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Effort Incentives and On-the-Job Search: An Alternative Role for Efficiency Wages in Employment Contracts

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Abstract We study an infinitely repeated principal-agent relationship with on-the-job search. On-the-job search is modeled as a dimension of the agent’s effort vector that has no effect on output, but raises his future outside option. The agent’s incentives to search are increasing in the degree to which a higher outside option improves his gains from trade. Search also increases the agent’s cost of effort thus generating an effort-substitution problem between work and search effort. We show that the principal can mute search incentives by offering an efficiency wage contract. Due to effort substitution, efficiency wages increase the agent’s work effort incentives for a given bonus scheme. Thus, efficiency wages serve as a complement rather than as a substitute to piece rates. Our results provide a new rationale for the use of efficiency wages as an incentive device and hence greatly extend the set of environments in which efficiency wages are predicted to be useful as an incentive device. Our findings thus also contribute to the explanation of empirically observed inter-industry variation in the size and composition of worker compensation.

Keywords: Moral Hazard, Efficiency Wages, On-the-job search.

JEL Classification Numbers: C73; D82; D86; J33.

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1 Introduction

Efficiency wage models offer a popular rationale for the argument that firms can overcome incentive problems by paying their employees high wages. The basic idea is that workers earn rents from jobs that pay sufficiently high wages. Since workers do not want to lose these rents, firms can motivate their employees to put forth effort by threatening to fire anybody who is caught shirking.\(^1\)

Even though the shirking model of efficiency wages is widely accepted as an explanation for a number of empirical labor market facts, the underlying theory actually imposes rather strong restrictions on the properties of the environments in which efficiency wages are effective as an incentive device. For instance, since efficiency wages rely on the threat of firing the agent in case of shirking, their use necessitates access to some monitoring technology. That is, the principal must be able to observe the agent’s effort choice perfectly with strictly positive probability. If instead the principal observes a noisy performance measure that provides only an imperfect signal of the agent’s effort choice, then efficiency wages require firing the agent on the equilibrium path implying inefficient output destruction. By contrast, incentive provision via piece rates allows the players to remain on the Pareto frontier even if performance measures are imperfect.\(^2\) An exception is the special case of subjective performance evaluation where the principal privately observes the agent’s performance. In that case, Fuchs (2007) shows that the optimal incentive contract does indeed take the form of an efficiency wage contract. However, in the canonical moral hazard framework with a publicly observed imperfect performance signal, efficiency wages are strictly dominated by performance pay.

The shirking model also provides no explanation for cases where firms supplement efficiency wage contracts with variable bonus components. In fact, extant studies view those two instruments as substitutes. Macleod and Malcomson (1993, 1998) (MM henceforth) study matching markets where firms compete for workers by offering self-enforcing incentive contracts. Because contracts are incomplete, credible incentives can only be provided if there exists a rent that either the firm or the worker stands to lose. MM show that piece rates are used when workers are in short supply, while efficiency wages emerge in the polar case when there are more workers than jobs. However, in equilibrium, it is either efficiency wages or piece rates that emerge as the unique incentive device.

Furthermore, the shirking model predicts that in general, efficiency wages are only effective as an incentive device in incomplete contracting environments, i.e. in situations where firms fail to enforce effort by the means of a formal contract. By contrast, if contracts can be based upon workers’ effort levels, incentive compatibility does no longer require the firm to leave a rent to the worker. Thus, in these cases, efficiency wages will no longer constitute the firm’s optimal instrument to motivate its workers. It is well-known though that firms may benefit from supplementing a formal contract by an implicit one when the contractible performance

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\(^1\)Main references for this shirking-model of efficiency wages are Calvo and Wellisz (1978, 1979), Shapiro and Stiglitz (1984) and Bowles (1985).

\(^2\)Notice that this is true even if this performance measure is not formally contractible. Still, in that case it is better to use a relational incentive contract than efficiency wages, see Levin (2003).
measure is distorted relative to some other measure that can however not be the basis of a formal contract. Yet, in these models, efficiency wages are generally again less desirable than performance pay.

Even though the use of efficiency wages is a well-documented empirical regularity in many labor markets, the above considerations indicate that the usefulness of efficiency wages as an instrument to motivate workers is restricted to rather specific situations. This paper provides an alternative justification for the use of efficiency wages as an instrument to motivate workers. We show that efficiency wages can improve effort incentives in a repeated moral-hazard problem with on-the-job search, where we understand on-the-job search as the agent engaging in costly search activities to influence his outside option. We model on-the-job search as a pure rent-seeking activity, i.e. from a joint-surplus perspective, it only increases the agent’s effort costs without improving productivity. Still, the agent is tempted to search as this raises his future bargaining position and thus his expected gains from future trade. In this setting, efficiency wages deter on-the-job search by reducing the responsiveness of the agent’s gains from trade to his outside option. Thereby, efficiency wages improve the effectiveness of a given piece rate system and hence serve as a complement rather than as a substitute to performance pay. Thus, the model can rationalize the use of efficiency wages even in settings where firms are unable to monitor their employees’ efforts but can base contracts on imperfect performance measures. Our results also provide an alternative justification for the use of efficiency wages in settings where firms already use formal contracts to provide effort incentives.

In principal-agent models, players’ outside options play a key role in shaping the set of contracts, both players are willing to agree on. Outside options specify players’ threat points and thus together with an allocation of bargaining power, determine the division of surplus among the two parties. In efficiency wage models, the agent’s outside option even determines the size of the wage necessary to induce effort. Given its prominent role in shaping the properties of incentive contracts, it is important to understand how outside options are determined and whether players can influence or even actively control them. Formally, an agent’s outside option represents his payoff if he does not trade with the principal. Typically, this payoff is interpreted as the agent’s next-best market alternative, i.e. as the next highest payoff that the agent can earn at some other firm. This payoff is generically assumed to be determined exogenously. By contrast, we argue that especially when analyzing long-term employment relationships,
the process that determines players’ outside options also impacts players’ behavior within the relationship and thereby the properties of the optimal contract.

The labor market differs from most other markets in that market participants typically face highly incomplete information regarding the location and identity of potential trading partners. Since outside options reflect the value of trade with alternative trading partners, a model of the formation process of outside options must take this frictions property into account. That is, outside options must first be located before they can be used in the bargaining process. For one-shot interactions, neglecting this search process may not pose a problem, but if we study long-term employment relationships, then the search process may well impact equilibrium behavior. Indeed, job search typically devours resources such as time, attention and cognitive effort. As these are the same resources that the agent also needs to work productively, searching for outside options likely impacts the agent’s productivity. Several papers endogenize the agent’s outside option by defining it as his value of being unemployed, with that value including the size of the unemployment benefits as well as the value of the prospect of finding a new job. However, this approach implicitly rules out on-the-job search, which is by no means without loss of generality. If an immediate job offer is better than being unemployed and having merely the prospects of finding new employment, then it is highly rational for workers who are engaged in a long-term employment relationship to search on the job. In fact, the practice of searching on the job is well-documented and by now extensively studied empirically. In most jobs, workers exert at least a little bit of on-the-job search such as maintaining a profile at some job platform or simply reading regular job newsletters.

The innovation of this paper is to include the agent’s decision problem whether and how much to search on the job in a repeated moral hazard problem representing a long-term employment relationship between a principal and an agent. Since on-the-job search draws on the same resources that the agent employs in the production process, the agent essentially faces a multitask problem as in Holmström and Milgrom (1991). In fact, while work effort increases the expected value of the bonus component of the agent’s compensation, search effort increases the likelihood of entertaining an alternative job offer in the next period, thereby improving the agent’s future bargaining position. Thus, the division of the agent’s resources among work and search effort is characterized by an effort substitution problem. The fundamental insight of the paper is that the principal can prevent inefficient on-the-job search by offering an efficiency wage contract. In fact, the benefits of search effort today are given by the increase in the expected future gains from trade. Thus, the principal can reduce these gains by offering an efficiency wage contract that leaves the agent a rent in case he entertains a low outside option.

The tradeoff between work and search effort faced by workers who search on the job has received surprisingly little attention both theoretically, as well as empirically. Although models of on-the-job search generally acknowledge that search is costly, few studies analyze the agent’s decision how much to search explicitly. Further, those models that do merely assume that

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7See Mortensen (1986) and Ridder and van den Berg (1997) for discussions of this issue.
8Shapiro and Stiglitz (1984) and Malcomson and Macleod (1993) use that avenue.
9See Topel and Ward (1992), Bontemps, Robin and van den Berg (1999) and Christensen et al. (2005).
workers face some cost-of-search function which is however unrelated to the amount of work effort. Yet, to the extent that the resources spent on search activities are also useful in the production process, on-the-job search costs should reflect the alternative value of these resources. The only study that we are aware of that examines this tradeoff is due to Burdett and Mortensen (1978) in which the authors attempt to unify search theory with the household’s time allocation problem. They analyze the decision problem of a household that needs to allocate his time budget among leisure, work and search activities. Thereby, Burdett and Mortensen (1978) endogenize the costs of search as the foregone utility of the alternative use of time in leisure or work. However, they only focus on the household’s problem and take the firm’s decision as given. Any implications regarding incentives and employers’ optimal response to this tradeoff are absent from their analysis.

Further, dynamic principal-agent models have become an increasingly popular tool to study incentives and compensation in long-term employment relationships characterized by incentive problems. However, the role of on-the-job search has yet to be studied in such a framework. In fact, the formation of endogeneous outside options in repeated agency models is no novelty. Several studies allow agents’ outside options to be affected by on-the-job activities. In Pendergast (1995) for instance, carrying out a given task increases an agent’s general skill level and thereby also the value of his outside option. As a result, in order to avoid the accompanied increase in the agent’s bargaining power, the principal may delegate too little authority to the agent. Similar problems are analyzed by Rajan and Zingales (2001), Kräkel (2005) and Zabojnik (2002). However, in none of these studies does the agent face an effort-substitution of the kind inherent in the tradeoff between work and search effort.

Incorporating search effort into a repeated moral hazard problem allows us to rationalize the use of efficiency wage contracts as a means to improve the effectiveness of performance pay in long-term employment relationships that are characterized by incentive problems. It thus justifies the use of efficiency wages even in jobs where firms cannot monitor their employees’ efforts. Further, while the model emphasizes the benefit of efficiency wages even in complete contracting environments, it is certainly not confined to such situations. In fact, under incomplete contracting, our results suggest that efficiency wages are likely even more beneficial, as they (indirectly) increase the value of the employment relationship, thereby enhancing the set of (implicit) bonus contracts that can be credibly promised by the principal.

The remainder of the paper is organized as follows. The next section lays out the basic framework. Section 3 solves a baseline model with exogenous search effort. Section 4 introduces search effort by endogenizing the probability distribution over outside options. Section 5 analyzes the benfit of efficiency wages in the presence of on-the-job search and the optimal composition of the agent’s incentive contract. Section 6 extends the framework to an incomplete contracting setting. Section 7 concludes.

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10 For instance Burdett (1978) and Christensen et al. (2005)
2 The Model

Consider a risk-neutral principal (she) and a risk-neutral agent (he). Both players are infinitely-lived and discount payoffs at the common factor $\delta \in (0, 1)$. At any date $t \in \{0, 1, 2, \ldots \}$ the two parties can trade. When they do so, the agent chooses a work effort level $e_t \in [0, \bar{e}]$ thereby generating a stochastic output $y_t \in \{y, \bar{y}\}$ with $y < \bar{y}$ for the principal. The probability that $y_t = y$ given work effort level $e_t$ is $f(e_t)$ with $f(0) = 0$, $f'(0) > 0$, $f''(0) \leq 0$ and $f(0) = 0$. By exerting work effort, the agent incurs private costs $c(e_t)$, with $c(0) = c'(0) = 0$, $c'(.) > 0$, $c''(.) > 0$ and $c'(\bar{e}) = \infty$. We define $S(e_t) \equiv E_y(y_t|e_t) - c(e_t)$ as the expected joint surplus that is generated per period by a work effort level of $e_t$. The first-best work effort level $e_{FB}$ maximizes the joint surplus and is defined by $f'(e_{FB})(\bar{y} - y) = c'(e_{FB})$. We write $S_{FB} \equiv S(e_{FB})$.

Before trade takes place, the principal offers a contract $(w_t, b_t, \bar{b}_t)$ to the agent. If the agent accepts the offer, he chooses a work effort level $e_t$ and subsequently $y_t$ realizes. After $y_t$ has realized, the agent is paid a fixed wage $w_t$ that is independent of $y_t$, as well as a bonus $b_t$ that is $b_t$ if $y_t = y$ ($y_t = \bar{y}$). Without loss of generality, we can restrict attention to $\bar{b}_t \leq 0$ and $\bar{b}_t \geq 0$.

If the agent rejects the offer, both parties consume their respective outside option. The principal’s outside option is $r_P$ for all $t$. The agent’s outside option $r_t^A$ is a random variable that can be either high $(r^h)$ or low $(r^\ell)$. We interpret $r^\ell$ as representing unemployment benefits and $r^h$ as being the per-period value of an alternative job. Throughout the paper, we will also refer to $r_t^A$ as the state of the game. The state realizes at the beginning of every period, before the principal issues his offer. The probability distribution of $r_t^A$ is as follows. If the agent has rejected the principal’s contract offer in period $t - 1$ and if $r_{t-1}^A = r^h$, the probability that $r_t^A = r^h$ is equal to one. In all other case, it is given by $\rho \in (0, 1)$. This specification implies that the agent cannot recall job offers that he has rejected in the past, though any outside offer that he accepts is renewed in the subsequent period. Further, we assume that trading is efficient in the following sense:

$$S_{FB} > r^P + r^h > r^P + r^\ell > 0.$$  (1)

Figure 1 summarizes the sequence of events for a representative period $t$.

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12 These assumptions ensure that the first-order approach is valid, see Rogerson (1985).

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Figure 1: Sequence of events in some period $t$. 
Players are concerned about their expected lifetime payoffs. Let \( I_t = 1 \) if the agent accepts the contract and 0 otherwise. Then, parties’ expected payoffs expressed as per-period averages are given by:

\[
\begin{align*}
\mu_t &= (1 - \delta)\mathbb{E} \sum_{t = 1}^{\infty} \delta^{t-t} \left[ I_{\tau} (w_{\tau} + b_{\tau} - c(e_{\tau})) + (1 - I_{\tau}) r_{A}^{\tau} \right], & \text{for the agent,} \\
\nu_t &= (1 - \delta)\mathbb{E} \sum_{t = 1}^{\infty} \delta^{t-t} \left[ I_{\tau} (y_{\tau} - w_{\tau} - b_{\tau}) + (1 - I_{\tau}) r_{P}^{\tau} \right], & \text{for the principal.}
\end{align*}
\]

Further, define \( s_t \equiv \mu_t + \nu_t \) as the joint per-period average payoffs. The analysis proceeds as follows. We first solve the model under the assumption that \( \rho \) is an exogenous parameter that cannot be influenced by either party. Subsequently, we endogenize the probability distribution of the agent’s outside option by introducing search effort.

### 3 Baseline Solution

We characterize optimal contracts using the equilibrium concept Perfect Public Equilibrium (PPE).\(^{13}\) In order to derive players’ equilibrium strategies we need to solve every stage backwards.

Consider some contract offer \((w, b, b)\) at date \(t\). Let \(u_t(e_t; w, b, b)\) denote the agent’s payoff from accepting that offer and choosing work effort level \(e_t\). Conditional on having accepted the offer, the agent will choose \(e_t\) as to maximize \(u_t(e_t; w, b, b)\). Thus, his incentive compatibility constraint is given by

\[
e_t \in \arg \max_e u_t(e; w, b, b). \quad \text{(IC}_t\text{)}
\]

Let \(u_t(w, b, b) \equiv u_t(e_t(w, b, b); w, b, b)\) denote the agent’s payoff from accepting the contract. Before trade takes place, the agent compares the payoffs from accepting the contract \((w, b, b)\) with the payoffs from rejecting it. Denoting the agent’s average per-period payoffs from rejecting a contract offer given \(r_{A}^{t}\) by \(\tilde{r}_{A}^{t}\), the distribution of \(r_{A}^{t}\) implies:

\[
\tilde{r}_{A}^{t} = \begin{cases} 
\tilde{r}^{h} = r^{h}, & \text{if } r_{A}^{t} = r^{h} \\
\tilde{r}^{\ell} = \frac{(1 - \delta)r^{\ell} + \delta \rho r^{h}}{1 - \delta(1 - \rho)}, & \text{if } r_{A}^{t} = r^{\ell}
\end{cases}
\]

The agent will thus accept the principal’s offer at date \(t\) if and only if:

\[
u_t(w, b, b) \geq \tilde{r}_{A}^{t}.
\]

Given \((\text{IC}_t)\) and \((\text{PC}_t)\), let \(\pi_t(w, b, b)\) denote the principal’s payoffs from offering the contract.

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\(^{13}\)PPE requires players’ strategies to be public and to form a Nash equilibrium after every public history. Notice that we can restrict attention to public strategies because the principal’s strategy is necessarily public, implying that the agent’s best reply is also a public strategy. See the textbook by Mailath and Samuelson (2006) for a proof of this result.
At any date \( t \), the principal will offer a contract as to maximize \( \pi_t(w, b, \overline{b}) \), subject to \( (PC_t) \) and \( (IC_t) \). In what follows, we will show that throughout the analysis, we can restrict attention to stationary contracts.

**Definition 1.** A contract \((w_t, b_t, \overline{b}_t)\) is stationary if on the equilibrium path, \((w_t, b_t, \overline{b}_t) = (w(r^A_t), b, \overline{b})\) for some \( w : \{r^h, r^l\} \to \mathbb{R} \) and \((\overline{b}, b) \in \mathbb{R}^2\).

A stationary contract thus involves a constant bonus scheme and a fixed wage component that reacts to the current state \( r^A_t \) according to the same rule in every period. Hence, a stationary contract specifies at most two different fixed wages that the agent can expect to receive during the course of his interaction with the principal.

Suppose that the contract \((w, b, \overline{b})\) offered in \( t \) induces continuation contracts in \( t + 1 \) with continuation payoffs as a function of \( y_t \) for the principal and the agent respectively given by \( \pi^i(y_t) \) and \( u^i(y_t) \) for \( r^A_{t+1} = r^i \in \{r^h, r^l\} \). Applying the first-order approach, \((IC_t)\) can be rewritten as follows:

\[
 f'(e_t) \left\{ (1 - \delta)(\overline{b} - b) + \delta \left( \rho [u^h(\overline{y}) - u^h(y)] + (1 - \rho) [u^l(\overline{y}) - u^l(y)] \right) \right\} = (1 - \delta)c'(e_t). \tag{3}
\]

**Proposition 1.** When \( \rho \) is exogenous, the principal’s (constrained) maximum payoff level can be attained by a stationary contract, where \((\overline{b} - b) = (\overline{y} - y)\) such that \( e_t = e^{FB} \) and \( w(r^A_t) \) such that \((PC_t)\) binds for all \( t \).

The formal proof of proposition 1 is in the appendix, yet the intuition behind it is straightforward. As is well-known, in a repeated moral hazard context, unlimited liability and risk neutrality allows the problem of generating a particular level of surplus to be separated from the problem of distributing it among the two parties.\(^{14}\) Thus, \( w_t \) can be used to make \((PC_t)\) binding, thereby minimizing the share of surplus admitted to the agent for any state \( r^A_t \). Then, payoff-maximization requires the principal to maximize the joint surplus, which is achieved by offering first-best incentives in every period. Finally, by offering a constant bonus scheme satisfying \((\overline{b} - b) = (\overline{y} - y)\), a binding \((PC_t)\) implies that the fixed wage changes only in response to the state. Thus, the principal’s equilibrium payoff level (from an \textit{ex ante} perspective) is given by

\[
 \pi(w(r^A_t), y, \overline{y}) = S^{FB} - [\rho \overline{r}^h + (1 - \rho)\overline{r}^l]\tag{4}
\]

Let us now turn to on-the-job search, i.e. to the case where the probability distribution of \( r^A_t \) is endogenously determined by the agent’s level of search effort.

\(^{14}\)See Levin (2003).
4 On-the-Job Search

The paradigm of frictional labor markets implies that the identification of job opportunities requires workers to engage in costly search activities.\textsuperscript{15} Within our context, the paradigm thus calls for the agent to exert some form of search effort in order to locate jobs paying $r^h$. In general, these search activities likely draw on the same resources that the agent uses to produce output for the principal, such as time, attention and cognitive capacity. In other words, on-the-job search will distract the agent from doing work for the principal. We account for these considerations by extending our baseline framework in the following way. In every period $t \in \{0, 1, 2, \ldots \}$, the agent can choose a level of search effort $z_t \in [0, e]$ in addition to work effort. Further, while the agent exerts work effort only if he trades with the principal, he chooses $z_t$ irrespective of whether $I_t = 1$ or 0, i.e. the agent searches both on-the-job as well as when unemployed. Exerting search effort in $t$ increases the probability of identifying an alternative job offer that can be used as a bargaining instrument in $t+1$. As before, whenever $I_t = 0$ plus $r_A^t = r^e$, the probability that $r_{t+1}^A = r^h$ is one, irrespective of $z_t$. In all other cases, the probability that $r_{t+1}^A = r^h$ is given by the function $\rho(z_t)$ with the properties $\rho'(\cdot) > 0$, $\rho''(\cdot) \leq 0$, $\rho(0) = 0$, $\rho(\bar{p}) \equiv \bar{p} < 1$ and $\rho'(0) = +\infty$. These properties imply that the agent will identify no alternative job opportunity if he does not exert at least a minimum of search effort ($\rho(0) = 0$), yet he will never be able to generate a job opportunity with certainty, even if he spends his entire effort endowment searching ($\bar{p} < 1$). Further, $\rho'(0) = +\infty$ ensures that the agent will exert at least a tiny bit of search effort.\textsuperscript{16,17}

We model the (opportunity) costs of search effort by letting the agent’s costs $c(\cdot)$ be a function of the total sum of effort $e_t + z_t$. Hence, the distraction of work effort due to on-the-job search is reflected by the marginal costs of work effort increasing with search effort. Notice that because condition (1) continues to hold, search effort is entirely inefficient. In the presence of search effort, the joint surplus in $t$ is given by $S(e_t, z_t) \equiv E(y|e_t) - c(e_t + z_t)$ and is therefore strictly decreasing in search effort. The socially efficient (first-best) search effort level $z^{FB}$ is thus equal to zero. However, since a higher outside option potentially improves the agent’s bargaining position, he may have an incentive to engage in inefficient search activities.

The introduction of search effort merely adds another effort dimension which is unobservable by the principal and which generates a noisy signal. Thus, we can continue applying PPE. However, before analyzing equilibrium contracts, observe that endogenizing $\rho(\cdot)$ also changes the agent’s rejection payoffs. When $r_A^t = r^e$ and $I_t = 0$, the agent does not merely passively consume her unemployment benefits, but also exerts search effort given the cost function $c(z_t)$. Thus, his optimal job search level under unemployment is the outcome of an optimization

\textsuperscript{15}See for instance Pissarides (2000) and Rogerson, Shimer and Wright (2005).
\textsuperscript{16}For instance, the agent may search only passively by registering for a job newsletter or an online job board.
\textsuperscript{17}Evidently, in a frictional labor market, principal and agent must also first locate each other, before they can trade, though our specification abstracts from the process that matches the two parties in the first place. However, abstracting from this process is without loss of generality as we can interpret our model as focusing on the interaction between principal and agent conditional on the two having located each other.
problem. Let \( z^u_t \) denote this outcome. The agent’s rejection payoffs are then given by

\[
\tilde{r}^A_t = \begin{cases} 
\tilde{r}^h = r^h, & \text{if } r^A_t = r^h \\
(1 - \delta) \left[ r^\ell - c(z^u_t) \right] + \rho(z^u_t) \delta r^h \\
1 - \delta (1 - \rho(z^u_t)), & \text{if } r^A_t = r^\ell
\end{cases}
\] (5)

We will return to the determination of \( \rho^u_t \) and \( \tilde{r}^\ell_t \) and their impact onto the equilibrium contract at a later instant.

Consider again some contract offer \((w, b, \bar{b})\) made by the principal in period \( t \) and let \( u_t(e_t, z_t; w, b, \bar{b}) \) denote the agent’s average per-period payoff from accepting the contract and choosing work effort \( e_t \) and search effort \( z_t \). Letting \( u^t(y_t) \) again denote continuation payoffs, we have:

\[
u_t(e_t, z_t; w, b, \bar{b}) = (1 - \delta) \left[ w + E_y(b|e_t) - c(e_t + z_t) \right] + \delta \left[ \rho(z_t) E_y(u^h|e_t) + (1 - \rho(z_t)) E_y(u^\ell|e_t) \right].
\] (6)

Conditional on accepting the contract, the agent will choose \( e_t \) and \( z_t \) as to maximize (6), thus yielding the following two incentive compatibility constraints:

\[
\begin{align*}
\frac{d}{d e_t} \left( (1 - \delta)(b - \bar{b}) + \delta \left( \rho(z_t) \left[u^h(y) - c(y) \right] + (1 - \rho(z_t)) \left[u^\ell(y) - c(y) \right] \right) \right) \\
= (1 - \delta)c'(e_t + z_t) \quad (IC^e_t)
\end{align*}
\]

\[
\begin{align*}
\frac{d}{d z_t} \left[ \rho(z_t) E_y(u^h|e_t) - E_y(u^\ell|e_t) \right] \\
= (1 - \delta)c'(e_t + z_t) \quad (IC^z_t)
\end{align*}
\]

Equations \((IC^e_t)\) and \((IC^z_t)\) characterize an effort-substitution problem as in Holmström and Milgrom (1991). As they show, an agent who needs to perform two distinct tasks will exert more (less) effort on a given task, the higher the benefits of performing that task (of performing the other task). In our framework, the agent performs work effort as well as search effort. While the benefits of work effort are determined by the bonus scheme, the benefits of search effort depend on how the continuation contract in \( t + 1 \) reacts to the state \( r^A_{t+1} \). Thus, the effort-substitution problem is created because the agent cares about his lifetime payoffs. \((IC^z_t)\) indicates that the agent will exert a positive level of search effort only if \( E_y(u^h|e_t) > E_y(u^\ell|e_t) \), i.e. only if a higher outside option increases his continuation payoffs. Define \( e_t(w, b, \bar{b}) \) and \( z_t(w, b, \bar{b}) \) as the effort levels implied by \((IC^e_t)\) and \((IC^z_t)\) given the contract offer \((w, b, \bar{b})\) and let \( u_t(w, b, \bar{b}) \equiv u_t(e_t(w, b, \bar{b}), z_t(w, b, \bar{b}); w, b, \bar{b}) \) denote the agent’s payoff from accepting the contract. Then, the agent’s participation constraint is as before given by

\[
u_t(w, b, \bar{b}) \geq \tilde{r}^A_t. \quad (PC_t)
\]

Further, let \( \pi_t(w, b, \bar{b}) \) denote the principal’s expected payoffs from offering a contract \((w, b, \bar{b})\)
satisfying \((IC^e_t), (IC^z_t)\) and \((PC_t)\). We have:

\[
\pi_t(w,b,\tilde b) = (1 - \delta) \left[ E_y(y|e_t(w,b,\tilde b)) - w - E_y(b|e_t(w,b,\tilde b)) \right] \\
+ \delta \left[ \rho(z_t(w,b,\tilde b))E_y(\pi^h|e_t(w,b,\tilde b)) + (1 - \rho(z_t(w,b,\tilde b))) E_y(\pi^\ell|e_t(w,b,\tilde b)) \right].
\]

(7)

The equilibrium contract offered at date \(t\) is characterized by the principal maximizing (7) subject to \((IC^e_t), (IC^z_t)\) and \((PC_t)\).

**Lemma 1.** If an equilibrium contract exists, then there exists an equilibrium contract that is stationary.

Lemma 1 simplifies the subsequent analysis. It implies that in the presence of on-the-job search, the principal can always maximize his payoffs by inducing a constant effort pair \((e(w,b,\tilde b), z(w,b,\tilde b))\) for all \(t\).

### 4.1 The Stationary Effort-Substitution Problem

Under a stationary contract, the incentive compatibility constraints simplify to:

\[
f'(e)(\bar b - \tilde b) = c'(e + z) \quad (IC^e)
\]

\[
\frac{\delta}{1 - \delta} \rho'(z)(u^h - u^\ell) = c'(e + z) \quad (IC^z)
\]

Denoting \(e^*\) and \(z^*\) as the solutions to the agent’s stationary effort-substitution problem, we can apply the implicit function theorem to derive the following comparative statics:

**Result 1.** Comparative Statics:

(i) \(\frac{\partial e^*}{\partial b} \geq 0, \frac{\partial z^*}{\partial b} \leq 0, \frac{\partial e^* + z^*}{\partial b} \geq 0\).

(ii) \(\frac{\partial e^*}{\partial u^\ell} \geq 0, \frac{\partial z^*}{\partial u^\ell} \leq 0, \frac{\partial e^* + z^*}{\partial u^\ell} \leq 0\).

(iii) \(\frac{\partial e^*}{\partial u^h} \leq 0, \frac{\partial z^*}{\partial u^h} \geq 0, \frac{\partial e^* + z^*}{\partial u^h} \leq 0\).

(iv) \(\frac{\partial e^*}{\partial \delta} \leq 0, \frac{\partial z^*}{\partial \delta} \geq 0, \frac{\partial e^* + z^*}{\partial \delta} \leq 0\).

As discussed above, under an effort-substitution problem, the effort spent on one task is increasing in the benefits from performing that task and decreasing in the benefits derived from performing the opposite task. The benefits of exerting work effort are given by the marginal increase in bonus payment that the agent expects to receive. Thus, increasing \(\bar b - \tilde b\) will increase \(e^*\) and reduce \(z^*\). Hence, our model shows that when workers search on the job, performance-pay does not only improve the agent’s work-effort incentives, it also induces the agent to substitute work for search effort, thereby reducing the deadweight loss created by search effort costs.

By contrast, the benefits from search are determined by the increase in the rent that the agent expects to receive under future employment contracts. First and foremost, these benefits are higher, the more patient the agent is, simply because the gains from searching today accrue
in the future. This result is intuitive. The more a worker cares about the future, the more will he be willing to incur immediate costs in order to improve future payoffs. The benefits from search are also higher the more the agent gains from entertaining a high relative to a low outside option, i.e. by the difference \( u^h - u^\ell \). Since we focus on compensation packages with a constant bonus scheme, this difference is ultimately determined by the fixed component \( w(r_t^A) \) of the contract that the principal offers to the agent. Thus, the effort-substitution problem implies that besides performance-pay, the principal can also influence the agent’s effort incentives via the fixed wage! We will show next that incentive contracts that improve the agent’s work-effort incentives will take the form of efficiency wage contracts.

### 4.2 Efficiency Wage Contracts

Notice that under a stationary contract, we can without loss of generality replace \( w(r_t^A) \) by \( u^i \) as the principal’s control variable. In particular, rearranging the agent’s per-period payoff derived under some stationary contract given the state \( r_t^A \), we have:

\[
w(r_t^A) = \begin{cases} 
    f(e^*)(\bar{b} - \bar{b}) - c(e^* + z^*) - \frac{1}{1 - \delta} \left[ (1 - \delta \rho(z^*)) u^h - \delta (1 - \rho(z^*)) u^\ell \right], & \text{if } r_t^A = r^h \\
    f(e^*)(\bar{b} - \bar{b}) - c(e^* + z^*) + \frac{1}{1 - \delta} \left[ \delta \rho(z^*) u^h - (1 - \delta (1 - \rho(z^*)) u^\ell \right], & \text{if } r_t^A = r^\ell 
\end{cases}
\]

(8)

Thus, the principal’s problem is to choose a quadruple \((\bar{b}, \bar{b}, u^\ell, u^h)\) as to maximize her average lifetime payoffs. Observe that ensuring participation requires that any stationary contract has \( u^h \geq r^h \) and \( u^\ell \geq \tilde{r}^\ell \). However, the agent’s effort incentives depend only upon the spread \( u^h - u^\ell \). Thus, the principal can increase the agent’s work-effort incentives and reduce his search-effort incentives either by reducing \( u^h \) or by increasing \( u^\ell \). As any reduction in \( u^h \) has a strictly positive first-order effect onto her profits, the principal will set \( u^h = r^h \) in any contract that she offers in equilibrium. Therefore, her problem reduces to finding the profit-maximizing triple \((\bar{b}, \bar{b}, u^\ell)\).

The principal thus uses two tools to steer the agent’s effort incentives, a standard piece rate component and a rent left to the agent in case he entertains a low outside option. Indeed, \((IC^c)\) implies that the principal can discourage any search activities if she sets \( u^\ell > \tilde{r}^\ell \). However, the agent’s effort incentives depend only upon the spread \( u^h - u^\ell \). Thus, the principal can increase the agent’s work-effort incentives and reduce his search-effort incentives either by reducing \( u^h \) or by increasing \( u^\ell \). As any reduction in \( u^h \) has a strictly positive first-order effect onto her profits, the principal will set \( u^h = r^h \) in any contract that she offers in equilibrium. Therefore, her problem reduces to finding the profit-maximizing triple \((\bar{b}, \bar{b}, u^\ell)\).

Further, notice that even though an increase in \( u^\ell \) reduces search and increases work effort, it also increases the principal’s expected wage costs. Therefore, the principal will in general opt to choose some \( u^\ell \) in the range \([\tilde{r}^\ell, r^h]\). Notice that even if \( u^\ell < r^h \), the principal still pays
the agent an efficiency wage at all dates $t$ with $r_t^A = r^t$, provided that $u^t > \tilde{r}^t$. To fix ideas, we introduce the following working definition for efficiency wage contracts:

**Definition 2.** An efficiency-wage contract is a stationary contract $(w(r_t^A), b, \tilde{b})$ such that for all $t$, $u_t(w(r_t^A), b, \tilde{b}) \geq \tilde{r}_t^A$ for $r_t^A \in \{r^h, r^f\}$ and $u_t(w(r_t^A), b, \tilde{b}) > \tilde{r}_t^A$ for at least one $r_t^A \in \{r^h, r^f\}$.

Thus, we define efficiency wage contracts as stationary contracts that leave the agent a positive rent in at least one state. With this definition, we keep with the literature in viewing efficiency-wage contracts as compensation contracts that (deliberately) pay the worker more than is necessary to induce participation. In fact, the notion of efficiency wages generally refers to situations where firms’ profits increase with the wages paid to its employees at least over some range.\(^{18}\) Whether wage contracts of the type defined by definition 2 are indeed efficient in this sense within our framework will be investigated in the next section where we analyze the relative effectiveness of performance-pay and efficiency wages as incentive devices.

## 5 Optimal Incentive Contracts

The stationary compensation contract $(w(r_t^A), b, \tilde{b})$ that the principal offers in equilibrium will maximize her expected per-period payoffs. Using relationship (8) and the fact that $u^h = r^h$ in equilibrium, the principal’s optimization problem can be stated as follows:

$$\max_{\{\bar{b}, b, u^f\}} \left[ E_y(y|e) - c(e + z) - \rho(z) r^h - (1 - \rho(z)) u^f \right]$$

s.t. $(IC^e), (IC^z), u^f \geq \tilde{r}^f$.

Differentiating the principal’s payoffs with respect to $b - \bar{b}$ and $u^f$ yields two first-order conditions that characterize the choice of the equilibrium contract:

$$f'(e)(\bar{y} - y) \frac{\partial e}{\partial (\bar{b} - b)} - \rho'(z)(r^h - u^f) \frac{\partial z}{\partial (\bar{b} - b)} - c'(e + z) \frac{\partial (e + z)}{\partial (\bar{b} - b)} = 0,$$

$$f'(e)(\bar{y} - y) \frac{\partial e}{\partial u^f} - \rho'(z)(r^h - u^f) \frac{\partial z}{\partial u^f} - c'(e + z) \frac{\partial (e + z)}{\partial u^f} - [1 - \rho(z)] = 0.$$  \(9\)

Equations (9) and (10) characterize the respective marginal impact of increasing either $\bar{b} - b$ or $u^f$ onto the principal’s profits. In equilibrium, the principal equates the marginal benefits of both incentive instruments. Hence, the (relative) intensity with which she uses either of the two instruments depends upon their relative impact onto her profits. This effect in turn depends at least in part on the sensitivity with which the agent’s work and search effort levels react to changes in either incentive device.

Consider a marginal increase in $\bar{b} - b$. Such an increase will raise the the agent’s work effort level and it will reduce his search activities. The benefits of increasing performance-pay are characterized by the first two expressions of equation (9). Performance-pay raises the

\(^{18}\)See for instance Katz (1986).
probability that output will be high and it decreases the probability that the principal will have to pay the agent his high outside option in the next period. Further, the comparative statics of the agent’s problem have shown that total effort rises (weakly) with the bonus. Hence, the costs of increasing performance-pay are given by the third term in (9), reflecting the associated reduction in total surplus due to an increase in total effort costs that the agent must be compensated for.

Now, consider a marginal increase in \( u^\ell \). The first three terms in (10) reflect the benefits of such an increase. First, just as performance-pay, increasing the efficiency wage raises the probability of a high output level and reduces the expected wage costs. Further, in contrast to a raise in performance-pay, the efficiency wage (weakly) reduces total effort thereby also reducing the effort costs. The only negative term in equation (10) is the fourth one which represents the first-order effect onto the principal’s wage costs.

To illustrate the principal’s tradeoff suppose for the moment that the agent’s reaction onto a change in \( b - \bar{b} \) is exactly the same as his reaction onto a change in \( u^\ell \), i.e. the respective derivatives coincide. In that case, the first three terms in both equations would be identical. Then, the only difference between the two equations would be the negative fourth term in equation (10). Hence, under the hypothetical case that bonus and efficiency wage have identical effects onto the agent’s effort levels, the principal would use only performance-pay and no efficiency wages to incentivize the agent. Intuitively, due to unlimited liability, any rise in \( b - \bar{b} \) can be recovered by adjusting the fixed wage. By contrast, increasing the rent to incentivize the agent implies by definition that the principal needs to give up some surplus. In equilibrium, however, the agent’s reaction will never be the same for both incentive instruments. Indeed, a necessary condition for this to hold would be that neither the bonus nor the efficiency wage would have an effect on total effort, which is evidently not possible. The profit-maximizing incentive mix will thus depend upon the relative sensitivity with which the agent reacts to a change in either incentive instrument. These in turn depend upon exogenous parameters such as \( r^h \), \( \delta \) and the search technology.

Consider an increase in \( r^h \), which may be interpreted as an improvement in labor market conditions or, from a cross-industry perspective, as an industry where a worker with a given ability level has better outside opportunities. A rise in \( r^h \) has two major effects onto the system of equations (9) and (10). First, since \( r^h \) raises the principal’s wage costs, it has a positive first-order effect onto the benefit of reducing search, i.e. the second term in both equations rises, increasing the benefit of both performance-pay and efficiency wages. Second, from the comparative statics of the agent’s behavior, we know that conditional on a given triple \((\bar{b}, B, u^\ell)\), a rise in \( r^h \) decreases work effort, while increasing search and total effort. Due to the concavity of \( f(e) \) and \( \rho(z) \) and the convexity of \( c(e + z) \), this raises \( f'(e) \) and \( c'(e + z) \) while reducing \( \rho'(z) \). Recall that both \( \bar{b} - \bar{b} \) and \( u^\ell \) raise \( e \) and reduce \( z \) but while \( \bar{b} - \bar{b} \) raises total effort costs, a rise in \( u^\ell \) reduces them. Hence, a rise in \( r^h \) tends to favor the use of efficiency wages over performance-pay, at least if we have no idea about the relative strength of the impact of either incentive device onto the agent’s behavior.
Next, consider an increase in $\delta$, i.e. consider some worker who is relatively more patient than another worker. Because that worker values future payoffs more strongly, he will exert relatively less work effort and relatively more search effort (and relatively more overall effort) for a given compensation package. Similar to $r^h$, if anything, these factors also tend to favor the use of efficiency wages over performance pay when workers are relatively more patient.

It should be noted that, in addition to the discussed first- and second-order effects of changes in the environmental parameters, there are also third-order effects, as $r^h$ and $\delta$ likely also affect higher-order derivatives of the agent’s optimal effort levels. In other words, the sensitivity with which the agent reacts to changes in the incentive structure will also crucially depend upon the environment. For instance, $(IC^z)$ indicates that the agent will respond more strongly to a change in the efficiency wage when $\delta$ is high, simply because any change in $u^\ell$ translates into a relatively larger change in the benefits from search than when $\delta$ is low. This is another source that favors the use of efficiency wages over performance-pay in high-patience environments. Further, the environment also influences other comparative statics, yet these effects generally depend upon third- and higher-order derivatives of $f(e)$, $\rho(z)$ and $c(e + z)$. Such derivatives, however, are notoriously difficult to interpret in an economically meaningful way.

The main message to carry away from the general model is that the absolute and relative use of efficiency wages and performance-pay as incentive tools depends upon their respective absolute and relative effectiveness in improving the principal’s profits. This effectiveness in turn depends first and foremost upon the response in the agent’s effort levels and this response in turn depends on the environment. Further, it should be noted that it is generally not guaranteed that we obtain an interior solution where the principal uses both incentive devices. For instance, we may easily come up with examples where the the marginal benefits of $b - b$ and $u^\ell$ decrease not only with an increased use of the respective incentive instrument but also with an increased use of the other incentive instrument. In that case, increasing the use of one incentive device will reduce the left-hand side of both equations (9) and (10) and it may happen that the left-hand side of one equation is greater than the other for any mix of the two incentive devices.

### 5.1 The Role of Unemployment Benefits

As a last step, let us assess the effect that a change in the level of unemployment benefits will have onto the agent’s behavior and thus onto the the surplus generated in equilibrium. Recall that the unemployment benefits determine the value of the agent’s outside option when the state is $r^\ell$. In particular, we had:

$$\tilde{r}^\ell_t = \frac{(1 - \delta) \left[ r^\ell - c(z_t^n) \right] + \rho(z_t^n)\delta r^h}{1 - \delta(1 - \rho(z_t^n))}. \tag{11}$$
On the equilibrium path, the agent will never reject an offer by the principal, hence without loss of generality, the agent’s (stationary) search behavior when unemployed, $z^u$, will satisfy:

$$z^u = \arg \max_z \left( 1 - \delta \right) \left[ r^\ell - c(z) \right] + \rho(z) \delta r^h \over 1 - \delta(1 - \rho(z))$$

(12)

We can easily prove the following result:

**Proposition 2.** A unit increase in $r^\ell$ raises $\tilde{r}^\ell_t$ by $\frac{1 - \delta}{1 - \delta(1 - \rho(z))}$.

Thus, an increase in the size of the unemployment benefits $r^\ell$ increases $\tilde{r}^\ell_t$, yet by less than one.

Now, to assess the effect of an increase in $r^\ell$ onto the contract that the principal offers in equilibrium, observe that $\tilde{r}^\ell$ appears only in the constraint $u^\ell \geq \tilde{r}^\ell$ in the principal’s optimization problem. Hence, a change in the unemployment benefits will only affect the equilibrium contract if this constraint binds, i.e. if the principal finds it optimal to use no efficiency wage to incentivize the agent! Thus, we have the following result:

**Corollary 1.** Whenever the principal pays an efficiency wage, a marginal increase in the unemployment benefits has no effect onto the equilibrium contract.

This resultss holds only for marginal changes in $r^\ell$. However, as long as the principal does not set $u^\ell = r^h$, a very large rise in unemployment benefits will eventually make the constraint $u^\ell \geq \tilde{r}^\ell$ binding. In that case, the high unemployment benefits force the principal to pay higher wages than he would find optimal to pay if unemployment benefits were lower. Further, observe that the agent does not care whether the level of $u^\ell$ is endogenously chosen by the principal or exogenously fixed by labor market regulation. In any case, a higher $u^\ell$ will lead him to reduce his search effort. Thus, in equilibrium, when $u^\ell \geq \tilde{r}^\ell$ binds, the principal will adjust the bonus scheme towards a change in $r^\ell$. In which direction the change will go is ambiguous, yet we know that the principal’s profits will fall as $r^\ell$ increases, because the binding constraint $u^\ell \geq \tilde{r}^\ell$ is tightened even further. However, because an increase in $r^\ell$ reduces search effort ceteris paribus, we do not know the effect onto welfare, i.e. onto the total value of the relationship. Indeed if the principal does not adjust the bonus scheme by much, a rise in unemployment benefits may even increase the value of the relationship, although it will certainly reduce the principal’s profits.
6 Incomplete Contracting

Throughout the analysis, we have assumed that the output produced by the agent is verifiable such that incentive contracts could be directly written on it. However, in many professions, performance is notoriously difficult to measure let alone to be verified by third parties. In such situations, effort incentives cannot be based on enforceable contracts, rather incentives must be provided using implicit contractual agreements, so-called relational contracts. Levin (2003) provides an extensive analysis of optimal incentive provision via such relational incentive contracts. He shows that if the interaction between principal and agent is repeated, the strength of the effort incentives that can be provided to the agent are bounded by the value of the relationship. In turn, the value of the relationship is determined by the relative value of what can be produced within the relationship compared to what each party can produce on its own, or with its next best alternative partner.

Our framework can be easily extended to such an incomplete contracting environment. We basically only need to add what Levin (2003) calls the dynamic enforcement constraint to the principal’s optimization problem. This constraint determines the maximum incentives that can be credibly provided by the principal within a given relationship. In our framework, this dynamic enforcement constraint reads as follows:

\[
(\bar{b} - \bar{b}) \leq \delta \left[ s(e(w(r_{t+1}^A), \bar{\delta}, \bar{b})) - E(r_{t+1}^A|\rho(z)) - r^P \right]
\]  

This condition shows that the incentives that are feasible and thus the surplus that can be generated depend upon the value of the relationship relative to the value of the parties’ outside options. A relaxation of this constraint increases the spread \((\bar{b} - \bar{b})\) that can be implemented and hence also increases the attainable surplus. Under these conditions, efficiency wages are likely to be even more useful. In particular, a reduction in search effort now has the additional benefit of reducing \(E(r_{t+1}^A|\rho(z))\), thereby relaxing (14) and improving the incentives that can be provided to the agent, which in turn increases the surplus.

Although a more thorough analysis analysis remains to be done\(^{19}\), the above considerations suggest that efficiency wages will be even more beneficial in occupations where performance is difficult to verify or even observe and where parties must thus rely on implicit, i.e. self-enforcing agreements.

\(^{19}\)Especially proving that we can again restrict attention to stationary agreements.
7 Conclusion

In this study, we have generated a new justification for the use of efficiency wages as an instrument to motivate workers. The rationale developed in this paper differs fundamentally from the classic shirking model in that it legitimizes the use of efficiency wage as a complement rather than as a substitute to performance pay.

To obtain these results, we have introduced on-the-job search to a repeated principal-agent relationship with moral hazard. Our main innovation has been to allow the agent to engage in search activities that improve his future outside options besides providing work effort for the principal. A key insight has been that the agent will only exert search effort as long as the payoff he receives under the employment relationship responds to the outside option that he entertains at any point in time. As search effort distracts the agent from the work he is actually suppose to do, the principal prefers to minimize the agent’s incentives to search. Our main result has been that the principal can reduce undesirable search effort by offering an efficiency wage contract to the agent. By paying an efficiency wage, the principal can reduce the responsiveness of the agent’s payoff to his outside option, thereby reducing his incentives to search. Our results greatly expand the class of environments in which efficiency wages are useful. Further, since industries generally differ with regard to the severity of incentive problems, workers’ level of patience or the competitiveness of the labor market, our model predicts that the optimal size and composition of workers’ compensation should differ across industries and jobs. Hence, our results contribute to explaining inter-industry differences in both total compensation as well as in the composition of the compensation of observationally similar workers. Finally, we find that under certain conditions, an increase in unemployment benefits may even have a positive impact on the productivity of the employment relationship. By contrast, in the classic shirking model of efficiency wages, such a rise worsens the agent’s incentives therefore decreasing the productivity of the relationship.
References


A Appendix

Proof of Proposition 1. Let $\pi_{t}^{\text{max}}$ denote the maximum payoff level that the principal can attain, given $r_{t}^{A}$, $(IC_{t})$ and $(PC_{t})$. Further, denote by $s_{t}^{\text{max}}$ the corresponding joint surplus and by $u_{t}^{\text{max}}$ the corresponding payoff to the agent such that $\pi_{t}^{\text{max}} = s_{t}^{\text{max}} - u_{t}^{\text{max}}$. Further, let $\pi'(y_{t})$ and $s'(y_{t})$ denote respectively the principal’s continuation payoff and the continuation surplus induced by the contract of interest for $r_{t+1}^{A} = r^{i} \in \{r^{h}, r^{\ell}\}$.

First, notice that under a (constrained) payoff-maximizing contract, $(PC_{t})$ can never be slack. To see this, suppose the opposite, $u_{t}(w_{t}, \tilde{b}_{t}, \tilde{b}_{t}) > \tilde{r}_{t}^{A}$. As $w_{t}$ does not appear in $(IC_{t})$, for given $\tilde{b}_{t}$, $\tilde{b}_{t}$ and $w_{t}(y_{t})$, changing $w_{t}$ does not affect the induced work effort level. Hence, the principal can always change $w_{t}$ in order to make $(PC_{t})$ without affecting $(IC_{t})$, thereby separating the problem of inducing participation from the problem of inducing a given effort level. Let $e_{t}^{\text{max}}$ denote the work effort level that maximizes the principal’s payoff in period $t$. Since $(PC_{t})$ will always bind, we have $u_{t}^{\text{max}} = r_{t}^{A}$ and thus

$$
\pi_{t}^{\text{max}} = s_{t}^{\text{max}} - \tilde{r}_{t}^{A} = (1 - \delta) [E(y|e_{t}^{\text{max}}) - c(e_{t}^{\text{max}})] + \delta [\rho E(s^{h}|e_{t}^{\text{max}}) + (1 - \rho)E(s^{\ell}|e_{t}^{\text{max}})] - \tilde{r}_{t}^{A}.
$$

Further, since $s_{t}^{\text{max}}$ is independent of $w_{t}$ for any $t$, maximizing payoffs requires $s'(y_{t}) = s_{t}^{\text{max}} = s^{\text{max}}$ for any $y_{t} \in \{y, \overline{y}\}$ and $r_{t+1}^{A} = r^{i}$. Suppose this is not true, i.e. there is at least one pair $(y_{t}, r^{i}) \in \{y, \overline{y}\} \times \{r^{h}, r^{\ell}\}$ such that $s'(y_{t}) < s_{t}^{\text{max}}$. Then, the principal could increase his payoffs without affecting either $(IC_{t})$ or $(PC_{t})$ by increasing $\pi'(y_{t})$ and thereby $s'(y_{t})$. In other words, any payoff-maximizing contract must entail payoff-maximizing continuation contracts. From $(PC_{t})$ and (14), it then follows that

$$
s_{t}^{\text{max}} = s^{\text{max}} = [E(y|e^{\text{max}}) - c(e^{\text{max}})].
$$

Thus, payoff-maximization requires maximizing the joint surplus which is obviously achieved by inducing $e^{\text{FB}}$ in every single period. Hence, our task is to find a stationary contract that induces $e^{\text{FB}}$, while making $(PC_{t})$ bind in each period.

Because $(IC_{t})$ is independent of the fixed wage, a stationary contract induces a constant level of work effort $e$ defined by:

$$
f'(e) \{ (\overline{b} - b) + \delta (\rho [u^{h}(\overline{y}) - u^{h}(y)] + (1 - \rho) [u^{\ell}(\overline{y}) - u^{\ell}(y)] ) \} = c'(e).
$$

Further, observe that a stationary contract requires $u^{i}(y_{t}) = u^{i}$ for all $i \in \{h, \ell\}$ and $y_{t} \in \{y, \overline{y}\}$. Hence, under a stationary contract, $(IC_{t})$ simplifies to

$$
f'(e) (\overline{b} - b) = c'(e).
$$

Now, in order to induce the work effort level $e$ defined by (16) by a stationary contract, consider
the alternative contract \((w(r_i^A), \tilde{y}, \tilde{b})\) with

\[
\bar{b}' - \bar{b} = (\tilde{b} - \bar{b}) + \delta \left( \rho \left[ u^h(\bar{y}) - u^h(y) \right] + (1 - \rho) \left[ u^i(\bar{y}) - u^i(y) \right] \right).
\] (18)

Evidently, replacing \((\bar{b}, \tilde{b})\) by \((\bar{b}', \tilde{b}')\) in (17) yields exactly the same work effort level as in (16), implying that the restriction \(u^i(y_t) = u^i\) places no restrictions whatsoever upon the induced effort level. Given this payoff-maximizing work effort level, making \((PC_t)\) bind is achieved by setting

\[
w(r_i^A) = \bar{r}_i^A - \bar{E}_y(b|e^*) + c(e^*) - \delta \left[ \rho \bar{r}_t^h + (1 - \rho) \bar{r}_t^f \right].
\] (19)

Thus, we have \(\pi_t(w(r_i^A), \bar{b}, \tilde{b}) = S(e) - \bar{r}_i^A\). Now, payoffs are maximized by maximizing \(S(e)\) which is obviously achieved by inducing the first-best effort level and setting \(\bar{b}' - \bar{b}' = \bar{y} - y\). \(\square\)

**Proof of Lemma 1.** First, observe that \(IC_t^e\) is independent of \(w\). Hence, as before, we can separate the problem of inducing a given work effort level from the problem of giving a particular payoff level, say \(u'\) to the agent by adjusting the fixed wage. It also follows from the proof of proposition 1 that we do not require variation in continuation payoffs over different realizations of \(y_t\) in order to provide work-effort incentives. Hence, without loss of generality, we can set \(u^i(\bar{y}) = u^i(y)\) for \(r_i^e \in \{ r^h, r^f \}\).

In the presence of on-the-job search, the principal’s payoffs at date \(t\) are now given by

\[
\pi_t = (1 - \delta) \left\{ E \left( \sum_{\tau=t}^{\infty} S(e_{t\tau}, z_{t\tau}) \right) - E \left( \sum_{\tau=t}^{\infty} u_{t\tau} \right) \right\}.
\] (20)

Looking at \((IC_t^e)\), (20) shows that we can no longer separate the minimization of the share of surplus admitted to the agent from the maximization of the joint surplus! To see this, notice that \(E \left( \sum_{\tau=t}^{\infty} S(e_{t\tau}, z_{t\tau}) \right)\) is maximized by setting \((e_t, z_t) = (e^{FB}, 0)\) for all \(t\), while \(E \left( \sum_{\tau=t}^{\infty} u_{t\tau} \right)\) is minimized by setting \(u_t = \bar{r}_i^A\) for all \(t\). However, by \((IC_t^e)\), making \((PC_t)\) bind for any \(t\) implies that \(z_t > 0\) and thus that maximization of joint surplus and minimization of the agent’s payoffs cannot be achieved at the same time.

Observe that by \((IC_t^e)\) and \((IC_t^z)\), a stationary compensation contract induces a constant effort pair \((e_t, z_t) = (e, z)\) for all \(t\). The reason is that neither \((IC_t^e)\) nor \((IC_t^z)\) depend upon \(r_i^A\). Just as \((IC_t^e)\), \((IC_t^z)\) is entirely forward looking in the sense that only the response of future contract and thus continuation payoffs onto future outside options matter for current search incentives. Thus, as long as the bonus scheme and the reaction of \(w_t\) to \(r_i^A\) are stationary, the induced work and search effort levels will be stationary as well.

To complete the proof, we show that a constant effort pair \((e, z)\) does indeed maximize the agent’s payoff in every period \(t\). The principal’s problem is to find a sequence of contracts \((w_t, b_t, \tilde{b}_t)\) that maximizes (20), subject to \((PC_t)\), \((IC_t^e)\) and \((IC_t^z)\). Thus, in contrast to the case with exogenous \(\rho(\cdot)\), now future contracts affect current behavior. In particular, consider some future date \(\tau > t\) where \(r_{\tau}^A = r^f\). Further, suppose that \(w_{\tau}\) is such that \(u_{\tau} > \bar{r}_t^f\). Then, reducing \(w_{\tau}\) does not violate \((PC_\tau)\) but increases the principal’s payoffs by reducing the share of surplus admitted to the agent. At the same time, however, reducing \(w_{\tau}\) when \(r_{\tau}^A = r^f\) increases
the spread in continuation payoffs $u^b - u^f$ in period $\tau - 1$, thereby increasing $z_{\tau-1}$ and reducing the surplus generated in $\tau - 1$. The payoff-maximizing sequence of contracts must balance these two effects. However, conditional on $(PC_t)$, $r^A_t$ has no direct effect onto $(PC_{\tau})$, $(IC_{e\tau})$ and $(IC_{z\tau})$ for any $\tau > t$. Thus, for given $r^A_t$, the principal faces the same problem for all $t$, thus proving that the payoff-maximizing contract induces a stationary effort pair $(e, z)$.

**Proof of Proposition 2.** We are looking for the marginal effect $\frac{\partial \tilde{r}^f}{\partial r^f}$. Since $\tilde{r}^f$ is a function of $z$, we have:

$$\frac{\partial \tilde{r}^f}{\partial r^f} = \frac{\partial \tilde{r}^f}{\partial z} \frac{\partial z}{\partial r^f}$$

Applying the envelope theorem, we have $\frac{\partial z}{\partial r^f} = 0$ and thus

$$\frac{\partial \tilde{r}^f}{\partial r^f} = \frac{1 - \delta}{1 - \delta (1 - \rho(z))}$$

(22)