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The Effect of On-net/Off-net Differentiation and Heterogeneous Consumers on Network Size in Mobile Telecommunications – An Agent-based Approach

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Abstract
I explore the effects of on-net / off-net differentiation on network sizes in mobile telecommunications when both rational and non-rational consumers coexist in the market. In particular, three different types of consumers are modeled: (1) fully informed rational (FIR) consumers who are perfectly informed about the true market shares of all networks and choose the network with the lowest expected cost of a call; (2) partly informed rational (PIR) consumers who only observe market shares within a circular sensing field and choose the network with the lowest expected cost of a call based on these observed market shares; and (3) non-rational (NR) consumers who choose the network with the highest market share among their immediate neighbors. Using an agent-based simulation approach and by systematical variation of four key parameters of the model, three key results emerge. First, if the share of FIR consumers is too high, all consumers will eventually join the initially larger network A. Second, if their share in the population is sufficiently large, NR consumers can prevent the growth of clusters of consumers subscribed to network B. Third, if the share of PIR consumers is high, clusters of consumers subscribed to network B can grow, thereby increasing network B’s market share, provided that the radius of their circular sensing field is small enough for the cluster size.

JEL codes: C63, K23, L14, L96
Keywords: on-net/off-net differentiation; tariff-mediated network effects; agent-based computational economics

1. Introduction
In the year 2007, KPN, the parent company of Germany’s second smallest network operator E-Plus, filed a complaint with the European Commission (EC) stating that the on-net/off-net differentiation practiced by its two largest competitors, T-Mobile and Vodafone, puts E-Plus at an unfair competitive disadvantage due to its smaller network size (KPN, 2007). The German Federal Cartel Office, entrusted by the EC with the investigation of KPN’s complaint, did not arrive at a final decision but stopped its investigation in the year 2009 due to insufficient evidence for anti-competitive effects of on-net/off-net differentiation (Haucap; Heimeshoff, and Stühmeier 2011; Haucap and Heimeshoff

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In contrast to that, in several other countries, the National Regulation Authorities have taken different measures in response to on-net/off-net differentiation in the mobile telecommunications market. Actions taken range from regulating mobile termination rates or the extent of on-net/off-net differentials to the fixing of price caps for off-net calls or even the prohibition of on-net/off-net differentiation (TMG 2011). Predominantly, these actions were implemented to protect small network operators and to stimulate competition. Theoretical justification for these actions mainly stems from the theoretical literature on the effects of on-net/off-net differentiation. As Harbord and Pagnozzi (2010) summarize in their survey of the extant theoretical literature, on-net/off-net differentiation “can be used strategically by incumbent operators to either prevent entry or to reduce competition from new entrants into their markets” (p. 7).

While the arguments supporting the anti-competitive effects of on-net/off-net differentiation developed in the theoretical literature critically hinge on consumers behaving fully rationally, evidence from empirical and experimental studies suggests that not all consumers are able to choose tariffs fully rationally (Claussen, Trüg, and Zucchini 2012). Instead, this literature has shown that at least some consumers are subject to different biases, such as, for instance, a “fallacy of dominant price vectors” (Bolle and Heimel 2005), an “on-net bias” (Haucap, Heimeshoff, and Grimm 2009), or a “price differentiation bias” (Haucap and Heimeshoff 2011).

If consumers do not choose their mobile tariff rationally, on-net/off-net differentiation can, in principle, be beneficial for small rather than large network operators. Due to their smaller network size, small network operators can set lower tariffs for on-net calls without losing much revenue while maintaining an off-net tariff at the level of their competitors (Haucap and Heimeshoff 2011). This strategically low on-net tariff, in turn, could attract, for instance, consumers with an on-net bias (due to the low on-net tariff) or consumers succumbing to a fallacy of dominant price vectors (due to the lower unweighted mean tariff). In fact, the strategic price setting behavior by small network operators to capture boundedly rational consumers has been put forward as a possible explanation for why, in many European countries, on-net/off-net differentiation has been introduced by the small mobile network operators, for instance E-Plus in Germany, Orange in Norway, Digicell in Ireland, or tele.ring in Austria (Haucap, Heimeshoff, and Grimm, 2009; Frontier Economics, 2004).

Hence, whether on-net/off-net differentiation is advantageous for large or for small network operators strongly depends on how consumers choose their network. However, hitherto it remains unclear how the presence of heterogeneous consumers, i.e. coexistence of fully rational and boundedly rational or even non-rational consumers, influences the effect of on-net/off-net differentiation on network sizes.

This paper sets out to fill this gap by answering the following question: How does on-net/off-net differentiation affect the size of a small and a large network if rational and non-rational consumers coexist in the mobile telecommunications market?
The remainder of this paper is structured as follows. In section two, I briefly review the theoretical, empirical, and experimental literature on on-net/off-net differentiation. In section three, I describe the methodology used and the setup of my simulation model. Section four contains a discussion of the strategy for model analysis and section five presents and discusses the results of the simulations. Section six summarizes and concludes the paper.

2. Related Literature

The effects of on-net/off-net differentiation have been explored both in the theoretical, empirical and experimental literature. While the theoretical literature predominantly investigates the competitive effects of on-net/off-net differentiation, the empirical and experimental literature focuses on how consumers actually behave in response to on-net/off-net differentiation.

One of the first theoretical papers which explored the competitive effects of on-net/off-net differentiation is the seminal work of Laffont, Rey, and Tirole (1998). Using a Hotelling setup with two networks offering two-part tariffs with on-net/off-net differentiation, Laffont, Rey, and Tirole show that when networks are symmetric, charging high (reciprocal) access markups, i.e. markups on the costs for terminating a call originating from a competitor’s network, intensifies competition as these translate into higher off-net prices. On the other hand, if firms are asymmetric, high access markups harm small entrants as these make off-net calls prohibitively costly for consumers subscribed to the entrant’s network which decreases consumers’ incentives to join the entrant’s network. Using the same setup, Gans and King (2001) find that negative access markups (“bill-and-keep”) can be used by incumbents to soften competition, a result also reported by Elliot (2004). Jeon, Laffont, and Tirole (2004) extend the framework of Laffont, Rey, and Tirole by including call externalities, i.e. consumers derive utility from being called, and the possibility to charge consumers for receiving calls. One key result of their analysis is that if networks differentiate between on-net and off-net calls, a “connectivity breakdown” can occur, i.e. the volume of off-net traffic goes to zero. In a similar vein, Hörlig (2007) shows that on-net/off-net differentiation can be used profitably as a predatory pricing device by a large network. In a multi-firm industry with logit demand, Calzada and Valetti (2008) submit that under the threat of additional entry, multiple incumbents have an incentive to collectively raise access markups to deter entrants. Hörlig (2009) generalizes the results of Gans and King to the case of n (symmetric) networks. Cabral (2011) studies a dynamic oligopoly game with constant fluctuation of consumers and finds that “if network effects are sufficiently strong, then the larger network tends to increase in its size” (p. 84). Finally, López and Rey (2012) studying the framework of Laffont, Rey, and Tirole show that if firms are asymmetric, it can be profitable for the larger network to foreclose the market by charging high access markups.

Among the first empirical studies on how consumers actually choose their network is the analysis by Corrocher and Zirulia (2009) who survey 197 Italian students regarding the determinants of mobile
tariff choice and various user characteristics. They identify a cluster of consumers “whose choice of a mobile operators is fairly random” (p. 73). Using data on a survey among 1044 students in Germany, Hauicap and Heimeshoff (2011) report that some consumers in their sample fail to weight prices for on-net and off-net calls with the appropriate calling probabilities, a behavior which the authors term “price differentiation bias”. Claussen, Trüg, and Zucchini (2012) study 969 tariffs in the German mobile telecommunications market covering the years 2004 to 2009. They analyze whether small or large networks are more likely to use on-net/off-net differentiation and whether the extent of on-net/off-net differentiation is greater for smaller or for larger networks. These scholars find that larger networks are significantly more likely to use on-net/off-net differentiation, which they interpret as evidence that larger networks strategically use on-net/off-net differentiation to gain a competitive advantage over smaller competitors.

The first experimental study on consumers’ behavior in response to on-net/off-net differentiation is an online-based consumer survey by Bolle and Heimel (2005) who note that a large fraction of mobile phone users suffers from a what the authors call a “fallacy of dominant price vectors”: consumers switch their network if the mean of the tariffs for on-net and off-net calls offered by a competitor is lower than the mean tariff of their current network, without weighting the tariffs for on-net and off-net calls with the appropriate calling probabilities. In a similar vein, Hauicap, Heimeshoff, and Grimm (2009) find in a laboratory-based experiment on tariff choices that some consumers exhibit an “on-net bias”, i.e. consumers choose the network with the cheapest tariff for on-net calls. And even in the absence of on-net/off-net differentiation, a significant fraction of consumers seems to be unable to choose the cost-minimizing contract, as results from an experiment of Barth and Graf (2011) indicate.

In summary, the theoretical literature by and large supports the notion that if consumers behave fully rationally, on-net/off-net differentiation negatively affects competition, whereas, on the other hand, both the empirical and the experimental literature strongly suggest that at least some consumers do not choose tariffs fully rationally (see also Claussen, Trüg, and Zucchini 2012). To integrate both views, my model allows for the coexistence of rational and non-rational consumers and explores how they influence the effect of on-net/off-net differentiation on network sizes.

3. The Model

To study the effects of on-net/off-net differentiation and heterogeneous consumers on network sizes, I use a relatively new methodology called agent-based computational economics (ACE), which is a tool for simulations in the social sciences. Following the definition of Axelrod and Tesfatsion (2006), I use the term agent-based computational economics to denote a “method for studying systems exhibiting

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2 The current version of the model is very preliminary. I plan to further extend the model in several directions in future versions. An overview over the additional elements to be included in the model is provided at the end of section three.
the following two properties: (1) the system is composed of interacting agents; and (2) the system exhibits emergent properties, that is, properties arising from the interactions of the agents that cannot be deduced simply by aggregating the properties of the agents.” (p. 1649). Following this definition, the system I study is the market for mobile telecommunication, the agents of the system are consumers deciding on which network to choose, and the emergent properties of the system are the market shares of the mobile networks. Interactions between the consumers take place in the form of (negative or positive) tariff-mediated network externalities that the decision of one consumer to join a specific network imposes on all other consumers. Negative externalities, for instance, arise if consumer X decides to switch from network A to B which, in turn, forces consumer Y, who wants to call X and is subscribed to network A, to make a (more expensive) off-net call instead of a (cheaper) on-net call, thereby increasing Y’s telephone bill. The distribution of market shares among the network operators is “emergent” because the decision of a single consumer to join a specific network is not only contingent on the tariffs she pays and how she calculates the attractiveness of the networks (=properties of the agent), but also on the decision making rules of all other consumers as well as on the actual market shares of all networks at the time of decision. Due to these mutual dependencies and feedback loops, inferring the final market shares of the networks from how consumers calculate the attractiveness of networks is impossible which makes the use of ACE particularly promising and attractive.

To describe the design of my model, I rely on the so-called ODD (“Overview”, “Design concepts”, and “Details”) protocol (Grimm et al. 2006; Grimm et al 2010; Railsback and Grimm 2012). This protocol has been developed by a group of researchers active mainly in the field of ecology in an attempt to provide a common standard of how agent-based models (ABMs) should be described in scientific publications. The main aim of the ODD protocol is “to make model descriptions more understandable and complete, thereby making ABMs less subject to criticism for being irreproducible” (Grimm et. al 2010, p. 2760).

The ODD protocol consists of three main sections, as illustrated by table 1: overview, design concepts, and details. The section overview, is again subdivided into three subsections. The subsection “purpose” very briefly states which question(s) the model intends to answer. The subsection “entities, state variables, and scales” describes the entities of the model, their characteristics and attributes, and how space and times is represented in the model. The “process overview and scheduling” subsection illustrates which entities perform which actions (processes) and in what order. However, in this subsection, the processes are only mentioned briefly, whereas they are fully described in the subsection “submodels”. The section design concepts contains a description of up to eleven different basic design concepts, such as, for instance, how individuals in the model adapt to changes in the environment (“adaptation”), which objectives they pursue (“objectives”) or what individuals know

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3 Of course, this rests on the assumption that the probability of making an on-net call corresponds to the network’s market share which is only the case if calls are randomly distributed across the networks.
about their environment ("sensing"). The third section, details, is composed of three subsections. The subsection “initialization” provides a description of the initial state of the world in \( t=0 \) and how this state is created. Whether or not the model uses time-series data as an input for the model\(^4\) is stated in the subsection “input data”. A comprehensive description of the more complex processes of the model is given in the “submodels” subsection.

<table>
<thead>
<tr>
<th>Overview</th>
<th>1. Purpose</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2. Entities, state variables, and scales</td>
</tr>
<tr>
<td></td>
<td>3. Process overview and scheduling</td>
</tr>
<tr>
<td>Design concepts</td>
<td>4. Design concepts</td>
</tr>
<tr>
<td></td>
<td>• Basic principles</td>
</tr>
<tr>
<td></td>
<td>• Emergence</td>
</tr>
<tr>
<td></td>
<td>• Adaptation</td>
</tr>
<tr>
<td></td>
<td>• Objectives</td>
</tr>
<tr>
<td></td>
<td>• Learning</td>
</tr>
<tr>
<td></td>
<td>• Prediction</td>
</tr>
<tr>
<td></td>
<td>• Sensing</td>
</tr>
<tr>
<td></td>
<td>• Interaction</td>
</tr>
<tr>
<td></td>
<td>• Stochasticity</td>
</tr>
<tr>
<td></td>
<td>• Collectives</td>
</tr>
<tr>
<td></td>
<td>• Observation</td>
</tr>
<tr>
<td>Details</td>
<td>5. Initialization</td>
</tr>
<tr>
<td></td>
<td>6. Input Data</td>
</tr>
<tr>
<td></td>
<td>7. Submodels</td>
</tr>
</tbody>
</table>

**Table 1: The ODD protocol (source: Grimm et al. 2010)**

In the following, I describe my model according to the ODD protocol (Grimm et al. 2006, Grim et al. 2010). In line with recommendations in the literature on agent-based modeling, I try to keep the design of the model as simple as possible (Axelrod 2005).

**Purpose.** The objective of the model is to answer the question how on-net/off-net differentiation affects the network sizes of a small and a large operator when both rational and non-rational consumers coexist in the market.

**Entities, state variables, and scales.** The model contains 1,000 consumers deciding about which mobile network they want to be subscribed to. Consumers are assumed to be either fully-informed and rational (FIR), partly-informed and rational (PIR), or non-rational (NR). Key characteristics of all consumers include their position on a two-dimensional grid, a dummy variable “network-a”, which takes on the value one if the consumer is currently subscribed to network A and zero if she is subscribed to network B\(^5\), and a variable “rule-number” indicating the consumer type (FIR, PIR, or NR). The market for mobile telecommunications is represented by a rectangle of 50 x 20 cells and

\(^4\) In ecological models, such input data might be annual rainfall and other weather conditions that are determined exogenously. In economic models, input data could comprise, for instance, data on stock prices or economic growth.

\(^5\) I assume full market participation, i.e. all consumers are always subscribed to one of the two networks.
each cell is occupied by exactly one consumer located at the center of the cell. The grid is toroidal, i.e. the world wraps both horizontally and vertically so that all consumers have exactly eight neighbors. The length of one time step is not explicitly defined. However it is shorter than the average duration of a mobile contract because in each time period only a fraction of consumers, specified by the variable “prob-of-switch”, is allowed to decide about whether to stay with the current network or switch to the competitor. The model automatically stops after 1,000 time periods.

**Process overview and scheduling.** In each period, five actions are executed successively. First, the computer draws a random sample of consumers who are allowed to decide on their network membership. The sample size is determined by the variable “prob-of-switch” which is set by the modeler. Second, all consumers of this sample execute the process “calculate-current-share-a” to calculate the current market share of network A as perceived by each consumer. Since I assume full market participation, the market share of network B is simply one minus the market share of network A. FIR consumers can observe the network membership of all consumers and can thus calculate the true market share. PIR consumers can only observe the network membership of consumers within a circular sensing field whose radius is specified by the modeler through the variable “sensing-radius”. NR consumers can only observe the network membership of their eight neighbors. Third, all selected consumers execute the process “calculate-attractiveness” to calculate a score for both networks which measures each network’s attractiveness to the respective consumer and depends on a consumer’s type. Fourth, each consumer of the sample executes the process “decide-switching” which induces the consumer to either stay with her current provider if the attractiveness of the current network is larger or equal to the attractiveness of the competing network or to switch to the competing network otherwise. Switching costs are assumed to be zero. Fifth, the process “do-plotting” is used to update the plot of market shares of both networks.

**Design concepts.** The focus of the model is on the sizes of networks A and B, which emerge endogenously from the interaction of the three different consumer types. The decision of each consumer to join a specific network depends on the market share of each network which in turn depends on the decisions of all other agents in previous periods. Due to this feedback loop, network sizes emerge in complex ways and cannot be inferred by simply considering initial market shares and consumers’ rule for calculating the attractiveness of the networks. Consumers adapt to their environment by either staying with their current network operator or by switching to the competing network. In doing so, consumers pursue the **objective** to be subscribed to the network which offers

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6 The idea that in each period only a certain fraction of consumers is allowed to decide about network membership closely resembles the concept of Calvo price-setting (Calvo 1983). The concept of Calvo price setting is a widely used concept in macroeconomics and assumes that firms are not able to constantly change their prices. Instead, under the Calvo pricing mechanism in each period only a fraction (1-θ) of all firms is allowed to adjust their prices.

7 Figure 8 in the appendix illustrates how different radii affect the number of consumers within a PIR consumer’s sensing field.

8 A detailed discussion of how the different consumer types calculate the attractiveness of networks is contained in the subsection „submodels“.
them the highest attractiveness score. When calculating the attractiveness of each network, consumers do not make any predictions about the future or about other consumers’ behavior. All consumers sense the tariffs for on-net and off-net calls of both networks. Moreover, FIR consumers sense the network membership of all other consumers, whereas PIR consumers sense the network membership of consumers within a circular sensing field and NR consumers only sense the network membership of their eight neighbors. Interaction takes place only indirectly through network externalities that arise from the decision of each consumer to join a specific network. The sole stochastic element of the model is the recurrent sampling of consumers which are allowed to decide about their network membership. The main observation of the model is the market share of networks A and B.

Initialization. The initialization of the model proceeds in four steps. First, 1,000 consumers are created by the computer and distributed across the rectangular grid so that each cell is occupied by exactly one consumer.

Second, each consumer is assigned to exactly one of the two networks so that the actual fraction of consumers subscribed to network A matches the market share of network A as specified by the modeler through the variable “initial-share-a”. To this end, the process “distribute-shares-in-clusters” is executed which distributes network membership such that clusters of consumers subscribed to network B occur. Network membership to network B is assigned clusterwise due to the following reason. If membership to network B was assigned randomly, the share of consumers subscribed to network A within the circular sensing field of PIR consumers would in expectation be equal to the actual market share of network A in the market. Therefore, in expectation, the market shares observed by PIR consumers would be equal to the true market shares and, hence, the behavior of PIR and FIR consumers would be similar.

In the third step of the initialization, the rules for how consumers calculate the attractiveness of networks are randomly assigned to all consumers by setting the variable “rule-number” to one for FIR consumers, to two for PIR consumers, and to three for NR consumers. The distribution is such that the fraction of FIR consumers corresponds to the value of the variable “share-fir”, the fraction of PIR consumers corresponds to the value of the variable “share-pir” and the remaining consumers are assigned to be NR consumers.

Fourth, the value for the variable prob-of-switch is initialized to 0.05 and kept constant throughout the simulation. Furthermore, the tariffs for on-net and off-net calls are set to two and four for network A and one and four for network B. Tariffs of both networks are also kept constant throughout the simulation. The lower on-net price for network B was chosen based on the idea stated earlier that small network operators are able to strategically use very low on-net tariffs to attract boundedly rational consumers. In addition to that, empirical evidence also suggests that the extent of on-net/off-net differentiation is slightly lower for small networks (Claussen, Trüg, and Zucchini 2012). The decision to keep the tariffs of both networks constant during one simulation run in the current version
of the model was made due to two reasons. First, it is very difficult to decide how both network operators should deal with the presence of PIR and NR consumers because this requires to answer several design-related questions, such as, for instance, should firms be aware of the fact that not all consumers are FIR, are firms themselves rational in their price setting behavior or do they also use heuristics, and how frequently are firms allowed to adjust their prices? However, answers to these questions would be somewhat arbitrary because the current theoretical literature on on-net/off-net differentiation disregards the existence of non-rational consumers. Second, my model focuses on the demand side of the market for mobile telecommunications. Its starting point is the empirical observation that in many countries the tariffs for on-net and off-net calls are differentiated. Taking this as given, the model then tries to investigate how the coexistence of rational and non-rational consumers influences the effect of on-net/off-net differentiation on network sizes in mobile telecommunications. It is easier to understand how the coexistence of rational and non-rational consumers affects network sizes under different market conditions if the supply side of the market is kept constant. However, in future versions of the model I plan to relax the assumption of fixed tariffs (see comments at the end of section 3).

Figure 1 shows an exemplary state of the market after completed initialization. Red cells are occupied by consumers subscribed to network A while blue cells accommodate consumers subscribed to network B. Membership to network B is distributed in ten clusters. FIR consumers are represented by a dot, PIR consumers by an x, and NR consumers by a triangle.

**Figure 1: Exemplary state of the market after initialization**

*Input Data.* The model does not use input data from external sources.

*Submodels.* In executing the process “calculate-attractiveness”, consumers who have been selected to decide about their network membership calculate an attractiveness score for both networks. For FIR consumers, attractiveness is defined as minus one times expected cost of a call using the true market shares, i.e., $\text{attractiveness}_i = -(M_{i}^\text{true} \cdot price_{i}^{\text{on-net}} + (1 - M_{i}^\text{true}) \cdot price_{i}^{\text{off-net}})$. PIR consumers also determine the attractiveness of a network by the expected cost of a call, albeit with the
restriction that these consumers are not able to observe the market shares in the whole market but only within their circular sensing field. Hence, FIR consumers calculate the attractiveness of a network as
\[ \text{attractiveness}_i = -\left( MS_i^{\text{observed}} \cdot \text{price}_i^{\text{on-net}} + \left( 1 - MS_i^{\text{observed}} \right) \cdot \text{price}_i^{\text{off-net}} \right). \]
NR consumers calculate the attractiveness of network i by the fraction of their eight neighbors who are subscribed to network i, i.e. 
\[ \text{attractiveness}_i = \frac{\text{(number of neighbors subscribed to i)}}{8}. \]

The process “distribute-shares-in-clusters” assigns membership to network B in clusters across the rectangular grid and proceeds in six steps. First, the variable “network-a” is set to one for all consumers. Second, the process calculates how many consumers subscribed to network B are needed to mirror the market share of network B as implicitly defined by the variable “initial-share-a”. This number is divided by the number of clusters specified by the variable “number-of-clusters” and subsequently rounded down to approximate the average number of consumers per cluster subscribed to network B which is also the average number of blue cells per cluster. Third, the necessary radius of each cluster is approximated by \( \text{radius} = \sqrt{\text{average number of blue cells per cluster}/2} \). Fourth, a number of consumers equal to the number of clusters is randomly chosen as the center of a cluster and all consumers within the calculated radius are assigned to network B (including the consumer located at the centre of the cluster). Fifth, the number of consumers actually subscribed to network B is compared to the required number of consumers calculated in step two. If the actual number of consumers subscribed to network B is too small (large) then the computer randomly picks the necessary number of consumers from network A (B) and assigns them to network B (A). In the sixth step, the colors of the cells are updated to correctly display the network membership of the consumer inhabiting the cell. It is easy to see that this procedure implies a negative relationship between the number of clusters and the size of each cluster. Table 2 summarizes the scheduling of the model.

<table>
<thead>
<tr>
<th>1. Initialization</th>
<th>1.1. Create and distribute 1,000 consumers</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1.2. Assign network membership clusterwise</td>
</tr>
<tr>
<td></td>
<td>1.3. Assign consumer types randomly</td>
</tr>
<tr>
<td></td>
<td>1.4. Initialize constant values</td>
</tr>
<tr>
<td>2. Simulation</td>
<td>2.1. Draw random sample of consumers allowed to decide on network membership</td>
</tr>
<tr>
<td></td>
<td>2.2. Selected consumers update perceived market shares</td>
</tr>
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<td></td>
<td>• FIR consumers sense true market shares</td>
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<td>• PIR consumers only sense market shares within circular sensing field</td>
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<td>• NR consumer only sense market shares among their eight neighbors</td>
</tr>
<tr>
<td></td>
<td>2.3. Selected consumers calculate attractiveness of both networks</td>
</tr>
</tbody>
</table>
|                   | • FIR consumers use 
\[-(MS_i^{\text{true}} \cdot \text{price}_i^{\text{on-net}} + \left( 1 - MS_i^{\text{true}} \right) \cdot \text{price}_i^{\text{off-net}})\]
|                   | • PIR consumers use 
\[-(MS_i^{\text{observed}} \cdot \text{price}_i^{\text{on-net}} + \left( 1 - MS_i^{\text{observed}} \right) \cdot \text{price}_i^{\text{off-net}})\]
|                   | • NR consumers use \((\text{number of neighbors subscribed to i})/8\) |
|                   | 2.4. Selected consumers switch network if attractiveness of competing network is higher than the attractiveness of the current network |
3. Observation

Table 2: Scheduling of the model

Planned extensions of the current model. The current version of the model is only preliminary. Future versions of the model will be extended in several directions. These include:

- **Implementation of calling clubs.** As has been shown in the extant literature, the network membership of close friends and family members has a great influence on a consumer’s decision to join a specific network (Gabrielsen and Vagstand 2008; Birke and Swann 2006). Hence, I plan to extend the model so that each consumer is assigned to a (probably varying) number of other consumers (“friends”) that consumers call more frequently than the rest of the population. When calculating the attractiveness of a network based on the expected cost of a call, FIR and PIR consumers will then put a higher weight on the network membership of their friends and, hence, the importance of market shares of both networks will decrease.

- **Dynamization of prices.** In the current version of the model, the assumption that tariffs for on-net and off-net calls are fixed is very strong. I plan to relax this assumption in future versions of the model so that both networks react to changes in their market shares by adjusting their tariffs.

4. Analysis of the model

For the analysis of agent-based models, Railsback and Grimm (2012) recommend to identify the key variables which have the greatest impact on the outcome of the model and to systematically vary these variables to understand the model’s behavior. In the present model, six variables are likely to have an influence on the emergence of network sizes. These are (1) the design of the rules for how consumers calculate the attractiveness of networks, (2) the tariffs for on-net and off-net calls, (3) the initial market share of network A, (4) the fraction of FIR, PIR, and NR consumers, (5) the size of the clusters as determined by their number, and (6) the sensing radius of PIR consumers. While the influence of the first four variables is rather self-evident, the reason why the last two variables influence network sizes might be less obvious. To see the influence of the cluster size and the sensing radius, consider the following example. Suppose, a PIR consumer subscribed to network A is located directly at the border of a cluster of consumers subscribed to network B. When deciding whether or not to switch her network, this consumer calculates the market share of network B (and similarly of network A) based on the fraction of consumers subscribed to network B (A) within her circular sensing field. If her sensing radius is small, this consumer will not sense beyond the border of the adjacent cluster (see left side of figure 2). Accordingly, approximately half of the consumers within her sensing field will be subscribed to network B. Depending on the tariffs for on-net and off-net calls, this might induce the consumer to switch to network B. If, however, the adjacent cluster of blue consumers is small or, equivalently, the consumer’s sensing radius is large, she will sense beyond the borders of the adjacent...
cluster (see right side of figure 2). This, in turn, will decrease network B’s market share as observed by this consumer which will make it less likely that she switches to network B. Therefore, both the number of clusters and the sensing radius of PIR consumers influence network sizes.

Instead of varying all six variables, I decided to keep both the design of the consumer types and the tariffs of both networks constant in the analysis for the following reasons. I did not vary the design of the consumer types because the focus of the paper is on investigating how the coexistence of rational and non-rational consumers influences the effect of on-net/off-net differentiation under different market conditions. Hence, I concentrated on varying market conditions rather than the design of rational and non-rational consumers. Moreover, I kept the tariffs of the networks constant across the different simulation runs because the extent of on-net/off-net differentiation affects the critical market share of network A below (above) which FIR and PIR consumers will chose network B (A). Hence, variation of tariffs would also require using different values for the initial market share of network A because these are chosen so that one value is close to the critical market share and the other value is relatively far-off. Accordingly, varying the tariffs across simulations would significantly complicate the analysis and the presentation of results which is why I refrained from varying the tariffs in the current version of the model.

Due to these considerations, the analysis focuses on four variables which will be varied systematically: (1) the initial market share of network A, (2) the share of FIR, PIR and NR consumers, (3) the number of clusters, and (4) the sensing radius of PIR consumers. However, since these four variables are continuous, the possible parameter space is too large to explore entirely. To solve this problem I used a rather low and a rather high value for each variable except for the fraction of FIR, PIR, and NR consumers for which I used four different values. Figure 3 shows the different values of the variables chosen for the analysis.

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9 With the tariffs for on-net and off-net calls set to two and four for network A and one and four for network B, FIR and PIR consumers join network B if the true / observed market share of network A is below 60%.
In the analysis I used the values 65% and 85% for the initial market share of network A. Since the tariffs for on-net and off-net calls are set to two and four for network A and one and four for network B, respectively, the expected cost of a call is lower for network A as soon as its market share exceeds 60%. A market share of 65% for network A was chosen to study a market in which network A’s initial market share is close to this threshold. I chose the upper limit of 85% to meet two objectives: First, network A owns a large part of the market and, second, there are enough consumers initially subscribed to network B so that the clusters of consumers subscribed to network B is sufficiently large even if the number of clusters is high. To study the effect of differing shares of FIR, PIR, and NR consumers, I distinguished between four different cases: (1) a market with 50% FIR, 25% PIR, and 25% NR consumers; (2) a market with 0% FIR, 50% PIR, and 50% NR consumers; (3) a market with 0% FIR, 75% PIR, and 25% NR consumers; and (4) a market with 0% FIR, 25% PIR, and 75% NR consumers. I chose these four cases to study market conditions in which a large fraction of consumers behaves fully rational (case a), in which no consumer behaves fully rationally but shares of PIR and NR consumers are symmetric (case b) or asymmetric (cases c and d). The number of clusters was somewhat arbitrarily set to two and ten, respectively, to ensure that clusters are neither too large nor too small. Finally, for the sensing radius of PIR consumers the values one and five were used to study conditions in which PIR consumers either sense even less than NR consumers or much more than FIR consumers.10

The combination of all values for each of the four variables results in $2\times4\times2\times2 = 32$ simulation scenarios. To minimize the influence of stochastic elements of the model on the results, I ran 100 repetitions for each of the 32 scenarios and averaged market shares for network A and B across the 100 repetitions for each of the 1,000 periods.

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10 When the sensing radius of PIR consumers increases, they approach FIR consumers in their behavior. In the extreme case, when the sensing radius is sufficiently large so that the circular sensing field comprises the whole market, PIR behave like FIR consumers.
Planned extensions of the current analysis. Similarly to the model design, I plan to extend the current analysis in future versions of the model. In particular, I plan to:

- Increase the number of repetitions for each of the 32 scenarios to further reduce the influence of stochastic elements in the model.
- Study the effect of additional values for each of the four variables to create multivariate contour plots (for an example see Railsback and Grimm 2012, p. 295-297) thereby exploring the possible parameter space more thoroughly and maybe identifying critical values and tipping points for the four variables.
- Systematically vary the spread between on-net and off-net tariffs as well as the difference in tariffs between both networks to investigate how the extent of on-net/off-net differentiation affects market outcomes.

5. Results and Discussion

Tables 3 through 10 in the appendix show the detailed results of the simulations. For each of the 32 scenarios, I averaged the market share for network A across the 100 repetitions for each period and plotted the average market share to visualize its course for an average run. Furthermore, for each scenario, I calculated the minimum, maximum, mean, and standard deviation of network A’s market share in the last period (t=1,000) to illustrate the average market outcome and its volatility for each scenario. In the following, I present the results for each of the 32 scenarios. In line with my research question, I subdivide the presentation of the results into four cases: A) share of FIR 50%, PIR 25%, and NR 25%; B) share of FIR 0%, PIR 50%, and NR 50%; C) share of FIR 0%, PIR 75%, and NR 25%; and D) share of FIR 0%, PIR 25%, and NR 75%.

Case A: FIR 50%, PIR 25%, NR 25%. As table 3 illustrates, network A substantially increases its market share if 50% of consumers are fully-informed and rational and the sensing radius of FIR consumers is one. This increase results in two possible market outcomes: either all consumers eventually subscribe to network A or only small clusters of PIR and/or NR consumers subscribed to network B “survive”. If small clusters remain in the market, network B’s market share ranges between 1.1% and 5.2%. These results hold irrespective of whether network A initially has a high or low market share and whether consumers subscribed to network B are initially concentrated in two or ten clusters. The two possible market outcomes can be explained by the fact that with the given values for network A’s initial market share and the tariffs of both networks, FIR consumers have an incentive to join network A. Hence, FIR consumers who are initially part of a cluster of consumers subscribed to network B will eventually join network A. This, in turn, will decrease the homogeneity of these clusters, as illustrated by figure 4. As a result, the observed market share of network A of PIR consumers who are either part of the cluster or close-by will increase. If this increase is sufficiently high, i.e. if the fraction of FIR consumers within a cluster is sufficiently high, these PIR consumers
have an incentive to join network A which further reduces the homogeneity of the cluster. Hence, FIR consumers within a cluster of consumers subscribed to network B can trigger a chain reaction with leads to the two aforementioned market outcomes. Small clusters of consumer subscribed to network B remain in the market only if, accidently, several PIR and/or NR consumers subscribed to network B inhibit adjacent cells so that they stabilize each other’s observed market shares due to their small sensing radius. Figure 5 shows an example of a market in which three clusters of PIR/NR consumers survive.

Table 4 shows that if the sensing radius of FIR consumers increases to five, there is a unique market outcome: all consumers eventually subscribe to network A. The reason why in these four scenarios no small clusters of consumers subscribed to network B survive is the large sensing radius of FIR consumers. A FIR consumer within such a small cluster senses beyond its border and, hence, the observed market share of network A for this consumer is large enough so that she will switch to network A. Accordingly, the cluster shrinks and eventually disappears altogether.

**Case B: FIR 0%, PIR 50%, NR 50%**. Table 5 shows that the market share of network A hardly changes if the sensing radius of PIR consumers is one and the number of initial clusters is two, regardless of the initial distribution of market shares. If network A’s initial market share is 65% (85%), on average its market share increases to 68.6% (86.6%) in period 1,000, with a standard deviation of 0.02 (0.01), as can be inferred from the left side of table 5. If the initial number of clusters increases to ten, network A on average loses 0.4% market share if it starts out with a market share of 65% (upper right cell of table 5) or gains 6.8% market share if its initial market share is 85% (lower right cell of table 5). The explanation for network A’s small loss in the case of 65% initial market
share is that in some cases the ten clusters of consumers subscribed to network B are large enough so that several clusters merge, as illustrated in figure 6. When the initial market share of network A is 85%, however, the ten clusters are not large enough to merge but instead some of the clusters disappear so that eventually only small clusters of consumers subscribed to network B remain in the market.

However, these results change if the sensing radius of PIR consumers is set to five, as displayed in table 6. If network A’s initial market share is 65%, network B increases its market share in 72 out of 100 repetitions when the initial number of clusters is two and in 83 out of 100 cases when the number of clusters is ten. In the remaining cases, the initial (random) arrangement of clusters is such that clusters cannot merge and, hence, market shares remain almost constant, with a slight increase for network A. However, if the clusters of consumers subscribed to network B can merge, cluster growth does not continue until all consumers are subscribed to network B. Instead, on average, cluster growth stops at a market share for network B of 55.2% (initially two clusters) or 63.2% (initially ten clusters). Closer inspection of these cases reveals that clusters grow and merge until vertical frontiers between consumers subscribed to network A and B arise which remain stable and consist of NR consumers. Such a market outcome is displayed in figure 7. The reason for why NR consumers form a barrier to cluster growth is rather straightforward. Consider a NR consumer subscribed to network A located exactly next to a vertical frontier of consumers subscribed to network B. If this consumer calculates the attractiveness of both networks, she will only consider her eight neighbors, five of which will be

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11 This small increase is due to those consumers who are initially subscribed to network B without being part of a cluster. These consumers will usually switch to network A during the simulation.
12 Very rarely, in the scenario with the initial number of clusters set to ten, the random arrangement of NR consumers is such that no vertical frontiers arise and, hence, clusters grow until all consumers are subscribed to network B. This occurred in 3 out of 100 repetitions.
subscribed to network A. Hence, this consumer will decide to stay with network A and, accordingly, the frontier of consumers subscribed to network B will not grow beyond this consumer.

Slightly different results emerge if the initial market share of network A increases to 85% which decreases the average size of clusters of consumers subscribed to network B. When there are initially ten clusters, network A always takes over the entire market. Due to network A’s high initial market share and the large number of clusters, each individual cluster is too small for PIR consumer’s sensing radius of five which is why PIR consumers also have an incentive to join network A. On the other hand, when the number of clusters is two, network A takes over the entire market in only 42 out of 100 repetitions. In further 43 repetitions, A’s market share increases and reaches 89.8% on average, whereas in the remaining 15 repetitions, A’s market share decreases to a mean of 74.0%. These three different outcomes are possible because in some cases, the arrangement of PIR and NR consumers is such that clusters shrink until they become “too-small” for PIR consumers’ sensing radius so that eventually the clusters completely disappear. In other cases, clusters do not underrun this threshold so that PIR consumers within the cluster are not able to sense beyond its borders and, hence, are willing to stay with network B, a situation comparable to the one displayed in figure 4. Finally, it is also possible that the two clusters are initialized rather close together to that the two clusters can merge which, in turn, results in a reduction of network A’s market share.

**Case C: FIR 0%, PIR 75%, NR 25%**. The results of case B illustrate that if the number of NR consumers is sufficiently large, they act as a barrier to cluster growth. Therefore, case C investigates the effect of an increase in the share of PIR consumers to 75%. Table 7 shows that setting the sensing radius of PIR consumers to one and the initial market share of network A to 85% results in almost unchanged market shares. At the end of a simulation, network A’s mean market share is 84.4% if the initial number of clusters is two and 87.4% if the number of clusters is ten. If, on the other hand, the initial market share of network A is 65%, network A’s market share decreases to an average of 59.6% (43.8%) with initially two (ten) clusters. The smaller decrease of A’s average market share for the scenario with two clusters is due to the fact that in some repetitions, the two clusters are too far apart so that they cannot merge and, hence, market shares hardly change at all, whereas in the case with ten clusters, mergers among several clusters always occur.

![Figure 7: Emergence of vertical frontiers of NR consumers stopping cluster growth](image)
When the sensing radius increases to five, the market becomes much more dynamic. As can be inferred from the upper half of table 8, if network A starts out with a market share of 65%, this share decreases drastically to an average value of 0.7% in the case of initially two clusters and 1.1% in the case of initially ten clusters. In the former case, in all but one of the 100 repetitions, eventually all consumers join network B, while in the latter case network B takes over the market in 98 out of 100 repetitions. Increasing the sensing radius of PIR consumers to five enables clusters to grow beyond their borders. Since the share of NR consumers is low, they only rarely stop cluster growth and, consequently, clusters grow until, eventually, all consumers are subscribed to network B. However, these results no longer hold if network A’s initial market share increases to 85% because this decreases the average size of the clusters and, in many cases, enables PIR consumers to sense beyond the borders of the clusters (see lower half of table 8). Setting the initial number of clusters to ten leads to all consumers sooner or later joining network A while in the case with initially two clusters, again, three different market outcomes are possible. In 60 out of 100 simulations, network A takes over the entire market whereas in 8 repetitions network B does so. In additional 15 repetitions, the market share of network A in period 1,000 is below 10% and it is highly likely that in these cases the remaining consumers would eventually also have subscribed to network B had the simulation not stopped automatically after 1,000 periods. Which of these three results materializes depends on whether or not clusters of consumers subscribed to network B are able to merge to larger entities and whether the remaining areas of consumers subscribed to network A between these merged entities become “sufficiently small”. If this latter condition is fulfilled, the sensing field of PIR consumers who are located in the areas between the merged clusters and subscribed to network A also includes consumers from the surrounding clusters. This might induce these consumers to also join network B so that clusters further grow.

Case D: FIR 0%, PIR 25%, NR 75%. In the scenarios with a large fraction of NR consumers, the market shows only moderate dynamics as long as the sensing radius of PIR consumers is small. Table 9 depicts that if network A starts with a market share of 65% (85%), its average market share at the end of a simulation is 72.1% (87.8%) with initially two clusters and 72.4% (95.9%) with initially ten clusters. The standard deviation for network A’s market share in period 1,000 ranges between 0.01 and 0.03 which is further evidence for the low market dynamics in these four scenarios.

Almost the same holds true when the sensing radius of PIR consumers increases to five (see table 10). When network A’s initial market share is 65%, its market share on average slightly increases to either 69.1% when the initial number of clusters is two or 75.4% when the number of clusters is ten. However, rarely, the initial arrangement of clusters is such that network B increases its market share as clusters merge. This occurs in 15 (6) out of 100 repetitions if the initial number of clusters is two (ten). Only in the scenario with an initial market share for network A of 85% and ten clusters (see lower right cell of table 10) on average significant deviations from the initial distribution of market shares occur. Except for one repetition eventually all consumers join network A.
These almost unchanged market outcomes in most of the eight scenarios of case D can be explained by the large fraction of NR consumers which prevent cluster growth (as has already been discussed). Only as the clusters of consumers subscribed to network B underrun a critical size, a process of continuous shrinkage sets in so that eventually all consumers subscribe to network B.

6. Summary and conclusion

The extant literature argues that on-net/off-net differentiation unfolds anti-competitive effects due to tariff-mediated network effects which benefit large network operators and harm smaller entrants or even prevent entry altogether (Harbord and Pagnozzi 2010). While these results critically hinge on consumers behaving fully rational, evidence from both empirical and experimental studies on consumers’ tariff choice suggests that not all consumers are capable of choosing their tariff fully rationally (Claussen, Türg, and Zucchini 2012; Haucap and Heimeshoff 2011). Therefore, this paper explores the effect of on-net/off-net differentiation on network sizes in mobile telecommunications when both rational and non-rational consumers coexist in the market.

To incorporate different types of consumer behavior in one model, I rely on a relatively new methodology called agent-based computational economics (ACE) which is characterized by two key features. First, individuals (agents) in the model are heterogeneous with respect to their characteristics and rules for behavior and act autonomously according to their own objective function. Second, interaction among individuals is usually local, i.e. individuals usually only sense their immediate environment and exclusively interact with other nearby individuals. These two features lead to so-called emergent system-level behavior, which implies that the properties of the system cannot be inferred from the properties of the individuals it is composed of.

In the model, the market for mobile telecommunications is represented by a rectangular grid of 1,000 cells with each cell occupied by exactly one consumer. Consumers are subscribed to exactly one of two networks which offer differentiated linear tariffs for on-net and off-net calls. Tariffs of both networks are fixed during a simulation and set so that the smaller network has a lower on-net tariff while both off-net tariffs are equal. Consumers are heterogeneous with respect to how they calculate the attractiveness of a network. In particular, I distinguish between three different types of consumers: (1) fully-informed rational (FIR) consumers, (2) partly-informed rational (PIR) consumers, and (3) non-rational (NR) consumers. FIR consumers observe the true market shares of both networks and calculate the attractiveness of a network based on the expected cost of a call.13 In contrast to that, PIR consumers are not able to observe the true market shares but instead they only observe the market shares within a circular sensing field. PIR consumers use these observed market shares to calculate the attractiveness of both networks based on the expected cost of a call. NR consumers calculate the attractiveness of a network based on the fraction of their eight neighbors who are subscribed to the

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13 Following Laffont, Rey, Tirole (1998), I assume a balanced calling pattern, i.e. the probability of making an on-net call is equal to the network’s market share.
respective network. Each simulation run lasts for 1,000 periods and in each period a fixed fraction of consumers is allowed to decide whether to stay with the current network or switch to the competitor.

To study the effect of on-net/off-net differentiation and heterogeneous consumers on network sizes, I systematically vary four key variables of the model. These include (1) the fraction of FIR, PIR, and NR consumers, (2) the initial distribution of market shares, (3) the number of clusters of consumers initially subscribed to network B, and (4) the radius of the circular sensing field of PIR consumers. For the distribution of consumer types I use four different parameter values, whereas for the latter three variables I use two values each. Hence, the analysis consists of 4x2x2x2=32 scenarios. For each scenario I run 100 repetitions and, subsequently, average market shares for each period to minimize the effect of stochastic elements of the model.

The results of the 32 scenarios offer three key insights. First, if the fraction of FIR consumers is too large, all consumers eventually join network A and, hence, network B exits the market. In short, this is due to the fact that all FIR consumers will eventually join network A which, in turn, decreases the homogeneity of clusters of consumers subscribed to network B (see figure 4). This, on the other hand, increases the market share of network A within PIR consumers’ circular sensing field and induces them to also join network A. The resulting chain reaction leads to all consumers eventually joining network A.

Second, if the share of NR consumers is too high, they act as a barrier to cluster growth. This can be explained by the fact that if a NR consumer subscribed to network A is located adjacent to a front of consumers subscribed to network B, only three of her eight neighbors are subscribed to network B which, however, is not enough to make her switch the network. If the share of NR consumers is large enough, they can create vertical barriers beyond which clusters of consumers subscribed to network B cannot grow (see figure 7).

Third, if the share of PIR consumers is high enough, it is in principle possible for the small network B to substantially increase its market share or even take over the market completely. However, this only occurs if the size of the clusters is large enough for the sensing radius of PIR consumers. If, however, PIR consumers are able to sense beyond the borders of clusters of consumers subscribed to network B, then network B’s market share in their circular sensing field may be reduced below a critical threshold so that these consumers have an incentive to join network A (see figure 2).

However, it is important to note that these results are only preliminary. In the future, I plan to extend both the model and the analysis in several directions to also include, for instance, calling-clubs whose members call each other more frequently than the rest of the population or to make the tariffs of both networks responsive to changes in the market shares.
References


Appendix

radius 1.0
radius 2.9
radius 4.0
radius 5.0
radius 1.5
radius 3.0
radius 4.2
radius 2.0
radius 3.2
radius 4.3
radius 2.3
radius 3.7
radius 4.5

Figure 8: Sensing field of PIR consumers for different radii
Table 3: Average market shares of network A for 50% FIR, 25% PIR, 25% NR, and sensing radius = 1
Table 4: Average market shares of network A for 50% FIR, 25% PIR, 25% NR, and sensing radius = 5
<table>
<thead>
<tr>
<th>FIR 40%</th>
<th>2 Clusters</th>
<th>10 Clusters</th>
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<tr>
<td>PIR 50%</td>
<td>Sensing Radius 1</td>
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<tr>
<td>NR 50%</td>
<td>MS(A) 65%</td>
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<tr>
<td>Min(t=1000): 62.1%</td>
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<tr>
<td>Mean(t=1000): 68.6%</td>
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<td>Min(t=1000): 51.5%</td>
<td>Max(t=1000): 72.5%</td>
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<td>Mean(t=1000): 64.6%</td>
<td>Std(t=1000): 0.04</td>
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<td>MS(A) 85%</td>
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<td>Min(t=1000): 81.2%</td>
<td>Max(t=1000): 91.1%</td>
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<tr>
<td>Mean(t=1000): 91.8%</td>
<td>Std(t=1000): 0.02</td>
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Table 5: Average market shares of network A for 0% FIR, 50% PIR, 50% NR, and sensing radius = 1
Table 6: Average market shares of network A for 0% FIR, 50% PIR, 50% NR, and sensing radius = 5
<table>
<thead>
<tr>
<th>FIR 0%</th>
<th>2 Clusters</th>
<th>10 Clusters</th>
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<tr>
<td>PIR 75%</td>
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<tr>
<td>NR 25%</td>
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<tr>
<td>MS(A) 65%</td>
<td>![Graph 1]</td>
<td>![Graph 2]</td>
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<tr>
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<td>Max(t=1000): 68.6%</td>
<td>Min(t=1000): 12.6%</td>
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<td>MS(A) 85%</td>
<td>![Graph 3]</td>
<td>![Graph 4]</td>
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<td>Max(t=1000): 92.1%</td>
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<td>Mean(t=1000): 84.4%</td>
<td>Std(t=1000): 0.02</td>
<td>Mean(t=1000): 87.4%</td>
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Table 7: Average market shares of network A for 0% FIR, 75% PIR, 25% NR, and sensing radius = 1
Table 8: Average market shares of network A for 0% FIR, 75% PIR, 25% NR, and sensing radius = 5
### Table 9: Average market shares of network A for 0% FIR, 25% PIR, 75% NR, and sensing radius = 1

<table>
<thead>
<tr>
<th>FIR 4%</th>
<th>PIR 25%</th>
<th>NR 75%</th>
<th>MS(A) 65%</th>
<th>MS(A) 85%</th>
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<tr>
<td></td>
<td>Sensing Radius 1</td>
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<td>Mean(t=1000): 72.4%</td>
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<td>Std(t=1000): 0.02</td>
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</table>

Mean Market Share Network A

Min(t=1000): 67.6%  Max(t=1000): 79.9%
Mean(t=1000): 72.1%  Std(t=1000): 0.03

Min(t=1000): 64.7%  Max(t=1000): 78.2%
Mean(t=1000): 72.4%  Std(t=1000): 0.02

Min(t=1000): 92.2%  Max(t=1000): 100%
Mean(t=1000): 95.9%  Std(t=1000): 0.02

Min(t=1000): 85.7%  Max(t=1000): 92.1%
Mean(t=1000): 87.8%  Std(t=1000): 0.01
Table 10: Average market shares of network A for 0% FIR, 25% PIR, 75% NR, and sensing radius = 5