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Conference Paper
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Suggested Citation: Molenda, Inga; Sieg, Gernot (2013) : Residential Parking in Vibrant City Districts, Beiträge zur Jahrestagung des Vereins für Socialpolitik 2013: Wettbewerbspolitik und Regulierung in einer globalen Wirtschaftsordnung - Session: Urban Economics II, No. E10-V2

Provided in Cooperation with:
Verein für Socialpolitik / German Economic Association

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Residential Parking in Vibrant City Districts

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Abstract

Living downtown has advantages because it allows for a convenient access to a variety of shopping and leisure activities and disadvantages because it implies difficulties to find a parking spot when parking capacity is scarce. We formally model the trade-off between privilege parking for residents and economic vitality in terms of the product variety offered in a vibrant city district and identify situations in which assigning on-street parking spaces to residential parking is an optimal policy, both from a welfare and from the residents’ perspective. Beforehand, we make clear that privilege parking for residents is unlikely to constitute a first best policy with regard to parking cost minimization and ensuring an efficient level of economic vitality.

JEL classification: R41; R48; D61; D72
Key words: residential parking, urban vitality, love of variety, local decision-making

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1 Introduction

Downtown areas and other vibrant districts of European cities are often both commercial and residential areas. Residents, a variety of retail stores and restaurants, and visitors from outside the district all add to their vibrancy. For car drivers, the downside of living in, working in, or visiting such a district is the struggle to find an individually suitable parking spot. Because many downtown areas and their surrounding districts of European cities were developed when car ownership was not as common as it is nowadays, its residents often lack sufficient private parking capacity so that they are dependent on public parking spaces. Residents usually experience a high disutility from searching for a parking spot in “their” neighborhood and from possibly not being able to park in close proximity to their homes, so they often favor parking regulations that privilege them. However, residents of vibrant city districts normally also enjoy the variety of stores and restaurants in their neighborhood and they know that non-resident customers are relevant to the variety offered and that parking policies like the establishment of residential parking areas can influence their visits negatively.

Still and Simmonds (2000) report results of both attitudinal studies and land-use/transport models that support the argument that economic vitality of urban centers is sensitive to the provision of parking. They emphasize the concerns local authorities often have when deciding on parking policies: retail is important to local residents, and maintaining the economic vitality of urban centers requires expenditures of shoppers from the outside as well.

A non-resident visits a vibrant city district to shop if the associated private benefit exceeds the associated private cost. In a setting where more shoppers add to more variety and with that to utility gains on other people’s side but, on the other hand, also induce a parking cost increase due to more competition for a suitable parking spot, either too many or too few non-residents might visit the district in an unregulated equilibrium, depending on the magnitude of the overall external effect.

Assuming the absence of further market distortions, a first-best solution contains both the minimization of the aggregate parking costs and the guarantee of an
efficient number of non-resident shoppers. However, its implementation can be difficult for several reasons: the solution includes parking fees and/or subsidies, but might be not feasible in case of heterogeneous residents and heterogeneous non-resident shoppers with regard to their preferences for variety and their valuation of parking cost. Furthermore, parking subsidies can provoke undesired behavior such as visits by non-residents to earn the subsidy but without any intention to shop. And in case that it is optimal that residents pay for parking, it is unlikely that they actually do so, either because of their opposition or because of urban development measures that a municipality applies.

As soon as residents have privileges on public parking capacities, the municipalities apply some kind of a residential parking policy. In Germany, for example, there are basically two different residential parking policies: the residents are either exempted from paying the charged parking fees or they are exclusively entitled to use a certain share of on-street parking spaces, say all parking spaces on one side of the road. In both cases, the residents need a residential parking permit that is issued by the municipal road traffic departments for an administrative fee of about €30 per annum.\textsuperscript{1} Important to note is that such a permit gives a special parking right to the holder but does not guarantee that the right can be exercised.

In our analysis, we discuss a residential parking policy according to which a certain share of on-street parking spaces is reserved for residents as an alternative to the first-best policy that might prove elusive. We reveal in which case assigning on-street spaces to residential parking can be reasonable in principle. And whilst taking into account that there is a trade-off between parking privileges for residents and economic vitality in terms of the product variety offered and valued by residents and visitors, we determine the optimal share of residential parking spaces. Additionally, because parking policies are decided on a local level and local voters are residents (Arnott, 2011), we further analyze the optimal share of on-street spaces allocated to residential parking from the residents’ perspective only and we

\textsuperscript{1}Applicants for residential parking permits have to meet a number of requirements: they normally have to be the owner of the car for which the permit is valid, they can apply for one permit only or they must not have a private parking space.
find that it exceeds the one that is optimal from the welfare perspective. With regard to meeting the two objectives minimization of parking cost and ensuring an efficient number of non-resident shoppers, such a residential parking policy is certainly inferior to the first-best policy.

Since transport economists have recognized that parking is a crucial element in urban transportation, parking has received increasing attention in the economic literature. Willson (1995) and Shoup (1999, 2005) discuss planning standards such as Minimum Parking Requirements with regard to urban sprawl, automobile use, and the accompanying social costs. Furthermore, many publications address cruising for on-street parking in downtown areas, both in isolation and in the context of general traffic congestion as well as in absence and in presence of an private off-street market (e.g., Glazer and Niskanen (1992); Arnott and Rowse (1999, 2009); Anderson and de Palma (2004); Shoup (2005); Calthrop and Proost (2006); Arnott and Inci (2006)). These studies recommend parking fees that reflect the social cost of parking as an efficient solution, at least if there is no off-street market. In presence of an off-street market, adjusting the on-street parking fee to the off-street price is found to be beneficial since it reduces cruising for parking. This positive effect is empirically observed by van Ommeren et al. (2012) for the Netherlands where parking fees on- and off-street are quite similar. To overcome the opposition of different parties that arises when the introduction of or an increase in on-street parking fees is discussed, Shoup (2005, p. 398) suggests the implementation of benefit districts where the parking revenue “is spent to clean the streets, plant street trees […] and ensure public safety.” In this context, he also addresses residential parking by contrasting the establishment of pure residential parking districts with a parking policy that “taxes foreigners living abroad” while residents park for free. Van Ommeren et al. (2011) emphasize the inefficiencies that can result from such a policy. For the residents of Amsterdam, they estimate a marginal willingness to pay of about €10 per day and find that it exceeds the actual tariff for a permit considerably but that it is lower than the parking fee that non-residents pay, which implies an inefficient use of the parking spaces. They also point to the further efficiency losses due to cruising for parking when the fee that non-residents pay
for parking can be assumed to reflect the social parking cost. To our knowledge, however, residential parking has not yet been analyzed in the context of the trade-off between parking cost minimization and valued product variety offered in a city district that accommodates both residents and different types of businesses.

2 Model

We look at a city’s vibrant residential and commercial district located at 0 on a [0, 1] interval. The residents of the district are homogeneous and denoted by \( r \). The number of residents is fixed and normalized to 1. The stores located in the district are denoted by \( s \). Outside of the district, non-residents, the mass of which is also 1, live uniformly distributed on a [0, 1] interval.

\[ \text{residents} \]
\[ \text{stores} \]

\[ 0 \hspace{1cm} \text{non-residents} \hspace{1cm} 1 \]

Figure 1: Spatial model structure

Stores and Product Variety. Each store offers a single variety of a differentiated product. Although products are heterogeneous, we assume that each store sells a unit of its product at the exogenously given price \( p \). With regard to their cost structure, the stores are assumed to be homogeneous. Marginal cost are zero but each store has to bear an entry cost \( \epsilon = E(s) \), which rises the more stores enter \( (\epsilon' = dE/ds > 0) \), and that with an either constant or increasing rate \( (\epsilon'' \geq 0) \). Such an assumption can be justified by the district’s limited spatial capacity and the ensuing difficulties to find an adequate location the more stores enter.

Both residents and non-residents value product variety and each resident and each non-resident who visits the district buys one unit of each product offered. The number of visiting non-residents is denoted by \( v \) so that the profit function of
each store $i$ is

$$\Pi_i(v, E(s)) = p \cdot [1 + v] - E(s). \quad (1)$$

A store $i$ establishes in the district as long as $\Pi \geq 0$. For the marginal entrant, $\Pi = 0$ holds, thus there is no more entry as soon as $p \cdot [1 + v] = E(s) = \epsilon$ applies. By means of the inverse function of $\epsilon = E(s)$, $s = E^{-1}(\epsilon)$, we find the no-profit number of stores as a function of the number of visitors

$$s(v) = E^{-1}(\epsilon) = E^{-1}(p \cdot [1 + v]) \quad (2)$$

with $ds/dv = p \cdot [E^{-1}]' = p/E' > 0$ and $d^2 s/dv^2 = -p \cdot E''/[E']^3 \leq 0$.

The net utility that a resident or a visiting non-resident gains from shopping or, more in general, from the variety offered in the district is $\tilde{u}_j(s)$ with $j = r, nr$.\(^2\) We assume that it is $\tilde{u}_j(0) = 0$ and that the marginal utility from variety of an additional store is constant or decreasing positive, thus it is $d\tilde{u}_j/ds > 0$ and $d^2 \tilde{u}_j/ds^2 \leq 0$. And as the number of stores, and with that the variety, increases when an additional non-resident decides to visit the district, his or her decision involves a positive external effect. Thus, formally it is $\tilde{u}_j(s(v))$ with $d\tilde{u}_j/dv = d\tilde{u}_j/ds \cdot ds/dv > 0$.

**Travel Cost.** When traveling to the district, non-residents have to bear per-unit car travel cost of $t$. We assume that non-residents have no alternative to traveling by car and that they do not carpool but travel alone.

**Parking Capacity and Costs.** We assume that each resident owns a car but does not have a private parking space. Thus, both residents and visiting non-residents have two possibilities to park their cars in the district: either on-street in direct proximity to the residential houses and stores or in public parking lot, which is assumed to be slightly away from the residential houses and stores.\(^3\) Because of that, residents and visitors do not consider the parking possibilities as perfect

\(^2\) We neglect $p$ as a function argument since $p$ is exogenous.

\(^3\) Employees working at the districts’ different stores are assumed to demand neither on-street nor off-street parking.
substitutes. In fact, we assume that a driver $j = r, nr$ does not face costs when parking on-street but that a resident bears a cost of $c_r$ and a non-resident who visits the district of $c_{nr}$ when parking in the parking lot. These costs $c_r$ and $c_{nr}$ comprise the driving cost to the parking lot and the walking cost to the residential houses and stores and back again.

\[
\begin{array}{|c|}
\hline
\text{residents} \\
\text{stores} \\
\text{on-street parking} \\
\text{parking lot} \\
\hline
\end{array}
\]

Figure 2: Structure of city district located at 0

We assume that the provision of the parking capacities does not entail costs. The number of on-street parking spaces $x_s$ is fixed and normalized to 1. For the parking lot’s capacity $x_l$, we assume $x_l \geq 1$. Both capacities combined, there is sufficient parking space for all potential parkers. We abstract from temporal variation in demand but assume that residents and visitors arrive at the district at the same time. In absence of any parking regulations, a driver $j = r, nr$ gets an on-street space by a random-rationing rule, as in Calthrop and Proost (2006). The probability for that is

\[
\rho = \frac{x_s}{r + v} = \frac{1}{1 + v}. \tag{3}
\]

We assume that the municipality has implemented an efficient parking guidance system and that there is sufficient road capacity. Thus, cruising for parking (e.g. Glazer and Niskanen (1992); Arnott and Rowse (1999, 2009); Anderson and de Palma (2004); Shoup (2005); Arnott and Inci (2006)) or the accompanying social cost are not considered in our model.

From equation (3), it becomes obvious that there are precisely enough on-street spaces available for all residents’ cars in case of $v = 0$ so that a resident gets an on-street spot with $\rho = 1$. However, rivalry in consumption emerges if $v > 0$. In
this case, each driver \( j = r, nr \) in the district faces a probability of having to park in the lot of \( 1 - \rho > 0 \) and expects a parking cost of

\[
p_{cj}(v) = c_j \cdot [1 - \rho] = c_j \cdot \left[ \frac{v}{1 + v} \right] \quad \text{with} \quad \frac{dp_{cj}}{dv} = \frac{c_j}{(1 + v)^2} > 0.
\]  

(4)

Thus, the visit of an additional non-resident does not only involves a positive but also a negative external effect since it leads to an increase in both the parking cost that a resident and a visitor expects.

In what follows, we discuss two parking regimes, namely the regimes \( pf \) (for “parking fees”) and \( rp \) (for “residential parking”) with regard to their ability to ensure both an optimal allocation of on-street parking spaces and an optimal number of visitors \( v^* \). The regime \( pf \) constitutes the benchmark but our focus in this paper lies on regime \( rp \). For reasons of clarity and comprehensibility, the explicit design of each parking regime \( k = pf, rp \) is explained directly before its analysis. Note at this point only that dependent on the applied parking regime \( k \), each resident experiences a full parking cost of \( pc_{r,k} \) and each visitor of \( pc_{nr,k} \).

**Overall and Aggregate Utilities.** In summary, each resident gains an overall utility from living in the vibrant district of \( u_{r,k} = \tilde{u}_r(s(v)) - pc_{r,k} \) and a non-resident who lives at \( w \in [0, 1] \) gains an overall utility from visiting the vibrant city district of \( u_{w,k} = \tilde{u}_{nr}(s(v)) - t \cdot w - pc_{nr,k} \). We assume that non-residents gain an alternative utility of zero when they do not visit the district. Thus, non-resident \( w \) visits the district if \( u_{w,k} \geq 0 \).

For the analysis of the different parking regimes \( k \), we need the aggregate utilities. The residents gain an aggregate utility of

\[
U_{r,k} = r \cdot u_{r,k} = 1 \cdot [\tilde{u}_r(s(v)) - pc_{r,k}]
\]  

(5)

and the visitors, and simultaneously all non-residents, of

\[
U_{nr,k} = \int_0^v u_{w,k} \, dw + \int_v^{1-v} 0 = \tilde{u}_{nr}(s(v)) \cdot v - \frac{t}{2} \cdot v^2 - pc_{nr,k} \cdot v.
\]  

(6)

Adding up \( U_{r,k} \) and \( U_{nr,k} \) gives welfare

\[
W_k = \tilde{u}_r(s(v)) - pc_{r,k} + \tilde{u}_{nr}(s(v)) \cdot v - \frac{t}{2} \cdot v^2 - pc_{nr,k} \cdot v.
\]  

(7)
3 Parking Fees

3.1 Optimal Fees/Subsidies

Optimal parking fees should both allocate on-street parking spaces to the group with the higher cost of using the parking lot and subsidize or tax the visitors to remedy the possible market failure of excess or insufficient entry. Thus, in a first best world the parking fees ensure that the explained externalities of an additional visitor are taken into account. So suppose that under parking regime \( k = pf \) the municipality charges the lump sum fees \( f_s \) for on-street parking and \( f_l \) for parking in the lot, which can be interpreted as subsides in case they are negative. With regard to the cost of using the parking lot, we first consider case (a) that a resident bears a higher cost than a non-resident and proceed with the reverse case (b).

In case (a), it is \( c_r > c_{nr} \) and optimal when residents park on-street and visitors use the parking lot. Thus, the fees \( f_s^a \) and \( f_l^a \) have to induce that (i) visitors prefer to park in the lot, (ii) residents prefer to park on-street and (iii) the pricing system leads to a number of visitors \( v^* \) that maximizes welfare. Self-selection (i) is ensured if \( pc_{nr,pf}^a = c_{nr} + f_l^a < f_s^a \) and self-selection (ii) if \( pc_{r,pf} = f_s^a \leq c_r + f_l^a \), which implies that both (i) and (ii) are ensured if \( f_s^a = c_r + f_l^a \).

Hence, the fee \( f_s^a \) prevents that the visit of an additional non-resident involves a negative externality in terms of an increase in expected parking cost as it is clear who parks where from the beginning. The fee (subsidy) \( f_l^a \) that the visitors pay (receive) for parking in the lot should be used to ensure the optimal number of visitors \( v^* \). The parking fees are expenditures for those who park in the district but revenues for the municipality (or it is the other way round) and therefore do not change welfare. In this case, welfare is

\[
W^a = \tilde{u}_r(s(v)) + \tilde{u}_{nr}(s(v)) \cdot v - \frac{t}{2} \cdot v^2 - c_{nr} \cdot v.
\]  

(8)

The optimal number of visitors \( v^* \) is found when \( dW^a/dv = 0 \), or when

\[
\left[ \frac{d\tilde{u}_r}{ds} + v \cdot \frac{d\tilde{u}_{nr}}{ds} \right] \frac{ds}{dv} + \tilde{u}_{nr}(s(v)) = t \cdot v + c_{nr}.
\]  

(9)

\[\text{To keep the notation as simple as possible, we drop the index } pf \text{ in the following analysis of optimal parking fees at all relevant welfare measures.}\]
However, the number of visitors can not be enforced directly by a social planner but results from individual decisions of the non-residents. Recall that non-residents gain an alternative utility of zero when they do not visit the district, thus non-resident \( w \in [0, 1] \) visits the district in case (a) if

\[
u_w^a = \tilde{u}_{nr}(s(v)) - t \cdot w - pc_{nr}^a = \tilde{u}_{nr}(s(v)) - t \cdot w - c_{nr} - f_l^a \geq 0. \tag{10}
\]

For the indifferent non-resident \( w = v \), who at the same time defines the total number of visitors, condition (10) is binding, so that

\[
f_l^a = \tilde{u}_{nr}(s(v)) - t \cdot v - c_{nr} \tag{11}
\]

has to hold. Inserting (9) in (11) gives

\[
f_l^a = \tilde{u}_{nr}(s(v^*)) - t \cdot v^* - c_{nr} \tag{12}
\]

which describes the negative parking fee or the parking subsidy that each non-resident receives when visiting the district and which is equivalent to the positive external effect that the last and optimal visitor causes. Thus, by subsidizing each visitor with \( f_l^a \) and by charging each resident the fee \( f_s^b = c_r + f_l^a \), the first best solution can be achieved.

If case (b) applies, thus \( c_r < c_{nr} \) is true as suggested by van Ommeren et al. (2011), parking costs are minimized if visitors park on-street and all on-street parking spaces are utilized. Therefore, the fees (or subsidies) \( f_s^b \) and \( f_l^b \) have to induce that (i) visitors prefer to park on-street and (ii) residents are indifferent between the two options. Self-selection is ensured if \( pc_{nr}^b = f_s^b = c_r + f_l^b = pc_{nr}^b \), which looks similar to the self-selection condition in case (a) but here it implicates that visitors park on-street instead of in the lot. The fee \( f_s^b \) should be used to ensure the optimal number of visitors \( v^* \). Welfare in this case is

\[
W^b = \tilde{u}_r(s(v)) + \tilde{u}_{nr}(s(v)) \cdot v - \frac{t}{2} \cdot v^2 - c_r \cdot v. \tag{13}
\]

The welfare functions \( W^a \) and \( W^b \), given by equations (8) and (13), distinguish by the last term: in case of \( c_r > c_{nr} \), an additional visitor parks in the lot and
therefore reduces welfare by $c_{nr}$ while he or she parks on-street in case of $c_r < c_{nr}$ and thereby crowds out a resident to the parking lot at the cost of $c_r$.

The optimal number of visitors $v^*$ is found when $dW^b/dv = 0$, or when

$$\frac{dW^b}{dv} = \left[ \frac{d\tilde{u}_r}{ds} + v \cdot \frac{d\tilde{u}_{nr}}{ds} \right] \frac{ds}{dv} + \tilde{u}_{nr}(s(v)) - c_r = t \cdot v. \quad (14)$$

Again, whether or not a non-resident visits the district is his or her individual decision. In case (b), non-resident $w \in [0,1]$ gains an overall utility of

$$u^b_w = \tilde{u}_{nr}(s(v)) - t \cdot w - p_c^b = \tilde{u}_{nr}(s(v)) - t \cdot w - f^b_s \quad (15)$$

that is equal to zero for the indifferent visitor $w = v$. Thus,

$$f^b_s = \tilde{u}_{nr}(s(v)) - t \cdot v \quad (16)$$

has to hold and inserting (14) in (16) gives

$$f^b_s = - \left[ \frac{d\tilde{u}_r(s(v^*))}{ds} + v^* \cdot \frac{d\tilde{u}_{nr}(s(v^*))}{ds} \right] \frac{ds(v^*)}{dv} + c_r, \quad (17)$$

which is the optimal on-street parking fee (or subsidy). In this case, the fee that ensures $v^*$ not only comprises the positive but also the negative externality of last and optimal visitor. Those of the residents who park in the lot receive $f^b_l = f^b_s - c_r$, thus an amount of the positive externality that the last and optimal visitor causes by which their cost of parking in the lot is reduced.

### 3.2 Discussion

In our model, we assume that there are two homogeneous types of car drivers (residents and visitors) and two parking possibilities (on-street and a public lot) in the district. Therefore, second degree price discrimination is possible. In reality, however, the implementation of such a parking regime may be difficult. First of all, it is more likely that the visitors are heterogeneous with regard to both their preference for product variety and their cost of using the parking lot. Hence, it is almost impossible to efficiently tax/subsidize their visits by on- or off-street parking fees, especially when these are also used as an instrument to allocate
the on-street parking spaces to drivers with the highest cost of using the parking lot. This purpose becomes even more difficult to fulfill if the residents are also heterogeneous regarding their cost of using the lot. Secondly, there could be more than only two parking possibilities in the district and its surroundings, all of which with different usage costs so that minimizing these costs and ensuring an efficient number of customers from the outside is difficult or impossible.

Moreover, as some of the parking fees are or might be parking subsidies, parking costs can become parking revenues and this might have undesired effects which require further regulations. For example, people from outside might visit the district without any intention to shop but to earn the subsidy. Furthermore, if there were residents who heretofore used their private parking capacity could have an incentive to use public parking capacities instead. More in general, car ownership could become more attractive for residents and therefore increase.

If the optimal on-street parking fees are positive, it is very likely that residents of downtown areas or other vibrant city districts oppose such a policy in “their” neighborhood. Basically, of course, residents do not automatically hold parking privileges in case the on-street parking capacity is public. Normally, however, the municipalities pursue parking polices whereby residents are exempted from paying for parking, whether due to the residents’ influence or due to urban development measures.

As soon as municipalities confer special parking rights to residents, they pursue a residential parking policy. A policy that exempts residents from paying the parking fees but that meets the two objectives - minimization of parking costs and ensuring an efficient number of visitors - is difficult. In case (a), self-selection (ii) might not work. Dependent on the magnitude of the parking subsidy that visitors receive for parking in the lot, it could also pay off for residents to park there. In case (b), self-selection (ii) only works if residents receive a subsidy in the amount of their cost of using the parking lot. Unless visitors can be distinguished from residents, this subsidy has to be considered when determining the on-street parking fee that visitors face and that ensures self-selection (i). But then, this on-street fee is contradictory to one that ensures the optimal number of residents.
4 Residential Parking

4.1 Parking Regime

Because of the difficulties of a residential parking policy according to which the municipality exempts residents from paying the parking fees but tries to meet the two objectives, we discuss a policy according to which the municipality reserves a certain share of on-street parking spaces exclusively for residents but does not charge parking fees at all. In principle, the reservation of on-street parking spaces for residents minimizes the aggregate parking costs only if residents bear a higher cost of using the parking lot than non-residents, thus if \( c_r > c_{nr} \) holds, and if all on-street parking spaces are reserved for residents. This, however, implicates the subsidization of non-resident visits and subsidies can provoke undesired behavior. To encourage non-residents to visit the district without paying a parking subsidy, it might be optimal to reserve a share smaller than one for residents. However, in this case neither the aggregate parking cost are minimal nor the welfare-maximizing number of visitors is ensured, so that such a residential parking policy can only be inferior to a policy that includes parking fees/subsidies for all parkers.

Recall that in case of unregulated parking, the probability to find a vacant on-street spot was \( \rho = 1 / (1 + v) \). Rivalry in consumption emerged if \( v > 0 \) and each member of both groups faced a probability of having to park in the parking lot of \( 1 - \rho > 0 \). Under parking regime \( k = rp \), the municipality assigns the share \( \alpha \in [0, 1] \) of on-street parking spaces to residential parking only. If \( \alpha = 0 \), none of the on-street spaces are reserved for residents and if \( \alpha = 1 \), all of the on-street spaces are reserved for residents. Thus, each resident gets an on-street parking spot with a probability of

\[
\rho_r = \frac{1 - \alpha}{1 + v} + \alpha = \frac{1 + \alpha \cdot v}{1 + v} \tag{18}
\]

whereas a non-resident who visits the district is able to park on-street with a probability of

\[
\rho_{nr} = \frac{1 - \alpha}{1 + v} \tag{19}
\]

The probabilities of having to park in the parking lot are the converse probabilities.
$1 - \rho_r$ and $1 - \rho_{nr}$. Hence, each resident expects a parking cost of

$$pc_r(v, \alpha) = c_r \cdot [1 - \rho_r] = c_r \cdot \frac{v \cdot [1 - \alpha]}{1 + v}$$

(20)

with

$$\frac{\partial pc_r}{\partial v} = \frac{c_r \cdot [1 - \alpha]}{[1 + v]^2} \geq 0 \quad \text{and} \quad \frac{\partial pc_r}{\partial \alpha} = -\frac{c_r \cdot v}{1 + v} < 0,$$

whereas each visitor of the district expects a parking cost of

$$pc_{nr}(v, \alpha) = c_{nr} \cdot [1 - \rho_{nr}] = c_{nr} \cdot \frac{v + \alpha}{1 + v}$$

(21)

with

$$\frac{\partial pc_{nr}}{\partial v} = \frac{c_{nr} \cdot [1 - \alpha]}{[1 + v]^2} \geq 0 \quad \text{and} \quad \frac{\partial pc_{nr}}{\partial \alpha} = \frac{c_{nr}}{1 + v} > 0.5$$

4.2 Equilibrium with Residential Parking

We assume that each resident has a parking permit which indicates that he or she is entitled to park in the declared residential parking spaces. The permit is issued as soon as someone settles in the district and is free of charge. From an empirical point of view, the fees for residential parking permits are very low. In Germany, for example, the fee for an annual parking permit is about €30, implying a daily cost of about €0.08, which in our opinion is negligible. For the residents of Amsterdam, van Ommeren et al. (2011) examine a marginal willingness to pay of €10 per day for an on-street parking permit. Thus, from a theoretical point of view, each of the homogeneous residents buys a parking permit if its fee does not exceed his or her willingness to pay. Possible expenditures for parking permits do not affect the welfare as they correspond to the revenues of the municipality and are returned in some way. If, however, the price for a parking permit exceeds each resident’s willingness to pay, there is no demand for such permits and an allocation of parking spaces to residential parking does not happen as it constitutes a waste of space.

To calculate the optimal share of residential parking spaces $\alpha$, we analyze the free entry equilibrium at first and determine the equilibrium number of shops $s^e$ and visitors $v^e$ as a function of $\alpha$.

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*Again, we drop the index $rp$ at all measure to keep the notation as simple as possible.*
Under regime $rp$, a non-resident living at $w \in [0, 1]$ gains an overall utility of $u_w = \tilde{u}_{nr}(s) - t \cdot w - p c_{nr}(v, \alpha)$. For the indifferent non-resident $w = v$, $u_w = 0$ holds and defines a function $v(s)$ with
\[
\frac{dv}{ds} = -\frac{\partial u_w / \partial s|_{w=v}}{\partial u_w / \partial v|_{w=v}} = -\frac{d\tilde{u}_{nr}/ds}{-t - c_{nr}/[1 + v]^2} > 0.
\] (22)
Since it is $\tilde{u}_{nr}(0) = 0$, a non-resident visits the district only if there is a positive number of shops. Furthermore, it is
\[
\frac{d^2v}{ds^2} = -\frac{\partial^2 u_{nr}/\partial s^2|_{w=v}}{[1 + v]^2} \leq 0.
\] (23)
Recall from section 2 that the number of stores is given by
\[
s(v) = E^{-1}(e) = E^{-1}(p \cdot [1 + v])
\]
with $ds/dv = p \cdot [E^{-1}]' = p/E' > 0$ and $d^2s/dv^2 = -p \cdot E''/[E']^3 \leq 0$.

Let $\tilde{u}_{nr}(E^{-1}(p)) > \alpha \cdot c_{nr}$ and $\tilde{u}_{nr}(E^{-1}(2p)) < t + c_{nr} \cdot [1 + \alpha]/2$. Then there exists an interior equilibrium $0 < s^e$ and $0 < v^e < 1$. The first condition ensures that the buying power of the residents leads to such a variety that a visiting the district pays off for at least some non-residents. The second condition makes sure that the combined buying power of the residents and all non-residents does not, due to the stores’ cost of entry, result in a variety so that it is worth visiting the district for all non-residents.

Figure 3\(^6\) shows the visitors’ free entry condition $u_w = 0$, which is fulfilled if $v = v(s)$ and the shops’ free entry condition $\Pi = 0$, which is fulfilled if $s = s(v)$. The intersection of $v(s)$ and $s(v)$ determines the equilibrium number of visitors $v^e$ and of stores $s^e$.

In equilibrium, $v^e$ and $s^e$ depend on different parameters like the parameter that indicates the non-residents’ valuation of product variety, the per-unit travel cost, the cost that a non-resident has to bear when he or she has to park in the parking

\(^6\)The figure with its three graphs is based on the results of our numerical example in section 6. To be more specific, we set $c_{nr} = 1/5$. Furthermore, we set $\alpha = 1/10$ to get $v_1(s)$ and $\alpha = 9/10$ to get $v_2(s)$. The behavior of $s(v)$ directly results from our assumptions in the example. However, the behavior of $v(s)$ and $s(v)$ also holds in general.

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Figure 3: Free entry equilibrium

lot and the specific entry cost of each store. Since our focus lies on a residential parking policy that assigns on-street parking spaces to residential parking only, we treat all those exogenous parameters as constant, except \( \alpha \in [0, 1] \). Thus, we define the equilibrium number of visitors as a function \( v^e(\alpha) \) and the equilibrium number of stores as a function \( s^e(\alpha) \).

We use the condition \( u_w = \tilde{u}_{nr}(s) - t \cdot w - p c_{nr}(v, \alpha) = 0 \) for the indifferent non-resident \( w = v \) to calculate

\[
\frac{dv}{d\alpha} = -\frac{\partial u_w/\partial \alpha}{\partial u_w/\partial v|_{w=v}} = -\frac{-c_{nr}/[1+v]}{-t - c_{nr}/[1+v]^2} = -\frac{c_{nr} \cdot [v + 1]}{c_{nr} + t \cdot [v + 1]^2} < 0. \tag{24}
\]

The sign of equation (24) entails a downward shift of \( v(s) \), as shown in figure 3. Because it is \( \partial \Pi/\partial \alpha = 0 \), \( s(v) \) does not shift, thus

\[
\frac{dv^e}{d\alpha} < 0 \tag{25}
\]

holds. In the following, we assume \( -1 < dv^e/d\alpha < 0 \). Since it is \( ds/dv > 0 \), it follows that

\[
\frac{ds^e}{d\alpha} < 0. \tag{26}
\]
In the free entry equilibrium, non-resident $w \in [0, 1]$ who visits the district gains an overall utility of
\[
u^e_w = \tilde{u}^e_{nr}(s^e(\alpha)) - t \cdot w - pc^e_{nr}(v^e(\alpha), \alpha) = \tilde{u}^e_{nr}(s^e(\alpha)) - t \cdot w - \frac{c_{nr} \cdot [\alpha + v^e(\alpha)]}{1 + v^e(\alpha)} \quad (27)
\]
and each resident gains a utility of
\[
u^e_r = \tilde{u}^e_{r}(s^e(\alpha)) - pc^e_{r}(v^e(\alpha), \alpha) = \tilde{u}^e_{r}(s^e(\alpha)) - \frac{c_{r} \cdot [1 - \alpha] \cdot v^e(\alpha)}{1 + v^e(\alpha)}. \quad (28)
\]

Lemma 1 If the share of residential parking spaces increases, both the residents’ and non-residents’ utility from variety decreases, thus it is
\[
d\tilde{u}^e_{j} = ds^e \cdot \frac{d\tilde{u}^e_{j}}{d\alpha} < 0 \text{ with } j = r, nr. \quad (29)
\]
Furthermore, an increase in the share of residential parking spaces results in an increase in the parking cost a non-resident expects,
\[
dpc^e_{nr} \cdot ds^e = \frac{\partial pc^e_{nr}}{\partial v^e} dv^e + \frac{\partial pc^e_{nr}}{\partial \alpha} > 0 \quad (30)
\]
but to a decrease in the expected parking cost of a resident,
\[
dpc^e_{r} \cdot ds^e = \frac{\partial pc^e_{r}}{\partial v^e} dv^e + \frac{\partial pc^e_{r}}{\partial \alpha} < 0. \quad (31)
\]

4.3 Welfare Analysis

In case of parking regime $rp$, welfare is
\[
W^e = \tilde{u}^e_{r}(s^e(\alpha)) - pc^e_{r}(v^e(\alpha), \alpha)
+ \tilde{u}^e_{nr}(s^e(\alpha)) \cdot v^e(\alpha) - \frac{t}{2} \cdot [v^e(\alpha)]^2 - pc^e_{nr}(v^e(\alpha), \alpha) \cdot v^e(\alpha). \quad (32)
\]
To find the optimal share of residential parking spaces $\alpha^*$, we derive $W^e$ with respect to $\alpha$ and obtain after some rearrangements
\[
dW^e = d\tilde{u}^e_{e} \cdot ds^e - \frac{\partial pc^e_{r}}{\partial v^e} dv^e - \frac{\partial pc^e_{r}}{\partial \alpha} + v^e \cdot \left[\frac{d\tilde{u}^e_{nr} \cdot ds^e - \partial pc^e_{nr} dv^e - \partial pc^e_{nr, rp}}{ds^e \cdot d\alpha - \partial v^e \cdot d\alpha - \partial \alpha} \right] + dv^e \cdot \left[\tilde{u}^e_{nr} - t \cdot v^e - pc^e_{nr}\right].
\]

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The term \[\tilde{u}_{er} - t \cdot v^e - pc_{nr}\] describes the marginal visitor’s utility which equals zero in the visitor’s optimum. Thus, we find

\[
\frac{dW^e}{d\alpha} = \frac{d\tilde{u}_r}{d\alpha} + v^e \cdot \frac{d\tilde{u}_{nr}}{d\alpha} - \frac{dpc_r}{d\alpha} + v^e \cdot \left( \frac{d\tilde{u}_r}{d\alpha} - \frac{dpc_r}{d\alpha} \right) - \partial pc_{nr} \frac{dv^e}{d\alpha} - \partial pc_{nr} \frac{d\alpha}{d\alpha} - \partial pc_{nr} \frac{d\alpha}{d\alpha} - \partial pc_{nr} \frac{d\alpha}{d\alpha} - \partial pc_{nr} \frac{d\alpha}{d\alpha} .
\]

which we set equal to zero.

**Proposition 1** The optimal share of residential parking spaces \(\alpha^*\) is found when the decrease in both the residents’ and visitors’ utility from variety due to a decrease in the number of stores equals the savings in overall parking cost, thus when

\[
\frac{d\tilde{u}_r}{d\alpha} + v^e \cdot \frac{d\tilde{u}_{nr}}{d\alpha} = \frac{dpc_r}{d\alpha} + v^e \cdot \frac{dpc_{nr}}{d\alpha} .
\]

To actually realize savings in overall expected parking cost,

\[
\frac{dpc_r}{d\alpha} + v^e \cdot \frac{dpc_{nr}}{d\alpha} = \frac{v^e(\alpha)[c_r + c_{nr} \cdot v^e(\alpha)][1 - \alpha]}{[1 + v^e(\alpha)]^2} + \frac{c_{nr} - c_r \cdot v^e(\alpha)}{1 + v^e(\alpha)} .
\]

has to be negative, which is only possible if

\[
\frac{c_r}{c_{nr}} > \left[ 1 + \frac{[1 + v^e(\alpha)] \cdot v^e(\alpha) \cdot [1 - \alpha]}{v^e(\alpha)[1 + v^e(\alpha)] - v^e(\alpha)[1 - \alpha]} \right] .
\]

Thus, inequality (36) defines a necessary condition for a positive share of residential parking spaces to be optimal. Because the right hand side of (36) is less than one visitors may bear as well a somewhat lower as a higher parking cost in the outside lot than residents. Note, however, that (36) is not sufficient for residential parking to be welfare-enhancing since it yet does not ensure \(dW^e/d\alpha\) to be positive.

### 4.4 Local Decision-Making

In reality, decisions on parking regulations are normally made on a local level (Arnott, 2011). Business owners may oppose regulations that prevent potential customers from visiting the district. In our model, however, we assume that business owners are non-residents and that businesses are perfectly mobile;
thus we omit business lobbying efforts. As residents are assumed to be homogena-
neous, individual preference equals collective preference. Thus, we simply derive
\( U^r_e = 1 \cdot [\tilde{u}^r_e(s^e(\alpha)) - pc^e_r(v^e(\alpha), \alpha)] \) with respect to \( \alpha \) and set it equal to zero to
determine the preferred share of residential parking spaces. It is
\[
\frac{dU^r_e}{d\alpha} = \frac{\partial \tilde{u}^r_e}{\partial s^e} \frac{ds^e}{d\alpha} - \frac{\partial pc^e_r}{\partial v^e} \frac{dv^e}{d\alpha} - \frac{\partial pc^e_r}{\partial \alpha} = \frac{d\tilde{u}^r_e}{d\alpha} - \frac{dpc^e_r}{d\alpha} = 0
\]
and thus, the optimal share of residential parking spaces from the residents’ persp-
tive \( \alpha^*_r \) is found when the decrease in the residents’ utility from variety due
to a decrease in the number of stores equals the residents’ savings in expected
parking cost, thus when
\[
\frac{d\tilde{u}^r_e}{d\alpha} = \frac{dpc^e_r}{d\alpha}.
\]
Even if there is a ballot and both residents and visitors are entitled to vote, one
of the residents is the median voter and, according to the median voter theorem,
\( \alpha^*_r \) is the adopted policy.

**Proposition 2** If local authorities only consider the preferences of the district’s
residents but leave out the preferences of non-residents as potential visitors in the
decision on residential parking, or if there is a ballot on it, the resulting share of
residential parking spaces exceeds the optimal share from the welfare perspective,
i.e. \( \alpha^*_r > \alpha^* \), whenever the welfare optimal share of residential parking spaces is
positive but less than one.

Proof: See appendix.

Residents prefer a higher share of on-street parking spaces to be reserved for
them than it is optimal from the welfare perspective because they ignore the
negative effects their decision has on the visitors’ overall utility in terms of a lower
utility from variety and higher expected parking cost.

### 4.5 A Numerical Example

The following example shall illustrate our general results. We assume that resi-
dents and non-residents share the same preference for variety. More precisely, it is
\(\hat{u}_r(s) = \hat{u}_{nr}(s) = [\mu - p] \cdot s\). Recall that the price each store charges its customers is exogenously given. We assume that it is \(p = 1/2\). Furthermore, we set \(\mu = 1\).

With that, it is \(\hat{u}_r(s) = \hat{u}_{nr}(s) = s/2\). The per-unit travel cost a non-resident faces when visiting the district is \(t = 1\). Altogether it follows that

\[
u_r = \hat{u}_r(s) - p c_r(v, \alpha) = \frac{1}{2} \cdot s - \frac{c_r \cdot [1 - \alpha] \cdot v}{1 + v} \quad (39)
\]

and

\[
u_w = \hat{u}_{nr}(s) - t \cdot w - p c_{nr}(v, \alpha) = \frac{1}{2} \cdot s - w - \frac{c_{nr} \cdot [v + \alpha]}{1 + v}. \quad (40)
\]

Non-resident \(w \in [0,1]\) visits the district if \(u_w \geq 0\). We assume that \(0 < c_{nr} < 1/4\), which implies that even some non-residents visit the district if all on-street spaces are assigned to residential parking. For the indifferent non-resident \(w = v\), it is \(u_w = 0\), so that the number of visitors as a function of the number of stores is

\[
v(s) = \frac{s + \sqrt{s - 2 - 2c_{nr}}^2 + 8 \cdot [s - 2c_{nr} \alpha] - 2 - 2c_{nr}}{4}. \quad (41)
\]

With regard to the entry cost that each store faces, we assume \(E(s) = e \cdot s\) with \(e = 1\), which defines

\[
s(v) = \frac{1}{2} \cdot [1 + v]. \quad (42)
\]

Both \(v(s)\) and \(s(v)\) are shown in Figure 3 for \(c_{nr} = 1/5\) and \(\alpha = 1/10\) as well as \(c_{nr} = 1/5\) and \(\alpha = 9/10\). Inserting (41) in (42) gives the equilibrium number of stores

\[
s^e = s^e(\alpha) = \frac{\sqrt{1 + c_{nr} + c_{nr}^2 - 3c_{nr} \alpha} + 1 - c_{nr}}{3} \quad (43)
\]

and inserting (43) then in (41) gives the equilibrium number of visitors

\[
v^e = v^e(\alpha) = \frac{2\sqrt{1 + c_{nr} + c_{nr}^2 - 3c_{nr} \alpha} + 1 - 2c_{nr}}{3}, \quad (44)
\]

also shown in Figure 3.

With the explicit results for \(s^e\) and \(v^e\), we can determine the explicit expressions for \(\hat{u}_j^e(s^e)\) and \(p c_j^e(v^e, \alpha)\) with \(j = r, nr\), and thus for \(dW^e/d\alpha\) and \(dU^e_r/d\alpha\) (see equations (33) and (37)). Solving \(dW^e/d\alpha = 0\) for \(\alpha\) gives

\[
\alpha = \frac{c_{nr}^2 [21 + 16c_{nr}] + 3c_r^2 [7 + 16c_{nr} \cdot [1 + c_{nr}]] - 2c_r c_{nr} [29 + 8c_{nr} \cdot [1 + 4c_{nr}]]}{16(3c_r - 2c_{nr})^2 c_{nr}}. \quad (45)
\]
Thus, the optimal share of residential parking spaces from the welfare perspective is

$$\alpha^*(c_r, c_{nr}) = \begin{cases} 
0 & \text{if } c_r \leq \xi \\
\alpha & \text{if } \xi < c_r < \tilde{c} \\
1 & \text{if } c_r \geq \tilde{c}
\end{cases} \quad (46)$$

with

$$\xi = \frac{c_{nr} \left[ 29 + [20 - 32 c_{nr}] \sqrt{1 + c_{nr} + c_{nr}^2 + 8 c_{nr} [1 + 4 c_{nr}]} \right]}{21 + 48 c_{nr} [1 + c_{nr}]}$$

and

$$\tilde{c} = \frac{16 c_{nr}^2 - 7 c_{nr}}{12 c_{nr} - 3}.$$ 

If the decision on the share of residential parking spaces is made on a local level, we simply have to solve \(dU^e_r/d\alpha = 0\) for \(\alpha\). We get an optimal share of residential parking spaces from the residents’ perspective of

$$\alpha^*_r(c_r, c_w) = \begin{cases} 
(16 + 7/c_{nr} + 16 c_{nr} - (6 c_r + c_{nr})/c_r^2) / 48 & \text{if } c_r \leq \tilde{c} \\
1 & \text{if } c_r > \tilde{c}
\end{cases} \quad (47)$$

with \(\tilde{c} = c_{nr}/[1 - 4 c_{nr}].\)

Figure 4: Optimal shares of residential parking spaces from both perspectives

Figure 4 illustrates our results for \(\alpha^*\) and \(\alpha^*_r\) when \(c_{nr} = 1/8\). If \(c_r < \xi\), assigning some but not all on-street parking spaces to residential parking is only optimal from
the residents' perspective. If $\zeta < c_r < \hat{c}$, however, also a social planner assigns
some but not all of the on-street spaces to residential parking, albeit fewer than the
residents would do. From the residents’ perspective, exclusive residential parking
in on-street spaces is optimal if $\hat{c} < c_r$, whereas a share of residential parking
spaces smaller than one is welfare optimal as long as $c_r < \hat{c}$. Only if $\bar{c} \leq c_r$, also a
social planner assigns all on-street parking spaces to residential parking.

In our example, we assumed that residents and non-residents are homogeneous
in their preference for variety, for which we used a numerical description. As a
result, the optimal shares of residential parking spaces seem to be independent of
this preference. To show that this is of course not the case, allow for the case that
residents and non-residents differ in their preference for variety. We still assume
that $\tilde{u}_{nr} = [\mu - p] \cdot s = [1 - 1/2] \cdot s$ but for a resident’s utility from variety, we
assume $\tilde{u}_r = [\lambda - p] \cdot s = [\lambda - 1/2] \cdot s$ with $\lambda \geq \mu = 1$. Thus, the utility a resident
gains from product variety is at least as high as the one a non-resident who visits
the district gains. The calculations of the equilibrium values are as usual, the only
difference is that some of them, as the residents’ aggregate overall utility and the
welfare, contain the variable $\lambda$. Solving $dW^e/d\alpha = 0$ for $\alpha$ in this case gives

$$\alpha = \frac{[5 - 16 c_{nr}] c_{nr}^2 - 2 c_{nr} [11 + 8 c_{nr} [1 + 4 c_{nr}]] c_r + 3 [7 + 16 c_{nr} [1 + c_{nr}]] c_r^2}{16 c_{nr} [2 c_{nr} - 3 c_r]^2}$$

$$+ \frac{c_{nr} [7 + 8 c_{nr} - 3 \lambda] - 9 c_r \lambda}{4 [2 c_{nr} - 3 c_r]^2}$$

(48)

with

$$\frac{d\alpha}{d\lambda} = \frac{c_{nr} [7 + 8 c_{nr} - 6 \lambda] - 9 c_r}{4 [2 c_{nr} - 3 c_r]^2} < 0,$$

(49)
at least for $c_r \geq c_{nr}$. From the sign of equation (49), we can deduce that the opti-
mal share of residential parking spaces from the welfare perspective is the lower the
more the residents value the variety offered in their neighborhood. The same rela-
tionship applies of course for the optimal share of residential parking spaces from
the residents’ perspective as the variable $\lambda$ is part of a resident’s utility. Further-
more, we can proceed on the assumption that there also is a negative relationship
between the non-residents’ (gross) utility from variety and the optimal share of
residential parking spaces from the welfare (but not the residents’) perspective.
5 Conclusion

Many cities provide residential parking permits for residents who live in downtown areas and other city districts where on-street parking capacity is scarce. These permits allow residents to park for free in their neighborhood while non-residents pay for parking or allow residents to park in on-street spaces that are reserved for their exclusive use. Within the context of a formal model of a vibrant city district whose residents and visitors appreciate the offered product variety but might experience inefficient high parking costs, we focused on the latter alternative and analyzed the trade-off between the more-convenient-parking-effect on the residents’ side due to residential parking and the loss-of-variety-effect due to fewer shoppers coming from outside the district. We determined both the share of residential parking spaces that is optimal from a welfare and from the residents’ perspective, and found that in case the decision is made locally more than welfare-optimal parking spaces are assigned to residential parking. This possible latter result can be assessed critically insofar as the discussed residential parking policy is already not the first best solution with regard to parking cost minimization and ensuring an efficient number of non-resident shoppers. A first best solution includes price discriminated parking fees (or subsidies) but might be difficult to implement.

In this study, we assumed that the provision of the on- and off-street parking capacities does not entail costs and that drivers do not cruise for a parking spot. An important extension of our residential parking analysis would relax these assumptions. With regard to negative cruising externalizes, the first best solution can provide a remedy whereas a policy according to which a share of on-street parking spaces is reserved for residents is likely to exacerbate the problem.

A further and ambitious approach for future research is a public choice analysis, including political lobbying. We assumed that retail stores, restaurants and other businesses are perfectly mobile and therefore indifferent to parking regulations that may have a negative effect on visits of non-resident customers. This assumption rarely holds in reality and therefore, businesses oppose residential parking. Thus, the resulting parking regime depends on the institutional design that the city’s or city district’s municipality uses to determine its parking policy.
Appendix

Proof of Proposition 2

To prove $\alpha^* \leq \alpha_r^*$, where $\alpha^* = \alpha_r^*$ is only possible if $\alpha^* = 0$ or $\alpha^* = 1$, it is sufficient to show $dW^e/d\alpha < dU^e_r/d\alpha$ for $\alpha \in (0,1)$. Using solutions (33) for $dW^e/d\alpha$ and (37) for $dU^e_r/d\alpha$, respectively, it holds that $dW^e/d\alpha < dU^e_r/d\alpha$ if and only if

$$v^e \left[ \frac{d\tilde{u}^e_{nr}}{d\alpha} - \frac{dpc^e_{nr}}{d\alpha} \right] = v^e \cdot \left[ \frac{d\tilde{u}^e_{nr}}{d\alpha} - c_{nr} \cdot \frac{[1 - \alpha]v^e'(\alpha) + v^e(\alpha) + 1}{1 + v^e(\alpha)^2} \right] < 0. \quad (50)$$

Recall from Lemma 1 that $d\tilde{u}^e_{nr}/d\alpha < 0$ and $dpc^e_{nr}/d\alpha > 0$ holds in our analysis. With that, the sign of the term in brackets of (50) is negative. It is $v^e > 0$ and thus $dW^e/d\alpha < dU^e_r/d\alpha$ holds for all $\alpha \in (0,1)$. □
References


