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Why Voting? A Welfare Analysis

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Why Voting? A Welfare Analysis

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Abstract

Voting is commonly applied in collective decision making, but at the same time it is criticized for being inefficient. We address this apparent conflict and consider committees deciding collectively between accepting a given proposal and maintaining the status quo. Committee members are privately informed about their valuations and monetary transfers are possible. We solve for the social choice function maximizing utilitarian welfare, which takes monetary transfers to an external agency explicitly into account. For regular distributions of preferences, we find that it is optimal to exclude monetary transfers and to decide by qualified majority voting.

Keywords: Voting; Collective decision making; Mechanism design; Private information

JEL: D71, D82

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Why is voting predominant in collective decision making? A common view is that often it is immoral to use money. This view is plausible, for example, when deciding who should receive a donated organ or whether a defendant should be convicted. However, it explains less convincingly why shareholders vote on new directors at the annual meeting, why managing boards of many companies make important operative decisions by voting, or why hiring committees vote when deciding on a new appointment. Indeed, voting is criticized for its inefficiency, and the economic literature argues that collective decisions can be improved if transfers are used to elicit preference intensities. But redistributing these transfers within the group introduces incentive problems, while wasting them reduces welfare. We model these considerations explicitly, and show that voting maximizes welfare.

Our analysis closely follows standard models of collective decision making: A finite population of voters decides collectively whether to accept a given proposal or to maintain the status quo. Agents are privately informed about their valuations and have quasi-linear utilities. Monetary transfers are feasible as long as they create no budget deficit and agents are willing to participate in the decision process. In contrast to much of the literature, we consider a utilitarian welfare function that takes monetary transfers to an external agency into account. We then investigate which strategy-proof social choice function maximizes this aggregate expected utility.

Our main result is that the optimal anonymous social choice function is implementable by qualified majority voting. Under such schemes, agents simply indicate whether they are in favor or against the proposal, and the proposal is accepted if the number of agents being in favor is above a predetermined threshold. This implies that, even though it is possible to use monetary transfers, it is optimal *not* to use them. Specifically, we show that any anonymous decision rule that relies on monetary transfers wastes money to such an extent that it is inferior to voting. It follows that it is *not* possible to improve upon voting without giving up reasonable properties of the social choice function. Our result thereby justifies the widespread use of voting rules in practice, and provides a link between mechanism design theory and the literature on

political economy.

Our finding that voting performs well from a welfare perspective stands in sharp contrast to the previous literature, which suggests to implement the value-maximizing public decision. However, this does not achieve the first-best because it induces budget imbalances (see, e. g., Green and Laffont 1979). While it is traditionally assumed that money wasting has no welfare effects, we consider a social planner that cares about aggregate transfers. This approach seems reasonable for at least two reasons: First, a social planner might be interested in implementing the decision rule that maximizes the agents' expected utility, which in turn depends on the payments they have to make. Second, groups often choose the rule by which they decide themselves, and when making this choice they take the payments they have to make into account. Hence, our approach provides an explanation for which decision rules are likely to prevail in practice.

Our result, that transfer-free voting schemes dominate more complex decision rules, follows from two basic observations. In a first step, we analyze the transfers that are necessary to implement a given decision rule. Incentive compatibility fixes the payment function up to a term that only depends on the reports of all other agents. We show that the requirements of (a) no money being injected and (b) all agents being willing to participate in the decision procedure, entirely fix the payment functions for any anonymous decision rule. In particular, it turns out that if money is necessary to induce truthful reporting then it has to be wasted. As an application, this implies that any anonymous social choice function is implementable with a balanced budget if and only if it can be implemented by qualified majority voting. In a second step, we then analyze the trade-off between increasing efficiency of the public decision and reducing the waste of monetary resources. For regular distribution functions, we show that this trade-off is solved optimally by not using money at all. This implies that the optimal social choice function is implementable by qualified majority voting. We also characterize the minimum number of votes that is optimally required for the adoption of the proposal.

Related Literature

Formal analyses of the question “should we use monetary transfers or not?” are rare; to the best of our knowledge, the only attempts are arguments that voting mechanisms are easy and perform well for large populations (Ledyard and Palfrey 2002), and that voting rules are coalition-proof (Bierbrauer and Hellwig 2012). We complement these papers by arguing that voting is optimal from a utilitarian perspective.

The fact that the optimal decision scheme does not use transfers relates our work to the analysis of optimal collective decision rules when monetary transfers are not feasible. This literature was initiated by Rae (1969), who compares utilitarian welfare of different voting rules and shows that simple majority voting (where a proposal is accepted if at least half of the population votes for it) is optimal if preferences are symmetric across outcomes. Recently, this approach was generalized to include more general decision rules (Azrieli and Kim 2012), to allow for correlated valuations (Schmitz and Tröger 2012) and to consider more than two alternatives (Gershkov, Moldovanu and Shi 2013).

Barbera and Jackson (2004) study a model where agents not only vote on a given proposal, but in a first stage decide on which voting rule to use in the second stage. They argue that only “self-stable” rules, i. e., voting rules that would not be changed once in place, are likely to prevail. If agents are ex-ante symmetric, only voting rules that maximize utilitarian welfare satisfy this condition. We contribute to this branch of the literature by showing that, in our setting, the exclusion of money is not costly.

Our insight that monetary transfers are not necessarily welfare-increasing relates our work to studies that exclude monetary transfers but allow for costly signaling. These studies assume that signaling efforts are wasteful and cannot be redistributed. It is shown that the welfare-maximizing allocation of private goods relies only on prior information and completely precludes wasteful signaling (Hartline and Roughgarden (2008), Yoon (2011), Condorelli (2012), Chakravarty and Kaplan (2013); see McAfee and McMillan (1992) for a res-

ult in a similar vein). In contrast, we allow for monetary transfers from and between agents and show that in a public good setting similar economic trade-offs arise.

An extensive literature in mechanism design studies allocation problems when monetary transfers are feasible. While VCG mechanisms implement the value-maximizing public decision (Groves 1973), this comes at the cost of budget imbalances that cannot be redistributed without distorting incentives (Green and Laffont 1979, Walker 1980).¹ Therefore, these mechanisms achieve the first-best only under the assumption that the social planner does not care about monetary resources. An opposite approach, where the budget is required to be exactly balanced, is pursued in Laffont and Maskin (1982).

The budget imbalances of VCG mechanisms might be less severe if they were quantitatively negligible in practical applications. This argument has been put forward by Tideman and Tullock (1976), who conjecture that wasted transfers are not important for large populations² and VCG mechanisms therefore approximate the first-best. In Section III we discuss how our result relates to this observation.

A small literature, which also considers money burning to be welfare-reducing, studies the allocation of a private good. Miller (2012) shows that the optimal mechanism never allocates efficiently and in some cases wastes monetary resources. If there are only two agents and the distribution functions are regular then the optimal mechanism transfers money and has a balanced budget (Drexler and Kleiner 2012, Shao and Zhou 2012). In contrast, the optimal social choice function in the present paper does not use money.

Finding the optimal social choice function involves understanding which part of the payments can be redistributed without distorting incentives (see also the work of Cavallo 2006). Our focus on anonymous social choice functions for a public good setting allows us to solve this problem.

¹For an approach using a weaker equilibrium concept see d'Aspremont and Gerard-Varet (1979). Note that the equivalence between dominant strategy and Bayes-Nash incentive compatible mechanisms established by Gershkov, Goeree, Kushnir, Moldovanu and Shi (2013) does not hold in this model as the budget is constrained ex-post.

²This claim was formally verified by Green and Laffont (1977).

The paper is structured as follows: We present the model in Section I, derive our main result in Section II and provide a short discussion of the result in Section III.

I Model

We consider a population of N agents³ deciding collectively on a binary outcome $X \in \{0, 1\}$. We interpret this as agents deciding whether they accept a proposal (in which case $X = 1$) or reject it and maintain the status quo ($X = 0$). Given a collective decision X , the utility of agent i is given by $\theta_i \cdot X + t_i$, where θ_i is the agent's valuation for the proposal and t_i is a transfer to agent i .⁴ Each agent is privately informed about his valuation, which is drawn independently from a type space $\Theta := [\underline{\theta}, \bar{\theta}]$ according to a distribution function F with positive density f . To make the problem interesting we assume that $\underline{\theta} < 0 < \bar{\theta}$.⁵ Both type space and distribution function are common knowledge. Let Θ^N denote the product type space consisting of complete type profiles with typical element $\theta = (\theta_i, \theta_{-i})$.

A social choice function in this setting determines for which preference profiles the proposal is accepted and which transfers are made to the agents. Formally, a *social choice function* is a pair $G = (X^G, T^G)$ consisting of a *decision rule*

$$X^G : \Theta^N \rightarrow \{0, 1\}$$

and a *transfer rule*

$$T^G : \Theta^N \rightarrow \mathbb{R}^N$$

such that, for any realized preference profile θ , $X^G(\theta)$ is the decision on the

³For convenience, we also write N for the set of agents $\{1, \dots, N\}$.

⁴Our analysis applies to costless projects as well as to costly projects with a given payment plan, in which case the valuation of agent i is interpreted as her net valuation taking her contribution into account. Also note that the analysis accommodates more general utility functions: Take any quasi-linear utility function such that the utility difference between $X = 1$ and $X = 0$ is continuous and strictly increasing in θ_i . Redefining the type to equal the utility difference, we can proceed with our analysis without change.

⁵The analysis directly extends to cases where $\underline{\theta} = -\infty$ and/or $\bar{\theta} = \infty$.

public outcome and $T_i^G(\theta)$ is the transfer received by agent i . A social choice function is *feasible* if, for any realization of preferences, no injection of money from an external agency is necessary, i. e., if

$$\sum_{i \in N} T_i^G(\theta) \leq 0. \quad (\text{F})$$

In many situations agents have the outside option to abstain from the decision process and leave the decision to the other agents. It is then without loss of generality to consider social choice functions that ensure participation in the following sense: If agent i leaves the decision process, the social choice function chooses some alternative $\underline{X}_i(\theta_{-i})$. Then the social choice function satisfies *universal participation* (see, e. g., Green and Laffont 1979) if, given this outside option, all agents prefer to participate in the decision process.⁶

$$\theta_i X^G(\theta) + T_i^G(\theta) \geq \theta_i \underline{X}_i^G(\theta_{-i}). \quad (\text{UP})$$

This constraint is weaker than the requirement that every agent derive utility of at least zero (often called individual rationality). For instance, majority voting satisfies universal participation but in general it is not individually rational.

Definition 1. *We call a decision rule X^G anonymous if it is independent of the agents' identities, i. e. if, for each permutation $\pi : N \rightarrow N$ and corresponding function $\hat{\pi}(\theta) = (\theta_{\pi(1)}, \dots, \theta_{\pi(N)})$, it holds that $X^G(\theta) = X^G(\hat{\pi}(\theta))$ for all θ .*

A social choice function is anonymous if the associated decision rule is anonymous.

This is a weak notion of anonymity, requiring only that the names of the agents do not affect the public decision. However, focusing on anonymous social choice functions is a potentially severe restriction.⁷ Nonetheless, it is

⁶We note that our analysis does not depend on any particular form of the function \underline{X}_i . This outside option could also depend on the privately observed valuation of agent i without any change in the analysis.

⁷For example, it excludes the use of "sampling Groves mechanisms" (Green and Laffont

often reasonable to impose anonymity as many fairness concepts build on this assumption (e. g., equal treatment of equals). This requirement also has a long tradition in social choice theory, see for example, Moulin (1983).⁸

We are interested in social choice functions that are *strategy-proof*, i. e., for which there exists a mechanism and an equilibrium in dominant strategies for the strategic game induced by this mechanism such that, for any realized type profile, the equilibrium outcome corresponds to the outcome that the social choice function stipulates. Requiring social choice functions to be strategy-proof is a standard approach in social choice theory (see, e. g., Moulin 1983).⁹

Throughout the paper we focus on anonymous and feasible social choice functions that are strategy-proof and satisfy universal participation. Which social choice function should a utilitarian planner choose? Given that the value-maximizing decision cannot be implemented with a balanced budget, a utilitarian planner should implement the second-best, i. e., maximize utilitarian welfare given by

$$U(X^G, T^G) := \mathbb{E}_\theta \left[\sum_{i=1}^N [\theta_i X^G(\theta) + T_i^G(\theta)] \right],$$

where the expectation is taken with respect to the prior distribution of θ . The assumption that the planner perfectly knows the prior distribution of types, although being very common in the literature on mechanism design, might be too strong in some settings. Note however, that the optimal social choice function derived in Theorem 1 does not depend on the exact distribution of types. Moreover, as we focus on robust implementation, misspecifications do

1979), where a VCG mechanism is used for a subset of the population and the budget surplus is redistributed to non-sampled agents.

⁸Note that this assumption would be without loss of generality if we allowed for stochastic decision rules. Given any social choice function (X^G, T^G) , apply this function after randomly permuting the agents. This defines a new social choice function $(\tilde{X}^G, \tilde{T}^G)$ that is anonymous and achieves the same utilitarian welfare. While this new rule treats all agents equally ex-ante, it is possible that agents with the same valuations are treated very differently after the uncertainty about the randomization is resolved.

⁹Bierbrauer and Hellwig (2012) show for the model we consider that strategy-proofness is equivalent to robust implementation in the spirit of Bergemann and Morris (2005).

not affect incentives and hence the performance of the optimal social choice function is not very sensitive to slight misestimations of the distribution of types.

II Results

To implement a given social choice function, we invoke the revelation principle (Gibbard 1973). It follows that we can focus without loss of generality on direct revelation mechanisms in which it is a dominant strategy for agents to report their valuations truthfully. Hence, a mechanism is given by a tuple (x, t) , where $x : \Theta^N \rightarrow \{0, 1\}$ maps reported types into a collective decision and, for each agent i , $t_i : \Theta^N \rightarrow \mathbb{R}$ maps reported types into the payment received by that agent. The requirement that a social choice function be strategy-proof translates to

$$\theta_i x(\theta_i, \theta_{-i}) + t_i(\theta_i, \theta_{-i}) \geq \theta_i x(\hat{\theta}_i, \theta_{-i}) + t_i(\hat{\theta}_i, \theta_{-i}) \quad \text{for all } \theta_{-i}, \theta_i, \hat{\theta}_i. \quad (\text{IC})$$

A mechanism is *qualified majority voting (with threshold k)*, if $x(\theta) = 1$ if and only if $|\{i : \theta_i \geq 0\}| \geq k$ and if in no case monetary transfers are made, i. e., $t_i(\theta) = 0$ for all i and θ .

Definition 2. *A distribution function F has monotone hazard rates if the hazard rate $\frac{f(\theta_i)}{1-F(\theta_i)}$ is non-decreasing in θ_i for $\theta_i \geq 0$ and the reversed hazard rate $\frac{f(\theta_i)}{F(\theta_i)}$ is non-increasing in θ_i for $\theta_i \leq 0$.*

This assumption is well-known from the literature on optimal auctions and procurement auction design; it is satisfied by many commonly employed distribution functions, for example by the uniform, (truncated) normal, and exponential distributions.

We are now ready to state our main result.

Theorem 1. *Suppose F has monotone hazard rates. Then the optimal social choice function is implementable by qualified majority voting with threshold*

$\lceil k \rceil$, where

$$k := \frac{-N \mathbb{E}[\theta_i | \theta_i \leq 0]}{\mathbb{E}[\theta_i | \theta_i \geq 0] - \mathbb{E}[\theta_i | \theta_i \leq 0]}.$$

That is, the optimal decision rule does not rely on monetary transfers at all and can be implemented using a simple indirect mechanism where each agent indicates whether she is in favor of or against the proposal. It is accepted if more than $\lceil k \rceil$ voters are in favor.¹⁰ The following example illustrates how voting mechanisms compare to the first-best and the best VCG mechanism.

Example 1. Let $N = 2$ and θ_i be independently and uniformly distributed on $[-3, 3]$ for $i = 1, 2$. If valuations were publicly observable the first-best could be implemented, which would yield welfare $U_{FB} = \frac{1}{2} \mathbb{E}[\theta_1 + \theta_2 | \theta_1 + \theta_2 \geq 0] = 1$. The best VCG mechanism is the pivotal mechanism, which gives welfare $U_{VCG} = \frac{1}{2}$ (see the Appendix). In contrast, unanimity voting, that is, accepting the proposal if and only if both agents have a positive valuation, yields welfare $U_{UV} = \frac{1}{4} \mathbb{E}[\theta_1 + \theta_2 | \theta_1 \geq 0, \theta_2 \geq 0] = \frac{3}{4}$. Hence, the welfare loss due to private information is twice as large under the best VCG mechanism as compared to unanimity voting.

The broader implications of Theorem 1 are discussed in Section III and a formal proof is provided in the Appendix. In the following, we build some intuition for this result.

As a first step, Lemma 1 characterizes direct mechanisms that are strategy-proof. It shows that the transfer of every type is determined by the decision rule up to a term that only depends on the reports of the other agents. Since this term changes the transfers of an agent without affecting his incentives, we call it “redistribution payment.”

As a second step, we show that, for any anonymous social choice function, positive redistribution payments are not feasible and therefore all collected payments have to be wasted (Lemma 2). In general, it is easy to build strategy-proof and budget-balanced social choice functions by ignoring one agent in the public decision and awarding him all payments by the other agents. Anonymity

¹⁰See also Nehring (2004), Barbera and Jackson (2006).

not only rules out this possibility, but one can prove that any mechanism which has positive redistribution payments is necessarily asymmetric.

Given that money cannot be redistributed in anonymous social choice functions, there is a direct trade-off between improving the decision rule and reducing the outflow of money. We show, as a third step, that this conflict is resolved optimally in favor of no money burning. To gain some intuition, fix a type profile of the other agents, θ_{-i} . Strategy-proofness implies that there is a cutoff θ_i^* such that the proposal will be accepted if the type of agent i is above θ_i^* . To solve for the optimal decision rule we need to find the optimal cutoff. Assume that the sum of valuations $\sum_{j \neq i} \theta_j + \theta_i^*$ is negative. Marginally increasing the cutoff leads to a rejection of the proposal which in this case increases efficiency (with a positive effect on welfare proportional to $f(\theta_i^*)$). On the other hand, strategy-proofness implies that agents with a type above the cutoff make a payment equal to the cutoff. Increasing the cutoff increases these payments (with a corresponding negative effect on welfare proportional to $1 - F(\theta_i^*)$). Monotone hazard rates imply that if the positive effect outweighs the negative effect at θ_i^* and therefore it is beneficial to marginally increase the cutoff, then it is optimal to set the cutoff to the highest possible value. Symmetric arguments imply that it is optimal to set all cutoffs either equal to zero or to the boundary of the type space, and hence that the optimal mechanism can be implemented by a voting rule.

Finally, the optimal number of votes required in favor of a proposal is given by the smallest integer number k such that the expected aggregate welfare of a proposal, given that k out of N voters have a positive valuation, is positive. Hence, the optimal threshold required for qualified majority voting depends on the conditional expected values given that the valuation is either positive or negative. Simple majority voting is optimal if valuations are distributed symmetrically around 0. If, however, opponents of a proposal are expected to have a stronger preference intensity, then it is optimal to require a qualified majority that is larger than simple majority.

As an easy consequence, Lemma 1 and 2 allow a characterization of the set of strategy-proof social choice functions that have a balanced budget.

Corollary 1. *A feasible and anonymous social choice function satisfying universal participation has a balanced budget if and only if it is implementable by qualified majority voting.*

In comparison to this corollary, Theorem 1 allowed for a larger class of social choice functions that potentially waste money. While we determine the optimal social choice function in this larger class in the theorem, this corollary characterizes any implementable social choice function in the smaller class of budget-balanced social choice functions. A closely related result has been obtained by Laffont and Maskin (1982), who in addition require weak Pareto efficiency but do not impose participation constraints.

III Discussion

This paper shows that utilitarian welfare, which takes transfers into account, is maximized by using qualified majority voting. Our result resolves the apparent conflict between the widespread use of such mechanisms in practice and the intuition that accounting for preference intensities can improve collective decisions. In particular, we show that the costs of accounting for preference intensities outweigh the benefits and the VCG mechanism is inferior to voting. In contrast, Tideman and Tullock (1976) argue that payments vanish as the number of agents gets large and hence the VCG mechanism should be used instead of voting. However, while it is generically true that the VCG mechanism approximates the first-best if the population is large enough, this is not sufficient for being superior to voting. In fact, voting also approximates the first-best. Moreover, for any fixed population, it turns out that voting provides a higher expected welfare. More generally, Theorem 1 indicates that being welfare-inferior to voting is not a problem of the VCG mechanism, but that it is in fact not possible to improve upon voting under the normative requirements of robust implementation and equal treatment of equals.

Classical social choice theory suggests that decisions should depend on the average willingness-to-pay in the population, i. e., a proposal should be accepted if the average willingness-to-pay is positive. In contrast, decision rules

considered in political economy and implemented in practice typically depend only on the number of agents with a positive willingness-to-pay. By taking an optimal mechanism design approach we are able to reconcile mechanism design theory with social choice practice and the literature on political economy.

An important question in this respect concerns the robustness of our results to alternative specifications of the decision problem. First, if one considers more general problems with more than two possible outcomes, the results will crucially depend on the restrictions imposed on preferences.¹¹ Second, it would be interesting to relax some of the restrictions we imposed on the social choice functions. While it appears that relaxing universal participation does not change the spirit of our results, our analysis depends crucially on the assumption of anonymity.

¹¹For example, for quadratic utilities and a continuum of alternatives, the efficient allocation rule can be implemented with a balanced budget (Groves and Loeb 1975).

Appendix

Proof of Example 1. Welfare of the pivot mechanism can be expressed as the difference between the welfare of the first-best and the transfers needed to implement the efficient decision:

$$U_{VCG} = U_{FB} - \frac{4}{36} \int_{-3}^0 \int_0^{-\theta_1} (-\theta_2) d\theta_2 d\theta_1 = \frac{1}{2}$$

Here, we used the fact that transfers are symmetric in the four regions $\{\theta \mid \theta_i \geq 0, \theta_j \leq 0, \theta_i + \theta_j \leq 0\}$ and zero everywhere else.

The following lemma is a standard characterization of strategy-proof mechanisms.

Lemma 1. *A mechanism is strategy-proof if and only if, for each agent i ,*

1. $x(\theta_i, \theta_{-i})$ is non-decreasing in θ_i for all θ_{-i} and
2. there exists a function $h_i(\theta_{-i})$, such that for all θ ,

$$\theta_i x(\theta_i, \theta_{-i}) + t_i(\theta_i, \theta_{-i}) = h_i(\theta_{-i}) + \int_0^{\theta_i} x(\beta, \theta_{-i}) d\beta. \quad (\text{A.1})$$

Equation (A.1) suggest the following definition:

Definition 3. *Agent i is pivotal at profile θ , if $\theta_i x(\theta) \neq \int_0^{\theta_i} x(\beta, \theta_{-i}) d\beta$.*

A necessary condition for agent i to be pivotal at θ is that $x(\theta) \neq x(0, \theta_{-i})$. If agent i is not pivotal at a given profile (θ_i, θ_{-i}) then her payment equals $h_i(\theta_{-i})$. If she is pivotal at this profile, her transfer is reduced by $\theta_i x(\theta) - \int_0^{\theta_i} x(\beta, \theta_{-i}) d\beta$.

Lemma 2. *Suppose a mechanism (x, t) is anonymous. Then $h_i(\theta_{-i}) = 0$ for all i and θ_{-i} .*

Proof. The proof consists of two steps.

Step 1: For all i and θ_{-i} , there exists θ_i such that no agent is pivotal at (θ_i, θ_{-i}) .

Note that all agents that are pivotal at profile θ submit reports of the same sign: If $x(\theta) = 1$ then monotonicity implies that $x(0, \theta_{-i}) = 1$ for all agents i with $\theta_i < 0$ and hence only agents with positive reports can be pivotal (and similarly for $x(\theta) = 0$).

Fix an arbitrary agent i and a report profile $\theta_{-i} \in \Theta^{N-1}$. Suppose without loss of generality that $x(0, \theta_{-i}) = 1$ and that all agents that are pivotal at $(0, \theta_{-i})$ submit positive reports (if no agent is pivotal at this profile, we are done; if $x(0, \theta_{-i}) = 0$ analogous arguments hold). We show that no agent is pivotal at profile $\theta := (\theta_{j^*}, \theta_{-i})$, where $j^* \in \arg \max_j \theta_j$. Monotonicity implies that $x(\theta) = x(0, \theta_{-i}) = 1$ and hence agent i is not pivotal. Anonymity implies that agent j^* is not pivotal. The claim is proved if we can show that if j is not pivotal at θ and $\theta_{j'} \leq \theta_j$, then j' is not pivotal at θ . Assume to the contrary that j' is pivotal at θ , i.e. $x(\theta) = 1$ and $x(0, \theta_{-j'}) = 0$. If $\hat{\pi}_{j,j'} : \Theta^N \rightarrow \Theta^N$ is the function permuting the j -th and j' -th component, then $\hat{\pi}_{j,j'}[(0, \theta_{-j})] \leq (0, \theta_{-j'})$. From monotonicity it follows that $x(\hat{\pi}_{j,j'}[(0, \theta_{-j})]) = 0$ and symmetry implies that $x(0, \theta_{-j}) = 0$, contradicting the assumption that j is not pivotal at θ .

Step 2: For all i and θ_{-i} we have $h_i(\theta_{-i}) = 0$.

Universal participation immediately implies that an agent with valuation 0 gets a weakly positive utility, i.e., $0 \cdot x(0, \theta_{-i}) + t_i(0, \theta_{-i}) \geq 0$. This implies $h_i(\theta_{-i}) \geq 0$ for all i, θ_{-i} . To obtain a contradiction, suppose that there exists an agent j and a report profile $\theta_{-j} \in \Theta_{-j}$ such that $h_j(\theta_{-j}) > 0$. By step one, we can choose θ_j such that no agent is pivotal at $\theta := (\theta_j, \theta_{-j})$, implying by (A.1) that $\sum_i t_i(\theta) = \sum_i h_i(\theta_{-i}) > 0$, contradicting (F).

The following lemma shows how utilitarian welfare of a social choice function can be expressed as the sum of two terms. The first only depends on the allocation rule, and the second consists of the redistribution payments.

Lemma 3. *Let (x, t) be an incentive compatible direct mechanism for social*

choice rule $G = (X^G, T^G)$ and define

$$\psi(\theta_i) = \begin{cases} \frac{-F(\theta_i)}{f(\theta_i)} & \text{if } \theta_i \leq 0, \\ \frac{1-F(\theta_i)}{f(\theta_i)} & \text{otherwise.} \end{cases} \quad (\text{A.2})$$

Then we have

$$U(X^G, T^G) = \int_{\Theta^N} \left[\sum_{i \in N} \psi(\theta_i) \right] x(\theta) dF^N(\theta) + \sum_{i \in N} \int_{\Theta^{N-1}} h_i(\theta_{-i}) dF^{N-1}(\theta_{-i}).$$

Proof. Note that for all θ_{-i} ,

$$\begin{aligned} & \int_{\underline{\theta}}^{\bar{\theta}} \left[\int_0^{\theta_i} x(\beta, \theta_{-i}) d\beta \right] f(\theta_i) d\theta_i \\ &= \left[\int_0^{\bar{\theta}} x(\beta, \theta_{-i}) d\beta \underbrace{F(\bar{\theta})}_{=1} - \int_0^{\underline{\theta}} x(\beta, \theta_{-i}) d\beta \underbrace{F(\underline{\theta})}_{=0} \right] - \int_{\underline{\theta}}^{\bar{\theta}} x(\theta_i, \theta_{-i}) F(\theta_i) d\theta_i \\ &= \int_0^{\bar{\theta}} \frac{1-F(\theta_i)}{f(\theta_i)} x(\theta_i, \theta_{-i}) dF(\theta_i) + \int_{\underline{\theta}}^0 \frac{-F(\theta_i)}{f(\theta_i)} x(\theta_i, \theta_{-i}) dF(\theta_i) \\ &= \int_{\underline{\theta}}^{\bar{\theta}} \psi(\theta_i) x(\theta_i, \theta_{-i}) dF(\theta_i), \end{aligned} \quad (\text{A.3})$$

where the first equality follows from integrating by parts, the second from rearranging terms and the third from the definition of Ψ .

Now rewrite

$$\begin{aligned} U(X^G, T^G) &= \int_{\Theta^N} \sum_{i \in N} [\theta_i x(\theta) + t_i(\theta)] dF^N(\theta) \\ &= \sum_{i \in N} \int_{\Theta^{N-1}} \int_{\underline{\theta}}^{\bar{\theta}} \left[\int_0^{\theta_i} x(\beta, \theta_{-i}) d\beta + h_i(\theta_{-i}) \right] dF(\theta_i) dF^{N-1}(\theta_{-i}) \\ &= \int_{\Theta^N} \left[\sum_{i \in N} \psi(\theta_i) \right] x(\theta) dF^N(\theta) + \sum_{i \in N} \int_{\Theta^{N-1}} h_i(\theta_{-i}) dF^{N-1}(\theta_{-i}), \end{aligned}$$

where the first equality follows by definition, the second from Lemma 1 and the third by plugging in equation (A.3).

For any subset $S \subseteq N$ of the agents, define the corresponding *orthant* as $\mathcal{O}_S = \{\theta \in \Theta^N \mid \theta_i \geq 0 \text{ if } i \in S, \theta_i \leq 0 \text{ if } i \notin S\}$.

Lemma 4. *Suppose that $\psi(\theta)$ is non-increasing in θ and $\int \psi(\theta) dF^N(\theta) < \infty$. Let \mathcal{O}_S be the orthant corresponding to some subset of agents S . Then the problem*

$$\begin{aligned} \max_x \quad & \int_{\mathcal{O}_S} \psi(\theta) \cdot x(\theta) dF^N(\theta) \\ \text{s. t. } \quad & x \text{ is non-decreasing in } \theta \\ & 0 \leq x(\theta) \leq 1 \end{aligned}$$

is solved optimally either by setting $x^(\theta) = 1$ or $x^*(\theta) = 0$.*

Proof. Suppose to the contrary that there exists a function $\hat{x}(\theta)$ that achieves a strictly higher value. Let $a_i := \inf\{\theta_i \mid (\theta_i, 0_{-i}) \in \mathcal{O}_S\}$, $b_i := \sup\{\theta_i \mid (\theta_i, 0_{-i}) \in \mathcal{O}_S\}$ and define $x^{(1)}(\theta_1, \theta_{-1}) := \frac{1}{F(b_1) - F(a_1)} \int_{a_1}^{b_1} \hat{x}(\beta, \theta_{-1}) dF(\beta)$. This function is constant in θ_1 , feasible for the above problem given that \hat{x} is feasible and, by Chebyshev's inequality, for all θ_{-1} ,

$$\begin{aligned} & \int_{a_1}^{b_1} \psi(\theta_1, \theta_{-1}) \hat{x}(\theta_1, \theta_{-1}) dF(\theta_1) \\ & \leq \int_{a_1}^{b_1} \psi(\theta_1, \theta_{-1}) dF(\theta_1) \frac{1}{F(b_1) - F(a_1)} \int_{a_1}^{b_1} \hat{x}(\theta_1, \theta_{-1}) dF(\theta_1) \\ & = \int_{a_1}^{b_1} \psi(\theta_1, \theta_{-1}) x^{(1)}(\theta_1, \theta_{-1}) dF(\theta_1). \end{aligned}$$

Since this inequality holds pointwise, we also have

$$\int_{\mathcal{O}_S} \psi(\theta) \hat{x}(\theta) dF^N(\theta) \leq \int_{\mathcal{O}_S} \psi(\theta) x^{(1)}(\theta) dF^N(\theta).$$

Iteratively defining $x^{(j)}(\theta_j, \theta_{-j}) = \frac{1}{F(b_j) - F(a_j)} \int_{a_j}^{b_j} x^{(j-1)}(\beta, \theta_{-j}) dF(\beta)$ for $j = 2, \dots, N$ we get a function $x^{(N)}(\theta)$ that is constant in θ . Repeatedly applying

Chebyshev's inequality along every dimension, we get

$$\int_{\mathcal{O}_S} \psi(\theta) \hat{x}(\theta) dF^N(\theta) \leq \int_{\mathcal{O}_S} \psi(\theta) x^{(N)}(\theta) dF^N(\theta).$$

Since the objective function is linear in x , the constant function $x^{(N)}$ is weakly dominated by either $x^* \equiv 1$ or $x^* \equiv 0$, contradicting the initial claim.

Proof of Theorem 1. Lemma 2 and Lemma 3 together imply that for any anonymous social choice function $G = (X^G, T^G)$ it holds that

$$U(X^G, T^G) = \int_{\Theta^N} \left[\sum_{i \in N} \psi(\theta_i) \right] x(\theta) dF^N(\theta),$$

where ψ is defined in (A.2) and x is the decision rule of the corresponding strategy-proof direct revelation mechanism. Lemma 4 then implies that the optimal allocation rule is constant and equal to 0 or 1 in each orthant. Symmetry of the problem implies that the optimal choice depends only on the number of agents with positive types.

Hence, it remains to determine the optimal cutoff for qualified majority voting. Let k solve

$$k\mathbb{E}[\theta_i \mid \theta_i \geq 0] + (N - k)\mathbb{E}[\theta_i \mid \theta_i \leq 0] = 0.$$

Then the expected aggregate valuation, given that $k' < k$ agents are in favor of the proposal, is negative. Therefore, it is optimal to accept the proposal if and only if at least $\lceil k \rceil$ agents have a positive valuation.

Proof of Corollary 1. Lemma 2 implies that for any social choice function satisfying the requirements of the corollary, one cannot redistribute money back to the agents. Lemma 1 then implies that any budget balanced social choice function must be constant in each orthant. Monotonicity and anonymity then imply that these social choice functions can be implemented by qualified majority voting.

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