β is the usual discount factor). As it is standard in this framework, the third equation is a Taylor rule with interest rate smoothing and contemporaneous reactions to inflation and output. It goes without saying that the smoothing coefficient μ_r is contained in the unit interval.

The same property for ϕ_π and ϕ_y in the first two equations of (NK) is less obvious, which is the reason why it is explicitly scheduled in the equation block. As a matter of fact, the common specifications of the IS equation are based on a habit persistence parameter χ between 0 and 1 that would give rise to a composite coefficient $\phi_y = 1/(1+\chi) \geq 1/2$. Concerning the Phillips curve, the two main proposals to establish a positive coefficient on lagged inflation $\hat{\pi}_{t-1}$ are an introduction of rule-of-thumb price setters (following Galí and Gertler, 1999), or of an indexation rule on the part of the firms that currently are not permitted to re-optimize their price (following Smets and Wouters, 2003, and Christiano et al., 2005). In both cases the resulting coefficient ϕ_π on expected inflation must exceed one-half (roughly), too.¹⁶

While the standard microfoundations are based on zero inflation and the absence of output growth in the steady state, there is more recent work relaxing these assumptions. The price to be paid for this step towards realism are (much) more complicated rela-

concludes from his estimations that still “considerable uncertainty remains about the true source of movements in the Federal Reserve’s inflation target”.

¹⁵ In the verbal discussions we may nevertheless omit mentioning the ‘gap’ and, for instance, simply speak of inflation.

¹⁶ Despite their wide acknowledgement, these stories do not perfectly meet the high and rigorous standards of New-Keynesian modelling. Two examples of well-established authors who rather characterize them as an *ad hoc* amendment are Fuhrer (2006, p.50) and Rudd and Whelan (2005, pp. 20f), which is the more detailed version of Rudd and Whelan (2007, p.163, fn 7).

tionships.¹⁷ System (NK) in its desirable clarity could nevertheless still be viewed as an approximation to the more ambitious generalizations in this field of macroeconomic theory, arguing that a good understanding of its properties should come in handy before engaging in more specialized versions.

The only microfounded approach we know of that admits a dominance of backward-looking behaviour, in the sense that ϕ_π or ϕ_y are also permitted to be less than one-half, is that of Branch and McGough (2009).¹⁸ Their paper sets up an economy in which rational expectations coexist with boundedly rational expectations. Introducing a set of axioms for consistent heterogeneous expectations and their aggregation, the authors derive a purely forward-looking Phillips curve and IS equation where, however, the expectation operator is a weighted average of the two types of expectations. Thus, in the present notation, ϕ_π and ϕ_x can be interpreted as the population shares of the firms and households, respectively, that entertain rational expectations.¹⁹ Moreover, the exact form in (NK) with just one lagged inflation rate and output gap on the right-hand side is obtained if the group of the non-rational agents is supposed to have static expectations, that is, if (perhaps for lack of a better or more reliable idea) they do not expect the value of the variable from their last observation to change until the next quarter.

The foundation proposed by Branch and McGough (2009) appears to be an attractive alternative to motivate a general formulation like (NK); in particular, if estimations would prefer lower values of ϕ_π or ϕ_y outside the conventional range. Note that (NK) would even be well-defined if ϕ_π or ϕ_y were zero and all of the firms or households, respectively, were purely backward-looking. On the other hand, besides the fact that this economy is still stationary and does not allow for long-run inflation, and besides thinking of less naive than just static expectations, a critical point of the approach is a justification of why the population shares ϕ_π and ϕ_y should remain fixed over time. Extensions of (NK) taking this issue into account are easily conceivable but beyond the scope of the present paper.²⁰

¹⁷ Contributions with positive trend inflation are Bakshi et al. (2003), Sahuc (2006), Ascari and Ropele (2007), Ireland (2007), and Cogley and Sbordone (2008), among others. Mattesini and Nisticò (2010) incorporate positive trend growth.

¹⁸ Fuhrer (2006) is a study of a Phillips curve similar to that in (NK) which likewise does not require, or imply, that ϕ_π (in the present notation) is positively bounded away from zero. In his discussion of the inflation persistence effects that strongly favours low values of ϕ_π , however, the author does not care about a rigorous structural interpretation of these situations.

¹⁹ To be scrupulous, one of the axioms implies that the so-called rational agents are not fully rational in the conventional sense (Branch and McGough, 2009, p. 1045).

²⁰ In another paper, Branch and McGough (2010), the population shares are modelled as endogenously changing over time according to a measure of evolutionary fitness, which includes a (relatively higher) cost of forming rational expectations. It is, however, another question beyond the scope of the present paper whether this conceptual progress would also yield a significantly better econometric performance.

Turning to the stochastic part of the model, each of the three core equations is augmented by an exogenous shock process. In the Phillips curve, serial correlation in the random shocks is included as a possible additional source of inflation persistence. This is in contrast to a common estimation practice that from the outset assumes either white noise shocks in a hybrid Phillips curve, or purely forward-looking price setting behaviour in combination with autocorrelated shocks.²¹ Here we refrain from this prior decision and try to find out whether the data indicate a certain tendency.

For symmetry, the IS equation should be treated in the same way (although this point seems to have been somewhat neglected in the literature). The shocks in the interest rate reaction function, i.e. the Taylor rule, are i.i.d. and, as it is standard in the three-equations framework, persistence is only supposed to be brought about by the lagged rate of interest. The white noise innovations $\varepsilon_{z,t}$, for $z = i, \pi, y$, are mutually uncorrelated and normally distributed with variance s_z^2 . Certainly $0 \leq \rho_\pi, \rho_y < 1$ in the AR(1) processes in the last two equations of (NK).

Over the relevant range of the numerical parameters, determinacy of (NK) will prove to be no problem.²² Under this condition, there are two uniquely determined matrices $\Omega, \Phi \in \mathbb{R}^{3 \times 3}$ (Ω being a stable matrix) such that, with respect to $x = (i, y, \pi)'$, $v = (v_i, v_y, v_\pi)'$, $\varepsilon = (\varepsilon_i, \varepsilon_y, \varepsilon_\pi)'$ and $N \in \mathbb{R}^{3 \times 3}$ the diagonal matrix with entries $(0, \rho_y, \rho_\pi)$, the reduced-form solution to (NK) is given by an ordinary ‘backward-looking’ stochastic difference equations system:

$$\begin{aligned} x_t &= \Omega x_{t-1} + \Phi v_t \\ v_t &= N v_{t-1} + \varepsilon_t \end{aligned} \tag{6}$$

Calculation of the matrices Ω and Φ is a routine matter in New-Keynesian economics, the details of which can therefore be omitted. Equation (6) is here quoted for a better comparison with the structure of our Old-Keynesian model, which is presented next.

3.2. *The Old-Keynesian model*

The Old-Keynesian model that we want to put forward as an alternative to the New-Keynesian model (NK) abjures the representative and intertemporally optimizing agents and their rational expectations. The central notion is rather that of an average business sentiment of firms that either have an optimistic or pessimistic attitude towards the fixed capital investment decisions they face. Jointly with the dynamic law that is to govern its changes over time, this sentiment variable is meant to capture some of the meaning

²¹ In similar models to ours, examples of i.i.d. shocks in a hybrid Phillips curve are Lindé (2005), Cho and Moreno (2006) or Salemi (2006), while the purely forward-looking models studied by, e.g., Lubik and Schorfheide (2004), Del Negro and Schorfheide (2004), Schorfheide (2005) permit some persistence in the shock process. These references have been chosen from the discussion in Schorfheide (2008; see p. 421, Table 3).

²² Determinacy means that, given a sequence of the random shocks, the model has a unique solution that remains bounded over time.

of Keynes' famous 'animal spirits' in a highly stylized manner. These issues and the specification details of this and other model elements are discussed in greater depth in Franke (2011a). Here we content ourselves with a short recapitulation of the model. In addition, we introduce a few extensions in order to put it on a similar footing to the New-Keynesian model.

The present model will deviate from Franke (2011a) in three ways: (i) the continuous-time formulation is transformed into discrete time, where the time unit is again a quarter; (ii) the Taylor rule is the same as in (NK), which slightly extends the version without interest rate smoothing in Franke (2011a); (iii) this model, which is still deterministic, will be investigated first, but subsequently random shocks will be included as well.

To begin with the model description, it has already been indicated that the central dynamic variable is the business sentiment, or the business climate, in the firm sector. Denoting it by b_t , it is predetermined in period t and given by the difference between the optimistic and pessimistic firms in this period, scaled by their total number. Hence $-1 \leq b_t \leq 1$; the extreme values -1 and $+1$ are attained if all firms are pessimistic and optimistic, respectively; and $b_t = 0$ if there are as many optimists as pessimists. The optimistic firms let their capital stock increase at a given and common high rate of growth, the pessimistic firms at a given and common low rate of growth. This yields the aggregate capital growth rate as a linearly increasing function of the business climate variable.

Total output (or the output-capital ratio) is determined by fixed investment (or the aggregate capital growth rate, respectively) *via* a multiplier relationship in a temporary IS equilibrium. Specifying the output gap y_t as the percentage deviations of this output-capital ratio from the value that would prevail in a balanced state $b_t = 0$, y_t is easily seen to be directly proportional to b_t with a proportionality factor $\eta > 0$. Lastly, for the general case, we add a normally distributed demand shock $\varepsilon_{y,t}$ to this relationship. Since the core of the model should contain sufficient mechanisms to generate any desired degree of persistence, we forego the option of serial correlation in this and the other shock processes. Thus, given b_t in period t , we have

$$y_t = \eta b_t + \varepsilon_{y,t} \quad (7)$$

Our Phillips curve looks like a purely forward-looking New-Keynesian curve except for one crucial conceptual difference: it does not refer to expected inflation for the next quarter but to a so-called inflation climate π_t^c , which is the firms' general aggregated assessment of inflation over a longer time horizon.²³ This inflation climate is treated as a predetermined variable, too. Complementing it with supply or cost push shocks $\varepsilon_{\pi,t}$, the Phillips curve thus reads,

²³ Franke (2007) discusses the ideas underlying this variable at greater length and also compares its 'microfoundations' with heterogenous firms to the New-Keynesian way of deriving a Phillips curve.

$$\pi_t = \pi_t^c + \kappa y_t + \varepsilon_{\pi,t} \quad (8)$$

Output and inflation being determined by (7) and (8), the nominal interest rate i_t is subsequently obtained from the Taylor rule with interest rate smoothing. Just as the Phillips curve, we write it here in level form. To this end, we now explicitly introduce the central bank's target rates of inflation and interest, π^* and i^* , which are supposed to be constant. This gives us

$$i_t = \mu_i i_{t-1} + (1-\mu_i) [i^* + \mu_\pi (\pi_t - \pi^*) + \mu_y y_t] + \varepsilon_{i,t} \quad (9)$$

A little bit sloppily, eqs (7)–(9) can be said to constitute the static part of the model. The second part describes the updating of the two climate variables b_t and π_t^c . To begin with the simpler rule, the inflation climate is assumed to adjust, instantaneously or with some delay, to a benchmark that is given by a weighted average of current inflation and target inflation,

$$\pi_{t+1}^c = \pi_t^c + \alpha_\pi [\gamma \pi^* + (1-\gamma) \pi_t - \pi_t^c] + \chi_{\pi y} \varepsilon_{y,t} \quad (10)$$

Certainly, both the adjustment speed α_π and the weight γ are contained in the unit interval, $0 \leq \alpha_\pi, \gamma \leq 1$. The higher γ , the stronger the confidence of firms that inflation will soon return to its target. For this reason, and for a more formal argument given in Franke (2007, p. 22) or Franke (2011a, Section 3), the parameter γ can be interpreted as representing the credibility of the central bank.²⁴

Another influence on the inflation climate in (10) may be exerted by the demand shocks. That is, positive demand shocks $\varepsilon_{y,t}$ in (7) could also induce the firms to expect generally higher inflation rates, not necessarily immediately but with some delay.²⁵ As a consequence, the actual rate of inflation π_t is now affected by two shocks, directly by $\varepsilon_{\pi,t}$ in the Phillips curve and indirectly, from the previous period *via* the climate variable π_t^c , by $\chi_{\pi y} \varepsilon_{y,t-1}$. It may also be said that positive values of $\chi_{\pi y}$ indicate ‘crossover effects’ of the demand shocks, from output to inflation.

The motivation for this higher flexibility of the model springs from the structure of system (6), which solves the New-Keynesian model. If the present model, when com-

²⁴ Complementarily, $(1-\gamma)$ can be interpreted as measuring the inflation persistence in the Phillips curve; see Franke (2007), p. 22. Assimilating an idea from de Grauwe’s (2010) bounded rationality version of the New-Keynesian three-equations model, a more ambitious modelling could introduce two types of inflation attitude between which the firms may switch according to some rule. If one attitude is given by π^* and the other by π_{t-1} , the role of the coefficient γ in (10) would be taken over by a long-run time average of the share of target inflation believers. The charm of this generalization is that such a central bank credibility would be endogenously determined. On the other hand, for the present model we will be interested in ‘what the data say’ on a specific numerical value of our fixed γ .

²⁵ Within a one-sector world, this would be an argument of self-fulfilling expectations. It would perhaps appear less artificial in a multi-sectoral setting, when firms in one sector learn about positive demand effects in another sector, notice that these firms increase inflation for the goods they sell, and then expect some spill-over effects from there to the other sectors of the economy, including their own.

pleted, is analogously decomposed into a deterministic part and a linear stochastic part of the form $\tilde{\Phi}\varepsilon_t$ for a suitable matrix $\tilde{\Phi}$ with some zero rows, the 3×3 nonzero core of $\tilde{\Phi}$ would be a diagonal matrix when $\chi_{\pi y} = 0$, whereas the entries in the matrix Φ in (6) are all nonzero. It could thus be said that the shock-free version of eq. (10) puts the stochastic structure of the present model at a certain disadvantage in comparison with its New-Keynesian competitor. Admitting $\chi_{\pi y} > 0$ is meant to make up for this. In this respect it may be noted that in the matrix $\tilde{\Phi}$ many zero entries would still remain; on the other hand, the nine entries in Φ are not all independent, either. Nonetheless, in the end we will leave it to the estimations whether or not they call for a positive value of $\chi_{\pi y}$.

The adjustments of the business climate b_t are conceptually more involved. They are based on the uniform transition probabilities prob_t^{-+} and prob_t^{+-} with which, within the current quarter t , a single pessimistic (optimistic) firm switches to an optimistic (pessimistic) investment attitude. Assuming a sufficiently large number of firms, the probabilistic elements become negligible at the macro level. The changes in the business climate b_t are then easily seen to be described by the following deterministic adjustment equation,

$$b_{t+1} = b_t + (1-b_t)\text{prob}_t^{-+} - (1+b_t)\text{prob}_t^{+-} \quad (11)$$

The probabilities in (11) are functions of a so-called feedback index f_t , where higher values of f_t increase the probability of switching from pessimism to optimism. Assuming symmetry and linearity with respect to *relative* changes, the transition probabilities of the firms are given by

$$\begin{aligned} \text{prob}_t^{-+} &= \text{prob}^{-+}(f_t) = \min\{1, \alpha_b \exp(f_t)\} \\ \text{prob}_t^{+-} &= \text{prob}^{+-}(f_t) = \min\{1, \alpha_b \exp(-f_t)\} \end{aligned} \quad (12)$$

The coefficient α_b measures the general responsiveness of the transition probabilities to the arrival of new information, as it is summarized by f_t . It may thus be characterized as a flexibility parameter (Weidlich and Haag, 1983, p.41). Note that even in the absence of active feedback forces in the index f_t , or when the different feedback variables behind f_t neutralize each other such that $f_t = 0$, the individual firms will still change their attitude with a positive probability. These reversals, which can occur in either direction, are ascribed to idiosyncratic circumstances, and their probability per quarter is given by $\text{prob}_t^{-+} = \text{prob}_t^{+-} = \alpha_b > 0$.

In the determination of the feedback index f_t , two components are distinguished. The first grasps the idea of herding, saying that the probability of switching from pessimistic to optimistic increases as the population share of the firms that are already optimistic increases. The second component can be conceived of as a counterpart of the central stabilizing effect in the so-called new macroeconomic consensus, which is the real interest rate transmission mechanism of monetary policy. It is correspondingly assumed that

optimism in the investment decisions tends to decline as the real interest rate increases. Combining the components in a linear way, we have $f_t = \phi_b b_t - \phi_i (i_t - \pi_t - \rho^*)$ so far, where $\phi_b, \phi_i \geq 0$ and ρ^* is the long-run equilibrium real rate of interest (from which i^* in the Taylor rule derives as $i^* = \rho^* + \pi^*$).

In addition, we want to allow for a possible influence of the random shocks $\varepsilon_{\pi,t}$ in the Phillips curve. Viewing them as cost pushes, positive shocks somewhat darken the prospects of the firms' future profitability and so tend to weaken their optimism. Introducing another coefficient $\chi_{f\pi} \geq 0$ and augmenting the equation above by this effect, which in the end is a crossover random effect from inflation to output, the following functional expression for the feedback index f_t is postulated:

$$f_t = \phi_b b_t - \phi_i (i_t - \pi_t - r^*) - \chi_{f\pi} \varepsilon_{\pi,t} \quad (13)$$

Equation (13) completes the description of the general model. It has a simple recursive structure. There are three predetermined variables at the beginning of period t : b_t , π_t^c and i_{t-1} . From them, eqs (7), (8), (9) compute successively the output gap, the inflation rate, and the interest rate for that quarter. Subsequently, the climate variables for the next quarter are obtained: π_{t+1}^c from (10) and b_{t+1} from, in that order, (13), (12), (11).

In the following we will speak of model (7)–(13) as a model of sentiment dynamics and use the acronym SD for that.²⁶ Our estimations will be concerned with three different versions: the deterministic one; a stochastic versions with only the diagonal shocks, so to speak; and the model in its full generality, which includes the crossover shock effects. For easier reference, we denote these cases as:

$$\begin{aligned} \text{SD-1:} & \text{ eqs (7)–(13) with } \varepsilon_{z,t} \equiv 0 \quad (z = i, \pi, y); \\ \text{SD-2:} & \text{ eqs (7)–(13) with } \varepsilon_{z,t} \sim N(0, \sigma_z^2), \quad \chi_{\pi y} = \chi_{f\pi} = 0; \\ \text{SD-3:} & \text{ eqs (7)–(13) with } \varepsilon_{z,t} \sim N(0, \sigma_z^2), \quad \chi_{\pi y}, \chi_{f\pi} \geq 0. \end{aligned} \quad (\text{SD})$$

The option of incorporating herding with $\phi_b > 0$ is certainly an alluring feature of the model, especially if high values of ϕ_b destabilize the steady state position and it is the nonlinear component in eq. (12) that keeps the dynamics within bounds (see Franke, 2011a, for a detailed analysis of this property). However, the socio-psychological dimensions behind the herding coefficient let its assumed constancy appear somewhat doubtful. It seems perhaps more likely that ϕ_b is varying over the business cycle and, in particular, it may be heavily affected by special exogenous events. Of course, ϕ_b is treated here as a constant for the sake of simplicity, but it is an open question if, or what kind of, formal estimations would be able to identify a time-varying coefficient, even if it indeed played a stronger role over certain episodes in the real world.

²⁶ Although the model will later be compared to the New-Keynesian model, an acronym for 'Old-Keynesian' might be in dispute as a matter of good or bad taste.

In this perspective we may distinguish between a strong and a weak version of the model. The strong version prevails if ϕ_b is markedly positive, so that we could informally speak of a “significant” contribution of herding to the dynamics. A confirmation of this Old-Keynesian disequilibrium component with its appeal to the ‘animal spirits’ would be a conceptually attractive result. On the other hand, a model without herding, i.e. with ϕ_b equal or close to zero, may be labelled a weak version of (SD). Here the only direct feedback on the business sentiment and thus the output gap is the real rate of interest, which together with a Taylor rule and a sort of expectations-augmented Phillips curve is the central stabilizing feedback channel in the so-called new macroeconomic consensus (NMC). Apart from the different treatment of expectations, (SD) is then on a similar footing to the New-Keynesian model. In this case the main question is for the relative moment matching performance of the rational expectations and our purely backward-looking variant of NMC.

4. Estimations of the Great Inflation period

4.1. Preliminaries

The estimations of both the New-Keynesian and Old-Keynesian models are concerned with three observable macroeconomic variables: prices and output of the private sector, and the nominal interest rate set by the central bank. Following usual practice, our empirical counterpart of the latter is the federal funds rate. Considering the economic background of the models, however, it seems appropriate to us to deviate from much of the estimation literature and work with price and output data from the firm sector rather than the GDP magnitudes (the firm sector being essentially nonfinancial corporate business). Appendix A3 gives the data source from which output and the price deflators have been obtained, and a URL from which our gap variables can be directly downloaded.

The modelling has assumed a given trend behaviour and so it is only straightforward to employ a deterministic trend concept for the empirical variables. As it should also reflect the business cycle frequencies of the fluctuations, we choose the convenient Hodrick-Prescott filter with the standard smoothing parameter $\lambda = 1600$ for quarterly data; the same for output, inflation and the interest rate. The total sample period covers the years from 1960 to 2007.²⁷ However, over these years one observes great changes in the general variability of the three variables and partly also in the qualitative profiles of their cross-covariances, which not only holds true for the variables in level but also in gap form. This makes it necessary to subdivide the period into two subsamples. They are commonly referred to as the periods of the Great Inflation (GI) and the Great Moderation (GM), where we specify the former by the interval 1960:1–1979:2 (78 observations) and the

²⁷ The Hodrick-Prescott trend itself is computed over a longer period, to avoid end-of-period effects.

latter by 1982:4–2007:2 (99 observations). The time inbetween is excluded because of its idiosyncrasy (Bernanke and Mihov, 1998). The need of the subdivision becomes obvious from the autocovariance diagrams below. To give an immediate and sufficient example, the standard deviation of the annualized firm inflation gap in GI is twice as high as in GM: 2.12% versus 1.04%.

The estimations in the present section concentrate on the Great Inflation period. This section will be rather extensive since most of the additional methodological issue arising in the course of the discussion of the results will be dealt with here. Section 5, which covers the Great Moderation, can then be somewhat shorter.

The main focus of our investigations will be on the Old-Keynesian model, which has not been subjected to a formal estimation before. Concerning the deterministic part of this sentiment dynamics, there are 10 parameters to estimate: four determining the output gap dynamics ($\alpha_b, \phi_b, \phi_i, \eta$), three determining the inflation dynamics ($\kappa, \alpha_\pi, \gamma$), and the three policy parameters in the Taylor rule (μ_i, μ_π, μ_y). For the simple stochastic version (SD-2), the standard deviations of the three shock variables are to be added, and for the more elaborate version with the cross-over shock effects there are two more coefficients, namely, $\chi_{\pi y}$ and $\chi_{f\pi}$ in eqs (10) and (13).

A first problem with the parameters should be considered right at the beginning. If prob_t^{-+} , prob_t^{+-} in (12) were linear functions of the feedback index f_t , the flexibility parameter α_b could not possibly be identified: multiplying it by an arbitrary number and dividing ϕ_b, ϕ_i and $\chi_{f\pi}$ by the same number would not alter anything. Although the exponential function introduces a nonlinearity in these transition probabilities, it is more of a global nature. Locally around $f_t=0$ its curvature is not very pronounced, so that an identification of α_b may remain an arduous task. We therefore prefer to fix this coefficient exogenously. To this end, recall its interpretation in the remark on eq. (12), according to which in the hypothetical absence of other influences a firm, for idiosyncratic reasons, would switch its attitude with a probability α_b per quarter. In this respect let us assume that this kind of switching would occur every two years (8 quarters) on average. Hence,

$$\alpha_b = 1/8 = 0.125 \quad (14)$$

Regarding the steady state parameters, target inflation π^* is set equal to 2.5% and the associated equilibrium rate of interest i^* equal to 5% (both rates being annualized as already mentioned above), which is just a matter of scaling.

4.2. *The deterministic version of the Old-Keynesian model*

The estimations begin with the most elementary sentiment dynamics (SD-1) without any random forces. An obvious steady state position is given by $y=0$, $\pi=\pi^*$ and $i=i^*$. For the continuous-time model with the simplified Taylor rule ($\mu_i = 0$), in Franke (2011a) a broad range of parameters with a sufficiently high herding coefficient ϕ_b (in the present

notation) was shown to exist that ensure uniqueness of this equilibrium, render it locally repelling, and give rise to a unique and globally attractive limit cycle. This feature carries over to the present formulation in discrete time and the slightly more general Taylor rule (9). Such a limit cycle can be viewed as the model's representative business cycle.²⁸ Our first MSM estimation then searches for a numerical parameter combination such that the 78 autocovariances from the list in (4) that it induces are as close as possible to their empirical counterparts. The solution of the corresponding minimization problem (1) with $J = J^{(78)}$ is reported as Scenario SD-1a in Table 1.

Before trying to assess whether a minimal loss of $J^{(78)} = 71.45$ is more indicative of a good or a bad match, we should have a look at the cycles thus generated. The time series of the output gap y_t and the inflation rate π_t are shown in the upper two panels in Figure 1. With a bit of more than five years, they may perhaps exhibit acceptable amplitudes and an acceptable period, but the pattern of the cyclical motions is clearly unsatisfactory. Output in reality simply does not crawl along a ceiling for roughly two years, then suddenly drops down on a floor and proceeds creeping there for another two years. It is thus also superfluous to comment on the tent-shape pattern of the inflation rate.

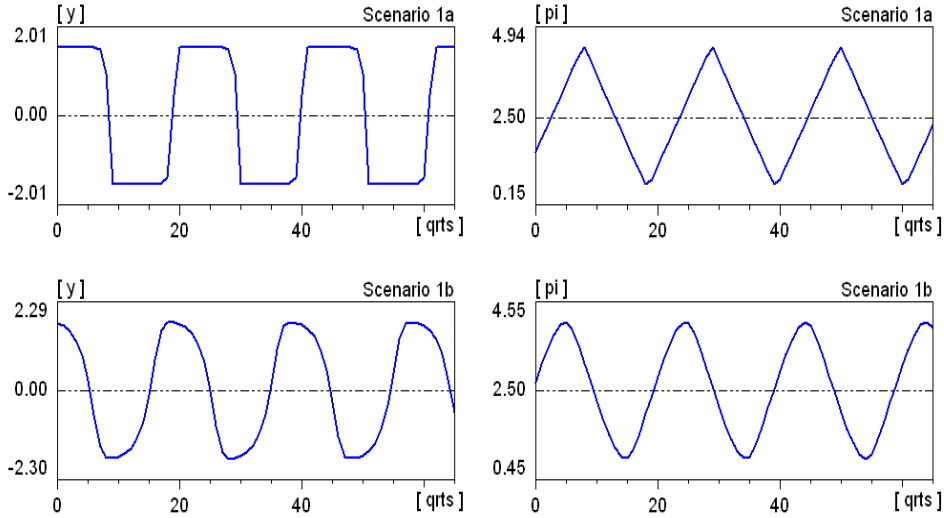


Figure 1: Time series y_t, π_t resulting from Scenario SD-1a and SD-1b.

Responsible for this behaviour is the fact that the constraints in the transition probabilities, $\text{prob}_t^{-+}, \text{prob}_t^{+-} \leq 1$ in (12), become binding over these stages. If we look at the specification of the feedback index f_t in eq. (13) then, owing to the high values of

²⁸ To be more precise, there is a unique one-dimensional manifold P in the three-dimensional space towards which all (non-degenerate) trajectories converge in the sense that they move on P in the limit, although the limit motion itself may not be strictly periodic but only quasi-periodic.

	Scenario Model SD						Model NK	
	1a	1b	2a	2b	3a	3b	a	b
ϕ_b	4.937	0.726	0.563	0.564	0.000	0.159	—	—
ϕ_i	13.834	2.962	0.067	0.726	0.238	0.307	—	—
η	1.545	1.841	99.729	3.850	8.606	5.336	—	—
κ	0.241	0.243	0.245	0.454	0.620	0.657	0.113	0.000
α_π	1.000	1.000	1.000	0.356	0.440	0.375	—	—
γ	0.000	0.000	0.065	0.000	0.000	0.000	—	—
μ_i	0.757	0.717	0.000	0.394	0.748	0.772	0.435	0.936
μ_π	1.481	1.460	1.208	1.578	1.504	1.564	1.374	3.695
μ_y	0.792	0.627	0.000	0.000	0.773	0.869	0.086	4.977
σ_y	—	—	0.000	0.351	0.238	0.045	0.428	0.543
σ_π	—	—	0.520	0.963	1.340	1.525	0.429	1.822
σ_i	—	—	0.401	0.331	0.000	0.000	0.000	0.283
$\chi_{\pi y}$	—	—	—	—	0.576	3.074	—	—
$\chi_{f\pi}$	—	—	—	—	0.430	0.551	—	—
$J^{(78)}$	71.45	75.15	47.70	68.81	32.22	33.35	37.13	107.45
$J^{(79)}$	999.00	75.15	47.70	68.92	32.22	33.38	—	—
$J^{(82)}$	—	—	333.04	98.53	46.78	43.08	286.68	119.04

Table 1: Estimations of models SD and NK (Great Inflation).

Note: Bold face figures indicate the type of loss function for which the scenario is optimal. High values of $J^{(79)}$ are truncated at 999. Underlying NK-a, NK-b and the four stochastic scenarios of SD is the same random seed \bar{c} : among 1000 estimations with different random seeds, this \bar{c} yields the median loss $J^{(82)}$ for Scenario 3b.

the (stabilizing) coefficient ϕ_i on the real rate of interest and the (destabilizing) herding coefficient ϕ_b , f_t actually becomes so large in modulus that it takes quite a while for it to return to more moderate values; and if it eventually does, it only takes two or three quarters until, with signs reversed, f_t soars to similarly high levels again. Apart from the unrealistic time series pattern, the resulting extreme probabilities are not very convincing, either.

These observations suggest introducing an additional moment m_{79} into the objective

function for the estimations. It considers the, as we may call them, “excess transition probabilities” $\alpha_b \exp(\pm f_t) - 1$ from (12) and penalizes the occurrence of positive values so heavily that the loss minimization procedure better seeks to avoid them, even at the cost of a worse match of the other 78 moments. Formally we proceed in three steps. First, m_{79} is specified as the average excess transition probability over the simulation horizon S (with respect to a given time path of the feedback index f_t , which in turn, of course, depends on the parameters θ and possibly a random seed c); second, its empirical counterpart is set equal to zero, to conform to the notation of the loss function (1); and third, we incorporate the penalty in the new diagonal element of the now (79×79) weighting matrix W , for which a value of 1000 turns out to be perfectly suitable. Thus,

$$\begin{aligned} m_{79} &= \frac{1}{S} \sum_{t=1}^S \max\{0, \alpha_b \exp(f_t) - 1\} + \max\{0, \alpha_b \exp(-f_t) - 1\} \\ m_{T;79,79}^{emp} &= 0 \\ W_{79,79} &= 1000 \end{aligned} \tag{15}$$

The correspondingly augmented loss function is designated $J^{(79)}$,

$$\begin{aligned} J^{(79)} : \quad & \text{loss function constituted by the 78 autocovariances and weights} \\ & \text{from (2), (4), plus moment } m_{79} \text{ with weight } W_{79,79} \text{ from (15)} \end{aligned} \tag{16}$$

Applying the new function $J^{(79)}$ to Scenario SD-1a quantifies its deficient time series features; the value we compute is so unacceptably high that in Table 1 we arbitrarily truncated it at 999.

The re-estimation of the deterministic model with $J^{(79)}$, which forms our Scenario 1b, confirms that the model can do better. As the table shows, adding the new criterion somewhat deteriorates the original matching, i.e. the loss $J^{(78)}$ from the autocovariances increases from 71.45 to 75.15. However, this seems a relatively low price for the total success concerning the excess transition probabilities, which have practically vanished (the 79th component of the loss is practically zero). Comparing the parameters in Scenarios 1a and 1b it is seen that the general improvement is essentially brought about by considerably lower values of the two sentiment parameters ϕ_b and ϕ_i . They nevertheless balance in a certain way, such that the (repelling) instability of the steady state is maintained. The smooth oscillations of Scenario 1b that we obtain are documented in the lower two panels of Figure 1. The slight asymmetry in the output gap proves that the nonlinearities in function (12) for the transition probabilities do take some effect in the outer regions of the state space.

On this sound basis we can now ask what is behind the pure number of the minimized loss $J^{(79)} = 75.15$. Figure 2 presents the profiles of the nine auto- and cross-covariances of the three single variables i_t , y_t , π_t over a lag horizon of 20 quarters, which—it may be taken into account—is longer than the eight quarters underlying the estimations

themselves. The empirical covariances are given by the dotted lines with the shaded area of a 95% confidence band around them.²⁹ The profiles generated by Scenario 1b are plotted as the thin (blue) lines. At a glance and taking the confidence bands as a guideline, the matching can already be reckoned quite satisfactory. While there is a certain moderate tendency to leave the confidence band at the higher lags, within the first eight lags we observe only three cases with stronger deviations in this respect, all of which at a zero lag. That is, these are the simulated variances of the interest rate in the upper-left panel, of the output gap in the central panel, and of the inflation rate in the lower-right panel, where all of these moments are too low.

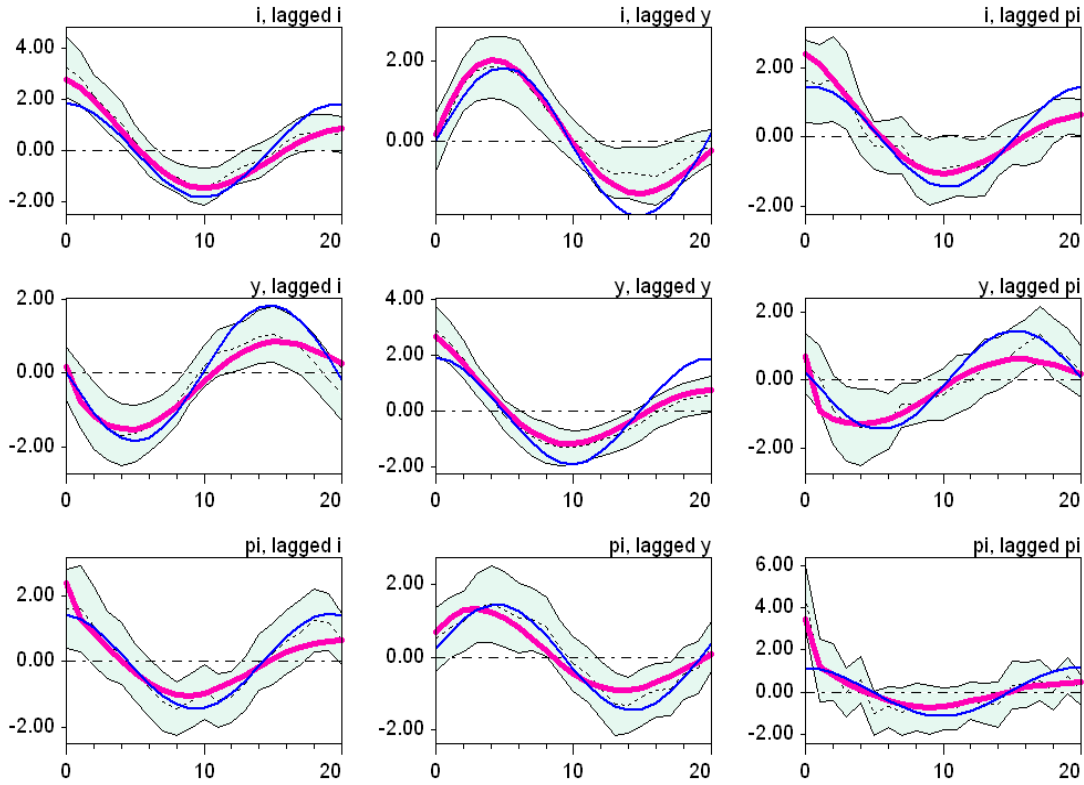


Figure 2: Auto- and cross-covariance profiles of the sentiment dynamics (SD-1b) and (SD-3b) (GI).

Note: Thin (blue) lines indicate Scenario 1b, bold (red) lines Scenario 3b. Shaded areas are the bootstrapped 95% confidence bands around the empirical moments (dotted lines).

Table 2 reports selected t -statistics of our different estimations. With respect to Scenario 1b it shows that, in terms of this criterion, the first two “violations” are not very

²⁹ The lower and upper bounds are the 2.5% and 97.5% quantiles from the bootstrap procedure described in Appendix A1. Besides, these confidence intervals tend to be narrower than those derived from a Newey-West estimator of the covariance matrix of the empirical moments.

serious. The most critical point of the deterministic model is rather its inability to match the variance of the inflation rate or, more precisely, to trace out the sudden drop of $\text{Cov}(\pi_t, \pi_{t-h})$ from $h = 0$ to $h = 1$; since afterwards the changes in the autocovariances remain relatively limited, the estimation decides to “sacrifice” the matching of the zero lag. It will have to be seen if the stochastic versions of the model can fare better in this respect.

As a secondary aspect we note in Table 2 that there is essentially one moment with which Scenario 1b pays for the more appropriate time series pattern *vis-à-vis* Scenario 1a. This is the variance of the output gap with its deterioration to $t_i = -2.30$, which the estimation of Scenario 1a had managed to keep inside the confidence interval. Regarding the autocovariances of the inflation rate, Scenario 1a and 1b share the same problem.

	Scenario Model SD						Model NK	
	1a	1b	2a	2b	3a	3b	a	b
<u>Covariance</u>								
(i_t, i_t)	-2.32	-2.38	-0.69	-0.11	-0.90	-0.84	-0.76	-0.46
(i_t, π_t)	-0.20	-0.22	1.08	1.02	1.28	1.31	1.01	-0.25
(y_t, y_t)	-1.41	-2.30	-0.64	-1.22	-0.37	-0.57	-0.34	-0.56
(y_t, π_{t-1})	-0.72	-0.64	-1.04	0.04	-2.21	-2.05	-0.84	-1.60
(π_t, y_{t-3})	-0.12	-0.07	0.05	-0.71	0.14	-0.01	0.10	-2.89
(π_t, π_t)	-4.42	-4.40	-3.49	-3.60	-1.87	-1.33	-3.57	0.65
<u>Raggedness</u>								
$R(i_t)$	—	—	2.31	2.07	0.17	0.33	-1.82	-0.65
$R(y_t)$	—	—	-0.12	0.31	1.61	0.75	-0.14	-0.14
$R(\pi_t)$	—	—	-16.73	-5.02	-3.45	-3.00	-15.86	-3.25
$J^{(78)}$	71.45	75.16	47.70	68.81	32.22	33.35	37.18	107.45
$J^{(79)}$	999.00	75.16	47.70	68.92	32.22	33.38	—	—
$J^{(82)}$	—	—	333.04	98.53	46.78	43.08	290.37	119.04

Table 2: Selected t -statistics of the estimations of models SD and NK (GI).

Note: Bold face figures indicate the most important shortcomings of the corresponding estimation.

4.3. The simple stochastic version

In this subsection we activate the demand shocks $\varepsilon_{y,t}$ in the output equation (7), the cost push shocks $\varepsilon_{\pi,t}$ in the Phillips curve (8), and the monetary policy shocks $\varepsilon_{i,t}$ in the Taylor rule (9). Hence the standard deviations σ_y , σ_π , σ_i have to be estimated as three additional parameters (the crossover effects *via* $\chi_{\pi y}$ and $\chi_{f\pi}$ are still switched off). This also means that we have to choose a random seed c . As remarked on eq. (5) above, we adopt the same $c = \bar{c}$ that will yield a median loss in our best estimation further below. The outcome from the minimization of our new loss function $J^{(79)}$ is reported as Scenario 2a in Table 1.

Comparing the loss $J^{(79)} = 47.70$ to that of Scenario 1b, it is seen that the random shocks can indeed achieve a nonnegligibly better match. More specifically, Table 2 with the t -statistics shows us a considerable improvement in the variances of the interest rate and the output gap, both of which are now inside the confidence intervals, and a more moderate improvement in the most critical moment, the variance of the inflation rate. Nonetheless, the latter is still clearly outside the confidence interval. Apart from that, note also that again there is no problem with excessively high transition probabilities as $J^{(78)} \approx J^{(79)}$ in Table 1.

If we look at the parameter estimates themselves, the extraordinarily high value of the proportionality factor η leaps to the eye. It is explained by the low value of ϕ_i in the sentiment dynamics which, in combination with the other coefficients, generates extremely narrow fluctuations of the business climate b_t (since f_t stays close to zero and so prob_t^{-+} and prob_t^{+-} in (11) differ only marginally). The high η then takes care that the thus induced fluctuations of the output gap y_t in eq. (7) are wide enough. While in this way the motions of the observable variables may appear acceptable, the implausible behaviour of the nonobservable b_t leaves an unpleasant aftertaste.

Instead of thinking about any immediate consequences of how to deal with this problem, we widen the horizon for the empirical regularities that we want the model to reproduce. The two middle panels in Figure 3 document the well-known fact that the empirical oscillations of the output gap y_t as a level variable are relatively smooth, whereas the motions of the inflation gap—as basically a first differences series of the (log) prices—are much more irregular. The upper two panels in the figure take the simulations of Scenario 2a and extract a time interval of the same length for the two variables. This example is sufficiently representative to illustrate that this scenario cannot account for the different time series patterns. It does not succeed in this respect even though the estimation gets along without any demand shocks (i.e., $\sigma_y = 0$ in Table 1). The output gap is nevertheless affected more indirectly by the monetary policy shocks, which take effect on the business climate *via* the feedback index f_t . They actually turn out to be so strong that the kind of raggedness in y_t and π_t looks about the same. Thus, compared to the empirical series, the simulated y_t seem to exhibit a similar degree of raggedness, and the simulated π_t are

clearly too smooth.

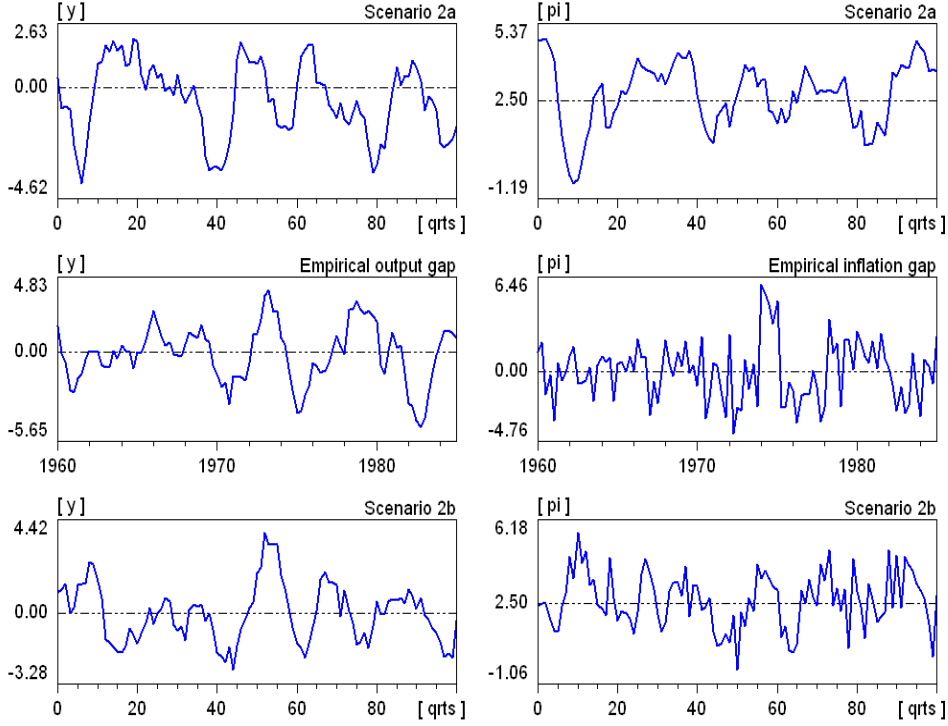


Figure 3: Empirical time series and y_t, π_t resulting from Scenarios 2a and 2b.

The human eye has no problem making these qualitative assessments. For a formal treatment, however, an unambiguous summary statistic is needed. To this end, let us begin with a straightforward characterization which would call a time series extremely ragged if it, from one observation to another, continuously alternates between increasing and decreasing. It appears “fairly” ragged if this occurs rather frequently, and the series is almost perfectly smooth if it steadily increases or decreases with the possible exception of some infrequent turning points. Furthermore, a time series where, for example, a strong increase between two observations is followed by a weak decline is at that stage less ragged than a series where the opposite changes are of a similar order of magnitude. These verbal descriptions of what our eye perceives without further reflection lead us to measure the raggedness of a time series $\{x_t\}_{t=1}^N$ ($N = T, S$) in the following way,³⁰

$$R_N(x_t) := \frac{\sum_{t=3}^N \max[0, -(x_t - x_{t-1})(x_{t-1} - x_{t-2})]}{\sum_{t=3}^N |(x_t - x_{t-1})(x_{t-1} - x_{t-2})|} \quad (N = T \text{ or } S) \quad (17)$$

³⁰ Strictly speaking, the notation $R_N(x_t)$ is slightly incorrect, but a correct one such as $R(\{x_t\}_{t=1}^N)$ would look rather cumbersome.

Clearly, this statistic is independent of the length and scale of the time series, and it can vary between unity and zero, indicating perfect raggedness and perfect smoothness, respectively. The index N for the sample period T or S can be omitted in the following since it will be easily understood from the context.

As a time average, $R(x_t)$ is an ordinary summary statistic that can be effortlessly added to the previous moments for a more ambitious moment matching. Also their variances can be bootstrapped from the empirical data in the same way as those in eq. (2) for our 78 autocovariances. Hence we may augment the present estimations by including the three moments

$$m_{80} = R(i_t), \quad m_{81} = R(y_t), \quad m_{82} = R(\pi_t) \quad (18)$$

which gives rise to the loss function $J^{(82)}$,

$$J^{(82)} : \quad \text{loss function constituted by } J^{(79)} \text{ plus moments } m_{80}, m_{81}, m_{82} \quad (19)$$

from (17) with their weights determined by (2)

Endowed with a measure of raggedness, Table 2 now reassures us that the output gap in Scenario 2a displays an even perfect behaviour in this respect. The interest rate is less satisfactory, and the t -statistic of the inflation rate points out a serious failure.

The next question therefore is if the model can alleviate this mismatch by minimizing the augmented loss function $J^{(82)}$. The answer is given by the outcome of Scenario 2b. First of all, this estimation brings about a higher degree of raggedness for the inflation rate, although it is still markedly lower than in empirical inflation. On the other hand, the raggedness in i_t and y_t is similar to Scenario 2a (see Table 2 and the illustration in the two bottom panels in Figure 3). Unfortunately, this improvement comes at the cost of a sizeable deterioration in the matching of the autocovariances, with $J^{(78)} = 68.81$ *versus* 47.70 for Scenario 2a. Actually, Scenario 2b thus falls back to a level that is not very much better than in the deterministic model version. The deterioration is a relatively general phenomenon; there is no single moment that is mainly responsible for it (this does not only hold for the few moments shown in Table 2). In addition, the slightly higher value of $J^{(79)}$ *versus* $J^{(78)}$ for SD-2b indicates rare cases where the transition probabilities prob_t^{-+} or prob_t^{+-} hit their upper bound.

Turning to the estimated parameters in Table 1, we see a couple of differences from the previous estimations. First and most importantly, the high value in Scenario 2a for η and the low value value for ϕ_i with their dubious implications for the sentiment dynamics do not carry over. Second, the slope parameter κ in the Phillips curve, which was very stable in the first three scenarios, has almost doubled. This goes along with a structural change in the adaptive inflation climate entering the Phillips curve. Before, we had $\alpha_\pi = 1$ and $\gamma \approx 0$, so that the climate was (nearly) equal to the rate of inflation most recently observed, $\pi_{t+1}^c = \pi_t$, whereas now, with $\alpha_\pi \approx 1/3$, its adjustments occur in a truly gradual manner. The credibility coefficient γ of the central bank remains nevertheless zero.

Third, Scenario 2b has a moderate interest rate smoothing in the Taylor rule *versus* no smoothing, $\mu_i = 0$, in Scenario 2a and considerably stronger smoothing in the deterministic cases. The zero responsiveness of the interest rate to output is maintained (even though it is again in stark contrast to the deterministic cases). Fourth, the present estimation differs from Scenario 2a in that it can no longer do without random perturbations on the demand side ($\sigma_y > 0$), and it also requires stronger effects of the cost post shocks.

In sum, as far as the simple stochastic model is concerned, these features render Scenario 2b more trustworthy than Scenario 2a.

4.4. *The stochastic version with crossover shock effects*

Although Scenario 2b is more satisfactory than Scenario 2a, can we be happy enough with its overall goodness-of-fit? Instead of further meditating on this question, we postpone it. We rather take a next step and augment the model by admitting the crossover random shock effects. That is, we add the coefficients $\chi_{\pi y}$ in (10) and $\chi_{f\pi}$ in (13) to the list of the parameters. This gives us model version SD-3 and a total of 14 parameters to estimate. We alternatively employ the loss functions $J^{(79)}$ and $J^{(82)}$, without and with the attempted matching of the raggedness in i_t , y_t , π_t , the minimization of which gives rise to Scenario 3a and 3b, respectively.

The corresponding two columns in Table 1 reassure us that both coefficients $\chi_{\pi y}$ and $\chi_{f\pi}$ come out with the correct positive sign, and they testify to a (very) strong improvement of these two estimations over the simple stochastic version of the model. The two parameters are also both remarkable in that the much better match of the autocovariances that they produce simultaneously implies a good match of the raggedness statistics, too. That is, as evidenced by the small difference in $J^{(79)}$ for Scenario 3a and 3b, now only a very low price has to be paid in terms of a deterioration of the autocovariances if the model is additionally desired to reproduce the raggedness of the empirical time series.

Comparing the profiles of the autocovariances from Scenario 3b to those of the deterministic Scenario 1b in Figure 2, it can generally be stated that the former remain over longer lags in the empirical confidence bands, even though the estimation itself with its lag horizon of 8 quarters has not called for that. The most important achievement of $\chi_{\pi y}$ and $\chi_{f\pi}$ is they allow the model to trace out the initial sudden drop in the autocovariances of the inflation rate: the entire profile is now inside the confidence band. The other moments are not too seriously affected by this enhancement except one: $\text{Cov}(y_t, \pi_{t-1})$ in the output-inflation nexus—but only this single covariance—slides out of the confidence band with a t -statistic of -2.05 (see Table 2). This is a somewhat strange outlier since we had no indication of any problems with this moment before.

Regarding the t -statistics of the three raggedness moments reported in Table 2, we can be fully satisfied with $R(i_t)$ and $R(y_t)$, and we see a further improvement for the

inflation rate. However, with $R(\pi_t) = 0.91$ the empirical raggedness is so high that the best we can get is a t -statistic of -3.00 for Scenario 3b (from a simulated $R(\pi_t)$ that after all is already as high as 0.80).

Our inspection of the single moments thus comes to just two moments the matching of which shows a deficiency. In both cases, however, it is not overly worrying, either. In sum, we can now be happy enough with the overall match of the model.

Despite this feeling of success, we should nevertheless check the numerical parameter estimates. The most critical one is the herding coefficient ϕ_b : it is rather low in Scenario 3b and even completely vanishes in Scenario 3a. Hence the herding component is no longer supported by the data and it is essentially only the real rate of interest that feeds back on the business climate. Referring to the distinction at the end of Section 3, what comes out of the estimation is the weak version of the Old-Keynesian model, but not the strong version.

As a consequence of the zero or low value of ϕ_b , both scenarios do not generate endogenous cycles in their deterministic core. The long-run equilibrium is (not only locally but also) globally asymptotically stable, so that the random shocks are necessary to keep the system in motion. Convergence occurs, however, in a cyclical manner and after a one-time shock it would take the economy about 12–14 years to return to the steady state. This feature still shines through in the stochastic simulations in that at least the output gap displays a pronounced cyclical behaviour, similar in kind to what is shown in Figure 3.

The reduced role of herding in the business sentiment dynamics has a bearing on the two other parts of the model, the Phillips curve as well as the Taylor rule. Compared to Scenario 2b, the slope parameter κ in the Phillips curve rises once again by more than one-third, although the degree of sluggishness in the adjustments of the inflation climate (the coefficient α_π) and the credibility of the central bank (which is totally missing, i.e. $\gamma=0$) have largely remained the same.

The policy parameters in the Taylor rule look more familiar now: there is a considerable degree of interest rate smoothing, the central bank pays considerable attention to the output gap, and μ_π is close to Taylor’s reference value of 1.50. Unfamiliar and actually economically dubious is the absence of monetary policy shocks. There are three ways to cope with it if $\sigma_i=0$ is deemed unacceptable. First, as in many empirical studies on the Taylor rule, the quarterly inflation rates in (9) may be replaced with four-quarter inflation. This reduction in the variability of the interest rate might broaden the scope for random shocks. Second, it is a well-known problem that estimations could assign a spurious role to a lagged interest rate in the Taylor rule because it helps pick up the central bank’s serially correlated measurement errors (Lansing, 2002) or episodes where the central bank has to react to financial market perturbations (Rudebusch, 2002). It would therefore be worthwhile to set μ_i equal to zero and add an AR(1) shock process in (9), which will certainly not be degenerate.

A third option would require the least effort. It maintains the present form of the Taylor rule and treats the noise σ_i as an exogenous parameter. After all, the increase in the loss brought about by moderately positive levels of σ_i is not very dramatic. More specifically, information from outside the model could be used to introduce a lower bound for σ_i . The goodness-of-fit achieved by the corresponding re-estimations would only be slightly worse than for Scenario 3b; the naked eye would hardly be able to notice any differences in the autocovariance profiles.³¹

Lastly, it may be mentioned that the difference in the values for $\chi_{\pi y}$ in Scenario 3a and 3b is no reason for concern. The much higher value in Scenario 3b is easily explained by the small demand shocks prevailing here. The product $(\chi_{\pi y} \sigma_y)$ is actually almost the same in the two scenarios.

The little discussion of the parameter estimates can be briefly summarized by saying that, despite the reduced or even missing significance of herding, the numerical parameter values still make economic sense; or they easily can make good sense if the remarks on σ_i are taken into account. Therefore the successful matching of our moments that we have pointed out before rests on solid economic grounds.

4.5. *Estimation of the New-Keynesian model*

The point of departure of this paper was the finding from previous research that the elementary three-equations model (NK) is remarkably successful in reproducing the autocovariance profiles of its three variables i_t, y_t, π_t . The obvious question is now how this compares to the matching of our Old-Keynesian model in the last two Scenarios 3a and 3b, which were equally claimed to be a good success. Such a comparison is even more meaningful as the *a priori* possible herding effect was not confirmed by these estimations. Hence the New-Keynesian and the Old-Keynesian models are on a similar footing and could be viewed as two variants of the New Macroeconomic Consensus. That is, they include similar arguments centring around the real rate of interest, while the expectations involved in them are specified in different ways: in a so-called forward-looking manner in the New-Keynesian model and—in that language—a purely backward-looking manner in the Old-Keynesian approach.

Let us then reconsider the estimation of the New-Keynesian model on the 78 autocovariance moments. The linear structure of the model is very convenient in this respect because it saves us from the sample variability across different simulation runs. In fact, the closed-form solution of such rational expectations models takes the form of a vector autoregression, so that its autocovariances can be computed analytically without having to simulate the model (they are the asymptotic moments, corresponding to an infinite

³¹ Apart from this it is worth noting that the “aggregate” noise level $\sigma_i + \sigma_y + \sigma_\pi$ in the economy is similar across the Scenarios 2b, 3a, 3b, only the distribution across the three random sources varies.

simulation size in MSM; see Appendix A4 for further details). Column NK-a in Table 1 reproduces the estimation in Franke (2011b), which is the outcome from the minimization of the present loss function $J^{(78)}$. Its value of 37.13 is a bit higher than in Scenarios SD-3a and SD-3b, but the difference will not be reckoned significant. So far it can therefore be stated that the New-Keynesian and the Old-Keynesian score about equally well in matching the empirical autocovariances.

Nevertheless, the close values of $J^{(78)}$ in these estimations are just an overall evaluation, they do not mean that also the entire autocovariance profiles of NK-a and SD-3a are similar. The most conspicuous example is the following. We already know that the main weakness of SD-3a is the relatively strong underestimation of the empirical $\text{Cov}(y_t, \pi_{t-1})$. Model NK-a, on the other hand, has no problem at all with that moment. Instead, Table 2 indicates that it cannot overcome a serious underestimation of the variance of the inflation rate, which is not perfect but still acceptable in SD-3a.

Regarding the additional moments that we consider, moment m_{79} does not apply to the New-Keynesian model and can be omitted here. We can thus directly turn to the three raggedness statistics. Although there is no analytical expression for them and we cannot get around simulating them, their sample variability over our long simulation horizon of $S = 10,000$ quarters is so limited that effects from different random seeds can be neglected. Column NK-a in Table 2 shows the implications of this estimation for the time series patterns: $R(y_t)$ is perfect, $R(i_t)$ is still tolerable, but $R(\pi_t)$ points out that the inflation rate is much too smooth as to claim it could mimic the behaviour of the empirical series to any decent degree. This failure is also reflected in the dramatically higher value of the augmented loss function $J^{(82)}$ in Table 1.³² Hence the original estimation of model NK yields a good matching of the autocovariances, whereas it is very unsatisfactory when it comes to the raggedness of the inflation rate.

It may be noted in passing in Table 1 that the estimation of those parameters that the New- and Old-Keynesian model have in common are very dissimilar in NK-a and SD-3a. The only two exceptions are the complete absence of noise in the Taylor rule ($\sigma_i = 0$) and the policy coefficient on inflation μ_π , which in both estimations is also not too different from Taylor's benchmark value 1.50. The relatively low value of the Phillips curve noise level σ_π in NK-a (0.429 *versus* 1.340 in SD-3a) is certainly the most immediate explanation for the insufficient raggedness of inflation in this estimation (in both cases the supply shocks, as a feature that is remarkable in itself, exhibit no serial correlation, so that the levels σ_π are directly comparable).

As we did for the three versions of the Old-Keynesian model, we should finally include the raggedness moments in the loss function and re-estimate model NK accordingly. This procedure, which gives rise to Scenario NK-b, is successful insofar as the ragged-

³² In order not to change or extend the numbering in the superscript of J , $J^{(79)}$ may be identified with $J^{(78)}$ for the New-Keynesian model.

ness statistics are similarly good to those in SD-3b (see Table 2). In all other respects, however, NK-b is inferior to SD-3b. First of all, the minimal value of $J^{(82)}$ is distinctly higher (119 *versus* 43). Second, the extended minimization seriously deteriorates the previous matching of the autocovariances; $J^{(78)}$ increases from 37 to 107. Third, the policy coefficients on inflation and output, μ_π and μ_y , are heavily affected by the re-estimation and their high values are no longer fully credible.³³

A comparison of the New-Keynesian model and the most elaborate version of the Old-Keynesian model can now be briefly summarized as follows. Both models are almost equally successful in reproducing the autocovariances of their state variables i_t , y_t , π_t . The performance in this dimension can indeed be said to be rather convincing. However, the autocovariances and the raggedness $R(\pi_t)$ of the inflation rate can hardly be reconciled in the New-Keynesian model. Either it produces a good match of the former and a bad match of $R(\pi_t)$, or the other way around. By contrast, in the Old-Keynesian model the two types of moments are largely compatible; a good match of one type can go along with at least an acceptable match of the other type. Hence in one single sentence, if we are more ambitious concerning the features that a model should be able to reproduce, the Old-Keynesian model with the crossover random shock effects model does a better job than the New-Keynesian three-equations model.

4.6. *Confidence intervals and an assessment of the goodness-of-fit*

The discussion was so far mainly concerned with how well in general and in detail the model-generated moments can match the empirical statistics. The Old-Keynesian upshot of this analysis was Scenario SD-3b, for which we already expressed our satisfaction. Coming to terms with the complete absence of monetary policy shocks and (more or less) the herding mechanism, the numerical parameters make good economic sense as well. This notwithstanding, it should not be forgotten that they are point estimates and we do not know how precise they are. This is the issue that we turn to now.

Under standard regularity conditions, the parameter estimates $\hat{\theta}$ from minimizing the loss function (1) are consistent and asymptotically follow a normal distribution around the (pseudo-) true parameter vector. There is furthermore an explicit expression for estimating the corresponding covariance matrix (Lee and Ingram, 1991, p.202), from which the standard errors of the single components $\hat{\theta}_i$ are obtained as the square roots of the diagonal elements. Apart from the general question concerning the extent to which the asymptotic result may carry over to small samples, and the possibility that the local shape of the loss function around $\hat{\theta}$ (which enters the calculation of the covariance matrix)

³³ They origin with an identification problem, which we do not discuss here. For completeness, the estimates of the other parameters in NK-a and NK-b are reported in Appendix A5. Incidentally, in NK-b the dynamic IS equation becomes more backward-looking than in NK-a, while the Phillips curve becomes (much) more forward-looking.

may be different from the global behaviour, we face the more elementary inconvenience of corner solutions. Trivially, a parameter cannot be distributed around an estimate on a lower boundary, as it is the case for γ and σ_i . A symmetric distribution is also quite unlikely to prevail if the estimate is ‘close’ to the boundary, which seems to be the case for σ_y and perhaps also ϕ_b .

These problems can be dealt with by having recourse to a bootstrap procedure. In this respect it is only natural to make use of the bootstrap that served us to obtain the variances of the empirical moments for the weighting matrix W in eq. (2) above. From there we already have a large set of artificial moments on which, rather than on the single empirical moment vector m_T^{emp} , a model can be re-estimated just as many times as we want. In this way we get a frequency distribution for each of the parameters and can easily compute the confidence intervals from them. We only have to factor in that the original loss function (1) should be suitably demeaned, such that if the bootstrapped moments coincide with the empirical moments and the simulation is run with the random seed underlying the original MSM, the re-estimation would yield the original parameters with a zero loss (the finer details of this recentring have been provided in Section 2.2).

Note that in this way two types of variability are taken into account. First, we allow for the variability in the generation of the data in the real world, which is captured by the bootstrapping of the empirical moments as just mentioned. Second, we allow for the sample variability in the model simulations by carrying out each estimation on the bootstrapped moments with a different random seed.

For the re-estimations themselves, the minimization procedure for the loss function can start out from the original estimates. This saves us the exploratory stage of the minimization procedure and we can directly apply the simplex algorithm. It nevertheless turns out that the algorithm has to be re-initialized several times until the minimum has been sufficiently approximated, which is a very time-consuming endeavour. We therefore limit this investigation to our Old-Keynesian showcase scenario, i.e. SD-3b, where 1000 re-estimations should be enough for the present purpose (they can easily take up to almost two days on a standard personal computer).

Figure 4 shows the resulting frequency distributions of the 14 parameters (their estimated density functions, to be exact). The shaded areas indicate the 95% range of the bootstrap distribution, that is, they are delimited by the 2.5% and 97.5% quantiles. We see that not only the parameters that gave rise to a corner solution have a skewed distribution (γ and σ_i) but also ϕ_b and σ_y , which confirms that their benchmark estimates happened to be ‘close’ to a corner solution indeed. Perhaps somewhat unexpectedly, the latter also holds true for $\chi_{\pi y}$.

There are other parameters for which, although they may have a more or less symmetric bootstrap distribution, the median values deviate to some extent from the estimates in Table 1. In Figure 4, the more pronounced cases are illustrated by the vertical dotted lines for the medians, which may be compared to the short thick bars for the original

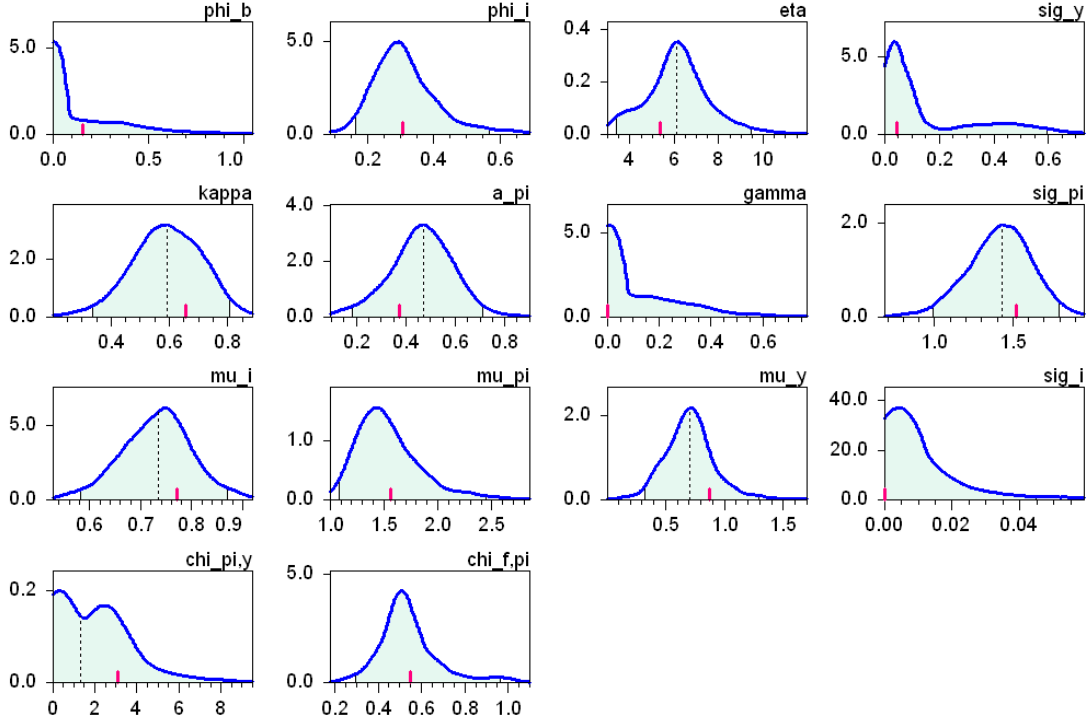


Figure 4: Frequency distributions of the re-estimated parameters for SD-3b (GI).

Note: The shaded areas represent 95% of the estimates. The dotted vertical lines in selected panels indicate the median values, the short (red) vertical bars the estimates from Table 1.

estimates. This phenomenon could be an implication of a bias in the estimations, according to which the bootstrap distribution may not be centred around the (pseudo-) true value of the parameter but around this value plus a bias term. As a consequence, the standard 95% interval given by the 2.5% and 97.5% quantiles would provide a distorted information.

Besides reporting the numerical values of the medians of the bootstrap distribution, Table 3 presents more suitable confidence intervals for the single parameters, whenever they are meaningful. The recipe for this is the concept of Hall's percentile confidence interval, which corrects for a possible bias and thus ensures asymptotic accuracy (see Appendix A2 for the details). Of course, the upper and lower bounds of them must remain within the admissible range. This applies to ϕ_i , η , κ , α_π , σ_π , μ_i , μ_π , $\chi_{\pi f}$. Specifically, taking also the shape of their distribution into account, the (pseudo-) median seems to be underestimated to some degree for κ , σ_π and μ_i , while for η and α_π it seems to be overestimated. In the cases for which the feasibility condition is violated, we resort to the standard percentile intervals. This concerns the parameters ϕ_b , σ_y , γ , μ_π , σ_i and $\chi_{y\pi}$.³⁴

³⁴ Regarding μ_π , we require that the Taylor principle be respected, i.e., this coefficient should not

	ϕ_b		ϕ_i		η		σ_y	
Median GI:	0.01	0.00 0.68	0.30	0.02 0.43	6.12	3.30 9.38	0.06	0.00 0.59
	κ		α_π		γ		σ_π	
Median GI:	0.59	0.52 0.99	0.47	0.06 0.58	0.01	0.00 0.54	1.43	1.17 1.97
	μ_i		μ_π		μ_y		σ_i	
Median GI:	0.74	0.64 0.93	1.49	1.08 2.44	0.71	0.18 1.16	0.01	0.00 0.34
	$\chi_{\pi y}$		$\chi_{f\pi}$					
Median GI:	1.33	0.00 8.68	0.52	0.05 0.72				

Table 3: Median values and confidence intervals from the re-estimations of SD-3b (GI).

Note: The figures shifted up- and downwards are the upper and lower bounds of the confidence intervals; the standard percentile intervals for ϕ_b , σ_y , γ , μ_π , σ_i , $\chi_{\pi y}$, and Hall's intervals for the other parameters.

By and large, one might feel that the parameters are not too precisely estimated.³⁵ In particular, the confidence intervals for the policy coefficients on the inflation and output gap, μ_π and μ_y , are wider than what would usually considered to be a reasonable range for them. We would have also preferred a slightly narrower range for the slope coefficient κ in the Phillips curve. On the other hand, all of their values are definitely higher than the (very) low estimates of κ for the New-Keynesian model (which incidentally are quite in line with many other estimations in the literature). For most of the other parameters, which relate to the latent variables in the model, it does not seem so easy to assess what would be deemed a decent width of the confidence intervals. In any case, unless one does not want to exogenously calibrate some of the 14 parameters, it would be desirable to include additional moments in the estimations in order to obtain a greater precision.

Evidently, the bootstrap distributions in Figure 4 are the marginal distributions. They are generally not independent of each other, that is, they do not tell us whether or to

fall short of unity.

³⁵ Perhaps it would be more adequate to de-emphasize the more extreme estimates and report the 90% confidence intervals.

what extent high values of one parameter go along with high or low values of some other parameters. This information is briefly summarized by the pairwise correlation coefficients of the parameter re-estimates. Actually, surprisingly many of these 13×12 relationships are relatively weak. The strongest connection prevails between the three policy parameters μ_i , μ_π , μ_y , whereas other noteworthy relationships seem to have an almost accidental character (except the one between κ and σ_π in the Phillips curve). For completeness, the correlation coefficients signalling the tightest relationships are listed in Table 4.

	μ_π	μ_y	α_π	σ_π	ϕ_b	ϕ_i	η
μ_i	0.69	0.69	-0.53				
μ_π		0.64					
κ				0.46	-0.46		
$\chi_{\pi f}$						0.59	-0.65

Table 4: Main correlations between the re-estimated parameters of SD-3b (GI).

A by-product of the re-estimations is that they serve for a general evaluation of the model's goodness-of-fit. It is well-known that, under the null hypothesis that the model is true, the minimized value of the loss function represents a statistic that is asymptotically chi-square distributed—provided, it has to be added, that the weighting matrix entering the loss function is optimal (Lee and Ingram, 1991, p.204). This standard J test for overidentifying restrictions is not applicable here since the latter supposition is not satisfied. However, we can employ the minimized losses J^b from the bootstrap experiment as our test distribution ($b = 1, \dots, 1000$). The benchmark estimation of SD-3b in Table 1 would have to be rejected at the conventional significance level if its estimated loss $\hat{J} = \hat{J}^{(82)} = 43.08$ exceeds the 95% quantile $J_{0.95}$ of the distribution of the J^b . Actually, however, we obtain $J_{0.95} = 60.76$. Hence model SD-3b passes the bootstrapped J test without any problem.

We can even take one step further and construct a p -value of the model to characterize its goodness-of-fit by a single number. It is given by the value of p that equates the $(1-p)$ -quantile of the distribution $\{J^b\}$ to \hat{J} , which says that if \hat{J} were employed as a benchmark for model rejection, then p is the error rate of falsely rejecting the null hypothesis that the model is the true data generation process. Clearly, the higher this p -value the better the fit. Here we calculate,

$$p\text{-value of SD-3b for GI} = 21.5\% \quad (20)$$

The bootstrap distribution $\{J^b\}$ and the critical quantiles are illustrated in Figure 5. Recall that $\hat{J} = 43.08$ is the median loss of 1000 estimations on the empirical moments

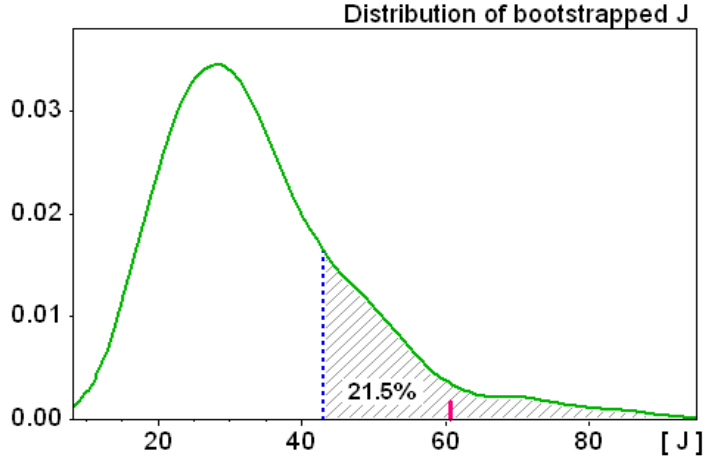


Figure 5: Distribution $\{J^b\}$ of the bootstrapped losses for SD-3b (GI).

Note: The short (red) vertical bar represents the 95% quantile of $\{J^b\}$, $J_{0.95} = 60.76$, the dotted (blue) line the loss $\hat{J} = 43.08$ from the original estimation in Table 1.

with different random seeds. If instead we consider their 97.5% quantile as a relatively unfavourable outcome, which gives rise to $J = 45.87$, we still get a p -value of 17.7%.

Nevertheless, despite the high matching quality of our Old-Keynesian model as it is indicated by the p -value in (20), it may not be forgotten that this evaluation is conditional on the specific choice of the moments that the model is required to match.

5. Estimations of the Great Moderation period

5.1. Equivalent matches in the deterministic Old-Keynesian model

When turning to the estimations for the Great Moderation period, it is only natural to take the same steps as for the Great Inflation, though occasionally in a more compact form. Accordingly, let us again begin with the deterministic version of the Old-Keynesian model, where now we skip the most elementary estimation and include the smoothing condition (15) in the set of moments right from the start. That is, we employ $J^{(79)}$ to estimate SD-1.

During the minimization, the first couple of trials with simulating annealing drew our attention to a fundamental problem that we face for GM, namely, that not all of the parameters can be sufficiently identified. It is particularly serious for the deterministic model, but in a somewhat weaker form the stochastic version will not be able to escape it, either. Regarding the present model version SD-1 we can most suitably concentrate on two parameters: the adjustment speed α_π in the updating of the inflation climate and the policy coefficient μ_π on inflation in the Taylor rule. Both of them cannot be pinned

down, which means we can exogenously fix them at arbitrary values over a wide range, estimate the remaining eight parameters, and nevertheless find nearly identical values of the minimized loss.

	$\alpha_\pi = 0.250$			$\alpha_\pi = 0.750$			$\alpha_\pi = 1.000$	$\alpha_\pi = 0.050$
ϕ_b	1.772	1.773	1.768	1.785	1.785	1.788	1.793	1.770
ϕ_i	1.225	1.225	1.222	1.232	1.231	1.231	1.237	1.220
η	1.292	1.291	1.289	1.288	1.287	1.285	1.290	1.296
κ	0.446	0.446	0.444	0.358	0.361	0.357	0.325	0.495
γ	0.763	0.764	0.757	0.694	0.697	0.688	0.638	0.207
μ_i	0.800	0.792	0.783	0.799	0.791	0.781	0.790	0.792
μ_π	1.000	1.500	2.000	1.000	1.500	2.000	1.500	1.500
μ_y	1.279	0.986	0.701	1.273	0.980	0.691	0.972	0.987
$J^{(79)}$	102.27	102.27	102.30	102.36	102.42	102.45	102.41	102.16

Table 5: A continuum of almost equivalent matches for SD-1 (GM).

Note: The parameters α_π and μ_π are exogenously fixed as indicated. Setting $\alpha_\pi = 0.00$ yields $J^{(79)} = 106.34$.

Table 5 documents this phenomenon for a number of selected values for α_π and μ_π . Fixing α_π at 0.25 and 0.75, respectively, it is seen that in each case an increase of μ_π from 1.00 to 2.00 yields a systematic increase of $J^{(79)}$, which, however, can only be said to be negligible. As far as variations of α_π for a given value of μ_π are concerned, we obtain a decline of $J^{(79)} = J^{(79)}(\alpha_\pi)$ from the higher to the lower values of α_π , which likewise is hardly worth mentioning. Only the rise of $J^{(79)}$ as α_π decreases from 0.05 down to zero is a bit more noticeable (see the note on the table).

Certainly, these exogenous variations of α_π and μ_π induce variations in the estimates of the other coefficients. They are quite limited for the parameters governing the output adjustments, i.e., for ϕ_b , ϕ_i and η . A *ceteris paribus* rise of μ_π causes a moderate decline in the other two policy coefficients μ_i and μ_y , while *ceteris paribus* changes of α_π have almost no effect on them. Things are the other way around for the parameters κ and γ in the inflation dynamics. They remain essentially unaffected by changes in μ_π , while both of them moderately decline when α_π rises from zero to unity—with the exception of γ which tends to become fairly low as α_π gets close to zero.

One can therefore choose almost any value of α_π and μ_π which, or the implications of which, one likes and obtain practically the same match. This is econometrically unsatisfactory, but it may not be unwelcome to the economist. For example, he or she may

prefer to set μ_π at the famous and credible Taylor benchmark value $\mu_\pi = 1.50$, or he or she may have a feeling for reasonable values of the adjustment speed α_π or the slope in the Phillips curve κ . These are elements of calibration which invoke other information or just common sense, and despite the scepticism of econometricians they are often found in empirical macroeconomic research. The lack of identification could also be regarded as a price for flexible modelling with a rich description, which does not wish to limit too early the number of meaningful structural parameters.

On the other hand, a positive side of the almost equivalent matches is a certain scope for the model that a smaller subset of the parameters might succeed in reproducing some additional empirical regularities. Incidentally, this reasoning does not apply to estimations employing a one and only likelihood function, whereas it is a fairly natural argument (or hope) in a moment matching approach.

In evaluating the general goodness-of-fit of scenario SD-1 for the Great Moderation, we can compare the measure of $J^{(79)} \approx 102$ from Table 5 to the corresponding result for the Great Inflation, which in Table 1 was reported to be $J^{(79)} = 75.15$. Although this is a quantitative deterioration, the covariance profiles for GM still do not appear to be too bad after all; see the thin (blue) lines in Figure 6.³⁶ The main differences from the thin (blue) lines in Figure 2 are the larger deviations, from the empirical statistics, of the model-generated $\text{Cov}(y_t, y_{t-h})$ for $h = 0, 1$ and $\text{Cov}(y_t, \pi_{t-h})$ for $h = 2, 3, 4$ (i.e., larger in terms of the width of the confidence bands). In many other respects, the profiles for GM and GI are not remarkably different. It may, in particular, be observed that in both cases the model does not manage to mimic the sudden initial drop in the autocovariances of the inflation gap.

5.2. *Estimations of the simple stochastic model*

Though being less severe than in the previous section, the identification problem carries over to the stochastic versions of the Old-Keynesian model. Here it is useful to consider the adjustment speed α_π as the only freely varying parameter. While, as will be seen, the minimized loss is no longer essentially constant, its changes will nevertheless remain rather limited over almost the entire range of α_π . At least, these changes can be easily dominated by variations arising from the sample variability, if the estimations have different random seeds underlying.

Let us first study this phenomenon in the stochastic model SD-2a without the cross random effects, which is estimated with $J^{(79)}$. The upper panel in Figure 7 is based on two different random seeds and depicts the resulting estimated losses as a function of α_π . For both random seeds there is a clear minimum value of these $\hat{J}^{(79)}$ at $\alpha_\pi = 0.02$.

³⁶ For completeness, these profiles result from the estimation with $\alpha_\pi = 0.25$ and $\mu_\pi = 1.50$ in Table 5. However, they are virtually indistinguishable from the profiles of the other parameter combinations in the table.

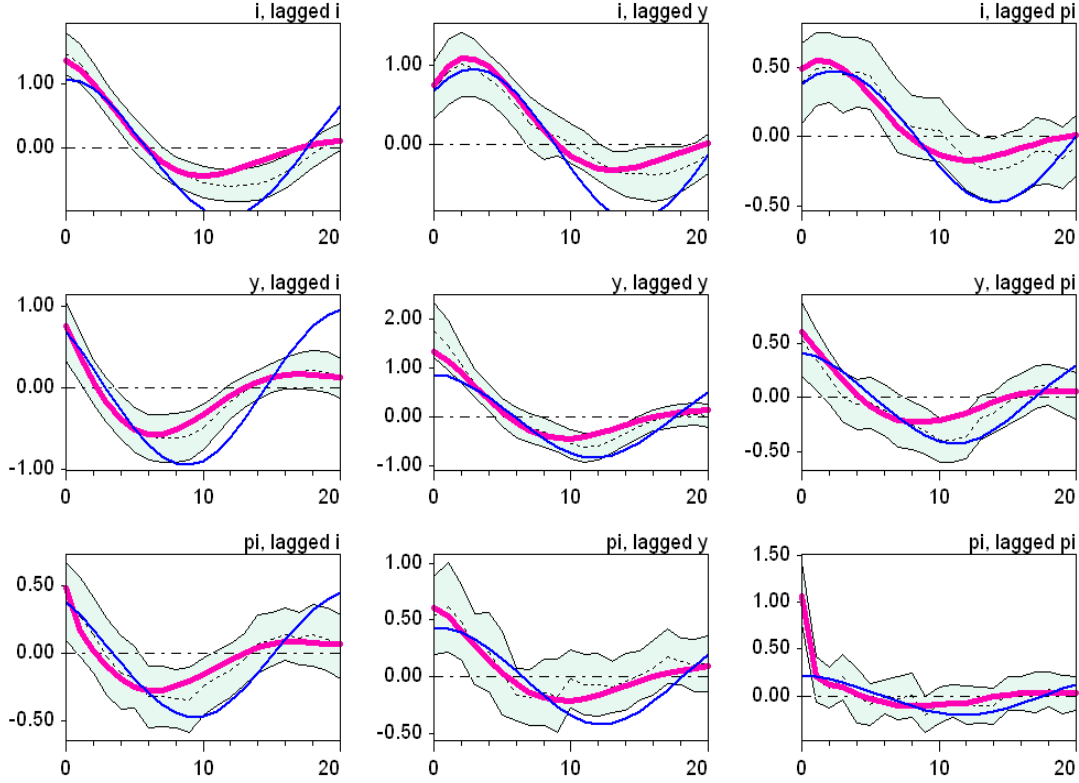


Figure 6: Auto- and cross-covariance profiles of the sentiment dynamics (SD-1) and (SD-3b) (GM).

Note: Thin (blue) lines indicate Scenario SD-1 (see Table 5 and footnote 36), bold (red) lines Scenario SD-3b(2) from Table 6. Shaded areas are the bootstrapped 95% confidence bands around the empirical moments (dotted lines).

However, we are not perfectly happy with this situation. First, the minimum is less outstanding than a first glance at the function might suggest. A zero adjustment speed and a speed $\alpha_\pi = 0.02$ will certainly be regarded as being essentially the same. Hence, if we take the upper function in the panel as an example, the associated losses of 43.5 and 41.5, respectively, should not be classified as being very different. Besides, referring to the autocovariance profiles of these two estimations, the naked eye has considerable difficulty recognizing any differences between them. Taking this evaluation for granted, it has to be concluded from the function $\alpha_\pi \mapsto \hat{\mathcal{J}}^{(79)}(\alpha_\pi)$ in Figure 7 that (almost) *all* of the admissible adjustment speeds α_π give rise to a matching that is equally acceptable.

A second point is that, as shown in the bottom panel of Figure 7, the estimated credibility $\hat{\gamma}$ of the central bank exhibits an excessive sensitivity near $\alpha_\pi = 0.02$. Shortly after that value the coefficient rises from zero to $\hat{\gamma} = 0.85$ at $\alpha_\pi = 0.10$, say. Hence one may not have too much faith in the estimation with $\alpha_\pi = 0.02$. In particular, if it is

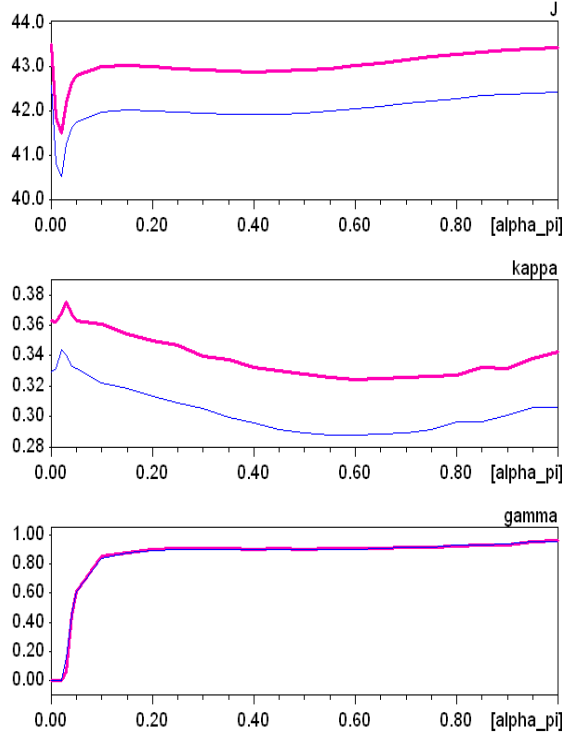


Figure 7: Estimations of SD-2a from two different random seeds under exogenous variations of α_π (GM).

Note: In the bottom panel, the functions $\alpha_\pi \mapsto \hat{\gamma}(\alpha_\pi)$ from the two random seeds almost coincide.

believed that the central bank credibility should be somewhat higher in GM than in GI, where it was close to zero, one may prefer higher adjustments speeds α_π that lead to higher estimates $\hat{\gamma}$.

The estimates of the other coefficients are fairly robust to the variations of α_π . Fixing α_π at the extreme values 0.02 and 1.00, respectively, the results in the first two columns of Table 6 may serve as a representative example. They are based on the same random number seed that gives rise to upper relationships in Figure 7 (actually, it yields the median loss in our showcase estimation later on). The middle panel of Figure 7 shows the moderate variations of the slope coefficient κ over the entire range of α_π . Apart from γ , this parameter exhibits the relatively strongest reactions, but the panel also shows that they are not monotonic. The second example represented by the thin (blue) lines in the figure indicate that the phenomena just described do not seem to be due to a special random number sequence.

Returning to the goodness-of-fit of the estimations, we may compare the SD-2a(1) and SD-2a(2) to the estimation SD-2a in Table 1 for GI, where we obtained $J^{(79)} = 47.70$.

	Scenario Model SD						Model NK	
	2a(1)	2a(2)	2b(1)	2b(2)	3b(1)	3b(2)	a	b
ϕ_b	1.171	1.171	1.255	1.262	0.630	0.596	—	—
ϕ_i	0.115	0.116	0.152	0.124	0.266	0.172	—	—
η	10.583	9.892	9.419	10.832	3.692	5.803	—	—
κ	0.368	0.343	0.350	0.323	0.559	0.438	0.027	0.031
α_π	0.020	1.000	0.020	1.000	0.001	1.000	—	—
γ	0.000	0.955	0.001	0.973	0.998	0.999	—	—
μ_i	0.774	0.783	0.748	0.761	0.801	0.804	0.801	0.807
μ_π	1.546	1.648	1.396	1.422	1.001	1.000	1.197	1.215
μ_y	0.871	0.875	0.886	0.944	1.225	1.241	1.240	1.297
σ_y	0.919	0.948	0.169	0.161	0.003	0.019	0.556	0.373
σ_π	0.829	0.847	0.854	0.911	0.649	0.423	0.328	0.457
σ_i	0.248	0.237	0.313	0.305	0.331	0.333	0.307	0.335
$\chi_{\pi y}$	—	—	—	—	5.403	40.664	—	—
$\chi_{f\pi}$	—	—	—	—	0.932	0.932	—	—
$J^{(79)}$	41.49	43.44	56.02	59.61	28.25	26.63	41.16	45.64
$J^{(82)}$	71.45	74.41	56.70	60.25	28.62	27.54	65.17	59.20

Table 6: Estimations of models SD and NK (Great Moderation).

Note: The parameter α_π is exogenously fixed. Bold face figures indicate the type of loss function for which the scenario is optimal. Underlying the estimations is the same random seed, which yields a median estimate of the loss $J^{(82)}$ for Scenario 3b(2).

It is thus seen that the simple stochastic version of the model produces a similarly good match for the two sample periods, at least as far as the more elementary set of moments is concerned.

The match of SD–2a(1) and SD–2a(2) can furthermore be contrasted with that of the New-Keynesian model, as it was estimated in Franke (2011b). The (partial) reproduction of the latter result in column NK–a in Table 6 shows a similar performance of the Old- and New-Keynesian model; none of them is superior to the other in the present context.³⁷ It

³⁷ As concerns the notation when comparing $J^{(79)}$ for SD–2a and NK–a, footnote 32 may be taken

is interesting to note that nevertheless some of the common parameters come out quite distinct. In the first instance, these are the slope κ in the Phillips curve and the two noise levels for output and inflation, σ_y and σ_π .

Let us next consider the raggedness moments. Trivially, this increases the loss function for SD-2a(1), SD-2a(2) and NK-a. However, the increase of $\hat{J}^{(82)}$ *versus* $\hat{J}^{(79)}$ is much more moderate than it was for SD-2a and NK-a in GI (cf. the last row in Table 1). The main reason for this is the lower empirical raggedness of the inflation gap in GM and so, with $R(\pi_t) = 0.84$ *versus* $R(\pi_t) = 0.91$ in GI, the models have less difficulty reproducing it. For example, while SD-2a and NK-a in GI are burdened with t -statistics higher than 15 in modulus (see Table 2), the estimations SD-2a(1) and SD-2a(2) in GM imply a perfect match with t -statistics of -0.07 and -0.28 , respectively; on the other hand, the t -statistics for $R(y_t)$ are worse with 5.39 and 5.40. Not the single statistics but the overall effect is similar for NK-a, which gives rise to values of -4.83 for $R(\pi_t)$ and 0.85 for $R(y_t)$.

These observations suggest that when now all of our moments are included in $J^{(82)}$, we will get a better performance than in GI. For the corresponding model versions SD-2b, we proceed in the same way as for SD-2a. The most important result is that these estimations are fraught with the same identification problem as before. Employing the same random seeds as in Figure 7, the functions $\hat{J}^{(82)} = \hat{J}^{(82)}(\alpha_\pi)$ are even, except for the higher scale, an almost exact reproduction of the functions $\hat{J}^{(79)} = \hat{J}^{(79)}(\alpha_\pi)$ in that figure. Also the functions for the two parameters κ and γ resemble closely those in Figure 7. A diagram like Figure 7 would therefore be superfluous for SD-2b.

Regarding the presentation of the numerical parameter estimates and the level of the total loss, we again choose the adjustments $\alpha_\pi = 0.02$ and $\alpha_\pi = 1.00$; see columns SD-2b(1) and SD-2b(2) in Table 6. The parameter values from these estimations are generally not too different from the previous ones, with the exception of the noise σ_y in the output adjustments. Consequently this leads to a considerable deterioration in the matching of the autocovariances, as indicated by the increase in $J^{(79)}$. In exchange, the minimized values of $\hat{J}^{(82)}$ are only marginally higher than $J^{(79)}$. Hence the lower level of σ_y primarily contributes to a better matching of the raggedness of the output gap, the t -statistics of which drop from around 5.40 for SD-2a to approximately 0.30. Thus all three raggedness moments are now exceptionally well reproduced in SD-2b.

How does the New-Keynesian model perform under the enriched loss function? The main details of the estimation of this model with $J^{(82)}$ are reported in the last column of Table 6 as NK-b. Concerning the overall goodness-of-fit, we see that again the Old- and New-Keynesian models are equally good. There are only differences in that NK-b is better than SD-2b in matching the autocovariances ($J^{(79)} = 45.64$ *versus* losses of more

into into account for the latter. The complete parameter estimates of NK-a as well as NK-b are reported in Appendix A5.

than 56), and SD-2b is better in matching the raggedness of the dynamic variables.

Back to the Old-Keynesian model, the general evaluation of SD-2b(1) is the same as for SD-2a(1): it provides the best match but the extremely low values of α_π and γ pose a severe credibility problem. As remarked above, because of the minor losses associated with them one may therefore decide on higher values of these parameters, where $\alpha_\pi = 1.00$ in SD-2b(2) represents the polar case. The latter is also worth presenting since the corresponding estimate of γ is so close to unity. Actually, note that in the limit, for $\alpha_\pi = 1$ and $\gamma = 1$, eqs (8) and (10) governing the inflation dynamics simplify to a special Phillips curve where the variations in expectations or the inflation climate, for that matter, play no longer any role. By virtue of the maximum central bank credibility, this Phillips curve is also perfectly anchored on the target rate of inflation π^* ,

$$\pi_t = \pi^* + \kappa y_t + \varepsilon_{\pi,t} + \chi_{\pi y} \varepsilon_{y,t-1} \quad (21)$$

(the coefficient $\chi_{\pi y}$ is presently still equal to zero). To complete this reasoning, the increase in $\hat{J}^{(82)}$ will be negligible when we fix γ directly at one and estimate the model once more; the loss just rises from 60.25 in SD-2b(2) to 60.28. We should like to point out that the parsimony of eq. (21), which frees us from the two parameters α_π and γ , could be regarded as a solution to the original identification problem. After the discussion above we know that the decline in the matching that we trade off for this is easily acceptable.

5.3. Estimations of the full stochastic model

It remains to include the cross random effects into the stochastic model. We abstain from the estimations without the raggedness moments, which would have been version SD-3a, and proceed straight to the minimization of the loss function $J^{(82)}$, which gives us SD-3b. Again, we exogenously vary α_π over the entire unit interval and estimate the other parameters, which by now total 13. The functions of the resulting losses $\hat{J}^{(82)} = \hat{J}^{(82)}(\alpha_\pi)$ for the same two random seeds as above are depicted in the top panel of Figure 8.

Also here, for both cases a minimum is obtained at a low value of α_π . For the random seed that gives rise to the bold (red) line in the diagram, it is this time as extreme as $\alpha_\pi = 0.001$. The corresponding parameter estimates are given in column SD-3b(1) in Table 6. This estimation is even less trustworthy than its counterparts SD-2a(1) and SD-2b(1) above. In contrast to them, the obvious expectation dramatically fails that a *ceteris paribus* lowering of α_π to precisely zero should produce no substantial differences: $J^{(82)}$ jumps from 28.62 to a value higher than 400. Strangely enough, re-estimating the model with $\alpha_\pi = 0$ reduces the loss to the previous order of magnitude, $\hat{J}^{(82)} = 29.47$, without any greater changes in the parameters. Because of the excessive sensitivity of the loss function in that region of the parameter space, estimation SD-3b(1) will thus better be discarded.

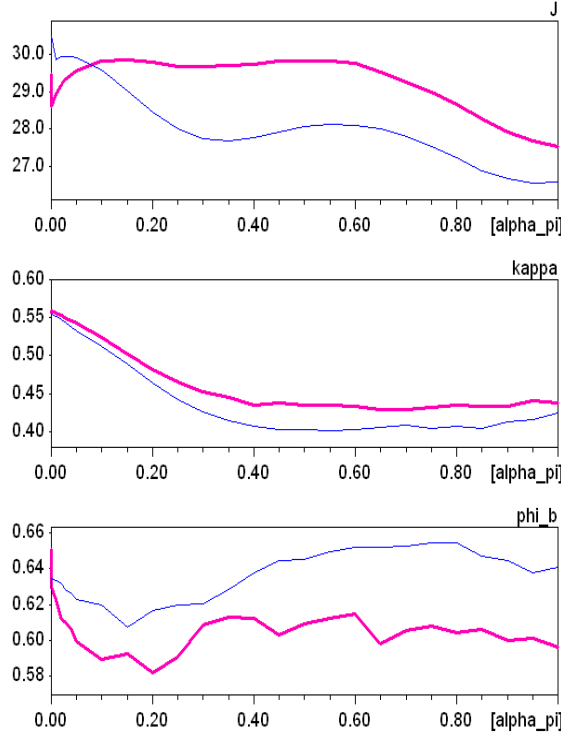


Figure 8: Estimations of SD-3b from two different random seeds under exogenous variations of α_π (GM).

As it was observed for the previous estimations, the reactions of the minimized losses to the variations in α_π are again fairly limited, so that the identification problem persists. Now, however, the function $\hat{J}^{(82)} = \hat{J}^{(82)}(\alpha_\pi)$ decreases rather than increases from a certain and relatively low level of α_π on. As a result, the function attains another minimum at the upper bound $\alpha_\pi = 1.00$, and at least for the two cases shown in Figure 8 it happens to be lower than the first one. As here the corresponding estimate $\hat{\gamma}$ is practically equal to one, we have two reasons for settling down on this situation: first the above argument of the parsimony of the thus (approximately) implied Phillips curve (21), and second the fact that it even constitutes a global minimum.

The outcome is estimation SD-3b(2) in Table 6, which we put forward as our showcase estimation for the Great Moderation. To justify the particular random seed it has underlying we add that after running 1000 estimations with α_π fixed at unity it was selected as the seed yielding the median loss.³⁸

The second panel in Figure 8 plots the estimates $\hat{\kappa} = \hat{\kappa}(\alpha_\pi)$ of the Phillips curve slope parameter. Unlike the function in Figure 7, it remains essentially constant from

³⁸To make sure, it may be repeated that the same random seed has then been employed for the previous estimations in Table 6 and for the bold (red) lines in both Figure 7 and 8.

$\alpha_\pi = 0.40$ on and does not rise again after its initial decline. The estimates $\hat{\gamma} = \hat{\gamma}(\alpha_\pi)$ are close to one over practically the entire range of α_π (cf. estimation SD-3b(1)). As this makes a graph uninteresting, the bottom panel of 8 presents the induced variations of $\hat{\phi}_b = \hat{\phi}_b(\alpha_\pi)$, from which we see that throughout and quite independently of α_π herding plays a certain role; not a very striking one but still stronger than in GI. The other parameter estimates behave in similar ways, except for the intensity $\hat{\sigma}_y$ of the supply shocks, which is inversely related to α_π .

If we consider the absolute level of the loss in SD-3b(2), we observe that including the crossover random effects as they are represented by the two parameters $\chi_{\pi y}$ and $\chi_{f\pi}$ achieves a sizeable improvement over model SD-2b and thus also over the New-Keynesian model NK-b. Again, as in GI, in the context of the full stochastic version the Old-Keynesian model can be said to outperform the New-Keynesian model.

There is nevertheless a slight difference of GM from GI. In GI, both SD-3b and NK-b yield a similar good match of the raggedness moments and the superiority of SD-3b originates with the autocovariance moments. In GM, the superiority of SD-3b is more evenly distributed across both types of moments. Apart from that, a comparison of the corresponding minimized losses shows for all models a tendency for a better matching in GM than GI.

For a finer analysis we can additionally ask which of the two parameters $\chi_{\pi y}$ and $\chi_{f\pi}$ provides a stronger contribution to the improvement by the full model. The answer is $\chi_{f\pi}$, that is, the crossover effects from the supply shocks $\varepsilon_{\pi,t}$ to the business climate in eq. (13) are more important than the demand shocks $\varepsilon_{y,t}$ in (indirectly) the Phillips curve. If $\chi_{f\pi}$ is fixed at zero and $\chi_{\pi y}$ is treated as a free parameter, the loss $\hat{J}^{(82)} = 60.25$ in SD-2b(2) falls to $\hat{J}^{(82)} = 59.35$ only. Conversely, if $\chi_{\pi y}$ is fixed at zero and $\chi_{f\pi}$ is freely estimated, a reduction to $\hat{J}^{(82)} = 36.62$ is obtained. Nonetheless, adding *now* $\chi_{\pi y}$ as a free parameter succeeds in a nonnegligible further improvement down to $\hat{J}^{(82)} = 27.54$.

At the end, the precision of the parameter estimates in SD-3b(2) should be examined. Of course, the same bootstrap re-estimations are applied as for GI in Section 4.6. There are only two differences. First, we commit ourselves to the polar case of the scenario where α_π and γ are both fixed at unity. Second, we draw on our explorations where we found a strong tendency of the policy coefficient μ_π toward one (and even lower values if this lower boundary was dropped). Hence we also fix $\mu_\pi = 1.00$ right away. The other details of the re-estimations have been described above, so we can directly turn to the marginal frequency distributions in Figure 9 for the 11 remaining parameters. The median values and the confidence intervals derived from them are reported in Table 7, which may also be compared with Table 3 for GI.

A by-product of these re-estimations was that they enable us to characterize the overall goodness-of-fit by the handy statistic of a single p -value for the model. Equation (20) thus established a value of 21.5% for GI. As there the benchmark scenario gave rise to a median loss of $J^{(82)} = 43.08$, whereas now SD-3b(2) yields $J^{(82)} = 27.54$, a considerably

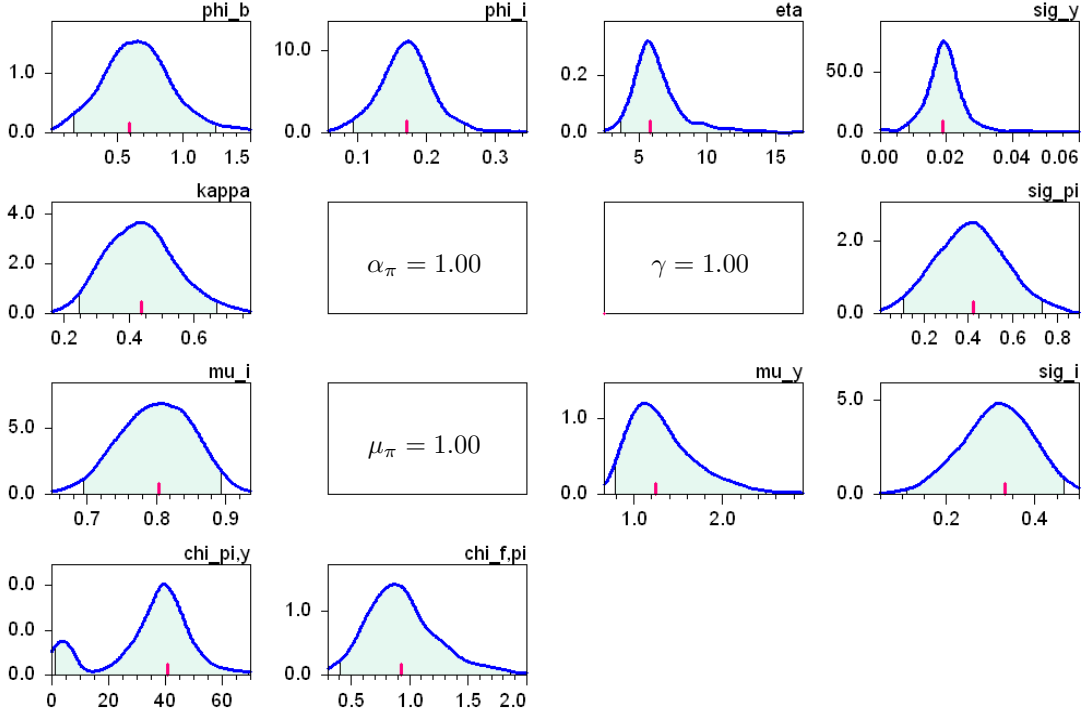


Figure 9: Frequency distributions of the re-estimated parameters for SD-3b(2) (GM).

Note: The shaded areas represent 95% of the estimates. The short (red) vertical bars indicate the estimates from Table 6.

higher p -value will be expected for the GM benchmark scenario. Actually, we obtain,

$$p\text{-value of SD-3b(2) for GM} = 90.0\% \quad (22)$$

One might suspect that (22) is in fact too good to be true. A possible explanation of it could be that our setting of $\alpha_\pi = \gamma = \mu_\pi = 1.00$ is overly restrictive and causes unduly high losses in the bootstrap distribution $\{J^b\}$. To check this, we drop the constraint and repeat the entire re-estimation experiment with 14 free parameters. By construction, the new bootstrapped losses will be somewhat lower. Nevertheless, the resulting reduction in the p -value is fairly limited: what we get in this way is a new p -value of 88.5%.

In sum, we do not take the p -value in (22) literally but content ourselves with the conclusion that it is indicative of a very good or even excellent match. Then, however, we should not forget to add that this holds with respect to the moments as we have chosen them. It goes without saying that by including additional moments in the estimations which are qualitatively different from the ones employed so far, the matching statistic would tend to deteriorate. The present moments, however, represent our present ambitions of what dynamic properties a small-scale Old-Keynesian model should be able to reproduce.

	ϕ_b		ϕ_i		η		σ_y	
Median GI:	0.65	0.17 1.24	0.17	0.09 0.26	5.95	3.70 11.67	0.02	0.01 0.28
	κ		α_π		γ		σ_π	
Median GI:	0.44	0.21 0.63	1.00	— —	1.00	— —	0.41	0.11 0.73
	μ_i		μ_π		μ_y		σ_i	
Median GI:	0.80	0.70 0.89	1.00	— —	1.25	0.79 2.49	0.32	0.20 0.56
	$\chi_{\pi y}$		$\chi_{f\pi}$					
Median GI:	38.0	12.1 80.1	0.91	0.40 1.86				

Table 7: Median values and confidence intervals from the re-estimations of SD-3b(2) (GM).

Note: The figures shifted up- and downwards are the upper and lower bounds of the confidence intervals; the standard percentile intervals for ϕ_b , η , σ_y , μ_y , σ_i , $\chi_{f\pi}$, and Hall's intervals for the other parameters.

6. Conclusion

The point of departure of the present paper was contemporary macroeconomic theory and the rich estimation literature it has brought forward. We expressed our preference for the method of simulated moments, according to which a set of model-generated summary statistics should come as close as possible to their empirical counterparts. Briefly saying, it is a more transparent estimation approach than the currently dominant likelihood procedures. In the context of the New-Keynesian stochastic three-equations models of the New Macroeconomic Consensus, a result from recent research was particularly remarkable: referring to the profiles of the autocovariances of output, inflation and the interest rate up to eight quarterly lags, such a model was found to provide a superb match. If unorthodox theory is to compete with mainstream theory and gain more acceptance, it has to understand this success as a challenge and try to develop models of similar complexity with a similar goodness-of-fit.

Taking up a fresh Old-Keynesian model (Franke, 2011a) from the literature, the paper claims to be a contribution that can fulfill these needs. Treating the periods of the so-

called Great Inflation (GI) and Great Moderation (GM) separately, the deterministic version exhibits an already respectable match in both of them, though it does not yet come up to the New-Keynesian model. This becomes possible if monetary policy as well as demand and supply random shocks are introduced. A simple stochastic version is limited to the direct shock effects, while a more detailed one adds two crossover effects. As a result, these two versions are able to achieve a similar good matching to the New-Keynesian model.

The paper also extends the estimation framework with the additional requirement that the abovementioned three model variables should reproduce the different degrees of raggedness in the empirical series. We established that at this level the elaborate Old-Keynesian model clearly outperforms the New-Keynesian model. This comes, however, at the ‘price’ of weakening the attractive herding component in the Old-Keynesian model; or it may even completely disappear so that, as in the New-Keynesian model, the real interest rate channel becomes the main dynamic mechanism.

This discussion emphasizes the excellent matching properties of the Old-Keynesian model. Regarding the parameter estimates that we obtain, certain problems should not be concealed. One is the possible absence of monetary policy shocks in GI. It was indicated that this shortcoming could be remedied by employing a slightly different (and perhaps more palatable) variant of the Taylor rule which replaces the lagged interest rate with serially correlated policy shocks.

More fundamental are possible identification problems, especially in GM. Three remarks may be in order here. First, the situation can be accepted as it is, regarding the lack of identification as a price that one may have to pay for flexible modelling with a richer structural description. Second, although denounced by econometricians, some parameters may be fixed exogenously. This will, in particular, be satisfactory or even welcome if they constitute a polar case that leads to a more parsimonious version of the original model, without deteriorating the previous matching too much. Third, from a positive point of view one has some reason to hope that, if there is a continuum of parameter combinations with no great differences in the fit, a suitable subset of them will be able to reproduce several additional moments not yet included in our estimations.

These are some immediate problems for future research. For the time being, however, we are satisfied with the moment matching properties of the Old-Keynesian model and its competitiveness *vis-à-vis* the New-Keynesian rival model.

Appendix A1: Bootstrap distribution of the empirical second moments

The following procedure is a modification of the block bootstrap, which seems more appropriate to us for the present problem. As will become clear in a moment, it may be called a period–cum–history sampling bootstrap. Although it is straightforward, we have so far not found another example of its application in the literature.

The estimations are concerned with the auto- and cross-covariances of the three macroeconomic variables x_t , π_t , r_t over a sample period of size T . For two representative empirical variables p_t , q_t from the ‘true’ data generation process of the real world ($p, q = x, \pi, r$), the sample covariance $\gamma_{pq}(h) := \text{Cov}(p_t, q_{t-h})$ with lag $h \geq 0$ is given by

$$\gamma_{pq}(h) = (1/T) \sum_{t=1+h}^T (p_t - \bar{p})(q_{t-h} - \bar{q}), \quad h = 0, 1, \dots, H,$$

$$\text{where} \quad \bar{p} = (1/T) \sum_{t=1}^T p_t, \quad \bar{q} = (1/T) \sum_{t=1}^T q_t$$

(H being the maximal lag in the covariances we are dealing with.) With a view to the bootstrap procedure specified in a moment, it is convenient to define the set of the time indices

$$I^o = \{1, 2, \dots, T\}$$

and rewrite this empirical covariance as

$$\gamma_{pq}^{emp}(h) = (1/T) \sum_{t \in I^o} (p_t - \bar{p})(q_{t-h} - \bar{q}) \quad (q_\tau = 0 \text{ if } \tau \leq 0)$$

(the superscript ‘*emp*’ has been added for greater clarity.)

Bootstrapping summary statistics that involve lagged values of the dynamic variables is usually carried out as a block bootstrap of the time series data. This is not an entirely satisfactory procedure because the independence between the randomly selected single blocks cannot reproduce the dependence structure of the original sample, a phenomenon known as the join-point problem. In addition, the variability of various moments may thus be increased (cf. Andrews, 2004, p.674).

While these are serious problems in likelihood or dynamic regression estimations,³⁹ they can be circumvented in the present moment matching approach. To put up a bootstrap sample b , we need not form a new series of consecutive data points and compute the moments from them, but we can sample directly from the time indices: alternatively to I^o they give us a new set I^b on which we can base the same calculations as above. Accordingly, a bootstrap sample in our approach is constituted by T random draws with replacement from the set I^o (each time index having the same probability $1/T$). Repeating this B times, we have $b = 1, \dots, B$ index sets

$$I^b = \{t_1^b, t_2^b, \dots, t_T^b\}$$

from which analogously to the empirical magnitudes we can subsequently obtain the bootstrapped moments

³⁹ For which Andrews (2004) proposes the concept of a block-block bootstrap.

$$\gamma_{pq}^b(h) = (1/T) \sum_{t \in I^b, t > h} (p_t - \bar{p}^b)(q_{t-h} - \bar{q}^b), \quad h = 0, 1, \dots, H; \quad b = 1, \dots, B;$$

$$\text{where} \quad \bar{p}^b = (1/T) \sum_{t \in I^b} p_t, \quad \bar{q}^b = (1/T) \sum_{t \in I^b} q_t$$

Note that while in an empirical covariance $\gamma_{pq}^{emp}(h)$ exactly h of the T terms in the sum are vanishing, there may be more or less such zero terms in a bootstrapped covariance $\gamma_{pq}^b(h)$. In this way also the first h time indices $1, \dots, h$ can contribute to the variability in these moments, although this effect might not appear fully appropriate. It could be avoided by fixing $t_h^b = h$ for $h = 1, \dots, H$ in the index set I^b and sampling only from the remaining time indices $H+1, \dots, T$. Such a device might indeed be acceptable for lags h close to the maximum lag H , but would unduly restrict the bootstrap variability of the covariances at shorter lags, for which more data than from period $H+1$ onward are available.⁴⁰

Having completed the calculations of all of the $\gamma_{pq}^b(h)$, we can for each such moment compute the variance

$$\text{Var}[\gamma_{pq}(h)] = (1/B) \sum_{b=1}^B [\gamma_{pq}^b(h) - \bar{\gamma}_{pq}(h)]^2, \quad \bar{\gamma}_{pq}(h) := (1/B) \sum_{b=1}^B \gamma_{pq}^b(h)$$

As made explicit in (2), the reciprocals form the diagonal of the weighting matrix W in the loss function. The square roots of these variances, i.e. the bootstrapped standard errors, can also be used to compute t -statistics in order to assess the deviations of the model-generated from the empirical moments; see eq. (3).

Appendix A2: The standard percentile and Hall's percentile confidence interval

Let a collection $\{\hat{\theta}^b: b = 1, \dots, B\}$ of parameter re-estimates be given, as stated in (5). With respect to a significance level $\alpha = 0.05$, let $\hat{\theta}_{i,L}$ be the estimate from (5) such that only a fraction $\alpha/2$ of all the bootstrap estimates $\hat{\theta}_i^b$ are less than this value, and likewise $\hat{\theta}_{i,H}$ the estimate that is exceeded by only $\alpha/2$ of the bootstrap estimates. The standard percentile confidence interval is then given by

$$CI_S(\theta_i) = [\hat{\theta}_{i,L}, \hat{\theta}_{i,H}] \tag{A1}$$

(the index S indicating that (A1) is regarded as the standard method.) If the original estimate $\hat{\theta}_i$ from (1) lies on the boundary of the admissible set of the parameters, Θ , and $\hat{\theta}_{i,L}$ (or $\hat{\theta}_{i,H}$) coincides with it, then $\hat{\theta}_{i,H}$ (or $\hat{\theta}_{i,L}$) itself will be the $(1-\alpha/2)$ -quantile (the $\alpha/2$ -quantile, respectively).

⁴⁰ Another option would be to use the data prior to the sample period for the lagged covariances. We experimented with this alternative but found that already at medium lags the covariances of the Great Moderation period were somewhat strangely influenced by the preceding data.

Although (A1) is a straightforward specification, it has to be taken into account that it may not have the desired coverage probability. In particular, if $\hat{\theta}_i$ is a biased estimate of θ_i^o , the bootstrap distribution may be asymptotically centred around θ_i^o plus a bias term. $CI_S(\theta_i)$ is then a $(1-\alpha)\%$ confidence interval for the latter quantity and may thus have a grossly distorted range as a confidence interval for θ_i^o .

An alternative to (A1) that fixes this problem is Hall's percentile confidence interval, which essentially is defined as

$$[2\hat{\theta}_i - \hat{\theta}_{i,H}, 2\hat{\theta}_i - \hat{\theta}_{i,L}] \quad (\text{A2})$$

It is based on the idea that the bootstrap distribution $(\hat{\theta}_i^b - \hat{\theta}_i)$ approximates the distribution $(\hat{\theta}_i - \theta_i^o)$. This implies that $\text{Prob}(\hat{\theta}_{i,L} - \hat{\theta}_i < \hat{\theta}_i - \theta_i^o < \hat{\theta}_{i,H} - \hat{\theta}_i) \approx \text{Prob}(\hat{\theta}_{i,L} - \hat{\theta}_i < \hat{\theta}_i^b - \hat{\theta}_i < \hat{\theta}_{i,H} - \hat{\theta}_i) = 1-\alpha$, and the first probability expression is easily seen to be equal to $\text{Prob}(2\hat{\theta}_i - \hat{\theta}_{i,H} < \theta_i^o < 2\hat{\theta}_i - \hat{\theta}_{i,L}) = \text{Prob}(\theta_i^o \in CI_H(\theta_i))$. Hence Hall's percentile method is asymptotically correct.

It can, however, happen that $2\hat{\theta}_i - \hat{\theta}_{i,H}$ falls short of a lower bound $\theta_{i,aL}$ of the admissible range of the parameter (something which by construction is not possible with the standard percentile interval). The lower end of the confidence interval may then be set equal to $\theta_{i,aL}$. Similarly so if $2\hat{\theta}_i - \hat{\theta}_{i,L}$ exceeds an upper bound $\theta_{i,aH}$ of the admissible range. We leave such a modification of (A2) aside since in these cases it seems more meaningful to resort to (A1).

Appendix A3: Data sources

It was convenient for us to extract the quarterly price and output time series from the database `fmdata.dat` in the zip file `fmfp.zip` that is provided by Ray Fair for working with his macroeconometric model. It is a plain textfile downloadable from <http://fairmodel.econ.yale.edu/fp/fp.htm>. The acronyms to identify real output and the price deflator, respectively, are GDP_R and GDP_D for the GDP data, and Y and PF for the firm sector (which essentially is nonfinancial corporate business). These explanations can be found in Appendix A.4, Table A.2., of the script *Estimating How The Macroeconomy Works* by R.C. Fair, January 2004, which can be downloaded from <http://fairmodel.econ.yale.edu/rayfair/pdf/2003a.pdf>.

The gap series $\hat{\pi}_t$, x_t , \hat{r}_t constructed from these data (detrending done by Hodrick-Prescott with smoothing parameter $\lambda = 1600$) can be downloaded from http://www.bwl.uni-kiel.de/gwif/downloads_papers.php?lang=en.

Appendix A4: The second moments of the New-Keynesian model

The $(n \times n)$ square matrices Ω and Φ that constitute the reduced-form solution (6) of system (NK) can be determined by using the method of undetermined coefficients ($n = 3$

presently). They are successively obtained as the (unique, under determinacy) solutions to the following two matrix equations (I_n being the $(n \times n)$ identity matrix),

$$\begin{aligned} A \Omega^2 + B \Omega + C &= 0 \\ (A \Omega + B) \Phi + A \Phi N + I_n &= 0 \end{aligned}$$

As indicated, our aim in the moment matching estimation is that the stochastic process (6) reproduces the autocovariances of the empirical counterparts of the variables in the vector $x_t \in \mathbb{R}^n$. It is convenient in this respect that (6) is essentially a first-order vector autoregression (VAR). The theoretical autocovariances can thus be easily obtained from the closed-form expressions given, e.g., in Lütkepohl (2007). We only have to adjust the notation by changing the dating of the shocks and rewrite (6) as

$$\begin{bmatrix} x_t \\ v_{t+1} \end{bmatrix} = \begin{bmatrix} \Omega & \Phi \\ 0 & N \end{bmatrix} \begin{bmatrix} x_{t-1} \\ v_t \end{bmatrix} + \begin{bmatrix} 0 \\ I \end{bmatrix} \varepsilon_{t+1} \quad (\text{A3})$$

With $z_t = (x'_t, v'_{t+1})'$, $D = (0 \ I)'$, $u_t = D \varepsilon_{t+1}$, and A_1 the $(2n \times 2n)$ matrix on the right-hand side associated with the vector $(x'_{t-1}, v'_t)' = z_{t-1}$, eq. (A3) can be more compactly written as

$$z_t = A_1 z_{t-1} + u_t, \quad u_t \sim N(0, \Sigma_u), \quad \Sigma_u = D \Sigma_\varepsilon D' \quad (\text{A4})$$

The (asymptotic) contemporaneous and lagged autocovariances of this VAR(1) are given by the matrices

$$\Gamma(h) := E(z_t z'_{t-h}) \in \mathbb{R}^{K \times K}, \quad K = 2n, \quad h = 0, 1, 2, \dots \quad (\text{A5})$$

Following Lütkepohl (2007, pp. 26f), their computation proceeds in two steps. First, $\Gamma(0)$ is obtained from the equation $\Gamma(0) = A_1 \Gamma(0) A'_1 + \Sigma_u$, which yields

$$\text{vec } \Gamma(0) = (I_{K^2} - A_1 \otimes A_1)^{-1} \text{vec } \Sigma_u \quad (\text{A6})$$

(the symbol ' \otimes ' denotes the Kronecker product and invertibility is guaranteed since A_1 is clearly a stable matrix). Subsequently the Yule-Walker equations are employed, from which the lagged autocovariances are recursively obtained as

$$\Gamma(h) = A_1 \Gamma(h-1), \quad h = 1, 2, 3, \dots \quad (\text{A7})$$

Appendix A5: The parameter estimates of the New-Keynesian model

	GI		GM	
	a	b	a	b
ϕ_y	0.466	0.226	0.000	0.000
τ	0.112	0.192	0.289	0.283
ρ_y	0.000	0.130	0.000	0.366
ϕ_π	0.326	0.823	0.567	0.612
κ	0.113	0.000	0.027	0.031
ρ_π	0.000	0.000	0.000	0.000
μ_i	0.435	0.936	0.801	0.807
μ_π	1.374	3.695	1.197	1.215
μ_y	0.086	4.977	1.240	1.297
σ_y	0.428	0.543	0.556	0.373
σ_π	0.429	1.822	0.328	0.457
σ_i	0.000	0.283	0.307	0.335
$J^{(79)}$	37.13	107.45	41.16	45.64
$J^{(82)}$	286.68	119.04	65.17	59.20

Table 8: Estimations of the New-Keynesian model.

Note: Bold face figures indicate the type of loss function for which the scenario is optimal.

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