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Arrovian aggregation of MBA preferences: An impossibility result

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Arrovian aggregation of MBA preferences: An impossibility result

Frederik Herzberg

ABSTRACT. Cerreia-Vioglio, Ghirardato, Maccheroni, Marinacci and Siniscalchi (*Economic Theory*, 48:341–375, 2011) have recently proposed a very general axiomatisation of preferences in the presence of ambiguity, viz. Monotonic Bernoullian Archimedean (MBA) preference orderings. We investigate the problem of Arrovian aggregation of such preferences — and prove dictatorial impossibility results for both finite and infinite populations. A novel proof methodology for special aggregation problems, based on model theory (in the sense of mathematical logic), is employed.

Key words: ambiguity; Ellsberg paradox; Knightian uncertainty; Monotonic Bernoullian Archimedean (MBA) preferences; Arrovian social choice; impossibility result; model theory (mathematical logic); ultrafilter; ultraproduct

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CONTENTS

1. Introduction	2
2. Monotonic Bernoullian Archimedean (MBA) preferences	2
3. The question of Arrovian aggregation of MBA preferences	4
4. Arrovian aggregation of MBA preferences is impossible	4
5. Discussion	5
6. Proofs	6
References	10

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1. Introduction

The study of preferences in the presence of risks with unknown probability distribution (also known as second-order risk, Knightian uncertainty or ambiguity) has been a very active area of research in recent decades. Interest in axiomatising such preferences goes back at least to the seminal paper by Ellsberg [4]; since then, various authors have proposed axiomatisations of increasing generality, in particular Gilboa and Schmeidler [9], Föllmer and Schied [6, 7] as well as Maccheroni, Marinacci and Rustichini [18].

An even greater level of generality was recently achieved by Cerreia-Vioglio, Ghirardato, Maccheroni, Marinacci and Siniscalchi [3] with their proposal of axiomatising preferences in the presence of ambiguity as Monotonic Bernoullian Archimedean (MBA) preferences and their extensive study of these preferences. This axiomatisation is distinguished by a comparatively low complexity, as measured by the number of quantifier alternations, which is the common complexity measure for formulae in mathematical logic.

One consequence is that the question whether a rational aggregation — in the sense of Arrovian social choice theory — of such preferences is possible, can be answered relatively quickly and comprehensively using tools from model theory (a branch of mathematical logic): Based on the model-theoretic approach to abstract aggregation theory pioneered by Lauwers and Van Liedekerke [15] and more recently further developed by Herzberg and Eckert [12, 13], we shall show that under some standard assumptions (a strong version of the independence axiom, the Pareto principle, and conditions ensuring the richness of the domain), non-dictatorial Arrovian aggregation of MBA preferences is impossible, neither for finite nor infinite populations.

Our realisation of this proof methodology requires the set of states of the world to be finite. There is the hope, however, that the use of infinitary model theory (cf. e.g. Keisler [14]) will provide the means to eventually overcome this limitation.

We shall review MBA preferences in Section 2, motivate the aggregation problem for MBA preferences in Section 3, formally state the formal results in Section 4 and conclude with a brief discussion in Section 5. The proofs of the main results are contained in Section 6.

2. Monotonic Bernoullian Archimedean (MBA) preferences

In the following, we briefly describe the axiomatisation of Monotonic Bernoullian Archimedean (MBA) preferences as well as the main representation theorem of Cerreia-Vioglio, Ghirardato, Maccheroni, Marinacci and Siniscalchi [3]. Herein, as was already pointed out in the introduction, we have to confine ourselves presently (for technical reasons) to the special case where there are only finitely

many states of the world. The framework is along the lines of Savage [19].

Let hence S be a finite set, and let Δ be the set of probability measures on the power-set algebra of S . Let X be a convex subset, with more than one element, of an arbitrary vector space. An *act* is a function $f : S \rightarrow X$. The *mixture* of two acts is defined in the obvious way: For all $\alpha \in [0, 1]$ and $f, g : S \rightarrow X$, the mixture $\alpha f + (1 - \alpha)g$ is defined point-wise as $s \mapsto \alpha f(s) + (1 - \alpha)g(s)$. (This is again an act as X is convex.) Clearly, any element $x \in X$ can itself be viewed as an act, viz. the constant act $s \mapsto x$.

MBA preferences are special preference relations among acts:

DEFINITION 1. Consider a binary relation \succsim on X^S with symmetric part \sim and asymmetric part \succ . The relation \succeq is called a Monotonic, Bernoullian and Archimedean (MBA) preference relation if and only if it satisfies all of the following:

- (1) \succsim is non-trivial,¹ complete and transitive;
- (2) for all acts f, g , if $f(s) \succsim g(s)$ for all $s \in S$, then $f \succsim g$ (Monotonicity Axiom);
- (3) for all $x, y, z \in X$ and $\lambda \in (0, 1]$, if $x \succ y$, then $\lambda x + (1 - \lambda)z \succ \lambda y + (1 - \lambda)z$ (Bernoullian or Risk Independence Axiom);
- (4) for all acts f, g, h , if $f \succ g \succ h$, then there exist $\alpha, \beta \in (0, 1)$ such that $\alpha f + (1 - \alpha)h \succ g \succ \beta f + (1 - \beta)h$ (Continuity or Archimedean Axiom).

MBA preferences admit a beautiful representation theorem (Cerreia-Vioglio, Ghirardato, Maccheroni, Marinacci and Siniscalchi [3, Proposition 1]) that substantially generalises the well-known representation theorems for maxmin expected-utility preferences (Gilboa and Schmeidler [9]) or variational preferences (Maccheroni, Marinacci and Rustichini [18]). This representation theorem (and to some extent also its predecessors) requires the notion of monotonic normalised continuous functionals. Let Γ be an interval $\subseteq \mathbf{R}$, and consider a functional $I : \Gamma^S \rightarrow \mathbf{R}$. We call I *monotonic* if and only if $I(a) \leq I(b)$ whenever $a \leq b$ (point-wise on S); I is called *normalised* if and only if I maps the constant function $s \mapsto \alpha$ to α for all $\alpha \in \Gamma$. In light of the norm equivalence on finite-dimensional vector spaces, the map I is called *continuous* if it is continuous with respect to any norm on the finite-dimensional $\mathbf{R}^S \supseteq \Gamma^S$, e.g. the Euclidean norm.

THEOREM 2 (Cerreia-Vioglio *et al.*). A binary relation \succsim on X^S is an MBA preference relation if and only if there exist a non-constant affine function $u : X \rightarrow \mathbf{R}$ and a monotonic, normalised, continuous functional $I : u[X]^S \rightarrow \mathbf{R}$ such that for all $f, g : S \rightarrow X$,

$$f \succsim g \Leftrightarrow I(u \circ f) \geq I(u \circ g).$$

¹In the sense that not for all acts f, g , $f \succsim g$.

The function u and the functional I are unique up to affine monotone transformation.

3. The question of Arrovian aggregation of MBA preferences

Moving to the aggregate level, consider a population in which all individuals rank their available acts under ambiguity according to MBA preference relations. One may ask: Is it possible to merge ('aggregate') these preferences into a single MBA preference relation, in a way that respects certain responsiveness axioms? Put slightly differently: Is there a rational mechanism, other than dictatorship, by which a population of individuals holding MBA preferences can agree on a single aggregate MBA preference ordering? If the answer were affirmative, this would, for instance, entail a microeconomic foundation for the macroeconomic use of MBA preferences at the aggregate level, e.g. multiplier preferences (cf. Hansen and Sargent [10]) employed by a social planner.

If one employs responsiveness axioms that are derived from Arrovian social choice theory *and restricts oneself to finite populations only*, one may expect the answer to be negative, on account of Arrow's [1] impossibility theorem. For the case of infinite populations — which has some potential interest for microfoundations in light of the continua-of-agents models used in macroeconomics —, one may hope that such an aggregation is feasible, on account of Fishburn's [5] possibility theorem.

As it turns out, however, the Archimedean axiom moves the problem towards the setting of Campbell's [2] impossibility theorem for aggregating continuous preference relations. Thus, there is a rather general result about Arrovian aggregation of MBA preferences — but, unfortunately, it is negative.

4. Arrovian aggregation of MBA preferences is impossible

In order to state the result, let us denote by \mathcal{P} the set of MBA preferences, and let us fix an arbitrary non-empty set N , henceforth called the *population*. A typical element of \mathcal{P}^N (*profile*) will be denoted by $\underline{P} = (P_i)_{i \in N}$. Whenever P is a preference ordering, we write P^\succ for its asymmetric part.

Let us introduce the following terminology.

- An *MBA aggregator* is a map with domain $\subseteq \mathcal{P}^N$ and range $\subseteq \mathcal{P}$.
- An MBA aggregator F is *systematic* if and only if for every \underline{P} in the domain of F and all $f, f', g, g' : S \rightarrow X$,

$$\{i \in N : f P_i g\} = \{i \in N : f' P_i g'\} \Rightarrow (f F(\underline{P}) g \Leftrightarrow f' F(\underline{P}) g').$$

- An MBA aggregator F is *Paretian* if and only if for every \underline{P} in the domain of F and all $f, g : S \rightarrow X$, if $f P_i g$ for all $i \in N$, then $f F(\underline{P}) g$.
- An MBA aggregator is *dictatorial* if and only if there exists some *dictator* $d \in N$ such that for every \underline{P} in the domain of F and all $f, g : S \rightarrow X$,

$$f F(\underline{P}) g \Leftrightarrow f P_d g.$$

- An MBA aggregator F is *universal* if and only if its domain is \mathcal{P}^N .
- An MBA aggregator is *weakly universal* if and only if there are MBA preference relations $\succsim_1, \succsim_2, \succsim_3$ such that
 - (1) $f \succsim_1 g, f' \succsim_1 g', f \succsim_2 g, f' \prec_2 g', f \prec_3 g, f' \succsim_3 g'$ for some acts f, f', g, g' , and
 - (2) the domain of F contains $\{\succsim_1, \succsim_2, \succsim_3\}^N$.
- An MBA aggregator F is said to have a *large domain* if and only if it is weakly universal, N is infinite and there exists a profile $(P_i)_{i \in N}$ in its domain and some acts f, g, h such that the set

$$\{i \in N : (\alpha f + (1 - \alpha)h) P_i^\succ g P_i^\succ (\beta f + (1 - \beta)h)\}$$

is finite for all $\alpha, \beta \in (0, 1)$ and such that $f F(\underline{P})^\succ g F(\underline{P})^\succ h$.

Thus, having a large domain means that the domain contains a profile of MBA preferences that fails to be equicontinuous at some acts f, g, h in a rather strong sense (viz. that not even one of its (infinite) subsequences is equicontinuous) and that these acts can be chosen in a non-degenerate manner (viz. such that $f \succ g \succ h$ holds in aggregate).

The notion of weak universality is indeed weaker than universality:

REMARK 3. *If S contains at least two distinct elements, then any universal MBA aggregator is weakly universal.*

The main finding of this paper is the following impossibility result:

- PROPOSITION 4. (1) *If N is finite, any weakly universal, Paretian, systematic MBA aggregator is dictatorial.*
- (2) *If N is infinite, any Paretian, systematic MBA aggregator with a large domain is dictatorial.*

5. Discussion

The axiomatisation of preferences in the presence of ambiguity in terms of MBA preferences by Cerreia-Vioglio *et al.* is remarkable in its great mathematical generality and elegant, simple logical form. The low degree of formal complexity permits the direct application of an aggregation-theoretic methodology that employs tools from mathematical logic. A relatively general impossibility theorem, encompassing both finite and infinite electorates, follows.

Hence, the aggregation of MBA preferences cannot follow Arrovian lines, not even on infinite populations. New avenues may open if one significantly weakens or jettisons independence-like conditions (such as the strong independence notion of systematicity — also known as neutrality — in this paper): For the very special case of von Neumann–Morgenstern preferences, it has long been known that mere Paretian aggregation is possible even for finite electorates (Harsanyi [11]) while Arrovian aggregation is not (Le Breton [16]). The Paretian aggregation of a large class of preference relations, including the Gilboa–Schmeidler maxmin expected-utility preferences, has been investigated by Gajdos, Tallon and Vergnaud [8].

However, the cost of thus abandoning an Arrovian approach to aggregation theory is, of course, that one leaves behind the most firmly established framework of social choice theory. One can hardly argue that a mere Paretian aggregate of MBA preference orderings should be seen as a compelling microeconomic foundation of a macroeconomic use of MBA preference relations at the aggregate level. In view of the impossibility results of this paper, it appears unlikely that MBA preferences at the aggregate level can be given a microeconomic justification.

6. Proofs

PROOF OF REMARK 3. It is enough to construct $\succsim_1, \succsim_2, \succsim_3 \in \mathcal{P}$ and $f, g : S \rightarrow X$ such that $f \sim_1 g$, $f \succ_2 g$ and $f \prec_3 g$. Then one can simply put $f' := g$ and $g' := f$ and \mathcal{P}^N satisfies the requirements on the domain of a weakly universal aggregator.

The construction of such $\succsim_1, \succsim_2, \succsim_3 \in \mathcal{P}$ and $f, g : S \rightarrow X$ is straightforward, thanks to the representation theorem of Cerreia-Vioglio *et al.* [3, Propositions 1] (Theorem 2).

Let $x_0, x_1 \in X$ be two distinct outcomes, and let $u : X \rightarrow \mathbf{R}$ be a non-constant affine function; without losing generality, $u(x_0) < u(x_1)$. Let $s_0, s_1 \in S$ be distinct, too, and choose f, g such that $f(s_0) = x_0$, $f(s_1) = x_1$, $g(s_0) = x_1$, $g(s_1) = x_0$. Let $\Delta(s_0, s_1)$ be the set of all probability measures on S that are actually concentrated on $\{s_0, s_1\}$, and let us define I_1, I_2, I_3 as follows:

$$\begin{aligned} I_1 : \quad a &\mapsto \min_{p \in \Delta(s_0, s_1)} \int a \, dp \\ I_2 : \quad a &\mapsto u(x_1) + \min_{p \in \Delta(s_0, s_1)} \int (a - u \circ f) \, dp \\ I_3 : \quad a &\mapsto u(x_1) + \min_{p \in \Delta(s_0, s_1)} \int (a - u \circ g) \, dp \end{aligned}$$

Now, clearly for all $h : S \rightarrow X$,

$$\begin{aligned} \min_{p \in \Delta(s_0, s_1)} \int u \circ h \, dp &= \min_{q \in [0,1]} (qu \circ h(s_0) + (1-q)u \circ h(s_1)) \\ &= u \circ h(s_1) + \min_{q \in [0,1]} q (u \circ h(s_0) - u \circ h(s_1)) \\ \min_{p \in \Delta(s_0, s_1)} \int -u \circ h \, dp &= \min_{q \in [0,1]} (-qu \circ h(s_0) - (1-q)u \circ h(s_1)) \\ &= -u \circ h(s_1) + \min_{q \in [0,1]} q (u \circ h(s_1) - u \circ h(s_0)), \end{aligned}$$

which can be used to verify that I_2, I_3 are normalised (that I_1 is normalised and that all of them are continuous and monotonic is obvious), and in a similar vein for all $h, \tilde{h} : S \rightarrow X$,

$$\begin{aligned} &\min_{p \in \Delta(s_0, s_1)} \left(\int u \circ h \, dp - \int u \circ \tilde{h} \, dp \right) \\ &= \min_{q \in [0,1]} \left(qu \circ h(s_0) + (1-q)u \circ h(s_1) - qu \circ \tilde{h}(s_0) - (1-q)u \circ \tilde{h}(s_1) \right) \\ &= u \circ h(s_1) - u \circ \tilde{h}(s_1) \\ &\quad + \min_{q \in [0,1]} q \left(u \circ h(s_0) - u \circ \tilde{h}(s_0) - u \circ h(s_1) + u \circ \tilde{h}(s_1) \right) \end{aligned}$$

From here, we can deduce for our above choices of f, g , exploiting $u(x_0) < u(x_1)$:

$$\begin{aligned} I_1(u \circ f) &= u(x_1) + u(x_0) - u(x_1) = u(x_0) \\ I_1(u \circ g) &= u(x_0) + 0 = u(x_0) \\ I_2(u \circ f) &= u(x_1) + 0 = u(x_1) \\ I_2(u \circ g) &= u(x_1) + u(x_0) - u(x_1) + 0 = u(x_0) \\ I_3(u \circ f) &= u(x_1) + u(x_1) - u(x_0) + 2(u(x_0) - u(x_1)) = u(x_0) \\ I_3(u \circ g) &= u(x_1) + 0 = u(x_1) \end{aligned}$$

Thus,

$$\begin{aligned} I_1(u \circ f) &= I_1(u \circ g) \\ I_2(u \circ f) &> I_2(u \circ g) \\ I_3(u \circ f) &< I_3(u \circ g) \end{aligned}$$

and therefore

$$f \sim_1 g, \quad f \succ_2 g, \quad f \prec_3 g$$

if $\succsim_1, \succsim_2, \succsim_3$ are the preference relations represented by $I_1 \circ u, I_2 \circ u, I_3 \circ u$, respectively. \square

PROOF OF PROPOSITION 4. For the proof of the main result, a purely technical reformulation is required. The model-theoretic approach to aggregation theory assumes that all relations involved are defined on the same set, in our case the set of acts. This means, in

particular, that we need to find a way of viewing scalars weights as acts and hence the mixture operator as a ternary operation on acts.

In order to achieve this, fix two distinct elements of $x_0, x_1 \in X$. For each $\alpha \in [0, 1]$, we identify α with the constant act $\alpha x_0 + (1 - \alpha)x_1$. It is easy to verify that $\alpha x_0 + (1 - \alpha)x_1$ is invertible. Therefore, the mixture operator can be viewed as mapping a triple — consisting of a constant act of the form $\alpha x_0 + (1 - \alpha)x_1$ and two other acts — to another act, viz. the one defined by $\alpha f + (1 - \alpha)g$. In addition, the definition of an MBA preference relation invokes the notion of a constant act and of the evaluation of an act at a certain state of the world s .

In this vein, all the axioms describing an MBA preference relation can be formulated using

- the strict preference relation symbol (denoted by R in the following),
- the predicate of being a constant act (denoted by C),
- the predicate of being an act of the form $\alpha x_0 + (1 - \alpha)x_1$ (denoted by I),
- for each state of the world s , the formal projection operator π_s that assigns to any act f the constant act $f(s)$,
- the formal mixture operator m , whose interpretation is that to any triple consisting of two acts f, g and an act of the form $\alpha x_0 + (1 - \alpha)x_1$ assigns $\alpha f + (1 - \alpha)g$.

and the usual logical connectives and quantifiers of first-order predicate logic.

For purely technical reasons we need to find a formalisation of the Archimedean axiom which does not contain negations; therefore, the strict preference relation symbol has to be used as a fundamental symbol. Since the preference orderings are assumed to be complete, we can recover the weak preference relation P and the symmetric part Q of P from R by introducing

$$fPg \equiv \neg gRp, \quad fQg \equiv fPg \wedge gPf.$$

In this formalisation, the MBA preference axioms are either universal, (i.e. of the form $(\forall f)(\forall g)(\forall h)\varphi$ where φ is quantifier-free) or — in the case of the Archimedean axioms — universal-existential (i.e. of the form $(\forall f)(\forall g)(\forall h)(\exists f')(\exists g')\phi$ where ϕ is quantifier-free. Of course, any preference ordering P among acts gives rise to a first-order structure — in the sense of model theory — whose domain is the set of acts and which interprets $R, C, I, (\pi(s))_{s \in S}, m$.

Using an extension, due to Herzberg and Eckert [13], of the model-theoretic approach to aggregation theory originally pioneered by Lauwers and Van Liedekerke [15], one can now prove that neither for finite nor for infinite electorates, there can be any non-dictatorial aggregators satisfying the conditions of the theorem. At the heart of the proof in both cases are the following two crucial facts. First, every

weakly universal systematic Paretian aggregator maps any profile \underline{P} in its domain to the restriction, to the set of acts, of the ultraproduct of (the sequence of first-order structures obtained from) \underline{P} with respect to some ultrafilter \mathcal{D}_F [13, Lemma 3.5, Lemma 3.8]. Secondly, if an ultrafilter contains a finite set, it must be *principal*, i.e. the system of supersets of some singleton — in other words, it must be of the form $\{C \subseteq N : d \in C\}$ for some $d \in N$. Therefore, if an ultrafilter contains a finite set, one can infer that F is a dictatorship: In order to verify that d is the dictator ruling under F , it is enough to note that F is the ultraproduct construction with respect to \mathcal{D}_F restricted to the set of acts (Herzberg and Eckert [13, Lemma 3.8]), whence it follows that actually

$$F(\underline{P}') = P'_d$$

for all profiles \underline{P}' in its domain.

Hence, in order to prove the dictatorial nature of F , it is sufficient to verify that indeed there is a finite subset of N among the elements of the ultrafilter \mathcal{D}_F . In the case of a finite electorate, this is trivial, because $N \in \mathcal{D}_F$ by the definition of an ultrafilter and N is finite by assumption.

In the case of an infinite electorate, more work and an additional assumption in the guise of the above large domain condition is needed. Let us first note that in the above logical formalisation, the Archimedean Axiom reads

$$\forall f \forall g \forall h \exists a \exists b \left(I a \wedge I b \wedge \left(\begin{array}{c} (f R g \wedge g R h) \rightarrow \\ (m(a, f, h) R g \wedge g R m(b, f, h)) \end{array} \right) \right)$$

Let us now fix acts f, g, h and a profile \underline{P} as in the large domain condition and consider the formula

$$\phi[a, b] \equiv I a \wedge I b \wedge f R g \wedge g R h \wedge m(a, f, h) R g \wedge g R m(b, f, h),$$

which has the — from a model-theoretic vantage point rather nice — property that it is free of universal quantifiers, negations and disjunctions, which we shall exploit shortly.

If \succsim is an arbitrary preference ordering, we shall write $\succsim \models \phi[a, b]$ (read: $\phi[a, b]$ holds for \succsim) if and only if it holds when the asymmetric part \succ of \succsim is inserted for R in $\phi[a, b]$, in other words, if and only if indeed a, b are acts of the form $a = \alpha x_0 + (1 - \alpha)x_1$, $b = \beta x_0 + (1 - \beta)x_1$ for $\alpha, \beta \in (0, 1)$ and

$$f \succ g \succ h, \quad \alpha f + (1 - \alpha)h \succ g \succ \beta f + (1 - \beta)h.$$

Observe that for any preference ordering P that satisfies the Archimedean Axiom as well as $f P^\succ g P^\succ h$, we have the existence of some $\alpha, \beta \in (0, 1)$ such that

$$(1 - \alpha)h \succ g \succ \beta f + (1 - \beta)h.$$

In particular, in our new notation, this means that

- (1) for all $P \in \mathcal{P}$, if $fP^\succ gP^\succ h$, then for some a, b , $P \models \phi[a, b]$.

By Herzberg and Eckert [13, Lemma 3.8], the aggregate $F(\underline{P})$ of any profile $\underline{P} = (P_i)_{i \in N}$ in the domain of F can be seen as the restriction of the ultraproduct of \underline{P} — more precisely: of the sequence of formal structures corresponding to the P_i — to X^S with respect to some ultrafilter \mathcal{D}_F on N .

Combining this model-theoretic description of $F(\underline{P})$ with the fact that $\phi[a, b]$ is free of universal quantifiers, negations and disjunctions, the inductive proof (in formula complexity) of the fundamental theorem on ultraproducts (Łoś's theorem [17]) shows: for all a, b , $F(\underline{P}) \models \phi[a, b]$ if and only if the collection of those i with $P_i \models \phi[a, b]$ is an element of the ultrafilter \mathcal{D}_F . Now, by our observation (1), there must be acts a, b such that $F(\underline{P}) \models \phi[a, b]$, because (i) $F(\underline{P}) \in \mathcal{P}$ (F being an MBA aggregator) and (ii) our choice of f, g, h, \underline{P} ensures that $fF(\underline{P})^\succ gF(\underline{P})^\succ h$. It follows that indeed $\{i \in N : P_i \models \phi[a, b]\} \in \mathcal{D}_F$ for some acts a, b .

On the other hand, by our choice of f, g, h and \underline{P} as in the large domain condition, the coalition $\{i \in N : P_i \models \phi[a, b]\}$ is actually finite. Hence, we have shown that among the elements of \mathcal{D}_F there is indeed a finite subset of N . As was explained before (when proving the result for the finite case), this entails that (\mathcal{D}_F is principal and hence) F is a dictatorship.

□

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