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Efficient Bargaining in a Dynamic Macroeconomic Model

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Efficient Bargaining in a Dynamic Macroeconomic Model

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January 25, 2013

Abstract

This paper analyzes the implications of bilateral bargaining over wages and employment between a producer and a union representing a finite number of identical workers in a monetary macroeconomic model of the AS–AD type with government activity. Wages and aggregate employment levels are set according to an efficient (Nash) bargaining agreement while the commodity market is cleared in a competitive way. It is shown that, for each level of union power, measured by the share it obtains of the total production surplus, efficient bargaining implies no efficiency loss in production. However, due to the price feedback from the commodity market and to income-induced demand effects, all temporary equilibria with a positive labor share are not Nash bargaining-efficient with respect to the set of feasible temporary equilibrium allocations.

The dynamic evolution of money balances, prices, and wages is analyzed being driven primarily by government budget deficits and expectations by consumers. It is shown that for each fixed level of union power, the features of the dynamics under perfect foresight are structurally identical to those of the same economy under competitive wage and price setting, i. e. for small levels of government demand, there exist two balanced paths generically, one of which with high employment and production is always unstable while the other one may be stable or unstable.

Keywords: Efficient Bargaining, Union Power, Aggregate Demand–Aggregate Supply, Government Deficits, Perfect Foresight, Dynamics

JEL Classification: C78, D61, E24, E25, E31, E42,

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1 Introduction

In most industrialized countries negotiations between workers unions and syndicates of producers about wage levels and employment conditions occur regularly. However, the discussion of the consequences of such bargaining agreements is often restricted to the labor market alone, neglecting consequences for the economy as a whole. This implies that intersectoral effects within the economy are often not discussed or are assumed to be small or negligible. From a general-equilibrium perspective, such interactions between factor and output markets are ignored to a large degree, although their analysis constitutes one of the central issues for a macroeconomic analysis in contrast to a microeconomic or partial-equilibrium analysis.

Two sets of issues concerning the role of bargaining need to be examined within a closed monetary macroeconomic model. The first addresses the consequences of union power or different forms of bargaining solutions on the temporary equilibrium outcome within a given period, i. e. describing and evaluating the intersectoral effects between the labor market and the rest of the economy. The questions to be answered are those on the role of bargaining on the level of prices and wages and their allocative consequences in each period in comparison to the competitive equilibrium. The second set deals with the implications of bargaining and union power for the dynamic evolution of the economy again in comparison to the competitive benchmark under perfect foresight or rational expectations.

The theoretical literature of bargaining between groups (as opposed to other wage–employment-determining procedures¹) takes primarily a general microeconomic perspective where cooperative aspects in wage and employment negotiations are well recognized and studied extensively (see for example McDonald & Solow 1981; Blanchard & Fischer 1993; Landmann & Jerger 1999; Gerber & Upmann 2006). However, many of them ignore the simultaneity of the determination of employment and output, eliminating cross-market effects, or equivalently said, the simultaneous determination of the level of total income/expenditure and its distribution. Thus, significant spillovers between markets or from the income distribution on the general-equilibrium or macroeconomic level are rarely discussed or analyzed, which reduces the validity of their results as contributions to macroeconomics.

The literature discussing *efficient bargaining* taking a macroeconomic perspective is not too sizable with few of the contributions analyzing the role of efficient bargaining for the spillovers across markets or for the dynamics in a monetary macroeconomic model. McDonald & Solow (1981) study noncompetitive wagesetting in partial-equilibrium models with capacity-constrained, fully unionized labor markets with one firm and one union. Inter alia, they analyze the cases of the monopolistic union (with the right to manage of the firm) as well as two types of efficient bargaining over wages and employment using the symmetric Nash resp. the Kalai–Smorodinsky bargaining solutions. The agents’ objective functions are the profit of the firm resp. the expected excess indirect utility of the representative union member. Indirect utility is measured in nominal wages for a constant reservation wage,² which typically is not derived endogenously.

Booth (1996) and Landmann & Jerger (1999) are two prominent presentations addressing and

¹such as efficiency wages, contract theory, search theory, matching theory, etc.

²There are some contributions dealing with specific dynamic or policy issues within nonmonetary models of capital accumulation, as for example Devereux & Lockwood (1991); Kaas & von Thadden (2004); Gerber & Upmann (2006); Koskela & Puhakka (2006). Gertler & Trigari (2009) presents an interesting combination of a market with matching and staggered Nash bargaining in an empirically oriented model.

discussing the efficient bargaining solution explicitly in a format which is closest to the one used here. Booth (1996) slightly extends the setting by McDonald & Solow (1981) by applying the generalized Nash bargaining solution while analyzing bargaining over wages alone. This leaves the employment decision to the firm which corresponds to the so called right-to-manage model. Its modeling generalizes the monopolistic-union model and shows that the resulting outcome is not Nash efficient in a static partial-equilibrium setting.

Concerning the macroeconomic perspective, Blanchard & Fischer (1993) derives some additional general implications and extensions of the allocative results as in McDonald & Solow (1981), from which it concludes that the presence of bargaining would lead to less employment fluctuations. However, it stops short of a full macroeconomic or dynamic embedding. This leaves open of how the model is to be appended to determine the price level, inflation, and dynamics under uncertainty. Some other contributions suggest closing the model via an expectations-augmented Phillips curve or adapting the NAIRU construction to arrive at a closed dynamic model. While this may be a convenient short cut, its consistency with the microeconomic structures underlying the labor market (for example justifying the assumption of a fixed nominal reservation wage!) is somewhat unclear. Other contributions like Lye, McDonald & Sibly (2001) employ different closing procedures for which they use to achieve empirically testable results concerning unemployment and inflation.

This paper starts from the general observation that there exist distinct cross-market feedbacks in a micro-based AS–AD model of a monetary macroeconomy under bargaining which stands a test with the general competitive case. These consist of significant cross-price effects from bargaining into the output market as well as income distribution effects inducing associated changes of aggregate demand, which imply positive or negative aggregate output/employment effects. Thus, a full comparison between the allocative (i. e. comparative-statics) properties as well as their dynamic implications relative to the competitive model can be carried out. In fact, this can be done for the same micro-based AS–AD model without requiring additional assumptions regarding the monetary closure of the model.

The paper presents a complete comparative-statics analysis with respect to union power characterizing the cross-market interactions through the temporary price feedback. It analyzes the efficient bargaining solution for the labor market, which is the most cooperative structure and solution concept from a bargaining point of view. While the literature agrees that this solution concept might be empirically the most unlikely, its theoretical implications for the macroeconomy must be examined as a benchmark model, to determine in particular its properties of efficiency and optimality which the literature seems to assign to it.³

In addition, the dynamic consequences for allocations and the stability of the evolution under perfect foresight and efficient bargaining are derived. This is carried out for the situation of constant bargaining power over time. It is apparent that a full dynamic analysis of efficient bargaining should allow the two parties to adjust their procedures over time and take the possibilities of repeated or sequential negotiation into account. For such repeated negotiations occurring in macroeconomic systems, static game theory again does not provide modeling approaches at a satisfactory level to be applied suitably to labor markets. The issues to be solved in the dynamic setting of repeated negotiations open a wide range of unsolved problems as to the dynamic setting of the negotiation. Again, with the cross-market feedbacks playing a major qualitative role, the negotiations and their procedures will have an influence on the

³For other noncompetitive solution concepts in the labor market see Böhm (2010). The so called right-to-manage model is analyzed in Böhm & Claas (2012b).

dynamic evolution of the economy. Such issues are left for future research.⁴

2 The Labor Market with Efficient Bargaining

Consider an economy in discrete time with three markets: a labor market, a commodity market, and a money market, and three sectors: a consumption sector, a production sector, and the public sector consisting of a central government and a central bank.⁵

2.1 The Public Sector

The government demands the produced commodity at a level $g \geq 0$ to produce public goods and services. These are assumed to be pure public goods providing a constant level of utility each period to each type of consumer. In addition, consumer preferences are assumed to be additively separable with respect to the level of the public good so that these do not induce marginal or behavioral effects by consumers.

To finance its consumption (the public good's production), the government levies a proportional tax on profits at the rate $0 \leq \tau_\pi \leq 1$ and on wages at the rate $0 \leq \tau_w \leq 1$. Since the government parameters are assumed to be given parametrically in each period,⁶ in general, the government budget is not balanced since its revenues and expenditures are endogenously determined. Therefore, the central bank creates/destroys the amount of money according to the need of the government arising from the unbalanced budget. Since money is the only intertemporal store of value held by consumers, any increase (decrease) of the amount of money required to balance the budget of the government is equivalent to the amount of savings (changes of the amount of money held by the private sector) in any given period.

2.2 The Production Sector

The nonstorable commodity is produced from labor only by a single profit-maximizing firm.⁷ The stock of capital does not depreciate and is assumed to be constant through time. Production possibilities in any period are described by a differentiable, increasing, and strictly concave production $F : \mathbb{R}_+ \rightarrow \mathbb{R}_+$, $L \mapsto F(L)$ satisfying the Inada conditions. At a given nominal wage rate $w \geq 0$ for labor and a sales price $p \geq 0$ for the commodity, a production decision L implies current profits $\Pi(p, w, L) := pF(L) - wL$. All profits are paid to consumers, who are the owners or the shareholders of the firm. There is no intertemporal decision making of the firm, i. e. there is no need to retain profits or to hold money. Therefore, the firm's objective is to maximize profits.

⁴Selten & Güth (1982) seem to be the only authors who have addressed the sequential wage bargaining issue in a dynamic context, however, only within a reduced-form macroeconomic model of the multiplier–accelerator type.

⁵The model is a standard version of an AS–AD model with fiat money whose consumption sector consists of cohorts of overlapping generations of different consumer types (see for example Böhm 2010).

⁶To save on notation, we omit the government parameters g , τ_w , and τ_π , wherever possible.

⁷This assumption is made for simplicity only, the extension to multiple homogeneous firms organized in a producers association is straightforward, but leaving all results of this paper qualitatively unchanged.

Under competitive conditions with prices and wages given, the behavior of the firm in each period in the two markets would induce the usual profit-maximizing labor demand function

$$h_{\text{com}}\left(\frac{w}{p}\right) := \arg \max_{L \geq 0} \{pF(L) - wL\} = (F')^{-1}\left(\frac{w}{p}\right)$$

and the commodity supply function $F(h_{\text{com}}(w/p))$. Given the Inada conditions for F , labor demand $h_{\text{com}} : \mathbb{R}_{++} \rightarrow \mathbb{R}_{++}$ is strictly monotonically decreasing, surjective, and thus invertible.

In noncompetitive situations, in particular under bargaining, pairs (L, w) of employment and wage levels have to guarantee nonnegative profits $\Pi(p, w, L) \geq 0$ for the producer. Therefore, the zero-profit contour implies the participation constraint for the producer

$$w \leq p \frac{F(L)}{L} =: W_{\Pi}(p, L),$$

which defines his reservation wage as a function $W_{\Pi}(p, L)$ of prices and a positive employment level.

2.3 The Consumption Sector

The consumption sector consists of overlapping generations of two types of homogeneous consumers. There are $n_w \geq 1$ workers and $n_s \geq 1$ shareholders in each generation, both of which live for two consecutive periods. The size and composition of the two groups is constant through time implying that at any one time, there are $n_s + n_w$ young resp. old consumers.

Each shareholder consumer receives net profits only in the first period of his life. His intertemporal preferences are assumed to be homothetic so that he spends the proportion $0 < c(p^e/p) < 1$ of his net income in the first period and saves the rest in the form of money to be spent on consumption in the second period where $p^e > 0$ is the consumers' forecast for the future commodity price.

Each worker supplies labor in the first period of his life to consume in the second period only. His preferences with respect to planned future consumption $c^e \geq 0$ and work $\ell \geq 0$ when young are described by an intertemporal utility function of the form $u(\ell, c^e) := c^e - v(\ell)$ where $v : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ measures the disutility from labor. The function v is assumed to be continuously differentiable, strictly monotonically increasing, strictly convex, with $v(0) = v'(0) = 0$ and $\lim_{\ell \rightarrow \infty} v'(\ell) = \infty$.

Given a wage rate $w > 0$, an employment level ℓ , and a wage tax τ_w , each worker saves his total nominal net wage income $(1 - \tau_w)w\ell$ in the form of money to be spent on consumption in the second period of his life. With given price expectations p^e , his planned future consumption satisfies $p^e c^e = (1 - \tau_w)w\ell$. Therefore, under competitive conditions and price expectations p^e , his utility-maximizing labor supply is given by

$$\arg \max_{\ell \geq 0} \left\{ u\left(\ell, (1 - \tau_w) \frac{w}{p^e} \ell\right) \right\} = (v')^{-1}\left((1 - \tau_w) \frac{w}{p^e}\right),$$

which is a continuous, strictly monotonically increasing, and surjective (invertible) function of the expected future value of the current nominal wage. As a consequence one obtains the aggregate competitive labor supply as

$$N_{\text{com}}\left(\frac{w}{p^e}\right) := n_w \ell = n_w (v')^{-1}\left((1 - \tau_w) \frac{w}{p^e}\right)$$

which has a global inverse given by

$$\frac{w}{p^e} = S_{\text{com}}(L) := \frac{1}{1 - \tau_w} v' \left(\frac{L}{n_w} \right).$$

Given the price expectations $p^e > 0$, his reservation wage for noncompetitive situations can be defined as follows. The labor market participation constraint of a worker for an acceptable nonzero employment–wage situation (ℓ, w) must provide a utility at least as high as not working when young. In other words, (ℓ, w) must be a solution of

$$u(0, 0) = 0 \leq u(\ell, c^e) = u \left(\ell, (1 - \tau_w) \frac{w}{p^e} \ell \right) = (1 - \tau_w) \frac{w}{p^e} \ell - v(\ell).$$

This implies the lower bound of the individually acceptable wage rate, i. e. his *reservation wage*, as

$$\frac{w}{p^e} = \frac{1}{1 - \tau_w} \frac{v(\ell)}{\ell}, \quad \ell > 0, \quad (1)$$

which is a strictly increasing function of the employment level. If workers are treated equally in all aggregate employment situations with level L (share total employment equally), one obtains the reservation wage from equation (1) as a function of the aggregate employment level L

$$\frac{w}{p^e} = S(L) := \frac{n_w}{L(1 - \tau_w)} v \left(\frac{L}{n_w} \right),$$

implying a useful relationship between the reservation wage and the wage under competitive conditions⁸

$$S_{\text{com}}(L) = E_v(L/n_w) S(L) > S(L) \quad \text{for all } L.$$

2.4 Efficient Bargaining and Employment

The union is perceived of as an aggregate agent representing all workers. Since all workers have identical characteristics, the union's bargaining will be concerned with the determination of the wage level w and the aggregate level of employment L , assuming that all workers are treated equally, i. e. each is paid the wage w with individual employment level L/n_w .

The framework chosen for the wage bargaining between the union and the producer as a wage determination device consists of an application of a bargaining solution to the *simultaneous* determination of the aggregate employment level L and of the wage rate w in each period under the assumption that the negotiating parties are both price takers in the commodity market. With this choice it is possible to discuss best the role of bargaining in temporary general equilibrium and compare the outcomes with the competitive case.

Under efficiency considerations, choosing the Nash bargaining solution is one possibility although in the repeated or dynamic context this may not be fully convincing. In other words, the producer and the union treat the commodity price as given, implicitly assuming that their bargaining decision has no influence on the induced equilibrium price in the short run. Moreover, both parties objective is to reach an efficient bargaining solution in the particular period

⁸For any function f we denote its elasticity at x as $E_f(x)$.

of negotiations. They do not consider possible intertemporal consequences of their negotiations on aspects of future wages, employment, or future payoffs. In other words, their bargaining perspective is myopic without regard to intertemporal effects of their negotiations, in spite of the fact that they are operating in a market economy with a past and a future. Thus, a *temporary equilibrium with efficient bargaining* will be defined and analyzed. This is given by a competitive price level p which equalizes aggregate supply and aggregate demand of the commodity market at which the levels of employment and wages induce the desired efficient bargaining solution between the union and the producer in each period.

Therefore, the result of the bargaining procedure in each period between the union and the producer consists of a joint decision with respect to the employment level L and the wage rate w where the producer's goal is to maximize its net profit while the union tries to maximize the aggregate excess wage bill for the workers. Let $\Pi(p, w, L) = pF(L) - wL$ denote the net profit and $\Omega(p^e, w, L) := wL - p^e S(L)L$ the excess wage bill, which are strictly concave functions in L . Given price expectations and commodity price $(p^e, p) \gg 0$, a bargaining agreement (L, w) is called *individually rational* if Π and Ω are nonnegative. An *efficient bargaining agreement* between the union and the employer is defined in the usual way.

Definition 2.1 *Given $(p^e, p) \gg 0$, an employment–wage pair $(L, w) \in \mathbb{R}_+^2$ is called efficient if there exists no other pair (L', w') such that*

$$\Pi(p, w', L') \geq \Pi(p, w, L) \quad \text{and} \quad \Omega(p^e, w', L') \geq \Omega(p^e, w, L)$$

with at least one strict inequality.

In order to characterize efficient agreements, one may define the associated Lagrangean function

$$\Lambda(w, L, \kappa) := \Omega(p^e, w, L) + \kappa (\Pi(p, w, L) - \bar{\Pi})$$

for any positive level $\bar{\Pi}$ and obtain from the first-order conditions for an interior solution $(L, w) \gg 0$

$$pF'(L) = p^e(S(L) + S'(L)L). \quad (2)$$

This condition defines a unique efficient employment level as a function h of the expected rate of inflation p^e/p , i. e. $L = h(p^e/p)$, for all levels of net profit $\bar{\Pi}$, due to the strict concavity of Ω and Π in L . This result is well-known from the literature. For the model here between the union and the producer, this implies that the determination of an efficient bargaining solution can be divided into two steps: the choice of the level of employment which depends on the market data upstream and downstream and the determination of the wage which then turns out to become the central point in the bargaining procedure of sharing the net gains.

Employment under Efficient Bargaining

Rewriting equation (2) and using $S(L) + S'(L)L = S_{\text{com}}(L)$, one obtains the explicit inverse of the employment function as

$$\frac{p^e}{p} = \frac{F'(L)}{S(L) + S'(L)L} = \frac{F'(L)}{S_{\text{com}}(L)} = h^{-1}(L), \quad (3)$$

which shows that h is a strictly monotonically decreasing function. It indicates also that the employment level chosen by the two bargaining parties is the same as the one which would result in equilibrium under a perfectly competitive labor market.

As a consequence of the results of the previous paragraph, the employment decision under efficient bargaining also turns out to be equivalent to the standard textbook representation of a bilateral monopoly between the union and the producer maximizing the joint net gain. For a given pair of price expectations and commodity price $(p^e, p) \gg 0$, this is given by

$$\Pi(p, w, L) + \Omega(p^e, w, L) = pF(L) - wL + wL - p^e S(L)L = pF(L) - p^e S(L)L$$

which is a function of the employment level alone. Thus, it is necessary that an efficient bargaining decision maximizes $pF(L) - p^e S(L)L$, which induces a first-order condition identical to (2), implying the same solution $L = h(p^e/p)$. Therefore, the employment decision under efficient bargaining coincides also with the one of a cooperative bilateral monopoly. In this interpretation, the labor market has been eliminated: the employment decision corresponds to an internal decision of a union–producer monopoly while the decision for the wage rate becomes a cost sharing issue.

This separability of the employment and the wage decision can be portrayed geometrically in the associated employment–wage space (see Figure 1). For $L > 0$, an acceptable wage must be such that $\Pi \geq 0$ and $\Omega \geq 0$, i. e.

$$w \leq p \frac{F(L)}{L} = W_{\Pi}(p, L) \quad \text{and} \quad w \geq p^e S(L) =: W_{\Omega}(p^e, L),$$

inducing the two status-quo wage functions W_{Π} and W_{Ω} which correspond to the reservation wage of the producer and of the union respectively. The area between the two functions in Figure 1 defines the set of individually rational employment–wage pairs. The set of efficient employment–wage choices under bargaining are those on the contract curve shown as the bold red line. Geometrically speaking, each point on the contract curve is a tangency point of an

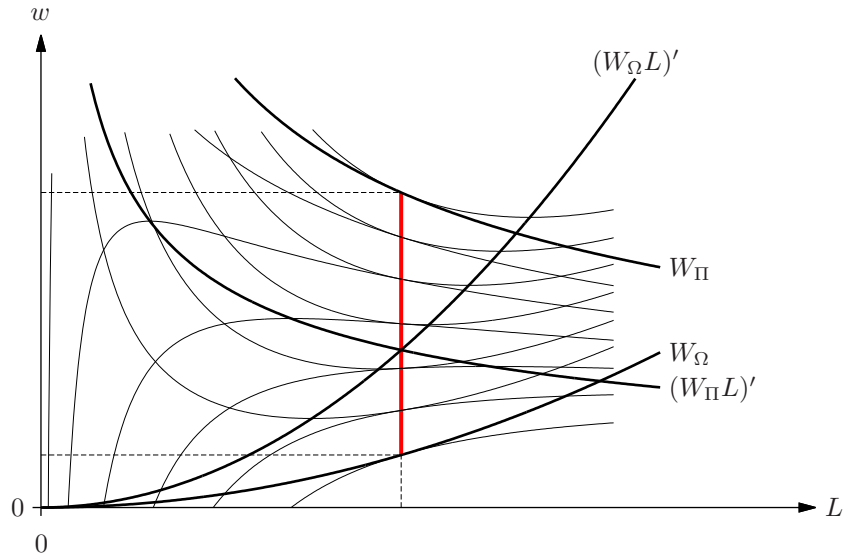


Figure 1: Employment under efficient bargaining

isoutility and of an isoprofit curve (the thin lines). Since all iso-utility/iso-profit curves are of the form

$$W_{\bar{\Pi}}(L) = \frac{pF(L) - \bar{\Pi}}{L} \quad \text{resp.} \quad W_{\bar{\Omega}}(L) = p^e S(L) + \frac{\bar{\Omega}}{L}$$

for all levels $\bar{\Pi}$ and $\bar{\Omega}$, the tangency condition $W'_{\bar{\Pi}}(L) \stackrel{!}{=} W'_{\bar{\Omega}}(L)$ implies

$$\frac{pF'(L)L - W_{\bar{\Pi}}(L)L}{L^2} \stackrel{!}{=} p^e S'(L) - \frac{W_{\bar{\Omega}}(L) - p^e S(L)}{L}.$$

Rewriting condition (3) using the two reservation wage functions, one obtains an intuitive and interesting relationship

$$W_{\Omega}(p^e, L) = p^e S(L) = \frac{E_F(L)}{E_S(L) + 1} \frac{pF(L)}{L} = \frac{E_F(L)}{E_S(L) + 1} W_{\Pi}(p, L) \quad (4)$$

for the relative shares depending on the elasticities of the reservation wage functions, which also characterizes the bargaining level of employment. This stipulates that the ratio between the two status-quo values should correspond to the ratio of their respective elasticities.

The Wage Rate under Efficient Bargaining

Given $(p^e, p) \gg 0$ and $L = h(p^e/p) > 0$, the bargaining decision between the two parties concerning the wage rate now constitutes a standard *bargaining game with constant transfers* with status-quo point $(0, 0)$ since $\Pi + \Omega = pF(L) - p^e S(L)L = W_{\Pi}(p, L)L - W_{\Omega}(p^e, L)L$ is a constant sum. For such games, the bargaining power between the two parties is measured by a number $0 \leq \lambda \leq 1$, which defines the relative share of the total gain to be allocated to the party having bargaining power λ . Thus, for a total gain $\Pi + \Omega = W_{\Pi}(p, L)L - W_{\Omega}(p^e, L)L$, the weights $(\lambda, 1 - \lambda)$ determine a linear redistribution among the two agents. Therefore, with $L > 0$ and $0 \leq \lambda \leq 1$ given, an application of the generalized Zeuthen solution⁹ implies choosing the bargaining wage as a convex combination of the two reservation wage levels W_{Π} (when $\Pi = 0$) and W_{Ω} (when $\Omega = 0$) with the same weights

$$W(p^e, \lambda, p, L) := \lambda W_{\Pi}(p, L) + (1 - \lambda) W_{\Omega}(p^e, L), \quad L = h(p^e/p). \quad (5)$$

Substituting (5) into the utility and into the profit functions yields the payoff vector (Π, Ω) of the bargaining solution

$$\begin{aligned} \begin{pmatrix} \Pi(p, W(p^e, \lambda, p, L), L) \\ \Omega(p^e, W(p^e, \lambda, p, L), L) \end{pmatrix} &= \begin{pmatrix} pF(L) - W(p^e, \lambda, p, L)L \\ W(p^e, \lambda, p, L)L - p^e S(L)L \end{pmatrix} \\ &= (W_{\Pi}(p, L) - W_{\Omega}(p^e, L))L \begin{pmatrix} 1 - \lambda \\ \lambda \end{pmatrix} = (pF(L) - p^e S(L)L) \begin{pmatrix} 1 - \lambda \\ \lambda \end{pmatrix}. \end{aligned} \quad (6)$$

For given (p^e, p) , Figure 2 displays the range of the mapping (6) for different values of the parameter λ , revealing its linear impact on the payoff distribution. A similar linear relationship holds for the role of λ on the bargaining wage. Finally, substituting (4) into the bargaining wage function (5), one finds that the equilibrium bargaining wage

$$W(p^e, \lambda, p, L) = \left(\frac{E_F(L)}{E_S(L) + 1} + \lambda \left(1 - \frac{E_F(L)}{E_S(L) + 1} \right) \right) \frac{pF(L)}{L} \quad (7)$$

⁹For such bargaining problems, the generalized Zeuthen solution for half-space games coincides with the generalized Nash solution, yet requiring less properties (see Peters 1992).

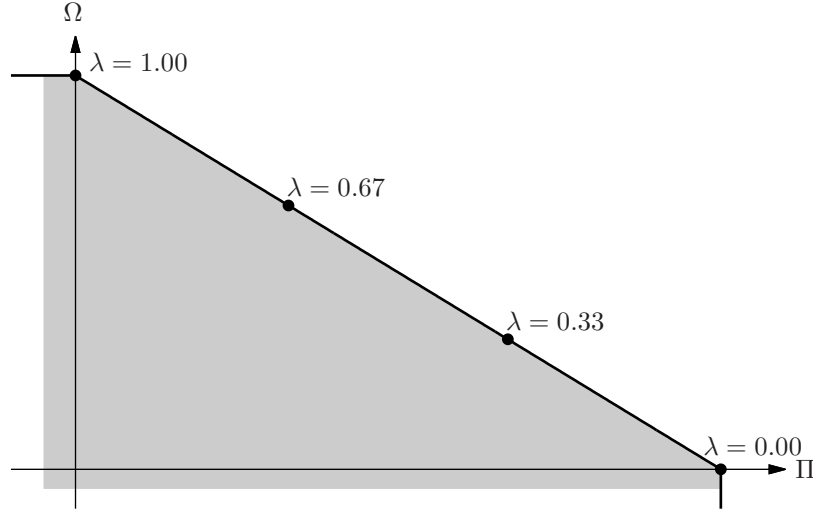


Figure 2: The impact of the bargaining power λ on the equilibrium payoff

is a multiple of average productivity, and that the equilibrium real wage

$$\frac{W(p^e, \lambda, p, L)}{p} = \frac{1}{E_F(L)} \left(\frac{E_F(L)}{E_S(L) + 1} + \lambda \left(1 - \frac{E_F(L)}{E_S(L) + 1} \right) \right) F'(L)$$

is a positive multiple of the marginal product of labor (with $L = h(p^e/p)$). Both equations show clearly how the bargaining parameter interacts with the elasticities of the two reservation wage functions.

Relative Union Power

As was seen above, an efficient bargaining solution $(L, w) = (h(p^e/p), W(p^e, \lambda, p, h(p^e/p)))$ is defined parametrically for a given $0 \leq \lambda \leq 1$ measuring the “bargaining power”. Thus, the model does not provide a fully endogenous determination of the bargaining power between the union and the producer. However, the efficient level of employment is independent of λ , implying that union–employer negotiations do guarantee productive efficiency. Therefore, the bargaining parameter λ determines exclusively the redistribution of revenue between the two parties, i.e. the share of wages and profits in total revenue.

It is intuitively clear (and also evident from the geometry of Figure 1) that there must be a unique bargaining level for which the parties agree on the competitive wage. This one equalizes marginal revenue resp. marginal cost $((W_\Pi L)'$ resp. $(W_\Omega L)')$. Geometrically speaking, this corresponds to the wage where the respective iso-utility and iso-profit curves are horizontal. Let the unique λ for which this condition holds be denoted by λ_{nat} , the “natural” λ . It is the solution of either

$$W(p^e, \lambda, p, L) \stackrel{!}{=} \frac{\partial(W_\Pi(p, L)L)}{\partial L} \quad \text{or} \quad W(p^e, \lambda, p, L) \stackrel{!}{=} \frac{\partial(W_\Omega(p^e, L)L)}{\partial L},$$

where $L = h(p^e/p)$. Inserting the definition of $W(p^e, \lambda, p, L)$ into the first equation gives

$$\lambda_{\text{nat}} W_\Pi(p, L) + (1 - \lambda_{\text{nat}}) W_\Omega(p^e, L) = \frac{\partial(W_\Pi(p, L)L)}{\partial L} = pF'(L) = E_F(L)W_\Pi(p, L).$$

Exploiting (4) then implies

$$\lambda_{\text{nat}}(L) = \frac{E_F(L)E_S(L)}{E_S(L) + 1 - E_F(L)}, \quad L = h(p^e/p).$$

In other words, $\lambda_{\text{nat}}(L)$ is determined by the elasticities E_S and E_F derived from labor supply and from production, respectively. Therefore, with isoelastic functions F and S , $\lambda_{\text{nat}}(L)$ is constant and independent of other data of the economy.

The wage share of total revenue under bargaining power λ can be computed in a similar manner.

$$\frac{wL}{py} = \frac{W(p^e, \lambda, p, L)}{W_{\Pi}(p, L)} = \frac{E_F(L)}{E_S(L) + 1} + \lambda \left(1 - \frac{E_F(L)}{E_S(L) + 1} \right) \in \left[\frac{E_F(L)}{E_S(L) + 1}, 1 \right] \quad (8)$$

This implies a profit share of total revenue as

$$\frac{\pi}{py} = 1 - \frac{wL}{py} = (1 - \lambda) \left(1 - \frac{E_F(L)}{E_S(L) + 1} \right). \quad (9)$$

Note that the wage share resp. the profit share for $\lambda_{\text{nat}}(L)$ is $E_F(L)$ resp. $1 - E_F(L)$, as expected, since at $\lambda_{\text{nat}}(L)$ the factor shares in total output must be equal to the respective elasticities of the production function F .

Underemployment and Overemployment

Since the bargaining solution $(L, w) = (h(p^e/p), W(p^e, \lambda, p, h(p^e/p)))$ is a *joint* agreement between the two agents, there can neither be any *involuntary* unemployment nor overemployment. In other words, any difference between $L = h(p^e/p)$ and the desired labor supply $N_{\text{com}}(w/p^e)$ has to be interpreted as a measure of a *voluntary* deviation from the competitive labor supply of the workers, which is a *supply side* measure. Similarly, any difference between L and the desired competitive employment $h_{\text{com}}(w/p)$ by the producer would be a *demand side* measure of *voluntary* deviation relative to the competitive regime.

Here, the voluntary *underemployment rate* will be defined in the usual way as

$$U = \mathcal{U} \left(L, \frac{w}{p^e} \right) := \frac{N_{\text{com}}(w/p^e) - L}{N_{\text{com}}(w/p^e)} = 1 - \frac{L}{N_{\text{com}}(w/p^e)}, \quad (10)$$

which measures the gap between the amount of labor which is actually traded (i. e. worked) and which would be supplied by the workers under competitive conditions at the given wage level. Since the rate of unemployment is defined for all expected real wages and all levels of labor, U defined in (10) can also be negative. This occurs if w/p^e is relatively low or L is relatively high. We interpret negative rates of underemployment as overemployment (or overtime).

3 Temporary Equilibrium with Efficient Bargaining

It is now straightforward to close the model and determine the macroeconomic implications of a temporary equilibrium under wage bargaining. The data at the beginning of an arbitrary period are aggregate money balances $M > 0$ held by old consumers, expected prices for the future

period $p^e > 0$, and the bargaining parameter $0 \leq \lambda \leq 1$, plus the parameters of the government (g, τ_w, τ_π) . Then, a temporary equilibrium with efficient bargaining is defined by a pair of prices and wages $(p, w) \gg 0$ such that the price p clears the commodity market competitively while the wage w equals the one set by the union and the producer in the bargaining solution. Associated with the equilibrium is the equilibrium allocation which consists of a pair of feasible employment and output levels $(L, y) = (L, F(L)) \gg 0$.

Since all agents in the economy – consumers, the producer, and the government – are assumed to be price takers in the commodity market, finding a temporary equilibrium is equivalent to finding a price p which equalizes aggregate demand and aggregate supply where aggregate demand has to be appropriately adjusted to the income distribution induced by the bargaining result.

3.1 The Role of Union Power in Temporary Equilibrium

Aggregate Supply and Aggregate Demand

The bargaining wage $W(p^e, \lambda, p, L)$ and the employment level $L = h(p^e/p)$ were derived as homogeneous functions of price expectations and prices in the previous section where the employment decision turned out to be independent of the bargaining parameter λ . Therefore, given a pair of price expectations and prices $(p^e, p) \gg 0$, let $\theta^e := p^e/p$ denote the expected inflation factor (one plus the inflation rate). Then, the aggregate commodity supply function is defined by

$$AS : \mathbb{R}_{++} \rightarrow \mathbb{R}_{++}, \quad AS(\theta^e) := F(h(\theta^e)),$$

a function of expected inflation alone which is globally invertible and differentiable. Since $h'(\theta^e) < 0$, one has $AS'(\theta^e) < 0$ so that, for any given price expectation $p^e > 0$, aggregate supply is a strictly increasing function of the commodity price

$$\frac{d AS(p^e/p)}{d p} > 0.$$

In contrast, the bargaining power λ has an influence on the income distribution and thus on aggregate demand. Since there are four different private consumers plus the government generating aggregate demand, the income distribution between profits and wage income and the total income generated determine aggregate demand.

The assumptions concerning the overlapping-generations structure of consumers imply that all current net wages are saved and a proportion $0 \leq c(\theta^e) \leq 1$ of current net profits is consumed by young shareholders. Therefore, aggregate real demand in any period is the sum of total real money balances $m := M/p$, government demand g , plus the demand by shareholders which is a function of aggregate profits. Thus, given money balances, price expectations, the bargaining weight, and prices (M, p^e, λ, p) , the income-consistent aggregate demand y must solve

$$\begin{aligned} y &= m + g + c(\theta^e)(1 - \tau_\pi) \frac{\pi}{p} \\ &\stackrel{(9)}{=} m + g + c(\theta^e)(1 - \tau_\pi)(1 - \lambda) \left(1 - \frac{E_F(L)}{E_S(L) + 1} \right) y \end{aligned}$$

with $L = h(\theta^e)$. Therefore, one obtains as the income-consistent aggregate demand function

$$y = D(m, \theta^e, \lambda) = \frac{m + g}{1 - c(\theta^e)(1 - \tau_\pi)(1 - \lambda)(1 - \frac{E_F(h(\theta^e))}{E_S(h(\theta^e))+1})}, \quad (11)$$

which is of the usual multiplier form with respect to money balances and government demand. Observe that aggregate demand is homogeneous of degree zero in (M, p^e, p) . Therefore, for given λ , it is a function of real money balances and of the expected rate of inflation. Obviously, $\partial D / \partial m > 0$, i.e. real balances have a positive effect on demand, and $\partial D / \partial \lambda < 0$, i.e. higher bargaining power by the union decreases profits and thus consumption demand by shareholders. In addition, if $\partial D / \partial \theta^e \geq 0$, then the demand is strictly decreasing in the commodity price p , i.e. $d D(M/p, p^e/p, \lambda) / dp < 0$ is negative. This property holds in particular when the savings proportion by shareholders is nondecreasing and when the reservation wage and the production function are isoelastic.

Therefore, given a bargaining weight $0 \leq \lambda \leq 1$ and any pair $(M, p^e) \gg 0$ of money balances and price expectations, the temporary equilibrium is given by a price p which clears the commodity market, i.e.

$$D\left(\frac{M}{p}, \frac{p^e}{p}, \lambda\right) = AS\left(\frac{p^e}{p}\right). \quad (12)$$

Concerning existence and uniqueness, one has the following immediate result.

Lemma 3.1 *Let the aggregate supply function AS be globally invertible with $AS'(\theta^e) < 0$, and assume that $\partial D / \partial m > 0$, $\partial D / \partial \theta^e \geq 0$ hold. Then, for every $(M, p^e) \gg 0$ and $0 \leq \lambda \leq 1$, there exists a unique positive temporary equilibrium price $p > 0$ solving equation (12).*

The uniqueness follows from the fact that the excess demand function is strictly monotonically decreasing. Figure 3 portrays the equilibrium situation in the usual aggregate demand–aggregate supply diagram of the commodity market. As a consequence of Lemma 3.1, one

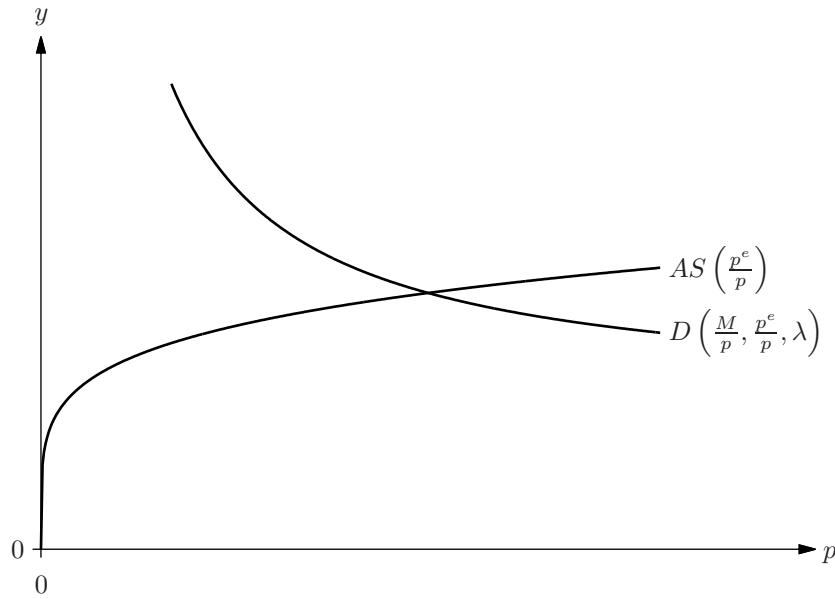


Figure 3: The temporary equilibrium price

obtains the following proposition characterizing temporary equilibrium.

Proposition 3.1 *There exist differentiable mappings $\mathcal{P} : \mathbb{R}_{++}^2 \times [0, 1] \rightarrow \mathbb{R}_{++}$ and $\mathcal{W} : \mathbb{R}_{++}^2 \times [0, 1] \rightarrow \mathbb{R}_{++}$, called the price law and the wage law respectively such that*

1. *the unique positive temporary equilibrium price is given by*

$$p = \mathcal{P}(M, p^e, \lambda), \quad (13)$$

2. *the unique positive temporary equilibrium wage is defined by*

$$w = \mathcal{W}(M, p^e, \lambda) := W \left(p^e, \lambda, \mathcal{P}(M, p^e, \lambda), h \left(\frac{p^e}{\mathcal{P}(M, p^e, \lambda)} \right) \right),$$

3. *\mathcal{P} and \mathcal{W} are homogeneous of degree one in (M, p^e) , for given λ .*

Properties of the Price Law

Applying the implicit function theorem to (12) with respect to M , one obtains the effect of an increase of money balances

$$\frac{\partial \mathcal{P}}{\partial M} = \frac{\frac{1}{\mathcal{P}} \frac{\partial D}{\partial m}}{-\frac{p^e}{\mathcal{P}^2} F' h' + \frac{M}{\mathcal{P}^2} \frac{\partial D}{\partial m} + \frac{p^e}{\mathcal{P}^2} \frac{\partial D}{\partial \theta^e}} > 0$$

with an elasticity

$$0 < E_{\mathcal{P}}(M) = \frac{\partial \mathcal{P}}{\partial M} \frac{M}{\mathcal{P}} = \frac{\frac{M}{\mathcal{P}} \frac{\partial D}{\partial m}}{-\frac{p^e}{\mathcal{P}} F' h' + \frac{M}{\mathcal{P}} \frac{\partial D}{\partial m} + \frac{p^e}{\mathcal{P}} \frac{\partial D}{\partial \theta^e}} < 1. \quad (14)$$

Thus, the temporary equilibrium price is a strictly increasing and strictly concave function of money balances since prices are nonnegative. Applying the implicit function theorem to (12) once more, one obtains a positive expectations effect on prices

$$\frac{\partial \mathcal{P}}{\partial p^e} = -\frac{\frac{1}{\mathcal{P}} F' h'}{-\frac{p^e}{\mathcal{P}^2} F' h' + \frac{M}{\mathcal{P}^2} \frac{\partial D}{\partial m} + \frac{p^e}{\mathcal{P}^2} \frac{\partial D}{\partial \theta^e}} > 0$$

with an elasticity

$$E_{\mathcal{P}}(p^e) = \frac{\partial \mathcal{P}}{\partial p^e} \frac{p^e}{\mathcal{P}} = \frac{-\frac{p^e}{\mathcal{P}^2} F' h'}{-\frac{p^e}{\mathcal{P}^2} F' h' + \frac{M}{\mathcal{P}^2} \frac{\partial D}{\partial m} + \frac{p^e}{\mathcal{P}^2} \frac{\partial D}{\partial \theta^e}} < 1, \quad (15)$$

which is also less than one, implying that equilibrium prices are a strictly increasing and strictly concave function in price expectations. Together this implies that the price law \mathcal{P} is strictly concave and homogeneous of degree one in (M, p^e) , with a representation of the form $p = p^e \mathcal{P}(M/p^e, 1, \lambda)$ which is strictly increasing and strictly concave in M/p^e .

Output and Employment

Given the price law, one obtains the associated temporary equilibrium allocation consisting of the levels of output and employment as functions of the same data (M, p^e, λ) , i. e.

$$\begin{aligned} y &= \mathcal{Y}(M, p^e, \lambda) := F \left(h \left(\frac{p^e}{\mathcal{P}(M, p^e, \lambda)} \right) \right) \quad \text{and} \\ L &= \mathcal{L}(M, p^e, \lambda) := h \left(\frac{p^e}{\mathcal{P}(M, p^e, \lambda)} \right), \end{aligned} \quad (16)$$

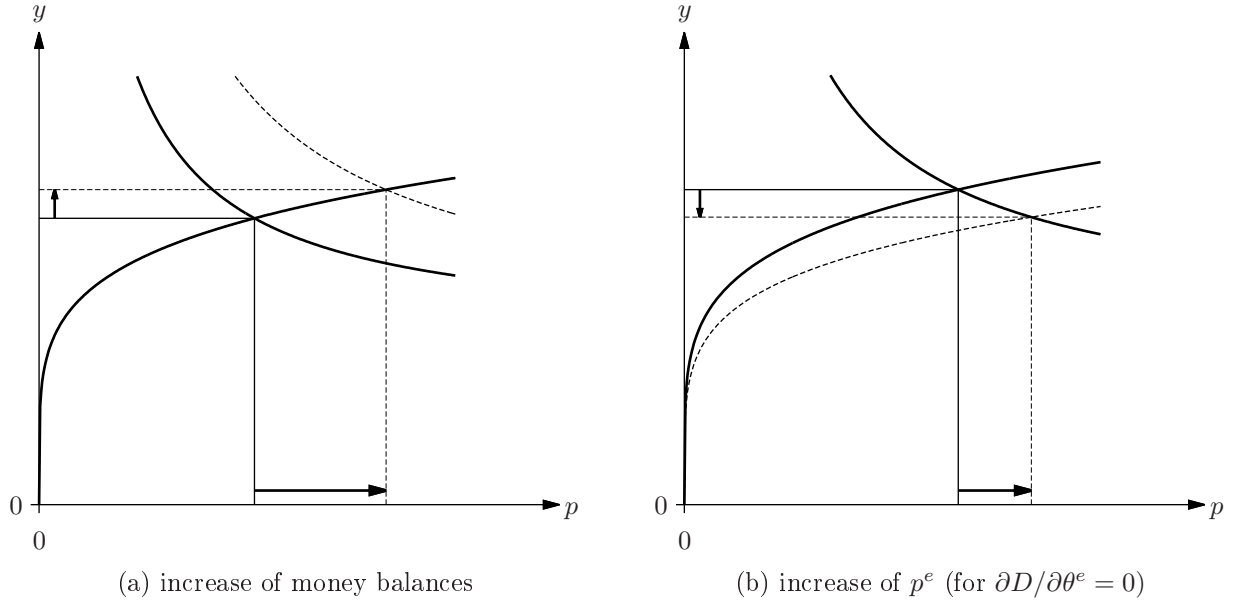


Figure 4: Comparative-statics effects of money balances and price expectations

which are homogeneous of degree zero in (M, p^e) . Using (14) and $0 < E_F(L) < 1$, one obtains the corresponding elasticities of money balances on employment and output as

$$E_{\mathcal{L}}(M) = -E_h(\theta^e)E_{\mathcal{P}}(M) > 0 \quad \text{and} \quad E_{\mathcal{L}}(M) > E_F(L)E_{\mathcal{L}}(M) = E_Y(M) > 0.$$

Thus, higher money balances imply higher equilibrium prices but also higher levels of employment and output. Similarly, applying property (15), $0 < E_F(L) < 1$, and the relationship

$$E_{\mathcal{L}}(p^e) = \underbrace{E_h(\theta^e)}_{<0} \underbrace{(1 - E_{\mathcal{P}}(p^e))}_{\in(0,1)} < 0$$

yields

$$E_{\mathcal{L}}(p^e) < E_F(L)E_{\mathcal{L}}(p^e) = E_Y(p^e) < 0.$$

Thus, output and employment decline with higher price expectations. Therefore, combined with the zero-homogeneity of the employment law and output law, this confirms the tradeoff between money balances and expectations for a constant level of output and employment. Figure 4 displays the comparative-statics results for changes of money balances and price expectations.

Properties of the Wage Law

In contrast to the above results, the comparative-statics effects of the wage law cannot be signed in general since several diverse effects interact in a nonlinear way. This can be seen partially from the form of the wage law equation

$$w = \mathcal{W}(M, p^e, \lambda) = \lambda W_{\Pi}(\mathcal{P}(M, p^e, \lambda), \mathcal{L}(M, p^e, \lambda)) + (1 - \lambda)W_{\Omega}(p^e, \mathcal{L}(M, p^e, \lambda)), \quad (17)$$

which shows an interaction of the effects of the price law and the employment law in the definition. However, it is possible in some special situations to determine the effects under

more restricted conditions. Writing the wage as the associated mark-up over the reservation wage of the workers (or equivalently as a mark-down from the reservation wage of the producer)

$$\begin{aligned} w &= \left(1 + \lambda \frac{E_S(\mathcal{L}(M, p^e, \lambda)) + 1 - E_F(\mathcal{L}(M, p^e, \lambda))}{E_F(\mathcal{L}(M, p^e, \lambda))} \right) W_\Omega(p^e, \mathcal{L}(M, p^e, \lambda)) \\ &= \left(\lambda + (1 - \lambda) \frac{E_F(\mathcal{L}(M, p^e, \lambda))}{E_S(\mathcal{L}(M, p^e, \lambda)) + 1} \right) W_\Pi(\mathcal{P}(M, p^e, \lambda), \mathcal{L}(M, p^e, \lambda)), \end{aligned}$$

one observes that the state variables exert their influence on wages via a primary effect through the price and employment laws and a secondary effect through the respective elasticities, which determine the mark-up. Therefore, in situations where the effect of the state variable on the mark-up is small and can be neglected, the wage effect has the same sign as the employment effect, i. e.

$$\text{sgn } E_W(M) = \text{sgn } E_S(L)E_{\mathcal{L}}(M) > 0$$

$$\text{sgn } E_W(p^e) = \text{sgn } (E_{\mathcal{P}}(p^e) - (1 - E_F(L))E_{\mathcal{L}}(p^e)) > 0$$

In this case, wages increase with money balances and with price expectations. This indicates, however, that wages can also fall when employment increases.

The effect of the state variables on the real wage can be determined using the same procedure. Writing the real wage as

$$\frac{w}{p} = \left(\frac{\lambda}{E_F(\mathcal{L}(M, p^e, \lambda))} + \frac{1 - \lambda}{E_S(\mathcal{L}(M, p^e, \lambda)) + 1} \right) F'(\mathcal{L}(M, p^e, \lambda)), \quad (18)$$

one finds that it can be written as a positive multiple of average labor productivity or of the marginal product of labor respectively. Therefore, *for given* λ , due to the concavity of the production function with average productivity declining in L , output and employment always move in the opposite direction as the real wage with respect to the state variables (M, p^e) , provided that the elasticities are constant or do not change too much.

The Role of Union Power

Since the parameter λ does not influence aggregate supply, the assumption $\partial D / \partial \theta^e \geq 0$ implies

$$\text{sgn } \frac{\partial \mathcal{P}}{\partial \lambda} = \text{sgn } \frac{\partial D}{\partial \lambda} < 0.$$

Therefore, an increase of union power has a negative effect on the temporary equilibrium price, i. e. the elasticity with respect to union power $E_{\mathcal{P}}(\lambda) < 0$ is negative. Therefore, an increase in union power induces a reduction of prices, output, and employment. Using the properties of the employment law (16) one has

$$E_{\mathcal{L}}(\lambda) = -E_h(\theta^e)E_{\mathcal{P}}(\lambda) < 0 \quad \text{and} \quad E_{\mathcal{L}}(\lambda) < E_F(L)E_{\mathcal{L}}(\lambda) = E_Y(\lambda) < 0. \quad (19)$$

Figure 5 portrays the effects of changes of union power on equilibrium prices, showing that there exists a strong nonlinear feedback from the bargaining power to equilibrium prices, output, and employment. Thus, while the wage bargaining procedure assumes price-taking behavior

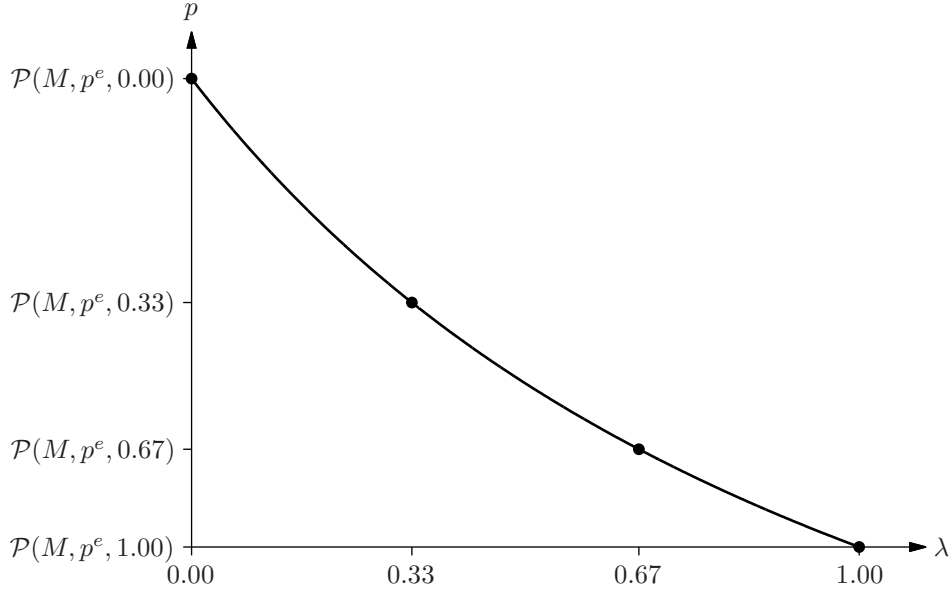


Figure 5: Range of equilibrium prices $\mathcal{P}(M, p^e, \lambda)$ for λ from 0 to 1

on behalf of both parties inducing a perceived wage increase under increased union power, the level λ of union power has a negative indirect or spillover effect on the equilibrium price which operates through a negative income effect on aggregate demand.

The bargaining power λ enters in multiple but opposite ways into the wage equation (17), similar to money balances and price expectations (M, p^e) . This implies that, in general, the overall effect of union power on the equilibrium wage cannot be signed. However, the effect of λ on the real wage can be determined using the same technique as above. Rewriting the real wage equation (18) in the form of (7) as

$$\frac{w}{p} = \left(\frac{E_F(\mathcal{L}(M, p^e, \lambda))}{E_S(\mathcal{L}(M, p^e, \lambda)) + 1} + \lambda \left(1 - \frac{E_F(\mathcal{L}(M, p^e, \lambda))}{E_S(\mathcal{L}(M, p^e, \lambda)) + 1} \right) \right) \frac{F(\mathcal{L}(M, p^e, \lambda))}{\mathcal{L}(M, p^e, \lambda)},$$

one finds that it must increase with union power whenever the wage is nonincreasing or when the effect of λ on the elasticities can be neglected.

3.2 Inefficient Redistribution under Efficient Bargaining

The negative feedback of union power on prices, output, and employment derived in (19) indicates that, from a macroeconomic point of view, a strong union under efficient bargaining may not guarantee an overall efficient allocation in temporary equilibrium. In other words, given the data of the economy (M, p^e, λ) , output is maximal when $\lambda = 0$ and minimal when $\lambda = 1$. This suggests that the bargaining procedure will never attain the global maximal surplus in the economy unless $\lambda = 0$.

To investigate the role of the bargaining power more closely, consider the payoff vector (Π, Ω) in temporary equilibrium, which is obtained by substituting the price law $\mathcal{P}(M, p^e, \lambda)$ from (13) and the wage law from (16) into the payoff vector (6). This yields

$$\begin{pmatrix} \Pi(M, p^e, \lambda) \\ \Omega(M, p^e, \lambda) \end{pmatrix} := \left(\mathcal{P}(M, p^e, \lambda) F(\mathcal{L}(M, p^e, \lambda)) - p^e S(\mathcal{L}(M, p^e, \lambda)) \mathcal{L}(M, p^e, \lambda) \right) \begin{pmatrix} 1 - \lambda \\ \lambda \end{pmatrix}.$$

Thus, the efficient bargaining solution at the temporary equilibrium is a linear one-to-one redistribution of the total net surplus

$$\mathbf{\Pi}(M, p^e, \lambda) + \mathbf{\Omega}(M, p^e, \lambda) = \mathcal{P}(M, p^e, \lambda)F(\mathcal{L}(M, p^e, \lambda)) - p^e S(\mathcal{L}(M, p^e, \lambda))\mathcal{L}(M, p^e, \lambda), \quad (20)$$

implying a marginal rate of substitution between $\mathbf{\Pi}(M, p^e, \lambda)$ and $\mathbf{\Omega}(M, p^e, \lambda)$ equal to minus one. Taking the derivative of (20) with respect to λ , one finds that

$$\begin{aligned} & \frac{d}{d\lambda}(\mathbf{\Pi}(M, p^e, \lambda) + \mathbf{\Omega}(M, p^e, \lambda)) \\ &= F(\mathcal{L}(M, p^e, \lambda)) \frac{\partial \mathcal{P}(M, p^e, \lambda)}{\partial \lambda} + \underbrace{\frac{d}{dL}(pF(L) - p^e S(L)L)}_{\stackrel{(3)}{=}0} \frac{\partial \mathcal{L}(M, p^e, \lambda)}{\partial \lambda} \\ &= F(\mathcal{L}(M, p^e, \lambda)) \frac{\partial \mathcal{P}(M, p^e, \lambda)}{\partial \lambda} < 0 \end{aligned} \quad (21)$$

has a negative sign. Thus, the aggregate equilibrium surplus is a strictly decreasing function with a global maximum at $\lambda = 0$. Geometrically speaking, this implies that the bargaining possibility frontier for all $0 < \lambda \leq 1$ in temporary equilibrium is strictly below the minus one tradeoff line at $\mathbf{\Pi}(M, p^e, 0) + \mathbf{\Omega}(M, p^e, 0)$.

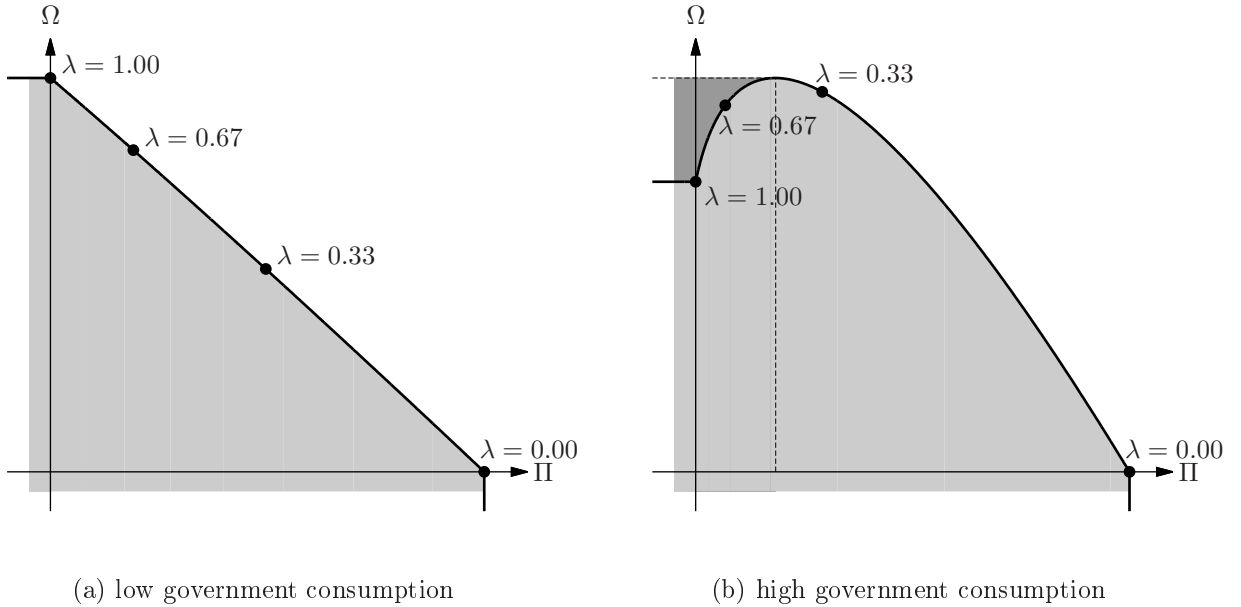


Figure 6: The payoff frontier: the role of union power and government consumption

It follows from (21) that the return to the producer $(1 - \lambda)(\mathbf{\Pi}(M, p^e, \lambda) + \mathbf{\Omega}(M, p^e, \lambda))$ is decreasing in λ while the influence on the wage bill cannot be signed in all cases. In fact, it may be increasing or decreasing depending on the data. Figure 6 displays the equilibrium payoff frontier for two different levels of government consumption, taking the feedback into account. Both panels show that the distribution of wealth is not linear in λ . The right panel describes a situation where the wage bill is declining with union power in some circumstances.

Finally, the two properties of declining aggregate surplus (21) and the linearity of the payoffs *for given* λ imply that the bargaining solution is not Nash efficient from a macroeconomic perspective at the equilibrium price $\mathcal{P}(M, p^e, \lambda)$ for all $\lambda > 0$. The argument is given geometrically

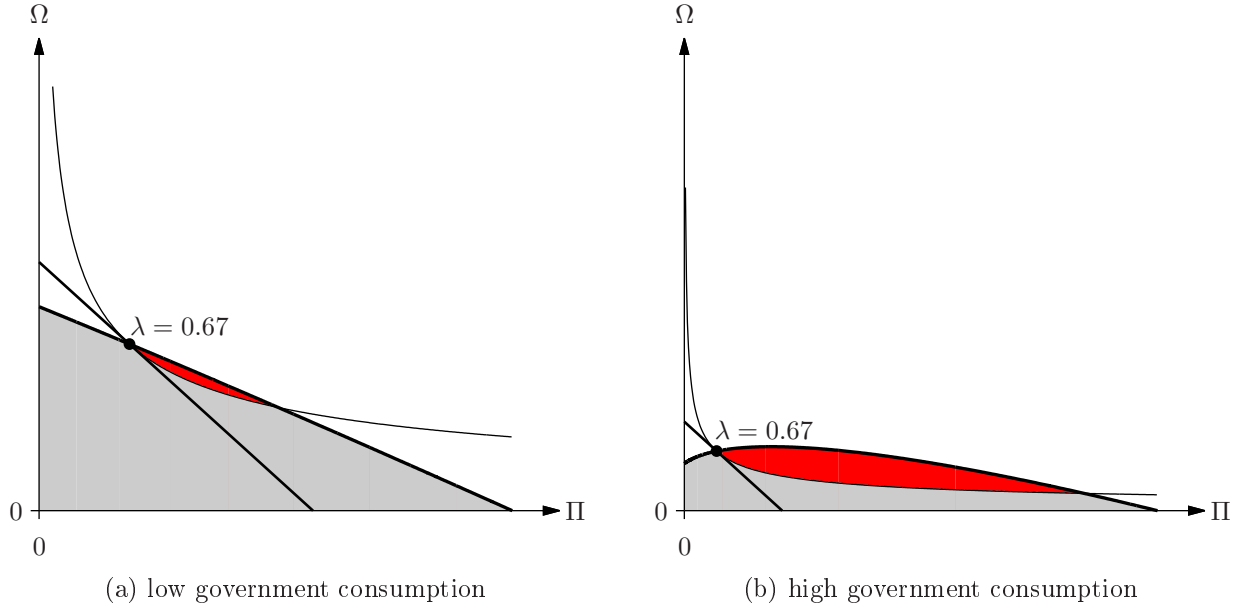


Figure 7: No efficient Nash bargaining solution under price feedback: the better set (red)

using Figure 7. Since both groups are price takers in the commodity market, they assume that its price is given and unaffected by their wage setting for given λ . Thus, the negotiating parties have a *perceived payoff frontier* with slope minus one while the slope of the true bargaining frontier is less than one in absolute value at every point. Figure 7 displays the level curve of the Nash bargaining solution for a given λ , which must have slope minus one at the equilibrium payoff. Since the slope of the bargaining frontier is flatter or even positive, the upper contour set of the Nash product intersects the feasible bargaining set, indicated geometrically by the red regions. Hence, there exists a lower λ and a redistribution at the equilibrium price $p = \mathcal{P}(M, p^e, \lambda)$ which improves the Nash product. Note, however, that each such improvement is again suboptimal as long as $\lambda > 0$.

3.3 Summary

For a general discussion of the role of bargaining as a wage determination device, one should note first that temporary equilibria with efficient bargaining exist and they are unique under the same set of assumptions as in other cases of wage setting with price flexibility and market clearing. Thus, efficient bargaining by itself cannot be the cause for involuntary unemployment. In particular, the temporary equilibrium induced by λ_{nat} coincides with the equilibrium with competitive behavior on both markets making the competitive outcome supportable by efficient bargaining.

From a macroeconomic point of view, however, the most striking result is that higher union power directed toward a desired and successful redistribution from profits to wages in temporary equilibrium always causes lower employment and lower output. This uniform negative impact of union power on employment and total output has additional allocative consequences. With constant exogenous demand (government demand plus money balances), an increase of union power implies lower profits and lower effective demand by young shareholders. Production becomes less attractive to producers even if the income distribution (i.e. the profit share in

output) stays constant, but the demand multiplier decreases. In other words, aggregate output to be distributed for private and public consumption declines with higher union power.

Therefore, if total output or aggregate private consumption in temporary equilibrium is considered as a welfare proxy, it would not be desirable to have a strong union imposing a high level of λ . However, the redistribution due to a higher wage bill implies higher savings and demand for money by workers inducing higher expected consumption in the second period. Thus, higher union power also induces an increase of real wealth for workers and higher expected indirect utility. Thus, young shareholders partly pay the bill of high union power through reduced consumption in both periods. Nevertheless, this increase always incurs a macroeconomic cost of lower total output.

Finally, it was shown that an efficient bargaining procedure between the participants in the labor market alone does not lead to an efficient outcome with respect to the objective of the bargaining when the remaining market is competitive. Generally speaking, this reconfirms the typical features of results known from Second-Best Theory, which say that noncompetitive or deviant behavior in one market alone while all others are competitive does not guarantee Second-Best allocations if there are spillovers between markets. Notice that this result equally applies to the competitive temporary equilibrium. In other words, even the fully competitive temporary equilibrium is not efficient with respect to the bargaining criterion, due to the price feedback. Thus, the exogenous parametric setting of the negotiating power of one side of the market induces only an efficient allocation with respect to the *perceived* feasible bargaining set, and which is inefficient with respect to general equilibrium feasibility. Thus, an efficient level of bargaining power would have to be determined endogenously.

From a general welfare perspective, however, it is not clear whether this inefficiency implies also suboptimality and failure to satisfy a Second-Best property since both criteria are applied to a comparative-statics analysis of allocations in temporary equilibrium at given money balances and expectations. Therefore, for the *dynamic* macroeconomic perspective taken here with overlapping generations of consumers, the Second-Best failure may not seem to be of such primary importance. Moreover, the welfare issue becomes even more complex for sequences of temporary equilibria and requires further criteria and investigations, also with respect to stationary states. What they imply for the dynamic development will be analyzed partly in Section 4.

4 Dynamics of Monetary Equilibrium

So far the characteristics of equilibria under bargaining were discussed for an arbitrary given period t with initial money balances M_t held by the private sector, expected prices for the next period by consumers $p_{t,t+1}^e$, and by the union power λ_t . Applying the respective mappings from the previous section, one obtains the uniquely defined levels of all other equilibrium values under competitive conditions in the commodity market, i. e. the price level, the negotiated wage level, the induced levels of output and employment (p_t, w_t, y_t, L_t) as well as the relative share of wages over profits $\lambda_t/(1 - \lambda_t)$. Thus, the triple $(M_t, p_{t,t+1}^e, \lambda_t)$ uniquely describes the state of the economy at any given time inducing all other variables.

The emphasis of the dynamic analysis in this section consists in exhibiting the major monetary mechanisms through fiscal deficits assuming that union power is constant over time and given

exogenously at some level $0 \leq \lambda \leq 1$. This implies that both sides of the bargaining process behave myopically paying no attention to intertemporal aspects. Since λ has a significant impact on most economic variables in every period, like output, incomes, prices, and consumption, which are relevant for welfare, it would be desirable to determine the level of union power in every period endogenously derived, for example, from criteria of optimal sequential (or recurrent) bargaining. An alternative approach could use different forms of adaptive intertemporal procedures (applying an agent-based approach) using aspiration levels and comparing them with the sequence of economic data achieved. Here we restrict the analysis to the myopic case without providing a justification what level of union power λ would be reasonable to be assumed, leaving such questions to be addressed in future research.¹⁰

4.1 Dynamics of Money Balances

Under constant union power λ , the dynamic development of the economy will be described completely by characterizing the evolution of the two state variables money balances and expected prices $(M_t, p_{t,t+1}^e)$, implying a two-dimensional state space \mathbb{R}_{++}^2 . Therefore, an analysis of the dynamic evolution of the economy requires the description of how money balances evolve and of how expectations are formed. The comparative-statics analysis in the previous section has shown that the size of the price feedback from bargaining varies with union power, a fact which originates from the impact of the bargaining power on the income distribution which in turn influences aggregate demand. Therefore, it is not unexpected that the dynamics of money balances depends on the bargaining power as well.

Given the fact that consumers live for two periods only, old consumers do not save so that aggregate private savings consists of money balances held by young consumers. Given the assumption that young workers do not consume in the first period of their live while shareholders consume a proportion $0 < c(\theta_{t,t+1}^e) < 1$ of profits, it follows that aggregate savings $S_t = M_{t+1}$ is given by

$$M_{t+1} = S_t := \underbrace{(1 - \tau_w)w_t L_t + (1 - c(\theta_{t,t+1}^e))(1 - \tau_\pi)\pi_t}_{\geq 0}.$$

Using income consistency $p_t y_t = M_t + p_t g + c(\theta_{t,t+1}^e)(1 - \tau_\pi)\pi_t$ and substituting the consumption expenditures of shareholders, one obtains

$$\begin{aligned} M_{t+1} &= M_t + p_t y_t \left((1 - \tau_w) \frac{w_t L_t}{p_t y_t} + (1 - \tau_\pi) \frac{\pi_t}{p_t y_t} \right) + p_t (g - y_t) \\ &= M_t + p_t g - \underbrace{\left(1 - (1 - \tau_w) \frac{w_t L_t}{p_t y_t} - (1 - \tau_\pi) \frac{\pi_t}{p_t y_t} \right) p_t y_t}_{\text{tax revenue}} \end{aligned}$$

as the equation for the evolution of money balances where for each variable the value of the corresponding equilibrium mapping evaluated at $(M_t, p_{t,t+1}^e, \lambda)$ must be substituted. The term in parenthesis can be interpreted as the average tax rate on total income implying the definition

$$\tilde{\tau}(M_t, p_{t,t+1}^e, \lambda) := 1 - (1 - \tau_w) \frac{w_t L_t}{p_t y_t} - (1 - \tau_\pi) \frac{\pi_t}{p_t y_t} \quad (22)$$

¹⁰The contribution by Selten & Güth (1982) treating a simplified macro model is one of the rare attempts of an integration of sequential bargaining into a fully dynamic general-equilibrium analysis. Others have discussed intertemporal issues in more explicit macro models, for example Blanchard & Fischer (1993).

using the respective equilibrium mappings. Writing the shares of wages and profits as elasticities as in (8) and (9), respectively, i. e. $B := E_F(h(\theta_{t,t+1}^e))$ and $C := 1/E_S(h(\theta_{t,t+1}^e))$ for short, and using the fact that $\theta_{t,t+1}^e \equiv p_{t,t+1}^e/\mathcal{P}(M_t, p_{t,t+1}^e, \lambda)$, one obtains

$$\begin{aligned}\tilde{\tau}(M_t, p_{t,t+1}^e, \lambda) &= 1 - (1 - \tau_w) \left(\frac{BC}{C+1} + \lambda \left(1 - \frac{BC}{C+1} \right) \right) - (1 - \tau_\pi)(1 - \lambda) \left(1 - \frac{BC}{C+1} \right) \\ &= \left(\frac{BC}{C+1} + \lambda \left(1 - \frac{BC}{C+1} \right) \right) \tau_w + (1 - \lambda) \left(1 - \frac{BC}{C+1} \right) \tau_\pi \\ &= \tau_\pi + \left(\frac{BC}{C+1} + \lambda \left(1 - \frac{BC}{C+1} \right) \right) (\tau_w - \tau_\pi).\end{aligned}$$

This equation exhibits the role of the different parameters in a transparent way. First, $\tilde{\tau}$ is between zero and one and is a convex combination of the two tax rates on wages τ_w and on profits τ_π . The weights for given union power λ are determined by the elasticities of the labor supply function and of the production function. Second, for constant elasticities $0 < B < 1$ and $C > 0$, union power λ has a positive (negative) impact on the average tax rate if and only if the tax rate on wage income is higher (lower) than the rate on profit income.

To conclude, the dynamic law (mapping) for money balances \mathcal{M} can be written in the usual format

$$M_{t+1} = \mathcal{M}(M_t, p_{t,t+1}^e, \lambda) := M_t + \mathcal{P}(M_t, p_{t,t+1}^e, \lambda) (g - \tilde{\tau}(M_t, p_{t,t+1}^e, \lambda) \mathcal{Y}(M_t, p_{t,t+1}^e, \lambda)) \quad (23)$$

showing that money balances change over time if and only if the government runs a nonzero deficit.

4.2 Dynamics with Perfect Foresight

For the description of consumer expectations, we analyze the situation under perfect foresight.¹¹ A sequence $\{p_{t,t+1}^e, p_t\}_{t=0}^\infty$ of prices and expectations will be said to have the *perfect-foresight property* if $p_{t,t+1}^e = p_{t+1}$ (or equivalently $p_{t-1,t}^e = p_t$) holds for all t . In order to guarantee that, for any period t , the actual price p_t coincides with its associated prediction $p_{t-1,t}^e$, the condition

$$p_{t-1,t}^e \stackrel{!}{=} p_t = \mathcal{P}(M_t, p_{t,t+1}^e, \lambda) \quad (24)$$

must hold for any t . This defines implicitly a functional relationship between any two successive forecasts, i. e. determining of how a perfect forecast should be chosen as a function of the previous forecast. Suppose for the moment that (24) has a unique solution, i. e. \mathcal{P} has an inverse with respect to $p_{t,t+1}^e$ defining a mapping $\psi^* : \mathbb{R}_+^2 \times [0, 1] \rightarrow \mathbb{R}_+$ by

$$p_{t,t+1}^e = \mathcal{P}^e(M_t, p_{t-1,t}^e, \lambda) =: \psi^*(M_t, p_{t-1,t}^e, \lambda).$$

Then, the forecast $p_{t,t+1}^e$ chosen in this way in period t is the unique prediction to make the forecast of period $t+1$ perfect. Since the information to make this forecast is available at the beginning of period t , any agent or forecasting agency can use the function \mathcal{P}^e to make such a perfect forecast. The fact that the mapping \mathcal{P} does not depend on time indicates that this logic

¹¹Böhm & Claas (2012a) analyzes the dynamics with rational expectations in the presence of technology shocks.

can be used in *every* period t so that the function $\psi^* \equiv \mathcal{P}^e$ is a valid perfect prediction rule in *every* period. This reasoning is the essential motivation to define the concept of a *perfect predictor* as a stationary mapping, i.e. a time-invariant forecasting rule which, if applied in every period, induces perfect foresight along any orbit.¹²

To show that such a perfect predictor exists for the bargaining model under the price law \mathcal{P} (from Proposition 3.1), let us assume for the remainder of this section that we can neglect the expectations effect in aggregate demand. This occurs, for example, when the production function and the labor supply function are isoelastic and when the propensity to consume c of the shareholder is constant.¹³ Then one obtains the following proposition.

Proposition 4.1 *Let the equilibrium condition (12) be of the form*

$$D\left(\frac{M}{p}, \lambda\right) = AS\left(\frac{p^e}{p}\right)$$

and assume that AS is globally invertible. There exists a unique globally defined perfect predictor $\psi^ : \mathbb{R}_+^2 \times [0, 1] \rightarrow \mathbb{R}_+$ given by*

$$p_{t,t+1}^e = \psi^*(M_t, p_{t-1,t}^e, \lambda) \equiv \mathcal{P}^e(M_t, p_{t-1,t}^e, \lambda) := p_{t-1,t}^e AS^{-1}\left(D\left(\frac{M_t}{p_{t-1,t}^e}, \lambda\right)\right)$$

inducing the equality $p_t = p_{t-1,t}^e$ for every t .

The statement of the proposition makes the previous heuristic reasoning precise, which is essentially the argument of the proof. Therefore, for any given λ , choosing the predictor ψ^* and substituting into the dynamic law (23) for money balances $M_{t+1} = \mathcal{M}(M_t, \psi^*(M_t, p_{t-1,t}^e, \lambda), \lambda) =: \mathcal{M}_{\psi^*}(M_t, p_{t-1,t}^e, \lambda)$ leads to the two-dimensional dynamical system in money balances and expectations defined by

$$\begin{pmatrix} M_{t+1} \\ p_{t,t+1}^e \end{pmatrix} = \begin{pmatrix} \mathcal{M}_{\psi^*}(M_t, p_{t-1,t}^e, \lambda) \\ \psi^*(M_t, p_{t-1,t}^e, \lambda) \end{pmatrix} = \begin{pmatrix} \mathcal{M}(M_t, \psi^*(M_t, p_{t-1,t}^e, \lambda), \lambda) \\ \psi^*(M_t, p_{t-1,t}^e, \lambda) \end{pmatrix}$$

whose orbits induce the perfect-foresight property.

The average tax rate (22) under perfect foresight, rewritten as

$$\tilde{\tau}(M_t, \psi^*(M_t, p_{t-1,t}^e, \lambda), \lambda) = \tilde{\tau}\left(\frac{M_t/p_{t-1,t}^e}{\psi^*(M_t/p_{t-1,t}^e, 1, \lambda)}, 1, \lambda\right) =: \tilde{\tau}_{\psi^*}\left(\frac{M_t}{p_{t-1,t}^e}, \lambda\right),$$

together with $y_t = D(M_t/p_{t-1,t}^e, \lambda)$ yields a two-dimensional system under perfect foresight in the usual format

$$\begin{pmatrix} M_{t+1} \\ p_{t,t+1}^e \end{pmatrix} = \begin{pmatrix} \mathcal{M}_{\psi^*}(M_t, p_{t-1,t}^e, \lambda) \\ \psi^*(M_t, p_{t-1,t}^e, \lambda) \end{pmatrix} = \begin{pmatrix} M_t + p_{t-1,t}^e \left(g - \tilde{\tau}_{\psi^*}\left(\frac{M_t}{p_{t-1,t}^e}, \lambda\right) D\left(\frac{M_t}{p_{t-1,t}^e}, \lambda\right)\right) \\ p_{t-1,t}^e AS^{-1}\left(D\left(\frac{M_t}{p_{t-1,t}^e}, \lambda\right)\right) \end{pmatrix}. \quad (25)$$

Finally, since all orbits have the perfect-foresight property, one can substitute the corresponding values for all t implying that the system (25) can be rewritten equivalently as

$$\begin{pmatrix} M_{t+1} \\ p_{t+1} \end{pmatrix} = \begin{pmatrix} \mathcal{M}_{\psi^*}(M_t, p_t, \lambda) \\ \psi^*(M_t, p_t, \lambda) \end{pmatrix} = \begin{pmatrix} M_t + p_t \left(g - \tilde{\tau}_{\psi^*}\left(\frac{M_t}{p_t}, \lambda\right) D\left(\frac{M_t}{p_t}, \lambda\right)\right) \\ p_t AS^{-1}\left(D\left(\frac{M_t}{p_t}, \lambda\right)\right) \end{pmatrix}. \quad (26)$$

¹²See Böhm & Wenzelburger (1999), Böhm (2010) for a general discussion of perfect predictors.

¹³The general case can be dealt with using Lemma 3.1.

Thus, the fact that there exists a globally defined perfect predictor (Proposition 4.1) implies that the perfect-foresight dynamics of the economy are generated by an *explicit* forward recursive two-dimensional system in the space of money balances and *actual* prices (M, p) for any given level λ of bargaining power.

4.3 Steady States and Stability

Let $(M, p) \gg 0$ be a steady state of the system (26) inducing constant money balances and constant prices (zero inflation) with a balanced government budget, and real balances $m = M/p$. This implies that the two conditions

$$g = \tilde{\tau}_{\psi^*}(m, \lambda)D(m, \lambda) \quad \text{and} \quad 1 = AS^{-1}(D(m, \lambda)) \quad (27)$$

hold simultaneously. Monotonicity, homogeneity, and continuity of aggregate demand in (M, p) imply that there exists a continuum of fixed points since every positive multiple of (M, p) is a fixed point of (26) as well. Geometrically speaking this implies that the set of positive steady states consists of the half-line in the state space \mathbb{R}_{++}^2 with slope m . Notice that the condition (27) requires $g = \tilde{\tau}_{\psi^*}(m, \lambda)AS(1)$, imposing equality of two values which are determined structurally by separate and independent features of the economy (see for example (22)). Thus, in the space of parameters of the economy, condition (27) can hold only on a set of measure zero, implying that perfect-foresight steady states with a balanced government budget do not exist generically.

For the local stability of such (rare) fixed points, one obtains from the Jacobian of the system¹⁴

$$J = \begin{pmatrix} 1 - \tilde{\tau}_{\psi^*}(m, \lambda)E_D(m)\frac{AS(1)}{m} & \tilde{\tau}_{\psi^*}(m, \lambda)E_D(m)AS(1) \\ \frac{E_D(m)}{E_{AS}(1)}\frac{1}{m} & 1 - \frac{E_D(m)}{E_{AS}(1)} \end{pmatrix}$$

with trace

$$\text{tr } J = 2 - \tilde{\tau}_{\psi^*}(m, \lambda)E_D(m)\frac{AS(1)}{m} - \frac{E_D(m)}{E_{AS}(1)}$$

and determinant

$$\det J = 1 - \frac{E_D(m)}{E_{AS}(1)} - \tilde{\tau}_{\psi^*}(m, \lambda)E_D(m)\frac{AS(1)}{m} = \text{tr } J - 1.$$

Therefore, the two eigenvalues ν_1 and ν_2 are

$$\nu_{1,2} = \frac{\text{tr } J \pm \sqrt{(\text{tr } J)^2 - 4\det J}}{2} = \frac{\text{tr } J \pm (\text{tr } J - 2)}{2}$$

implying $\nu_1 = \text{tr } J - 1 = \det J$ and $\nu_2 = 1$. Since

$$\nu_1 = \text{tr } J - 1 = 1 - \underbrace{\frac{\tilde{\tau}_{\psi^*}(m, \lambda)AS(1)}{m}}_{=g/m} \underbrace{\frac{E_D(m)}{m/(m+g)}}_{=m/(m+g)} - \frac{E_D(m)}{E_{AS}(1)} = \underbrace{\frac{m}{m+g}}_{\geq 0} - \underbrace{\frac{E_D(m)}{E_{AS}(1)}}_{< 0} > 0,$$

¹⁴For simplicity, it is assumed that the average tax rate is independent of real balances. This occurs in the isoelastic example or under one common tax rate.

both eigenvalues are nonnegative, which excludes the possibility of cycles. To establish an upper bound for ν_1 , note that

$$\nu_1 = E_D(m) \left(1 - \frac{1}{E_{AS}(1)} \right) = \frac{m}{m+g} \left(1 - \frac{BC - (C+1)}{BC} \right) = \frac{m}{m+g} \frac{C+1}{BC}.$$

Thus, for $g > 0$, the ratio of elasticities $(C+1)/(BC)$ should be sufficiently small for stability while instability is likely for small B . Figure 8 displays the situation with a continuum of steady

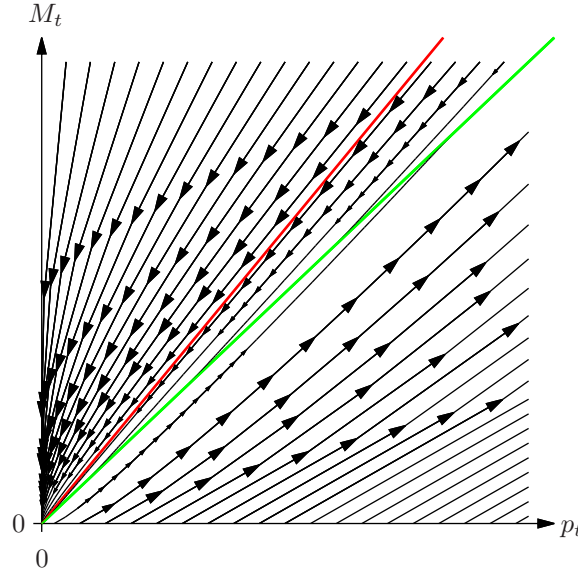


Figure 8: Convergence to a continuum of steady states

states under the parameterization given in Table 1. The green half-line is the set of steady states of (26) while the red half-line indicates the boundary of the basin of attraction, corresponding to an unstable balanced path (see the next section below). A numerical simulation for the values in Table 1 shows that all orbits starting within the basin of attraction (the area to the

A	B	C	τ_w	τ_π	λ	c	n_w
1.00	0.99	0.99	0.50	0.25	0.50	0.99	1

Table 1: Parameterization used in Figure 8

lower right of the red line) converge to a positive fixed point on the green line, whereas all paths originating in the triangle to the upper left of the red line converge to the origin with prices converging more slowly than money balances.

4.4 Dynamics of Real Money Balances under Perfect Foresight

Since in general fixed points of (26) do not exist, the economically interesting situations to be analyzed are those when money and prices expand or contract at the same rate, implying constant levels of real money balances together with constant allocations. Such orbits are called balanced paths.

Definition 4.1 *An orbit $\{(M_t, p_t)\}_{t=0}^\infty$ of the system (26) is called a balanced path if for all t one has $m_t := M_t/p_t = M_{t+1}/p_{t+1} = m_{t+1}$.*

Balanced paths can be identified with half-lines in the state space \mathbb{R}_{++}^2 . Exploiting the homogeneity of the two mappings describing the money dynamics and the price dynamics, (26) induces a one-dimensional system describing the dynamics of real balances, given by

$$m_{t+1} = \mathcal{F}(m_t) := \frac{\mathcal{M}_{\psi^*}(M_t, p_t, \lambda)}{\psi^*(M_t, p_t, \lambda)} = \frac{m_t + g - \tilde{\tau}_{\psi^*}(m_t, \lambda) D(m_t, \lambda)}{AS^{-1}(D(m_t, \lambda))}. \quad (28)$$

Positive fixed points of (28) are associated with positive balanced paths of (26). It is straightforward to show that $\mathcal{F}(m_t)$ is strictly increasing and strictly convex for all m_t . This implies that there exist at most two fixed points and that the dynamics are monotonic (no cycles). For the isoelastic example, the time-one map of real money balances has an explicit form.

Proposition 4.2 *Let the aggregate supply function be isoelastic with elasticity $-\frac{BC}{C+1-BC} < 0$. Assume that aggregate demand is of the form*

$$D(m, \lambda) = \frac{m + g}{\tilde{c}(\lambda)}, \quad (29)$$

where $1/\tilde{c}(\lambda)$ is the demand multiplier from (11), and that the average tax rate under perfect foresight $\tilde{\tau}_{\psi^*}(\lambda)$ introduced in (22) is independent of real balances.

Then, (28) has the isoelastic form

$$\begin{aligned} \mathcal{F}(m_t) &= \frac{m_t + g - \tilde{\tau}_{\psi^*}(\lambda) D(m_t, \lambda)}{AS^{-1}(D(m_t, \lambda))} = \frac{(\tilde{c}(\lambda) - \tilde{\tau}_{\psi^*}(\lambda)) \frac{m_t + g}{\tilde{c}(\lambda)}}{AS^{-1}\left(\frac{m_t + g}{\tilde{c}(\lambda)}\right)} \\ &= \frac{\tilde{c}(\lambda) - \tilde{\tau}_{\psi^*}(\lambda)}{AS^{-1}(1)} \left(\frac{m_t + g}{\tilde{c}(\lambda)} \right)^{\frac{C+1}{BC}}. \end{aligned} \quad (30)$$

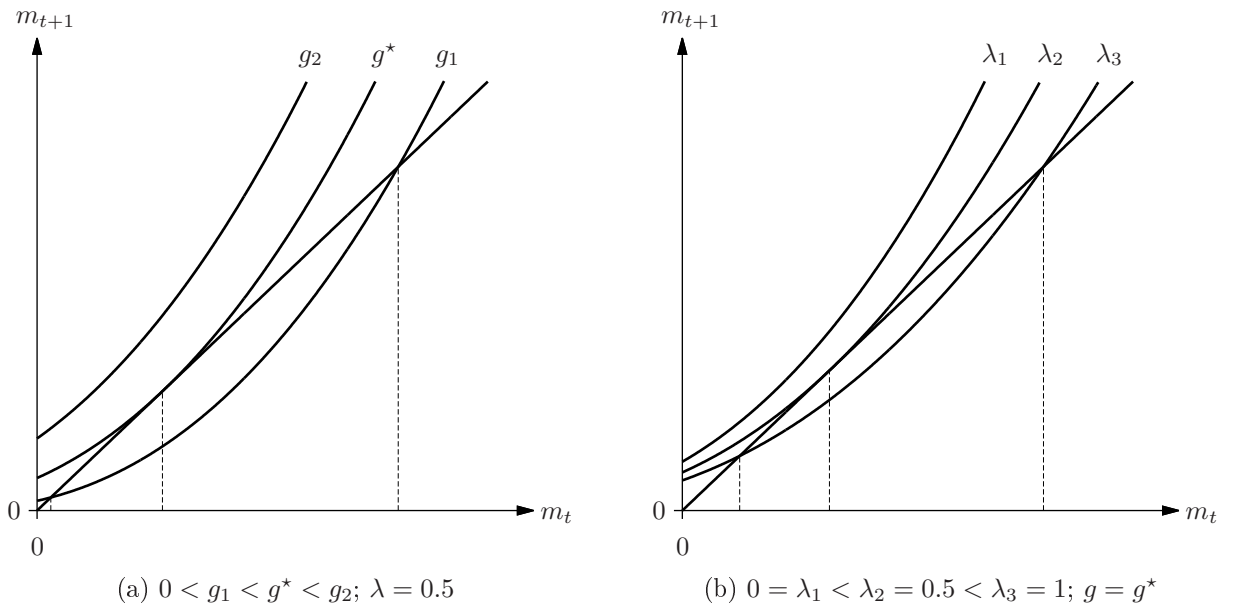


Figure 9: The role of government consumption and of union power

The analysis of the dynamics of (28) is now straightforward. For each (g, λ) , the mapping \mathcal{F} has an elasticity greater than one in $m + g$ with $\mathcal{F}(0) = 0$ for $g = 0$. Thus, \mathcal{F} is strictly convex

and increasing with $\lim_{m \rightarrow \infty} \mathcal{F}(m)/m = \infty$ while government consumption $g > 0$ induces a horizontal shift of its graph. Therefore, there exists a critical level $g^* > 0$ such that \mathcal{F} has no fixed points for $g > g^*$, exactly one fixed point for $g = g^*$, and two positive fixed points for $0 < g < g^*$, see Figure 9 (a). Similarly, for fixed g , a change of union power induces a family

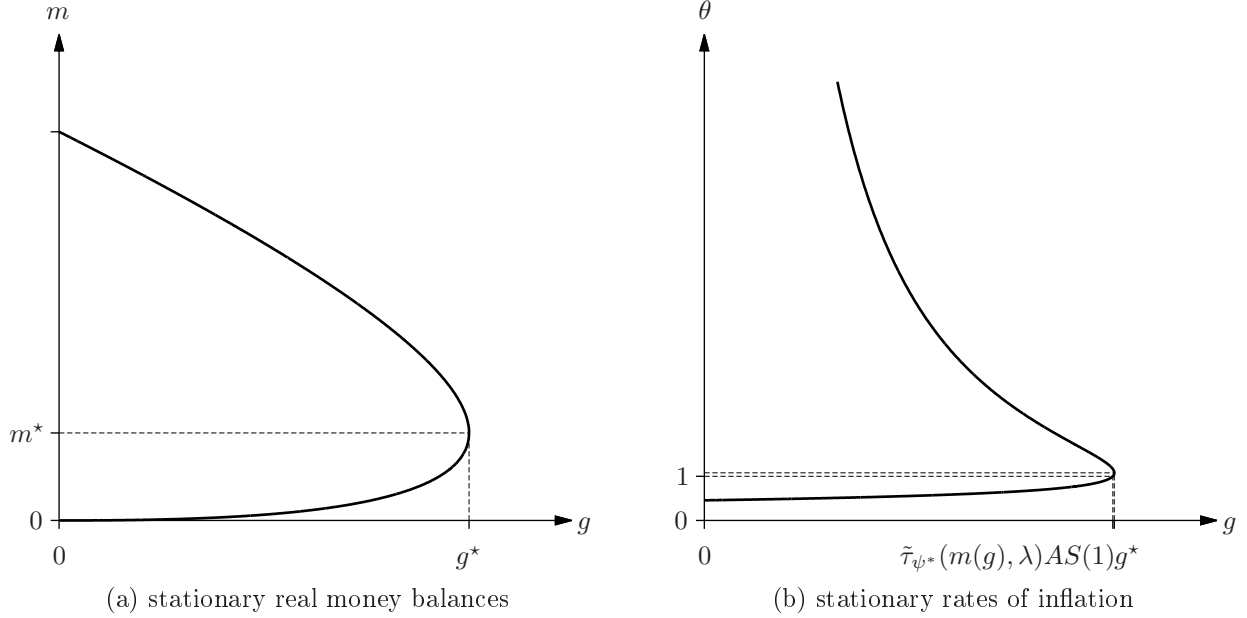


Figure 10: Stationary states for parameters as in Table 1; λ constant

of convex time-one maps through its influence on the demand multiplier and the average tax rate. Figure 9 (b) displays the situation where union power has an overall negative effect on the dynamics of real balances which occurs when, for example, $\tau_\pi = \tau_w$.

For any given pair $(g, \lambda) \geq 0$, the rate of inflation at a fixed point m is a strictly monotonically decreasing function $\theta = AS^{-1}(D(m, \lambda))$ in real money holdings. Therefore, the lower fixed point is associated with a higher rate of inflation than the upper one. If $g \rightarrow 0$, real money balances at the lower fixed point tend to zero, which implies that the equilibrium rate of inflation tends to infinity. Thus, for small g the lower balanced path always has positive inflation. However, this does not imply that the upper one is always associated with deflation since $\theta^* = AS^{-1}(D(m^*, \lambda))$ associated with g^* may be larger or smaller than one. Hence, by continuity, there may exist deflationary steady states as well for some g close to g^* , see for example Figure 10 (b).

If two fixed points exist, by convexity and monotonicity of \mathcal{F} , the lower one is asymptotically stable with the basin of attraction being the half-open interval between zero (included) and the upper fixed point (excluded), see Figure 11.

The linearity of aggregate demand in $m + g$ implies for any fixed point $m = \mathcal{F}(m)$

$$\frac{m}{m+g} = \frac{\tilde{c}(\lambda) - \tilde{\tau}_{\psi^*}(\lambda)}{\tilde{c}(\lambda)} \frac{1}{AS^{-1}\left(\frac{m+g}{\tilde{c}(\lambda)}\right)}.$$

In addition, one finds for the isoelastic case (30)

$$\mathcal{F}'(m) = E_{\mathcal{F}}(m) = \frac{C+1}{BC} \frac{m}{m+g} = \frac{C+1}{BC} \frac{\tilde{c}(\lambda) - \tilde{\tau}_{\psi^*}(\lambda)}{\tilde{c}(\lambda)} \frac{1}{AS^{-1}\left(\frac{m+g}{\tilde{c}(\lambda)}\right)}. \quad (31)$$

Figure 11 displays the stability/instability properties in the case of two fixed points.

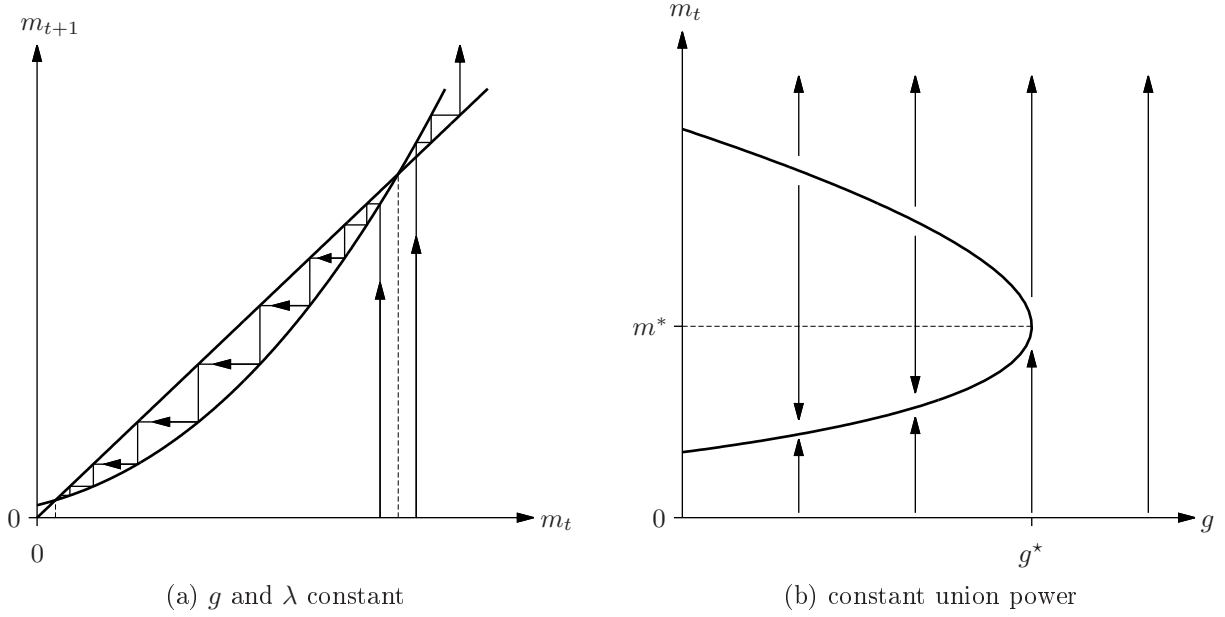


Figure 11: Stability and convergence

4.5 Stable Balanced Paths

It is well-known from models of economic growth that stability and convergence of the ratio of two variables is only a necessary condition for convergence of an orbit to a balanced path in the two-dimensional state space. In other words, stability in real money balances does not imply convergence to the balanced path.¹⁵ For any $(M_t, p_t) \in \mathbb{R}_{++}^2$, let $\Delta_t := M_t - mp_t = (m_t - m)p_t$ denote the distance from the balanced path m for any t . Convergence of an orbit $\{(M_t, p_t)\}_t$ to the balanced path requires that this distance converges to zero in addition to the fact that $\lim_{t \rightarrow \infty} m_t = m$. This notion of convergence in the two-dimensional state space allows for permanent inflation resp. deflation (and thus an unbalanced governmental budget) when there exists a ray or half-line through the origin along which the system (26) moves in a balanced fashion.

Definition 4.2 Let $m = M'_t/p'_t > 0$ denote the level of real money balances associated with a balanced path $\{(M'_t, p'_t)\}_{t=0}^\infty$. An orbit $\{(M_t, p_t)\}_{t=0}^\infty$ of the dynamical system (26) is said to converge to the balanced path m if m_t converges to m and $\Delta_t = M_t - mp_t = (m_t - m)p_t$ converges to zero for $t \rightarrow \infty$.

A balanced path will be called (*asymptotically*) *stable* if orbits of the system (26) converge (*asymptotically*) in the sense of Definition 4.2. For any balanced path $m > 0$, one can write

$$\Delta_{t+1} = (m_{t+1} - m)p_{t+1} = \frac{m_{t+1} - m}{m_t - m} \frac{p_{t+1}}{p_t} (m_t - m)p_t = \frac{m_{t+1} - m}{m_t - m} \frac{p_{t+1}}{p_t} \Delta_t.$$

Since $p_{t+1}/p_t = AS^{-1}(D(m_t, \lambda))$, the dynamical system (28) in real money balances induces the two-dimensional dynamical system in (m, Δ) given by

$$\begin{pmatrix} m_{t+1} \\ \Delta_{t+1} \end{pmatrix} = \begin{pmatrix} \mathcal{F}(m_t) \\ \frac{\mathcal{F}(m_t) - m}{m_t - m} AS^{-1}(D(m_t, \lambda)) \Delta_t \end{pmatrix}. \quad (32)$$

¹⁵see Deardorff (1970); Böhm (2009); Pampel (2009)

Thus, a stable fixed point $(m, 0)$ of the system (32) implies convergence to the balanced path in the sense of Definition 4.2. The two eigenvalues of the Jacobian of (32) evaluated at a fixed point $(m, 0)$ are

$$\frac{\partial m_{t+1}}{\partial m_t}(m, 0) = \mathcal{F}'(m) \quad \text{and} \quad \frac{\partial \Delta_{t+1}}{\partial \Delta_t}(m, 0) = \mathcal{F}'(m)AS^{-1}(D(m, \lambda)),$$

which are both positive. Since the upper balanced path is associated with $\mathcal{F}'(m) > 1$, it can never be stable. Therefore, the lower one is stable if and only if $\mathcal{F}'(m)AS^{-1}(D(m, \lambda)) < 1$.

For the isoelastic case, (31) implies

$$\mathcal{F}'(m)AS^{-1}\left(\frac{m+g}{\tilde{c}(\lambda)}\right) = \frac{\tilde{c}(\lambda) - \tilde{\tau}_{\psi^*}(\lambda)}{\tilde{c}(\lambda)} \frac{C+1}{BC}$$

which is less than one if and only if

$$\frac{\tilde{c}(\lambda) - \tilde{\tau}_{\psi^*}(\lambda)}{\tilde{c}(\lambda)} < \frac{BC}{C+1}, \quad (33)$$

a condition relating the tax-adjusted multiplier $\tilde{c}(\lambda)$ to the elasticity of the aggregate supply function. Both terms of the inequality are positive and less than one.

In order to evaluate the significance of the condition (33), observe first that both sides of the inequality are independent of m and g . The value of the right-hand side of the inequality is determined exclusively by the parameters of the supply side. In principle, any value is possible so that for any given bargaining power λ stable as well as unstable balanced paths occur for large open sets of parameters. Since $\tilde{c}(\lambda)$ is increasing in λ , a stable situation for given $BC/(C+1)$ may be changed into an unstable one when union power λ increases. Figure 12

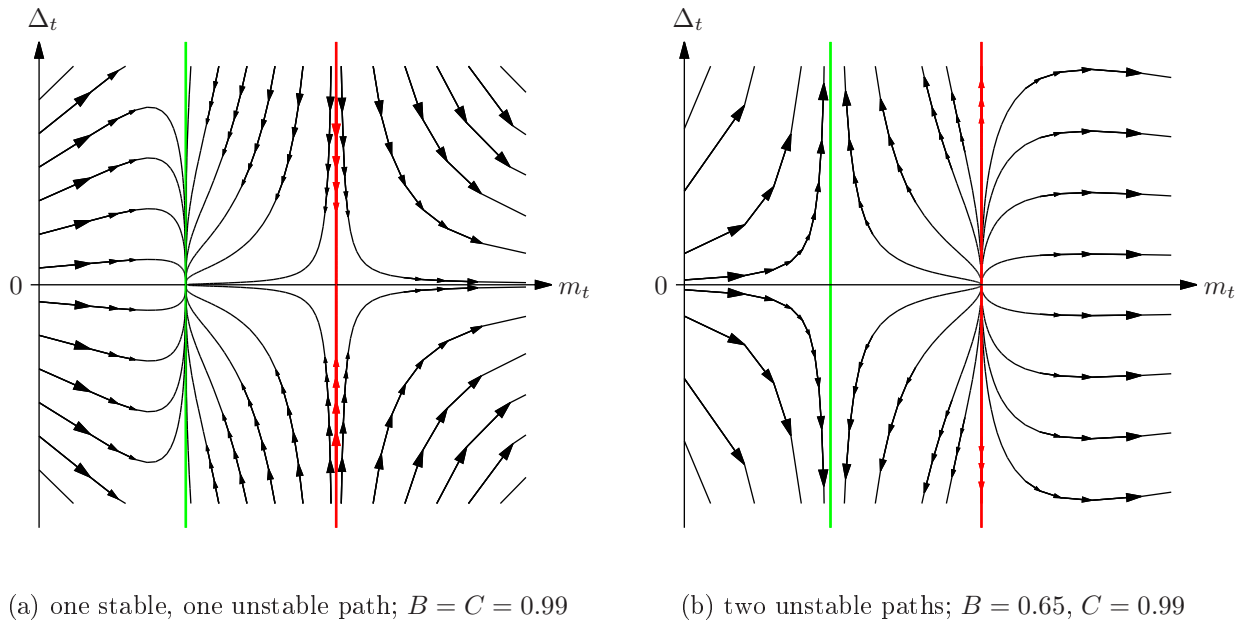


Figure 12: Stability of balanced paths in (m, Δ) -space; $\lambda = 0.5$

displays the phase portraits of the two cases, showing the stable case in panel (a), where the lower one is a sink and the upper one is a saddle. Panel (b) displays an unstable situation

where the lower steady state is a saddle and the upper steady state is a source. To each fixed point of the system (32) corresponds an associated balanced path in the state space (M, p) . Each of the phase portraits of Figure 12 have corresponding expanding orbits in the state space. Their phase portraits are shown in Figure 13 displaying the numerical results for the two parameterizations. Panel (a) shows that, for the parametrization given in Table 1, all paths with initial real money holdings below the level of the unstable steady state of (28) converge to a balanced path with the corresponding slope (level of real balances) in the state space of the system (26). In contrast, panel (b) indicates that both balanced paths are unstable when the value of B is decreased.

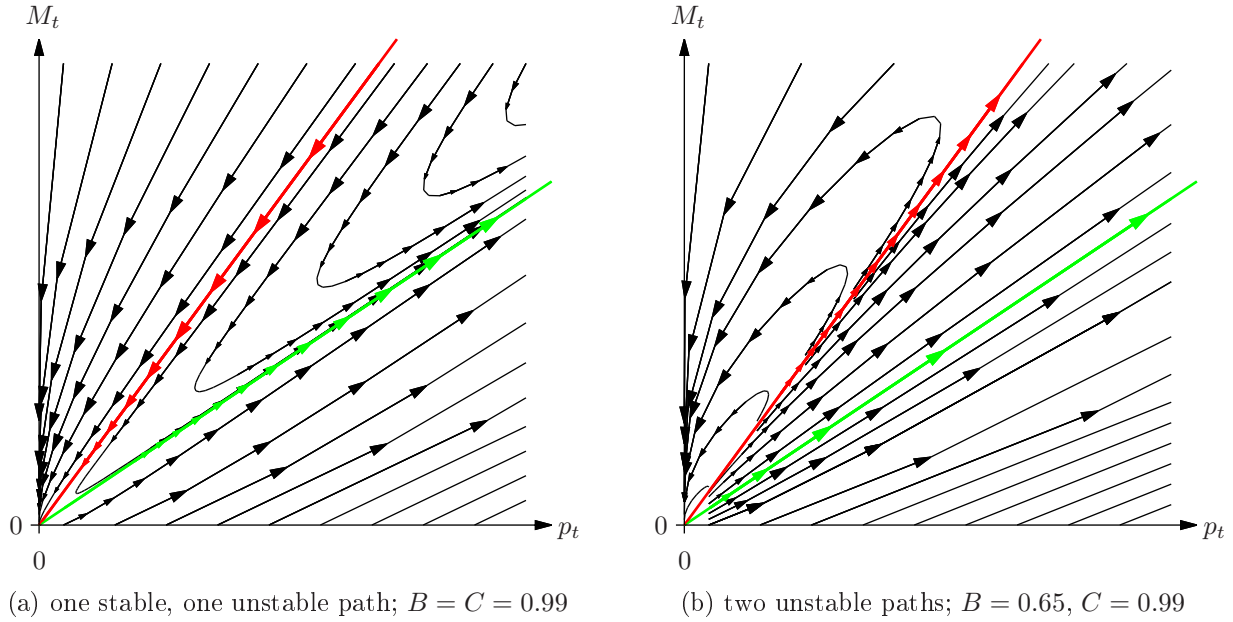


Figure 13: Stability of balanced paths in the state space; $\lambda = 0.5$

Finally, the critical levels (bifurcation values) of each parameter can be determined at which the lower balanced path loses stability. Given the parametric separability of the two sides of the inequality for the stability condition (33), there exist large open regions of stability and instability in parameter space. Figure 14 (a) displays the boundaries between the stability region (white) and instability region (dark gray) for five equidistant levels of union power increasing from $\lambda = 0$ (left) to $\lambda = 1$ (right). The boundaries in the light gray area show the bifurcation curves, i.e. the stability tradeoff between the parameters B and C for different λ . Subfigure (b) summarizes the role of the three parameters, indicating that for $BC/(C+1) > 0.5$ the lower balanced path is stable for all values of λ .

One may also want to know whether governmental parameters have an influence on the stability of the lower balanced path. Since the size of government demand g has a strong impact only on the location of the two balanced paths but no influence on the convergence, the government may obtain some control over the stability through the two tax rates τ_w and τ_π . Figure 15 displays some associated bifurcation curves. All four diagrams show that an increase in union power may destabilize an otherwise stable balanced path.

In summary, the dynamic analysis has shown one more time that the size of union power plays a major role for the evolution of the economy in different respects. One of the decisive impacts is on the level of stationary output and employment through its negative role on the demand

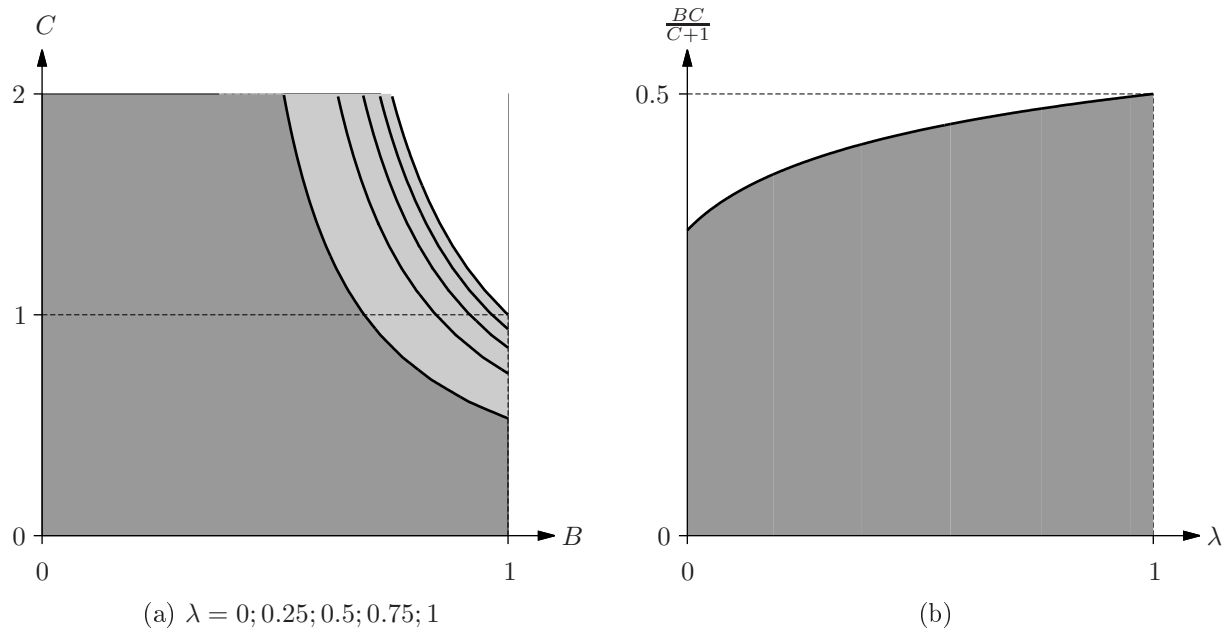


Figure 14: Two-parameter bifurcations: role of labor market parameters and union power

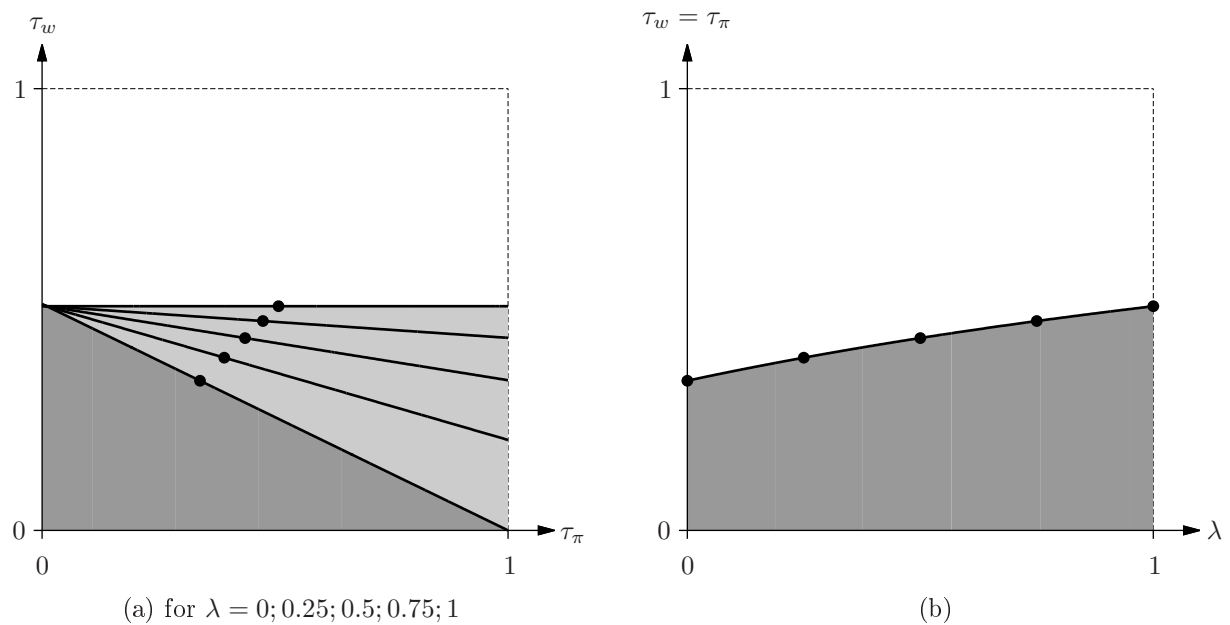


Figure 15: Two-parameter bifurcations: role of taxation and union power

multiplier/aggregate demand implying consequences for the level of stationary money balances and output. This indicates that high levels of union power may be unattractive from a stand point of economic performance while low levels may prevent existence (see Figure 9) or stability (see Figure 14 and 15) .

5 Summary and Conclusion

Two main issues were investigated in this paper. The first one dealt with the *allocative consequences* of efficient bargaining arrangements between a union and a producer association over wage and employment levels on the temporary equilibrium of a macroeconomy as compared to the competitive or other noncompetitive equilibria. It was shown that, for any level of bargaining power, the temporary equilibrium with efficient bargaining in the labor market induces well defined temporary equilibria, one of which is the competitive equilibrium. Note that the two extreme situations of maximal or minimal union power do not coincide with the allocations under monopolistic union or a monopsonistic producer syndicate. Most importantly, an efficient bargaining solution in the labor market combined with a competitive output market induces sizable cross-market effects within the macroeconomy which offset the efficiency feature built into the bargaining concept at given market prices. Thus, equilibria under efficient bargaining are only Second-Best optimal. In other words, contrary to common understanding and to economic folklore derived from partial-equilibrium models, efficient bargaining between a union and producers' syndicate in the labor market does not generate the desired efficiency expected for the macroeconomy as a whole. Moreover, it was shown that economic activity, i. e. output and employment, declines with an increase of union power depending on demand conditions. Thus, high bargaining power leads to low employment and low output in temporary equilibrium at all states, and it may even lead to low nominal wages in certain cases. Therefore, a high *relative* income distribution of wages to profits forced upon an economy by a powerful union comes at the cost of low real economic activity, an outcome which makes a strong union not desirable from a general welfare point of view.

The second part of the investigation concerned the dynamics of the economy under perfect foresight. It was shown that structurally a monetary macroeconomy with efficient bargaining and constant union power behaves in the same way dynamically as under competition in both markets (see Böhm 2010). Existence and stability of balanced states were shown to depend in the same way on the government parameters and the consequences implied by the budget deficit. For the example with isoelastic functions in both sectors, it was shown that the stability conditions are completely determined by the elasticities in both sectors and by union power. In this case, all orbits are monotonic, and underemployment or overemployment levels are constant over time. These results extend to situations with stochastic shocks in production or demand. In other words, the properties of rational-expectations equilibria are also structurally identical to those of perfect competition. Economies with efficient bargaining behave much like competitive ones when union power is constant over time, where the latter controls the income distribution, however, with decisive effects on all aspects of the macroeconomy.

Finally, it should be noted that two of the assumptions maintained throughout could be contested on several grounds. The first postulating constant bargaining power at all times may be questioned since it has a weak microeconomic justification in an intertemporal context. Since renegotiations occur between the same parties in every period, it would be necessary to take the intertemporal linkages of a dynamic economy into account in the bargaining procedure. Selten & Güth (1982) presents such a sequential bargaining solution in a dynamic nonmonetary economy. Blanchard & Fischer (1993) discusses some of these intertemporal issues without analyzing a complete dynamic model. A full dynamic analysis with efficient sequential bargaining still needs to be done. A second desirable modification would be of removing the efficiency requirement in the bargaining process to one where negotiations are only over wages while the employment levels are determined through the market. This modification introduces the right-

to-manage principle into the macroeconomy bringing the model closer to empirically observed negotiations and mechanisms as well as to many partial-equilibrium treatments in the literature. Both extensions imply an endogenous determination of the union power. Such adjustments of the measure of bargaining power over time increase the potential for interesting employment and output cycles and other tradeoffs of the dynamic evolution of the macroeconomy.

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