

Stijepic, Damir

**Conference Paper**

## Successive Technical Change and the Demand for Skill

Beiträge zur Jahrestagung des Vereins für Socialpolitik 2013: Wettbewerbspolitik und Regulierung in einer globalen Wirtschaftsordnung - Session: Technological Change, No. F10-V1

**Provided in Cooperation with:**

Verein für Socialpolitik / German Economic Association

*Suggested Citation:* Stijepic, Damir (2013) : Successive Technical Change and the Demand for Skill, Beiträge zur Jahrestagung des Vereins für Socialpolitik 2013: Wettbewerbspolitik und Regulierung in einer globalen Wirtschaftsordnung - Session: Technological Change, No. F10-V1, ZBW - Deutsche Zentralbibliothek für Wirtschaftswissenschaften, Leibniz-Informationszentrum Wirtschaft, Kiel und Hamburg

This Version is available at:

<https://hdl.handle.net/10419/79870>

**Standard-Nutzungsbedingungen:**

Die Dokumente auf EconStor dürfen zu eigenen wissenschaftlichen Zwecken und zum Privatgebrauch gespeichert und kopiert werden.

Sie dürfen die Dokumente nicht für öffentliche oder kommerzielle Zwecke vervielfältigen, öffentlich ausstellen, öffentlich zugänglich machen, vertreiben oder anderweitig nutzen.

Sofern die Verfasser die Dokumente unter Open-Content-Lizenzen (insbesondere CC-Lizenzen) zur Verfügung gestellt haben sollten, gelten abweichend von diesen Nutzungsbedingungen die in der dort genannten Lizenz gewährten Nutzungsrechte.

**Terms of use:**

*Documents in EconStor may be saved and copied for your personal and scholarly purposes.*

*You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.*

*If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.*

# Successive Technical Change and the Demand for Skill

Preliminary and Incomplete, February 2013

Damir Stijepic

*Department of Money and Macroeconomics, Goethe University Frankfurt, Germany*

---

## **Abstract**

Skill-Biased Technical Change is one of the most prominent explanations for the rise in wage inequality in the United States over the last decades. However, the explanation is challenged for several reasons. In this paper, I propose an alternative type of technical change, where new technologies are initially adopted only by a fraction of firms (henceforth referred to as Successive Technical Change). I show that the implications of Successive Technical Change - in a heterogeneous firms model with search frictions - are in line with a broad set of stylized facts I derive from the Current Population Survey and the Economic Census of the United States. In particular, the model is consistent with the polarization of within-group wage distributions and the revenue distribution, the rise in the skill premium, and the increase of the firm size wage premium of college-graduates relative to the one of non-college-graduates. Perhaps the most interesting prediction of the model is that - depending on the state of the economy - a policy that fosters technology adoption at small and medium sized firms may decrease inequality.

*Keywords:* wage inequality, technical change, heterogeneous firms, frictional labor markets, on-the-job search

*JEL:* F16, J31

---

*Email address:* damir@stijepic.com (Damir Stijepic)

## 1. Introduction

Since the 1980s the United States has experienced a substantial increase in wage inequality.<sup>1</sup> Skill-Biased Technical Change is one of the most prominent explanations. However, international trade and institutional factors have as well been argued to be important. In this paper, I propose an alternative explanation: Successive Technical Change. With this term I refer to a situation where a new technology is initially adopted only by a fraction of firms. As time passes, all firms may eventually upgrade to the new technology. However, the key idea is that technical change is happening successively. To illustrate the explanatory power of Successive Technical Change, I construct a heterogeneous firms model with search frictions in the labor market, where technical change is endogenously successive due to fixed adoption costs. The resulting model is consistent with a broad set of stylized facts I derive from the Current Population Survey and the Economic Census of the United States. In particular, the period since the late 1980s was marked by a polarization of within-group wage distributions and the revenue distribution, a rise in the skill premium, and an increase of the firm size wage premium of college-graduates relative to the one of non-collage-graduates. While the model provides a reasonably good fit to the data, it has as well distinct policy implications. In contrast to Skill-Biased Technical Change, Successive Technical Change is skill-neutral in the long run provided that the new technology is eventually fully adopted. Revenue dispersion, within-group inequality, and the firm size wage premia are as well unaffected by Successive Technical Change in the long-run provided that all firms upgrade to the advanced technology. Furthermore, depending on the state of the economy a policy that fosters technology adoption at small and medium sized firms may decrease inequality. This stands as well in contrast to a standard Skill-Biased Technical Change formulation.

The proposed structural model is essentially a wage-ladder model, where workers climb the ladder upwards by finding employers that offer higher wages and move downwards if laid off. In particular, I consider an on-the-job search model with heterogeneous firms à la Burdett and Mortensen (1998) with two extensions: First, I introduce a possibility for firms to upgrade their productivity ex post by incurring a fixed cost. Therefore, additionally to the exogenous dispersion in productivity of the original Burdett and Mortensen (1998) framework, the model, that I present in this paper, exhibits an endogenous component as well. Comparative

---

<sup>1</sup>See Lemieux (2008) for a documentation of inequality patterns and a discussion of possible explanations.

statistics across steady states allow to assess the effects of technical change. Secondly and similarly as in Holzner and Launov (2010), I consider two types of workers and allow firms to discriminate by worker types. Therefore, my framework allows a comprehensive analysis of wage inequality patterns: within-group inequality, between-group inequality, but as well differences in within-group inequality across worker types.

As in the Burdett and Mortensen (1998) model the most productive firms find it profitable to pay higher wages and therefore are able to attract and retain a higher steady state labor force. In equilibrium only a fraction of and only the most productive firms self-select into the more advanced technology for sufficiently high fixed costs of a technology upgrade. In such an equilibrium the productivity dispersion exceeds the one that would prevail under an adoption rate of zero. The higher productivity dispersion translates into a higher revenue dispersion, which in turn translates into higher within-group wage inequality. This is the mechanism that generates the positive effect of Successive Technical Change on wage inequality.

I relate the heterogeneous effect of Successive Technical Change on different skill groups to differences in relative upward mobility defined as the share of job-to-job transitions in overall worker-employer separations. On the one hand, empirical evidence suggests substantial heterogeneity across skill groups in the sense that college-graduates exhibit a consistently higher upward mobility relative to non-college-graduates over the entire sample period (see Section 2 and Fallick and Fleischman (2004)). On the other hand, the relative upward mobility is a key determinant of a worker's earnings prospects in the on-the-job search literature. In a nutshell, according to the model high-skill workers, whose mobility exceeds the one of low-skill workers, are initially more likely to profit from Successive Technical Change relative to low-skill workers, since they are more likely to find the better-paid jobs at the high-productivity firms that adopt the new technology.

This paper is, of course, related to the literature on Skill-Biased Technical Change and its more sophisticated form of endogenously skill-biased directed technical change by Acemoglu (2002). A recent assessment of the empirical content of Skill-Biased Technical Change can be found in Acemoglu and Autor (2011). Conceptually the key difference is that Successive Technical Change is not based on technology that is per se skill-biased, it is the difference in adoption rates among heterogeneous firms that generates the skill-bias. Note that Skill-Biased Technical Change and Successive Technical Change are not exclusive in the sense that the technology that is adopted successively may be skill-biased. Empirically the key difference lies in the predicted pattern of the rise in wage in-

equality. While the standard Skill-Biased Technical Change formulation predicts an increase in inequality along the entire wage distribution, the rise in overall inequality under Successive Technical Changes is the result of an increase in top-end wage inequality. Empirical evidence attributes the rise in inequality in the 1990s and the new millennium mostly to rising upper-tail wage inequality. Furthermore, the literature suggests that the increase in low-end wage inequality alongside top-end wage inequality in the 1980s is mainly due to a decrease in the minimum wage (e.g., DiNardo et al., 1996; Lee, 1999).

Furthermore, this paper is related to the literature on international trade and wage inequality insofar as it identifies large and very productive firms as the driving force behind the increase in wage inequality. See Helpman et al. (2010, 2012) and the references therein for an overview of the literature. Indeed, the results of this paper carry over to an environment with horizontally differentiated product markets and fixed costs of exporting. In such a specification access to foreign markets plays a similar role as a productivity upgrade under the current setup: Exporting and adoption of more advanced technologies both lead to higher revenue dispersion and finally to higher wage inequality. I do not try to disentangle the effect of technological change, international trade, or any other source that may lead to disparities between small and large firms, but rather interpret technical change in a more general way. Note that the literature suggest a close link between exporting and productivity improvements (e.g., Lileeva and Trefler, 2010; Melitz and Trefler, 2012).

One of the prediction of the model is that differences in the the mobility of high-skill and low-skill workers generates a positive relation between the share of high-skill workers and firm size. Furthermore, Successive Technical Change has only a positive effect on the skill premium as long as the skill-size relation is positive. A negative skill-size relation actually results in a decrease of the skill premium in response to Successive Technical Change. To my best knowledge there is only one other paper that establishes a theoretical link between the skill composition of firms and the aggregate evolution of the skill premium: Holmes and Mitchell (2008). They established that under a positive skill-size relation market expansions lead to an increase in the skill premium and otherwise to a decrease. However, Holmes and Mitchell (2008) assume a different micro-structure and address different stylized facts.

The paper is structured as follows. Section 2 documents stylized facts on worker mobility, wages and revenues. I present the structural model in Section 3 and discuss the qualitative implication of the model in Section 5. Section 7 concludes. A description of the data and robustness exercises are available in

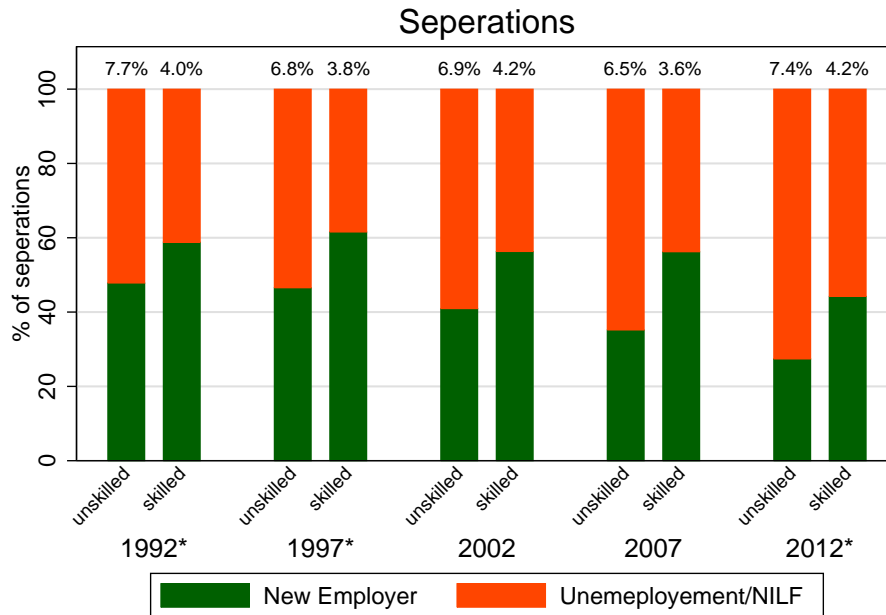


Figure 1: Worker Mobility.

Appendix A. Auxiliary results, derivations and proofs to all propositions in the text are in Appendix B.

## 2. Empirical Evidence

In this section I take a look at data of the United States over the period of the late 1980s up to the present and formulate a set of stylized facts on inter-firm worker mobility, differences in skill composition, wage policies and revenues across firm size classes, the skill premium and within group wage inequality. I rely on two data sources: the Current Population Survey (CPS) and the Economic Census of the United States. For further information see the Data Appendix.

*Stylized Fact I: Worker Mobility* The first stylized fact is that the share of job-to-job transitions in employer-worker separations is consistently higher for high-skill workers relative to low-skill workers over the sample period. Figure 1 depicts the share of employer-worker separations that are followed by transitions to new employers and into unemployment, respectively, and the overall separation rate as measured in percent of employment. First, low-skill workers exhibit consistently higher employer-worker separations rates: While the separation rate for low-skill

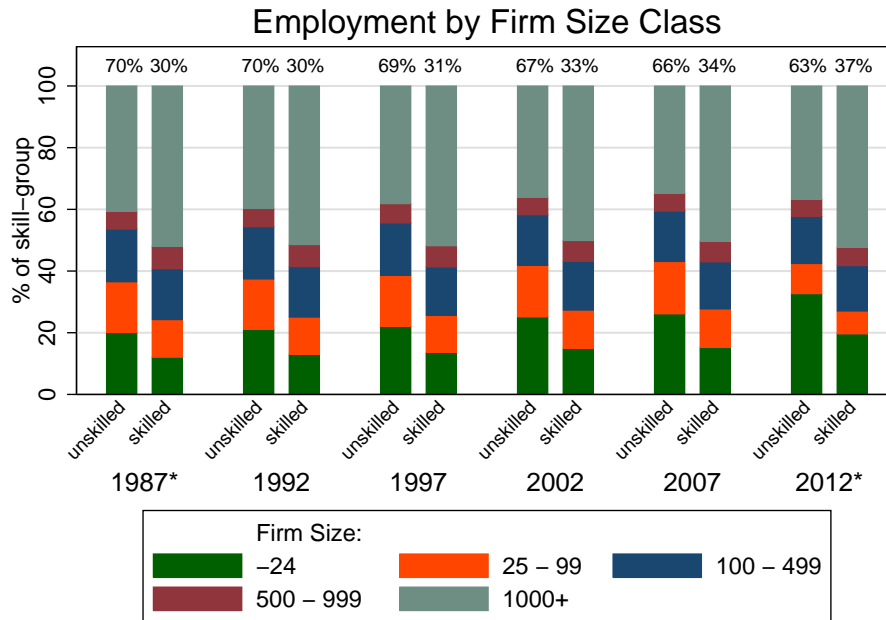


Figure 2: Employment Share.

workers ranges between 6.5% to 7.5%, the one for high-skill workers is about 3.7% to 4.5%. Secondly, while high-skill workers exhibit typically less job-to-job transitions as measured in percent of employment, they have a disproportionately lower separation rate into unemployment. Therefore, the share of job-to-job transitions in employer-worker separations is higher for high-skill workers relative to low-skill workers: For instance, the share of job-to-job transitions in employer-worker separations was 55% for high-skill workers relative to 40% for low-skill workers in the years 2000 to 2004.

*Stylized Fact II: Skill Composition by Firm Size Class* The second stylized facts is that the higher relative upward mobility of high-skill workers corresponds to a higher share of high-skill workers at larger (in terms of employment) firms. This is illustrated in Figure 2. For instance, the share of high-skill workers employed at firms with 1000 employees and more was about 50% compared to 35% for low-skill workers in the years 2000 to 2004. Furthermore, while the share of skilled workers is increasing over the sample period from 30% to 37% percent of overall employment, the share of high-skill workers and low-skill workers employed at firms with 1000 employees or more is relatively stable with 60-65% and

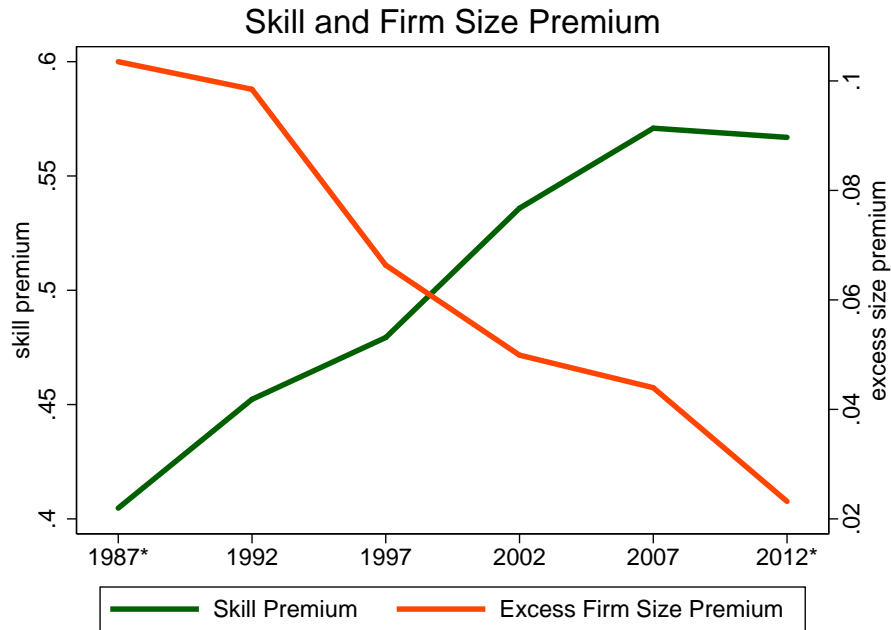


Figure 3: Skill and Wage Premium

50%, respectively.

*Stylized Fact III: Skill and Firm Size Wage Premia* The third stylized fact is that the increase in the skill premium over the sample period is accompanied by a decrease in the firm size wage premium of low-skill workers relative to the one for high-skill workers. First note that the raw wage premium of firms of 1000 and more employees over all other firms amounts to around 25%. Figure 3 shows how the excess firm size wage premium, i.e. the raw wage premium of firms with 1000 and more employees over all other firms for unskilled workers minus the corresponding wage premium for skilled workers, and the skill premium evolve over the sample period. On the one hand, the excess firm size wage premium is decreasing steadily from 10% to 2% over the whole sample period. On the other hand, the skill premium is increasing from 40% to 55%.

*Stylized Fact IV: Within-Group Inequality* The fourth stylized fact is that the increase in the skill premium and the decrease in the firm size excess premium over the sample period are accompanied by an increase in the upper tail wage inequality for both high- and low-skill workers. Figure 4 depicts the evolution of



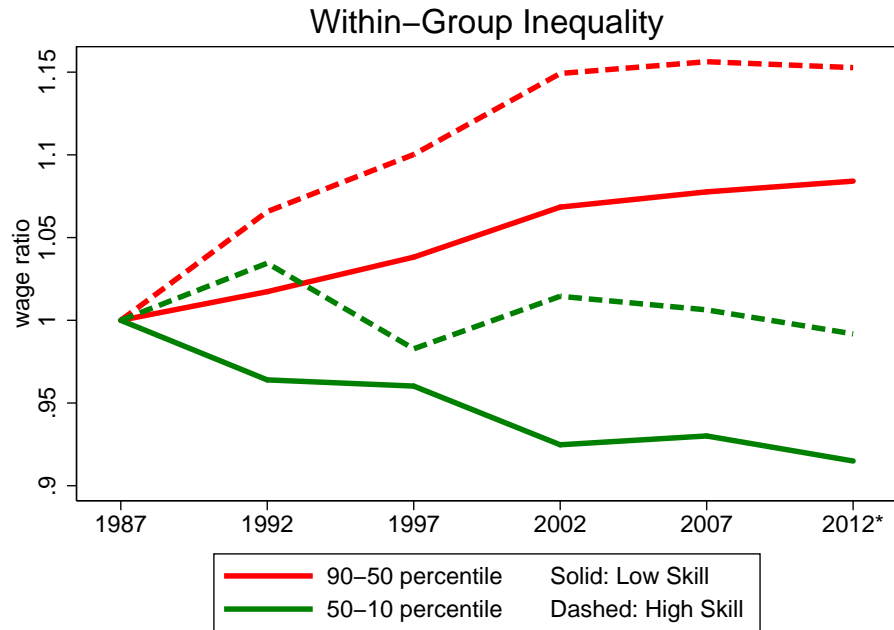


Figure 4: Within-Group Inequality

the 90-to-50 and 50-to-10 quantile wage ratios for high- and low-skill workers. For both groups upper tail wage inequality is increasing over the sample period: for high-skill workers by 15% and for low-skill workers by 8%. Lower tail wage inequality is decreasing by 10% for low-skill workers, and is rather stable for high-skill workers over the sample period.

*Stylized Fact V: Revenue Dispersion* The fifth and last stylized fact is that revenues per employee at large firms are increasing relative to revenues at small firms. Figure 5 shows how differences in revenues per employee between large and small firms evolve over time for different definitions of large firms. For all definitions of large firms, i.e. at least 25, 100, 500 or 1000 employees, the revenues per employee at large firms are increasing relative to other firms. Depending on the definition the increases ranges from 30% to 40%. Another interesting feature of the data is that revenue dispersion among large firms according to the different definitions is widening during the first half of the sample period, but closing again thereafter.

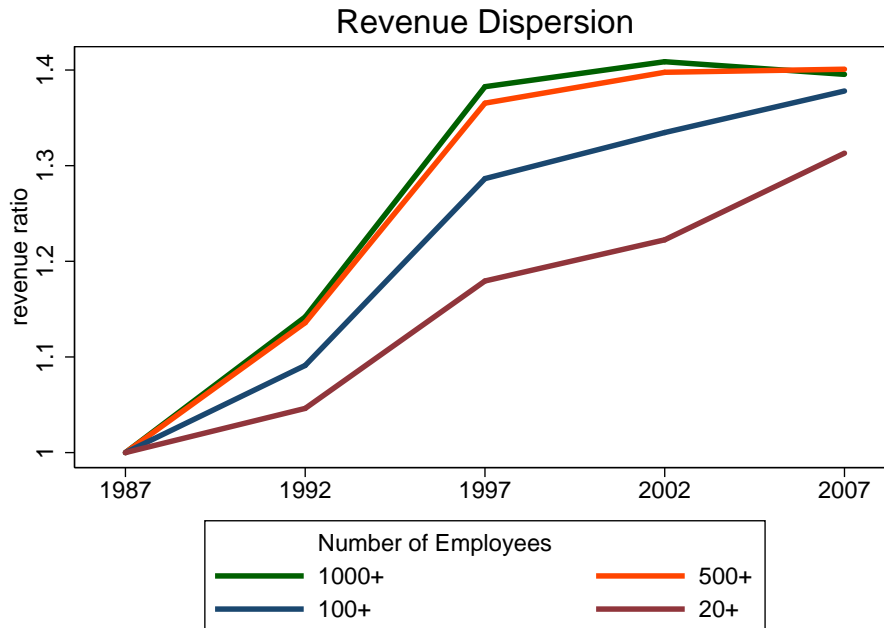


Figure 5: Revenue Dispersion

### 3. Framework

The model is in essentially an on-the-job search model with heterogeneous firms à la Burdett and Mortensen (1998) with two extensions: First, I introduce a possibility for firms to upgrade their productivity ex post by incurring a fixed cost. Therefore, additionally to the exogenous dispersion in productivity of the original Burdett and Mortensen (1998) framework, the model, that I present in this paper, exhibits an endogenous component as well. Secondly and similarly as in Holzner and Launov (2010), I consider two types of workers and allow firms to discriminate by worker types. Therefore, my framework allows a comprehensive analysis of wage inequality patterns: within-group inequality, between-group inequality, but as well differences in within-group inequality across worker types. While worker types may vary by essentially all characteristics, I focus here on differences in mobility.

### 3.1. The Economy

Let the economy consist of one sector with one homogeneous final good and let that final good be the numeraire.<sup>2</sup> There is a competitive fringe of potential firms that can choose to enter the economy by committing to an infinite stream of payments,  $f_e > 0$ .<sup>3</sup> Once a firm incurs the sunk entry cost, it observes its total factor productivity,  $A$ , which is drawn from a Pareto distribution,  $\Gamma_{A_e}(A) = 1 - (A_e/A)^z$ .<sup>4</sup> After a firm observes its productivity, it decides whether to exit the economy given fixed cost of production,  $f_d > 0$ . Each firm, indexed by  $j$ , operates a Cobb-Douglas production technology, where the two factor inputs are high-skill labor,  $l_H$ , and low-skill labor,  $l_L$ .<sup>5</sup> Therefore, its output,  $y(j)$ , is determined by

$$y(j) = A(j)l_H(j)^\theta l_L(j)^{1-\theta}, \quad \theta \in (0, 1), \quad (1)$$

where  $\theta$  is the share parameter. We assume the elasticity of substitution to be equal to one for illustrative purposes, but an additional feature of this specification is that such a production function does not allow for skill-biased technical change in its standard formulation. Therefore, this helps illustrate the novelty of the mechanism presented in this paper: The mechanism is not based on technology that is per se skill-biased, it is the difference in adoption rates among heterogeneous firms that generates the skill-bias. In what follows I use the terms output and revenues interchangeably.

There are two worker types - high-skill workers,  $H$ , and low-skill workers,  $L$  - and there is a continuum of each type of workers. All workers are ex-ante identical, risk-neutral, and equally productive conditional on type.<sup>6</sup> In the following

---

<sup>2</sup>While I assume a homogeneous goods product market for the sake of simplicity, the theoretical results hold as well with horizontally differentiated product markets.

<sup>3</sup>Firms' entry and exit decisions, as well as the technology upgrade decision are modeled in a similar way as in the Melitz (2003) heterogeneous firm model of international trade.

<sup>4</sup>The motivation for this assumption is twofold: First, estimations of similar models find the productivity distribution in most cases to be reasonably close to Pareto (e.g., Bontemps et al., 2000). Secondly, the Pareto distribution is scale invariant, and hence, the ex ante productivity distribution of active firms, i.e. before eventual technology upgrades, is always Pareto in equilibrium (irrespective of the zero profit productivity cutoff). Therefore, the results are not driven by changes in the ex ante productivity distribution of active firms, but by differences in the ex post technology choice.

<sup>5</sup>The equilibrium that I consider requires the production function to be supermodular (see Appendix B.3).

<sup>6</sup>We assume risk-neutrality of suppliers for the sake of simplicity. Our theoretical results hold irrespective of workers being risk-neutral or risk-averse.

I suppress the worker type subscript to save on notation. Information about job opportunities arrives only occasionally at rate  $\lambda > 0$ . Workers have no bargaining power and may only accept or reject a firm's wage offer.<sup>7</sup> Firms are not allowed to condition their offers on a worker's outside option, however, firms may offer different wages to each worker type.<sup>8</sup> Once a match between a firm and a worker is formed, it is at risk of being dissolved at an exogenous rate  $\delta > 0$ . Additionally, separation occurs endogenously if a worker obtains another job offer and decides to accept it. Therefore, a worker may be employed or unemployed. In the first case, she receives the wage offered by the respective firm, and in the latter case I assume that the worker enjoys no utility at all.<sup>9</sup> Therefore, workers climb the wage ladder upwards by finding employers that offer higher wages, and move downwards if laid off. I allow mobility to differ by worker type, in particular I assume the high-skill workers to be more mobile compared to low-skill workers, i.e.  $\lambda_H/\delta_H > \lambda_L/\delta_L$ . This is in line with empirical evidence presented in Section 2. As it will become clear from derivations later in the text, a worker's upward mobility,  $\lambda$ , relative to her downward mobility,  $\delta$ , is the key determinant of her earnings prospects. Workers' time preference rate is denoted by  $\rho$ .

### 3.1.1. Labor Market

Similarly as in Burdett and Mortensen (1998), the optimal behavior of workers is as follows: When information about new job opportunities arises, workers quit their current job and move to the new one, provided that the new wage offer exceeds the current one. Given a flow utility of zero, unemployed workers accept any positive wage offer.<sup>10</sup> See Appendix B.1 for further details. Now I characterize the steady state equilibrium in the labor market. Let  $N$  denote the steady-state equilibrium measure of firms in the sector,  $M$  the measure of workers, and  $U$  the measure of unemployed workers. In steady state, the flow into employment,  $\lambda U$ ,

---

<sup>7</sup>Shimer (2006) studies bargaining in a similar setup. Cahuc et al. (2006) allow for non-zero bargaining power of workers in the Postel-Vinay and Robin (2002) model.

<sup>8</sup>Here we follow Burdett and Mortensen (1998). Postel-Vinay and Robin (2002) consider a setup, where firms condition their wage offers on a worker's outside option and incumbent firms may match outside offers.

<sup>9</sup>I refrain from explicitly modeling a home production technology, and preferences over the home production good and the other goods available in the economy, since this is not the focus of my analysis. However, in the quantitative analysis I take account of minimum wage constraints.

<sup>10</sup>Strictly speaking, I assume unemployed workers to accept new offers if indifferent to doing so, whereas I assume employed workers to accept new offers only if strictly preferred.

equals the flow into unemployment,  $\delta(M - U)$ , and therefore

$$U = \frac{M}{1 + k}, \quad (2)$$

where  $k \equiv \lambda/\delta$ . Let  $G(\cdot)$  denote the steady state wage offer distribution and let  $H(w)$  denote the steady state proportion of workers receiving a wage no greater than  $w$ . In steady state, the flow of unemployed workers into firms offering a wage no greater than  $w$ ,  $\lambda G(w)u$ , equals the flow out into unemployment,  $\delta H(w)(M - U)$ , and into higher paid jobs,  $\lambda(1 - G(w))H(w)(M - U)$ , and therefore

$$H(w) = \frac{G(w)}{1 + k\bar{G}(w)}. \quad (3)$$

Firms with a labor force of  $l$  offering a wage  $w$  lose workers when they separate into unemployment,  $\delta l$ , or are poached by other firms that offer higher wages,  $\lambda\bar{G}(w)l$ . Firms attract workers who are unemployed,  $(\lambda/N)U$ , or poach workers from firms that offer lower wages,  $(\lambda/N)(M - U)H(w)$ . Therefore, a firm's steady state labor force for a given wage offer,  $w$ , is

$$l(w) = \frac{k}{b(1 + k\bar{G}(w))^2}, \quad (4)$$

where  $b \equiv N/M$  denotes labor market tightness.<sup>11</sup> All three channels, i.e. separation, retention and attraction are affected by a firm's wage strategy. Since firms are in all dimension identical except for the offered wage from the perspective of workers, a firm attains a higher steady state size by offering a higher wage.

### 3.1.2. Product Market

After having observed its productivity, a firm chooses whether or not to produce and which wage offer to post. A firm that decides to produce faces following optimization problem<sup>12</sup>

$$\pi(j) \equiv \max_{w_H, w_L} \left\{ A(j)l_H(w_H)^\theta l_L(w_L)^{1-\theta} - w_H l(w_H) - w_L l(w_L) - f_d \right\}. \quad (5)$$

The optimization problem of the firm consists in the trade-off induced by the ambivalent effect of wages on profits: On the one hand, higher wages decrease profits per worker, on the other hand, it allows the firm to attract and retain more workers.

<sup>11</sup>Following standard practice we implicitly impose a law of large numbers at the individual firm's level and treat firm size as deterministic.

<sup>12</sup>Following standard practice we assume firms to maximize steady state profits.

### 3.2. Technical Change

I consider technical change in form of Successive Technical Change. After entering the market, firms may choose to upgrade their productivity by a multiplicative factor,  $Y_x > 1$ . However, operating this more advanced technology is assumed to be associated with a fixed cost,  $f_x > 0$ . In equilibrium and for sufficiently high operating fixed costs,  $f_x$ , only a fraction of firms finds it profitable to adopt the new technology. However, as fixed costs decrease more and more firms will find it profitable to upgrade their technology. Hence the term Successive Technical Change. Under the Successive Technical Change specification a firm's production function reads

$$y(j) = Y(j)A(j)l_H(j)^\theta l_L(j)^{1-\theta}, \quad (6)$$

where  $Y(j) = Y_x$  if a firm adopts the more advanced technology and 1 otherwise. Successive Technical Change has distinct implications for within-group and between-group inequality, which are discussed in Section 5.

## 4. Equilibrium Characterization

In this Section I characterize the equilibrium of the model. First note following two propositions:

**Proposition 1 (Rank Equilibrium I).** *There is a zero-profit cutoff for productivity,  $A_d$ , such that a firm drawing a productivity below  $A_d$  exits without producing. Similarly, there is an adoption cutoff for productivity,  $A_x$ , such that a firm drawing a productivity below  $A_x$  does not find it profitable to adopt the more advanced technology.*

Therefore, the firm's decision whether or not to produce and whether or not to adopt the advanced technology takes a standard form.

**Proposition 2 (Rank Equilibrium II).** *Firms of equal productivity have the same wage policies, and more productive firms offer higher wages relative to less productive firms to each worker type.*

Given Propositions 1 and 2, we index firms henceforth by productivity,  $A$ . In the remainder of this section it is solved for the zero-profit productivity cutoffs  $A_d$ , the technology adoption productivity cutoff  $A_x$ , the mass of firms  $N$ , and the inverse wage distribution  $H^{-1}(q)$ .

#### 4.1. Productivity Cutoffs and Product Market Clearing

The least productive firms that is active in the market pays the workers' reservation wage of zero, such that the productivity cutoff  $A_d$  reads

$$y(A_d) = f_d. \quad (7)$$

The technology adoption cutoff,  $A_x$ , is given by

$$(Y_x - 1) Y(A_x) = f_x, \quad (8)$$

where we use the fact that the marginal firm's wage and thus its firm size do not depend on its technology adoption decision.<sup>13</sup> Finally, we require the product market to clear, i.e.

$$N\bar{y} = (M_H - U_H)\bar{w}_H + (M_L - U_L)\bar{w}_L + Nf_d + \bar{\Gamma}_{A_d}(A_x)Nf_x + \frac{1}{\bar{\Gamma}_{A_d}(A_d)}Nf_e, \quad (9)$$

where bars denote the respective averages. This determines the mass of firms,  $N$ .

#### 4.2. Wages

Given Proposition 1, firms' technology adoption variables,  $Y(A)$ , equals  $Y_x$  if  $A \geq A_x$  and 1 if  $A < A_x$ . Therefore, given Propositions 2, the inverse wage distribution for each worker type allows following representation

$$H_i^{-1}(q) = 2f_d\theta_i b_i \left( \frac{1 + k_i}{1 + k_i q} \right)^2 \int_0^q \frac{\hat{y}_i(x)}{1 + k_i x} dx, \quad (10)$$

for  $q \in [0, 1]$ , and  $i \in \{H, L\}$ , and where  $\theta_L = 1 - \theta$ , and  $\theta_H = \theta$ .  $\hat{y}_i(q)$  denotes output relative to the least productive firm and is given by

$$\hat{y}_i(q) = \hat{Y}_i(q) \bar{q}^{-1/z} (1 + k_i q)^{2+1/z} \left( \frac{1 + k_j}{1 + k_j + (k_i - k_j)q} \right)^{2\theta_j}, \quad (11)$$

where  $j \in \{H, L\}$  and  $j \neq i$ . Furthermore,  $\hat{Y}_i(q)$  denotes the technology adoption variable relative to the least productive firm, i.e.  $\hat{Y}_i(q)$  equals  $Y_x/Y(\underline{A})$  if  $q \geq \Gamma_x/(1 + k_i \bar{\Gamma}_x)$  and 1 otherwise. See Appendix B.2 for details. Wages are increasing in labor market tightness  $b$  as a result of a standard supply argument.

<sup>13</sup>Note that this is the case as the marginal cost of additional labor input exhibits a discontinuity at  $A_x$ .

## 5. Theoretical Results

In this section I discuss the qualitative implications of the model. First, I characterize the micro-structure of the economy, before I describe how Successive Technical Change affects the economy.

**Proposition 3 (Skill Composition).** *High-productivity firms are larger in terms of employment and have a larger share of high-skill workers relative to low-productivity firms in equilibrium. Therefore, the economy exhibits a positive skill-size relation.*

More productive firms offer higher wages and can therefore attract and retain a higher steady labor force (see Proposition 2). As the mobility of high-skill workers exceeds the one of low-skill workers, i.e.  $k_H > k_L$ , high-skill workers are more likely to find the better-paid jobs at high-productivity firms. Since the high-productivity firms are as well the large firms in equilibrium, the economy exhibits a positive skill-size relation. This establishes the theoretical counterpart to the empirical evidence presented in Section 2 on the differences in mobility between skill-groups (Stylized Fact I), the skill-composition of firms (Stylized Fact II), and the well-documented firm size wage premium. In the following I use the concept of Lorenz dominance in order to assess the effects on inequality.<sup>14</sup> Lorenz dominance is a general concept and consistent with lower inequality according to a wide class of inequality measures, most prominently the Gini-Coefficient. Furthermore, it proves convenient to define three stages of the economy: a *pre-adoption* stage, in which no firm adopts the new technology, a *partial-adoption* stage, in which only a fraction of firms adopts the new technology, and a *full-adoption* stage, in which all active firms operate the new technology.

**Proposition 4 (Successive Technical Change and Inequality).** *Successive Technical Change increases initially revenue dispersion, within-group inequality, the firm size premium for high-skill workers relative to the one for low-skill workers, and the skill premium. However, as the economy reaches the full-adoption stage, all of the previously stated statistics assume their initial values, i.e. the full-adoption and pre-adoption stage are identical with respect to the given set of statistics.*

---

<sup>14</sup>Lorenz dominance is closely related to second-order stochastic dominance. For two distributions  $F(\cdot)$  and  $G(\cdot)$  with equal means following statements are equivalent: (i)  $G(\cdot)$  Lorenz dominates  $F(\cdot)$ , and (ii)  $F(\cdot)$  second-order stochastically dominates  $G(\cdot)$ .



Successive Technical Changes increases the productivity dispersion among active firms relative to the pre-adoption stage provided that not all firms upgrade to the new technology. The increased productivity dispersion translates through the production function (6) into higher revenue dispersion. Furthermore, the adoption of the new technology increase the marginal revenue for the respective firms. Therefore, workers are now more valuable to firms and hence, firms a willing to offer higher wages to the workers. However, as it is only the high-productivity firms that adopt the new technology and high-productivity firms are at the same time the high-wage firms, wage inequality increases for each worker type. The increase in the firm size premium for high-skill relative to the one of low-skill workers results from the relatively higher mobility of high-skill workers. Since high-skill workers are more likely to change employers, the competition between firms for workers is fiercer. In other words, given the higher mobility of high-skill workers, workers are able to appropriate a larger share of the match surplus. This generates an extra wage premium at the largest firms relative to low-skill workers in the partial-adoption stage relative to the pre-adoption stage. The increase in the skill premium is a direct implication of the changes in the firm size wage premium, and the fact that high-skill workers a more likely to be employed at upgrading firms. Finally, the pre-adoption stage and the full-adoption stage do not differ according to the here discussed statistics, as the statistics are scale invariant and the pre- and full-adoption stage differ only by a multiplicative constant (e.g. equation (10)).

In an environment where a new technology is adopted initially only by a fraction of firms, and provided that all firms adopt the new technology before the implementation of even more advanced technology, the model predicts a cyclical pattern for the skill-premium and the other statistics discussed here. In contrast to Skill-Biased Technical Change ,Successive Technical Change is skill-neutral between two technology adoption phases. However, more complex patterns may arise if the adoption of several technologies overlaps. Therefore, in contrast to Skill-Biased Technical Change, Successive Technical Change is skill-neutral in the long run provided that the technology is eventually fully adopted. Another interesting prediction of the model is that the effect of Successive Technical Change on within-group inequality and the other statistics (with the exception of the skill premium), is robust to other trends that the economy might be undergoing simultaneously. In particular, the results here hold irrespective of changes in fixed costs of production or entry, and the relative supply of high-skill labor, since those variables affect only the scale of the variables in equilibrium. However, this is not the

case for the skill premium, which is given by

$$\frac{\bar{w}_H}{\bar{w}_L} = \frac{\theta M_L(1 + k_H) \int_0^1 \frac{\bar{x}\hat{y}(x)}{1+k_H\bar{x}} dx}{(1 - \theta)M_H(1 + k_L) \int_0^1 \frac{\bar{x}\hat{y}(x)}{1+k_L\bar{x}} dx}. \quad (12)$$

Following a standard supply argument, the skill-premium is decreasing in the relative supply of high-skill workers. Therefore, the positive effect of Successive Technical Change on the skill premium may be offset by a sufficiently large positive trend in the relative supply of high-skill labor.

## 6. Quantitative Analysis

Coming soon.

## 7. Conclusion

In this paper I explore the implication of Successive Technical Change in a heterogeneous firms framework with search frictions in the labor market. I show that Successive Technical Change increases initially the productivity dispersion among firms, which translates into higher revenue dispersion, and finally into higher within-group wage dispersion. Provided that the upward mobility of high-skill workers exceeds the one of low-skill workers - as it is supported by empirical evidence -, Successive Technical Change leads as well to an increase in the skill premium. This increase in the skill premium is associated with a decrease in firm size premium of low-skill workers relative to the one of high-skill workers, and reflects high-productivity firms increased demand for high-skill labor after upgrading to the more advanced technology. This is not driven by technology-skill complementarities as usually postulated, but the higher mobility of high-skill workers relative to low-skill workers. It is the high degree of mobility that makes highly productive and therefore highly competitive firms compete more intensively for the high-skill factor input.

## References

- Acemoglu, D., October 2002. Directed technical change. *Review of Economic Studies* 69 (4), 781–809.
- Acemoglu, D., Autor, D., 2011. Skills, Tasks and Technologies: Implications for Employment and Earnings. Vol. 4 of *Handbook of Labor Economics*. Elsevier, Ch. 12, pp. 1043–1171.
- Bontemps, C., Robin, J.-M., van den Berg, G. J., May 2000. Equilibrium search with continuous productivity dispersion: Theory and nonparametric estimation. *International Economic Review* 41 (2), 305–58.
- Burdett, K., Mortensen, D. T., 1998. Wage differentials, employer size, and unemployment. *International Economic Review* 39 (2), pp. 257–273.
- Cahuc, P., Postel-Vinay, F., Robin, J.-M., 2006. Wage bargaining with on-the-job search: Theory and evidence. *Econometrica* 74 (2), 323–364.
- DiNardo, J., Fortin, N. M., Lemieux, T., September 1996. Labor market institutions and the distribution of wages, 1973-1992: A semiparametric approach. *Econometrica* 64 (5), 1001–44.
- Fallick, B., Fleischman, C. A., 2004. Employer-to-employer flows in the u.s. labor market: the complete picture of gross worker flows. *Finance and Economics Discussion Series 2004-34*, Board of Governors of the Federal Reserve System (U.S.).
- Helpman, E., Itskhoki, O., Redding, S., 2010. Inequality and unemployment in a global economy. *Econometrica* 78 (4), 1239–1283.
- Helpman, E., Muendler, M., Itskhoki, O., Redding, S., 2012. Trade and inequality: From theory to estimation. mimeo.
- Holmes, T. J., Mitchell, M. F., 2008. A theory of factor allocation and plant size. *RAND Journal of Economics* 39 (2), 329–351.
- Holzner, C., Launov, A., January 2010. Search equilibrium and social and private returns to education. *European Economic Review* 54 (1), 39–59.
- Lee, D. S., August 1999. Wage inequality in the united states during the 1980s: Rising dispersion or falling minimum wage? *The Quarterly Journal of Economics* 114 (3), 977–1023.
- Lemieux, T., January 2008. The changing nature of wage inequality. *Journal of Population Economics* 21 (1), 21–48.
- Lileeva, A., Trefler, D., August 2010. Improved access to foreign markets raises plant-level productivity... for some plants. *The Quarterly Journal of Economics* 125 (3), 1051–1099.
- Melitz, M. J., 2003. The impact of trade on intra-industry reallocations and aggregate industry productivity. *Econometrica* 71 (6), pp. 1695–1725.

- Melitz, M. J., Trefler, D., Spring 2012. Gains from trade when firms matter. *Journal of Economic Perspectives* 26 (2), 91–118.
- Postel-Vinay, F., Robin, J.-M., Nov 2002. Equilibrium wage dispersion with worker and employer heterogeneity. *Econometrica* 70 (6), 2295–2350.
- Shimer, R., May 2006. On-the-job search and strategic bargaining. *European Economic Review* 50 (4), 811–830.

## **Appendix A. Data and Robustness**

Coming soon.

## Appendix B. Auxiliary Results and Proofs

### Appendix B.1. Optimal Behavior of Workers

At a moment in time, each worker is either employed or unemployed. In the first case, the worker receives the wage offered by the respective firm, and in the latter case the worker obtains a flow utility of zero. The Hamilton-Jacobi-Bellman equation of a worker receiving a wage of  $w$ ,  $V(w)$ , reads

$$\rho V(w) = w + \lambda \int \max [V(w), V(x)] - V(w) dG(x) + \delta (V_u - V(w)), \quad (\text{B.1})$$

where  $V_u$  is the value of being unemployed, and  $G(\cdot)$  denotes the steady state wage offer distribution. The equation states, that the opportunity cost of new information while working is equal to the obtained wage plus the expected capital gain attributed to receiving new information about job opportunities minus the capital loss resulting from exogenous match destruction. The Hamilton-Jacobi-Bellman equation of an unemployed worker,  $V_u$ , reads

$$\rho V_u = \lambda \int \max [V_u, V(x)] - V_u dG(x). \quad (\text{B.2})$$

The equation states, that the opportunity cost of new information while unemployed is equal to the expected capital gain attributed to receiving information about job opportunities. It can be shown that the value function of an employed workers is increasing in the wage and that the reservation wage of the unemployed equals zero under the given setup. See Burdett and Mortensen (1998) and Bontemp et al. (2000) for further details. Therefore, optimal behavior of workers is as described in Section 3.1.1.

### Appendix B.2. Equilibrium Wages

In this section we derive the inverse wage distribution (10). First note, that by Propositions 2 and 1 the first order conditions with respect to  $w_H$  and  $w_L$  of the maximization problem (5) read

$$\theta_i y(A) \frac{l'_i(w_i(A))}{l_i(w_i(A))} - w_i(A) l'_i(w_i(A)) - l_i(w_i(A)) = 0 \quad (\text{B.3})$$

where  $i \in \{H, L\}$ ,  $\theta_H = \theta$ ,  $\theta_L = 1 - \theta$ , and where primes denote the partial derivative with respect to the argument  $w_H$  and  $w_L$ , respectively. Using (4) and rearranging the first order condition (B.3) yields

$$w'_i(A) + w_i(A) \frac{2k_i \gamma(A)}{1 + k_i \bar{\Gamma}(A)} = \frac{\theta_i y(A)}{l_i(A)} \frac{2k_i \gamma(A)}{1 + k_i \bar{\Gamma}(A)}. \quad (\text{B.4})$$

This is a linear differential equation and admits following solution with the boundary condition  $w(\underline{A}) = 0$ :<sup>15</sup>

$$w_i(A) = 2\theta_i b_i (1 + k_i \bar{\Gamma}(A))^2 \int_{A_d}^A \frac{\gamma(x)y(x)}{1 + k_i \bar{\Gamma}(x)} dx \quad (\text{B.5})$$

To simplify the expression we use the zero-profit condition (7), properties of the Pareto distribution, and apply a change of variable formula

$$w_i(\Gamma) = 2(1 + k_i \bar{\Gamma})^2 \theta_i b_i f_d \int_0^\Gamma \frac{\hat{y}(x)}{1 + k_i \bar{x}} dx, \quad (\text{B.6})$$

for  $\Gamma \in [0, 1]$ , and where

$$\hat{y}(\Gamma) = \hat{Y}_P(\Gamma) \bar{\Gamma}^{-1/z} \left( \frac{1 + k_H}{1 + k_H \bar{\Gamma}} \right)^{2\theta} \left( \frac{1 + k_L}{1 + k_L \bar{\Gamma}} \right)^{2(1-\theta)}, \quad (\text{B.7})$$

and  $\hat{Y}_P(\Gamma) = Y_x/Y(0)$  if  $\Gamma \geq \Gamma_{A_d}(A_x)$  and 1 otherwise. Substituting for  $\Gamma$  with (3) and applying a change of variable formula one obtains the inverse wage distribution (10).

### Appendix B.3. Proofs

**PROOF OF PROPOSITION 1.** Let  $\tilde{y}(A)$  denote the equilibrium output of a firm of productivity  $A$  scaled by its productivity  $Y(A)A$ , and let  $W(A)$  denote the costs of the respective firm. Profit maximization of firms implies following chain of inequalities if  $A > A'$

$$\begin{aligned} AY(A)\tilde{y}(A) - W(A) &\geq AY(A')\tilde{y}(A') - W(A') \\ &> A'Y(A')\tilde{y}(A') - W(A') \geq A'Y(A)\tilde{y}(A) - W(A). \end{aligned} \quad (\text{B.8})$$

Subtracting the last inequality from the first one yields

$$(A - A')Y(A)\tilde{y}(A) \geq (A - A')Y(A')\tilde{y}(A'). \quad (\text{B.9})$$

Given fixed costs of a technology upgrade, a firm's decision to adopt a new technology depends positively on a firm's total production. Therefore,

$$Y(A) \geq Y(A') \text{ and } \tilde{y}(A) \geq \tilde{y}(A') \text{ if } A > A', \quad (\text{B.10})$$

from which Proposition 1 follows.  $\square$

<sup>15</sup>It is optimal for the least productive firm to offer a wage of zero, as otherwise the least productive firm could increase its profit by decreasing its wage offer (e.g., Bontemps et al., 2000).

PROOF OF PROPOSITION 2. This proof builds on Bontemps et al. (2000) and Holzner and Launov (2010), and requires the production function to be supermodular, i.e.

**Definition 1 (Supermodularity).** A function  $f : R^k \rightarrow R$  is supermodular if

$$f(x \vee y) + f(x \wedge y) \geq f(x) + f(y) \quad \forall x, y \in R^k, \quad (\text{B.11})$$

where  $\vee$  denotes the component-wise minimum and  $\wedge$  the component-wise maximum of  $x$  and  $y$ .

The Cobb-Douglas production function used in the main text satisfies supermodularity. We begin by establishing an intermediate results, from which the Proposition will follow.

**Lemma 5.** *An equilibrium of the wage posting game satisfies following condition*

$$w_H(A) \geq w_H(A') \text{ and } w_L(A) \geq w_L(A') \text{ if } A > A'. \quad (\text{B.12})$$

PROOF. We proceed by a proof by contradiction. Assume  $w_H(A) > w_H(A')$  and  $w_L(A) < w_L(A')$ , and note that factor input is non-decreasing in the offered reward. Therefore,

$$\begin{aligned} 0 &< \tilde{y}(w_H(A), w_L(A)) - \tilde{y}(w_H(A'), w_L(A)) \\ &\leq \tilde{y}(w_H(A'), w_L(A)) - \tilde{y}(w_H(A'), w_L(A')) < 0, \end{aligned} \quad (\text{B.13})$$

where  $\tilde{y}(A)$  denotes the equilibrium output of a firm of productivity  $A$  scaled by its productivity  $Y(A)A$ . The first and last inequality follow from equation (B.10) and optimality of firms' wage offers. The second inequality results from supermodularity of the production function.  $\square$

Finally, given Lemma 5, it remains to be shown that all firms of the same productivity offer the same wage. We achieve this last task, by showing that the continuity of the productivity distribution,  $\Gamma$ , leaves no room for wage dispersion among firms of the same productivity. But first note that the support of each cross sectional wage distribution  $H_i$  with  $i \in \{H, L\}$  is necessarily connected in equilibrium, as otherwise firms may increase profits by lowering wage offers (see Bontemps et al., 2000). We proceed by a proof by contradiction. With no loss of generality assume that the set of productivity values, for which the optimal wage is not a singleton, is given by  $[A_0, A_1]$ , where  $A_0 < A_1$ . Furthermore, as those



optimal wage sets do not intersect and are connected (Lemma 5), the segment of the real line of admissible wage values  $[0, \infty)$  can only be divided into countably many intervals. Therefore, this establishes the countability of a segment of the real line, i.e.  $[A_0, A_1]$ , which provides the desired contradiction.  $\square$

PROOF OF PROPOSITION 3. Using equation (4), Proposition 2, and rearranging yields

$$s = \frac{k_H m_H}{k_H m_H + k_L m_L \left( \frac{1+k_H \bar{\Gamma}}{1+k_L \bar{\Gamma}} \right)^2}, \quad (\text{B.14})$$

i.e. the share of high-skill workers at a firm with rank  $\Gamma$  in the productivity distribution of active firms. From the last expression it follows that  $\frac{\partial s}{\partial \Gamma} > 0$  as  $k_H > k_L$ .  $\square$

PROOF OF PROPOSITION 4. Proposition 4 consists of five claims:

**Claim 1** Within-group inequality of each worker type in the partial-adoption stage exceeds the one in the pre-adoption stage.

**Claim 2** The skill premium in the partial-adoption stage exceeds the one in the pre-adoption stage.

**Claim 3** Revenue dispersion in the partial-adoption stage exceeds the one in the pre-adoption stage.

**Claim 4** The excess firm size wage premium in the partial-adoption stage exceeds the one in the pre-adoption stage.

**Claim 5** All the previously stated statistics do not differ between the pre- and the full-adoption stage.

I now prove each of the claims in turn.

*Proof of Claim 1* Lorenz Dominance is defined as follows:

**Definition 2 (Lorenz Dominance).** Let  $H_A(w)$  and  $H_T(w)$  be two cumulative distribution functions and denote their mean values as  $\mu_{H_A}$  and  $\mu_{H_T}$ , respectively.  $H_A$  Lorenz dominates  $H_T$  iff  $L(H_A, q) \geq L(H_T, q)$  for all  $q \in [0, 1]$  and for some  $q$  with strict inequality, where  $L(H, q) = \frac{1}{\mu_H} \int_0^{H^{-1}(q)} w dH(w)$ .

As the wage functions are given by equation (10) in terms of quantiles of the respective cross sectional distribution,  $H$ ,  $L(H, q)$  is simply  $\int_0^q H^{-1}(x)dx \Big/ \int_0^1 H^{-1}(x)dx$ . With  $Y_x > 1$  Proposition 4 follows directly from the last expression.  $\square$

*Proof of Claim 2* Using equations (3) and (B.6) to derive the average wage for each worker type yields

$$\bar{w}_i = \int_0^1 2(1 + k_i \bar{\Gamma})^2 \theta_i b_i f_d \int_0^\Gamma \frac{\hat{y}(x)}{1 + k_i \bar{x}} dx \frac{1 + k_i}{(1 + k_i \bar{\Gamma})^2} d\Gamma. \quad (\text{B.15})$$

Changing the order of integration and rearranging results in

$$\begin{aligned} \bar{w}_i &= 2\theta_i b_i (1 + k_i) f_d \int_0^1 \frac{\bar{x} \hat{y}(x)}{1 + k_i \bar{x}} dx \\ &\quad + (Y_x - 1) 2\theta_i b_i (1 + k_i) f_d \int_{\Gamma_x}^1 \frac{\bar{x} \hat{y}(x)}{1 + k_i \bar{x}} dx \equiv \bar{w}_L^0 + \bar{w}_L^{-0} \end{aligned} \quad (\text{B.16})$$

for  $\Gamma_x \in (0, 1)$  and where  $\hat{y}(x) \equiv \hat{y}(x)/Y_p(x)$  and  $\Gamma_x \equiv \Gamma_{A_d}(A_x)$ . Proposition 4 claims

$$\bar{w}_H / \bar{w}_L > \bar{w}_H^A / \bar{w}_L^A \quad \text{if } \Gamma_x \in (0, 1), \quad (\text{B.17})$$

which is equivalent to

$$\bar{w}_H^{-A} / \bar{w}_H^A > \bar{w}_L^{-A} / \bar{w}_L^A \quad \text{if } \Gamma_x \in (0, 1). \quad (\text{B.18})$$

Using the definitions of  $w_i^0$  and  $w_i^{-0}$  the last expression reads

$$\int_{\Gamma_x}^1 \frac{\bar{x} \hat{y}(x)}{1 + k_H \bar{x}} dx \Big/ \int_0^1 \frac{\bar{x} \hat{y}(x)}{1 + k_H \bar{x}} dx > \int_{\Gamma_x}^1 \frac{\bar{x} \hat{y}(x)}{1 + k_L \bar{x}} dx \Big/ \int_0^1 \frac{\bar{x} \hat{y}(x)}{1 + k_L \bar{x}} dx \quad (\text{B.19})$$

The inequality follows from  $k_H > k_L$ , which completes the proof.  $\square$

*Proof of Claim 3* Follows directly from the definition of revenues and  $Y_x > 1$ .  $\square$

*Proof of Claim 4* Consider a high-productivity firm with rank  $\Gamma_u$  in the productivity distribution of active firms that adopts the new technology, i.e.  $1 > \Gamma_u > \Gamma_x$ ,

and a low-productivity firm of rank  $\bar{\Gamma}_d$  that does not upgrade its technology, i.e.  $\Gamma_x > \bar{\Gamma}_d > 0$ . By equation (B.6) it follows for the relative wages

$$\frac{w_i(\Gamma_u)}{w_i(\bar{\Gamma}_d)} = \frac{(1 + k_i \bar{\Gamma}_u)^2 \int_0^{\Gamma_u} \frac{\hat{y}(x)}{1+k_i \bar{x}} dx}{(1 + k_i \bar{\Gamma}_d)^2 \int_0^{\bar{\Gamma}_d} \frac{\hat{y}(x)}{1+k_i \bar{x}} dx} \quad (\text{B.20})$$

Therefore,

$$\begin{aligned} \frac{w_H(\Gamma_u)}{w_H(\bar{\Gamma}_d)} - \frac{w_L(\Gamma_u)}{w_L(\bar{\Gamma}_d)} &= \frac{w_H(\Gamma_u)}{w_H(\bar{\Gamma}_d)} \Big|_{\Gamma_x=0} - \frac{w_L(\Gamma_u)}{w_L(\bar{\Gamma}_d)} \Big|_{\Gamma_x=0} \\ &+ (Y_x - 1) \left( \frac{(1 + k_H \bar{\Gamma}_u)^2 \int_{\Gamma_x}^{\Gamma_u} \frac{\hat{y}(x)}{1+k_H \bar{x}} dx}{(1 + k_H \bar{\Gamma}_d)^2 \int_0^{\bar{\Gamma}_d} \frac{\hat{y}(x)}{1+k_H \bar{x}} dx} - \frac{(1 + k_L \bar{\Gamma}_u)^2 \int_{\Gamma_x}^{\Gamma_u} \frac{\hat{y}(x)}{1+k_L \bar{x}} dx}{(1 + k_L \bar{\Gamma}_d)^2 \int_0^{\bar{\Gamma}_d} \frac{\hat{y}(x)}{1+k_L \bar{x}} dx} \right), \end{aligned} \quad (\text{B.21})$$

where  $\hat{y}(x) \equiv \hat{y}(x)/Y_P(x)$ . The term in parentheses is positive for  $k_H > k_L$ , which completes the proof.  $\square$

*Proof of Claim 5* As the pre- and full-adoption stages differ only by a multiplicative constant and all statistics discussed here are scale invariant, Claim 5 follows immediately.  $\square$