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Persistence in the price-to-dividend ratio and its macroeconomic fundamentals∗

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Abstract

Persistent accelerations of the log price-to-dividend ratio (PtDR) have attracted a lively discussion in the literature. In this paper we estimate a continuous time-varying long-run state of the PtDR by means of a state space model, and test the joint significance of three macroeconomic fundamentals. While consumption risk, risk sharing among households, and the demographic structure of the population affect the long-run PtDR jointly, consumption risk is the dominating factor in shaping the variations in the long-run state of the PtDR throughout the entire last century.

Keywords: Price-to-dividend ratio, consumption risk, risk sharing, demographics, nonlinear state space model, particle filtering

JEL Classification: E44, G01, G12

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1 Introduction

Aggregate stock prices have risen to unprecedented levels in the late 1990s relative to any fundamental values. The logarithmic price-to-dividend ratio (PtDR), for example, has increased by 24% from 1980 to 2012. Even after a substantial decline since its peak at 2000, its level is still far away from its historical values (see the left panel of Figure 1). In light of such observations a natural interest arises in the potential determinants of the persistent increase in the PtDR. The macroeconomic literature proposes a persistent decline in expected stock returns or a persistent increase in dividend growth rates as contributors, where most empirical evidence has pointed to factors that affect the former. For example, declines in the volatility of consumption growth rates (Bansal and Yaron; 2004; Bansal, Khatchatrian and Yaron; 2005; Lettau, Ludvigson and Wachter; 2008; Bansal, Kiku and Yaron; 2010) and improvements in the degree of risk-sharing among households and regions (Lustig and Van Nieuwerburgh; 2005, 2006, 2010) could lead to declines in the equity premium. Expected stock returns can be also influenced by demographic structures of the population (Geanakoplos, Magill and Quinzii; 2004; Favero, Gozluklu and Tamoni; 2011), persistent changes of taxation and regulatory systems (McGrattan and Prescott; 2005), or gradual entry of new participants to stock markets (Vissing-Jorgensen; 2002; Guvenen; 2009).

The persistent increase in the PtDR could reflect the persistent effect of macroeconomic shocks over a long period. Structural breaks in the mean of the PtDR have been suggested to model the persistence of the PtDR in the literature. For example, Lettau et al. (2008) adopt a two-state regime switching model for the volatility and the mean of consumption growth, which supports the pattern of structural breaks in the PtDR. Adjusting the PtDR by its mean with structure shifts, Lettau and Van Nieuwerburgh (2008) show that the adjusted PtDR explains variations in stock returns in a significant and stable manner. Favero et al. (2011) provide evidence on a slowly evolving mean of the PtDR. They show that both the PtDR and a demographic variable, the middle-aged to young ratio, are jointly significant in long-horizon predictive regressions for stock returns. The predictive power of the demographic variable may come from its impact on the slowly evolving mean of the PtDR.

In this paper we model the persistence in the PtDR by means of a latent long-run state of the PtDR explicitly, and analyse its relationship with three macroeconomic influences simultaneously. A continuous time-varying long-run state can be seen as a generalization of constant means with structural breaks. The latter is a special case of the former. Estimating a time-varying long-run state allows the data to provide information to their full extent. Herwartz, Rengel and Xu (2012) also show that the PtDR adjusted by its continuous long-run trend has better predictive content for future stock returns than the PtDR adjusted by its mean with structural breaks. Following Herwartz and Xu (2012),
we adopt a nonlinear state space model with a latent financial state variable reflecting the time-varying long-run state of PtDR. This model is based on a modified version of the present-value model proposed by Campbell and Shiller (1988). We employ US data with the sample period from 1913 to 2010. It covers several economic turning points, the Great Depression in 1929, the first and second world war, the Black Monday in 1987, the dot-com bubble from 1995 till 2000, and the recent financial crisis that emerged with the failure of Lehman in 2008. Estimation results confirm that the long-run state of the PtDR is indeed time varying (see the right panel of Figure 2). Moreover, the estimated long-run state displays some ‘stepwise’ movement, which resembles the pattern for the estimated mean with structural breaks as those diagnosed by Lettau and Van Nieuwerburgh (2008).

An important implication of considering a continuous long-run path of the PtDR is that multiple influences on the long-run PtDR can be analyzed simultaneously. This enables robust evidence on the significance of each determinant suggested in the literature. By means of a vector error correction model we investigate the endogenous relationship between the long-run state of the PtDR and three macroeconomic factors: the consumption volatility, the degree of risk sharing among households through the market for housing-collateralized debt, and the demographic structure of the population. Markets dislike economic uncertainty, and a decreasing consumption volatility (also interpreted as consumption risk) can lead to a decline in equity premia (Bansal and Yaron; 2004). Changes in the consumption volatility can affect asset prices through the stochastic discount factor in consumption-based asset pricing models (Lettau et al.; 2008). The second variable measures the degree of risk sharing through the market for housing-collateralized debt by the ratio of housing wealth to human wealth (Lustig and Van Nieuwerburgh; 2005, 2006). This variable accounts for spillover effects from housing to equity prices, which are important in the context of the recent developments in US markets. When house prices increase, the fostered collateral value of housing weakens the borrowing constraints of households. Risk sharing among households is strengthened. The dispersion of consumption growth across households is less sensitive to aggregate consumption growth shocks, and this lowers the risk premium in stock markets (Lustig and Van Nieuwerburgh; 2005). The third fundamental is the ratio of the middle-aged to young population. As Geanakoplos et al. (2004) demonstrate theoretically a small ratio of the middle-aged to young population goes along with excess demand for consumption, and the asset prices should decline to encourage saving and clear the market. Thus, we expect a positive relationship between the middle-aged to young population and the long-run PtDR.

Our estimation results and impulse response analysis show that while significant influences from all three factors on the long-run state of the PtDR are confirmed, consumption risk plays the most important role. Consumption risk alone can explain three quarters of the variations in the long-run PtDR that can be explained by our model. Graphical illustrations from simple static regressions confirm that consumption risk governs movements
in the long-run PtDR in a stable manner throughout the sample that covers almost an entire century. This result is robust for alternative measures of the consumption risk. In contrast, risk sharing among households and the demographic factor are found to influence the long-run state of the PtDR only in particular subsamples. The degree of risk sharing could explain the long-run trend of the PtDR after the 1940s, but not before. During the 1920s there was an expansion of residential housing. This was reflected in a pyramid-shaped ratio of the housing wealth to human wealth ratio between 1920 and 1940. However, during this time period the long-run state of the PtDR remained relatively low and constant. Regarding the middle-aged to young ratio, it shared similar trends as the long-run state of the PtDR before 1960 and after 1990, but not in between. Since the 1960s the baby boom generation entered into the young population and, thus, invokes a decline of the ratio of the middle-aged to young population. When the baby boom generation became middle-aged since the 1980s, this ratio was driven up. Therefore, between 1960 and 1990 the ratio of the middle-aged to young population displays a remarkable U-shape. However, during this period the long-run state of the PtDR remained relatively stable and unaffected by changes of the demographic pattern.

Section 2 introduces the present value model that captures the persistence in the PtDR. Section 3 shows the estimates of the state space model. In Section 4 we investigate the linkage between the time-varying long-run state of the PtDR and its potential macroeconomic fundamentals by means of a cointegration analysis. Section 5 concludes. A detailed description of the data and a sketch of the particle filtering approach are provided in the Appendix.

## 2 Persistence in the PtDR

We consider S&P500 data from 1871 to 2010.\(^1\) The PtDR is shown in the left hand side of Figure 1. It seemed to vary around 2.8 before 1950. After this period it has increased substantially and stayed at levels above 3. Since the 1990s a further dramatic increase occurred. The PtDR increased to an unprecedented level up to 4.5 at its peak. Sample means from rolling windows show that the PtDR doesn’t have a constant mean, as can be seen from the right hand side panel of Figure 1. Apart from eyeballing, unit root tests confirm the nonstationarity of the PtDR.\(^2\)

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1. It is downloaded from the URL of Robert J. Shiller, [http://www.econ.yale.edu/~shiller/data.htm](http://www.econ.yale.edu/~shiller/data.htm). The sample consists of annual S&P500 index quotes, the corresponding dividends, a 3-month US Treasury Bill rate and the CPI.
2. Detailed results on unit root and stationarity testing are available from the authors upon request, see also the related literature (Campbell; 1999; Herwartz and Morales-Arias; 2009; Park; 2010).
Given the nonstationarity of the PtDR, the present-value model introduced by Campbell and Shiller (1988) is not fully suitable to approximate the PtDR, as confirmed by Campbell (2008). Let $P_t$ and $D_t$ denote the stock price and corresponding dividend, respectively. The log-return, realized at the end of period $t + 1$, $r_{t+1} = \ln(P_{t+1} + D_{t+1}) - \ln(P_t)$, can be formulated as a nonlinear function of the PtDR, $\eta_t = \ln(P_t) - \ln(D_t)$, as

$$r_{t+1} = -\eta_t + \ln(\exp(\eta_{t+1}) + 1) + \Delta d_{t+1}. \quad (1)$$

Throughout, lower case letters denote natural logarithms of the corresponding upper case quantities, and $\Delta$ is shorthand for the first difference operator such that $\Delta d_t = d_t - d_{t-1}$.

If the PtDR were stationary, a linear approximation around its constant mean could be applied to (1). Due to the observed persistence of the PtDR this approach is hardly justified. Following Herwartz et al. (2012), a latent time-varying long-run state of PtDR ($\tilde{\eta}_t$) is used as the expansion point for the first-order Taylor approximation of the one-step-ahead stock returns in (1),

$$r_{t+1} \simeq \kappa(\rho_t) - \eta_t + \rho_t \eta_{t+1} + \Delta d_{t+1}, \quad (2)$$

where $\rho_t \equiv 1/(1 + \exp(-\tilde{\eta}_t))$ and $\kappa(\rho_t) \equiv -\ln(\rho_t) - (1 - \rho_t) \ln(1/\rho_t - 1)$. The time-varying long-run state of the PtDR ($\tilde{\eta}_t$) is captured in $\rho_t$, which represents the time-varying mean ratio of the stock price to the sum of the stock price and the dividend. Taking the conditional expectation and iterating (2) forward provides the following log-linear present value formulation of the PtDR

$$\eta_t \simeq \frac{\kappa(\rho_t)}{1 - \rho_t} + \sum_{i=1}^{\infty} \rho_t^{i-1} E_t[\Delta d_{t+i} - r_{t+i}] + \lim_{i \to \infty} \rho_t^i E_t[\eta_{t+i}]. \quad (3)$$

The above equation decomposes the PtDR into three components: a time-varying deterministic term (as a function of $\rho_t$), the present value of future return-adjusted dividend growth rates discounted at the time-varying rate ($\rho_t$), and a terminal value. The essential difference between the present value model in Campbell and Shiller (1988) and equation (3) is that while $\rho_t = \rho$ is constant in the former, $\rho_t$ is time varying in the latter. The long-run state of the PtDR influences PtDR dynamics through both the deterministic term and the valuation of expected future cash flows in equation (3).

Assume a stochastic error term $\epsilon_t \sim N(0, \sigma^2)$ capturing potential rational bubbles and other influences in $\lim_{i \to \infty} \rho_t^i E_t[\eta_{t+i}]$. Substituting $E_t$ by the objective expectation given the information set at the end of period $t$ ($\tilde{E}_t$), equation (3) is transformed into the

$$\eta_t \simeq \frac{\kappa(\rho_t)}{1 - \rho_t} + \sum_{i=1}^{\infty} \rho_t^{i-1} \tilde{E}_t[\Delta d_{t+i} - r_{t+i}] + \lim_{i \to \infty} \rho_t^i \tilde{E}_t[\eta_{t+i}].$$

Similar as those in Lettau and Van Nieuwerburgh (2008), the following assumptions are adopted to derive equation (3): $E_t(\eta_{t+i}) = \eta_t$ for all $i \geq 1$; $E_t(\kappa_{t+i}) = \kappa_t$ for all $i \geq 1$; $E_t(\rho_{t+i}\eta_{t+i+1}) = E_t(\rho_{t+i})E_t(\eta_{t+i+1})$, with $i \geq 1$. 

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measurement equation of the state space model,

\[ \eta_t = \frac{\kappa(\rho_t)}{1 - \rho_t} + \sum_{i=1}^{\infty} \rho_t^{i-1} \tilde{E}_t[\Delta d^e_{t+i} - r^e_{t+i}] + \epsilon_t. \]  

(4)

Superscripts \( e \) symbolize excess dividend growth rates \( \Delta d^e_{t+i} = \Delta d_{t+i} - r^f_{t+i} \), or excess returns \( r^e_{t+i} = r_{t+i} - r^f_{t+i} \), where \( r^f_t \) is the risk-free interest rate. Consistent with the persistence of the PtDR, the (financial) state equation of the model is formalized by means of a bounded random walk, i.e.

\[ \rho_t = \rho_{t-1} + u_t, \quad u_t = e_t + \xi_t - \xi_{-t}. \]  

(5)

The financial state process \( \rho_t \) is initialized with \( \rho_0 \), which is later treated as a model parameter. The error term \( u_t \) comprises \( e_t \sim N(0, \sigma^2_e) \), \( \xi_t := [0 - (\rho_{t-1} + e_t)]I\{\rho_{t-1} + e_t < 0\} \) and \( \xi_{-t} := [(\rho_{t-1} + e_t) - 1]I\{\rho_{t-1} + e_t > 1\} \), where \( I(\cdot) \) denotes an indicator function, and \( \xi_t \) and \( \xi_{-t} \) are regulators which limit the random walk in the range of 0 and 1. One may argue that a stationary first order autoregressive process with an autoregressive parameter close to unity can also capture the persistence in the long-run state of the PtDR and, thus, \( \rho_t \). However, if a bounded stationary first order autoregressive state process is used as a state equation, the resulting log-likelihood value is much lower than the one with the bounded random walk process. A more detailed discussion of the bounded non-stationary process can be found in Cavaliere and Xu (2012).

3 Estimation

Due to its nonlinearity, the state space model described in the last section is estimated by means of a particle filtering procedure based on 3000 trajectories (Cappé, Godsill and Moulines; 2007). The details of this approach are provided in the Appendix. To obtain realizations of the objective expectations about the future excess dividend growth rates \( \tilde{E}_t[\Delta d^e_{t+i}] \) and excess returns \( \tilde{E}_t[r^e_{t+i}] \) in (4), we follow Campbell and Shiller (1988), and apply simple multistep forecasts from a small dimensional first order VAR. The historical dimension of 141 years in our data set might include periods of structural breaks. Thus, we use recursive VAR forecasts from a rolling window instead of expanding windows (e.g. in Campbell and Vuolteenaho; 2004). We choose 42 years as the size of rolling windows since this size is superior to others according to the log-likelihood evaluation of the state space model and might reflect some long run economic patterns commonly referred to as Kondratieff cycles (e.g. Solomou; 2008).  

The smallest possible specification of the first order VAR would include the PtDR

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4 Since the first 42 observations serve as initialization period the resulting sample covers nearly an entire century (1913 to 2011).
series \((\eta_t)\), the excess dividend growth \((\Delta d_t^e)\) and the excess return \((r_t^e)\). We estimate a VAR comprising in addition inflation, \(\pi_t\), and a linear local trend \(t\). As proposed by Campbell and Vuolteenaho (2004) the consideration of inflation is suggested to account for effects of money illusion on equity prices. Considering potential local trends, we include a deterministic linear trend.\(^5\)

Table 1 about here

The estimates of model parameters are provided in Table 1. The left hand side panel of Figure 2 depicts the estimated financial state process. The time-varying financial state is clearly different from it’s fixed counterpart.

Figure 2 about here

Since the state space framework nests the standard model of Campbell and Shiller (1988) relying on a constant mean of the PtDR, Table 1 also includes the corresponding estimates as comparison. The log-likelihood statistics indicate the superiority of the model formulating the time-varying long-run state. The right hand side panel of Figure 2 displays the PtDR, its sample mean, the implied long-run state of PtDR, which is calculated from the estimated financial state process \((\hat{\eta}_t = \ln(1/\hat{\rho}_t - 1))\). Moreover, we display mean estimates obtained after accounting for structural breaks. The PtDR (black line) has evolved around the estimated long run state (grey solid line). Comparing the simple sample mean (black dashed line) with the time-varying long-run state (grey solid line), the former overstates the long-run state in the sample period before the 1990s. This exaggeration is due to the extremely high values of the PtDR since the 1990s. Overall, the constant mean fails to accurately describe the long-run pattern of the PtDR. It is worth mentioning that the estimated long-run state displays some ‘stepwise’ movement. A similar pattern for the mean of the PtDR with structural breaks in 1954 and 1994 is diagnosed by Lettau and Van Nieuwerburgh (2008). To compare the time varying state with this result we similarly apply the structural break test of Bai and Perron (1998, 2003) and display the corresponding means (grey dashed line) in the right panel of Figure 2.\(^6\) In this sense, our time-varying long-run state model can be regarded as a generalization of the structural break model. The nonlinear model is more flexible, and provides a smoothly evolving long-run state of the PtDR. However, there are clear differences between the

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\(^5\) Non-stationarity of the PtDR causes explosive paths in forecasts of excess return and excess dividend growth rates in 1998, 1999 and 2000. In these three years we restrict the coefficient of PtDRs’ lagged value in the VAR equation of the PtDR and reestimate the system by means of EGLS until all eigenvalues of the characteristic polynomial are smaller than unity in modulus.

\(^6\) The test is robust to serial correlation and heteroscedasticity, the trimming is 5% of the sample. The test statistics for testing 0 against 1 or 2 breaks are \(supF(1|0) = 25.52\) and \(supF(2|0) = 45.39\), for the test of 1 against 2 breaks the statistic is \(supF(2|1) = 20.38\). For all statistics the null hypothesis of no break can be rejected with 1% significance. The timing of breaks 1954 and 1995 differs slightly from those diagnosed in Lettau and Van Nieuwerburgh (2008).
results from our model and those from the structural break model. As can be seen from the right hand side panel of Figure 2, around 1940 the long-run state of the PtDR reached its trough. Since then, it has followed an upward sloping trend which reached its peak shortly after 2000. However, even when two structural breaks are considered, means of the PtDR for the period before 1954 and after 1994 are biased upwards compared with the estimated long-run state in the right panel of Figure 2.

The estimation results confirm that the long-run state of the PtDR is time-varying. The log-likelihood estimates reveal the superiority of the time-varying long-run state model compared with the constant mean model of the PtDR. Compared to the smoothly evolving long-run state of the PtDR, the structural break model may provide biased estimates of the state of the PtDR.

4 Stock markets and macroeconomic variations

In this section, the linkage between the long-run state of the PtDR and potential macroeconomic determinants is investigated empirically. The analysis is carried out with the financial state (\( \hat{\eta}_t \)), which represents the underlying long-run state of the PtDR (\( \hat{\rho}_t \)).\(^7\) First, we discuss the considered macroeconomic factors briefly. A detailed description of the macroeconomic variables is given in the Appendix. In the second place we provide a cointegration analysis to assess the explanatory content of rival factors. In the literature so far, only single factors have been used one at a time, which might lead to misinterpretation. We investigate potential factors simultaneously by means of a higher dimensional system. This provides a robust analysis of the significance of distinct influences. At last, we look at the robustness of diagnostic results with regard to alternative measures employed to approximate the considered long term determinants of the PtDR.

4.1 Macroeconomic variations

We concentrate on three important macroeconomic influences that have been documented to affect equity premia: The consumption risk, the degree of risk sharing among households through markets for housing-collateralized debt, and the demographic structure of the population.

Consumption risk. Macroeconomic volatility, in particular consumption risk, influences asset valuation, as shown theoretically by Bansal and Yaron (2004). Markets dislike economic uncertainty and a decreasing consumption risk is consistent with a decline in equity premia. This may explain the strong surge of the PtDR in the 1990s. To measure consumption risk we employ the historical series of real per capita consumption recently

\(^7\) If \( \hat{\eta}_t \) instead of \( \hat{\rho}_t \) is used, the results are qualitatively similar.
collected by Barro and Ursua (2008). Thus, we include the entire past century for the US market while so far in the literature only post war periods are considered (Bansal et al.; 2005; Lettau et al.; 2008).

By means of rolling time windows of size $W$ we determine the purely data-driven risk measure proposed in Bansal et al. (2005) as

$$cr_t^W = \ln \left( \sum_{i=0}^{W-1} |co_{t-i}| \right), \quad (6)$$

where $co_t$ denotes the centered growth rate of per capita consumption.\(^8\) To choose an appropriate window size $W$ we evaluate a potential long-term relation between the financial state derived in Section 3 and the consumption risk measured with window sizes $W = 3, \ldots, 30$. The long term static relation is

$$\hat{\rho}_t = \beta_1 + \beta_2 cr_t^W + v_t, \quad (7)$$

where $v_t$ denotes a residual term. It turns out that $W = 15$ maximizes the $R^2$ of the static regression. A window length of 15 years is well embedded in the range of durations of low frequency economic cycles that have become prominent in the literature (Juglar cycles: 9 to 10 years; Kitchen cycles: 7 to 11 years; Kuznets swings: around 20 years; Kondratieff cycles: 45 to 60 years (e.g. Solomou; 2008; Miligate; 2008)).\(^9\) Considering an overlapping-generation model, Geanakoplos et al. (2004) also argue that agents often consider a 20 year horizon to incorporate demographic trends in long term asset price expectations. For further analysis we rely on $cr_t^W$ with a time window $W$ of 15 years, which provides a long term prospective on the macroeconomic uncertainty.

**Risk sharing.** The degree of risk sharing among households through markets for housing-collateralized debt can be formalized by the housing wealth to human wealth ratio (Lustig and Van Nieuwerburgh; 2005, 2006). When house prices increase, the fostered collateral value of housing weakens the borrowing constraints of households. Risk sharing among households is strengthened. The dispersion of consumption growth across households is less sensitive to aggregate consumption growth shocks, and this lowers the risk premium in stock markets (Lustig and Van Nieuwerburgh; 2005). This channel also accounts for spill-over effects from housing to equity prices, which are important in the context of the recent developments in US markets. Following Lustig

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\(^8\) We do not detect any significant pattern of serial correlation in $co_t$. The $p$-values of respective Ljung-Box statistics including 5 and 10 lags are 0.525 or 0.404, respectively.

\(^9\) To account for the robustness of results related to the respective window sizes in $cr_t^W$ alternative values for $W$ are also considered below. Further, to account for the dependence of results on the measurement of consumption risk, we alternatively apply the Exponential Weighted Moving Average (EWMA) or RiskMetrics approach which is widely applied in volatility measurement as a second simple data-driven filter (Morgan; 1996).
and Van Nieuwerburgh (2005, 2006) we approximate the ratio of housing wealth to human wealth by the ratio of the outstanding home mortgages to labor income \((\text{moy}_t)\) and the ratio of residential real estate wealth to labor income \((\text{rwy}_t)\).\(^{10}\) Historical quotes of outstanding home mortgages and residential real estate dating before 1945 are available from the US Census Bureau. More recent data is available from the Federal Reserve System. Data for labor income is acquired from the Bureau of Economic Analysis.

**Demographics.** Demographic factors and the respective composition of market participants may govern the variations in the PtDR. Adopting a theoretical model, Geanakoplos et al. (2004) demonstrate that agents’ incentives for holding equity vary over the life cycle. While the younger population intends to consume and willingly borrows for this purpose, the middle aged population concentrates more on saving and consumes these savings after retirement. One can characterize the overall shape of the population pyramid by the middle-aged to young ratio \((\text{my}_t)\), i.e. the ratio of the population aged 40-49 to the 20-29 year old. Geanakoplos et al. (2004) show that when \(\text{my}_t\) is small, there is excess demand for consumption by young and retired agents, and asset prices should decline to encourage saving and clear the market. Favero et al. (2011) also demonstrate the joint significance of \(\text{my}_t\) and the PtDR in long-horizon predictive regressions for stock returns. This is consistent with the view that a slowly moving mean of the PtDR could be driven by \(\text{my}_t\). Since our model provides a continuous time-varying path of the long-run PtDR, we can test the relationship between \(\text{my}_t\) and the long-run PtDR explicitly. For the \(\text{my}_t\) process we employ data since 1900, which are available from the US Census Bureau.

### 4.2 Cointegration analysis

In this section, we analyze the relationship between the financial state process \(\tilde{\rho}_t\) reflecting the long-run state of the PtDR and considered macroeconomic factors: The consumption growth volatility \(\left(c_{\text{r}}W_t\right)\), the degree of risk sharing represented by the housing wealth to human wealth ratio \((\text{rwy}_t, \text{moy}_t)\) and the middle aged to young ratio \((\text{my}_t)\). We depict these four factors in Figure 3. Comparing the consumption risk (upper left panel) to the estimated financial state in Figure 2, the former is almost a reversed mirror image of the latter. Macroeconomic uncertainty has continuously decreased since the 1940s, which is consistent with the continuous increases in the financial state of the stock markets for the same period. Looking at the two variables, \(\text{rwy}_t\) (lower left panel) and \(\text{moy}_t\) (upper right panel), representing risk sharing through markets for housing-collateralized debt, there were extraordinary peaks in the 1930s and the 2000s. The first peak in the house price had been built up through rapid expansion of residential housing markets in the

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\(^{10}\)In contrast, Lustig and Van Nieuwerburgh (2005, 2006) quantify the housing wealth to human wealth ratio as the residual from the cointegration relation between the logarithmic housing and human wealth variable to determine a stationary measure.
1920s. This house price bubble has been deflated since the stock market crash in 1929. The second peak corresponds to the recent housing prices that emerged with easy credit market conditions and ended with the subprime mortgage crisis in 2007. While the increases in $rwy_t$ and $moy_t$ seem to be consistent with the increases of the financial state in stock markets since the 1940s, the variations before this period are not corresponding to each other. Looking at the lower right hand side panel of Figure 3, the middle-aged to young ratio has shown a remarkable U-turn since the 1960s. This is mainly under the influence of the baby boom after the second world war. Since the 1960s the baby boom generation had entered into the statistic of the young population, reducing thereby $my_t$. Due to the same reason, the ratio has been increasing since the 1980s when the baby boom generation became middle-aged.

Before we turn to cointegration analysis and complex VECM models, it turns out to be quite useful to look at the explanatory content offered by static regressions. Figure 4 shows the fit of static regressions between the financial state and its potential factors:

$$\hat{\rho}_t = \beta_0 + \beta_1 x_t + v_t,$$

(8)

where $x_t \in \{cr^W_t, moy_t, rwy_t, my_t\}$ and $v_t$ is an error term. A simple regression with consumption risk ($cr^W_t$) fits the financial state in stock markets astonishingly well, as can be seen from the upper left panel of Figure 4. The movements of the financial state and the consumption risk are strongly linked over the entire past century. In contrast, the housing wealth to human wealth ratios ($moy_t, rwy_t$) and the middle-aged to young ratio ($my_t$) may only fit to the financial state in stock markets in certain subsamples. Fitted curves with $moy_t$ and $rwy_t$ deviate from the financial state ($\hat{\rho}_t$) in the stock markets from 1920 to 1940 (upper-right and lower-left panels). And the middle-aged to young ratio approximates the trend in $\hat{\rho}_t$ most accurately before 1960 and after 1990. The long-term state in stock markets does not show the U-turn between 1960 and 1990 as predicted by demographic structures.

Considering all three influences - consumption risk, risk sharing, demographics - together in the static regression $\hat{\rho}_t = \beta_0 + \beta_1 cr^W_t + \beta_2 rwy_t + \beta_3 my_t + v_t$, the obtained fit depicted in Figure 5 appears similar to the one from the bivariate regression with consumption risk (see upper left panel of Figure 4). It appears that consumption risk plays the dominating role in affecting the long-run PtDR throughout the last century. This conjecture is confirmed by the cointegration analysis.

11 Substituting $rwy_t$ by $moy_t$ to approximate the degree of risk sharing among households does not change results qualitatively.
As the first step of the cointegration analysis, we look at the individual characteristics of each variable by means of unit root tests. Unit root diagnostics for levels and first differences of \( \hat{\rho}_t, cr_t^W, moy_t, rwy_t \) and \( my_t \) are documented in Table 2. All tests indicate first order integration of \( \hat{\rho}_t, cr_t^W, moy_t \) and \( rwy_t \). Regarding \( my_t \) the KPSS test provides evidence for first order integration. The null hypothesis of stationarity is rejected with 10% significance for \( my_t \) while it cannot be rejected for \( \Delta my_t \). Although other unit root tests hint at stationarity of \( my_t \), we follow Favero et al. (2011) and treat \( my_t \) as a first order integrated process.\(^{12}\)

Cointegration tests are applied for each bivariate combination of the financial state with one of its potential determinants, and a four dimensional system comprising all determinants jointly. The former serve as a preliminary analysis for the latter. We consider four types of cointegration tests: The Johansen trace test, the Durbin-Watson (DW) test, the ADF and the KPSS type statistic proposed by Shin (1994). The latter three diagnostics are based on residuals from regressions (8) of \( \hat{\rho}_t \) on the potential determinants. While the distribution of the ADF and DW statistic are derived under the null hypothesis of no cointegration, the asymptotic distribution of the KPSS type test applies under the null hypothesis of cointegration. Diagnostic results are documented in Table 3.

While we cannot confirm bivariate cointegrations between the financial state and the considered measures of risk sharing among households (\( moy_t, rwy_t \)) and demographics (\( my_t \)), there is mixed evidence for cointegration between the financial state and consumption risk (\( cr_t^W \)). Although the Johansen trace test indicates a cointegration rank of zero for the latter, results from ADF and DW statistics support the presence of a common trend among the financial state and consumption risk. Considering the null hypothesis of cointegration, the KPSS statistic also indicates a cointegration relation between the time varying equity valuation and consumption risk.

The ambiguous evidence for cointegration relations in bivariate systems points to the prospect of cointegration in the higher dimensional system. Thus, we test for cointegrating relations in the four dimensional system comprising \( \hat{\rho}_t, cr_t^W, rwy_t \) and \( my_t \). Estimated cointegration parameters from a system with \( moy_t \) instead of \( rwy_t \) are similar. We focus our analysis on the VECM with \( rwy_t \) henceforth.

\(^{12}\)Evidence for a unit root is indicated by all four considered tests, except for the Phillips and Perron (1988) test, if we use the longest available series ranging from 1900 to 2049, where projections from 2010 to 2049 are included.
Distinct cointegration test statistics displayed in Table 4 provide evidence in favour of one cointegration relation. For the Johansen trace test, the AIC indicates a lag length of 5 and the BIC hints at two lags. As can be seen from the upper block of Table 4, a cointegration rank of unity is confirmed with 5% and 10% significance accordingly. Moreover, the KPSS statistic hints at a stable long term relation in the four dimensional system while the null hypothesis of no cointegration cannot be rejected by means of the ADF and DW test. The overall evidence supports one cointegration relation in the system.

Based on the positive evidence of a cointegration relation among $\hat{\rho}_t$, $cr_t^W$, $rwy_t$, and $my_t$, we estimate the corresponding vector error correction models. To reduce the estimation uncertainty, we apply a subset procedure to reduce the number of parameters. The cointegration parameters are estimated by means of the Johansen approach in the first step. Then restrictions on the adjustment coefficients and short-run parameters are imposed through a system testing procedure. Corresponding parameters with smallest absolute $t$-statistics are sequentially restricted to zero until all remaining $t$-ratios exceed 1.96 in absolute value. In the second step EGLS estimates for the restricted model are obtained. The respective parameter estimates are documented in the right hand side block of Table 5. Reported LM-tests show that there are no significant autocorrelations at lower orders in the residuals. We have a sound model for the considered variables.

To have a comprehensive picture on the influences of $cr_t^W$, $rwy_t$, $my_t$ on $\hat{\rho}_t$, we consider impulse response functions. Since we haven’t found any instantaneous correlation among error terms in the VECM, the forecast error impulse response analysis with one shock in one variable at a time is a sensible choice. Effects of standardized shocks in $\hat{\rho}_t$, $cr_t^W$, $rwy_t$, and $my_t$ on $\hat{\rho}_t$ over the next 20 years are depicted in Figure 6. Three conclusions can be derived from the VECM.

Firstly, significant long-run effects from all three factors - consumption risk, risk sharing through markets of housing-collateralized debt, and the demographic structure - on the long-run state of the PtDR can be confirmed. As can be seen from the right hand side block of Table 5, all estimated cointegration parameters are significant and have the expected sign. While consumption risk ($cr_t^W$) has a significantly negative influence on the financial state ($\hat{\rho}_t$), residential real estate wealth to labor income ($rwy_t$) and the middle-aged to young ratio ($my_t$) have significantly positive influences on $\hat{\rho}_t$. All three factors influence the long-run PtDR jointly. The significance of the adjustment coefficient for $\Delta\hat{\rho}_t$.

To calculate the standard deviation of the residuals in the equation of $my_t$ for scaling the impulse response, we modified outliers. Although there are only a few outliers, they are quite far away from the remaining residuals. We substitute these outliers with values about two standard deviations so that the obtained standard deviation does not overstate the true variations in the residual.
in Table 5 and the impulse response analysis in Figure 6 support this conclusion. As can be seen from the upper right panel in Figure 6, positive shocks in consumption volatility have significantly negative effects on the financial state in stock markets over a decade. The decline in equity premia reflects the decreasing macroeconomic uncertainty. Through improved risk sharing among households, benign shocks in housing markets drive down the risk premium and push up the stock prices over a long-run horizon (lower left panel). Also shocks in the demographic structure have very persistent effects on the long-run state of the PtDR. As can be seen from the lower right panel, after 20 years the effect is still quite persistent.

Secondly, consumption risk explains the most variations in the long-run PtDR. To facilitate the understanding about the influence of consumption risk, we adopted a 2-dimensional subset VECM for $\hat{\rho}_t$ and $cr^W_t$ for purposes of comparison. Corresponding results are provided in the left hand side panel of Table 5. Comparing these results with those from the 4-dimensional VECM in the right hand side panel, the cointegration parameter of $cr^W_t$ is of similar scale. The adjusted $R^2$ shows that using $cr^W_t$ alone explains about 32.9% of the variations in $\hat{\rho}_t$. Adding $rwy_t$ and $my_t$ into the system will only add another 10.7% (43.6% – 32.9%). Changes in the consumption growth volatility have the most important influence on the long-term PtDR. This confirms the observations from simple static regressions illustrated in Figure 4.

Moreover, consumption risk and demographic factors appear unaffected by its cointegration relation with the financial state in stock markets and the ratio of residential real estate wealth to labor income. The adjustment coefficients for $\Delta cr^W_t$ and $\Delta my_t$ are insignificantly different from zero (having t-statistics smaller than 1.96 in absolute value) and, thus, have been eliminated from the EGLS estimation. In addition, it is not surprising to see that the adjustment coefficient for $\Delta rwy_t$ is significant. The ratio of the residential real estate wealth to labor income not only represents the risk sharing channel but also reflects the housing premium. Since both equity premium and housing premium can be influenced by common factors such as the state of the economy, stock and housing markets can share the same stochastic trend. Due to the (informational) inefficiency in the housing markets, adjustments of $rwy_t$ may follow behind the variations in the stock markets.\(^{14}\)

Therefore, while the consumption risk, the risk sharing through markets of housing-collateralized debt, and the demographic structure all play a role in determining the long-run state of the PtDR, consumption risk is the most important factor.

\(^{14}\)Impulse response analysis confirms that the only shock that has an influence on $rwy_t$ is the one from $\hat{\rho}_t$.\]
4.3 Robustness analysis

Consumption growth risk is in the limelight of our results. This latent variable has been constructed through the approximation in (6) with $W = 15$. To check the robustness of our results, we consider three alternative measures of consumption risk. First, window sizes $W = 10, 20$ are used. Second, we replace the absolute growth rates of consumption by squared changes obtaining the following risk measures:

$$
\tilde{c}_t^W = \ln \left( \sum_{i=0}^{W-1} c_{t-i}^2 \right),
$$

(9)

$$
\tilde{c}_t^W = \ln \left( \sqrt{\sum_{i=0}^{W-1} c_{t-i}^2} \right).
$$

(10)

For both variants $\tilde{c}_t^W$ and $\tilde{c}_t^W$ the window size $W = 14$ is determined by means of $R^2$ statistics of a static regressions as (7). In the third place, similar to the approach taken by RiskMetrics (Morgan; 1996), we determine risk processes by means of exponential smoothing, i.e.\(^{15}\)

$$
cr_t^\lambda = \ln \left( \frac{1 - \lambda}{1 - \lambda^t} \sum_{i=1}^{\infty} \lambda^{i-1} |c_{t-i}| \right),
$$

(11)

$$
\tilde{c}_t^\lambda = \ln \left( \frac{1 - \lambda}{1 - \lambda^t} \sum_{i=1}^{\infty} \lambda^{i-1} c_{t-i}^2 \right),
$$

(12)

$$
\tilde{c}_t^\lambda = \ln \left( \sqrt{\frac{1 - \lambda}{1 - \lambda^t} \sum_{i=1}^{\infty} \lambda^{i-1} c_{t-i}^2} \right).
$$

(13)

While observations $c_{t-i}$ enter $cr_t^W$, $\tilde{c}_t^W$ and $\tilde{c}_t^W$ with equal weight, the exponential smoothing attaches higher weights to more recent observations. The decay factor $\lambda$ governs the persistence of risk processes. We use $\lambda = 0.92$ for $cr_t^\lambda$ and $\lambda = 0.89$ for $\tilde{c}_t^\lambda$ and $\tilde{c}_t^\lambda$, since they maximize the $R^2$ in corresponding static regressions similar to (7). Unit root diagnostics indicate nonstationarity of all considered alternative measures. This result is similar to the one from the benchmark approximation of consumption risk, $cr_t^W$, $W = 15$. Detailed unit root diagnostics for the alternative measures of the consumption risk are available from the authors upon request.

Table 6 about here

Cointegration diagnostics for bivariate combinations of the financial state ($\tilde{p}_t$) and

\(^{15}\)Andersen, Bollerslev, Christoffersen and Diebold (2006) propose the multiplicative adjustment factor $1/(1 - \lambda^t)$ to account for the truncation of RiskMetrics measures that is implied by the sample range.
alternative approximations of consumption risk are documented in Table 6. The KPSS statistics show throughout that all measures of consumption risk are cointegrated with the financial state process. While ADF and DW tests indicate no cointegration relation, they appear to suffer from low power under strong persistence of equilibrium errors. The particular importance of the alternative measures of consumption risk in describing the long-run PtDR is confirmed by eyeballing the static regression outcomes in Figure 7. We cannot diagnose a systematic shortcoming of alternative consumption risk processes in governing the financial state in stock markets. Therefore, we conclude that trends in consumption risk can explain a major part of movements in the long-term PtDR over the past century.

Figure 7 about here

5 Conclusions

The price-to-dividend ratio (PtDR) from 1913 to 2010 in the US is highly persistent. Shocks have persistent effects on this ratio over a long period. To model this feature, we consider a modified version of the present value model (Campbell and Shiller; 1988) containing a time-varying long-run state of the PtDR. To estimate this latent long-run state, we utilize the state space framework and apply the particle filtering approach. Estimation results show that the long-run state of the PtDR is indeed time-varying. The time-varying state model of the PtDR clearly outperforms the constant mean model according to the log-likelihood evaluations.

The feasibility of obtaining the latent long-run state of the PtDR enables us to test the joint significance of three macroeconomic influences: consumption risk, risk-sharing among households through markets of housing collateralized debt, and the demographic structure of the population. While all three factors play significant roles in determining the long-run trend of the PtDR, consumption risk is the dominant force. Consumption risk alone can explain three quarters of the variations in the long-run state of the PtDR that can be explained by our model. Moreover, it has shaped the long-run state of the PtDR throughout the last century in a stable manner. In comparison, the degree of risk-sharing could explain the long-run trend of PtDR after the 1940s, but not before. And the ratio of the middle-aged to the young population shared a similar trend as the long-run PtDR before 1960 and after 1990.

For future research it is interesting to compare the long-run states of the PtDR from different markets and find the common components in their variations. International risk sharing could be one potential (global) determinant. As Artis and Hoffmann (2008) have pointed out, international risk sharing has increased since financial markets became more integrated since the 1980s. This might have played an important role in determining
variations in the long-run PtDR of different markets in this period.
Appendix - Data description

Stock market indices and dividends. Annual series are provided by Robert J. Shiller and available from the internet.\textsuperscript{16} They contain the S&P500 index based on averages of closing prices in each January of a year and corresponding dividends for the period from 1871 to 2010. In contrast to Campbell and Shiller (1988) who define the return in period $t$ as realized at the beginning of period $t+1$ we define it in (1) as realized at the end of period $t$. Thus, we apply averages of daily closing prices in each December instead of average prices in January. The December closing prices are collected from a similar dataset provided by Shiller which is on a monthly frequency. For the period before 1926 Shiller calculated the price series by means of data of Cowles (1939). Annual dividends correspond to the sum of the four quarterly paid dividends within the corresponding year. For more details see Shiller (1992, 2005).

Interest rates and inflation. Similar to Campbell and Vuolteenaho (2004), we use a short term rate based on 3-month US Treasury Bills to approximate the risk-free rate. Interest rates are calculated backward to 1871 in a similar manner as in Shiller (2005). For the period before 1931 we use data from Homer (1963): For 1871-1899, from table 44, col.1, p. 319/320; for 1900-1919, from table 51, col.4, p. 364; for 1920-1930, from table 51, col.3, p. 366/367. For the periods from 1931 to 1933 and since 1934 we employ 3-month US Treasury Bill rates in terms of ‘auction high’ and ‘secondary market’ quotes, both published by the Board of Governors of the Federal Reserve System and available from the internet.\textsuperscript{17} These two series are available at a monthly frequency. To determine an annual series the geometric mean over the months within each year has been used.

Inflation series are extracted from the consumer price indices for all urban consumers provided by Robert J. Shiller.\textsuperscript{18} For the period before 1913 Shiller calculated the series by means of data in Warren and Pearson (1935).

Other macroeconomic variables. Annual quotes of real per capita consumption (1869 to 2006) are collected by Barro and Ursua (2008) and available from the net.\textsuperscript{19} To update this series until 2010 we use consumption data from the Bureau of Economic Analysis\textsuperscript{20} and population data available from World Development Indicators collected by the World Bank.\textsuperscript{21}

\textsuperscript{16}http://www.econ.yale.edu/~shiller/data.htm
\textsuperscript{17}http://www.federalreserve.gov/releases/h15/data.htm
\textsuperscript{18}http://www.econ.yale.edu/~shiller/data.htm
\textsuperscript{19}http://www.economics.harvard.edu/faculty/barro/data_sets_barro
\textsuperscript{20}http://www.bea.gov/national/nipaweb/SelectTable.asp
\textsuperscript{21}http://data.worldbank.org/data-catalog/world-development-indicators
The ratio of the 40-49 over the 20-29 year aged population is determined by annual population data from Datastream (period since 1950, ‘USPOP24Y’ for the 20-24 year old agents, ‘USPOP29Y’ for the 25-29, ‘USPOP44Y’ for the 40-44 and ‘USPOP49Y’ for the 44-49). Data for the period before 1950 are from the US Census Bureau.\textsuperscript{22}

For calculation of the ratio of outstanding home mortgages to labor income we use the series of outstanding home mortgages for the period before 1945 from the Bureau of the Census (Table N262)\textsuperscript{23} and for the more recent years the series of the Federal Board of Governors (Series: Z1/Z1/LA153165105.A).\textsuperscript{24} Labor income since 1929 is from the Bureau of Economic Analysis calculated as the sum of personal income and net transfer income.\textsuperscript{25} To be more precise we base on Table 2.1 ‘personal income and its disposition’ and take therein the sum of series 3, 6, 9 and 16 while we subtract series 24. For the period before 1929 the series of labor income is from a historical data collection in Maddison (2001).\textsuperscript{26}

Measures of residential real estate wealth for the period before (since) 1945 are from the Bureau of the Census, Table N197,\textsuperscript{27} (Federal Board of Governors, Series: Z1/Z1/FL155035015.A).\textsuperscript{28} The determination of the labor income series is analogous to that described above. For further details see Lustig and Van Nieuwerburgh (2005).

**Appendix - Particle filtering**

The state space model of the price-to-dividend ratio in (4) and (5) is highly nonlinear in the parameters and the maximization of the corresponding log-likelihood function is not tractable analytically. Using Monte Carlo approximation techniques it becomes possible to derive an approximative log-likelihood value by means of the so-called particle filtering approach. We apply the standard particle filter described in Cappé et al. (2007) (Algorithm 3, bootstrap filter) and an optimization technique based on the simplex search method of Lagarias, Reeds, Wright and Wright (1998) for parameter estimation that does not depend on gradient estimation. The particle filtering algorithm, specific for the state space model provided in Section 2, has the following form:

\textit{Step (1): Initialization (t=1).} Sample N particles \( \tilde{\rho}_1^{(i)} \sim N(\rho_0, \sigma_\epsilon^2), i=1,\ldots,N \) and

\begin{itemize}
  \item \textsuperscript{22}http://www.census.gov/popest/archives/pre-1980/PE-11.html
  \item \textsuperscript{23}http://www.census.gov/prod/www/abs/statab.html
  \item \textsuperscript{24}http://www.federalreserve.gov/releases/z1/
  \item \textsuperscript{25}http://www.bea.gov/iTable/index_nipa.cfm
  \item \textsuperscript{26}http://www.ggdc.net/maddison/Monitoring.shtml
  \item \textsuperscript{27}http://www.census.gov/prod/www/abs/statab.html
  \item \textsuperscript{28}http://www.federalreserve.gov/releases/z1/
\end{itemize}
determine importance weights

\[ \tilde{w}_1^{(i)} = \frac{1}{\sqrt{2\pi\sigma^2}} \exp \left( -\frac{1}{2} \left( \frac{\tilde{\epsilon}_1^{(i)}}{\sigma} \right)^2 \right). \]

Normalized weights are obtained as

\[ w_1^{(i)} = \frac{\tilde{w}_1^{(i)}}{\sum_{i=1}^{N} \tilde{w}_1^{(i)}} \]

**Step (2): Iteration (t=2,...,T).**

1. Select \( N \) particles according to weights \( w_1^{(t-1)} \). Set accordingly \( \tilde{\rho}_t^{(i)} = \tilde{\rho}_{t-1}^{(i)} \) (resampling)

2. For all particle draw

\[ \tilde{\rho}_t^{(i)} \sim N(\rho_{t-1}^{(i)}, \sigma^2), \ i = 1, ..., N, \]

and determine raw weights

\[ \tilde{w}_t^{(i)} = \frac{1}{\sqrt{2\pi\sigma^2}} \exp \left( -\frac{1}{2} \left( \frac{\tilde{\epsilon}_t^{(i)}}{\sigma} \right)^2 \right) \]

3. Normalize weights

\[ w_t^{(i)} = \frac{\tilde{w}_t^{(i)}}{\sum_{i=1}^{N} \tilde{w}_t^{(i)}} \]

4. go back to step ‘1’.

Averaging over non-normalized weights \( \tilde{w}_t^{(i)} \) yields estimates of the contribution of \( \epsilon_t \) to the Gaussian likelihood function, while averaging over draws \( \rho_{t}^{(i)} \) results in estimates of \( \rho_t \), e.g. \( \hat{\rho}_t = \frac{1}{N} \sum_{i=1}^{N} \rho_t^{(i)}, \ t = 1, ..., T. \)

A procedure called systematic resampling is used to compute uniform distributed random numbers to implement the resampling step. This technique is described in Robert and Casella (2005). Doucet and Johansen (2009) argue that such a technique reduces the noise introduced by resampling and it is commonly employed in the related literature.
References


Tables

<table>
<thead>
<tr>
<th>Time-varying $\rho$</th>
<th>Constant $\rho$ ($\sigma_c = 0$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho_0$</td>
<td>$\sigma_c$</td>
</tr>
<tr>
<td>0.950</td>
<td>$1.32E-03$</td>
</tr>
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**Table 1: Parameter estimates** The state space model with measurement equation (4) and a bounded random walk as state process (5) is estimated by means of the particle filtering approach. To forecast excess dividend growth rates ($\tilde{E}_t[\Delta d_{t+1}]$) and excess returns ($\tilde{E}_t[r_{t+1}]$), we use the VAR model of $\eta_t$, $\Delta d_t$, $r_t$, $\pi_t$, $t$, and the rolling window with size 42.

<table>
<thead>
<tr>
<th>$ADF_t$</th>
<th>$PP_t$</th>
<th>$DF_{GLS}$</th>
<th>$KP_{SS}$</th>
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<tr>
<td>$\hat{\rho}_t$</td>
<td>-0.645</td>
<td>-0.472</td>
<td>-0.773</td>
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<tr>
<td>$\Delta \hat{\rho}_t$</td>
<td>-3.634***</td>
<td>-5.152***</td>
<td>-3.670***</td>
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<tr>
<td>$cr_t^W$</td>
<td>-0.657</td>
<td>-0.682</td>
<td>-1.050</td>
</tr>
<tr>
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<td>-7.665***</td>
<td>-1.424</td>
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<td>-0.148</td>
</tr>
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<td>$\Delta moy$</td>
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<td>$\Delta rwy$</td>
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<tr>
<td>my</td>
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<td>-5.400***</td>
<td>-2.214**</td>
</tr>
<tr>
<td>$\Delta my$</td>
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<td>-4.010***</td>
<td>-4.190***</td>
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<tr>
<td>$\Delta^2 my$</td>
<td>-14.381***</td>
<td>-14.381***</td>
<td>-14.282***</td>
</tr>
</tbody>
</table>

**Table 2: Unit-root tests** The time-varying financial state in stock markets is denoted as $\hat{\rho}_t$. The degree of risking is reflected by the outstanding home mortgages to labor income ($moy_t$) and the ratio of residential real estate wealth to labor income ($rwy_t$). $my_t$ is the middle-aged to young ratio. Test regressions include a constant. $ADF$ refers to the Augmented Dickey-Fuller test where the lag selection criterion is the AIC. For $PP$, the test statistic considered in Phillips and Perron (1988), the spectral AR estimator is used to calculate the long run variance. $DF_{GLS}$ refers to the test proposed by Elliott et al. (1996) where the AIC is applied to determine the lag length. For the $PV$-test proposed by Perron and Vogelsang (1992) the innovation outlier model is applied and the lag length is determined by means of a t-test procedure. A Bartlett Kernel is applied in the $KP_{SS}$-test of Kwiatkowski et al. (1992). Significance at 1%, 5%, 10% level is denoted by ***; **; *, respectively.
Table 3: Cointegration tests for bivariate systems

A constant in the cointegration relations is included. The Johansen trace test statistic is denoted by ‘Trace’ and cointegration ranks by r, respectively. The columns labeled ‘Lag’ report the number of lagged differences included in the VECM. Quantities in boldface indicate the lag length proposed by the AIC, italic entries those proposed by the BIC. If both information criteria indicate the same lag length it is indicated as underlined quantity. While p-values are provided for Johansen trace tests, critical values at 5% significant level, denoted as c-value, are shown for DW, ADF, and KPSS. For Durbin-Watson (DW) statistic, the 5% quantile of the upper limiting distribution displayed in Sargan and Bhargava (1983) is provided. Additionally the simulated critical value form Engle and Granger (1987) is given in squared brackets. ADF denotes the Augmented Dicky-Fuller statistic based on residuals from static regressions, where the AIC is applied to determine the lag length. There the critical value from MacKinnon (2010) is given. KPSS denotes the cointegration test proposed by Shin (1994) applying a Bartlett Kernel.
<table>
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<th>Lag</th>
<th>Trace</th>
<th>p-value</th>
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<td>0.0727</td>
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<tr>
<td>r=1</td>
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<td>28.83</td>
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<table>
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<tr>
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<td>DW</td>
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<tr>
<td>ADF</td>
<td>−3.460</td>
</tr>
<tr>
<td>KPSS</td>
<td>0.122</td>
</tr>
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Table 4: Cointegration tests for the four dimensional system \((\hat{\rho}_t, cr^W_t, rwy_t, my_t)\). For Johansen trace tests, a constant is included. AIC indicates optimal lag length of 4 (differences) and BIC indicates 1 when up to 10 lags are searched. Further test specifications are identical to those described in Table 3. c-value denotes the critical value at 5% significant level.

<table>
<thead>
<tr>
<th>VECM of ((\hat{\rho}_t, cr^W_t))</th>
<th>VECM of ((\hat{\rho}_t, cr^W_t, rwy_t, my_t))</th>
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<td><strong>Cointegration vector</strong></td>
</tr>
<tr>
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<td>(\hat{\rho}_t)</td>
</tr>
<tr>
<td>(const)</td>
<td>(const)</td>
</tr>
<tr>
<td>(cr^W_t)</td>
<td>(cr^W_t)</td>
</tr>
<tr>
<td>(rwy_t)</td>
<td>(rwy_t)</td>
</tr>
<tr>
<td>(my_t)</td>
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</tr>
<tr>
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<td>(-1)</td>
</tr>
<tr>
<td>(0.929)</td>
<td>(0.912)</td>
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<tr>
<td>(-0.023)</td>
<td>(-0.018)</td>
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<tr>
<td>((574.351))</td>
<td>((322.169))</td>
</tr>
<tr>
<td>((-16.535))</td>
<td>((-14.348))</td>
</tr>
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<td><strong>Error correction parameters</strong></td>
<td><strong>Error correction parameters</strong></td>
</tr>
<tr>
<td>(\Delta \hat{\rho}_t)</td>
<td>(\Delta \hat{\rho}_t)</td>
</tr>
<tr>
<td>(\Delta cr^W_t)</td>
<td>(\Delta cr^W_t)</td>
</tr>
<tr>
<td>(\Delta rwy_t)</td>
<td>(\Delta rwy_t)</td>
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<tr>
<td>(\Delta my_t)</td>
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<tr>
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<td>(0.109)</td>
</tr>
<tr>
<td>((2.557))</td>
<td>((4.140))</td>
</tr>
<tr>
<td>(7.699)</td>
<td>(–)</td>
</tr>
<tr>
<td>((2.939))</td>
<td>((–9.550))</td>
</tr>
<tr>
<td>(–)</td>
<td>((–3.620))</td>
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<td><strong>R(^2) and [adjusted R(^2)]</strong></td>
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<tr>
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<td>([0.436])</td>
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<tr>
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<td>0.185</td>
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<tr>
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<td>([0.176])</td>
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<td><strong>p-values of serial correlation LM-test</strong></td>
<td><strong>p-values of serial correlation LM-test</strong></td>
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<tr>
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<td>lag order 4</td>
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<td>0.234</td>
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Table 5: Cointegration parameters for the two and four-dimensional system. For the four-dimensional VECM, a lag length of 4 is selected based on the AIC. Although a lag length of 3 is suggested by AIC for two-dimensional VECM, lag 4 is used to obtain a stable system. t-statistics are reported in parenthesis.
Table 6: Cointegration tests for alternative consumption risk measures $\text{cr}_t^W$, $\tilde{\text{cr}}_t^W$, and $\tilde{\text{cr}}_t^\lambda$ with $\bullet \in \{W, \lambda\}$ are defined in Section 4.3. The test specifications are identical to those described in Table 3. Critical values for the ADF, the DW and the KPSS statistic are -3.399, 0.404 and 0.314, respectively.

<table>
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<th>ADF</th>
<th>DW</th>
<th>KPSS</th>
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<td>0.376</td>
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<tr>
<td>$\tilde{\text{cr}}_t^W$, $W = 14$</td>
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<td>0.319</td>
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<tr>
<td>$\tilde{\text{cr}}_t^W$, $W = 14$</td>
<td>-2.209</td>
<td>0.319</td>
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<tr>
<td>$\text{cr}_t^\lambda$, $\lambda = 0.92$</td>
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<td>$\tilde{\text{cr}}_t^\lambda$, $\lambda = 0.89$</td>
<td>-2.725</td>
<td>0.294</td>
</tr>
<tr>
<td>$\tilde{\text{cr}}_t^\lambda$, $\lambda = 0.89$</td>
<td>-2.725</td>
<td>0.294</td>
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</table>
Figures

Figure 1: The price-to-dividend ratio The PtDR is depicted in the left panel. The sample ranges from 1871 to 2010. The right panel illustrates the overall sample mean of the PtDR (grey line) and sample mean from rolling windows (black line). Each window includes the most recent 15 years.

Figure 2: Time-varying financial state The left panel illustrates the estimated time-varying financial state $\hat{\rho}_t$ (black line) along with its time invariant counterpart (grey line). The right panel depicts the PtDR (black solid line), the sample mean (black dashed line), the estimated time-varying long-run state of the PtDR (grey solid line), and a mean process with two discrete shifts as in Lettau and Van Nieuwerburgh (2008) (grey dashed line).
Figure 3: Macroeconomic Influences The displayed consumption risk is measured by means of the benchmark specification $cr_t^W$ with $W = 15$. 
Figure 4: Fit of static regressions in bivariate systems. The fit of the following estimated static regressions of $\hat{\rho}_t$ is depicted. Upper left panel: $\hat{\rho}_t = 0.93 - 0.02c_r^W + \tilde{v}_t$ (with $W = 15$). Upper right panel: $\hat{\rho}_t = 0.94 + 0.04m_{oyt} + \tilde{v}_t$. Lower left panel: $\hat{\rho}_t = 0.92 + 0.03r_{wyt} + \tilde{v}_t$. Lower right panel: $\hat{\rho}_t = 0.92 + 0.04m_{yt} + \tilde{v}_t$.

Figure 5: Fit of the static regression in the four dimensional system. Results from the estimated static regression: $\hat{\rho}_t = 0.923 - 0.020c_r^W + 0.003r_{wyt} + 0.007m_{yt} + \tilde{v}_t$. 
Figure 6: Impulse response analysis Impulse responses of the financial state $\hat{\rho}_t$ of the stock markets (solid lines) to an innovations of size one standard deviation in the financial state ($\hat{\rho}_t$), the ratio of residential real estate wealth to labor income ($rwy_t$), and the middle-aged to young ratio ($my_t$). The dashed lines are the 95% Hall’s bootstrap confidence intervals.
Figure 7: The financial state and alternative consumption risk measures. The fit of the estimated static regressions of \( \hat{\rho}_t \) on alternative consumption risk measures is illustrated. The upper panel displays results conditioned on alternative window sizes for the benchmark measure \( cr^W_t \). The middle panel visualizes the fit of a further benchmark measure \( cr^W_t \) and those of the alternative measures \( \tilde{cr}^W_t \) and \( \tilde{\tilde{cr}}^W_t \). The exponential smoothing based measures \( cr^\lambda_t \), \( \tilde{cr}^\lambda_t \) and \( \tilde{\tilde{cr}}^\lambda_t \) are displayed in the lower panel. The bolded window sizes \( W \) or decay parameters \( \lambda \) denote those that maximize the \( R^2 \) in the respective static regressions.