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# Household Income Risk, Nominal Frictions, and Incomplete Markets\*

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## Abstract

This paper examines the effects of changes in uncertainty of household income on the macroeconomy. Households face substantial idiosyncratic income risk that is up to two orders of magnitude larger than total factor productivity uncertainty, very persistent and varies substantially over the business cycle. We build a New Keynesian model with heterogeneous agents, where changes in precautionary savings due to time-varying uncertainty depress aggregate activity. With countercyclical markups through sticky prices, increased precautionary savings lower aggregate demand and generate significant output losses as the economy is demand-driven in the short-run. The decline in output is more severe, if the central bank is constrained by the zero lower bound. Our results imply that household income uncertainty may be an important factor in explaining the persistent decline of consumption during the Great Recession.

**Keywords:** Incomplete Markets, Nominal Rigidities, Uncertainty Shocks.

**JEL-Codes:** E22, E12, E32

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# 1 Introduction

The Great Recession has led to a reconsideration of the role of uncertainty in business cycles. High uncertainty is regarded as one of the main factors holding back recovery. Increased uncertainty has been documented in various markets, but household income uncertainty is arguably most often blamed for the slow recovery. Households face substantial idiosyncratic income risk that is up to two orders of magnitude larger than that of total factor productivity (TFP) risk, very persistent and varies substantially over the business cycle. The seminal work by Storesletten et al. (2001) estimated that during an average NBER recession households' income risk is about 126% higher than at the peak of an expansion. The aim of this paper is to quantify the contribution of time-varying household income uncertainty to business cycle variations. We follow Basu and Bundick (2011) and Gornemann et al. (2012) in modeling the transmission of uncertainty shocks through nominal rigidities leading to time-varying markups.<sup>1</sup>

Our starting point is that market incompleteness induces households to hold precautionary savings. The extent of precautionary savings depends on the volatility of the household's income stream. If dispersion of income increases, households will save more for precautionary motives. Without nominal frictions the consequent drop in consumption simply triggers a decline in the real interest rate that is sufficient to equalize aggregate demand and supply. This overly simplistic neoclassical setup, however, ignores that price inertia is a robust feature of the data and that the nominal interest rate set by the central bank may not be market-clearing as, for instance, at the zero lower bound.

For this reason, the present paper extends the standard-incomplete markets model, as pioneered by Bewley (1980), Huggett (1993), and Aiyagari (1994), to incorporate time-varying idiosyncratic volatility and a nominal rigidity.<sup>2</sup> In this model, when faced with higher uncertainty, households' increased propensity to save decreases aggregate demand and prices. Depending on the reaction of mone-

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<sup>1</sup>There is also a literature that models cyclical variations in factor reallocation as transmission mechanism, e.g. Bachmann and Bayer (2011) or Bloom et al. (2012)

<sup>2</sup>Storesletten et al. (2001) explore the effect of time-varying income risk in a standard incomplete markets model analyzing the welfare costs of business cycles and argue that time variations in risk due to the business cycles generate considerable welfare costs. They do not explore the business cycle implications of time-varying risk.

tary policy, interest rates may not fall sufficiently in order to restore the flexible-price equilibrium. Therefore, following a drop in aggregate demand, aggregate output may decline as well.

We set up two versions of the model. First, we look at an economy where all capital is held by entrepreneurs who also receive all profits and who are perfectly insured among each other against idiosyncratic shocks. Workers, by contrast, only hold fiat money as an asset. In this economy, when labor income becomes more uncertain, workers desire to increase self-insurance by saving more which drives up money demand. Given the particular asset structure, the central bank is not able to supply workers with more real money by open market policy. The first version of the model hence captures an approximation of monetary policy at the zero lower bound. We show that in this setting, an uncertainty increase can have substantial depressing effects. A one standard deviation increase in uncertainty decreases aggregate activity on impact by 0.84% in the first quarter and by 3.1% over the first year. The economy recovers from this shock only fairly sluggishly after 20 quarters.

After analyzing the simplified version of the model, we relax the assumptions on the asset structure. Workers now own the physical capital in the economy, while entrepreneurs own all firms and perfectly insure amongst themselves against idiosyncratic profit risks. Workers trade the shares in the economy's capital stock as in Aiyagari (1994) or Krusell and Smith (1998) and the central bank can now intervene. We assume that the aggregate stock of capital is fixed so that fluctuations in savings lead to fluctuations in the relative price of capital. We explicitly model the transmission mechanism of monetary policy in that the central bank offers an interest rate on a nominal asset of which it supplies a negligible amount. The optimal household portfolio allocation links the return on the shares in the capital good, inflation and the nominal interest rate.

Upon an increase in uncertainty, households want to increase their savings and thereby drive up capital prices which lowers the return on capital. If the central bank does not decrease the return on the nominal asset quick enough, this yields an excess demand for the nominal asset and an excess supply of goods, which in turn drives down good prices. Nominal rigidities then cause consumption and output to fall, which even further drives down returns on the real asset. In this

version of the economy, a one standard deviation increase in uncertainty decreases aggregate activity on impact by  $XX\%$  and the economy recovers from this shock only fairly sluggishly after  $YY$  quarters.

Our modeling strategy focuses on how the capital market, and in particular the precautionary savings channel of an uncertainty shock, impact on the aggregate economy. For this reason, we abstract from a labor-leisure decision by households. As stressed in Basu and Bundick (2011), an increase in aggregate uncertainty operates in a standard New Keynesian model with capital through the labor market. If uncertainty about aggregate productivity increases, the representative household wants to insure against the higher income risk by producing more today, i.e. supplying more labor. As a result, wages and hence marginal costs for firms fall. If prices are rigid, firms' markups over marginal costs will increase and the demand for consumption and investment goods falls. Since in the New Keynesian model output is demand driven, a recession follows. We abstract from this labor market channel in order to isolate the effects stemming from precautionary savings.

In a similar vein, Mericle (2012) develops a model with incomplete markets, nominal rigidities and shocks to idiosyncratic uncertainty. Our contribution differs from Mericle (2012) by modeling strategy, solution method and focus. We explicitly model central bank policy, we solve for the full general (Krusell-Smith) equilibrium taking the dynamic evolution of heterogeneity into account, and focus on the quantitative implications of uncertainty shocks in the tradition of calibrated DSGE models of the business cycle. Mericle (2012), by contrast, focusses on the effects of the interaction of the zero-lower bound and idiosyncratic uncertainty in an environment, where because of the lower bound central bank policy is inactive, prices of consumption goods are entirely fixed and households hold out-of-equilibrium price expectations. While all this leads to strong and intuitive theoretical results that highlight important propagation channels, it limits the quantitative predictions.

The remainder of the paper is organized as follows: Section 2 develops our basic model where households only hold fiat money and monetary policy is inactive. Section 3 discusses the solution method. Afterwards, Section 4 modifies the model by replacing money by claims to the aggregate capital stock and introducing active monetary policy. Section 5 presents the calibration, Section 6 our numerical

results, and Section 7 concludes.

## 2 The Basic Model

We model an economy inhabited by two types of agents: Workers and Entrepreneurs. Workers supply labor and are subject to idiosyncratic shocks to their productivity. Workers have only access to fiat money as an asset for self-insurance. Entrepreneurs, by contrast, own firms which turn labor services, final goods and a fixed capital stock into intermediate goods and these into differentiated final consumption goods. The final goods producers are monopolistic price setters that are subject to a pricing friction à la Calvo Calvo (1983) and hence can only adjust their prices with some positive probability. Intermediate goods producers on the other hand are perfectly competitive.

### 2.1 Workers

There is a continuum of ex-ante identical worker-households (in short households) of measure one. Households are infinitely lived, derive felicity from consumption  $c_t$  and maximize the discounted sum of felicity:

$$V = E_0 \max_{\{c_{it}\}} \sum_{t=0}^{\infty} \beta_W^t u(c_{it}) \quad (1)$$

The utility function is twice continuously differentiable, increasing and concave in  $c_t$ , and takes CRRA form with risk aversion  $\xi$ :

$$u(c_t) = \frac{1}{1-\xi} c_t^{1-\xi}, \xi > 0.$$

Workers derive income from supplying labor. Households are endowed in each period with  $h_{it}$  efficiency units of labor, which evolves according to an AR(1)-process.

$$\log h_{it} = \rho_h \log h_{it-1} + \epsilon_{it}, \quad \epsilon_{it} \sim N(\mu, \sigma_{ht}) \quad (2)$$

Since we abstract from a labor leisure trade-off to isolate the precautionary savings

effect of income uncertainty, households provide all of their hours of labor  $\bar{N}$  and thus total labor input supplied is given by

$$N^S = \bar{N} \int h_j dj$$

We assume that markets are incomplete. Households can only trade in one non-state-contingent asset  $m_{it}$ . In our basic setup, this asset does not bear any interest and households can only hold non-negative amounts. Prices are denominated in units of this asset, i.e. the asset is fiat money. Hence, households maximize their utility subject to the following sequence of budget constraints.

$$P_t c_{it} + m_{it} = m_{it-1} + P_t w_t h_{it} \bar{N}, \quad m_{it} \geq 0, \quad (3)$$

where  $P_t = \left( \int p_{jt}^{1-\eta} dj \right)^{\frac{1}{1-\eta}}$  is the average price level,  $w_t$  is the real wage and  $c_{it}$  is household  $i$ 's demand of the bundled consumption good obtained from bundling varieties  $j$  of differentiated consumption goods according to a Dixit-Stiglitz aggregator

$$c_{it} = \left( \int c_{ijt}^{\frac{\eta-1}{\eta}} dj \right)^{\frac{\eta}{\eta-1}}$$

Each of these differentiated goods is offered at price  $p_{jt}$  such that the demand for each of the varieties is given by

$$c_{ijt} = \left( \frac{p_{jt}}{P_t} \right)^{-\eta} c_{it}.$$

We assume that the aggregate nominal money supply  $M_t$  is given through past emissions of money in exchange for government expenditures, but is fixed to  $\bar{M}$  in the short run.

Since households' saving decisions will be some non-linear function of a household's wealth and productivity, the price level  $P_t$  and therefore aggregate real money  $\tilde{M}_t = \frac{\bar{M}}{P_t}$  will be functions of the entire joint distribution  $\Theta_t$  of  $(\tilde{m}_t, h_t)$ . Consequently  $\Theta_t$  becomes a state variable of the household's planning problem once we express inflation as a function of  $\Theta_t$  and the exogenous states. Since we want to focus on movements in income risk, we assume that all other parameters of

the economy are time-fixed except for the variance of idiosyncratic income shocks, where we assume

$$\sigma_{ht}^2 = \bar{\sigma}^2 s_t, \quad \log s_t = \rho_s \log s_{t-1} + \nu_t, \quad \nu_t \sim N(0, \sigma_s), \quad (4)$$

where  $\bar{\sigma}^2$  is the steady state labor risk of the households and  $s$  shifts this risk.

Let the inflation rate in period  $t$  be  $\pi_t = \frac{P_t}{P_{t-1}}$ . We can then rewrite the household's budget constraint by eliminating the price level and replacing nominal money holdings  $m_{it}$  by real money holdings  $\tilde{m}_{it}$ :

$$c_{it} + \tilde{m}_{it} = \pi_t^{-1} \tilde{m}_{it-1} + w_t h_{it} \bar{N}, \quad \tilde{m}_{it} \geq 0. \quad (5)$$

The household's problem in recursive form accordingly reads

$$V(\tilde{m}, h; \Theta, s) = \max_{c, \tilde{m}'} u(c) + \beta_w EV(\tilde{m}', h'; \Theta', s') \quad (6)$$

subject to  $c + \tilde{m}' = \pi(\Theta, s)^{-1} \tilde{m} + w(\Theta, s) h \bar{N}$  and  $\tilde{m}' \geq 0$ . We define  $\tilde{m}^*$  as the corresponding policy function that describes households' money demand.

## 2.2 Entrepreneurs

Entrepreneurs own both the intermediate goods producing sector and the final goods production. We assume entrepreneurs can perfectly insure each other against the risk that Calvo-pricing introduces and can hence express profits from the production of intermediate goods as  $r_t K_t$ , where  $K_t = \bar{K}$  is fixed physical capital used in the production. Entrepreneurs do not have access to any assets beyond the insurance against pricing shocks. In particular, we assume that they cannot form capital or hold money. The former assumption reflects our focus on short-run fluctuations, while the latter is made for tractability reasons. It enables us to determine the price setting of entrepreneurs without having to take into account intertemporal decision making of the workers. Under these assumptions, the consumption of an entrepreneur is given by

$$c_t^E = r_t \bar{K} + E(\Pi_{jt}),$$



where  $\Pi_{jt}$  is the current profit of the  $j$ -th final goods producer. Given their preferences,

$$V = E_0 \sum_{t=0}^{\infty} \beta_E^t u(c_{it}^E) \quad (7)$$

entrepreneurs maximize over prices of final goods.

### 2.2.1 Final Goods Producers

Final goods producers buy intermediate goods at a price equalling the nominal marginal costs  $MC_t P_t$ , where  $MC_t$  are the real marginal costs at which the intermediate goods are traded due to perfect competition, and diversify them without the need of additional input factors. Final goods come in varieties uniformly distributed on the unit interval and each indexed by  $j \in [0, 1]$ . Resellers are monopolistic competitors and therefore can charge a markup over their marginal costs. They are, however, subject to a Calvo Calvo (1983) price setting friction and can only update their prices with probability  $\theta$ . They maximize the expected value of future discounted profits by setting today's price  $p_{jt}$  taking into account the price setting friction:

$$\max_{\{p_{jt}\}} \sum_{s=0}^{\infty} \theta^s E Q_{t,t+s} \Pi_{jt,t+s} = \sum_{s=0}^{\infty} \theta^s E Q_{t,t+s} Y_{jt,t+s} (p_{jt} - MC_{t+s} P_{t+s}) \quad (8)$$

$$s.t. : Y_{jt,t+s} = \left( \frac{p_{jt}}{P_{t+s}} \right)^{-\eta} Y_{t+s}$$

where  $Q_{t,t+s} = \beta_E^s E \left( \frac{u_c(c_{t+s}^E)}{u_c(c_t^E)} \right)^\xi$  is the stochastic discount factor of entrepreneurs.  $\Pi_{jt,t+s}$  are the profits and  $Y_{jt,t+s}$  is the production level in  $t+s$  of a firm  $j$  whose last price reset was in period  $t$ .

We obtain the following first order condition with respect to  $p_{jt}$ :

$$\sum_{s=0}^{\infty} \theta^s E Q_{t,t+s} Y_{jt,t+s} \left( \frac{p_{jt}^*}{P_{t-1}} - \underbrace{\frac{\eta}{\eta-1}}_{\mu} MC_{t+s} \frac{P_{t+s}}{P_{t-1}} \right) = 0 \quad (9)$$

where  $\mu$  is the static optimal markup.

We assume that individual resellers mutually insure idiosyncratic pricing risks so that we can solve the resellers' planning problem by log-linearizing around the zero inflation steady state without having to know the solution of the households problem as entrepreneurs and households do not interact on an asset market. This yields after some tedious algebra, see e.g. Galí (2008), the new Keynesian Phillips curve.

$$\log \pi_t = \beta E_t(\log \pi_{t+1}) + \kappa(\log MC_t + \mu) \quad (10)$$

where

$$\kappa = \frac{(1 - \theta)(1 - \beta\theta)}{\theta}.$$

The assumption of perfect insurance amongst entrepreneurs represents a short cut when solving for the equilibrium. A log-linear approximation of the first-order condition does not affect the workings of our model, because the main focus of our analysis lies on households' income risk and its effects on precautionary asset holdings.

### 2.2.2 Intermediate Goods Producers

Since we abstract from a household's labor-leisure choice and capital is fixed, we need to introduce another mechanism through which aggregate output can vary in the economy. A standard way would be to model capacity utilization. Varying utilization, however, has direct consequences for the variability of the marginal productivity of capital, which is substantially dampened. While this would not be of concern in our basic model, it would interfere with the transmission mechanism of uncertainty shocks in the extended version. For this reason, we proceed in a different way.

We assume that the intermediate goods producing sector operates a gross CRS production function instead of a value added production function, which combines pre-products  $X_t$  acquired on the final consumption goods market at price  $P_t$ , labor  $N_t$  and capital  $K_t = \bar{K}$ . Hence, total gross output of the intermediate goods sector is

$$Y_t = X_t^\alpha N_t^\beta K_t^{1-\alpha-\beta}.$$

Let  $MC_t$  be the relative price at which intermediate goods are sold to final

goods producers. The intermediate goods producers seek to maximize profits through their choice of the extend of pre-products used in production.

$$MC_t Y_t - X_t = MC_t X_t^\alpha N_t^\beta \bar{K}^{1-\alpha-\beta} - X_t$$

The optimal amount of pre-products is then given by

$$X_t^* = \alpha MC_t Y_t = (\alpha MC_t)^{\frac{1}{1-\alpha}} N_t^\gamma \bar{K}^{1-\gamma}; \quad \gamma := \frac{\beta}{1-\alpha}. \quad (11)$$

Once the optimal amount of pre-products used in production is determined, we can express GDP, which is equal to consumption in this setting, as

$$C_t = Y_t - X_t^* = \left[ (\alpha MC_t)^{\frac{\alpha}{1-\alpha}} - (\alpha MC_t)^{\frac{1}{1-\alpha}} \right] N_t^\gamma \bar{K}^{1-\gamma}. \quad (12)$$

It moreover implies that the intensity in which pre-products are used in production is pro-cyclical, which is in line with the data.

$$X_t/Y_t = \alpha MC_t.$$

The pro-cyclicality of pre-products represents a further amplification mechanism. A lower initial consumption demand for final goods  $C_t$  diminishes resellers' demand for intermediate goods. Since intermediate goods producers in turn employ final goods in their production process, they will also decrease their demand  $X_t$  for final goods. This interdependence of final and intermediate goods producers amplifies the initial decline in the demand for final goods and thus the output drop.

The real wage and the user costs of capital are given by the marginal products of labor and capital.

$$w_t = \beta \alpha^{\frac{\alpha}{1-\alpha}} MC_t^{\frac{1}{1-\alpha}} N_t^{\gamma-1} \bar{K}^{1-\gamma} \quad (13)$$

$$r_t + \delta = (1 - \alpha - \beta) \alpha^{\frac{\alpha}{1-\alpha}} MC_t^{\frac{1}{1-\alpha}} N_t^\gamma \bar{K}^{-\gamma} \quad (14)$$

## 2.3 Goods, Money and Labor Market

Given the real wage above, the labor market clears.

$$N_t^D = N_t^S$$

In our basic setting, monetary policy is passive and keeps the nominal amount of fiat money constant at  $\bar{M}$ . This means the money market clears, whenever

$$\frac{\bar{M}}{P_t} = \int \tilde{m}_t^*(\tilde{m}_{t-1}, h_t; \Theta_t, s_t) \Theta_t(\tilde{m}_{t-1}, h_t) d\tilde{m}_{t-1} dh_t =: A(\Theta_t, s_t)$$

such that the price level is given by

$$P_t = \frac{\bar{M}}{A(\Theta_t, s_t)}.$$

We can rewrite this condition in inflation form, i.e. as a sequence of market clearing conditions.

$$\tilde{M}_t := \int \tilde{m}_{t-1} \Theta_t(\tilde{m}_{t-1}, h_t) d\tilde{m}_{t-1} dh_t = \pi_t \int A(\Theta_t, s_t) \quad (15)$$

The goods market clears, whenever the money market clears due to Walras law.

## 2.4 Recursive Equilibrium

A *recursive equilibrium* in our basic model is a set of policy functions  $\{c^*, \tilde{m}^*\}$ , value functions  $V$ , pricing functions  $\{w, \pi\}$ , aggregate real money and labor supply functions  $\{\tilde{M}, N\}$ , distribution  $\Theta$  over individual money holdings and productivity, and a perceived law of motion  $\Gamma$ , such that

1. Given  $V$ ,  $\Gamma$ , prices, and distributions, the policy functions  $\{c^*, \tilde{m}^*\}$  solve the household's problem and given the policy functions  $\{c^*, \tilde{m}^*\}$ , prices and distributions, the value function  $V$  is a solution to the Bellman equation (6).
2. The labor, money and goods market clear, i.e. (13), (15), and (10) hold.
3. The actual law of motion and the perceived law of motion  $\Gamma$  coincide, i.e.  $\Theta' =$

$\Gamma(\Theta, s')$ .

### 3 Numerical Implementation

Of course the dynamic program (6) and hence the recursive equilibrium is not computable as it involves the infinite dimensional object  $\Theta$ .

#### 3.1 Krusell-Smith equilibrium

In order to turn this problem into a computable one, we assume that households predict future prices only on the basis of a restricted set of moments as in Krusell and Smith (1997, 1998). Specifically, we make the assumption that households condition their expectations on last period's average real money holdings  $\tilde{M}_t$  as in (15), the realized variance of idiosyncratic productivity  $var(h_{it})_t$  and the uncertainty state  $s$ . The reason behind this assumption is that from (15) one can infer the inflation rate in period  $t$ , given by  $\frac{\tilde{M}_{t-1}}{\tilde{M}_t}$ , and the distribution of inflation rates pins down the distribution of real returns on money holdings as can be seen from (5). Hence knowing the law of motion of  $\tilde{M}_t$  is sufficient for a household in making its savings decisions. If the optimal money demand function  $\tilde{m}^*$  is sufficiently close to linear where the mass of  $\Theta$  is, then we can expect approximate aggregation to hold with  $s_t$ ,  $var(h_{it})_t$ , and  $\tilde{M}_t$ .

While the laws of motion for  $s_t$  and  $var(h_{it})_t$  are pinned down by (2) and (4), households use the following log-linear forecasting rule for future real money holdings and hence aggregate prices, where the coefficients depend on the uncertainty state.

$$\log \tilde{M}_{t+1} = \beta_M^1(s_t) + \beta_M^2(s_t) \log \tilde{M}_t + \beta_M^3(s_t) var(h_{it})_t \quad (16)$$

Importantly, as in Krusell and Smith (1997) we need to find in each period the inflation rate that clears the money market. Concretely, this means the deposited rule (16) is used to solve for the household's policy functions. In this, the forecasting rule for real money holdings also implies expectations about  $\pi_{t+1}^e$ , which is

equal to

$$\frac{\tilde{M}_t}{\tilde{M}_{t+1}} = \exp \left[ (1 - \beta_M^2(s_t)) \log \tilde{M}_t - \beta_M^1(s_t) - \beta_M^3(s_t) \text{var}(h)_t \right].$$

Having solved for the policy functions of the household conditional on the forecasting rule (16), we then simulate  $n$  independent sequences of economies for  $t = 1, \dots, T$  periods, keeping track of the actual distribution  $\Theta_t$ . The initial distribution  $\Theta_1$  in each simulation equals the stationary one from a model without aggregate risk. We then calculate in each period  $t$  the optimal household policies for given inflation rates assuming that the household resorts to the policy functions derived under rule (16) from period  $t + 1$  onwards. After determining the market clearing inflation rate, we obtain next period's distribution  $\Theta_{t+1}$ . In doing so, we obtain  $n$  sequences of equilibria. The first  $T/2$  observations of each simulation are discarded to minimize the impact of the initial distribution. We next re-estimate the parameters of (16) from the simulated data and update the parameters accordingly. By using  $n = 100$  and  $T = 10000$ , it is possible to make use of parallel computing resources. Subsequently, we re-calculate policy functions and iterate until convergence in the forecasting rules.

The quality of approximation from (16) is relatively high. The minimal within sample  $R^2$  is 99.95% for  $\tilde{M}$ , which implies an  $R^2$  of still 90% for  $\pi'$ .<sup>3</sup> Also the out-of-sample performance, see Den Haan (2010), of the forecasting rule is good, see the Appendix for details.

### 3.2 Solving the household planning problem

In solving for the household's policy functions we apply an endogenous gridpoint method as originally developed in Carroll (2006) and extended by Hintermaier and Koeniger (2010), iterating over the first-order conditions. We approximate the productivity process by a discrete Markov chain with 21 states and time-varying transition probabilities, using the method proposed by Tauchen (1986). The stochastic volatility process is approximated in the same vein using 3 states.

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<sup>3</sup>The right hand side of the inflation forecast varies less, see Bachmann et al. (2013) for a similar point in investment vs. capital forecast in heterogeneous firm models.

The first order condition of the household problem can be written as

$$u_c(c) = \beta E \left\{ \frac{1}{\pi'(\tilde{M}, s', \text{var}(h)')} \frac{\partial V'}{\partial \tilde{m}'} \right\} + \lambda \quad (17)$$

where  $\lambda \geq 0$  is the multiplier of the potentially binding non-negativity constraint on asset holdings  $\tilde{m}'$ . Making use of the envelope theorem, we can replace the derivatives of the value function and obtain

$$u_c(c) = \beta E \left\{ \frac{1}{\pi'(\tilde{M}, s', \text{var}(h)')} u_c(c') \right\} + \lambda \quad (18)$$

where  $c'(\tilde{m}, h; \tilde{M}, s, \text{var}(h))$  is the consumption policy for the next period.

## 4 Extended Model

The model of Section 2 has two key shortcomings: First, there is no role for standard monetary policy through open market interventions or setting of some nominal interest rate. Second, the assumption of households not at all being able to use claims to the economy's stock of capital to smooth their consumption may be overly simplistic, even though it may actually be a relatively accurate description that the assets finally used for self-insurance bear little interest, see Kaplan and Violante (2011). In any case, the central bank is able to change the interest rate paid on liquid assets to some extent.

We therefore extend our model in the following way: Worker-households now own and trade shares in the fixed aggregate capital stock. The capital stock is rented out to intermediate goods producing entrepreneurs. Furthermore, workers can trade among themselves nominal bonds, of which the central bank offers an  $\epsilon$  amount. By setting the nominal interest rate on these bonds, the central bank can influence the portfolio choice of households.

While in the simple Bewley economy the central bank was forced to be inactive, it can now effectively conduct monetary policy using a Taylor (1993)-type interest rule.

An increase in uncertainty again induces workers to demand more assets for self-insurance. As a result, asset prices increase and consequently asset returns

decline in the aftermath. If the central bank does not cut back interest rates quick enough, this leads to an excess demand for bonds or excess supply of goods, respectively, and prices fall. Since prices are sticky, part of the demand decrease is accommodated by a decrease in supply and hence output falls driving down marginal costs.

This has a second round effect on capital returns, as discussed in Basu and Bundick (2011), driving down real asset returns further, which again depresses goods demand for a given interest rate set by the central bank. For this reason, the economy of our extended model may feature a larger demand decrease after an uncertainty shock than our basic economy. However, this will depend crucially on the strength of the central banks interest rate cut and on the steady state amount of self-insurance. If the central bank is able to aggressively cut interest rates, it can supply the economy with the needed increase in aggregate asset value without a deflationary episode.

## 4.1 The Household's Decision Problem

The household problem changes compared to the basic model in terms of the assets that are available for self-insurance. Households can use nominal one period zero-bonds  $b_{it}$  and capital shares  $k_{it}$ , such that their budget constraint now becomes

$$c_{it} + b_{it} \frac{1}{I_t} + q_t k_{it} = b_{it-1} \frac{1}{\pi_t} + (q_t + r_t) k_{it-1} + w_t h_{it} \bar{N} \quad (19)$$

where  $q_t$  is the price of a capital share,  $r_t$  is the rental rate of capital determined by (14), and  $I_t$  is the nominal discount (interest rate plus principal) on the zero bond promised in period  $t$ . Through the market clearing condition on the bonds market, this nominal interest rate is set by the central bank's Taylor rule, see 4.2. We assume that households cannot borrow.

$$b_{it} \geq 0, \quad k_{it} \geq 0 \quad (20)$$

It is key for the household to correctly predict the distribution of inflation  $\pi$ , the return on capital  $r$ , and the price of capital  $q$  as these determine the distribution of returns on the household's portfolio  $(k_{it}, b_{it})$  in period  $t + 1$ . As was the inflation



in our basic model, these prices are functions of the joint distribution  $\Theta$  of  $(k, b, h)$ . The recursive version of the household's problem takes the form

$$V(k, b, h; \Theta, s) = \max_{c, k', b'} u(c) + \beta EV(k', b', h'; \Theta', s') \quad (21)$$

subject to (20) and

$$c + b'/I(\Theta, s) + q(\Theta, s)k' = b/\pi(\Theta, s) + (q(\Theta, s) + r(\Theta, s))k + w(\Theta, s)h\bar{N}.$$

We can reduce the state space of the household's problem observing that the portfolio choice of the household does not actually constitute a state variable. What matters for the household's decisions is the amount of resources available, see Brandt (2009). Defining  $a$  to be the total non-human wealth of a household at the beginning of a period, we can re-write the household's problem as

$$V(a, h; \Theta, s) = \max_a u(c) + \beta \max_{\phi} E \{V(a' R^*(\phi), h'; \Theta', s')\} \quad (22)$$

subject to  $c = a - a' + wh$ , where  $\phi$  is the fraction of the portfolio invested in capital shares and

$$R(\phi; \Theta, \Theta', s, s') = (1 - \phi) \frac{I(\Theta, s)}{\pi'(\Theta', s')} + \phi \frac{q'(\Theta', s') + r'(\Theta', s')}{q(\Theta, s)}$$

is the stochastic gross return on the household's portfolio. Note that while  $a$  denotes wealth cum interest and dividends,  $a'$  is next period's wealth before interest and dividends are realized.

The portfolio choice is an intratemporal problem and optimal portfolio weights are based on current states  $(a, h, \Theta, s)$ . Formally,

$$\begin{aligned}
\phi^*(a, h; \Theta, s) &= \arg \max \{EV(a'R(\phi; \Theta, \Theta', s, s'), h'; \Theta', s')\}, \\
&\text{where the optimized portfolio returns are} \\
R^*(a, h; \Theta, \Theta', s, s') &:= R(\phi^*(a, h; \Theta, s); \Theta, \Theta', s, s') \\
&\text{and the asset policy is given by} \\
a^*(a, h; \Theta, s) &= \arg \max_{a'} \left\{ u(c) + \beta \max_{\phi} EV(a'R^*(a, h; \Theta, \Theta', s, s'), h'; \Theta', s') \right\}
\end{aligned} \tag{23}$$

The redefinition of the household's decision problem with available resources as state variable enables us to characterize the household's optimal choices by only two first-order conditions, namely the consumption Euler equation

$$u_c(c) = \beta E \{R^*(a, h; \Theta, \Theta', s, s')u_c(c')\} \tag{24}$$

and the optimal portfolio choice

$$E \left\{ \frac{q'(\Theta', s') + r'(\Theta', s')}{q(\Theta, s)} u_c(c') \right\} = E \left\{ \frac{I(\Theta, s)}{\pi'(\Theta', s')} u_c(c') \right\} \tag{25}$$

## 4.2 Central Bank

While the return on real assets is determined by the capital market, we assume that the central bank sets the gross nominal interest rate on bonds  $I$  according to the following Taylor (1993)-type rule:

$$\log \left( \frac{I}{I_{SS}} \right) = \phi_{\pi} \log \left( \frac{\pi}{\pi_{SS}} \right) + \phi_y \log \left( \frac{y}{y_{SS}} \right) \tag{26}$$

In line with the empirical literature, the central bank adjusts the gross nominal interest rate whenever inflation ( $\phi_{\pi} > 1$ ) or output ( $\phi_y \geq 0$ ) are not on target. The inflation target is assumed to be  $\pi_{SS}$  and the output target  $y_{SS}$  equals the value of output in the non-stochastic steady state.

As the central bank sets  $I$ , it determines demand for bonds and hence indirectly the demand for goods (i.e. all income minus excess demand for bonds). To clear the goods market, inflation  $\pi_t$  will adjust in each period. The government issues a

small amount  $\bar{b} = \epsilon$  of bonds (government debt), such that the sum over all bond holdings has to equal this quantity in every period. We assume that the interest payment on bonds is financed by a lump sum tax levied on the entrepreneurs.<sup>4</sup>

The bonds market clears if

$$\bar{b} = \int \phi^*(a^*(a, h; \Theta, s); \Theta, s) a^*(a, h; \Theta, s) \Theta(a, h) da dh \quad (27)$$

### 4.3 Recursive Equilibrium

A *recursive equilibrium* in our extended model is a set of policy functions  $\{c^*, a^*, \phi^*\}$ , value functions  $V$ , pricing functions  $\{w, \pi, q\}$ , aggregate capital and labor supply functions  $\{N, K\}$ , distribution  $\Theta$  over individual asset holdings and productivity, and a perceived law of motion  $\Gamma$ , such that

1. Given  $V$ ,  $\Gamma$ , prices, and distributions, the policy functions  $\{c^*, a^*, \phi^*\}$  solve the household's problem and given the policy functions  $\{c^*, a^*, \phi^*\}$ , prices and distributions, the value function  $V$  is a solution to the Bellman equation (22).
2. The labor, bond, capital and goods market clear, i.e. (13), (10), (26), and (27) hold.
3. The actual law of motion and the perceived law of motion  $\Gamma$  coincide, i.e.  $\Theta' = \Gamma(\Theta, s')$ .

### 4.4 Krusell-Smith Equilibrium

The recursive equilibrium is obviously in the extended model not computable as well. Since we look at short-term dynamics, the capital stock is assumed to be fix. This assumption can be interpreted as a special case of extremely high adjustment costs and simplifies the optimization problem of the intermediate goods sector, as the introduction of quadratic capital adjustment costs would otherwise lead to an additional state variable. While in the Bewley economy aggregate real money

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<sup>4</sup>This simplifies the household's planning problem, as we can work with one non-negativity constraint on real and nominal claims. It can be understood as an approximation to a setting in which workers can issue small amounts of bonds.

alters due to changes in the price level, the real value of the capital stock in the economy  $q\bar{K}$  now fluctuates according to the price level  $q$  of capital. These value fluctuations are driven by two sources: The self-insurance services that workers receive from the capital good fluctuate as uncertainty varies and the rental rate of capital fluctuates as firms' markup is changing. When making their investment decisions, workers need to predict the real value of capital  $\tilde{K}' = q'\bar{K}$  in the next period in order to predict asset returns. They condition their expectations on last periods capital value  $\tilde{K}$ , realized variance of idiosyncratic productivity  $var(h)$  and the uncertainty state  $s$ .

$$\log \tilde{K}' = \beta_K^1(s) + \beta_K^2(s) \log \tilde{K} + \beta_K^3(s) var(h) \quad (28)$$

Predictions about future prices are now made with an additional forecasting rule for inflation

$$\log \pi' = \beta_\pi^1(s') + \beta_\pi^2(s') \log \tilde{K}' + \beta_\pi^3(s') var(h)' \quad (29)$$

whereas the expected change in the price of capital  $q'$  can be directly obtained from the law of motion of the capital's value

$$q' = \frac{\tilde{K}'}{\bar{K}} \quad (30)$$

## 5 Calibration

To answer the question in how far time-varying uncertainty can account for business cycles, we first seek to closely calibrate the model to the U.S. economy - especially w.r.t. household income uncertainty and household wealth. The calibration sample ranges from 1984Q1 to 2008Q3. One period in the model refers to a quarter of a year. The choice of parameters as summarized in Tables 1 and 2 is explained next.

### 5.1 Households

The period utility function is of the constant relative risk aversion form, twice continuously differentiable as well as increasing and concave in  $c_t$ . It takes the

form:

$$u = \frac{1}{1 - \xi} c^{1-\xi}$$

where in the baseline calibration  $\xi = 3$ . The time-discount factor,  $\beta_W$ , is calibrated, jointly with other parameters, to achieve an annual money holdings to output ratio of 0.89. For the purpose of the paper, it is key to capture the uncertainty faced by households in the U.S. The idiosyncratic productivity process is calibrated toward that aim.

We adopt the conventional AR(1) process for idiosyncratic productivity, as the dynamics of individual earnings in the Panel Study of Income Dynamics (PSID) is quite well replicated by an autoregressive process. The algorithm by Tauchen (1986) is used to discretize the AR(1) process for the log of individual productivity with mean zero, persistence parameter  $\rho_h$  and a variance of the innovation of  $\sigma_t^2$ . The autocorrelation of annual earnings is chosen to be 0.95, which is within the range of existing empirical estimates (0.9 to 1).

The variance of innovation  $s$  follows an AR(1) process as well capturing the evolution of uncertainty. The parameters of the process are taken from Bayer and Juessen (2009), who estimate the autocorrelation to be  $\rho_s = .9457$ .<sup>5</sup> The variance accordingly fluctuates around the estimated average value of  $\sigma_h^2 = 0.0873$  - getting smaller by one-third at the state of minimal uncertainty and increasing by the same amount when uncertainty is highest.

Table 1: Productivity Process

Parameter	Value	Description
$\rho_h$	0.9873	Persistency of level shocks
$\sigma_h^2$	0.0873	Short run variance
$\rho_s$	0.9457	Persistency of variance
$\sigma_s^2$	0.0090	Variance of innovations to the variance

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<sup>5</sup>All values of the stochastic volatility processes are already adjusted for the quarterly calibration.

Table 2: Calibrated Parameters

Parameter	Value	Description
Households		
$\beta_W = \beta_E$	0.974	Time-discount factor
$\sigma$	3	Coefficient of relative risk aversion
Intermediate Goods		
$\alpha$	0.45	Share of pre-products
$\gamma = \beta/(1 - \alpha)$	2/3	Share of labor
Final Goods		
$\kappa$	0.0201	Price stickiness
$1/\exp(-\mu)$	39.56%	Markup
Monetary policy		
$M_s/Y$	3.56	Money to output ratio (qrtly.)

## 5.2 Intermediate Goods Producers

We parameterize the production function of the intermediate good producer according to the U.S. National Income and Product Accounts (NIPA). In the U.S. economy, total amount of pre-products used in production, the intermediate consumption, makes up roughly 45% of gross output. Hence, we set  $\alpha = 0.45$ . The labor and capital share (2/3 and 1/3) confirm with standard macroeconomic calibrations.

## 5.3 Final Goods Producers

Final good producers differentiate intermediate goods and thereby become price setters. We calibrate the price setting behavior to match the standard markup and price stickiness employed in the New Keynesian literature. The Calvo parameter  $\kappa$  implied by the New Keynesian Phillips curve is chosen in such a way to yield an average price duration of 8 quarters. The steady state marginal costs  $\exp(-\mu) = 0.7165$  and imply a markup of roughly 40%.

## 5.4 Central Bank

In the basic model economy, the steady state quarterly money to output ratio  $M_s/Y$  is calibrated to 3.46 in order to match the annual M2 to GDP ratio of

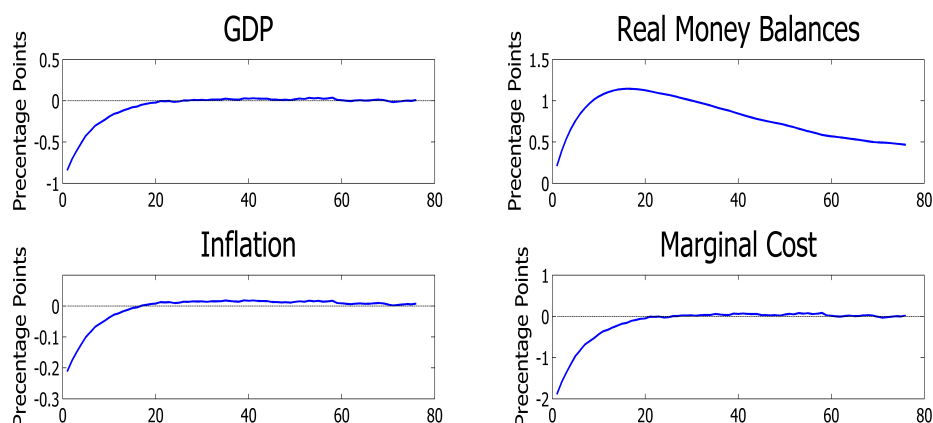
the US in 2011 of 0.89 (see World Development Indicators). M2 is given by the amount of currency held outside banks, demand deposits except from those owned by the central government, and the time, savings, and foreign currency deposits of resident sectors other than the central government.

## 6 Quantitative Results

### 6.1 Quantitative Impact of Uncertainty Shocks

Holding the calibrated parameters fixed, we analyze the effects of an exogenous increase in the uncertainty of individual household productivity. Figure 1 plots impulse responses of the endogenous variables of the model to an one standard deviation increase in the variance of idiosyncratic productivity.

Figure 1: Impulse Response Functions Basic Model: 1 Quarter Uncertainty Shock



In face of higher volatility in income, households reduce consumption to increase their precautionary real money holdings. Real money balances jump upwards on impact and rise for another 16 quarters, reaching a maximum increase of 1.46%. Afterwards they slowly converge to a higher equilibrium level due to the deflationary period after impact. As the nominal money supply remains fixed at  $\bar{M}$ , inflation  $\pi_t$  jumps down shifting the price level until the higher real money demand is met.

With Calvo price setting, only a fraction of final goods producers can adjust their prices. As a result, prices cannot decrease sufficiently to offset the decreased demand for final goods by households. With less final goods demanded, the final goods sector will in turn demand less intermediate goods.

In a model where capital cannot be adjusted, intermediate goods producers can only downscale their demand for labor and pre-products, i.e. change the extent to which production is vertically integrated. Since households have no disutility from working and supply all of their labor, real wages will fall in equilibrium while the total amount of labor used in the production remains unchanged. Intermediate goods producers will eventually only reduce their share of pre-products used in production, further depressing demand and prices for final goods. Since the perfectly competitive intermediate goods producers face lower marginal costs, the markup of final goods producers increases, who in turn reduce their production. So in fact, the recession is further amplified as the lower demand for intermediate goods by final goods producers will in turn once more reduce the demand for their own goods.

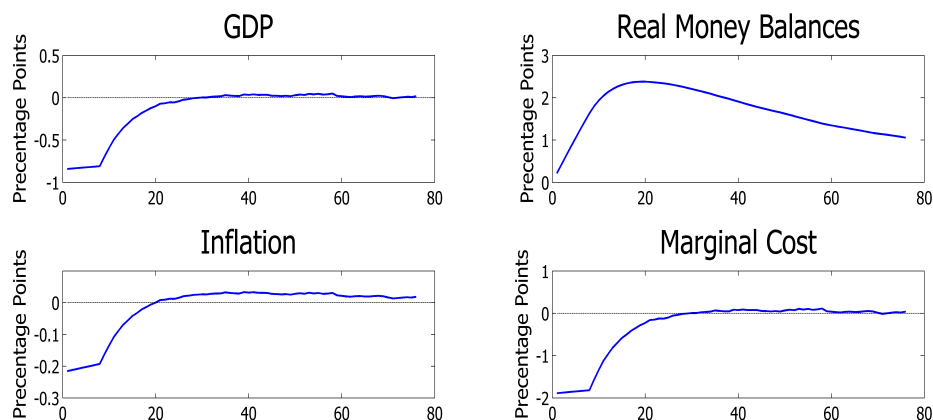
As a result, output falls on impact by 0.84% below its steady state level after a one standard deviation increase in income uncertainty and only recovers 20 quarters later. The output loss over the first four quarters combined totals 3.1%.

Figure 2 plots the effect of an extended period of 8 consecutive quarters of high uncertainty. The initial impact, of course, remains the same, but the overall output loss increases in comparison to Figure 1. The cumulative annual output loss is 4.15% in the first year of high uncertainty and 10.45% over a period of 5 years.

Figure 2 represents a lower bound for the effect of household income uncertainty during the Great Recession if monetary policy had been entirely passive. The stochastic volatility process for income uncertainty is estimated from U.S. data, which basically covers the era of the Great Moderation that arguably underestimates the increase in uncertainty during the Great Recession. Newer waves of the PSID covering the Great Recession are not available yet. Moreover, a one-standard deviation event might be a conservative estimate of the actual rareness and size of the uncertainty that came with the crisis.



Figure 2: Impulse Response Functions Basic Model: 8 Quarters High Uncertainty



## 6.2 Extended Model

*To be added*

## 7 Conclusion

This paper examines how changes in the uncertainty of household income affect the macroeconomy. We show that precautionary savings arising from market incompleteness can generate substantial output losses in a environment with countercyclical markups via sticky prices by merging the standard New Keynesian and the standard incomplete markets model. Calibrating the model to match income uncertainty estimated from the PSID, we find that a prolonged period of 8 consecutive quarters of increased income uncertainty leads to substantive output drops which may help to understand the slow recovery of the U.S. economy during the Great Recession.

At the same time the model points to the role of unconventional monetary policy. Even at the zero lower bound, additional money holdings have value to some agents in the economy as they provide self-insurance services. This value increases in times of high income uncertainty. Therefore, in our basic model setup Wallace neutrality does not hold. Even though the interest rate on money is zero, open market policy can help to reach the new equilibrium money holdings without

going through a deflationary period.

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