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Exchange Rate and Price Dynamics at the Zero Lower Bound


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We analyse nominal exchange rate and price dynamics after risk shocks with short-term interest rates constrained by the zero lower bound (ZLB). In an open-economy DSGE model, temporary risk shocks lead to shifts of the exchange rate and the price level if a central bank targets inflation. These shifts are amplified by the ZLB constraint. Empirical evidence for Switzerland supports the view that the responses of the exchange rate and the price level to a temporary risk shock are larger and more persistent if the ZLB is binding. Our theoretical discussion shows that alternative monetary policy rules – such as a price-level target or responding to the exchange rate level – are able to mitigate exchange rate and price fluctuations when the ZLB is binding.

**JEL classification:** C11, C32, E31, E37, E52, E58, F31

**Keywords:** Exchange rate and price dynamics; Zero lower bound on interest rates; Small open-economy DSGE model; Monetary policy strategy; Monetary transmission; Bayesian VAR; Sign restrictions.
1 Introduction

In the aftermath of the financial crisis and during the euro area debt crisis, many central banks in industrialised countries found themselves confronted with the zero lower bound on nominal interest rates (ZLB). As a consequence, these central banks were unable to respond to further contractionary shocks using short-term interest rates as their traditional monetary policy instrument (Jordan 2012). This was a serious restriction for small open economies, such as Switzerland, that were not at the heart of the crisis because their currencies tended to appreciate with changing risk perceptions. Between 2009 and 2011, Switzerland experienced several bouts of unusually sharp appreciation pressures (in some months around 20% year-on-year) followed by persistently negative inflation rates. The appreciation of the Swiss franc came to a halt only after the Swiss National Bank (SNB) introduced a minimum exchange rate against the euro in September 2011 (SNB 2011a).

The recent Swiss experience led us to ask whether there is a connection between the adverse effects of international risk premium shocks, the ZLB on nominal interest rates and deflationary pressures.

Indeed, there is a large literature showing that, when interest rates are constrained by the ZLB, the volatility of real and nominal variables increases because the central bank is not able to respond to shocks using its traditional instrument (see e.g. Fuhrer and Madigan (1997), Coenen et al. (2004), Reifsneider and Williams (2000), Wolman (2005), Williams (2009), Bodenstein et al. (2009), and Amano and Shukayev (2012)). However, most studies conclude that the ZLB is not a serious issue because it should only rarely be binding. There are three qualifications to this conclusion.

First, the probability of hitting the ZLB increases considerably if a country targets an inflation rate below 2% (Reifsneider and Williams 2000, Coenen et al. 2004). Second, (domestic) risk premium shocks are more likely to make the ZLB binding than other shocks (Amano and Shukayev 2012). Third, even if the ZLB is relatively often binding...
because of a very low average inflation rate, a central bank may avoid excessive volatility of macroeconomic outcomes if it follows a history-dependent monetary policy rule, such as a price-level target (Coibion et al. 2012). Despite the abundant work on the topic, to our best knowledge no paper has examined the impact of international risk premium shocks in a small open economy with a low inflation target. Moreover, because of lack of data with a binding ZLB, no paper has estimated impulse response functions in a VAR to these risk premium shocks when the ZLB is binding.

Our contribution therefore is to derive the response of exchange rates and prices to a risk premium shock in a small open economy which faces the ZLB. We show, theoretically and empirically, that the dynamic behaviour of the nominal exchange rate and the price level differ markedly with binding ZLB if the central bank targets a low inflation rate. However, these undesirable exchange rate and price fluctuations can be avoided to some degree by various history-dependent monetary policy rules. The intuition of the theoretical argument is based on Svensson (2009) who argues that exchange rate movements near the ZLB are driven by shifting price level expectations. We formalise this idea by simulating impulse response functions in a linearised open-economy DSGE model on which we impose a ZLB constraint. Empirically, we derive these impulse response functions from a Bayesian VAR in which the parameters are allowed to change between episodes in which the ZLB is binding and episodes in which it is not. Because the ZLB is only rarely binding a careful choice of the prior distribution is important. We do this by centering the prior distribution at the mean of the posterior distribution based on the sample in which the ZLB is not binding. The shocks in the BVAR are identified using sign-restrictions that are consistent with our open-economy DSGE model.

The theoretical findings show that the exchange rate moves with the expected future price level, conditional on other, exogenous variables. It follows that, if price level expectations are well anchored, temporary risk shocks only have a temporary impact on the nominal exchange rate. However, to the extent that price level expectations are shifted, temporary risk shocks do have a larger and more persistent effects. Such shifts are particularly likely with a binding ZLB because the central bank is unable to respond to negative risk shocks with lowering interest rates. If the central bank’s strategy is to set an inflation target, short-run deviations from the inflation target shift the expected future
price level and therefore the nominal exchange rate. Our key insight is that when the ZLB is binding, temporary risk shocks have a larger and more persistent effect on the price level and the nominal exchange rate. In contrast, if the central bank achieves to implement a price level target, the effect of risk shocks to the nominal exchange rate is less pronounced and only temporary. In addition, we analyse a modified inflation targeting rule where the central bank additionally responds to the level of the exchange rate. Even though the central bank attaches only a small weight to the exchange rate, such a rule is able to replicate some of the desirable features of the price-level targeting rule once the ZLB is binding. This is because current exchange rate movements reflect shifting price-level expectations when the ZLB is binding. However, if the ZLB is not binding, such a rule is more aggressive relative to a price-level targeting rule.

The empirical findings are based on an episode when the SNB achieved a low average inflation rate in accordance with its definition of price stability (between 0 and 2%). Using data from this episode we test our theoretical predictions that we derived under an inflation targeting rule. We find that, with a non-binding ZLB, consumer prices fall only temporarily after a negative risk shock. At the posterior median, the prices for tradables and non-tradables return to their initial level after 24 and 48 months, respectively. Moreover, the response of the exchange rate to this shock is temporary. After about one year, the exchange rate returns to its initial level. Consistent with a monetary policy strategy that is able to accommodate a risk shock, the interest rate differential between Switzerland and its main trading partners widens for about two years. However, with a binding ZLB, we find evidence that tradable and non-tradable prices change permanently after a temporary risk shock. But also, the exchange rate does not return to its initial level. Finally, we find some evidence that the responses of the exchange rate and the price level are somewhat larger for a given size of a risk shock when the ZLB is binding.

In our characterisation of the SNB’s policy framework, we use the term inflation targeting in a relatively loose and broad sense. As Baltensperger et al. (2007) note, most major central banks today could be classified as inflation targeters because they are committed on a long-run price stability objective which is usually defined in terms of the inflation rate. Formally, however, the SNB cannot be characterised as an inflation targeter (see Jordan et al. 2010). Two fundamental differences apply. First, price stability is defined as a range for the annual inflation rate instead of a specific number. Second, the SNB does not specify a time range for reaching its definition of price stability. Our empirical results are based on the premise that the SNB’s framework is more closely related with inflation targeting as we model it theoretically than with price-level targeting.
The paper is organised as follows. In the next section we discuss in a DSGE model the role of the nominal target for the dynamics of the exchange rate and the price level after a temporary risk shock with and without a ZLB constraint. Afterwards, we present the empirical methodology and our empirical results. Finally, we offer some concluding remarks.

2 A small open-economy DSGE model

Monetary policy influences exchange rates and prices not only by setting the current interest rate, but also through guidance on its future policy actions. In this section, we argue from a theoretical point of view, that such forward guidance is particularly relevant for exchange rate and price dynamics when the ZLB is binding. Specifically, if a central bank credibly announces to keep future inflation stable – i.e. it is an inflation targeter – temporary risk shocks have persistent effects on the expected future price level, and therefore on the nominal exchange rate. However, if the central bank targets a certain path for the price level, the nominal exchange rate reacts only temporarily and, furthermore, the reaction of the exchange rate on impact is less pronounced.

This result becomes intuitive by means of a simple manipulation of the uncovered interest rate parity (UIP), which we augment with a stationary risk premium (e.g. McCallum and Nelson 2000):

\[ i_t - i^*_t = e_{t+1|t} - e_t + \xi_t , \]  

where all variables are in logarithms and \( e_t \) denotes the nominal exchange rate (in terms of foreign currency units per unit of domestic currency), \( i_t \) the short-term nominal interest rate and \( \xi_t \) the risk premium. Expectations at time \( t \) about the nominal exchange rate one month ahead are denoted by \( e_{t+1|t} \). Foreign variables are labelled by an asterisk.

Iterating this equation forward for \( T \) periods and replacing the nominal exchange rate by the real exchange rate \( (q_t) \) and the domestic and foreign price levels \( (p_t, p^*_t) \) yields

\[ e_t = q_t + T[p_t + p^*_t - p_t + p^*_t] + \sum_{j=0}^{T-1} [e_{t+j|t} - (i_t + j|t - i^*_{t+j|t})] . \]  

(2)
It becomes apparent that the current nominal exchange rate moves one to one with expectations on the future domestic price level \( p_{t+T|t} \), all other things equal. If monetary policy successfully determines these expectations, one source of fluctuations in the current nominal exchange is removed. If a central bank steers expectations on the future inflation rate instead of the price level, temporary deviations from this inflation target may lead to shifts of the expected future price level, directly translating into a persistent shift in the current nominal exchange rate (see e.g. McCallum and Nelson 2000).

Of course, the analysis solely based on the UIP is incomplete, not specifying e.g. the precise impact of risk shocks on expectations of future interest rates. Therefore, we proceed with an analysis within a small open-economy DSGE model augmented by a ZLB constraint. The analysis shows that the intuition survives in this more rigorous setting. Indeed, at the ZLB, risk premium shocks lead to large and persistent shifts in the nominal exchange rate if the policy rule implements an inflation target, while the reaction is smaller and only temporary for a price level target.

Our DSGE model is based on Monacelli (2005), Justiniano and Preston (2010) and Bäurle and Menz (2008). The small economy features a representative household supplying labor, and investing in either domestic or foreign bonds. Moreover, the interest rate on foreign bonds is subject to a risk premium which affects an UIP condition as in (I). This risk premium is assumed to be a function of net foreign assets. Consumption goods are produced by a continuum of monopolistically competitive firms which are sold domestically and abroad. Moreover, a continuum of monopolistically competitive retailers import products from abroad and resell them to the consumers. Calvo (1983)-style price rigidities are introduced at the retail stage as well as at the import stage. It follows that the law-of-one price is violated in the short run. The economy is assumed to be small, such that the rest of the world is exogenously given.

Monetary policy is specified by means of a Taylor-type rule following Justiniano and Preston (2010), in which nominal interest rates respond to their own lag, the inflation rate, output as well as changes in output and changes in the

\footnote{This differs from McCallum and Nelson (2000) because they assume that foreign products are used as intermediate inputs in the production process for domestic consumption goods.}
exchange rate. We adapt this rule incorporating the ZLB by setting $i_t = \max(0, i_{t}^{nc})$ where $i_{t}^{nc}$ is the unconstrained interest rate (measured in levels instead of deviations from steady state), but simplify the rule so that interest rates do not respond to output growth and exchange rate changes.

To analyse the effect of the monetary policy regime, we solve the model for three alternative policy rules. In addition to a rule implementing an inflation target and price level target, respectively, we suggest a modified inflation targeting rule which attaches a small weight to the level of the nominal exchange rate in terms of deviations from steady state. This is an interesting case to study for various reasons. First, as discussed before, with binding ZLB an appreciation of the currency mainly mirrors falling price-level expectations. In such a situation, taking into account the level of the nominal exchange rate will make the central bank respond to a model-consistent forecast of the price level. In practice, this implies that the central bank would take into account a market based price level forecast and it does not have to deal with measuring price level expectations directly. Second, the inflation rate and exchange rate levels are probably more closely monitored by the general public than the price level and therefore such a rule may be more widely understood. All rules are assumed to be known to economic agents and perfectly understood. We therefore have the following three generalised Taylor-rules:

\[ i_{t}^{nc} = \bar{\pi} + \rho_i (i_{t-1}^{nc} - \bar{\pi}) + \psi_{\pi} \pi_t + \psi_y y_t + \varepsilon_{i,t} \quad (3) \]

\[ i_{t}^{nc} = \bar{\pi} + \rho_i (i_{t-1}^{nc} - \bar{\pi}) + \psi_{\pi} \pi_t + \psi_y y_t + \varepsilon_{i,t} \quad (4) \]

\[ i_{t}^{nc} = \bar{\pi} + \rho_i (i_{t-1}^{nc} - \bar{\pi}) + \psi_{\pi} \pi_t + \psi_e e_t + \psi_y y_t + \varepsilon_{i,t} \quad (5) \]

\footnote{Justiniano and Preston (2010) show that central banks in Canada, Australia and New Zealand do not respond to exchange rate changes. For Switzerland, Baurle and Menz (2008) find evidence that the central bank takes exchange rate changes into account.}

\footnote{This is inspired by McCallum (2006) who proposes to use as instrument a weighted average of the interest rate and, with a small weight attached, of the rate of depreciation. However, we stress the potential benefits of including the exchange rate level which may anchor price-level expectations at the ZLB. Other authors have proposed to set the level of the exchange rate instead of a short-term interest rate as a monetary policy instrument (see e.g. Svensson 2001). We see the latter as a way to escape a liquidity trap, which we rule out in our analysis. The rule analysed in this paper should be viewed as a way of avoiding prolonged periods with deflation and zero interest rates in the first place and may not be seen as a way of escaping a liquidity trap (see also McCallum 2006).}

\footnote{Of course we could think of other history-dependent rules. For example, Reifschneider and Williams (2004) propose a rule that makes up for the cumulative difference between $i_t$ and $i_t^{nc}$ in the future. Eggertsson and Woodford (2003) propose a price level target adjusted for the output gap. We do not claim that the rules analysed in this study are optimal in a small open economy from a welfare point of view. This would be an interesting topic to study in future research.}
The model is solved based on a log-linear approximation of the first-order conditions around a zero inflation steady state. Thus, the target inflation rate as well as the target price level are both zero. To account for the fact that a higher inflation rate makes the ZLB less likely to bind, we add a positive real interest rate and a positive inflation target acting as a buffer ($\bar{r} + \bar{\pi}$), such that the likelihood of hitting the ZLB becomes realistic.

The log-linear approximation is reiterated in the Appendix. A detailed derivation can be found in Justiniano and Preston (2010) or Bäurle and Menz (2008).

**Figure 1 — Impulse responses without ZLB**

![Impulse responses without ZLB](image)

Note: Quarterly impulse response functions after an exceptionally large risk premium shock of 2 standard deviations if we ignore the ZLB. All variables except the interest rate are measured in deviations from steady state.

Impulse response functions including the ZLB constraint are derived using the algorithm developed by Holden (2011) and extended by Holden and Paetz (2012). This algorithm can be used to simulate DSGE models and impulse response functions with inequality

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7We are grateful that the authors shared a Dynare code of the algorithm. We additionally simulated impulse responses using the deterministic solution of the model. The results are qualitatively identical.
constraints. The algorithm is designed to analyse local dynamics of linearised models around a particular steady state. In particular, we have to specify some horizon at which the ZLB will be non-binding anymore; we set this horizon to 20 years (80 quarters). We therefore have to ensure that the steady state, i.e. the nominal target, is reached eventually. Other authors achieve this by assuming that fiscal policy boosts aggregate demand to rescue the economy from falling into a deflationary spiral (Coenen et al. 2004). We take a short-cut for the inflation-targeting rule by allowing for a very small weight on the price level. This ensures that the equilibrium is unique and at some point in the future the ZLB will not be binding anymore. The same is accomplished automatically in our modified inflation targeting rule by including the exchange rate level with a small weight.

We calibrate the model using the posterior median coefficients and shock processes estimated by Bäurle and Menz (2008) for Switzerland. The steady state real interest rate amounts to 1.6% and the inflation target we set to 1%, which matches the Swiss experience over the past 20 years. Figure 1 shows the impulse responses to a risk shock if we do not impose the ZLB constraint. For inflation targeting and price-level targeting, the differences are marginal because the central bank is able to accommodate the risk shock. We therefore conclude that exchange rate and price dynamics are very similar with inflation targeting and price-level targeting if the ZLB is non-binding. This can be achieved because the interest rate falls into negative territory as prescribed by all three policy rules. Meanwhile, the modified inflation targeting rule leads to a more accommodative policy as the price level for imported items does not fall as much as under price-level targeting and the price level for domestic items increases more.

If we take the ZLB into account the dynamics change considerably (Figure 2). With binding ZLB and inflation targeting, the initial appreciation is twice as large as with non-binding ZLB. In addition, the domestic currency appreciates permanently. This

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*There are a few exceptions. Three calibrated parameters are set to the values used by Leist (2011) (the discount factor, the share of imported goods, and the coefficient of relative risk aversion) which yields more plausible impulse response functions of output to a risk shock. Moreover, we treat the foreign economy perfectly symmetric with the exception that the foreign interest rate is not subject to a ZLB constraint. The latter can be justified by results presented in Bodenstein et al. (2009); they show in a two-country model that the transmission of foreign demand shocks to the domestic economy does not greatly depend on whether the foreign economy is constrained by the ZLB. Note that using the estimates by Justiniano and Preston (2010) for Canada, Australia and New Zealand yields similar qualitative conclusions. Parameter values, and Dynare programs are available upon request.
Figure 2 — Impulse responses with ZLB

Note: Quarterly impulse response functions after an exceptionally large risk premium shock of 2 standard deviations if we take the ZLB into account. All variables except the interest rate are measured in deviations from steady state.
is mirrored in substantial declines of domestic and the imported prices. As the shock is of the same size as in the previous exercise, these different responses can be traced back entirely to the ZLB constraint. With a price-level target, the adverse effects of the shock can be avoided to a large degree. This can be traced back to the different interest rate responses. After the short-term interest rate moves away from the zero-line with inflation targeting, the price-level targeting rule prescribes zero interest rates for additional four quarters. Moreover, even after 14 quarters interest rates remain lower than with an inflation target. Because agents perfectly anticipate the future lower interest rates, the exchange rate appreciates only by a limited amount and the price level drops by much less and only temporarily. The modified inflation target rule, which may be easier to communicate, accomplishes similar dynamics as the price-level targeting rule. It also implies lower interest rates for a considerable period of time. However, compared to the price-level target, interest rates increase somewhat earlier, the exchange rate appreciates more strongly and the output loss is larger.

From the DSGE model we obtain several predictions regarding exchange rate and price fluctuations with and without a ZLB constraint and some of them we test empirically in the following sections. These testable predictions may be summarised as follows:

(i) If the ZLB is not binding, price and exchange rate dynamics are almost observationally equivalent under price-level targeting and inflation targeting.

(ii) At the ZLB, the long-run response of the exchange rate and the price level to a risk premium shock are more persistent.

(iii) At the ZLB, the initial responses of the exchange rate and the price level to a risk premium shock are larger.

3 A Bayesian vector autoregressive model for Switzerland

In the previous section, we showed that in a standard small open-economy DSGE model, a binding ZLB renders the impact of risk premium shocks on price and exchange rate levels larger in magnitude and more persistent, if the central bank credibly announces to keep future inflation stable. The aim of the following sections is to test these hypotheses empirically based on macroeconomic data for Switzerland in the framework of a Bayesian
vector autoregressive model (BVAR).

Switzerland fits our theoretical setup well as it is a very small and very open economy with a long-standing record of low inflation and low interest rates. Furthermore, since the adoption of flexible exchange rates in the early 1970s, Switzerland experienced three episodes with short-term interest rates very close to 0% (Figure 3). In line with the predictions of the DSGE model, these episodes were accompanied with appreciation pressures on the Swiss currency. In two of the three episodes, we observed bouts of unusually sharp appreciation pressures (in some months around 20% year-on-year) and a substantial widening of the inflation differential (around 5 pp). In the late 1970s, the appreciation was effectively stopped by introducing a minimum exchange rate against the German mark (see Bernholz 2007, pp. 180). In September 2011, the SNB introduced a minimum exchange rate, now against the euro (SNB 2011a). In the relatively short period from 2003 to 2004, the Swiss franc did not appreciate much. However, at the time, the SNB was strongly concerned about the strength of the Swiss franc and made several direct references to foreign exchange interventions without taking explicit action (Burkhard and Fischer 2009).

Although we obtained data since the late 1970s, we chose to focus on the period from 1994 to 2012. We ignore data before 1994 because inflation was on average substantially higher and therefore the period does not fit well our theoretical setup of a central bank targeting a low inflation rate at 1%. Since 1994, the SNB achieved a low average inflation rate at 0.8% and since 2000, the SNB explicitly defined price stability as a pre-specified range for annual CPI inflation (0%-2%) but no fixed time frame for reaching this range; it has to be met only in the medium to long term. We think that, of the three rules analysed in the previous section, the SNB’s monetary policy framework is most closely related to inflation targeting. We ignore that the SNB defined its operational target in

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9As such, Switzerland may be a more interesting case to study than, for example, Japan. Indeed, in a commentary to Svensson (2001), Glenn Stevens argued: “How open is Japan? And how small is it? In brief, the answer is “not very” in both cases.” Moreover, Switzerland is often referred to in discussions on the ZLB because of its low inflation and low interest rate record (see e.g. Bernanke et al. 2004, Svensson 2009, Amano and Shukayev 2012). However, with the exception of Burkhard and Fischer (2009), we are not aware of a study analysing ZLB episodes for Switzerland.

10The start date of this sample is inspired by Stulz (2007), who finds a significant downward shift in the mean of Swiss CPI inflation in mid-1993. Therefore, all data are from a low-inflation period.

11For a general discussion see Baltensperger et al. (2007), for a more precise characterisation of the SNB’s monetary policy framework, in particular for the key differences to explicit inflation targeting, see Jordan et al. (2010).
Figure 3 — Swiss zero-lower-bound episodes

Note: The short rate is the 3M Libor, the long rate the 10-year government bond yield; the nominal effective exchange rate and inflation differential are defined against Switzerland’s 24 largest trading partners.
terms of the 3M Libor only since the introduction of the new monetary policy framework in December 1999. Before, the SNB targeted monetary aggregates to keep prices stable. However, the two episodes of main interest with a binding ZLB since 1994 all occurred after the adoption of the new monetary policy framework.

Because the remaining two ZLB episodes are relatively short, we prefer to use a Bayesian estimation approach instead of a classical OLS-based estimator as the OLS estimator is known to be inadmissible, i.e. there are estimators with a lower forecast error variance. This shortcoming of the OLS estimator is particularly relevant when estimating a model with a reasonably large number of parameters but having at our disposal only a small sample. We use a diffuse prior for the long sample where the ZLB does not bind, but implement informative prior restrictions on the coefficients for the ZLB sample. Specifically, we exclude the interest rate in the ZLB sample, reflecting the belief that there is no relevant variation in interest rates at the ZLB. Furthermore, we assume that the remaining coefficients in the ZLB sample are similar to the respective coefficients in the period in which the ZLB does not bind. The degree of similarity, i.e. the tightness of the prior, is treated as additional hyper-parameter and, as such, determined by means of a posterior analysis.

A detailed technical description of the estimation approach and a formal argument why our approach is related to a TVP-VAR is given in the following sections. Subsequently, we describe the data and the identification scheme for the structural shocks relying on sign-restrictions derived from our DSGE model, before presenting the empirical results in Section 4.

3.1 The generic form of the model

We assume that data is generated by a vector autoregressive process:

\[ y_t = B_{0,t} + B_{1,t} y_{t-1} + \ldots + B_{p,t} y_{t-p} + Q_t \varepsilon_t \quad (6) \]

\[ \varepsilon_t \sim N(0, I_{n_t}) \quad , \quad (7) \]

\footnote{Following Assenmacher-Wesche (2008), we added a broad measure of money to our main specification as a robustness test; the SNB traditionally stressed the role of money also with the adoption of the new monetary policy framework in 2000.}
where \( y_t \) is a \( n_t \times 1 \) vector of endogenous variables and \( \varepsilon_t \) is an \( n_t \times 1 \) vector of exogenous shocks. Note that we allow the dimension of \( y_t \) to change over time. \( B_{0,t}, \ldots, B_{p,t} \) and \( Q_t \) are matrices containing the unknown parameters. The likelihood of the model is invariant to orthonormal transformations of \( Q_t \), the contemporaneous impact of shocks on observed variables. We therefore parameterise the likelihood function in terms of \( \Sigma_t = Q_t'Q_t \) and estimate this reduced form model. Only in a second step, we identify \( Q_t \) based on further restrictions derived from economic theory. Following Scholl and Uhlig (2008) we do not include a constant in our baseline specification, i.e. we set \( B_{0,t} \) to zero.

We assume that the parameters only change when the interest rate hits or leaves the ZLB. Thus, we define for the lags \( l = 1, \ldots, p \) \( B_{l,1} = B_{l,t \in \{ \text{ZLB not binding} \}}, \quad B_{l,2} = B_{l,t \in \{ \text{ZLB binding} \}}, \quad \Sigma_1 = \Sigma_{t \in \{ \text{ZLB not binding} \}}, \quad \Sigma_2 = \Sigma_{t \in \{ \text{ZLB binding} \}} \). We rewrite the system for each sample as follows, extending e.g. Giannone et al. (2012) to the case of parameters that switch between two regimes. First, define the matrices \( Y = [y_{p+1}, \ldots, y_T]' \), \( X = [1, y_{t-1}', \ldots, y_{t-p}'], \quad x = [x_{p+1}, \ldots, x_T]', \quad X = I_{n_t} \otimes x \), and \( e = vec([\varepsilon_{p+1}, \ldots, \varepsilon_T]') \). Then, split these matrices such that \( Y_1, X_1 \) and \( e_1 \) collect observations corresponding to the sample in which \( t \in \{ \text{ZLB not binding} \} \) and \( Y_2, X_2 \) and \( e_2 \) collect the observations of the ZLB episode in which \( t \in \{ \text{ZLB binding} \} \). Then, write the system as

\[
Y_i = X_i \beta_i + e_i
\]

for \( i \in \{1, 2\} \), where \( \beta_i = vec(B_i) \) with \( B_i = [B_{0,i}, \ldots, B_{p,i}]' \). Hence, the system can be written as a linear regression model and standard Bayesian methods for such models can be applied. The number of regressors is \( k_i = n_i p + 1 \) in each sample and the number of observations are denoted by \( T_1 \) and \( T_2 \) respectively.

We follow the bulk of the literature by selecting a natural conjugate prior distribution for the model parameters, setting

\[
\Sigma_i \sim IW(\Psi, \nu)
\]

\[
\beta_i | \Sigma_i \sim N(\beta_i, \Sigma_i \otimes \Omega_i)
\]
The posterior distribution can be shown to be

$$
\Sigma_i | y_i, X_i \sim IW(\Psi_i + \hat{\varepsilon}_i \hat{\varepsilon}_i, T_i - k_i + d_i) \quad (12)
$$

$$
\beta_i | \Sigma_i, y_i, X_i \sim N(\hat{\beta}_i, \Sigma_i \otimes (x'_i x_i + \Omega_i)) \quad (13)
$$

with $\hat{B}_i = (x'_i x_i + \Omega_i^{-1})^{-1}(x'_i y_i + \Omega_i^{-1} \hat{\beta}_i)$, $\hat{\beta}_i = vec(\hat{B}_i)$, $\hat{\varepsilon}_i = y_i - x_i \hat{B}_i$ and $\hat{\beta}_i$ being a $k_i \times n_i$ matrix obtained from reshaping $\beta_i$ suitably.

### 3.2 Parameterising the prior distribution

For the period in which the interest rate is not constrained by the ZLB, we assume a non-informative prior $\varphi(\beta_1, \Sigma_1) = |\Sigma_1|^{-\frac{n_1+1}{2}}$, which can be interpreted as an inverse Wishart distribution as in (10) with $\Psi_1 = 0$ and $d_1 = 0$. The resulting posterior distribution is

$$
\Sigma_1 | X_1, Y_1 \sim IW(\bar{\Psi}_1, \bar{d}_1) \quad (14)
$$

$$
\beta_1 | \Sigma_1, X_1, Y_1 \sim N(\bar{\beta}_1, \Sigma_1 \otimes \bar{\Omega}_1) \quad (15)
$$

with

$$
\bar{\beta}_1 = (X'_1 X_1)^{-1}x'_1 y_1 \quad (16)
$$

$$
\bar{\Omega}_1 = (X'_1 X_1)^{-1} \quad (17)
$$

$$
\bar{\Psi}_1 = (Y_1 - X_1 \bar{\beta}_1)'(Y_1 - X_1 \bar{\beta}_1) \quad (18)
$$

$$
\bar{d}_1 = T_1 \quad (19)
$$

For the ZLB episode, the number of parameters is rather large compared to the number of observations. In this case, parameterising the prior distribution, i.e. selecting $\beta_2$, $\Omega_2$, $\Psi_2$ and $d_2$, becomes critical. We approach this problem by implementing an a priori belief that the distribution of the parameters in the ZLB episode is similar to the one in the pre-lower bound episode for the parameters that do not involve the interest rate. Thus, from the posterior distribution of the pre-lower bound parameters, we select the appropriate elements, i.e. the ones that are not coefficients on the interest rate or coefficients in the
interest rate equation and parameterise the prior distribution \((10)\) for the ZLB parameter as follows:

\[
\beta_2 = Z_\beta \beta_1 \\
\Omega_2 = Z_\Omega \Omega_1 \lambda \\
\Psi_2 = Z_\Psi \lambda \Psi_1 \\
d_2 = \lambda d_1 ,
\]

where \(Z_\beta\), \(Z_\Omega\) and \(Z_\Psi\) are matrices selecting the appropriate rows and columns and \(\lambda\) is a parameter determining the tightness of the prior. Inspecting the prior distribution, we see that if \(\lambda\) becomes larger, the prior distribution becomes tighter around the pre-lower bound sample estimates.

We determine this prior weight, or scale of jump variance, by conducting a formal posterior analysis with respect to \(\lambda\). Specifically, we add a gamma prior for \(\lambda\) and simulate from its posterior distribution by introducing a Random-Walk Metropolis-Hastings step into the otherwise standard posterior sampling procedure for the VAR coefficients (see e.g. Giannone et al., 2012, for an application of the same idea to a Bayesian VAR with different sets of dummy observations). Thus, we produce draws from the posterior distribution using the following algorithm. Starting with initial parameters \(\beta_{i,0}, \Sigma_{i,0}, \lambda_{i,0}\) and \(\lambda_{i,0}\), we iterate \(j = 1, \ldots, J\) times over the following steps:

**Step 1:** Draw \(\beta_{i,j}\) from \((12)\) and \(\Sigma_{i,j}\) from \((13)\).

**Step 2:** Draw a candidate value \(\lambda^*\) from

\[
\lambda^* = \lambda_{j-1} + \zeta
\]

with \(\zeta \sim N(0, V)\), \(V\) being the scaled inverse hessian of the posterior density evaluated at the posterior mode of \(p(\lambda|y, X) \propto p(\lambda)p(y|\lambda)\).\(^{13}\) Accept the candidate values with probability

\(^{13}\)The marginal likelihood \(p(y|\lambda)\) can be derived analytically, see e.g. Giannone et al., 2012.
\[ \alpha = \min \left\{ 1, \frac{p(\lambda^*|y,X)}{p(\lambda_{j-1})} \right\} \]

The scale of \( V \) is set such that the acceptance rate is between 0.2 and 0.3.

**Step 3:** For each draw \( j \) and regime \( i \), produce \( J \) draws of \( Q_{i,j} \) using the method described in Uhlig (2005) and retain only the parameter draws for which the implied impulse response functions satisfy certain restrictions.

### 3.3 Interpretation of the prior in relation to TVP-VARs

Our approach shows striking similarities to a TVP-VAR specification in which the parameter jump only when the interest rates hits or leaves the ZLB. In standard TVP-VAR analysis, parameters are allowed to vary in each period. Usually, it is assumed that the parameters in period \( t \) equal the parameters in \( t-1 \) distorted by an exogenous shock vector. To reduce the degrees of freedom, it is necessary to impose parametric assumptions on this shock vector. Our training sample approach implements a similar idea. To see this, write \( \beta_2 \) as a function of \( \beta_1 \) and a shock vector \( \xi \),

\[ \beta_2 = \beta_1 + I(\tilde{i}_t \leq 0)\xi, \]

where \( \tilde{i}_t \) is the interest rate that would prevail if there was no ZLB. Further assume that \( \xi \sim N \left(0, \left(\frac{1}{\lambda} - 1\right)\text{Var}(\beta_1|y_1, X_1)\right) \). We then get

\[
\begin{align*}
E(\beta_2|y_1, X_1) &= E(\beta_1|X_1) \\
\text{Var}(\beta_2|y_1, X_1) &= \frac{\text{Var}(\beta_1|X_1)}{\lambda}
\end{align*}
\]

which exactly corresponds to the prior distribution described above. This establishes that training-sample priors are conceptually related to a TVP-VAR with occasional jumps. The relation refers to the fact that the jump in parameters is constrained by the distributional assumptions on the innovation term. This is the main difference to a pure regime switching approach, in which the prior and the posterior distribution of the parameters are independent across regimes. Inspecting the posterior distribution and equation (25), it becomes apparent that the scale of \( \lambda \) determines at the same time the weight of the
prior distribution relative to the information contained in the actual data and the scale of
the a priori variance of the shock to the parameter vector.

There are also differences to the TVP-VAR specifications used in the literature. First,
the fact that we assume that the jumps can only occur in specified periods reduces the
degrees of freedom considerably. While this may be seem to be restrictive, it allows us
to parameterize the distribution of the parameter innovations less tightly than necessary
in standard TVP-VAR analysis. We allow for fewer, but potentially larger parameter
changes. A second difference is that in our case, the parameters jump as a function of the
state of the economy. In TVP-VAR analysis, parameter innovations are independent of
observed variables. In our view, the latter is questionable when focussing on parameter
changes resulting from hitting a ZLB constraint.\footnote{A further difference is that the distribution of the innovation matrix is not a function of the data
observed in earlier periods in standard TVP-VAR analysis.}

\subsection*{3.4 Data and identification}

We include in our model the following variables:

\begin{equation}
\begin{aligned}
y_t &= \left[ p^*_t, \ y_t, \ (i_t - i^*_t), \ p^l_t, \ p^u_t, \ p^n_t, \ e_t \right].
\end{aligned}
\end{equation}

That is, it includes the foreign price level \(p^*_t\), Swiss GDP \(y_t\), the short-term interest rate
differential \((i_t - i^*_t)\), import prices at the docks \(p^l_t\), tradable and non-tradable consumer
prices \(p^u_t, p^n_t\) and the nominal effective exchange rate \(e_t\).\footnote{The information set is largely comparable to Stulz (2007) and popular among studies that use VARs
to examine the exchange rate pass-through to prices (see e.g. McCarthy 2007). We additionally control
for foreign interest rates because they appear in the UIP condition discussed in the previous section.}

We expect the response of prices to depend on whether they refer to imported, tradable or non-tradable goods. A
detailed description of the data is given in the Appendix.\footnote{We are grateful to Jonas Stulz for providing his code for calculating a monthly indicator of Swiss GDP.} All variables are included in
logarithms of the levels, except the interest rate differential which we leave untransformed.

The structural risk shock is identified by a sign restriction approach put forward
by Uhlig (2005) and applied by An and Wang (2011) in the context of estimating the
exchange rate pass-through for some industrial countries. To check the plausibility of our
restrictions, Table \ref{tab:impulse} shows the direction of the impulse response functions in the small

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|c|c|c|c|}
\hline
Variable & \(y_t\) & \(p^*_t\) & \((i_t - i^*_t)\) & \(p^l_t\) & \(p^u_t\) & \(p^n_t\) & \(e_t\) \\
\hline
\end{tabular}
\caption{Impulse response functions for different variables.}
\end{table}
open-economy DSGE model presented in Section 2.17

<table>
<thead>
<tr>
<th></th>
<th>$e_t$</th>
<th>$i_t$</th>
<th>$p_t$</th>
<th>$p_t^i$</th>
<th>$p_t^d$</th>
<th>$y_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Risk premium (−)</td>
<td>−</td>
<td>−</td>
<td>−</td>
<td>+</td>
<td>−</td>
<td>−</td>
</tr>
<tr>
<td>Monetary policy (+)</td>
<td>+</td>
<td>+</td>
<td>−</td>
<td>−</td>
<td>+</td>
<td>−</td>
</tr>
<tr>
<td>Preference (+)</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>−</td>
<td>−</td>
</tr>
<tr>
<td>Technology (−)</td>
<td>−</td>
<td>+</td>
<td>+</td>
<td>−</td>
<td>+</td>
<td>−</td>
</tr>
<tr>
<td>Cost push (+)</td>
<td>−</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>−</td>
<td>−</td>
</tr>
</tbody>
</table>

Note: Sign of initial impulse responses according to a small open-economy DSGE model presented in Section 2.17. Shaded cells denote restrictions we impose in the VAR model. $e_t$: exchange rate; $i_t$: short-term interest rate; $p_t$: consumer price index; $p_t^i$: prices imported goods; $p_t^d$: prices domestic goods; $y_t$: output; $e_t < 0$ denotes an appreciation of the domestic currency against the foreign currency.

The shaded cells contain restrictions we impose in the VAR model. In particular, a negative risk shock leads to an appreciation of the Swiss franc and therefore to lower import prices at the docks. Adding import prices allows to identify a risk shock without restricting the variables in which we are mostly interested in, that is tradable and non-tradable consumer prices. In addition, the DSGE model suggests that domestic short-term interest rates decline on impact to stabilise inflation. Therefore, we require that the interest rate differential becomes negative. In addition, we require that output declines while the Swiss franc appreciates. Because Switzerland is a small open economy, no sign restrictions are imposed on foreign variables. To distinguish a risk shock from other shocks abroad, we impose a zero restriction that foreign variables do not react contemporaneously.18 The sign restrictions derived from the DSGE model in Table 1 are chosen such that they all imply an appreciation of the domestic currency. Our sign restrictions identify a risk shock because they differ at least for one variable from the responses of other structural shocks in the DSGE model that lead to an appreciating domestic currency. In particular, the table shows that the risk premium shock is the only shock that leads to an appreciation of the currency and lower interest rates.

A popular identification strategy in the literature analysing exchange rate pass-through is to assume a Cholesky ordering of domestic variables.19 It is argued, that an exchange rate shock may affect import and consumer prices immediately. 17 They are largely compatible with the responses in other DSGE models for Switzerland; see, e.g. Bäurle and Menz (2008), Cuche-Curti et al. (2004), Leist (2011), and Rudolf and Zurlinden (2012).

18 Here, our strategy differs from An and Wang (2011) who restrict the foreign variables symmetrically to the domestic variables. We replicated their strategy as a robustness test.

19 e.g. Hahn (2003), Stulz (2007), McCarthy (2007).
for real activity and monetary policy. We replicated this identification strategy and our results proved robust. However, the DSGE model implies that all domestic variables may respond immediately and therefore we prefer our sign restriction approach over any particular Cholesky ordering.

4 Empirical results

4.1 Does inflation targeting look like price-level targeting when the ZLB does not bind?

First, we test our hypothesis whether the impulse response functions with non-binding ZLB are observationally similar for a central bank achieving a low inflation rate as we would theoretically expect for a price-level targeter. Figure 4 shows the impulse responses to an identified negative risk shock. The impulse responses are based on 800 draws from the posterior distribution of the reduced form VAR coefficients. For each of these draws we obtain 800 draws from the space of possible impulse vectors. Inference is based on those draws that satisfy our sign restrictions. These restrictions are imposed for $H = 6$ months.

All posterior draws of the impulse responses are normalised by the median response of the exchange rate at $H = 0$ so that we show the impulse responses consistent with an immediate 1% appreciation of the Swiss franc.

The median response of tradable prices is hump-shaped. After a 1% appreciation, tradables prices fall almost immediately by 0.1%. They reach a trough after twelve months and then gradually return to their initial level after about two years. We observe a similar hump-shaped pattern for non-tradables prices. The main difference is that the trough is reached later. The hump-shaped responses translate into the response of the aggregate CPI.

To understand the hump-shaped responses, it is useful to examine the responses of the other variables included in the model. These responses show that, with a non-binding ZLB, inflation targeting may also successfully stabilise the price level after a risk shock.

\footnote{According to Stulz (2007), a hump-shaped response is a common finding in studies on the exchange rate pass-through to import prices.}

\footnote{The response for the aggregate CPI is simply a weighted average of the responses for tradables (41.7\%) and non-tradables (58.3\%). The results remain qualitatively unchanged when including the aggregate CPI directly.}
The risk shock affects the exchange rate only temporarily. By construction, the Swiss franc appreciates on impact by 1%; subsequently, the exchange rate response returns to zero after one year. In line with an accommodating monetary policy stance, the temporary appreciation is accompanied by a widening interest rate differential. The median response suggests that Swiss interest rates fall relative to foreign interest rates by 10 bp before gradually returning to their initial level.

4.2 Are price and exchange rate movements more persistent when the ZLB binds?

Second, we test the hypothesis whether price and exchange rate dynamics are more persistent during episodes when interest rates are constrained by the ZLB. Figure 5 shows the price dynamics with a binding ZLB. The timing and magnitude of the response of tradables prices is similar in the first few months. However, the median response differs markedly at longer horizons. The effect of a 1% appreciation is persistent. According to the posterior median, tradables prices fall by 0.2% after 48 months. We observe a similar pattern for non-tradables but the median response is somewhat smaller. The response of the exchange rate is in line with inflation targeting that does not anchor price level
expectations. Although the Swiss franc normalises somewhat after the sign restrictions are removed, the median response is persistent and amounts to $-0.5\%$ after 48 months.

**Figure 5 — Impulse responses binding ZLB**

![Graphs showing impulse responses for different variables.](image)

Note: Posterior median impulse response functions to a negative risk shock. 80% and 50%-HPDI are given as shaded areas. The responses are normalised by the initial median response of the exchange rate.

Scholl and Uhlig (2008) emphasise that the median response can be misleading, because the underlying median impulse response does not come from any particular point in the parameter space (see also Fry and Pagan 2007). Therefore, to test whether the responses are indeed more persistent, the first panel in Table 2 shows the posterior probability of a negative price level and exchange rate response after 48 months. With a non-binding ZLB, observing a positive long-run response is almost as likely as observing a negative response. The probability that the response is negative after 48 months amounts to 60.5% for tradables and 63.3% for non-tradables prices. The probability that the exchange rate response is negative is only slightly higher at 65.4%. With a binding ZLB, the posterior distribution shifts towards negative values and therefore the probability of observing a negative response increases. The posterior probability of observing a negative response after 48 months amounts to 94.1% for tradables prices, 85.9% for non-tradables prices and 90.6% for the exchange rate.

In the second panel of Table 2 we document the posterior probability that we observe a trough of the response at a relatively long horizon. We conservatively define a persistent
Table 2 — Posterior probability

<table>
<thead>
<tr>
<th></th>
<th>Tradable CPI</th>
<th>Non-tradable CPI</th>
<th>Exchange rate</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Negative response at 48 months</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Non-binding ZLB</td>
<td>0.61</td>
<td>0.63</td>
<td>0.650</td>
</tr>
<tr>
<td>Binding ZLB</td>
<td>0.94</td>
<td>0.86</td>
<td>0.910</td>
</tr>
<tr>
<td><strong>Trough at or later than 48 months</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Non-binding ZLB</td>
<td>0.08</td>
<td>0.08</td>
<td>0.08</td>
</tr>
<tr>
<td>Binding ZLB</td>
<td>0.39</td>
<td>0.31</td>
<td>0.25</td>
</tr>
</tbody>
</table>

Note: The table shows whether the impulse responses are persistent or transitory. The first panel shows the posterior probability of a negative response after 48 months. The second panel shows the posterior probability that the trough (minimum) of the response occurs at 48 months (or possibly later).

response to reach a trough at a horizon of 48 months or later, the longest horizon for which we have calculated the impulse responses. With a non-binding ZLB, most of the troughs for tradable prices are observed between 0 and 18 months. But only 8% of all responses seem to be persistent. By contrast, the share of responses that are persistent amounts to 39% with a binding ZLB. For non-tradable prices, most of the troughs occur at a horizon between 18 and 36 months if the ZLB does not bind. Only 8% of all responses are persistent. With a binding ZLB, the share of persistent responses increases to 31%. Finally, most of the troughs of the exchange rate occur up to six months independently of the ZLB; this reflects our identifying restrictions. However, we see that only 8% of the responses are persistent with a non-binding ZLB. By contrast, 25% of the exchange rate responses reach a trough at 48 months.

We showed that temporary risk shocks lead to persistent responses of the exchange rate and the price level at the ZLB, but to only temporary responses if the ZLB is not binding. Our theoretical model is able to generate such responses by combining an inflation target with the ZLB on nominal interest rates. However, this result may be criticised on the ground that the risk premium itself may have become more persistent. More specifically, the unfolding financial crisis may have led to permanent changes in risk perception. As a consequence, the Swiss franc would have appreciated permanently. We now investigate

22 Many price-setting models imply a stronger response of prices to larger and more persistent shocks. For example, a menu-cost model predicts that firms, on average, react more quickly and more strongly to larger shocks. A similar implication has the model by Gopinath et al. (2010). With more volatile exchange rates fewer firms would engage in local-currency pricing which would lead to higher pass-through to import prices. Finally, Taylor (2000) shows that more persistent cost shocks lead to higher exchange rate pass-through.
whether empirical evidence favours this alternative interpretation, finding that there is little support for this channel to be the main driver of our results.

We therefore proxy the risk premium by the difference of the VIX in Switzerland and the US.\[23\] Moreover, we adapt the sign restrictions by assuming that a risk shock increases the uncertainty abroad by more than in Switzerland. Naturally, this would imply that the VIX in the US would increase more than in Switzerland and therefore we require the difference between the Swiss and US VIX to be negative while leaving the exchange rate unrestricted.

The impulse responses of selected variables of this alternative model are shown in Figure 6. We normalise the impulse responses by the median response of the VIX. Remarkably, the response of the VIX differential itself does not differ significantly whether the ZLB is binding or non-binding. It responds only temporarily to an identified risk shock and returns to zero after about 20 months. Moreover, the responses for the total CPI and the exchange rate are not greatly affected. We therefore conclude that a more persistent or more volatile risk premium is not the main driver of our results.

4.3 Are price and exchange rate movements larger when the ZLB binds?

Our third hypothesis derived from the DSGE model was that the initial response of the exchange rate and the price level to a risk premium shock are larger with binding ZLB. Without a direct measure of the risk premium, we cannot distinguish changes of the size of the shock from a change of the responsiveness to a given size of a shock. However, using our specification including the VIX differential may be used to analyse this question.

Table 3 shows the numerical values of the posterior median for the short-run and long-run impulse response functions with 80%-HPDI. For the CPI, we see that the initial response is not larger with binding ZLB than with non-binding ZLB. Indeed, the initial response is roughly zero in both cases. In the long-run, the response is negative if the ZLB is binding but there is much less evidence that the response is negative if the ZLB is non-binding. The results are even more clear cut for the exchange rate. The immediate response with binding ZLB is $-0.90\%$ when the ZLB is binding but only $-0.69\%$ if the ZLB is non-binding. Moreover, the 80%-HPDI clearly shifts downwards with binding

\[23\]This difference is relatively persistent but unit root tests suggest that it is stationary.
Figure 6 — Impulse responses (including VIX measure)

(a) Non-binding ZLB

(b) Binding ZLB

Note: Posterior median impulse response functions to a negative risk shock. 80% and 50%-HPDI are given as shaded areas in red (non-binding ZLB) and blue (binding ZLB). The responses are normalised by the initial median response of the VIX differential.

ZLB. In the long-run, the exchange rate response is still −0.67% with binding ZLB but only −0.21% with non-binding ZLB. Again, the 80%-HPDI shifts downwards although the posterior uncertainty is relatively high.

Table 3 — Size of short- and long-term responses (including VIX measure)

<table>
<thead>
<tr>
<th></th>
<th>Swiss CPI</th>
<th>Exchange rate</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Initial response</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Non-binding ZLB</td>
<td>0.001 [−0.028, 0.034]</td>
<td>−0.685 [−0.928, −0.417]</td>
</tr>
<tr>
<td>Binding ZLB</td>
<td>−0.006 [−0.044, 0.031]</td>
<td>−0.907 [−1.331, −0.485]</td>
</tr>
<tr>
<td><strong>Response after 48 months</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Non-binding ZLB</td>
<td>−0.031 [−0.196, 0.097]</td>
<td>−0.212 [−0.907, 0.316]</td>
</tr>
<tr>
<td>Binding ZLB</td>
<td>−0.105 [−0.282, 0.048]</td>
<td>−0.674 [−1.881, 0.405]</td>
</tr>
</tbody>
</table>

Note: Posterior median of the short-run (immediate) and long-run (48 months) response with 80%-HPDI in brackets. The responses are normalised by the initial median response of the VIX differential.
4.4 Robustness analysis

We conducted a wide range of robustness tests that are all available upon request. Three particular robustness tests are worth discussing in more detail. A first reservation with respect to our results may be that the SNB intervened in the foreign exchange market several times since early 2009. We therefore estimated the impulse response functions for the ZLB sample only in the first episode (3/2003 to 6/2004). This excludes the crisis period and the exchange rate interventions by the SNB since early 2009. The response of the CPI does not change much compared to our main specification. However, the response of the exchange rate is somewhat less persistent. Overall, this still supports our hypothesis that a binding ZLB leads to more persistent responses of the exchange rate and the price level.

Second, we included a constant to our model. This may be particularly important because we may overestimate the persistence of a stationary process if we fail to control for a constant term. However, we may also underestimate the persistence of a non-stationary process with zero mean if we add a constant by mistake. Therefore, the specifications including and excluding a constant may give us two extreme cases. As we would expect, the estimated impulse responses are somewhat less persistent with a binding ZLB, if we include a constant. However, the results still support the conclusion that temporary risk shocks have more persistent effects with a binding than with a non-binding ZLB.

Third, we varied the tightness of the prior for the ZLB coefficients. Recall that we use the scaled posterior distribution from the sample with a non-binding ZLB as the prior

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24 Qualitatively, most of our findings were not affected. We used the same recursive identification scheme as Stulz (2007); we included separately foreign and Swiss short-term interest rates instead of the interest rate differential; we included the interest rate also in the ZLB sample; we additionally included long-term interest rates; we added a broad measure of money; we estimated separate models with US and euro area data to approximate foreign variables; we excluded episodes with exchange rate interventions by the SNB from the sample; we estimated the model on a longer sample with a non-binding ZLB starting in 1985; we imposed symmetric restrictions on foreign variables; we varied the lag number and the horizon for the sign restrictions; we left GDP and the exchange rate unrestricted; we used an output gap measure instead of GDP; we used the official Swiss CPI.

25 From early 2009 until early 2010, the appreciation came to a halt because the SNB acted “[. . .] to prevent any further appreciation of the Swiss franc against the euro.” (SNB 2009, p. 9). Since early 2010, the SNB allowed the Swiss franc to appreciate somewhat as it aimed only to “[. . .] prevent an excessive appreciation of the Swiss franc against the euro” (SNB 2010, p. 9). In June 2010, the SNB stopped the foreign exchange interventions because “[. . .] the threat of deflation in Switzerland had largely disappeared” (SNB 2011b, p. 11). In the following, as the euro area debt crisis intensified, the Swiss franc appreciated to record highs in the Summer 2011. As a response to the strong appreciation, the SNB introduced a minimum exchange rate in September 2011 which is in effect to this day.
Note: Estimated posterior density function of impulse response function to a negative risk shock after 48 months with binding ZLB. The weight of the prior is varied between $\lambda \in [0.2, 0.6]$, where the latter roughly corresponds to the value estimated from the data. The probability that the exchange rate response is negative ranges from 76% ($\lambda = 0.2$) to 92% ($\lambda = 0.6$). For the aggregate CPI it ranges from 77% to 97%.

distribution for the ZLB sample. The tightness of this prior is treated as an additional parameter to be estimated from the data. It can be interpreted as the weight given to observations from the non-binding ZLB sample in the posterior distribution of the parameters in the ZLB sample. Figure 7 shows how the estimated posterior density of the impulse response functions after 48 months changes with various values for this weight. A value of $\gamma = 0.6$ roughly correspond to the weight estimated from the data. When we gradually reduce the weight of the prior, we see that the variance of the posterior distribution increases. However, even with a very small weight of the prior distribution ($\gamma = 0.2$), there is a 3/4 chance that the price level and the exchange rate are negative, 48 months after a temporary risk shock. This suggests that, while our specific choice of the prior helps to reduce the posterior uncertainty, it is not the main driver of our results.

5 Conclusions
Nominal exchange rate fluctuations are a major concern for central banks in small and open economies because they are seen as an important determinant of consumer price inflation. In Switzerland, unusually large exchange rate fluctuations have triggered bold policy action by the Swiss National Bank, such as a minimum exchange rate against the German mark in 1978 and a minimum exchange rate against the euro in 2011. As it
happens, these episodes coincided with very low short-term interest rates.

We offer a structural explanation of such unusually large shifts in the nominal exchange rate when the ZLB is binding. We show theoretically and empirically that, anchoring expectations on the inflation rate may lead to large and persistent responses of the exchange rate and the price level after a temporary risk shock. By contrast, our theoretical considerations suggest that taking into account the level of the nominal exchange rate or even anchoring expectations on a price level path instead of the inflation rate would alter the impact of temporary shocks; they have smaller and only temporary effects on the exchange rate and the price level even though interest rates may be constrained by the ZLB. However, in periods when the ZLB does not bind, these differences are negligible.

Our theoretical model formalised the idea that anchoring price level expectations may be particularly useful in a small and open economy with a preference for low inflation. On the one hand, a low inflation target increases the probability of hitting the ZLB. As a consequence, also the probability of large and persistent shifts of the nominal exchange rate increases. On the other hand, targeting a higher inflation rate may not be desirable because of welfare losses due to relative price distortions (see e.g. Khan et al. 2003, Coibion et al. 2012). Therefore, a central bank may need to actively manage price-level expectations when favouring a low average inflation rate, a relatively stable exchange rate, and an independent monetary policy strategy.
References


Appendix A: Log-linear approximation

To make the paper self-contained, this appendix reiterates the linearised first-order conditions using the notation of Baurle and Menz (2008) for the small open-economy model by Monacelli (2005). The only difference is that we do not express the nominal interest rate in terms of deviations from the steady state and therefore subtract its steady state value \((\bar{r} + \bar{\pi})\) and we adapted the monetary policy rule. All variables except the nominal interest rate are denoted in terms of deviations from steady state.\(^{26}\)

The model features an Euler equation relating consumption to future consumption, the real interest rate and preference shocks.

\[
c_t - hc_{t-1} = c_{t+1|t} - hc_t - \frac{1-h}{\sigma} (i_t - \bar{r} - \bar{\pi} - \pi_{t+1|t}) + \frac{1}{\sigma} \left( \varepsilon_{g,t} - \varepsilon_{g,t+1|t} \right) . \tag{27}
\]

Then, domestic goods market clearing requires that domestic output is related to consumption, the terms of trade, the law of one price gap, and foreign output.

\[
y_t = (1 - \alpha)c_t + \alpha \eta (2 - \alpha)s_t + \alpha \eta \psi_{F,t} + \alpha y^*_t \tag{28}
\]

In addition, we have Phillips-curve relationships for both, domestic and import price deflators.

\[
\pi_{F,t} - \delta_F \pi_{F,t-1} = \beta (\pi_{F,t+1|t} - \delta_F \pi_{F,t}) + \kappa_F \psi_{F,t} + \varepsilon_{cp,t} \tag{29}
\]

\[
\pi_{H,t} - \delta_H \pi_{F,t-1} = \beta (\pi_{H,t+1|t} - \delta_H \pi_{H,t}) + \kappa_H \psi_{H,t} + \kappa_H mc_t ,
\]

where real marginal costs are given by

\[
mc_t = \varphi y_t - (1 + \varphi)\varepsilon_{a,t} + \alpha s_t + \frac{1-h}{\sigma}(c_t - hc_{t-1}) \tag{30}
\]

Domestic CPI inflation is defined as

\[
\pi_t = \pi_{H,t} + \alpha \Delta s_t . \tag{31}
\]

The uncovered interest rate parity condition differs from the one that we discuss in section 2 because it includes a term for the log real net foreign asset position as a fraction of steady state domestic income (moreover, it is set up in terms of the inflation differential and the real interest rate):

\[
(i_t - \bar{r} - \bar{\pi}) - (i^*_t - \bar{r} - \bar{\pi}^*) = \pi_{t+1|t} - \pi^*_{t+1|t} + \Delta q_{t+1|t} - \chi a_t - \varepsilon_{rp,t} . \tag{32}
\]

The budget constraint requires:

\[
c_t + a_t = \frac{1}{\beta} a_{t-1} - \alpha (s_t + \psi_{F,t}) + y_t . \tag{33}
\]

\(^{26}\)Parameter values and Dynare programs to derive the results are available upon request.
The monetary policy rule differs in some respects from Baurle and Menz (2008), as described in the main part of the paper. The three different monetary policy strategies are formalised in the following modified Taylor rules:

- **Inflation targeting**: \( i_t = \bar{r} + \bar{\pi} + \rho(i_{t-1}^{nc} - \bar{r} - \bar{\pi}) + \psi_\pi \pi_t + \psi_y y_t + \varepsilon_{i,t} \) (34)

- **Price-level targeting**: \( i_t^{nc} = \bar{r} + \bar{\pi} + \rho(i_{t-1}^{nc} - \bar{r} - \bar{\pi}) + \psi_p p_t + \psi_y y_t + \varepsilon_{i,t} \)

- **Mod. inflation targeting**: \( i_t^{nc} = \bar{r} + \bar{\pi} + \rho(i_{t-1}^{nc} - \bar{r} - \bar{\pi}) + \psi_\pi \pi_t + \psi_e e_t + \psi_y y_t + \varepsilon_{i,t} \)

- **ZLB constraint**: \( i_t = \max(0, i_t^{nc}) \).

The price level is defined as

\[ p_t = p_{t-1} + \pi_t \] (35)

Some variables specific to the open economy are the law of one price gap (\( \psi_{F,t} \)), the terms of trade (\( s_t \)) and the nominal as well as the real exchange rate (\( e_t, q_t \)):

\[
\begin{align*}
\Delta \psi_{F,t} &= \Delta e_t + \pi^*_t - \pi_t - (1 - \alpha)(s_t - s_{t-1}) \\
\Delta s_t &= \pi_{F,t} - \pi_{H,t} \\
q_t &= \psi_{F,t} + (1 - \alpha)s_t
\end{align*}
\] (36)

Note that the foreign economy is exogenously given to the small domestic economy and it is set up symmetrically (except that we do not generalise the rule to price level targeting, and that we do not allow for a binding ZLB).
### Appendix B: Data

**Table 4 — Data**

<table>
<thead>
<tr>
<th>Label</th>
<th>Description</th>
<th>Source</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e_t$</td>
<td>Nominal effective exchange rate</td>
<td>SNB</td>
<td>Against 24 main trading partners of Switzerland.</td>
</tr>
<tr>
<td>$y_t$</td>
<td>Swiss GDP</td>
<td>SECO, authors’ calculations</td>
<td>Transformed to monthly frequency using the method described in <a href="Stulz2007">Stulz (2007)</a>. Seasonally adjusted.</td>
</tr>
<tr>
<td>$p_t^t$</td>
<td>Tradable consumer prices</td>
<td>SFSO, authors’ calculations</td>
<td>Tradable items are defined as those items that have an import share larger than 5%. CPI items not collected monthly are interpolated using a Kalman-filter approach (<a href="HuwilerKaufmann2013">Huwiler and Kaufmann (2013)</a>). Seasonally adjusted.</td>
</tr>
<tr>
<td>$p_t^n$</td>
<td>Non-tradable consumer prices</td>
<td>SFSO, authors’ calculations</td>
<td>Non-tradable items are defined as those items that have an import share smaller than 5%. CPI items not collected monthly are interpolated using a Kalman-filter approach (<a href="HuwilerKaufmann2013">Huwiler and Kaufmann (2013)</a>). Seasonally adjusted.</td>
</tr>
<tr>
<td>$p_i^t$</td>
<td>Import price index</td>
<td>SFSO</td>
<td>Import prices at the docks in domestic currency excluding indirect taxes.</td>
</tr>
<tr>
<td>$i_t$</td>
<td>Short-term interest rate</td>
<td>SNB</td>
<td>3M Libor</td>
</tr>
<tr>
<td>$i_t^*$</td>
<td>Foreign short-term interest rate</td>
<td>OECD MEI, BIS, authors’ calculations</td>
<td>Average of three-month euro area, US and Japanese short-term interest rates.</td>
</tr>
<tr>
<td>$p_t^*$</td>
<td>Foreign consumer prices</td>
<td>SNB, authors’ calculations</td>
<td>Calculated based on nominal and real effective exchange rates (see <a href="Stulz2007">Stulz (2007)</a>). Seasonally adjusted.</td>
</tr>
<tr>
<td>$vix_t$</td>
<td>Swiss volatility index</td>
<td>SNB, authors’ calculations</td>
<td>Implied volatility of SMI linked with implied volatility of DAX in 1/1999.</td>
</tr>
<tr>
<td>$vix_t^*$</td>
<td>S&amp;P 500 volatility index</td>
<td>Datastream</td>
<td>Implied volatility of S&amp;P 500.</td>
</tr>
</tbody>
</table>

Note: All data calculated by the authors are available upon request.

To obtain a measure of tradable and non-tradable consumer prices we assume that CPI items with an average import share larger than 5% are tradable. The import share was provided by the Swiss Federal Statistical Office. We then aggregate these items to an index of tradable and non-tradable prices using the official CPI expenditure weights. Before doing so, we interpolated the price series of some items using a Kalman-filter approach because they were collected only on a quarterly basis ([Huwiler and Kaufmann (2013)](HuwilerKaufmann2013)). Because of data availability restrictions, we cover 88.1% of the CPI basket at 2012 expenditure weights. However, the results do not change significantly if we use the official aggregate CPI.