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The Effects of Remedies on Merger Activity in Oligopoly*

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Abstract

We analyze the effects of structural remedies on merger activity in a Cournot oligopoly when the antitrust agency applies a consumer surplus standard. Remedies increase the scope for profitable and acceptable mergers, while divestitures to an entrant firm are most effective in this regard. Remedial divestitures are most attractive from a social welfare point of view, when the merging parties can extract the entire gains associated with the asset sale. We also show that the merging parties have strong incentives to search for the most efficient buyer and we identify instances so that a remedy rule induces strictly price-decreasing mergers. Finally, under incomplete information an efficient merger type is to be doomed to “overshoot” with its divestiture proposal in a pooling equilibrium which is also possible under separation.

JEL-Classification: L13, L41, K21

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1 Introduction

Remedies are increasingly used by antitrust agencies (in short: AA) in the US and EU to clear merger proposals which are otherwise subject to serious anticompetitive concerns (see FTC, 1999, EU, 2006, and OECD, 2011, for recent remedy reviews).¹

The US Horizontal Merger Guidelines and the EU Merger Regulation allow for remedial offers to address competitive concerns (see DOJ, 2010, and EU, 2004, respectively). The EU Remedy Notice states that “the most effective way to restore effective competition, apart from prohibition, is to create the conditions for the emergence of a new competitive entity or for the strengthening of existing competitors via divestiture” (EU, 2008, Article 22). Accordingly, remedies are offered by the merging parties to effectively protect competition and to remove any competition concern the AA may have.

The following principles in association with remedies are stated both in EU and US regulations concerning remedies (see, EU, 2008, and DOJ, 2011, respectively): *First*, the remedy is designed and proposed to the AA by the merging firms, while the AA can either reject or accept the offer.² *Second*, a remedial divestiture may go to an already existing competitor or to a new entrant firm. *Third*, the remedy must be proportional to the competitive concern (see EU, 2004, Article 30).³

Taking care of those features, we analyze the impact of remedies on (horizontal) merger activity in oligopoly. We consider remedies in the form of physical asset sales (“divestitures”).⁴

¹Early accounts of remedies are Parker and Balto (2000) and the volume edited by Leveque and Shelanski (2003).

²This is particularly true for fix-it-first remedies in the US and phase 1 merger proposals in the EU. The rules are somewhat different in the next stage of the merger processes in the US and the EU (see, for instance, Wood, 2003, for a comparison of the US and EU merger control systems and the role of remedies therein, and Farrell, 2003, who describes the remedy settlement as a bargaining process between the merging parties and the AA).

³We assume that the objective of the AA is to protect consumer interests. Recent Industrial Organization literature (e.g., Nocke and Whinston, 2010, 2012) takes the consumer surplus standard for granted. For instance, Whinston (2007) states that the AA’s “enforcement practice in most countries (including the US and the EU) is closest to a consumer surplus standard.”

⁴Remedies are distinguished into structural and behavioral remedies (see EU, 2008, and DOJ, 2011). Structural

We assume that a merger produces synergies which makes it desirable from a consumer perspective in the first place.⁵ We show that the possibility to clear a merger conditional on remedies enlarges the set of profitable and acceptable mergers (under a consumer surplus standard). In addition, the set of mergers is even further enlarged when we consider divestitures to entrant firms. We obtain endogenously that divestiture proposals are always proportional to the competitive concern. That is, a lower merger synergy level must induce a larger divestiture proposal. Comparing divestitures to an incumbent competitor and an entrant firm, the latter type allows to clear a merger with less asset sales which gives rise to a larger parameter range for successful mergers.

We show that the type of divestiture (either to a competitor or an entrant firm) critically depends on the merging firms' ability to extract rents from the purchaser of the assets. Most importantly, we show that the type of divestiture is optimal from a social welfare perspective, if the merging firms can extract the entire gains from trade. This result follows from noticing that a merger with remedies is always externality-free (i.e., leaves consumer surplus and outsiders' profits unchanged). It is then immediate that the merging parties make the socially optimal decision, when able to extract the entire gains from trade. If, however, rent-extraction is limited, then the divestiture either goes to an entrant firm in the absence of any bargaining power (which involves minimal divestitures) or to a competitor under a bidding scenario (as the competitor has always a larger maximum willingness to pay than an entrant firm).

The merger remedy guidelines of the DOJ distinguish between "fix-it-first remedies" and "post-consummation sales" (DOJ, 2011, pp. 22-25). The guidelines clearly favor an adequate fix-it-first remedy, while the post-consummation sale is much more restrictive (and costly) for the merging parties. Quite bluntly the remedy guidelines state: "For the parties, resolving a merger's competitive issue with an upfront buyer can shorten the divestiture process, provide more certainty about the transaction than if they (...) must seek a buyer for a package of assets post-consummation, and avoid the possibility of a sale dictated by the Division in which

remedies involve asset sales to counter anticompetitive effects of a merger, while behavioral remedies target the merged firms' after merger business conduct (see DOJ, 2011, p. 6).

⁵Our analysis is placed in a Cournot setting in which synergies are necessary to make consumers not worse off after the merger (see Farrell and Shapiro, 1990a; Spector, 2003; Verge, 2010).

the parties might have to give up a larger package of assets” (DOJ, 2011, p. 22). Entering into a consent decree is costly, full of uncertainty, and further burdened with a crown-jewel provision which has to be offered to make the remedy more attractive for potential buyers. Those additional costs create commitment value for the merging firms in the fix-it-first sales process because a failure to reach an agreement may make the entire merger unattractive. If the selling power becomes maximal, then our analysis shows that the merging firms select the social welfare maximizing purchaser of the assets.

We can also show that the merging firms have a strong incentive to search for the most efficient buyer as this tends to increase the feasible set of mergers and, at the same time, keeps the asset sales necessary to induce an approval at its lowest possible value. We also identify instances which lead to mergers under a remedy rule which are strictly price-decreasing. *First*, a divestiture to a competitor firm which is able to realize synergies may lead to lower prices, and *second*, sequential mergers may induce a series of (price-fixing) divestitures which may lead to a strictly price-decreasing merger among newly created firms. Finally, we also examine the case where the AA is uncertain about the merger type (in terms of its synergy level). We show that there exists either a pooling equilibrium or a separating equilibrium. In the former case, both high-cost and low-cost mergers are cleared. Interestingly, mergers of relatively inefficient types may now become profitable which would have never been cleared under complete information. A separating equilibrium exists only if the pooling equilibrium does not exist which occurs when the inefficient type cannot merge profitably. It may then be the case that the efficient type proposes a relatively large divestiture (above the full information level) to deter the inefficient type from mimicking its behavior. Hence, divestiture proposals are more often than not larger under incomplete information when compared with the full information case. With that, we provide a new rationale for the often mentioned overshooting phenomenon associated with structural divestitures.

Our paper contributes to the analysis of mergers in Cournot oligopoly when productive capital in an industry is fixed (Perry and Porter, 1985; Farrell and Shapiro, 1990a,b; McAfee and Williams, 1992). That approach was applied to structural remedies in Medvedev (2007), Verge (2010), and Vasconcelos (2010). Verge (2010) shows that under fairly general conditions a

re-allocation of productive assets through remedies cannot increase consumer surplus when synergies are absent. Medvedev (2007) shows for a three-firm oligopoly that remedies in association with merger synergies extend the scope for acceptable mergers. Vasconcelos (2010) analyzes remedies for the case of a four firm oligopoly when merger synergies are possible. Each firm owns one unit of capital and capital is indivisible. He assumes that the AA maximizes consumer surplus which is crucial when at least three firms are involved in a merger. In those instances he shows the possibility of an “over-fixing” problem associated with remedial divestitures (see also Farrell, 2003). The AA uses its power to restructure the industry optimally. Overfixing may have adverse effects because a firm may abstain from proposing a (socially desirable) merger with two other firms. Instead, the acquirer expects (correctly) that the AA will use its power to sell one of the acquired firms to the remaining competitor. Consequently, the acquirer may strategically propose a one-firm takeover which can be worse from a consumer point of view than allowing a takeover of two other firms. Hence, remedies may not serve consumer interest as the antitrust authority is “overshooting” in terms of consumer protection.

Cabral (2003) analyzes mergers in a differentiated industry with free entry. When assets are sold to an entrant firm as a remedy, then a “buy them off” effect follows which means that an entrant firm is dissuaded from opening a new store (or introducing a new product variant). That effect may work against the interest of consumers, who are better off the more variants are offered in the market.

Recently, the impact of remedies on the effectiveness of merger control has been examined empirically (see Duso et al., 2011, and Duso et al., 2012, for the EU and Clougherty and Seldeslachts, 2012, for the US).⁶ Those works use an event study approach which identifies the anticompetitive effect of a merger by abnormal stock market returns of competing firms. Overall, the results appear to indicate that an upfront-buyer remedy tends to restore the pre-merger competitive situation.

We proceed as follows. Section 2 presents the basic model. In Section 3 we conduct the merger analysis for three different merger control regimes depending on whether or not remedies

⁶Ormosi (2012) analyzes major EU merger cases and shows that remedial offers and efficiency claims are often strategic to avoid costly delay in litigation processes.

are feasible and the type of the purchaser (either a competing firm or an entrant firm). In Section 4 we compare the merger outcomes in the different merger control regimes with regard to profitable and approvable merger outcomes. In addition, we also analyze the social welfare effects. Section 5 presents three extensions. In Section 5.1 we show that the optimal remedy type (divestiture to competitor or to entrant firm) depends on the merged firm’s ability to extract rents from the buyer. Section 5.2 analyzes the merging parties’ incentives to search for the most efficient purchaser. Section 5.3 presents a dynamic merger game to show that remedies tend to support higher concentration outcomes which are strictly desirable from a consumer perspective. Finally, Section 6 concludes.

2 The Model

We analyze the effects of remedies in a Cournot oligopoly with homogeneous products. There are $n \geq 3$ symmetric firms indexed by $i \in I = \{1, \dots, n\}$. All firms produce a homogenous good with inverse market demand given by $p(X) = 1 - X$, for $X < 1$, where X is the sum of firms’ individual outputs, x_i ; i.e., $X := \sum_i x_i$. Firm i ’s production costs depend on its output level, x_i , and the capital, K_i , it uses for production. Total productive capital of the industry, \bar{K} , is fixed and fully distributed among the firms in the industry; i.e., $K_i > 0$ for all $i \in I$ and $\sum_{i \in I} K_i = \bar{K}$.⁷ Specifically, firm i ’s production cost is given by $C_i(x_i, K_i) = x_i^2/K_i$.^{8,9} We normalize K_i to one, so that each firm uses one unit of capital in the absence of a merger. It then follows that $\bar{K} = n$.

⁷We perform a short-run analysis which is appropriate as competition authorities typically make prediction only for the “foreseeable” future (see, e.g., DOJ, 2011, p. 31).

⁸The underlying idea is that a firm’s cost function depends critically on the amount of capital it owns, while overall capital in the industry is fixed. A merger then combines the capital of the former independent firms. In addition, marginal costs are increasing in the output level which mirrors the capacity constraint implied by the fixed capital assumption.

⁹That specification of the cost function is borrowed from Perry and Porter (1985) and it was used in works as Farrell and Shapiro (1990b) and McAfee and Williams (1992). Farrell and Shapiro (1990a) and Verge (2010) present a more general Cournot oligopoly framework, though our functional form remains as a special case of their analysis.

The benchmark solution is the n -firm Cournot oligopoly equilibrium which describes the market outcome before the merger.¹⁰ When all firms $i \in I$ maximize their profits $\pi_i = p(X)x_i - x_i^2$ simultaneously by choosing their outputs, we obtain that each firm produces $x^*(n) = 1/(n+3)$, realizes profits of $\pi_i^*(n) = 2/(n+3)^2$, while the market price is

$$p^*(n) = \frac{3}{n+3}. \quad (1)$$

The AA adheres to a consumer standard. A merger, therefore, is approved if and only if the price level is not larger after the merger when compared with the pre-merger equilibrium $p^*(n)$. We distinguish three different merger control regimes depending on whether or not remedies are possible and on the remedy type.¹¹

- *No-remedy* (in short: *NR*): When merger guidelines do not allow for a remedial divestiture, then the AA can either approve or block the merger proposal altogether.
- *Divestiture to entrant* (in short: *DE*): In this case merger control allows for an approval conditional on a divestiture of a share of the target firm's assets to an entrant firm that keeps the consumer price from rising.
- *Divestiture to competitor* (in short: *DC*): Merger control allows for an approval conditional on a divestiture to an incumbent competitor if it counters any price-increasing effects of the proposed merger.

We examine a bilateral merger with firm 1 being the acquirer and firm 2 the target firm. Firms 1 and 2 will merge if the merged entity's profit does not fall short of their pre-merger profits, $2\pi_i^*(n)$. A merger may lead to a synergy which is measured by the parameter $s \in [0, 1]$. The synergy rotates the cost function downward such that marginal costs for a given level of output come down. Precisely, the cost function of the merged firm M (which combines the assets of firms 1 and 2) is given by $C_M = sx_M^2/(K_1 + \sigma K_2)$ or, assuming $K_1 = K_2 = 1$, by

¹⁰We use asterisks (*) to indicate equilibrium values in the before merger benchmark.

¹¹Throughout our analysis we assume that the AA can only impose a remedy on the merging firms that the parties themselves proposed. This mirrors legal practice in the EU and in the US (see EU, 2008, and DOJ, 2011, respectively). See also EU (2006).

$C_M = sx_M^2/(1 + \sigma)$, where σ is the share of firm 2's capital which stays under control of the merged firm.¹² If s is close to one, the synergy of the merger is negligible, while for smaller values of s the merger's synergy becomes larger. Accordingly, $1 - \sigma$ is the share of firm 2's capital which goes as a divestiture to another firm which may be an existing competitor or a new entrant firm. An entrant firm E which obtains the divestiture $1 - \sigma$ operates with the cost function $C_E(x_E, \sigma) = x_E^2/(1 - \sigma)$, whereas an incumbent competitor (say firm $i = 3$) which gets the divestiture $1 - \sigma$ produces with the cost function $C_3(x_3, \sigma) = x_3^2/(2 - \sigma)$.

We analyze the following merger game. In the first stage, firm 1 proposes to merge with firm 2. Depending on the merger control regime it may offer a remedial divestiture to the AA which will confirm a merger proposal if and only if the price level does not increase after the merger.¹³ In the second stage, depending on the AA's observable decision, all independent firms compete in Cournot fashion.¹⁴

3 Merger Analysis

No-remedy (regime NR). Under the no-remedy regime, the AA can only clear or reject the merger proposal in its entire. Hence, if a merger is approved, then $\sigma = 1$ always holds. Firm M 's profit function is then given by $\pi_M = p(X)x_M - sx_M^2/2$, while the remaining $n - 2$ competitors (indexed by $j \in J = \{3, \dots, n\}$) have a profit function of the form $\pi_j = p(X)x_j - x_j^2$. Firms' simultaneous output choices give rise to a system of $n - 1$ first-order conditions with the following

¹²That is, we suppose that the acquirer will divest parts of the target firm's assets in case the AA requires a remedy to approve the merger. We could also assume that the authority requires to divest parts of the acquirer's assets which would not change the results of our analysis.

¹³This setting mirrors merger control practice in the EU and US, where the merging firms are expected to propose a remedial divestiture in case of competitive concerns.

¹⁴We note that the quadratic and multiplicative specification of firms' production costs ($C(x) = ax^2/b$ with $a \in [0, 1]$ and $b > 0$) always ensures an interior solution. When a firm's output becomes small, its marginal costs go to zero, while its marginal revenue must stay strictly positive ($p(X \geq 1) = 0$ can never be an equilibrium outcome).

solutions:

$$x_M^{NR} = \frac{3}{n(1+s) + s + 4} \text{ and} \quad (2)$$

$$x_j^{NR} = \frac{1+s}{n(1+s) + s + 4}, \text{ for all } j \in J. \quad (3)$$

Inserting the equilibrium values (2)-(3) into the inverse demand function, $p(X)$, we obtain the equilibrium price

$$p^{NR} = \frac{3(1+s)}{n(1+s) + s + 4}. \quad (4)$$

The equilibrium profit of the merged firm M is given by $\pi_M^{NR} = (x_M^{NR})^2(1+s/2)$ or, after inserting (2), by $\pi_M^{NR} = 9(1+s/2)/(n(1+s) + s + 4)^2$. Clearly, the merged firm's profit level decreases the smaller the synergy; i.e., $\partial\pi_M^{NR}/\partial s < 0$. The merger is profitable if the merged firm's profit is not smaller than the sum of the pre-merger profits. Evaluating the respective difference at the lowest possible synergy level (that is, $\pi_M^{NR}(s=1) - 2\pi^*$), we obtain the expression $27/[2(2n+5)^2] - 4/(n+3)^2$ which has only one positive zero at $n \approx 3.14$. It is then immediate that the profit differential $\pi_M^{NR} - 2\pi^*$ is strictly positive for $n=3$ and any $s \in [0, 1]$. For all $n \geq 4$, we can calculate the maximal value of $s^{\max}(n)$ such that the merger is profitable. Solving $\pi_M^{NR} - 2\pi^* \geq 0$ for s , we get

$$s \leq s^{\max}(n) := \frac{17 - 26n - 7n^2 + (3n+9)\sqrt{17 + 22n + 41n^2}}{16(n+1)^2} \text{ for } n \geq 4. \quad (5)$$

Inspecting the right-hand side of (5), we get that $s^{\max}(n)$ is monotonically decreasing. In the limit we get $\lim_{n \rightarrow \infty} s^{\max}(n) = (3\sqrt{41} - 7)/16 \approx 0.763$.

Lemma 1. *In a symmetric 3-firm Cournot oligopoly a two-firm merger is strictly profitable for all $s \in [0, 1]$. For $n \geq 4$, a bilateral merger is strictly profitable if $s \in [0, s^{\max}(n)]$, with $s^{\max}(4) < 1$, $\partial s^{\max}(n)/\partial n < 0$, and $\lim_{n \rightarrow \infty} s^{\max}(n) = (3\sqrt{41} - 7)/16 \approx 0.763$.*

From Lemma 1 it follows that a bilateral merger is always profitable independently of the number of firms whenever the synergy is large enough; i.e., $s \leq s^{\max}(\infty) \approx 0.763$ holds.

The price level does not increase after the merger if $p^* - p^{NR} \geq 0$. Inserting the equilibrium values (1) and (4), respectively, we obtain

$$p^* - p^{NR} = \frac{3(1-2s)}{(n+3)[n(1+s) + s + 4]} \geq 0$$

which is true for $s \leq 1/2$. Hence, an AA which applies a consumer welfare standard will block the merger whenever $s > s^* = 1/2$ and allow the merger for $s \leq s^* = 1/2$. Interestingly, the decision rule is independent of the pre-merger concentration level (i.e., it does not depend on the number of firms n). We summarize those results in the following lemma.

Lemma 2. *Suppose a no-remedy merger control regime (NR). Then only mergers which create relatively large synergies with $s \leq s^* := 1/2$ are cleared, while merger proposals with small synergy levels $s > s^*$ are blocked by the AA.*

We next allow for asset sales sought as a remedy for the increased market power resulting from a merger which is an issue if the merger's synergy parameter, s , is greater than one-half.

Divestiture to entrant (regime DE). With a remedy rule at hand the AA can make a merger proposal conditional on structural remedies. We assume that the AA accepts all remedial offers which off-set any price-increasing effect of the merger proposal. That is, the remedy is only relevant if the post-merger price is expected to be higher than the pre-merger price in the absence of a remedial divestiture. From Lemma 2, this is the case if $s > 1/2$. In those instances, the acquirer may offer a divestiture of a share of the target firm's capital, $1 - \sigma$, which suffices to fix the consumer price at the pre-merger level.

Consider the case of divestitures to a new entrant firm, E . The profit function of the merged firm M , the entrant firm E , and the remaining rival firms $j \in J$, are given by $\pi_M = p(X)x_M - sx_M^2/(1 + \sigma)$, $\pi_E = p(X)x_E - x_E^2/(1 - \sigma)$, and $\pi_j = p(X)x_j - x_j^2$, respectively. In a Cournot equilibrium the following first-order conditions must be fulfilled:¹⁵

$$p(X) - x_M - \frac{2s}{1 + \sigma}x_M = 0, \tag{6}$$

$$p(X) - x_E - \frac{2}{1 - \sigma}x_E = 0, \text{ and} \tag{7}$$

$$p(X) - x_j - 2x_j = 0, \text{ for all } j \in J. \tag{8}$$

¹⁵Using symmetry for all $j \in J$, $X = x_M + x_E + (n - 2)x_j$ must hold in equilibrium.

Solving this system of n equations, we obtain firms' equilibrium output levels

$$x_M^{DE} = \frac{3(2\sigma - \sigma^2 + 3)}{\phi(n)}, \quad (9)$$

$$x_E^{DE} = \frac{3(1 + 2s(1 - \sigma) - \sigma^2)}{\phi(n)}, \text{ and} \quad (10)$$

$$x_j^{DE} = \frac{\sigma(2 - 2s - \sigma) + 3(1 + 2s)}{\phi(n)}, \text{ for all } j \in J, \quad (11)$$

with $\phi(n) := 3n + 12s + 8\sigma + 2n\sigma - 8s\sigma - 7\sigma^2 - n\sigma^2 + 6ns - 2ns\sigma + 15$. Substituting the equilibrium values (9)-(11) into the inverse demand, $p(X)$, we get the post-merger price depending on the synergy level, s , and the divestiture, $1 - \sigma$, which yields

$$p^{DE}(s, \sigma; n) = \frac{3(3 - \sigma)(1 + 2s + \sigma)}{\phi(n)}. \quad (12)$$

The post-merger price (12) is not larger than the pre-merger price (1) if

$$\begin{aligned} p^{DE}(s, \sigma; n) - p^*(n) &\leq 0 \text{ or} \\ \frac{s(3 + \sigma) + \sigma(2\sigma - 1) - 3}{\phi(n)} &\leq 0. \end{aligned} \quad (13)$$

Differentiating the denominator of (13), $\phi(n)$, with respect to n , we obtain

$$\partial\phi(n)/\partial n = (3 - \sigma)(1 + 2s + \sigma)$$

which is always strictly positive. Evaluating $\phi(n)$ at the lowest possible value of n , we obtain $\phi(3) = 30s + 14\sigma - 14s\sigma - 10\sigma^2 + 24$, which is strictly positive for all admissible values of s and σ . Hence, the sign of (13) depends only on the numerator which yields the condition

$$s \leq s^{DE}(\sigma) := 1 - \frac{2\sigma^2}{3 + \sigma} \quad (14)$$

with $\partial s^{DE}(\sigma)/\partial\sigma < 0$, $s^{DE}(\sigma = 0) = 1$, and $s^{DE}(\sigma = 1) = 1/2$. Condition (14) mirrors the fact that a full divestiture ($\sigma = 0$) preserves the market structure, so that any merger with synergies $s \in [0, 1]$ must benefit consumers (with indifference holding at $s = s^{DE}(\sigma)$). A divestiture, $1 - \sigma > 0$, is only necessary for low synergy levels, $s > 1/2$. Taking the inverse of condition (14), we obtain the critical value

$$\sigma^{DE}(s) = \frac{1 - s + \sqrt{(1 - s)(25 - s)}}{4}, \text{ for } s \in (1/2, 1] \quad (15)$$

from which the divestiture level, $1 - \sigma^{DE}(s)$, follows which is necessary to leave the price level unchanged after the merger. Hence, for every merger with synergies, $s \leq 1$, there exists a unique critical value $\sigma^{DE}(s)$ such that the post-merger price level is not larger than the pre-merger price, whenever the divestiture is large enough (i.e., $1 - \sigma \geq 1 - \sigma^{DE}(s)$ is fulfilled). As a consequence, a merger proposal with a certain synergy level $s < 1$ can only pass the decision screen of the AA if at least the share $1 - \sigma^{DE}(s)$ of the target firm's capital is sold to an entrant firm. Finally, from (14) it follows that $s^{DE}(\sigma = 1) = 1/2$. Hence, any bilateral merger with sufficiently large synergies (i.e., $s \leq 1/2$) is non-price increasing.¹⁶

Given the AA's decision rule, we can now examine the profitability of a merger proposal. This is an issue when synergies are relatively small (i.e., $s > 1/2$), because the AA will then require a divestiture according to (15). Given $s > 1/2$, the merged firm's profit is $\pi_M^{DE} = [1 + s/(1 + \sigma)] (x_M^{DE})^2$ which must not fall short of the joint pre-merger profit level of firms 1 and 2, $2\pi^*$. Note that $\partial\pi_M^{DE}/\partial\sigma > 0$. Hence, the proposed divestiture, $1 - \sigma$, must fulfill condition (14) with equality. Calculating the profit differential $\pi_M^{DE} - 2\pi^*$ and substituting $s = s^{DE}(\sigma)$, we obtain

$$\pi_M^{DE} - 2\pi^* \Big|_{s=s^{DE}(\sigma)} = \frac{\sigma(27 - 5\sigma) - 18}{(3 - \sigma)^2 (3 + n)^2}. \quad (16)$$

The sign of the right-hand side of (16) is equal to the sign of the numerator.¹⁷ It is now easily checked that the profit differential is positive for all $\sigma \geq \bar{\sigma}^{DE} := (3/10)(9 - \sqrt{41}) \approx 0.78$. Inserting that value into (14) we get that any merger with synergies $s \leq \bar{s}^{DE} := (33\sqrt{41} - 157)/80 \approx 0.68$ remains profitable under the *DE*-regime. We summarize our results as follows.

Lemma 3. *Suppose a divestiture to entrant regime (DE). Then all mergers with relatively large synergies, $s \leq s^* := 1/2$, are approved without a remedy. For lower synergy levels $s \in (1/2, 1]$, only merger proposals with commitments to divest at least $1 - \sigma^{DE}(s)$, where $\sigma^{DE}(s)$ is given by (15), will be approved by the AA. Given the divestiture requirement $\sigma^{DE}(s)$, mergers remain profitable for all $s \in (1/2, \bar{s}^{DE}]$ with $\bar{s}^{DE} := (33\sqrt{41} - 157)/80$. Divestitures are decreasing in the merger's synergy and the maximum divestiture observed in equilibrium is obtained at*

¹⁶This result mirrors Lemma 2, so that structural remedies are only necessary for mergers with relatively small synergies; i.e., $s > 1/2$ holds.

¹⁷Note that the sign of the profit differential does not depend on the number of firms, n .

$$1 - \sigma^{DE}(\bar{s}^{DE}) \approx 0.22.$$

Divestiture to competitor (regime DC). Again, consider a merger between firms 1 and 2. A share of $1 - \sigma$ of firm 2's capital is possibly divested to an incumbent competitor firm $j \in J$, say firm 3. Define $L := J \setminus \{3\}$ as the set of the $n - 3$ remaining incumbent competitors. The profit functions of firm M , firm 3, and the remaining competitors $l \in L$, are then given by $\pi_M = p(X)x_M - s/(1 + \sigma)x_M^2$, $\pi_3 = p(X)x_3 - 1/(2 - \sigma)x_3^2$, and $\pi_l = p(X)x_l - x_l^2$, respectively. In equilibrium, the following first-order conditions must be fulfilled:¹⁸

$$p(X) - x_M - \frac{2s}{1 + \sigma}x_M = 0, \quad (17)$$

$$p(X) - x_3 - \frac{2x_3}{2 - \sigma} = 0, \text{ and} \quad (18)$$

$$p(X) - x_l - 2x_l = 0, \text{ for all } l \in L. \quad (19)$$

Solving this system of $n - 1$ equations, we obtain the equilibrium output levels

$$x_M^{DC} = \frac{3(3\sigma - \sigma^2 + 4)}{\psi(n)}, \quad (20)$$

$$x_3^{DC} = \frac{3(4s + \sigma - 2s\sigma - \sigma^2 + 2)}{\psi(n)}, \text{ and} \quad (21)$$

$$x_l^{DC} = \frac{8s + 3\sigma - 2s\sigma - \sigma^2 + 4}{\psi(n)}, \text{ for all } l \in L, \quad (22)$$

with $\psi(n) := 4n + 12s + 12\sigma + 3n\sigma - 6s\sigma - 6\sigma^2 - n\sigma^2 + 8ns - 2ns\sigma + 18$. Substituting (20)-(22) into $p(X)$, we obtain the post-merger price depending on the synergy level, s , and the divestiture requirement, $1 - \sigma$, which yields

$$p^{DC}(s, \sigma; n) = \frac{3(4 - \sigma)(1 + 2s + \sigma)}{\psi(n)}. \quad (23)$$

The post-merger price (23) is not larger than the pre-merger price (1) if

$$\begin{aligned} p^{DC}(s, \sigma; n) - p^*(n) &\leq 0 \text{ or} \\ \frac{9(\sigma^2 - \sigma + 4s - 2)}{\psi(n)} &\leq 0. \end{aligned} \quad (24)$$

Note that $\partial\psi(n)/\partial n = 4 + 3\sigma - \sigma^2 + 8s - 2s\sigma$, which is always strictly positive. Evaluating $\psi(n)$ at the lowest possible value of n , we obtain $\psi(3) = 30 + 36s + 21\sigma - 12s\sigma - 9\sigma^2$, which

¹⁸Using symmetry for all $l \in L$, $X = x_M + x_3 + (n - 3)x_l$ must hold in equilibrium.

is strictly positive for all admissible values of s and σ . Hence, the sign of (24) depends only on the numerator which yields the condition

$$s \leq s^{DC}(\sigma) := \frac{1}{4}(2 + \sigma - \sigma^2). \quad (25)$$

It is easily checked that $s^{DC}(\sigma)$ is strictly concave, obtains a unique maximum at $\sigma = 1/2$ with $s^{DC}(\sigma = 1/2) = \bar{s}^{DC} := 9/16 \approx 0.56$, and is equal to one-half at points $\sigma \in \{0, 1\}$. Hence, for any $s \in [1/2, \bar{s}^{DC}]$, there are two solutions $\sigma_1(s) = (1 - \sqrt{9 - 16s})/2$ and $\sigma_2(s) = (1 + \sqrt{9 - 16s})/2$, such that the price level does not change after a merger.

Given the AA's decision rule, we can now examine the profitability of a merger proposal. The merged firm's equilibrium profit is $\pi_M^{DC} = [1 + s/(1 + \sigma)](x_M^{DC})^2$ which must not fall short of the joint pre-merger profit level of firms 1 and 2, $2\pi^*$. Note that $\partial\pi_M^{DC}/\partial\sigma > 0$. Hence, the divestiture, $1 - \sigma$, must fulfill condition (25) with equality and the proposed divestiture must take the upper value of the solution $(\sigma_1(s), \sigma_2(s))$ for any $s \in (1/2, \bar{s}^{DC}]$. Define the upper value as

$$\sigma^{DC}(s) := \frac{1 + \sqrt{9 - 16s}}{2}. \quad (26)$$

Calculating the profit differential $\pi_M^{DC} - 2\pi^*$ and substituting $s = s^{DC}(\sigma)$ for $\sigma \geq 1/2$, we obtain

$$\pi_M^{DC} - 2\pi^* \Big|_{s=s^{DC}(\sigma), \sigma \geq 1/2} = \frac{23\sigma - 10 - 4\sigma^2}{(n + 3)^2 (4 - \sigma)^2}. \quad (27)$$

The denominator of the right-hand side of (27) is always strictly positive, and the numerator is strictly positive for $\sigma \in [1/2, 1]$. Hence, any price-fixing divestiture $1 - \sigma^{DC}(s) \in [0, 1/2]$ leaves the merger proposal profitable. We summarize our results in the next lemma.

Lemma 4. *Suppose a divestiture to competitor regime (DC). Then all mergers with relatively large synergies, $s \leq s^* := 1/2$, are approved without a remedy. For lower synergy levels $s \in (1/2, \bar{s}^{DC}]$, with $\bar{s}^{DC} = 9/16$, only merger proposals with commitments to divest at least $1 - \sigma^{DC}(s)$, where $\sigma^{DC}(s)$ is given by (26), are approved by the AA. In that area all merger proposals remain strictly profitable. Divestitures are decreasing in the merger's synergy level and the maximum divestiture observed in equilibrium is $1 - \sigma^{DC}(\bar{s}^{DC}) = 1/2$. For $s \in (\bar{s}^{DC}, 1]$, a merger is never approved.*

We are now in a position to summarize the impact of remedies in merger control and we can evaluate the welfare consequences of remedies. We do so in the next section.

4 Comparison of Results and Social Welfare

For synergy levels $s > 1/2$ the merging firms have to propose a divestiture to the AA to get the merger approved. We have shown that the merging parties will always propose a minimal divestiture which leaves the price level just at its pre-merger level. A price-fixing divestiture exists under regime DE if $s < \bar{s}^{DE}$, while such a remedy exists under regime DC only if $s < \bar{s}^{DC}$. Due to $\bar{s}^{DC} < \bar{s}^{DE}$, for $s \in (1/2, \bar{s}^{DC}]$ a merger is cleared under both regimes DE and DC . Comparison of the optimal divestitures $1 - \sigma^{DC}$ and $1 - \sigma^{DE}$ under regimes DC and DE , respectively, yields

$$[1 - \sigma^{DE}(s)] - [1 - \sigma^{DC}(s)] = \frac{1}{4} \left[1 + s - \sqrt{(1-s)(25-s)} + 2\sqrt{9-16s} \right]. \quad (28)$$

The right-hand side of (28) approaches zero as $s \rightarrow 1/2$ and decreases monotonically over $s \in (1/2, \bar{s}^{DC}]$. Hence, the optimal divestiture is smaller under regime DE than under regime DC . Moreover, the difference of the divestiture levels increases when the merger creates less synergies.

Lemma 5. *Suppose $s \in (s^*, \bar{s}^{DC}]$, so that a merger involves divestitures under regimes DE and DC . The minimal divestiture necessary to induce the AA to approve the merger proposal is strictly larger under regime DC than under regime DE ; i.e., $\sigma^{DC}(s) < \sigma^{DE}(s)$, while the difference of the divestiture levels increases when the synergy level decreases. Moreover, any merger with minimal divestitures is externality-free.*

The last part of Lemma 5 follows from noticing that minimal divestitures imply that the pre-merger price is not affected by the merger. The next proposition summarizes the analysis of the merger game under regimes NR , DE , and DC with regard to the equilibrium merger outcome and the equilibrium divestiture level.

Proposition 1. *Remedies increase the scope for mergers depending on the merger synergy level and the merger control regime.*

- i) If $s \in (0, s^*]$, then firms always merge and remedies are never used.*
- ii) If $s \in (s^*, \bar{s}^{DC}]$, then firms merge only if remedies are feasible. Under regime DE the divestiture is $1 - \sigma^{DE}(s)$ and under regime DC the divestiture is $1 - \sigma^{DC}(s)$, where $1 - \sigma^{DC}(s) > 1 - \sigma^{DE}(s)$ always holds.*

iii) If $s \in (\bar{s}^{DC}, \bar{s}^{DE}]$, then firms merge only under regime DE with divestiture $1 - \sigma^{DE}(s)$.

iv) If $s \in (\bar{s}^{DE}, 1]$, then a merger never occurs.

Proposition 1 shows that remedies increase the scope for profitable and acceptable mergers and that remedies to entrants are more effective than remedies to competitors in this regard. Part *iv*) also shows that remedies are not effective, when the synergies created by the merger become too small. In those instances, a remedy exists under regime DE , but the requirement is too restrictive, so that a merger is not profitable anymore.¹⁹

Social welfare. What are the effects of remedies on social welfare (i.e., the sum of consumer surplus and producer surplus which we abbreviate by W) when compared with a merger control regime which does not allow for remedies? Remedies are relevant in the area $s \in (s^*, \bar{s}^{DE}]$. If a remedy is used, then a merger is externality-free, because the merging parties always propose the minimal necessary divestiture which leaves the price level unchanged. It then follows that the first-order conditions of the outsider firms (either firms $j \in J$ under regime DE or firms $l \in L$ under regime DC) also remain unaffected by the merger (see (8) for regime DE and (19) for regime DC). As a consequence, the social welfare effect of remedies then only depends on a comparison of total production costs for the firms involved in the merger (firms 1 and 2) and the firm which is the beneficiary of the divestiture policy (either firm E under regime DE or firm 3 under regime DC). Let us call those firms the insiders. We obtain the following result.

Proposition 2. *Suppose $s \in (s^*, \bar{s}^{DE}]$, so that a merger is only cleared with remedies. Then there exists a critical value \hat{s} , with $s^* < \hat{s} < \bar{s}^{DC}$, such that the ordering of social welfare (which follows from a comparison of the insiders' total production costs) depends on the synergy parameter, s , as follows:*

i) If $s \in (s^*, \hat{s})$, then $W^{DC} > W^{DE} > W^*$.

¹⁹Proposition 1 allows us to discuss what would happen if the divestiture affects the synergy level negatively. Let $s(\sigma) = s + f(\sigma)$ be the merger synergy as a function of the assets which remain under control of the merged firm. Assume $\partial f / \partial \sigma < 0$ with $f(\sigma \rightarrow 0) = 1 - s$ and $f(\sigma \rightarrow 1) = 0$. The merged firm's cost function then becomes $\tilde{C}_M = [s + f(\sigma)] x_M^2 / (1 + \sigma)$. It is easily checked that the minimal divestiture is increased under regimes DE and DC . Moreover, under regime DE , it is no longer true that a price fixing remedy exists for any $s \leq 1$. Overall, the scope for profitable mergers with divestitures decreases significantly.

ii) If $s \in (\widehat{s}, \bar{s}^{DC}]$, then $W^{DE} > W^{DC} > W^*$.

iii) If $s \in (\bar{s}^{DC}, \bar{s}^{DE}]$, then $W^{DE} > W^{DC} = W^*$.

Proof. See Appendix.

Proposition 2 shows that a merger control regime which allows for remedies is always preferable from a social welfare perspective when compared with regime NR . Proposition 2 also mirrors the fact that total production costs tend to be lower the more equal the distribution of capital among firms becomes. If the divestiture is relatively small, then regime DC leads to the highest welfare level. The entrant firm's capital is so small in that case, such that insiders' production costs are larger under regime DE than under regime DC . However, part ii) of Proposition 2 also shows that there exists an interval for relatively large divestiture levels (associated with relatively small synergies), where regime DE outperforms regime DC . In that case, two effects tend to lower insiders' total production costs under regime DE when compared with regime DC : *Firstly*, the entrant firm gets a relatively large share of firm 2's productive capital, and *secondly*, the divestiture level is smaller under regime DE than under regime DC , so that a larger share of production goes to the merged firm which benefits from synergies.

Finally, part iii) of Proposition 2 shows that for lower synergy levels regime DE outperforms both regime DC and regime NR . In that area, a merger is only an equilibrium outcome under regime DE , whereas a merger cannot occur under regimes DC and NR .

Overall, those results indicate that it is not necessarily optimal that remedies go to an entrant firm. In contrast, Proposition 2 shows that this is only the case when divestitures have to be large. It then follows that the entrant firm obtains a sufficiently large share of the productive capital which has the positive effect of lowering its total production costs. If, however, the divestiture is relatively small, then a divestiture to an existing competitor is preferable from a social welfare perspective as this results in a more even distribution of the productive capital in the industry.

5 Extensions

In the following we discuss four extensions. *First*, we examine the optimal remedy type (divestiture to competitor or to entrant firm) depending on the merged firm's ability to extract rents from the buyer. *Second*, we analyze how the efficiency of the buyer (competitor or entrant) affects the set of equilibrium merger outcomes. *Third*, we show that remedies may support higher concentration and strictly lower prices in a dynamic merger game when firms can merge sequentially in pairs. *Fourth*, we analyze separating and pooling equilibria when the AA is uncertain about the synergy level associated with a merger proposal.

5.1 Endogenous Remedy Type

Part *ii*) of Proposition 1 shows that there is a range of synergy levels, with $s \in (s^*, \bar{s}^{DC}]$, such that both a divestiture to an entrant and a divestiture to a competitor constitute acceptable remedies for the AA. Which remedy is optimal from the merging firms' perspective? To answer this question, we distinguish three cases: *first*, selling the divestiture at a fixed price, *second*, auctioning off the right to buy the divestiture, and *third*, the case of perfect seller power, in which case the merging firms can make a take-it or leave-it proposal to a pre-selected buyer.

Selling at a fixed price. Assume that the divestiture is sold at a fixed price which does not exclude any potential buyer.²⁰ Then the merged firm selects the buyer which guarantees the highest after-merger profit level; i.e., it compares π_M^{DE} with π_M^{DC} .²¹ If we assume that the merged firm can optimally adjust the size of the asset sales, we can apply Lemma 5 which states that the optimal divestiture is strictly larger if sold to a competitor. Moreover, optimal asset sales guarantee that the pre-merger price p^* stays put after the merger. Independently of the divestiture type, the merged firm's first-order condition then becomes

$$p^* - x_M - \frac{2s}{1 + \sigma} x_M = 0,$$

²⁰Below (in the bidding context), we derive firms' willingness to pay. At this stage, it suffices to assume that the exogenous selling price is small (or even zero), as we want to determine the optimal divestiture type when the merged firm's ability to extract rents is limited. See Vasconcelos (2010) for a similar analysis.

²¹In the following, we indicate equilibrium values by the superscript *DE* if $\sigma = \sigma^{DE}(s)$ and by *DC* if $\sigma = \sigma^{DC}(s)$. Hence, $\pi_M^{DE} := \pi_M(\sigma^{DE}(s))$, $\pi_M^{DC} := \pi_M(\sigma^{DC}(s))$ and so on.

from which we obtain the after-merger output level

$$x_M = \frac{p^*}{1 + \frac{2s}{1+\sigma}} \quad (29)$$

which is decreasing in the divestiture level, $1 - \sigma$. From Lemma 5 we know that the divestiture is always larger if the buyer is an existing competitor; i.e., $\sigma^{DC}(s) < \sigma^{DE}(s)$ holds always. Thus, $x_M^{DE} > x_M^{DC}$ and consequently $\pi_M^{DE} > \pi_M^{DC}$. Therefore, the merging firms will propose a divestiture to an entrant if the assets are sold at a fixed price.

Bidding for the divestiture. Again, suppose $s \in (s^*, \bar{s}^{DC}]$, so that both an asset sale to an entrant firm or to a competitor can serve as a remedy. We assume again optimal divestitures as stated in Proposition 1. For simplicity, we take it for granted that the merged firm can extract the entire willingness to pay from the winning bidder; e.g., by setting a reserve price.²²

The gain of the entrant firm from acquiring the divestiture is $\pi_E^{DE} - R$, where R denotes the entrant's outside option which we normalize to zero. A competitor's maximum willingness to pay is given by $\pi_j^{DC} - \pi^*$, with $j \in J$, which is the net gain of acquiring the assets.²³

The solution of the bidding game follows from Proposition 2. If $s \in (s^*, \hat{s})$, then part *i*) Proposition 2 states that $W^{DC} > W^{DE}$ holds, from which it follows that the competitor has a larger willingness to pay than the entrant and that the merged firm also prefers to sell to a competitor. To see this, note that $W^{DC} > W^{DE}$ is equivalent to $\pi_j^{DC} + \pi_M^{DC} > \pi_M^{DE} + \pi_E^{DE} + \pi^*$, or

$$\pi_M^{DC} + [\pi_j^{DC} - \pi^*] > \pi_M^{DE} + \pi_E^{DE}. \quad (30)$$

The left-hand side of (30) is the maximum profit the merged firm might realize if it divests to a competitor, while the right-hand side is the merged firm's maximum total profit in case of a divestiture to an entrant firm. We have just shown that $\pi_M^{DC} < \pi_M^{DE}$ holds in the "selling at a fixed price" scenario. It then follows from (30) that $\pi_j^{DC} - \pi^* > \pi_E^{DE}$ must hold, so that a

²²This assumption can be relaxed if many potential entrants bid for the divestiture.

²³Note that a merger which is cleared with remedies $1 - \sigma^{DE}$ (if the buyer is an entrant firm) or with $1 - \sigma^{DC}$ (if the buyer is an existing competitor) is always externality-free (see Lemma 5). Hence, if the entrant firm acquires the divestiture, then all competitors' profits remain at their pre-merger level, π^* . Similarly, if a competitor, say $j = 3$, acquires the divestiture, then the remaining competitors' profits stay constant at the pre-merger level. It then follows that all competitors $j \in J$ have the same maximum willingness to pay for the divestiture.

competitor's maximum willingness to pay exceeds the entrant's maximal bid. Thus, a competitor wins the bidding and the merged firm realizes after-merger profits π_M^{DC} and extracts at least π_E^{DE} , up to the competitor's maximum willingness to pay $\pi_j^{DC} - \pi^*$ (left-hand side of (30)).

For $s \in (\widehat{s}, \bar{s}^{DC})$, part *ii*) of Proposition 2 states that the ordering of social welfare is reversed; i.e., $W^{DE} > W^{DC}$ holds. Hence, the inequality sign in (30) is now also reversed. By the same argument as before, the merged entity now prefers a divestiture to the entrant firm as this maximizes the sum of after-merger profits and the price at which the divestiture is auctioned off. However, straightforward calculations show that the entrant's maximum willingness to pay does never surpass a competitor's maximum bid; i.e., $\pi_j^{DC} - \pi^* > \pi_E^{DE}$ remains valid. We conclude, that a competitor will always post a higher bid than the entrant, so that the divestiture goes to a competitor when the divestiture is auctioned off.

Perfect selling power. If the merging firms can commit to make a take-it or leave-it offer to a pre-selected firm, it extracts all gains from trade. Hence, if (30) holds, then the divestiture is sold to a competitor. By Proposition 2, this is the case if $s \in (s^*, \widehat{s})$. Accordingly, for the remaining parameter values $s \in (\widehat{s}, \bar{s}^{DC}]$, condition (30) is reversed, so that the divestiture then goes to an entrant firm.

Proposition 3. *Suppose $s \in (s^*, \bar{s}^{DC}]$, so that both a divestiture to an entrant or to a competitor are possible. The outcome of the sales process depends critically on the selling mechanism.*

i) If the divestiture is sold at a fixed price which does not exclude any potential buyer, then the merged firm sells the divestiture to an entrant firm.

ii) If the divestiture is sold through an auction in which all buyers bid their maximum willingness to pay, then the divestiture goes to a competitor.

iii) If the merged firm can make a take-it or leave-it offer to a pre-selected buyer, then the divestiture is sold to a competitor for $s \in (s^, \widehat{s})$, and sold to an entrant firm for $s \in (\widehat{s}, \bar{s}^{DC}]$.*

Proposition 3 shows that the merged firm's ability to extract rents from the asset sale is critically determining the divestiture type. If, for some reason, potential buyers can avoid to get absorbed in a bidding race, so that rent extraction is severely limited, then a divestiture to an entrant firm should be most likely. In that instance, the merging parties minimize the amount of assets to be sold. If rent extraction is enhanced, for instance, when the asset sale is

structured through an auction-type selling process, then the divestiture should be expected to go to an existing competitor. In an auction, competitors' maximum willingness to pay always exceeds the maximum bid of an entrant firm. Finally, part *iii*) of Proposition 3 shows that the merged firm's divestiture decision is perfectly aligned with the social welfare maximizing rule (see Proposition 2), whenever the merged firm can commit to a take-it or leave-it offer to a pre-selected buyer. The merged firm is then able to extract the entire surplus created by the divestiture process. As the trade of divestitures is externality-free, it follows that the merged firm makes the socially optimal choice.

The message of Proposition 3 is that the merging parties should have a maximum of power in the asset sales process, because this must lead to a selection of the socially preferred buyer type. Intuitively, the merging parties maximize the gains from trade under the remedy constraint. As any merger with remedies is externality-free, it then follows that the socially optimal buyer is selected.

It is noteworthy that remedy guidelines mirror our findings. For instance, the merger remedy guidelines of the DOJ distinguish between "fix-it-first remedies" and "post consummation sales" (DOJ, 2011, pp. 22-25). Successful fix-it-first remedies eliminate the competitive concerns and allow the AA to clear the merger without the need to file the case in court. In contrast, post-consummation sales induce the AA to file the case in court to obtain a consent decree which allows to enforce and monitor the remedial provisions because of the court's contempt power. The guidelines clearly favor an adequate fix-it-first remedy, while the post-consummation sale is much more restrictive (and costly) for the merging parties. With regard to the fix-it-first remedy, the guidelines "provide the parties with the maximum flexibility in fashioning the appropriate divestiture" (DOJ, 2011, p. 22). Accordingly, the merging parties can adjust the divestiture freely, so that the assets can be "tailored to a specific proposed purchaser" (DOJ, 2011, p. 22). In contrast, if a consent decree is needed for a post-consummation sale, then the guidelines build up a credible threat of force. *First*, a package of assets to be divested must be identified in advance, and *second*, "crown-jewels" must be offered "to increase the likelihood that an appropriate purchaser will emerge" (DOJ, 2011, p. 24).

Those rules increase the commitment value of the merging parties when proposing an asset

sale to a potential purchaser to obtain a fix-it-first remedy. *First*, the guidelines give a maximum of flexibility in adjusting the asset sale to the competitiveness of the purchaser. *Second*, entering into a consent decree is costly, full of uncertainty, and further burdened with the crown-jewel provision. Those additional costs may make the entire merger unattractive, adding to the commitment value necessary to extract rents in the fix-it-first sales process.

5.2 Efficiency of the Buyer

We show that mergers become more likely when the purchaser can more efficiently employ the divested assets. We analyze the case of a competitor buyer and the case of an entrant buyer separately. In the former case, potential competitors may be heterogenous with regard to their ability to generate synergies when merging their businesses with the divested assets (“heterogenous competitors”). In the latter case entrant firms may differ concerning their efficiency (“heterogenous entrants”).²⁴

In our basic model a divestiture is only proposed for $s > 1/2$, which ensured that the divestitures implied by (15) and (26) are always strictly positive. Introducing the possibility that the buyer can be more efficient than in our basic model, gives rise to the problem that a merged firm with synergies s very close to $1/2$ may want to propose an infinitesimal small divestiture. Below we see that this is an issue when competitors are heterogenous. To determine the smallest possible divestiture, from now on we assume $\sigma \in [0, 1 - \varepsilon]$, with $\varepsilon > 0$ and arbitrarily small. We call the smallest possible divestiture an ε -divestiture.²⁵

Heterogenous competitors. Suppose that competitors have different abilities to generate synergies when merging their businesses with the divested assets. Let parameter t be a measure of that ability, so that a competitor of type t has the cost function $C_t = tx^2/(2 - \sigma)$ if it merges its assets with the divested assets $1 - \sigma$. It is natural to assume $s \leq t \leq 1$, so that the competitor

²⁴Motta, Polo, and Vasconcelos (2003) describe another concern which relates to collusive behavior after merger. Buyers may differ regarding their competitive behavior in the future. If the AA is not well informed, then the merging firms may want to select the least competitive type of buyer.

²⁵A small divestiture of productive capital can lead to synergies on the buyer’s side because it gives access to some essential inputs or intangible assets (e.g., brand name, business secrets, intellectual property rights).

buying the divestiture does not realize larger synergies than the merging firms. We can use the analysis of our basic model (regime DC) to solve for the new divestiture requirement $\sigma^{DC}(s, t)$, which takes the buyer's type into account. Using the system of first-order conditions (17)-(19), while noticing that the buyer firm has marginal costs $2tx/(2 - \sigma)$ instead of $2x/(1 + \sigma)$, we obtain a new requirement

$$\sigma^{DC}(s, t) = \begin{cases} \frac{1}{2} (1 + \sqrt{9 - 16st}), & \text{if } \frac{1}{2} (1 + \sqrt{9 - 16st}) < 1 - \varepsilon \\ 1 - \varepsilon, & \text{otherwise} \end{cases}, \quad (31)$$

from which the price-fixing divestiture rule $1 - \sigma^{DC}(s, t)$ follows. Obviously, a lower value of t (i.e., a more efficient buyer type) implies a smaller divestiture. Comparison with the former divestiture rule (26) shows that a lower value of t implies that the merging firms can achieve a price-fixing remedy with less divestitures. Solving for the maximal approvable synergy level, we obtain from (31) that all mergers with synergy parameters

$$s(\sigma, t) \leq s^{DC}(\sigma, t) := \frac{1}{4t}(2 + \sigma - \sigma^2) \text{ if } \frac{1}{2} (1 + \sqrt{9 - 16st}) < 1 - \varepsilon \quad (32)$$

can pass the decision screen of the AA. Again, comparison of (32) with condition (25) shows that $s^{DC}(\sigma, t = 1) = s^{DC}(\sigma)$, while $\partial s^{DC}(\sigma, t)/\partial t < 0$. Hence, a divestiture to a more efficient buyer (lower value of t) increases the scope for approvable mergers. The lowest approvable synergy level s is, again, reached at $\sigma = 1/2$, where $\bar{s}^{DC}(1/2, t) = 9/(16t)$, which increases when t is reduced. The smallest possible value of t is $t = s$. At this point, all mergers with synergy parameters $s \leq \bar{s}^{DC}(1/2, s) = 3/4$ are approvable (note the difference to $\bar{s}^{DC}(1/2) = 9/16$ according to Lemma 4).

For $t < 1$, we also obtain ε -divestitures which are strictly price-decreasing. The smallest possible parameter value of s such that a price-fixing divestiture (larger than an ε -divestiture) exists follows from $\lim_{\sigma \rightarrow 1} s^{DC}(\sigma, t) = 1/(2t)$. Hence, all merger proposals with synergy parameters $s \in (1/2, 1/(2t))$ are then cleared with an ε -divestiture. In all those instances, the post-merger price must be strictly smaller than the pre-merger price, so that consumers are better off after the merger.

We next check the profitability condition (27) for the divestiture condition (31), which gives

for $\pi_M^{DC}(\sigma^{DC}(s, t)) - 2\pi^*$ the expression

$$\frac{2 [\Psi_2 + \Psi_1(12s^2 + 13t^2s + 23s^2t + 15t^2)]}{[\Psi_3 + \Psi_1(nt + 3t - sn - 3s)] [3(s + t) + 8st + \Psi_1(t - s)] (n + 3)}, \quad (33)$$

with $\Psi_1 := \sqrt{9 - 16st}$, $\Psi_2 := 45t^2 + 39st^2 - 36s^2 - 69s^2t - 48s^2t^2 - 40t^3s + 32s^3t$, and $\Psi_3 := 9(s + t) + 24st + n(3t + 8st + 3s)$. The denominator of (33) is strictly positive as the terms in the first two brackets are strictly positive for all n . The zeros of the numerator are independent of n . The unique positive zero is then obtained at

$$\tilde{s}(t) = -\frac{209}{128}t - \frac{21}{32} + \frac{21}{128}\sqrt{41}t + \frac{9}{32}\sqrt{41}, \text{ for } 1/2 < t \leq \hat{t},$$

where \hat{t} follows from $\tilde{s}(\hat{t})\hat{t} = 9/16$.²⁶ If the efficiency of the buyer is maximal (i.e., $t = s$), then we obtain the largest value of the synergy parameter

$$\tilde{s}(t = s) = \frac{12(-7 + 3\sqrt{41})}{337 - 21\sqrt{41}} \approx 0.723,$$

such that a merger is both profitable and acceptable for all $s < \tilde{s}(t = s) \approx 0.723$. In that case, the merging firms must divest $1 - \sigma^{DC}(s, s) \approx 0.104$, which is much less than the maximal divestiture under the *DC* regime (50 percent; Lemma 4) and even much smaller than the maximal divestiture under regime *DE* (22 percent; Lemma 2).

Heterogenous entrants. Suppose that potential entrants differ with respect to their productive efficiency. Suppose an entrant of efficiency type r has the cost function $C_r = rx_r^2/(1 - \sigma)$, with $r \in [s, 1]$. That is, an entrant with a lower value of r has lower marginal costs if it acquires a divestiture $1 - \sigma > 0$. Substituting the entrant firm's marginal costs $2rx/(1 - \sigma)$ into the entrant's first-order condition under regime *DE* (7) and comparing the resulting price level with the pre-merger price, p^* , we get that all mergers with synergies

$$s \leq s^{DE}(\sigma, r) = \frac{1}{4r + \sigma - 1} [2 + r(1 + \sigma) - 2\sigma^2] \quad (34)$$

are approvable (where we assume $s > 1/2$). A smaller value of r reduces the minimal divestiture needed to fix the price at the pre-merger level.²⁷ In contrast to the heterogenous competitor

²⁶Precisely, we obtain $\hat{t} := 6(3\sqrt{41} - 7)/(209 - 21\sqrt{41}) \approx 0.983$. Note that for $t > \hat{t}$ the profit differential (33) is strictly positive.

²⁷Formally, $\partial s^{DE}(\sigma, r)/\partial r = -9(1 - \sigma^2)/(1 - 4r - \sigma)^2 < 0$.

case, there always exists a unique divestiture $1 - \sigma^{DE}(s, r) > 0$, for any $s > 1/2$ and $r \in [s, 1]$. From (34) we obtain the price-fixing divestiture rule

$$\sigma^{DE}(s, r) = \frac{1}{4} \left(r - s + \sqrt{16 + 8(r + s) + r^2 + s^2 - 34rs} \right),$$

which obtains at $r = s$ the value $\sigma^{DE}(s, s) = \sqrt{1 + s(1 - 2s)}$, which approaches one if $s \rightarrow 1/2$. We conclude that the merging firms have strict incentives to search for the most efficient entrant. Assuming $s > 1/2$, this follows from noticing that a merger under regime *DE* is only approved if divestitures are proposed which fix the price at the pre-merger level. A lower value of r unambiguously reduces the price-fixing divestiture. Inspecting the first-order condition of the merged firm (6) it is obvious that the merged firm must increase its output, which implies an increase in its profit level. At the same time the entrant firm reduces its output by the same amount, so that the price level stays put.²⁸ Hence, the merging firms want to propose a divestiture with the most efficient entrant type.

Overall, those results show that the merging firms have strong incentives to search for an efficient buyer (be it an incumbent competitor or an entrant firm). If successful, this increases the likelihood of a profitable and approvable merger. Interestingly, an ε -divestiture may become possible when the buyer is a competitor. In those instances the price level decreases, so that consumer surplus increases as well.

5.3 Remedies in Sequential Mergers

We have so far seen that remedies increase the scope for mergers, which already indicates that remedies can lead to more asymmetric market structures with higher measures of concentration. In this section we propose a sequential merger process to derive the ultimate equilibrium market structure, which can be expected when remedies are feasible.²⁹ We refer to our basic model and

²⁸Note that the incumbent competitors' first-order conditions (8) are not affected as the merger under the *DE* regime remains price-fixing for all $r \in [s, 1]$.

²⁹Sequential mergers were analyzed in Nilsson and Sorgard (1998), who show that merger outcomes are likely to be path-dependent. See Nocke and Whinston (2010, 2012) for recent contributions which identify conditions such that a myopic merger review is nevertheless subgame perfect.

we restrict the analysis to parameter values $s \in (\bar{s}^{DC}, \bar{s}^{DE}]$.³⁰ In that range the divestiture must go to an entrant firm to induce the AA to approve the merger.

We start with a symmetric four-firm Cournot oligopoly (see Vasconcelos, 2010, for an analogous setting). We invoke the assumption that synergies s can only be created once. Hence, if a firm is a result of a previous merger, then that firm cannot create synergies again.³¹ For simplicity, we assume that the realized synergy s is the same for all mergers. We suppose disjoint sets of possible mergers (see Nocke and Whinston, 2010). That is, the initial set of four firms $I = \{1, 2, 3, 4\}$ is divided into two subsets of two firms, say $I_1 = \{1, 2\}$ and $I_2 = \{3, 4\}$. Firms in those sets can merge sequentially (firstly, firms in set I_1 , and secondly, firms in set I_2). If firms in both sets found it optimal to merge, then two new merged firms M_1 (which is the merger of firms 1 and 2) and M_2 (which is the merger of firms 3 and 4) emerge. At the same time (as each merger must have been price-fixing) entrant firms E_1 and E_2 have entered the market. In the third stage of the merger game, we allow for any possible bilateral merger. Then, the merger formation process ends and firms compete in Cournot fashion. To simplify, we assume that firms have only once the opportunity to merge. Hence, if firms 1 and 2 (firms 3 and 4) do not merge in the first (second) stage of the game, then they cannot merge in the last stage of the game.

Note that this merger game always induces two mergers in the first two stages of the game. Hence, in the first stage, firm 1 proposes a merger with firm 2 with remedies to an entrant firm according to Proposition 1. Thus, an entrant firm enters with capital $1 - \sigma^{DE}(s)$. According to Lemma 5, the merger is externality-free. At the end of stage 1, we obtain a new set of firms which consists of the merged firm, M_1 , two incumbent competitors 3 and 4, and the entrant firm E_1 .

In the second stage, firms 3 and 4 can also propose a merger. The profitability of a merger between firms 3 and 4 follows from noticing that the first merger is externality-free. Firms 3 and 4, therefore, face the same decision problem (i.e., first-order conditions) as firms 1 and 2 in the first stage of the game. Hence, firms 3 and 4 will also find it profitable to propose a merger

³⁰Of course, our analysis remains valid if we consider $s \in (s^*, \bar{s}^{DC}]$. Then, by Proposition 3, a divestiture to an entrant can be expected when the merged firm's ability to extract rents from the purchaser is limited.

³¹This assumption ensures that divesting parts of a firm and merging these parts later on does not create additional synergies.

with exactly the same divestiture $1 - \sigma^{DE}(s)$ that was applied to the first merger. This remedy creates a new entrant firm E_2 .

We, therefore, obtain in the third stage a market structure with two large firms (M_1 and M_2) and two symmetric small firms E_1 and E_2 . It is obvious that a merger between one of the merged firms and an entrant firm cannot be approvable as such a merger does not create synergies anymore. Similarly, a merger of firms M_1 and M_2 is strictly price-increasing in the absence of a remedy which remains true if we consider a divestiture to either firm E_1 or E_2 . A merger between firms M_1 and M_2 with a price-fixing divestiture to a new entrant firm is not profitable. We are, therefore, left with a possible merger between entrant firms E_1 and E_2 .

Both entrant firms operate with the same size of capital $1 - \sigma^{DE}(s)$. In the third stage, we then obtain the following first-order conditions:

$$\begin{aligned} p(X) - x_M - \frac{8s}{5-s+\Psi_4}x_M &= 0 \text{ for firms } M_1 \text{ and } M_2, \text{ and} \\ p(X) - x_E - \frac{8}{3+s-\Psi_4}x_E &= 0, \text{ for firms } E_1 \text{ and } E_2, \end{aligned} \quad (35)$$

with $\Psi_4 := \sqrt{(1-s)(25-s)}$, where we substituted (15) into (6) and (7), respectively. If firms E_1 and E_2 merge to form a new firm F with combined capital $2[1 - \sigma^{DE}(s)]$, and cost function $C_F = sx_F^2/[2(1 - \sigma^{DE}(s))]$, then the merged firm's first-order condition becomes

$$p(X) - x_F - \frac{4s}{3+s-\Psi_4}x_F = 0. \quad (36)$$

Solving equations (35) and (36), we obtain the equilibrium outputs

$$\begin{aligned} x_F &= \frac{3s^3 - 3s^2\Psi_4 + 33s^2 + 196s - 24s\Psi_4 - 76 + 4\Psi_4}{12s^3 + 200s^2 + 688s - 224} \text{ and} \\ x_M &= \frac{3s^2\Psi_4 + 16s\Psi_4 - 4\Psi_4 - 3s^3 - 36 + 11s^2 + 132s}{12s^3 + 200s^2 + 688s - 224}, \text{ for } M_1 \text{ and } M_2. \end{aligned}$$

Computing the after-merger market price

$$p(s) = \frac{15s^3 + 145s^2 + 228s - 76 - 3s^2\Psi_4 - 8s\Psi_4 + 4\Psi_4}{12s^3 + 200s^2 + 688s - 224},$$

we get that this merger decreases the market price below the initial price level, p^* , for all $s \in (\bar{s}^{DC}, \bar{s}^{DE}]$. The merger must be profitable, as the merged entity produces more than both

entrant firms independently. Note finally, that the mergers in stages 1 and 2 remain profitable even though the market price is lower than in the absence of mergers.

We, therefore, have shown that the possibility of remedies can convert a symmetric market with four firms into a much more concentrated and asymmetric market with two large firms and one small firm. In the absence of remedies no merger could occur, so that concentration would be lower. However, consumers benefit from remedies as they allow to approve mergers which yield synergies otherwise not realizable.

Proposition 4. *Consider a four firm Cournot oligopoly and a dynamic merger game. Suppose $s \in (\bar{s}^{DC}, \bar{s}^{DE}]$. Without remedies the market structure is not affected by merger activities. With the introduction of remedies (in particular, divestitures to an entrant firm), the equilibrium market structure consists of two equally large firms and one small firm. Each merger realizes synergy s and the market price is strictly lower than in the absence of mergers.*

Proposition 4 is related to Nocke and Whinston (2010), where conditions are identified such that a forward looking merger control regime cannot do better than a myopic one. Our analysis shows that their result carries over when remedies are taken into account and subsequent mergers are possible between newly created competitors. In fact, allowing for subsequent mergers among entering firms may give rise to additional price reducing effects if synergies can be realized.

5.4 Incomplete Information

We now assume that the AA does not know the exact synergy level associated with a merger proposal. Again, we build on our basic model and we consider only remedies to an entrant firm. We suppose a two-type case setting. Let there be two types of synergy levels, $\theta = h, l$, among potential merger proposals: s_h (“high-cost” type) and s_l (“low-cost” type), with $s_l < s_h$ and $s_h, s_l \in I_1 := [1/2, (3\sqrt{41} - 7)/16]$.³²

A priori, the efficient type s_l occurs with probability ρ and the inefficient type s_h with counter

³²Restricting attention to the interval I_1 ensures that any merger proposal is strictly profitable (the upper bound follows from Lemma 1), while there may be merger proposals (with $s = 1/2$) which should be approved without a divestiture. Moreover, there can be merger proposals with $s > \bar{s}^{DE}$ which should be blocked under complete information (see Lemma 3).

probability $1 - \rho$. Given $s_\theta \in I_1$, there exists a minimum share of productive capital $\sigma_\theta^{\min} \in [0, 1]$ which has to stay within the merged firm to ensure profitability of the merger; i.e., σ_θ^{\min} follows from solving $\pi_M^{DE}(\sigma_\theta^{\min}, s_\theta) - 2\pi^* \geq 0$.³³ While the AA does not know the exact synergy level, the merging candidates know their exact synergy level s_θ , with $\theta = l, h$.³⁴

We analyze the same game as in our basic set-up with the following qualifications: *First*, a divestiture proposal is binding and cannot be reversed after the AA's decision. *Second*, only strictly profitable mergers are proposed.³⁵ *Third*, the AA approves the merger only if the expected post-merger price is not higher than the pre-merger price p^* .³⁶ We solve for subgame perfect Bayesian Nash equilibrium.

Let $p^{pm}(s_\theta)$ denote the post-merger price following a merger of type s_θ with divestiture $1 - \sigma_\theta$. Such a merger-proposal will be approved if and only if

$$p^* - E(p^{pm}) \geq 0 \text{ or} \quad (37)$$

$$b_h p^{pm}(s_h) + b_l p^{pm}(s_l) \leq p^*, \quad (38)$$

where the left-hand side of (38) denotes the expected post-merger price, given the AA's beliefs b_h and b_l about facing a merger proposal of either a high-cost type, s_h , or a low-cost type, s_l , respectively.

A priori beliefs of the AA are $b_l = \rho$ and $b_h = 1 - \rho$. There is only one instance, in which the AA can update its belief such that it can infer from the divestiture proposal the merger's type for sure. This can only be the case, when the divestiture is so large that only the low-cost type can profitably merge. If, in contrast, the divestiture is small enough so that both types find it profitable to merge, then the AA must stick to the a priori distribution of merger types.

The existence of a separating equilibrium follows from noting that a merger's profitability is

³³Such a σ_θ^{\min} exists as we assume $s_\theta \leq s^{\max}(\infty)$ (see Lemma 1), and since profitability is strictly monotonously decreasing in the divestiture's size (which follows from (29) when the divestiture is sold at a fixed price).

³⁴In addition, all competitors know the true synergy level after the merger is successfully completed.

³⁵That assumption simplifies the analyzes of the separating equilibrium below. Precisely, the high-cost type will not mimic the low-cost type type in case of indifference which allows us to easily calculate the size of the divestiture in the separating equilibrium.

³⁶The AA is assumed to be risk-neutral.

strictly increasing in the synergy level, s_θ . It then follows that the low-cost type can profitably merge with higher divestitures than the high-cost type; i.e., $\sigma_l^{min} < \sigma_h^{min}$ holds always. Hence, any merger proposal of a type s_θ will never incorporate a share of capital lower than σ_θ^{min} .

As the AA can, therefore, infer the merger proposal's type if the profitability condition holds only for the low-cost type, we can specify the following belief system of the AA. If the proposal σ is larger than σ_h^{min} , then the antitrust authority has beliefs $b_l = \Pr(s_\theta = s_l | \sigma > \sigma_h^{min}) = \rho$ and $b_h = \Pr(s_\theta = s_h | \sigma > \sigma_h^{min}) = 1 - \rho$, whereas beliefs are updated with $b_l = \Pr(s_\theta = s_l | \sigma \leq \sigma_h^{min}) = 1$ and $b_h = \Pr(s_\theta = s_h | \sigma \leq \sigma_h^{min}) = 0$, if the proposal σ falls short of σ_h^{min} .

Since the proposed divestiture may serve as a signalling device, we obtain pooling and separating equilibria in the incomplete information scenario.³⁷

Proposition 5. *There exists a unique solution σ^{po} so that (38) holds with equality. All equilibria which involve successful merger proposals are the following.*

i) If $\sigma^{po} > \sigma_h^{min}$, then σ^{po} constitutes the unique equilibrium; that is, there is only “pooling” and any merger type proposes a merger with divestiture $1 - \sigma^{po}$.

ii) If $\sigma^{po} \leq \sigma_h^{min}$ and $\sigma^{DE}(s_l) > \bar{\sigma}^{DE}$, only a separating equilibrium exists in which the efficient type proposes a successful merger; the divestiture then is of size $1 - \min\{\sigma^{DE}(s_l), \sigma_h^{min}\}$.

Proposition 5 states that a successful is either the result of a pooling equilibrium or of a separating equilibrium depending on whether or not the solution to condition (38) (holding with equality) leaves the high-cost type's merger profitable or not. If that profitability condition is fulfilled, then only a pooling equilibrium exists. The low-cost type could only induce separation by proposing a higher divestiture level than under the pooling equilibrium which can never be profitable as the merger will also be cleared with the lower pooling divestiture. Proposing a lower divestiture is also not a feasible strategy as this would

Inspection of (38) yields that the divestiture $1 - \sigma^{po}$ in a pooling equilibrium increases in s_h and p_h , while it decreases in s_l and p_l . It is then immediate that a pooling equilibrium is more likely if the synergy level of the high-cost type is not too low. Conversely, if the high-cost type's synergy level becomes so low that a merger is no longer profitable at σ^{po} , then a merger can only be the outcome of a separating equilibrium. In the latter instance, the low-cost type

³⁷The proof of the following proposition is provided in the Appendix.

proposes a divestiture $1 - \min\{\sigma^{DE}(s_l), \sigma_h^{min}\}$ which ensures that the high-cost type has no (strict) incentive to mimic the efficient type's proposal. We then obtain two possible scenarios in a separating equilibrium. Either the high-cost firm is sufficiently inefficient, so that it suffices to propose the full information price-fixing divestiture, $1 - \sigma^{DE}(s_l)$, or the high cost firm is efficient enough so that a merger proposal with divestitures $1 - \sigma^{DE}(s_l)$ remains profitable. In the latter instance, the low-cost firm has to propose a the "separating" divestiture level $1 - \sigma_h^{min}$ which deters the high-cost type from proposing a merger.

It follows from Proposition 5 a merger with $s_h \geq \bar{s}^{DE}$ can be an equilibrium outcome under pooling. Such a merger is never profitable under complete information but can become profitable if the AA is unsure about the merger type. In a pooling equilibrium the price-fixing divestiture is smaller than the complete information divestiture of the high-cost type and larger than the complete information divestiture of the low-cost type; i.e., $1 - \sigma^{DE}(s_h) > 1 - \sigma^{po} > 1 - \sigma^{DE}(s_l)$ holds. Hence, there may be instances where the high-cost type merger becomes profitable under incomplete information as the requested divestiture level is reduced.

It then also follows that the low-cost type must propose a larger divestiture in any pooling equilibrium when compared with the divestiture level under complete information. As the AA requires a too large divestiture from a low-cost type, it "overshoots" in those instances in terms of consumer protection. After a low-cost type merger has been cleared in a pooling equilibrium, the price is strictly smaller than before the merger.

The overshooting phenomenon can also occur in a separating equilibrium, whenever the low-cost type proposes a divestiture of $1 - \sigma_h^{min}$ that is larger than its full information divestiture level $1 - \sigma^{DE}(s_l)$. The following example, describes such a situation. Let assume a market with four firms and merger types $s_l = 1/2$ and $s_h = 7/10$, with a priori probabilities $\rho = 1/10$ and $1 - \rho = 9/10$. We then obtain profitability thresholds $\sigma_h^{min} \approx 0.80$ and $\sigma_l^{min} \approx 0.57$ for the high-cost type and the low-cost type, respectively. In addition, in a pooling equilibrium condition (38) must hold with equality which yields $\sigma^{po} \approx 0.78$. The low-cost firm will propose no divestiture under complete information; i.e., $\sigma^{DE}(s_l) = 1$. According to Proposition 5 there exists a separating equilibrium, in which the efficient type divests $1 - \min\{\sigma^{DE}(s_l), \sigma_h^{min}\} = 1 - \sigma_h^{min} = 1/5$, which means the AA overshoots in terms of consumer protection.

6 Conclusion

We analyzed the effects of remedies on merger activity in a standard Cournot oligopoly with homogeneous products under a consumer welfare standard. In general, remedies increase the scope for profitable mergers that do not harm consumers. Remedial offers must be larger when the merger's synergy level is smaller, which mirrors the proportionality principle in remedy regulations. Moreover, divestitures to an entrant firm are more effectively countering anticompetitive effects than divestitures to an existing competitor. The ability of the merging firms to extract the gains from trade of the asset sale is critical when the purchaser is endogenously determined. That ability is maximal when the merging parties can make a take-it or leave-it offer to a pre-selected buyer, in which case the socially preferred divestiture is chosen.

We have also shown that the merging firms have strong incentives to search for the most efficient buyer as this tends to increase the feasible set of mergers and, at the same time, keeps the asset sales necessary to induce an approval at its lowest possible value. We also identify instances which lead to mergers under a remedy rule which are strictly price-decreasing. *First*, a divestiture to an efficient competitor firm which is able to realize synergies may lead to lower prices, and *second*, sequential mergers may induce a series of (price-fixing) divestitures which create two new entrant firms. The entrant firms can then realize synergies by merging their businesses, which results in a strictly lower price when compared with the price which would prevail in the absence of remedies.

If the AA is uncertain about the merger's synergies, then the remedy proposal may be used as a signalling device.³⁸ If, however, the pooling equilibrium involves a divestiture which leaves the inefficient type merger profitable, then only a pooling equilibrium exists. We have shown that the AA always overshoots in terms of consumer protection in a pooling equilibrium since the low-cost type must divest more than under complete information. In a separating equilibrium overshooting can also occur in which case the low-cost type proposes a divestiture large than the full information divestiture to deter the high-cost firm from proposing a merger at all.

³⁸This idea is related to Lagerlöf and Heidhues (2005). They argue that the efficiency defense in merger control may induce the merging parties to undertake investments into the production of hard evidence to signal their synergy type.

Finally, introducing incomplete information also evokes new questions concerning the optimality of extreme options (Szalay, 2005); that is, the AA may abstain from producing any information on its own when remedies are possible, while it may have stronger incentives to do so when it must either clear or block the merger altogether. Moreover, in an incomplete information setting, the assessment of remedies may also depend on the broader institutional environment, which may vary between inquisitorial or adversarial (Dewatripont and Tirole, 1999).

Appendix

In this Appendix we provide the proof of Propositions 2 and 5.

Proposition 2. Suppose that a remedy is used under regimes DE and DC . Then, the comparison of regimes DE and DC with regime NR follows from a revealed preference argument. If a remedy is used, then it is always the smallest possible one which leaves the price level unaffected. Hence, the merger is always externality-free: both consumer surplus and the outsider firms' profits do not change in those instances. This follows directly from inspecting the outsider firms' first-order conditions (8) and (19) under regimes DE and DC , respectively. As the merging firms find it profitable to proceed with the merger and the beneficiaries of the remedy can increase their profits it must follow that social welfare increases under regimes DE and DC when compared with regime NR .

The comparison of social welfare under regimes DE and DC for synergy parameters $s \in (s^*, \bar{s}^{DC}]$ depends on a comparison of total production costs of the involved firms. Total production costs of firms M , E , and 3 under regime DE are

$$\Sigma^{DE} := C_M(x_M^{DE}, \sigma^{DE}) + C_E(x_E^{DE}, \sigma^{DE}) + C(x_3^{DE}) = \frac{s(x_M^{DE})^2}{1 + \sigma^{DE}} + \frac{(x_E^{DE})^2}{1 - \sigma^{DE}} + (x_3^{DE})^2,$$

while total production costs of firms M and 3 under regime DC are

$$\Sigma^{DC} := C_M(x_M^{DC}, \sigma^{DC}) + C(x_3, \sigma^{DC}) = \frac{s(x_M^{DC})^2}{1 + \sigma^{DC}} + \frac{(x_3^{DC})^2}{2 - \sigma^{DC}}.$$

We then get that the sign of $(W^{DC} - W^{DE})$ is equal to the sign of $(\Sigma^{DE} - \Sigma^{DC})$. Using the equilibrium outputs (9)-(11) and the divestiture level (15) under regime DE , and the equilibrium

outputs (20)-(22) and the divestiture level (26) under regime DC , we obtain

$$\begin{aligned} & 98s^2\Psi_4\Psi_5 - 25s\Psi_4\Psi_5 - 90s^3\Psi_4\Psi_5 + 61s^4\Psi_4\Psi_5 - 29s^5\Psi_4\Psi_5 \\ & - 3253s^4\Psi_5 + 556s^3\Psi_5 + 3267s^2\Psi_5 - 15\Psi_4\Psi_5 \\ & + 527s^5\Psi_4 + 29s^6\Psi_5 - 438s^5\Psi_5 - 8s^6\Psi_4 - 35s\Psi_4 - 86s\Psi_5 \\ & + 732s^3\Psi_4 - 1816s^2\Psi_4 + 645s^4\Psi_4 - 45\Psi_4 - 75\Psi_5 - \Psi_6 \\ \Sigma^{DE} - \Sigma^{DC} = & \frac{\hspace{10em}}{(-15 - ns^2 - 3s^2 - 5n - 18sn - 3\Psi_4 - 54s + ns\Psi_4 - n\Psi_4 + 3s\Psi_4)^2} \quad (39) \\ & \times (-3 - \Psi_5 - 11s + s\Psi_5)^2 / 4 \end{aligned}$$

with $\Psi_4 := \sqrt{(1-s)(25-s)}$, $\Psi_5 := \sqrt{9-16s}$, and $\Psi_6 := 225 - 11485s^4 - 4932s^3 + 2562s^5 + 853s^2 + 58s - 8s^7 + 631s^6$. Since the numerator of (39) is independent of n and since the denominator is strictly positive for all $n \geq 3$ and all $s \in [s^*, \bar{s}^{DE}]$, the zeros of (39) are independent of n and we obtain that (39) is strictly positive between the zeros $s = 1/2$ and $\hat{s} \approx 0.562425$ and strictly negative for $s \in (\hat{s}, \bar{s}^{DC})$. Note also that $\hat{s} < \bar{s}^{DC}$. We conclude that $\Sigma^{DE} > \Sigma^{DC}$, and hence, $W^{DC} > W^{DE}$ holds for (s^*, \hat{s}) , whereas $\Sigma^{DC} > \Sigma^{DE}$, and hence, $W^{DE} > W^{DC}$ holds for $(\hat{s}, \bar{s}^{DC}]$ (with equality holding at $s = \hat{s}$).

Proposition 5. First, we show that there exists always a unique solution to (37) with equality. Given synergy level $s_\theta \in \{s_l, s_h\}$ and a divestiture to an entrant firm, $1 - \sigma \in [0, 1]$, the post-merger price is given by

$$p^{pm}(s, \sigma) = \frac{3(-3 - 2\sigma - 6s + 2\sigma s + \sigma^2)}{-3n - 15 - 2n\sigma - 8\sigma - 6sn - 12s + n\sigma^2 + 7\sigma^2 + 2\sigma sn + 8\sigma s}.$$

We assume that $\sigma \in I_2 := [1/2, 1]$.³⁹ The expected post-merger price is strictly monotonously increasing in $s \in I_1$ and strictly monotonously increasing in $\sigma \in I_2$. We obtain $p^{pm}(s, \sigma = 1/2) = (60s + 45)/(15n + 32s + 20ns + 69)$ and $p^{pm}(s, \sigma = 1) = (3s + 3)/(n + s + ns + 4)$. For all n and all $s \in I_1$, the difference $p^{pm}(s, \sigma = 1/2) - p^*$ is negative and the difference $p^{pm}(s, \sigma = 1) - p^*$

³⁹We know from Lemma 3 that under a price fixing regime all equilibrium shares σ which stay under the control of the merged firm are given by $[0.678, 1]$ for $s \in I_1$, so that I_2 covers all equilibrium shares σ . Clearly, given the AA's decision rule, the equilibrium divestiture according to (38) will be lower than the price-fixing divestiture of the inefficient type s_h and higher than the price-fixing divestiture of the efficient type s_l as given by Lemma 3 and thus we may restrict our analysis to $\sigma \in I_2$.

is positive. Thus, for all beliefs $b_l \in [0, 1]$ and $b_h = 1 - b_l$, we obtain

$$b_h p^{pm}(s_h, \sigma = 1/2) + b_l p^{pm}(s_l, \sigma = 1/2) \leq p^{pm}(s_h, \sigma = 1/2) < p^*$$

and

$$p^* < p^{pm}(s_l, \sigma = 1) \leq b_h p^{pm}(s_h, \sigma = 1) + b_l p^{pm}(s_l, \sigma = 1).$$

Since the range of values of $p^{pm}(\sigma)$ on $\sigma \in I_2$ covers p^* and since $p^{pm}(\sigma)$ is continuous and strictly monotonously increasing, the intermediate value theorem implies a unique solution $\sigma^{po} \in [1/2, 1]$ at which condition (38) holds with equality for all admissible values of b_l , b_h , s_l , and s_h .

Second, we investigate in which case a pooling equilibrium exists. Assuming strict profitability holds for both merger types in σ^{po} ; i.e., $\sigma^{po} > \max\{\sigma_l^{min}, \sigma_h^{min}\} = \sigma_h^{min}$, proposing a merger with divestiture $1 - \sigma^{po}$ to an entrant constitutes a pooling equilibrium. Proposing a lower divestiture is no equilibrium action since all merger-types prefer divesting a lower share, so that the antitrust authority cannot update beliefs and thus rejects the merger proposal according to (37). Proposing a higher divestiture is no equilibrium action as well since the merger's profitability decreases in the divestiture's size, so that each firm has an incentive to deviate to a lower divestiture and divest only the minimal required share. If, however, $\sigma^{po} \leq \sigma_\theta$ for both $\theta \in \{l, h\}$, in equilibrium neither the efficient nor the inefficient type will propose a merger incorporating σ^{po} due to profitability concerns. Whereas in this case the inefficient merger would not have been proposed under full information, too, the more efficient merger may have been profitable under full information, but lost profitability due to the DE-regime with incomplete information. Finally, if $\sigma_l^{min} \leq \sigma^{po} < \sigma_h^{min}$, then only the efficient type proposes an admissible merger and thus we cannot obtain a pooling equilibrium.

Third, we investigate the existence of separating equilibria, in which an inefficient merger proposes to keep share σ_h of the second merger's capital and in which the efficient type proposes to keep share σ_l . Remember that in case a divestiture-proposal of an inefficient merger is approved, the same proposal by an efficient merger would be approved anyway since the required divestiture is higher if the type is less efficient. If σ_l is a profitable proposal only for the efficient type, i.e. $\sigma_l \leq \sigma_h^{min}$, then the antitrust authority can precisely infer the merger's type and update beliefs to $b_l = \Pr(s_\theta = s_l | \sigma_l \leq \sigma_h^{min}) = 1$ and $b_h = 0$, so that (37) reduces to the DE-regime's decision rule (see section 3).

We assume that (σ_l, σ_h) with $\sigma_l \neq \sigma_h$ constitutes a separating equilibrium in which both types propose a strictly profitable merger. If in a separating equilibrium (σ_l, σ_h) the efficient type's proposal is denied, then the inefficient type's proposal is denied, too. Else, since proposing σ_h fulfills the profitability condition for the inefficient and thus also for the efficient type, the latter has an incentive to deviate to σ_h . It cannot occur in a separating equilibrium that both proposals σ_l and σ_h are approved, since each merger would profitably deviate by proposing $\min\{\sigma_l, \sigma_h\}$, which may give a pooling, but no separating equilibrium. Thus, in a separating equilibrium necessarily either both type's proposals are denied or only the efficient type's proposal is approved. In the latter case, since the inefficient type must not have an incentive to deviate, we require that the efficient type's proposal σ_l fulfills $\sigma_l \leq \sigma_h^{min}$. We define $\sigma^{DE}(s_l) = 0$ for $s > \bar{s}^{DE}(s_l)$, indicating that for weak synergies no profitable, price-fixing merger with divestitures to an entrant is possible. Now, in case $\sigma^{DE}(s_l) > 0$, in a separating equilibrium the efficient type necessarily proposes $\min\{\sigma^{DE}(s_l), \sigma_h^{min}\}$, which gives the maximum share so that the proposal is approved and so that strict profitability holds only for the efficient type. The inefficient type proposes some strictly profitable, but denied σ_h or no merger at all. This gives a separating equilibrium in case no pooling equilibrium exists. If, however, a pooling equilibrium in which both types propose σ^{po} exists, due to $\sigma^{po} > \sigma_h^{min} \geq \min\{\sigma^{DE}(s_l), \sigma_h^{min}\}$ no separating equilibrium exists.⁴⁰ If $\sigma^{DE}(s_l) = 0$, in equilibrium both types propose a strictly profitable, but denied merger, or both propose no merger at all.

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⁴⁰ Assuming additional costs going along with a pooling equilibrium like delay or costs of governmental inspections equalizes just an upward shift of σ^{po} and is thus not worthwhile to be considered.

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