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Trade and the Firm-Internal Allocation of Workers to Tasks*

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Abstract

This paper looks inside the firm and investigates how trade alters the matching of worker-specific abilities and task-specific requirements. The outcome of this matching depends on how firms organize their recruitment process and how much they invest into the screening of applicants. In the open economy, the most productive firms start exporting. They increase their market share and therefore find it attractive to increase their screening investment, which improves the matching outcome. Things are different for non-exporters, whose market share shrinks in the open economy, lowering their incentive to invest to screen applicants. Due to this asymmetric response, access to trade raises the dispersion of labor productivity between heterogeneous producers, while at the same time increasing the average quality of worker-task matches and thus economy-wide labor productivity. The productivity-enhancing effect of endogenous adjustments in the firm-internal allocation of workers to tasks points to a so far unexplored channel through which gains from trade can materialize.

JEL codes: F12, F16, L23

Keywords: International Trade, Firm-Internal Labor Markets, Heterogeneity

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1 Introduction

In any industrialized economy, labor markets have to solve the complex problem of matching task-specific skill requirements and worker-specific abilities. The outcome of this matching process is typically not efficient. This is not only because some workers do not find a job at all. Rather, a significant share of workers cannot exploit full productivity because they are not matched with the best occupation (see Legros and Newman, 2002; Eeckhout and Kircher, 2011).¹ In recent years, this source of inefficiency has also sparked considerable attention in the trade literature. With an increasing general interest in the consequences of trade for *underemployment*, several authors have highlighted improvements in matching quality as a key aspect of gains from trade in terms of both welfare and employment (Amiti and Pissarides, 2005; Davidson, Matusz, and Shevchenko, 2008; Larch and Lechthaler, 2011). Thereby, the typical approach is to associate the quality of the matching process with its ability to match heterogeneous workers with heterogeneous firms in an efficient way, assuming implicitly that the production process covers just a single task with a specific skill requirement. However, this ignores the sophisticated structure of modern production processes and thus misses an important role of firms in reducing the requirement-ability mismatch by improving the assignment of workers to specific tasks within the boundaries of a single production entity.²

Studying the role of firms for matching workers with tasks and discussing how access to trade affects the matching outcome is the main purpose of this paper. Starting point of our analysis is a Melitz (2003) model, in which firms are heterogeneous due to differences in their productivity levels. As in Acemoglu and Autor (2011), we assume that production consists of a continuum of tasks that differ in their skill requirements. For performing these tasks, firms hire heterogeneous workers. Heterogeneity is horizontal in the sense that workers differ in their ability to perform specific tasks, while being equally productive over the whole range of activities.³ This implies that all workers have the same value to firms and lacking information about abilities of individual workers firms randomly draw their employees from the labor supply pool. This lack of information generates a source of mismatch between task-specific skill requirements and worker-specific abilities within

¹Usually the claim is that more able workers are employed by more productive firms, which is referred to by the term *positive assortative matching* in the literature. Compared to the large number of theoretical studies on positive assortative matching, there is surprisingly little empirical support for it (see Lentz and Mortensen, 2010).

²The idea that the quality of worker-task matches are important for firm performance at least dates back to work by Barron and Loewenstein (1985) and Barron, Black, and Loewenstein (1989). Meyer (1994) points to the relevance of optimal task assignment in the context of team production. Burgess, Propper, Ratto, von Hinke Kessler Scholder, and Tominey (2010) show that productivity losses from a mismatch of workers and tasks in teams can indeed be significant and that one important channel through which incentive payments to managers can improve the outcome of production units is the better assignment of workers to tasks. The relevance of the mechanism studied here can also be inferred from a recent study by Lazear and Shaw (2009), who emphasize that there is no evidence of extreme sorting of workers across firms as claimed by the literature on positive assortative matching, while there is clear evidence of sorting within the firm.

³This is similar to the labor supply assumption in Amiti and Pissarides (2005). However, in their model production consists of a single task, and hence each firm has an *ideal* worker. The view that worker ability is task-specific is well in line with recent empirical evidence on occupation-specific human capital (see, for instance, Kambourov and Manovskii, 2009; Sullivan, 2010).

the boundaries of a production unit.⁴ To reduce this mismatch, firms can invest into a screening technology for gathering some (imperfect) information about the abilities of their workforce. We model the screening investment in a rudimentary way, allowing for two possible interpretations that are common in the literature. On the one hand, screening may be part of the recruitment process as in Helpman, Itskhoki, and Redding (2010) and can help narrowing the pool of suitable applicants. On the other hand, screening may take place after the recruitment of workers, for instance, in the form of job rotation (cf. Li and Tian, 2012).⁵ In both interpretations, a higher investment provides better knowledge about the abilities of workers and therefore leads to a better match of these workers with the different tasks in the production process (cf. Pellizzari, 2011). The incentives to screen are more pronounced in larger firms, and hence there is an additional source of heterogeneity in our model, which is endogenous and reinforces heterogeneity of firms due to exogenous differences in firm productivity.

We use this model to shed new light on the consequences of trade for labor market outcome, thereby focussing on adjustments in the firm-internal labor market.⁶ To be more specific, we are interested in how trade affects underemployment arising from a mismatch between worker-specific abilities and task-specific skill requirements. To keep the analysis simple, we focus on trade between symmetric countries and consider the empirically relevant case, in which only the most productive firms export in the open economy (see, for instance, Bernard and Jensen, 1995, 1999). Having access to the export market, high-productivity firms can expand their market share in the open economy, which provides an incentive for these firms to screen their workforce more intensively, as this further improves the matching quality and thus lowers production costs. Low-productivity non-exporters, on the other hand, lose market share and thus lower their investment into the screening technology, which raises their production costs. By changing the cost structure, this asymmetric response to trade liberalization exerts a feedback effect on the entry/exit decision of firms in both the domestic and the export market, which is not present in other trade models with heterogeneous firms. Furthermore, it alters the productivity distribution of active firms by driving a wedge between matching efficiency of exporters and non-exporters. This provides an alternative to the ‘learning-by-exporting’ hypothesis for explaining the empirical finding that firms become more productive when entering the export market (see Fryges and Wagner, 2008).⁷ Finally, adjustments in the firm-

⁴The question of how workers are paired with tasks is also addressed in a sizable literature on worker assignment. However, assignment models usually presume perfect information of workers and firms and thus produce an efficient allocation of labor (see Dupuy, 2008, for a literature review). Assignment models with imperfect information include Sattinger (1995) and Shimer (2005). These studies focus on search frictions and analyze the assignment of workers to firms, while they do not shed light on the efficiency losses involved in a mismatch between worker-ability and task-specific skill requirements within the boundaries of the firm.

⁵The literature distinguishes three motives for job rotation: *employee learning* (job rotation as a training device); *employee motivation* (job rotation makes work more interesting) and *employer learning* (job rotation as a way to discover in which jobs different employees are best at). Our model is in line with the ‘employer learning’ view, which was first discussed by Ortega (2001). Empirical support for this motive is provided by Eriksson and Ortega (2006).

⁶According to Doeringer and Piore (1971) an *internal labor market* is “an administrative unit, such as a manufacturing plant, within which the pricing and allocation of labor is governed by a set of administrative rules and procedures. [...] This market] is to be distinguished from the *external labor market* of conventional economic theory where pricing, allocating and training decisions are controlled directly by economic variables” (pp. 1f).

⁷Greenaway and Kneller (2007) and Wagner (2007) summarize existing empirical evidence regarding the feedback

internal labor allocation process lower the *aggregate* mismatch between worker-specific abilities and task-specific skill requirements, thereby generating a productivity stimulus that reinforces the gains from trade in an otherwise identical Melitz (2003) model.

The firm-level adjustments to trade liberalization are not so different, in principle, from the adjustments in Helpman, Itskhoki, and Redding (2010). In their model, firms can invest into a screening technology in order to receive a more precise signal about the quality of applicants. More specifically, screening allows the firm to detect (and reject) applicants below a certain ability threshold. The higher the investment, the more effective is screening and the higher is the average ability of workers employed by the firm. The screening investment is endogenous and responds to trade in a similar way as the screening investment does in our model. It increases in exporting firms and shrinks in non-exporting ones. Aside from these similarities, there is a crucial difference between the focus of Helpman, Itskhoki, and Redding (2010) and the focus of this paper. Whereas Helpman, Itskhoki, and Redding (2010) study imperfections in the external labor market, we are interested in the firm-internal allocation of workers. To be more explicit, in our setting all workers are equally valuable to firms and only differ in their ability to perform specific tasks, whereas workers in Helpman, Itskhoki, and Redding (2010) differ in the productivity they can elicit in a firm of a specific type. Hence, there is an efficiency loss in the Helpman, Itskhoki, and Redding (2010) model, because firms are not matched with the *ideal* worker, while there is an efficiency loss in our setting, because workers do not perfectly fit the skill requirements of tasks they are performing within the boundaries of a firm.

By opening up the black box of production and modeling explicitly the firm-internal labor allocation process, our model not only identifies a new channel through which positive trade effects can materialize, but also contributes to a growing literature on the role of globalization for firm organization. A first line of research in this literature has pointed to the role of openness for the boundaries of firms (see Grossman and Helpman, 2002; Antràs, 2003; Antràs and Helpman, 2004; Conconi, Legros, and Newman, 2012). In contrast to these studies, we focus on the question how trade changes the organization of labor within these boundaries. This renders our analysis akin to Marin and Verdier (2008, 2012) who investigate the impact of trade on the hierarchy structure in firms and the incentives to empower human capital. The hierarchy structure of firms is also addressed by Caliendo and Rossi-Hansberg (2012) who analyze how access to exporting changes the number of layers of management.⁸ In contrast to all of these studies, we do not look on changes in the hierarchy structure but on matching quality, so that our findings are complementary to the results in this literature. Finally, the key mechanism discussed in this paper differs from a pure division of labor effect, which arises if there is a change in the number of tasks performed by a single worker (see Becker and Murphy, 1992). In our setting, it is not the number of tasks performed by a single worker but rather the matching of workers with these tasks that matters.

effects of exporting on firm productivity. Our reading of the literature is that there is some support for such a positive feedback effect, but not all existing studies can identify a significant impact.

⁸In a recent study, Sly (2012) investigates the composition of management teams and shows that trade can alter this composition significantly.

The remainder of the paper is organized as follows. In Section 2, we set up a baseline model with a perfect labor market and characterize the equilibrium in the closed economy. In Section 3, we consider trade between two symmetric countries, characterize the open economy equilibrium, and investigate how a movement from autarky to trade affects the allocation of labor ‘inside’ the firm as well as per capita income. We also shed light on the consequences of marginal trade liberalization. In Section 4, we extend the baseline model to one with search frictions in the hiring process and analyze how imperfections in the outside labor market alter our insights regarding the impact of trade on the firm-internal organization of workers. Section 5 provides a calibration exercise that allows us to quantify the impact of trade on welfare and underemployment. Section 6 concludes with a brief summary of the most important results.

2 The closed economy

2.1 Model structure

We consider an economy that is populated by an exogenous mass of workers L , who supply one unit of labor in a perfectly competitive labor market. There are two sectors of production: a perfectly competitive final goods industry that produces a homogeneous output good by assembling differentiated intermediate goods; and a monopolistically competitive intermediate goods industry that hires labor for its production of differentiated goods. Similar to Egger and Kreickemeier (2009, 2012), we represent the final goods technology by a constant-elasticity-of-substitution (CES) production function without external scale economies. To be more specific, we assume that the technology for producing final output Y is given by

$$Y = \left[M^{-\frac{1}{\sigma}} \int_{\omega \in \Omega} x(\omega)^{\frac{\sigma-1}{\sigma}} d\omega \right]^{\frac{\sigma}{\sigma-1}}, \quad (1)$$

where $x(\omega)$ denotes the quantity of intermediate good ω used in the final goods production, M is the Lebesgue measure of set Ω and represents the mass of available intermediate goods, and $\sigma > 1$ denotes the (constant) elasticity of substitution between different product varieties. Y serves as numéraire in our analysis, implying that the price index corresponding to the production function in Eq. (1) is equal to one, by assumption. Denoting by $p(\omega)$ the price of intermediate good ω , we can write total costs of producing output Y as follows: $\int_{\omega \in \Omega} p(\omega)x(\omega)d\omega$. Maximizing final goods profits with respect to $x(\omega)$, then gives intermediate goods demand

$$x(\omega) = \frac{Y}{M} p(\omega)^{-\sigma}. \quad (2)$$

At the intermediate goods level, there is a continuum of firms, each of them supplying a unique variety under monopolistic competition. Following Acemoglu and Autor (2011), we assume that intermediate goods production is a composite of different tasks. To be more specific, there is a continuum of tasks that is represented by the unit interval. The production technology is of the

Cobb-Douglas type and given by

$$x(\omega) = \phi(\omega) \exp \left[\int_0^1 \ln x(\omega, i) di \right], \quad (3)$$

where $x(\omega, i)$ is the production level of task i in firm ω and $\phi(\omega)$ is this firm's *baseline* productivity. Task $x(\omega, i)$ is performed (produced) by workers who are employed in a linear-homogenous production technology, which is the same for all tasks. To keep things simple, we assume that task-level output is equal to the effective labor input: the mass of workers performing the task multiplied by these workers' average productivity. The productivity of workers in performing a specific task differs, because workers differ in their abilities, whereas tasks differ in their skill requirements. To capture this in a tractable way, we assume that both workers and tasks are uniformly distributed along the unit interval, and the gap between ability and skill requirement is measured by the distance of a worker to the task in the unit interval.

In the hiring process firms have to solve the problem of matching specific workers with specific tasks, and this is essential because firms face an efficiency loss from mismatch if workers do not end up in those occupations in which they have the highest competence. The degree of mismatch depends on the *average* distance between workers and tasks in a firm's production process. To determine this average distance, we can first note that the expected distance when randomly assigning workers from interval $[0, b]$ to a task located at $t \in [0, b]$ is given by

$$dist(t) = \frac{1}{b} \left[\int_0^t (t - j) dj + \int_t^b (j - t) dj \right] = \frac{1}{b} \left(t^2 - tb + \frac{b^2}{2} \right), \quad (4)$$

where j gives the location of workers in the considered interval. Accordingly, the expected distance when drawing t randomly from interval $[0, b]$ amounts to

$$\widehat{dist} = \frac{1}{b^2} \int_0^b \left(t^2 - tb + \frac{b^2}{2} \right) dt = \frac{b}{3}. \quad (5)$$

From (5) it follows that the extent of mismatch crucially depends on the length of the interval, b . We interpret b as the amount of information firms have about the location of workers in the unit interval. Without screening, firms are uninformed about the specific abilities of their applicants. Hence, they hire workers by randomly selecting them from the labor supply pool at the common market-clearing wage rate w .⁹ This gives $b = 1$ and $\widehat{dist} = 1/3$.

However, firms do not have to accept this outcome. They can reduce the efficiency loss from mismatch by screening their applicants. Similar to Helpman, Itskhoki, and Redding (2010), we associate the implementation of a screening technology with a fixed cost expenditure $f_\mu = [1 + \mu(\omega)]^\gamma$ and assume that screening provides an imprecise signal about worker ability, with the quality of the signal increasing in screening effort $\mu(\omega)$. To be more specific, by screening with effort $\mu(\omega)$,

⁹Due to symmetry, all workers receive the same wage in equilibrium, irrespective of their location in the ability interval.

a firm can divide the ability interval into $1 + \mu(\omega)$ segments of equal length. Firms can then hire workers at the market-clearing wage rate, w , for a specific task by randomly selecting them from the respective ability segment, so that the average distance between worker-specific abilities and task-specific skill requirements reduces to $\widehat{dist}(\omega) = (1/3) [1 + \mu(\omega)]^{-1}$.¹⁰

At the firm level, efficiency of workers in the performance of tasks is inversely related to $\widehat{dist}(\omega)$ and denoted by $\kappa(\omega)$. In the interest of analytical tractability, we choose a specific functional form and capture the relationship between $\kappa(\omega)$ and $\widehat{dist}(\omega)$ by $\kappa(\omega) \equiv (1/3)\widehat{dist}(\omega)^{-1}$. This gives $\kappa(\omega) = 1 + \mu(\omega)$. Effective labor input at the task level is therefore given by $[1 + \mu(\omega)]l(\omega)$ and, since tasks enter production function (3) symmetrically, total output of firm ω can be written in the following way:

$$x(\omega) = \phi(\omega) [1 + \mu(\omega)] l(\omega). \quad (6)$$

According to (6), firm productivity consists of two parts: an exogenous baseline productivity $\phi(\omega)$, which captures the efficiency of coordinating the bundle of different tasks within the boundaries of the firm, and an endogenous productivity term $\kappa(\omega) = 1 + \mu(\omega)$, which captures how effectively the heterogeneous abilities of workers are used for performing the different tasks in the production process. Crucially, firms can increase their productivity by investing into a screening technology which improves the matching quality in the firm-internal labor allocation process and thus raises $\kappa(\omega)$.¹¹

The baseline productivity is drawn by firms in a lottery from the common Pareto distribution, $G(\phi) = 1 - \phi^{-\nu}$. To participate in this lottery, firms have to pay a fee f_e in units of final output Y . This investment allows just a single draw and is immediately sunk. After productivity levels are revealed, producers decide upon setting up a plant and starting production. This involves an additional fixed cost f (in units of final output) for setting up a local distribution network. Only firms with a sufficiently high baseline productivity will pay this additional fixed cost and start production, while firms with a low ϕ will stay out of the market. This two-stage entry mechanism is similar to Melitz (2003), with two main differences. On the one hand, we consider a static model variant along the lines of Helpman and Itskhoki (2010) and Helpman, Itskhoki, and Redding (2010). On the other hand, firms can install a screening technology for improving the quality of worker-task matches, by making an investment f_μ which is endogenous.

2.2 Equilibrium in the closed economy

After the lottery, the baseline productivity is revealed, and the firm either stays out of the market or it decides to produce, sets its employment level $l(\omega)$ and chooses its screening effort $\mu(\omega)$ to

¹⁰We ignore integer problems and, due to symmetry, suppress task indices.

¹¹This mechanism is not too different, in principle, from an *R&D* investment that lowers variable production costs (see, for instance, Eckel, 2009).

maximize profits

$$\pi(\omega) = p(\omega)x(\omega) - wl(\omega) - [1 + \mu(\omega)]^\gamma - f \quad (7)$$

subject to (2), (6), and a set of common non-negativity constraints. The (interior) solution to this maximization problem is given by the two first-order conditions:

$$\pi_l(\omega) = \frac{\sigma - 1}{\sigma} p(\omega) \phi(\omega) [1 + \mu(\omega)] - w = 0, \quad (8)$$

$$\pi_\mu(\omega) = \frac{\sigma - 1}{\sigma} p(\omega) l(\omega) \phi(\omega) - \gamma [1 + \mu(\omega)]^{\gamma-1} = 0. \quad (9)$$

Being interested in interior solutions, we must ensure that all firms find it attractive to implement a screening technology. Intuitively, this requires that the costs of screening the workforce must be small relative to production fixed costs f . To put it more formally, all firms find it attractive to screen their applicants at least a little bit if $(1 + f)(\sigma - 1) \geq \gamma$. Furthermore, to avoid that (all) firms make an infinitively high investment into screening, the additional costs of further increasing the screening effort must exceed the additional benefits of doing so at high levels of $\mu(\omega)$, which is the case if $\gamma > \sigma - 1$. In the appendix, we derive the two conditions and show that for the respective parameter domain, $\pi(\omega)$ has a unique interior maximum in (l, μ) -space.

With these insights at hand, we can proceed with rewriting first-order condition (8) as follows:

$$p(\omega) = \frac{\sigma}{\sigma - 1} \frac{w}{\phi(\omega) [1 + \mu(\omega)]}. \quad (10)$$

Hence, in line with textbook models of monopolistic competition, firms set prices as a constant markup on marginal costs, which in our setting are inversely related to the firms' screening effort $\mu(\omega)$. First-order condition (9) determines the profit-maximizing screening effort $\mu(\omega)$, and accounting for (6), we can reformulate the respective condition to

$$r(\omega) = \frac{\sigma \gamma}{\sigma - 1} [1 + \mu(\omega)]^\gamma, \quad (11)$$

where $r(\omega) = p(\omega)x(\omega)$ denotes revenues of firm ω . Eq. (11) establishes a positive relationship between firm-level revenues and screening expenditures. Combining (2), (10) and (11), we get:

$$\frac{r(\omega_1)}{r(\omega_2)} = \left(\frac{1 + \mu(\omega_1)}{1 + \mu(\omega_2)} \right)^\gamma, \quad \frac{r(\omega_1)}{r(\omega_2)} = \left(\frac{\phi(\omega_2) [1 + \mu(\omega_2)]}{\phi(\omega_1) [1 + \mu(\omega_1)]} \right)^{1-\sigma}. \quad (12)$$

These two expressions jointly determine relative screening effort and relative revenues of firms 1 and 2 as functions of these firms' baseline productivity ratio. This implies that heterogeneity of the two firms is fully characterized by their baseline productivity differential, and we can therefore use

productivity ϕ to index firms from now on. Hence, we can rewrite (12) in the following way:

$$\frac{1 + \mu(\phi_1)}{1 + \mu(\phi_2)} = \left(\frac{\phi_1}{\phi_2} \right)^{\frac{\sigma-1}{\gamma-\sigma+1}}, \quad \frac{r(\phi_1)}{r(\phi_2)} = \left(\frac{\phi_1}{\phi_2} \right)^{\frac{\gamma(\sigma-1)}{\gamma-\sigma+1}}. \quad (13)$$

Since $\gamma > \sigma - 1$ is a prerequisite for finite screening investment, we can conclude that in an interior equilibrium firms with higher ϕ -levels make higher revenues and choose a higher screening effort. This is well in line with evidence, for example, by Barron, Black, and Loewenstein (1987), who find a positive and significant relation between expenditures in screening workers and employer size. Furthermore, the model is also consistent with the finding that workers are more productive in larger firms (see Idson and Oi, 1999), pointing to the role of better matching quality for explaining this size differential.

To separate active from inactive firms we can characterize a marginal producer, who is indifferent between starting production and remaining inactive. We denote the productivity of this firm by ϕ^* , which we refer to by the term *cutoff productivity level*. The zero-cutoff profit condition, which characterizes this firm, is given by $r(\phi^*)/\sigma = f + [1 + \mu(\phi^*)]^\gamma$. We can combine this indifference condition with (11) to explicitly solve for screening effort and revenues of the marginal producer:

$$1 + \mu(\phi^*) = \left(\frac{f(\sigma-1)}{\gamma-\sigma+1} \right)^{\frac{1}{\gamma}}, \quad r(\phi^*) = \frac{\sigma\gamma f}{\gamma-\sigma+1}. \quad (14)$$

In view of (13) and (14), we can calculate average profits of active producers, $\bar{\pi}$. Defining $\xi \equiv \gamma(\sigma-1)/(\gamma-\sigma+1)$, we obtain¹²

$$\bar{\pi} = \frac{f\xi}{\nu - \xi}, \quad (15)$$

where $\nu > \xi$ is assumed to ensure a finite positive level of $\bar{\pi}$. Furthermore, free entry into the productivity lottery requires that, in equilibrium, the expected return to entry $(1 - G(\phi^*)) \bar{\pi}$ equals the participation fee f_e . Therefore, the free entry condition in our static model reads

$$\bar{\pi} = f_e(\phi^*)^\nu. \quad (16)$$

Together, Eqs. (15) and (16) determine $\bar{\pi}$ and ϕ^* . This completes the characterization of firm-level variables in the closed economy, and we can now turn to studying the main economy-wide variables of interest: welfare and underemployment, arising from the firm-internal mismatch of workers and tasks.

With just a single consumption good, utilitarian welfare equals per-capita income. Since aggregate profits equal total expenditures for the lottery participation fee and the price of final output equals one, according to our choice of numéraire, per-capita income equals wage rate w in our setting. To solve for the wage rate, we can combine $r(\phi^*) = p(\phi^*)x(\phi^*)$ and $Y = Mr(\phi^*)\nu/(\nu - \xi)$.

¹²Derivation details are deferred to the appendix.

Substituting (2) and (10) and accounting for (14)-(16), we can calculate

$$w = \frac{\sigma - 1}{\sigma} \left(\frac{\nu}{\nu - \xi} \right)^{\frac{1}{\sigma-1}} [1 + \mu(\phi^*)] \phi^* = \frac{\sigma - 1}{\sigma} \left(\frac{\nu}{\nu - \xi} \right)^{\frac{1}{\sigma-1}} \left(\frac{f\xi}{f_e(\nu - \xi)} \right)^{\frac{1}{\nu}} \left(\frac{f\xi}{\gamma} \right)^{\frac{1}{\gamma}}. \quad (17)$$

According to (17), our model gives rise to the somewhat counter-intuitive results that an increase in production fixed costs f provides a stimulus for per-capita income (and thus utilitarian welfare). The reason for this outcome is that firm entry exerts a negative externality on the output of incumbent firms, who end up being too small relative to the social optimum. In other models of monopolistic competition, this negative externality is counteracted by a positive externality due to stronger labor division in the production of final output (see Ethier, 1982), and the two externalities exactly offset when applying the technology in Matusz (1996). Final goods production does not give rise to an external scale effect in our setting, and hence the model considered here lacks a positive externality of firm entry, implying that the mass of producers deviates from the social optimum.¹³ Higher production fixed costs imply that firms must be more productive in order to survive in the market. This improves the composition of active producers, which is to the benefit of the representative consumer in our setting.

To obtain an economy-wide measure of mismatch between workers and tasks, we compute the *average* distance between task-specific skill requirements and worker-specific abilities. As formally shown in the appendix, this aggregate measure of mismatch is given by

$$u = \frac{1}{3[1 + \mu(\phi^*)]} \frac{\gamma(\nu - \xi)}{\gamma(\nu - \xi) + \xi} = \frac{1}{3} \left(\frac{\gamma - \sigma + 1}{f(\sigma - 1)} \right)^{\frac{1}{\gamma}} \frac{\gamma(\nu - \xi)}{\gamma(\nu - \xi) + \xi}, \quad (18)$$

where the second equality follows from (14). The existence of underemployment due to a mismatch of abilities and skill-requirements is the main difference between our setting and an otherwise identical Melitz (2003) framework with homogeneous workers and a single-task production technology. The source of underemployment also differs from other models that introduce search frictions into a Melitz framework (see, for instance, Helpman and Itskhoki, 2010; Helpman, Itskhoki, and Redding, 2010; Felbermayr, Prat, and Schmerer, 2011). In our setting, it is not the the existence of recruitment costs *per se* but rather the mismatch of worker-specific abilities and task-specific skill requirements in the production of goods that generates an inefficient allocation of labor and thus underemployment. This completes the analysis of the closed economy.

¹³Combining the labor market clearing condition with the constant markup rule, gives $wL\sigma/(\sigma-1) = Mr(\phi^*)\nu/(\nu-\xi)$, which in view of (14) and (17), can be solved for the mass of firms M :

$$M = \frac{\sigma - 1}{\sigma} \left(\frac{\nu}{\nu - \xi} \right)^{\frac{2-\sigma}{\sigma-1}} \left(\frac{f\xi}{\gamma} \right)^{\frac{1}{\gamma}} \left(\frac{f\xi}{f_e(\nu - \xi)} \right)^{\frac{1}{\nu}} \frac{L}{\xi f}.$$

3 The open economy

3.1 Basic structure and preliminary insights

In this section, we consider trade between two fully symmetric countries, whose economies are as characterized in the previous section. There are no impediments to the international transaction of final goods, whereas exporting of intermediates involves two types of costs: On the one hand, there are fixed costs $f_x > 0$ (in units of final output) for setting up a foreign distribution network and, on the other hand, there are iceberg transport costs, which imply that $\tau > 1$ units of intermediate goods must be shipped in order for one unit to arrive in the foreign economy. Both of these costs are also present in the Melitz (2003) framework and – in combination with the heterogeneity of firms in their baseline productivity levels – they generate self-selection of only the best (most productive) producers into exporting, provided that these costs are sufficiently high. However, the decision to start exporting is more sophisticated in our setting, because it influences a firm's optimal choice of screening effort and thus exerts a feedback effect on profits attainable in the domestic market. Hence, there is an interdependence between the decision to export and a firm's performance in its domestic market, which does not exist in Melitz (2003). Due to this interdependence, we have to distinguish between variables referring to exporters (denoted by superscript e) and non-exporters (denoted by superscript n). Furthermore, we use subscript x to refer to variables associated with foreign market sales of an exporter, while domestic variables are index free.

Holding economy-wide variables constant, access to exporting does not affect a non-exporter's profit-maximizing choice of $l(\phi)$ and $\mu(\phi)$ as characterized by (8) and (9). Things are different for an exporter, who realizes revenues $r^e(\phi)$ and $r_x^e(\phi) = \tau^{1-\sigma} r^e(\phi)$ in the domestic and foreign market, respectively, implying that in the open economy this firm's profit-maximizing choice of $\mu(\phi)$ is given by

$$(1 + \tau^{1-\sigma}) r^e(\phi) = \frac{\sigma\gamma}{\sigma - 1} [1 + \mu^e(\phi)]^\gamma \quad (19)$$

instead of (11). However, since condition (19) is structurally the same for all exporters, we can conclude that the ratio of screening effort and the ratio of total revenues in (13) remain unaffected in the open economy, when comparing two firms of the same export status (n or e) but differing productivity levels. In contrast, when comparing two firms with the same baseline productivity but differing export status, we obtain

$$\frac{1 + \mu^e(\phi)}{1 + \mu^n(\phi)} = (1 + \tau^{1-\sigma})^{\frac{1}{\gamma - \sigma + 1}} \quad \frac{r^e(\phi)}{r^n(\phi)} = (1 + \tau^{1-\sigma})^{\frac{\sigma - 1}{\gamma - \sigma + 1}}. \quad (20)$$

From the analysis of the closed economy we know that a firm's screening effort increases with its revenues. Since, all other things equal, exporting generates additional revenues from sales to foreign consumers, it renders screening more attractive, resulting in $\mu^e(\phi) > \mu^n(\phi)$. On the other hand, the higher screening effort under exporting improves the quality of worker-task matches and thus lowers unit production costs. This stimulates sales in both the domestic and the foreign market,

implying $r^e(\phi) > r^n(\phi)$ in Eq. (20). Hence, there is a positive feedback effect of exporting on domestic revenues, which raises the incentives of firms to serve foreign consumers.

Despite the additional complexity arising from the feedback effect that a firm's exporting decision exerts on its domestic profits, our model preserves key properties of the Melitz (2003) model, regarding the partitioning of firms by export status. To see this, we can make use of (11), (13), (14), (19), and (20) and write a firm's profit gain from exporting, $\Delta\pi(\phi) \equiv \pi^e(\phi) - \pi^n(\phi)$, as follows:

$$\Delta\pi(\phi) = \left[(1 + \tau^{1-\sigma})^{\frac{\xi}{\sigma-1}} - 1 \right] \left(\frac{\phi}{\phi^*} \right)^\xi f - f_x. \quad (21)$$

The profit differential in (21) increases in ϕ , and we can thus conclude that if the two trade cost parameters, f_x and τ , are sufficiently high, there is self-selection of only the most productive firms into exporting as in other applications of the Melitz model. This is the case we are focussing on in this paper, and we can therefore characterize a firm that is indifferent between exporting and non-exporting: $\Delta\pi(\phi) = 0$. We denote the (cutoff) productivity of this firm by ϕ_x^* , implying that firms with $\phi > \phi_x^*$ end up being exporters, while firms with $\phi < \phi_x^*$ end up being non-exporters. Solving $\Delta\pi(\phi_x^*) = 0$ for the ratio between the two productivity cutoffs ϕ_x^* and ϕ^* , we obtain

$$\frac{\phi_x^*}{\phi^*} = \left(\frac{f_x/f}{(1 + \tau^{1-\sigma})^{\frac{\xi}{\sigma-1}} - 1} \right)^{\frac{1}{\xi}}, \quad (22)$$

and there is partitioning of firms by export status if $\phi_x^*/\phi^* > 1$. Furthermore, we can use the productivity ratio in (22) to calculate the share of exporting firms in the open economy: $\chi \equiv [1 - G(\phi_x^*)]/[1 - G(\phi^*)] = (\phi_x^*/\phi^*)^{-\nu}$, and thus

$$\chi = \left\{ \frac{f}{f_x} \left[(1 + \tau^{1-\sigma})^{\frac{\xi}{\sigma-1}} - 1 \right] \right\}^{\frac{\nu}{\xi}}. \quad (23)$$

From (22) and (23), we can conclude that higher trade cost costs, i.e. a higher fixed exporting cost f_x or a higher iceberg transport cost parameter τ , raise the minimum productivity level that is necessary to render exporting an attractive choice, thereby lowering the share of exporters in the total population of active firms, χ . With this insights at hand, we are now equipped to solve for the open economy equilibrium.

3.2 The open economy equilibrium

The equilibrium in the open economy is characterized by a two-stage entry mechanism that is similar to the closed economy, but additionally involves the decision to start exporting or to sell exclusively to the domestic market (at stage 2). Access to the export market raises profits of the most productive producers, and this provides a stimulus for the average profit of active firms, which

in the open economy are given by¹⁴

$$\bar{\pi} = \frac{f\xi}{\nu - \xi} \left(1 + \chi \frac{f_x}{f} \right) \quad (24)$$

instead of (15). Combining Eq. (24) with the free entry condition in (16), we can calculate cutoff productivity ϕ^* relative to its closed economy counterpart (subscript a): $\phi^*/\phi_a^* = (1 + \chi f_x/f)^{1/\nu}$. Hence, opening up to trade with a symmetric partner country leads to an upward shift in the cutoff productivity level ϕ^* . The mechanism behind this effect is well understood from Melitz (2003). Access to exporting generates additional demand for labor, and hence firms at the lower bound of the productivity distribution have to leave the market in order to restore the labor market equilibrium. This points to an important asymmetry of how firms are affected by trade liberalization in a Melitz model. While the most productive firms experience a profit gain due to access to the export market, the least productive producers experience a profit loss due to stronger competition for scarce labor in the open economy.

To shed further light on the asymmetry in the firm-level response to trade, we can study how producers adjust their internal labor market in the open economy. We start with a closer look on non-exporting firms. Provided that the marginal firm in the market is not exporting, its screening effort remains to be given by (14). However, the new marginal producer has a higher baseline productivity than the marginal producer in the closed economy, and hence its screening effort is definitely lower than under autarky. Furthermore, since the link between the ratio of screening effort and the ratio of baseline productivities among non-exporting firms remains to be given by (13), it is clear that all non-exporting firms respond to the trade shock with a reduction in their screening effort. This is intuitive, as the sales level of non-exporting firms declines in the open economy, so that these firms are not willing to keep the (relatively) expensive screening technology they have installed in the closed economy. Contrasting the screening effort of a non-exporter in the closed and the open economy, we can compute:

$$\frac{1 + \mu^n(\phi)}{1 + \mu^a(\phi)} = \left(\frac{1}{1 + \chi f_x/f} \right)^{\frac{\xi}{\gamma\nu}} < 1. \quad (25)$$

Calculating the screening differential for an exporting firm, we obtain

$$\frac{1 + \mu^e(\phi)}{1 + \mu^a(\phi)} = \left(\frac{(1 + \tau^{1-\sigma})^{\frac{\nu}{\sigma-1}}}{1 + \chi f_x/f} \right)^{\frac{\xi}{\gamma\nu}} = \left(\frac{(1 + \chi^{\xi/\nu} f_x/f)^{\frac{\nu}{\xi}}}{1 + \chi f_x/f} \right)^{\frac{\xi}{\gamma\nu}}, \quad (26)$$

where the second equality follows from Eq. (23). Noting that $\nu > \xi$ holds by assumption, it is straightforward to show that $\mu^e(\phi) > \mu^a(\phi)$: A firm that starts exporting in the open economy realizes higher revenues and thus raises its screening effort relative to autarky. The differential impact of trade on screening in non-exporting and exporting firms is graphically depicted by Figure

¹⁴Derivation details are deferred to the appendix.

1 and summarized in Proposition 1.¹⁵

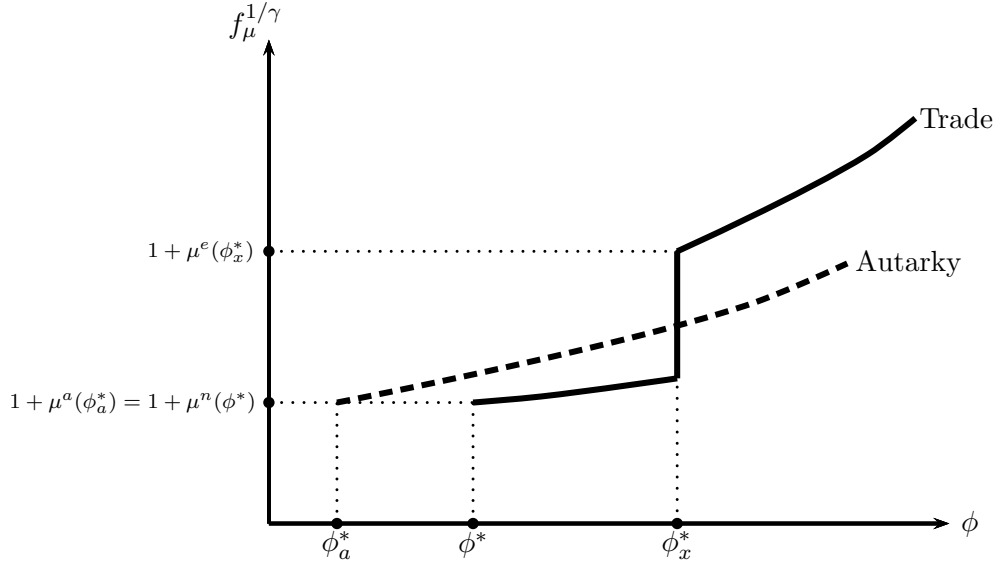


Figure 1: The impact of trade on firm-level screening effort

Proposition 1 *A country's opening up to trade, leads to an asymmetric response in the firm-internal allocation of workers to tasks. Whereas exporters expand their screening effort and thus improve the quality of worker-task matches, non-exporters reduce their screening effort and accept a larger mismatch between skill requirements and abilities in the performance of tasks.*

Proof. Analysis in the text. ■

Due to asymmetric firm-level consequences, it is clear that access to trade exerts counteracting effects on the general equilibrium variables of interest: wage rate (welfare) w and underemployment u . Similar to the autarky scenario, the wage rate in the open economy, can be derived by combining $r(\phi^*) = p(\phi^*)x(\phi^*)$ with the adding up condition $Y = M(1 + \chi f_x/f)r(\phi^*)\nu/(\nu - \xi)$. Substituting (2) and (10) – with $M(1 + \chi)$ presuming the role of M in the open economy – and accounting for (14), (16), and (24), we can calculate

$$w = \left(\frac{1 + \chi f_x/f}{1 + \chi} \right)^{\frac{1}{\sigma-1}} \left(1 + \chi \frac{f_x}{f} \right)^{\frac{1}{\nu}} w_a. \quad (27)$$

Hence, gains from trade are guaranteed if $f_x/f \geq 1$, while losses from trade cannot be ruled out if $f_x/f < 1$.¹⁶ Trade can be welfare-deteriorating in our setting, because under production technology (1) the outcome of decentralized firm entry is not socially optimal. To the extent that

¹⁵For illustrative purposes, we have assumed $\xi > \gamma$, whereas in general $\xi >, =, < \gamma$ is possible.

¹⁶For instance, with a parametrization of $\nu = 8$, $\sigma = 3$, $\tau = 1.5$, and $\gamma = 10$, there are losses from trade if $f_x/f \leq 0.77$ – with $f_x/f \geq 0.58$ establishing selection of only the most productive firms into export status, i.e. $\chi \in (0, 1)$.

trade aggravates the distortion of firm entry, the resulting welfare loss may outweigh the welfare stimulus from market integration (cf. Shy, 1988). In our setting, the existence of *net* gains from trade depends on the relative strength of two selection effects. On the one hand, there is selection of the best producers into exporting, which raises labor demand *ceteris paribus*. On the other hand, there is selection of the least productive firms out of the market, which lowers labor demand. The two selection effects are interdependent and their relative strength depends on fixed cost ratio f_x/f . If this fixed costs ratio is sufficiently high, it is the selection into exporting that dominates rendering the overall effect of trade on labor demand and thus welfare positive.

As outlined in Proposition 1, there are asymmetric firm-level effects of trade on the mismatch between abilities and skill requirements. Exporting firms increase their screening expenditures, and hence their matching outcome is improved. The opposite is true in non-exporting firms. However, there is an additional positive effect on economy-wide underemployment because labor is relocated towards exporting firms in the open economy and, due to this change in labor composition, the overall impact of trade on the average quality worker-task matches is positive. To see this, we can explicitly solve for our measure of underemployment in the open economy. As formally shown in the appendix, we get:

$$u = \frac{1 + a(\tau)\chi^{1+\xi/(\nu\gamma)}f_x/f}{1 + \chi f_x/f}u^a, \quad \text{with} \quad a(\tau) \equiv \frac{(1 + \tau^{1-\sigma})^{\frac{(\gamma-1)\xi}{\gamma(\sigma-1)}} - 1}{(1 + \tau^{1-\sigma})^{\frac{\xi}{\sigma-1}} - 1}. \quad (28)$$

Noting that $a(\tau) < 1$, it is immediate that $u < u_a$, which proves that trade reduces the *average* mismatch between task-specific skill requirements and worker-specific abilities, thereby lowering underemployment.

We can summarize the main insights from our analysis as follows.

Proposition 2 *Opening up to trade improves the average quality of worker-task matches, thereby reducing economy-wide underemployment due to a misallocation of workers to tasks. The impact of trade on welfare is not clear-cut in general. Only if fixed costs of exporting relative to production fixed costs f_x/f are sufficiently high, there are gains from trade in our setting.*

Proof. Analysis in the text. ■

We complete the analysis in this section by shedding light on the consequences of a marginal reduction in transport cost parameter τ . Such a decline increases expected income from exporting, and thus raises χ , according to (23), as well as average profit income $\bar{\pi}$, according to (24). On the other hand, there is a stimulus on labor demand, which enforces additional market exit at the lower bound of the productivity distribution and therefore leads to an upward shift in cutoff productivity ϕ^* . Furthermore, a marginal decline in the iceberg transport cost parameter augments the heterogeneity in screening effort between non-exporting and exporting producers, according to (20). With respect to adjustments in the wage rate, we can infer from (27) that $dw/d\tau < 0$ if $f_x/f > 1$. In this case, a gradual reduction in the iceberg transport cost parameter exerts a positive monotonic impact on welfare. In contrast, if $f_x/f < 1$, changes in τ need not exert a

monotonic impact on w . Finally, from the analysis above we know that a country's movement from autarky to trade with an arbitrary transport cost level unambiguously improves the average quality of worker-task matches. We can therefore safely conclude that a marginal decline in τ must lower u if transport costs have been large initially. In the appendix we show that this effect extends to the case where τ has already been low prior to the fall in the iceberg transport cost parameter, so that a gradual decline in τ reduces underemployment u monotonically.

4 A model variant with involuntary unemployment

In this section, we introduce search frictions as an additional source of inefficiency in the allocation of labor to show how mismatch between the abilities of workers and the skill requirements of tasks impact traditional forms of underemployment. For this purpose, we consider a competitive search model along the lines of Rogerson, Shimer, and Wright (2005), in which firms post wages and workers direct their search to the most attractive employer to queue for a job, there.¹⁷ The mass of matches between workers and jobs, m , depends positively on the number of applicants, s , and the number of open vacancies, v . In the interest of analytical tractability, we choose a Cobb-Douglas specification and write $m(s, v) = As^{1-\zeta}v^\zeta$, where $\zeta, A \in (0, 1)$ are the same for all producers.¹⁸ Measuring by $q \equiv s/v$ the queue length of workers applying for jobs, the probability of the firm to fill a specific vacancy is given by $\alpha_e(q) \equiv m(s, v)/v = Aq^{1-\zeta}$. In our static model, this equals the share of vacancies filled in the respective firm. The probability of a worker to be hired, when queuing for a job, is given by $\alpha_w(q) \equiv m(s, v)/s = Aq^{-\zeta}$. In the subsequent analysis we focus on interior solutions with $\alpha_e(q), \alpha_w(q) \in (0, 1)$. For which parameter domain such an interior solution is realized will be discussed below.

Setting unemployment compensation equal to zero and denoting by V the highest income a worker can expect when applying for a job at a different firm, queuing for vacancies in a firm with productivity ϕ is only attractive for the worker if $V \leq \alpha_w[q(\phi)]w(\phi)$. Since firms set the same wage for all workers in our setting (see above), additional workers apply for jobs in this firm as long as the inequality is strict. This lowers the probability of being hired by the firm, $\alpha_w[q(\phi)]$, and the adjustment process continues until the expected return of workers is the same in all active firms. Hence, $V = Aq(\phi)^{-\zeta}w(\phi)$ must hold in equilibrium, and the directed search mechanism therefore

¹⁷Rogerson, Shimer, and Wright (2005) provide an excellent overview of different search-theoretic approaches, their main advantages and disadvantages. In the context of heterogeneous firms, a competitive search model has also been considered by Ritter (2011) and Felbermayr, Impulliti, and Prat (2012).

¹⁸In a competitive search model it is not necessary to choose an ad hoc specification of the matching function. Instead, one can as well take the coordination problem of directed search seriously and provide a clean microfoundation of this problem by choosing an urn-ball matching function (see Peters, 1991, for an early contribution and King and Stähler, 2010, for an application in the context of trade). A disadvantage of this more advanced approach is its lower analytical tractability, and we therefore prefer treating the matching function as a black box as it is still common in the literature.

establishes a positive link between queue length $q(\phi)$ and the posted wage $w(\phi)$:

$$w(\phi) = \frac{q(\phi)^\zeta V}{A}. \quad (29)$$

The mass of vacancies set up by a firm with productivity ϕ , $v(\phi)$, is linked to this firm's employment level, $l(\phi)$, according to $v(\phi) = l(\phi)/[Aq(\phi)^{1-\zeta}]$. The costs of installing and advertising a vacancy are measured in units of final output and are given by $k > 0$.

With these insights at hand, we can write firm-level profits in the closed economy as follows:

$$\pi(\phi) = p(\phi)x(\phi) - \frac{q(\phi)^\zeta V}{A}l(\phi) - [1 + \mu(\phi)]^\gamma - f - \frac{kl(\phi)}{Aq(\phi)^{1-\zeta}}. \quad (30)$$

The firm sets $l(\phi)$, $q(\phi)$ and $\mu(\phi)$ simultaneously to maximize profits (30) subject to (2), (6) and a set of non-negativity constraints. The (interior) solution to this maximization problem is characterized by the following three first-order conditions:

$$\pi_l(\phi) = \frac{\sigma-1}{\sigma}p(\phi)\phi[1 + \mu(\phi)] - w(\phi) - \frac{k}{Aq(\phi)^{1-\zeta}} = 0, \quad (31)$$

$$\pi_q(\phi) = -\frac{l(\phi)\zeta q(\phi)^{\zeta-1}V}{A} + (1-\zeta)l(\phi)\frac{k}{Aq(\phi)^{2-\zeta}} = 0, \quad (32)$$

$$\pi_\mu(\phi) = \frac{\sigma-1}{\sigma}p(\phi)l(\phi)\phi - \gamma[1 + \mu(\phi)]^{\gamma-1} = 0. \quad (33)$$

Equations (29) and (32) jointly determine

$$q(\phi) = \frac{(1-\zeta)k}{\zeta V} \equiv q, \quad w(\phi) = \frac{(1-\zeta)k}{\zeta Aq^{1-\zeta}} \equiv w, \quad (34)$$

implying that all firms pay the same wage, irrespective of the prevailing productivity differences. This outcome is in line with models of random matching between workers and heterogenous firms, in which wages are determined by individual Nash bargaining. For instance, Felbermayr and Prat (2011, p. 286) point out that in their setting all firms pay the same wage, because “multiple-worker firms exploit their monopsony power until employees are paid their outside option [that] is constant across firms because it depends solely on aggregate outcomes.” This gives a prominent role to over-hiring in models with individual wage bargaining, which, however, is not present under wage posting. Instead, in our model the finding of a uniform wage level is a consequence of three model ingredients: linear hiring costs, the same outside option of workers with differing abilities, and the isoelastic demand structure.¹⁹

¹⁹There are different possibilities to modify the model such that it gives rise to the empirically well-documented pattern that larger, more productive firms pay higher wages. For instance, one could consider convex instead of linear recruitment costs, as suggested by Helpman and Itskhoki (2010). Alternatively, one could modify the wage setting process and assume that firms post fair wages, as in Egger and Kreickemeier (2009, 2012) and Amiti and Davis (2012). Finally, one could also give up the symmetry of firm-worker matches and instead assume that ability is firm-specific and employers can learn about this ability during the recruitment process by installing a screening technology, as suggested by Helpman, Itskhoki, and Redding (2010). While all of these modifications would allow for firm-specific wage payments, the costs of these extensions in terms of analytical tractability would be enormous, and we therefore

Combining (31) and (34) gives the modified price-markup rule

$$p(\phi) = \frac{\sigma}{(\sigma - 1)\phi[1 + \mu(\phi)]} \frac{k}{\zeta A q^{1-\zeta}}, \quad (35)$$

where marginal labor costs are augmented by recruitment expenditures. According to (33), revenues of a firm with productivity ϕ are still given by (11) and are thus the same as in the baseline model. This implies that the zero-cutoff profit condition remains to be given by (14), and hence neither $\bar{\pi}$ nor ϕ^* depend on the prevailing search frictions or the costs of establishing and posting vacancies, k .

With the firm-level variables at hand, we can now solve for the general equilibrium outcome in the closed economy. For this purpose, we first look at queue length q . Substituting (2) into $r(\phi^*) = p(\phi^*)x(\phi^*)$ and accounting for $Y = Mr(\phi^*)\nu/(\nu - \xi)$ gives $p(\phi^*)^{\sigma-1} = \nu/(\nu - \xi)$. Using (35) and noting that $\phi^*[1 + \mu(\phi^*)] = [(f\xi)/\gamma]^{1/\gamma} \{(f\xi)/[f_e(\nu - \xi)]\}^{1/\nu}$ follows from (14)-(16), we can derive

$$q = \left\{ \frac{k\sigma}{A\zeta(\sigma - 1)} \left(\frac{\nu - \xi}{\nu} \right)^{\frac{1}{\sigma-1}} \left(\frac{f_e}{f} \frac{\nu - \xi}{\xi} \right)^{\frac{1}{\nu}} \left(\frac{\gamma}{f\xi} \right)^{\frac{1}{\gamma}} \right\}^{\frac{1}{1-\zeta}}. \quad (36)$$

To solve for economy-wide unemployment \hat{u} , we can substitute $V = (1 - \hat{u})w$ into (29). Rearranging terms, yields $1 - \hat{u} = Aq^{-\zeta}$, which establishes the intuitive result that a larger queue length at individual firms leads to higher economy-wide unemployment. Accounting for q from (36), we can compute

$$1 - \hat{u} = \left\{ \frac{\zeta(\sigma - 1)}{k\sigma} \left(\frac{\nu}{\nu - \xi} \right)^{\frac{1}{\sigma-1}} \left(\frac{f}{f_e} \frac{\xi}{\nu - \xi} \right)^{\frac{1}{\nu}} \left(\frac{f\xi}{\gamma} \right)^{\frac{1}{\gamma}} \right\}^{\frac{\zeta}{1-\zeta}} A^{\frac{1}{1-\zeta}}. \quad (37)$$

Eq. (37) characterizes involuntary unemployment as one important aspect of *underemployment* and measures the efficiency loss due to search frictions. However, it does not capture the efficiency loss, arising from a mismatch between workers and tasks in the firm-internal allocation of labor. This form of underemployment can be measured by the average distance between task-specific skill requirements and worker-specific abilities and is represented by u . Crucially, the existence of search frictions does not impact the firm-internal labor allocation, and hence u remains to be given by (18) in the closed economy. This invariance result only holds in one direction. Changes in the firm-internal allocation of labor alter $1 + \mu(\phi^*) = (f\xi/\gamma)^{1/\gamma}$, and hence q and $1 - \hat{u}$, according to (36) and (37).²⁰

decided to stick to the more parsimonious model variant without wage differentiation.

²⁰Eqs. (36) and (37) can also be used for characterizing the parameter domain that establishes an interior solution with $\alpha_e(q), \alpha_w(q) \in (0, 1)$. More specifically, we can conclude that $\alpha_e(q) = Aq^{1-\zeta} < 1$ and $\alpha_w(q) = Aq^{-\zeta} < 1$ simultaneously hold if

$$A^{\frac{1}{\zeta}} < \frac{k\sigma}{\zeta(\sigma - 1)} \left(\frac{\nu - \xi}{\nu} \right)^{\frac{1}{\sigma-1}} \left(\frac{f_e}{f} \frac{\nu - \xi}{\xi} \right)^{\frac{1}{\nu}} \left(\frac{\gamma}{f\xi} \right)^{\frac{1}{\gamma}} < 1,$$

Finally, welfare in the closed economy is given by $(1 - \hat{u})w$, which, in view of (34), (36) and (37), can be expressed as

$$(1 - \hat{u})w = \frac{1 - \zeta}{\zeta} \frac{k}{q} = (1 - \zeta)A^{\frac{1}{1-\zeta}} \left(\frac{\zeta}{k}\right)^{\frac{\zeta}{1-\zeta}} \tilde{w}^{\frac{1}{1-\zeta}} = (1 - \zeta)(1 - \hat{u})\tilde{w}, \quad (38)$$

where \tilde{w} equals the wage rate in the benchmark model with a perfect labor market, given by (17). From (38) it is obvious that the existence of search frictions reduces per capita labor income and thus welfare in our setting. This completes the discussion of the closed economy.

We now turn to the open economy and shed light on the effects of trade for the two sources of underemployment. Thereby, we impose the same assumptions as in the baseline model and consider two symmetric countries, iceberg transport costs for shipping intermediate goods across borders and fixed exporting costs to generate selection of only the best firms into export status. With these assumptions at hand, we can now repeat the analysis of the closed economy step by step in order to derive the main variables of interest for the open economy. However, since the respective calculations are straightforward, we leave them to the interested reader and only summarize the main results from this analysis, here. From the closed economy, we know that the existence of labor market imperfection does not affect the allocation of workers to tasks, and hence our insights regarding the consequences of trade for the firm-internal mismatch remains unaffected by adding a search friction. This implies that the open economy level of u remains to be given by (28).

Furthermore, it is easily confirmed that the existence of a search friction does not alter Eqs. (19)-(21), therefore leaving the exporting decision unaffected. As a consequence, the share of exporting firms remains to be given by (23). Noting from (38) that per capita labor income in the more sophisticated model variant with search frictions is a convex function of the wage rate in the benchmark model with a perfect labor market, we can infer the welfare effects of trade by considering Eq. (27). To more specific, we can write

$$\frac{(1 - \hat{u})w}{(1 - \hat{u}^a)w^a} = \left(\frac{\tilde{w}}{\tilde{w}^a}\right)^{\frac{1}{1-\zeta}} = \left[\left(\frac{1 + \chi f_x/f}{1 + \chi}\right)^{\frac{1}{\sigma-1}} \left(1 + \chi \frac{f_x}{f}\right)^{\frac{1}{\nu}}\right]^{\frac{1}{1-\zeta}}. \quad (39)$$

Hence, the existence of search frictions does not change the welfare effects of trade in a qualitative way, but it magnifies the (positive or negative) welfare implications identified in Section 3. To understand, where the additional welfare effect comes from, it is worth noting that we can write

$$\frac{w}{w^a} = \left(\frac{q}{q^a}\right)^{\zeta-1} = \left(\frac{1 + \chi f_x/f}{1 + \chi}\right)^{\frac{1}{\sigma-1}} \left(1 + \chi \frac{f_x}{f}\right)^{\frac{1}{\nu}}, \quad (40)$$

according to (34) and (39). From (27) and (40) it follows that in the presence of search frictions the wage adjustments triggered by trade are of equal magnitude as in the benchmark model with a perfectly competitive labor market. Therefore, any additional welfare effect must come from

while the two probabilities are positive if $\zeta, k, A > 0$ (and $\nu > \xi$ as previously assumed).

adjustments in the employment rate. Accounting for

$$\frac{1 - \hat{u}}{1 - \hat{u}^a} = \left(\frac{q}{q^a} \right)^{-\zeta} = \left(\frac{(1 - \hat{u})w}{(1 - \hat{u}^a)w^a} \right)^\zeta = \left[\left(\frac{1 + \chi f_x/f}{1 + \chi} \right)^{\frac{1}{\sigma-1}} \left(1 + \chi \frac{f_x}{f} \right)^{\frac{1}{\nu}} \right]^{\frac{\zeta}{1-\zeta}}, \quad (41)$$

provides support for this conclusion. Eqs. (39)-(41) show that there is a direct link between employment, wage and welfare effects of trade in our setting. From Section 3 we know that lacking an external scale effect in the production of final goods, selection of exporters must be sufficiently strong in order for trade to provide a stimulus on aggregate labor demand and equilibrium wages. In this case, the price of the final good falls relative to the wage rate. This lowers the costs of installing and advertising vacancies relative to the costs of compensating workers, and thus alleviates the search friction with positive consequences for aggregate employment. Both of these effects contribute to a welfare gain. Things are different if selection effects are weak. In this case, it is possible that labor demand is dampened in the open economy, so that wages decline. However, if wages decline relative to the price of the final good, the establishment of new vacancies becomes less attractive, rendering the search friction more severe than under autarky, with adverse effects on economy-wide employment.

The following proposition summarizes the main insights from the analysis in this section.

Proposition 3 *The existence of search frictions does not alter our insights from the benchmark model regarding the impact of trade on the mismatch of workers and task in the firm-internal labor market. Furthermore, with search frictions, trade triggers wage and employment effects that go into the same direction. As a consequence, the welfare implications of trade, while not altered qualitatively, are reinforced in the model variant with search frictions.*

Proof. Analysis in the text. ■

We complete the discussion in this section by having a closer look on the specific role played by adjustments in the firm-internal allocation of workers for the impact of trade on welfare and economy-wide unemployment. For this purpose, it is worth noting that our model degenerates to one without screening if $\gamma \rightarrow \infty$. We can therefore infer insights upon the role played by the firm-internal labor allocation from differentiating (39)-(41) with respect to γ . More specifically, we can determine how changes in γ alter the employment and welfare effects of trade, by studying the sign of

$$\frac{d(w/w^a)}{d\gamma} = \frac{d(w/w^a)}{d\chi} \frac{d\chi}{d\gamma}. \quad (42)$$

Differentiating (23) with respect to γ gives

$$\frac{d\chi}{d\gamma} = -\frac{\nu\chi}{\gamma} \left\{ \frac{1}{\gamma - \sigma + 1} \frac{(1 + \tau^{1-\sigma})^{\frac{\xi}{\sigma-1}}}{(1 + \tau^{1-\sigma})^{\frac{\xi}{\sigma-1}} - 1} \ln(1 + \tau^{1-\sigma}) - \frac{1}{\gamma} \ln\left(\chi^{\frac{\xi}{\nu}}\right) \right\} < 0. \quad (43)$$

A higher γ implies that fixed costs are more responsive to changes in the screening effort. Accordingly, firms will adjust their screening effort less strongly when facing the opportunity of exporting, so that the fixed cost increase due to exporting is less pronounced, and hence the share of exporters increases ceteris paribus if γ goes up. On the other hand, the now lower wedge of screening effort eats up part of the productivity advantage of exporters relative to non-exporters, thereby lowering the incentives of firms to sell abroad. In our model, it is the second effect that dominates, so that a higher γ reinforces self-selection into exporting, and therefore implies a smaller share of exporting firms χ .

Furthermore, differentiating (40) with respect to χ yields

$$\frac{d(w/w^a)}{d\chi} = \frac{w/w^a}{\nu(1 + \chi f_x/f)(1 + \chi)} \left[\frac{\nu}{\sigma - 1} \left(\frac{f_x}{f} - 1 \right) + (1 + \chi) \frac{f_x}{f} \right]. \quad (44)$$

It is easily confirmed that the bracket term on the right-hand side of (44) is increasing in f_x/f , and hence wages increase monotonically in the share of exporting firms if f_x/f (and thus the selection effect) is sufficiently large. In line with our insights from Section 3, $f_x/f \geq 1$ is sufficient (not necessary) for a monotonically positive impact of an increase in χ on w/w^a . If such a monotonic effect exists, an increase in γ unambiguously lowers the positive wage, employment, and welfare effects of trade, and hence positive economy-wide effects would be underestimated if one ignores endogenous adjustments in the firm-internal allocation of workers to tasks. However, if the impact of a higher χ on w/w^a is non-monotonic, things are even more worrying, because in this case ignoring endogenous adjustments in the way workers are assigned to tasks may give wrong predictions regarding the existence of positive wage, employment and welfare effects of trade. The following proposition summarizes these results.

Proposition 4 *The ability of firms to adjust the quality of worker-task matches leads to weaker selection of firms into exporting, and thus a larger share of exporting firms. Provided that an increase in the share of exporting firms exhibits a positive monotonic impact on wages, adjustments in the firm-internal allocation of workers to tasks therefore strengthen the employment and welfare stimulus relative to a model where such adjustments do not exist. If the relationship between the share of exporting firms and wages is non-monotonic, adjustments in the firm-internal allocation of labor may reverse the employment and welfare effects of trade.*

Proof. Analysis in the text. ■

5 A calibration exercise

In this subsection, we aim at quantifying the effects of trade in our setting. For this purpose, we calibrate our model, using parameter estimates from the literature. A first set of useful parameter estimates is provided by Egger, Egger, and Kreickemeier (2011). Egger, Egger, and Kreickemeier (2011) structurally estimate the main parameters of a trade model with heterogeneous firms and labor market imperfections due to a fair-wage effort mechanism, using firm-level data from five

European countries – Bosnia and Herzegovina, Croatia, France, Serbia, and Slovenia – for the period 2000 to 2008. For our calibration exercise, we consider the parameter estimates for France, which hosts the majority of firms in the respective data-set. A first parameter available from the empirical application in Egger, Egger, and Kreickemeier (2011) is the elasticity of substitution, for which they report a value of $\sigma = 6.7$. This estimate is similar to other findings in the literature (see, for instance, Broda and Weinstein, 2006). Furthermore, using the structural relationship between revenues of exporting firms, Egger, Egger, and Kreickemeier (2011) estimate an analogon to ξ/ν , for which they report a value of 0.87, when relying on information for French firms. This is fairly close to the estimate of 0.83 reported by Arkolakis and Muendler (2010) for Brazilian firms.

Unfortunately, there are no direct estimates available for γ , and we are therefore not able to calculate the parameter values for γ and ν separately. However, from the formal discussion in Section 2 we can infer that existence of an interior solution requires a sufficiently high level of ν . With $\xi/\nu = 0.87$, ν must be larger than 6.5 in our calibration exercise. Since we cannot further confine the possible parameter values, we consider three parameter values are in line with this constraint and choose $\nu = 7$, $\nu = 9$ and $\nu = 11$ for our calibration exercise.²¹ Taking account of $\sigma = 6.7$ and $\xi/\nu = 0.87$, we can then calculate the corresponding γ -levels: $\gamma = 89.01$ for $\nu = 7$, $\gamma = 20.95$ for $\nu = 9$ and $\gamma = 14.10$ for $\nu = 11$.

A further variable of interest is the share of exporters, χ . Eaton, Kortum, and Kramarz (2011) report from official administrative statistics that 15 percent of French manufacturing firms were exporters in 1986. Egger, Egger, and Kreickemeier (2011) find a significantly larger share of exporters, using the Amadeus data-set. According to their data-base, 45 percent of French firms did export in the average year between 2000 and 2008. Since it is well known that the Amadeus data is biased towards large, incorporated firms, we consider the evidence provided by Eaton, Kortum, and Kramarz (2011) to be more reliable and accordingly set $\chi = 0.15$ in our calibration exercise. Recent empirical research aims at estimating compulsory measures of the iceberg trade cost parameter τ by employing information on observed international trade flows into a structural gravity equation. Existing results from this literature suggest setting $\tau = 1.5$ (see, for instance, McGowan and Milner, 2011; Novy, 2012). With the iceberg trade cost parameter and the share of exporters at hand, we can then compute a theory-consistent value of fixed cost ratio f_x/f . Using the parameter values from above, we obtain $f_x/f = 0.93$ if $\nu = 7$, $f_x/f = 0.96$ if $\nu = 9$, and $f_x/f = 0.98$ if $\nu = 11$. Finally, we follow common practice in the search literature and set $\zeta = 0.5$ (see Petrongolo and Pissarides, 2001, for supportive empirical evidence).

Table 1 summarizes the main insights from our calibration exercise and reports employment and welfare effects associated with a movement of France from autarky to its observed degree of openness: $\chi = 0.15$. From column 4 we see that gains from trade seem to be rather small in our setting, which at least partly may be explained by the absence of external scale economies in the

²¹These ν -values are well in line with shape parameters of the productivity distribution applied in other numerical applications of the Melitz (2003) model. For instance, Arkolakis and Muendler (2010) consider 5 and 8 as low and high values for the shape parameter, whereas Felbermayr and Prat (2011) consider a value of 9.23.

Table 1: Quantifying the impact of trade on welfare and underemployment

Parameter values			Changes in percent		
ν	γ	f_x/f	$\Delta(1-u)w$	$\Delta(1-\hat{u})$	Δu
7	89.01	0.93	3.45	1.71	-0.37
9	20.95	0.96	2.81	1.39	-1.54
11	14.10	0.98	2.45	1.22	-2.29

Notes: An exporter share of $\chi = 0.15$, an iceberg trade cost parameter of $\tau = 1.5$, a σ -value of 6.7, and a parameter ratio $\xi/\nu = 0.87$ have been considered for computing the figures in this table.

production of final goods. However, the welfare gains documented in Table 1 are in the range of welfare effects reported by Eaton and Kortum (2002), who set up a multi-country Ricardian model for 19 OECD economies and investigate how much countries in their data-set would lose if trade were entirely abolished. They compute losses ranging from 0.2 percent for Japan and 10.3 percent for Belgium, and for France they report a welfare loss of 2.5 percent. This is fairly close to the welfare loss from abolishing trade entirely when setting $\nu = 11$ in our model, which amounts to 2.4 percent.

The effects of trade on economy-wide employment are reported in Column 5. At a first glance, the employment effects may seem not sizable. However, it is noteworthy that evaluated at a current unemployment rate of about 10 percent, an employment effect of $\Delta(1-\hat{u}) = 1.22$ (for $\nu = 11$) implies that the observed degree of openness has lowered the French unemployment rate by 1.1 percentage points relative to autarky. Column 6 reports our calibration results for the impact of trade on the average mismatch in the firm-internal allocation of workers to tasks. In line with our theoretical result, this mismatch is reduced in the open economy and the more so, the larger is ν . This is intuitive, as we see from Column 2 that higher levels of ν are associated with smaller levels of γ and thus a smaller elasticity of screening costs in screening effort. As a consequence, for higher values of ν firms will adjust their screening effort more strongly to new exporting opportunities, leading to a more pronounced reduction in the economy-wide mismatch of workers and tasks in response to trade.

For providing insights on the extent to which adjustments in the firm-internal allocation of workers govern the employment and welfare implications of trade in our setting, we can contrast the findings in Table 1 with those from an otherwise identical model variant in which such adjustments are not feasible. For this purpose, we look at the limiting case of $\gamma \rightarrow \infty$, which, as outlined above, implies that producers do not screen their applicants – implying $\mu(\phi) = 0$ for all ϕ . Considering $\nu = 11$ as the preferred value for the shape parameter of the Pareto productivity distribution and thus setting $\gamma = 14.10$ and $f_x/f = 0.98$ according to Table 1, we can compute a theory-consistent share of exporters that corresponds to the parameter values at hand. We compute $\chi = 0.02$, which

confirms our insight from the formal analysis that higher levels of γ lower the share of exporters monotonically (see (43)). From the analysis in Section 4, we are warned that changes in the share of exporting firms need not exhibit a monotonic effect on employment and welfare if $f_x/f < 1$, which is the case in our exercise. It is therefore *a priori* not clear whether the movement of France from autarky to the observed degree of openness would have been beneficial in the absence of screening. In the numerical application, we can evaluate the welfare and employment effects of trade for the limiting case of $\gamma \rightarrow \infty$. For the preferred parametrization, this gives $\Delta(1 - \hat{u})\bar{w} = 0.28$ and $\Delta(1 - \hat{u}) = 0.14$, respectively. Therefore, eliminating the ability of firms to screen their workforce and endogenously adjust the quality of worker-task matches would not alter the welfare and employment effects of trade qualitatively, but it would lead to a significant decline of its beneficial consequences. Finally, recollecting from Section 4 that gains from trade in our model are a composite of positive employment and positive wage effects, we can ask which of these two partial effects is the more important source of welfare stimulus. The answer to this question is simple. Setting $\zeta = 0.5$, we obtain $\Delta(1 - \hat{u}) = \Delta w$, according to (40) and (41). Since Δw corresponds to the welfare effect of trade in the absence of search frictions, we can therefore conclude that disregarding labor market imperfections leads to a significant downward bias in the calibrated welfare effects of trade.

6 Concluding remarks

This paper sets up a model of heterogeneous firms along the lines of Melitz (2003) and enriches this workhorse of modern trade theory by associating production with a continuum of tasks that differ in their skill requirements. Furthermore, we assume that workers differ in their abilities to perform these tasks, and firms therefore face the complex problem of matching heterogeneous workers with heterogeneous tasks. To solve this allocation problem in a satisfactory, firms require information about worker ability and they can get this information by screening their applicants. Screening involves fixed costs and provides an imprecise signal about the ability of workers. The higher the investment into the screening technology, the better is the signal and the better is therefore the match between abilities of workers and skill requirements of tasks. Intuitively, firms that have a higher *ex ante* productivity install a better screening technology, so that heterogeneity of firms is reinforced by the endogenous investment into screening.

We use this framework to study the consequences of trade for welfare and underemployment, arising from the mismatch between workers and tasks. Assuming that only the best (most productive) firms self-select into exporting, trade exerts an asymmetric effect on the screening incentives of high- and low-productivity firms. High-productivity firms expand production due to exporting, and therefore find it attractive to install a better (more expensive) screening technology than in the closed economy. In contrast, low-productivity firms do not export and lose market share at home. In response, they lower their screening expenditures. Despite this asymmetry in firm-level adjustments to trade, we show that the average mismatch between worker-specific abilities and task-specific skill requirements unambiguously shrinks in the open economy. This points to a so

far unexplored channel through which trade can improve the labor market outcome and stimulate welfare.

In an extension to our baseline model, we consider imperfections in the external labor market due to search frictions. Relying on a competitive search model with wage posting, we show that this modification does not alter our insights regarding the consequences of trade for the firm-internal allocation of workers to tasks. However, due to adjustments in involuntary unemployment, there is now a second channel through which trade affects economy-wide underemployment. Whether more or less workers find a job in the open economy is in general not clear and depends on the strength of selection of firms into exporting. If fixed costs of exporting are high relative to domestic fixed costs, selection into exporting is strong and in this case trade increases welfare and lowers underemployment due to a higher matching efficiency inside and outside the firm. In a calibration exercise, we rely on parameter estimates for French firms to quantify the relative importance of adjustments in the firm-internal and the firm-external labor market. We find that both adjustments are important channels for gains from trade to materialize. For instance, eliminating the ability of firms to screen their applicants and to adjust the quality of worker-task matches endogenously would lower gains from trade by almost 90 percent, whereas disregarding improvements in the outside labor market would lower gains from trade by 50 percent when relying on the preferred parametrization of our model.

To put it in broader perspective, one can interpret our analysis as an attempt to widen the picture of underemployment and to show that positive labor market consequences of trade need not only materialize due to a reduction in involuntary unemployment. Rather efficiency gains may be triggered by adjustments in the firm-internal organization of labor and according to our results these gains may indeed be sizable. Of course, more research is needed before one can draw a definite conclusion about how trade affects the way labor is used in modern production. We hope that the insights from our analysis encourage such research.

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Appendix

Existence and uniqueness of a maximum of $\pi(\omega)$

Let us first assume that system (8) and (9) has a solution, i.e. $\pi(\omega)$ has a stationary point (l_0, μ_0) . Then, this stationary point is a strict local maximum if the Hessian matrix

$$H(\omega) = \begin{pmatrix} \pi_{ll}(\omega) & \pi_{l\mu}(\omega) \\ \pi_{\mu l}(\omega) & \pi_{\mu\mu}(\omega) \end{pmatrix} \quad (45)$$

of $\pi(\omega)$ is negative definite when evaluated at (l_0, μ_0) . $H(\omega)$ is negative definite if $\pi_{ll} > 0$ and $|H(\omega)| = \pi_{ll}(\omega)\pi_{\mu\mu}(\omega) - \pi_{l\mu}(\omega)^2 < 0$ hold. Twice differentiating $\pi(\omega)$ gives:

$$\pi_{ll}(\omega) = -\frac{\sigma-1}{\sigma^2} \frac{p}{x} \phi(\omega)^2 [1 + \mu(\omega)]^2 < 0, \quad (46)$$

$$\pi_{\mu\mu}(\omega) = \frac{\sigma-1}{\sigma^2} \frac{p}{x} \phi(\omega)^2 l(\omega)^2 - \gamma(\gamma-1) [1 + \mu(\omega)]^{\gamma-2}, \quad (47)$$

$$\pi_{l\mu}(\omega) = \pi_{\mu l}(\omega) = \left(\frac{\sigma-1}{\sigma} \right)^2 p(\omega) \phi(\omega) > 0. \quad (48)$$

We can therefore compute

$$|H(\omega)| = \frac{\sigma-1}{\sigma^2} \frac{\phi(\omega)^2 p(\omega)}{x(\omega)} \left\{ \frac{\sigma-1}{\sigma} (2-\sigma) r(\omega) + \gamma(\gamma-1) [1 + \mu(\omega)]^\gamma \right\}. \quad (49)$$

Evaluating the latter at (l_0, μ_0) , we can make use of (9) and set $r(\omega) = [\gamma\sigma/(\sigma-1)] [1 + \mu(\omega)]^\gamma$. This implies

$$|H(\omega)| = \frac{(\sigma-1)^2}{\sigma^3} \phi(\omega)^2 p(\omega)^2 [\gamma - (\sigma-1)] \quad (50)$$

and thus $|H| >, =, < 0$ if $\gamma >, =, < \sigma-1$. Therefore, $\gamma > \sigma-1$ gives a sufficient condition for a local maximum of $\pi(\omega)$ at stationary point (l_0, μ_0) .

We now show that system (8), (9) has a unique interior solution for all active producers if we impose the additional parameter constraint $(1+f)(\sigma-1) > \gamma$. For this purpose, it is worth noting that for any given $\mu(\omega)$, Eq. (8) has a unique solution in $p(\omega)$ which is represented by (10). Substituting this constant markup pricing rule into (9) and accounting for (2), allows us to define a function

$$F(\mu(\omega)) \equiv \frac{Y}{M} \left(\frac{w}{\phi} \frac{\sigma}{\sigma-1} \right)^{1-\sigma} - \frac{\sigma\gamma}{\sigma-1} [1 + \mu(\omega)]^{\gamma-\sigma+1}, \quad (51)$$

whose function value is equal to zero if first-order conditions (8) and (9) hold. It is easily confirmed that $F'(\cdot) < 0$ and $\lim_{\mu(\omega) \rightarrow \infty} F(\cdot) < 0$ hold if $\gamma > \sigma-1$ is assumed. Hence, accounting for constant markup pricing, $F(\mu(\omega)) = 0$ has a unique solution in $\mu(\omega)$ if $F(0) > 0$. In view of constant markup pricing, operating profits are a constant fraction $1/\sigma$ of firm-level revenues $r(\omega) = p(\omega)x(\omega)$. Since

the minimum possible fixed cost of production (without screening) equals $1 + f$, firms are only willing to start production if $r(\omega) \geq \sigma(1 + f)$. Accounting for (2) and (10), it follows that $r(\omega)$ is increasing in screening effort $\mu(\omega)$, so that

$$r(\omega) \geq \frac{Y}{M} \left(\frac{w}{\phi} \frac{\sigma}{\sigma - 1} \right)^{1-\sigma}. \quad (52)$$

Putting together, it follows that $F(0) \geq \sigma(1 + f) - \sigma\gamma/(\sigma - 1)$ must hold for all active firms, rendering $(\sigma - 1)(1 + f) > \gamma$ sufficient for $F(0) > 0$.

Summing up, we can therefore conclude that the profit-maximization problem in Section 2 has a unique interior solution (for active producers) if $(\sigma - 1)(1 + f) > \gamma$ and $\gamma > \sigma - 1$ simultaneously hold. *QED*

Derivation of Equation (15)

Aggregate revenues of all intermediate goods producers equal

$$R = M \int_{\phi^*}^{\infty} r(\phi) \frac{dG(\phi)}{1 - G(\phi^*)} = M \frac{f\gamma\sigma}{\gamma - \sigma + 1} \frac{\nu}{\nu - \xi}. \quad (53)$$

Dividing R by σ and subtracting fixed costs for operating the local distribution network, Mf , and for installing the screening technology,

$$M \int_{\phi^*}^{\infty} [1 + \mu(\phi)]^{\gamma} \frac{dG(\phi)}{1 - G(\phi^*)} = M \frac{f(\sigma - 1)}{\gamma - \sigma + 1} \frac{\nu}{\nu - \xi}, \quad (54)$$

gives aggregate profits $\Pi = M\xi f/(\nu - \xi)$. Dividing Π by M , we finally obtain (15). *QED*

Derivation of Equation (18)

Total distance of worker-specific abilities and task-specific skill requirements can be calculated by multiplying the average distance of a firm by this firm's employment level and aggregating the resulting expression over all firms. This gives total underemployment:

$$\begin{aligned} U &= M \int_{\phi^*}^{\infty} \frac{l(\phi)}{3[1 + \mu(\phi)]} \frac{dG(\phi)}{1 - G(\phi^*)} = M \frac{l(\phi^*)}{3[1 + \mu(\phi^*)]} \int_{\phi^*}^{\infty} \left(\frac{\phi}{\phi^*} \right)^{\frac{(\gamma-1)\xi}{\gamma}} \frac{dG(\phi)}{1 - G(\phi^*)} \\ &= M \frac{l(\phi^*)}{3[1 + \mu(\phi^*)]} \frac{\gamma\nu}{\gamma(\nu - \xi) + \xi}. \end{aligned} \quad (55)$$

Dividing U by economy-wide employment

$$L = M \int_{\phi^*}^{\infty} l(\phi) \frac{dG(\phi)}{1 - G(\phi^*)} = M l(\phi^*) \int_{\phi^*}^{\infty} \left(\frac{\phi}{\phi^*} \right)^{\xi} \frac{dG(\phi)}{1 - G(\phi^*)} = M l(\phi^*) \frac{\nu}{\nu - \xi}, \quad (56)$$

then gives *average* underemployment u in (18). *QED*

Derivation of Equation (24)

Total revenues in the open economy are given by

$$R = M \int_{\phi^*}^{\phi_x^*} r^n(\phi) \frac{dG(\phi)}{1 - G(\phi^*)} + M(1 + \tau^{1-\sigma}) \int_{\phi_x^*}^{\infty} r^e(\phi) \frac{dG(\phi)}{1 - G(\phi^*)}. \quad (57)$$

Substituting (20) and accounting for (14), we can calculate

$$R = M \frac{f\gamma\sigma}{\gamma - \sigma + 1} \left(1 + \chi \frac{f_x}{f}\right) \frac{\nu}{\nu - \xi}. \quad (58)$$

Dividing R by σ and subtracting fixed costs Mf , $M\chi f_x$, and

$$M \int_{\phi^*}^{\phi_x^*} [1 + \mu^n(\phi)]^\gamma \frac{dG(\phi)}{1 - G(\phi^*)} + M \int_{\phi_x^*}^{\infty} [1 + \mu^e(\phi)]^\gamma \frac{dG(\phi)}{1 - G(\phi^*)} = M \frac{f(\sigma - 1)}{\gamma - \sigma + 1} \left(1 + \chi \frac{f_x}{f}\right) \frac{\nu}{\nu - \xi}, \quad (59)$$

we get aggregate profits $\Pi = M\xi f(1 + \chi f_x/f)/(\nu - \xi)$. Dividing Π by M , finally gives (24). *QED*

Derivation of Equation (28)

Total underemployment in the open economy is given by

$$U = M \int_{\phi^*}^{\phi_x^*} \frac{l^n(\phi)}{3[1 + \mu^n(\phi)]} \frac{dG(\phi)}{1 - G(\phi^*)} + M(1 + \tau^{1-\sigma}) \int_{\phi_x^*}^{\infty} \frac{l^e(\phi)}{3[1 + \mu^e(\phi)]} \frac{dG(\phi)}{1 - G(\phi^*)}. \quad (60)$$

Noting that

$$M \int_{\phi^*}^{\phi_x^*} \frac{l^n(\phi)}{3[1 + \mu^n(\phi)]} \frac{dG(\phi)}{1 - G(\phi^*)} = M \frac{l^n(\phi^*)}{3[1 + \mu^n(\phi^*)]} \frac{\gamma\nu}{\gamma(\nu - \xi) + \xi} \left[1 - \chi \left(\frac{\phi_x^*}{\phi^*}\right)^{\frac{(\gamma-1)\xi}{\gamma}}\right] \quad (61)$$

while

$$\begin{aligned} M(1 + \tau^{1-\sigma}) \int_{\phi_x^*}^{\infty} \frac{l^e(\phi)}{3[1 + \mu^e(\phi)]} \frac{dG(\phi)}{1 - G(\phi^*)} \\ = M \frac{l^n(\phi^*)}{3[1 + \mu^n(\phi^*)]} \frac{\gamma\nu}{\gamma(\nu - \xi) + \xi} \chi \left(\frac{\phi_x^*}{\phi^*}\right)^{\frac{(\gamma-1)\xi}{\gamma}} (1 + \tau^{1-\sigma})^{\frac{(\gamma-1)\xi}{\gamma(\sigma-1)}}, \end{aligned} \quad (62)$$

we can further calculate

$$U = M \frac{l^n(\phi^*)}{3[1 + \mu^n(\phi^*)]} \frac{\gamma\nu}{\gamma(\nu - \xi) + \xi} \left\{1 + \chi \left[(1 + \tau^{1-\sigma})^{\frac{(\gamma-1)\xi}{\gamma(\sigma-1)}} - 1\right] \left(\frac{\phi_x^*}{\phi^*}\right)^{\frac{(\gamma-1)\xi}{\gamma}}\right\}. \quad (63)$$

Substituting (22) and (23), and accounting for the definition of $a(\tau)$ in (28), then gives

$$U = M \frac{l^n(\phi^*)}{3[1 + \mu^n(\phi^*)]} \frac{\gamma\nu}{\gamma(\nu - \xi) + \xi} \left[1 + a(\tau) \chi^{1 + \frac{\xi}{\nu\gamma}} \frac{f_x}{f} \right]. \quad (64)$$

Dividing U by economy-wide employment

$$\begin{aligned} L &= M \int_{\phi^*}^{\phi_x^*} l^n(\phi) \frac{dG(\phi)}{1 - G(\phi^*)} + M(1 + \tau^{1-\sigma}) \int_{\phi_x^*}^{\infty} l^e(\phi) \frac{dG(\phi)}{1 - G(\phi^*)} \\ &= M l^n(\phi^*) \frac{\nu}{\nu - \xi} \left(1 + \chi \frac{f_x}{f} \right) \end{aligned} \quad (65)$$

and noting that $\mu^a(\phi_a^*) = \mu^n(\phi^*)$, we obtain u in (28). *QED*

The impact of marginal trade liberalization on underemployment u

Let us first define $\rho(\tau) \equiv (1 + \tau^{1-\sigma})^{\frac{\xi}{\sigma-1}}$, with $\rho'(\tau) < 0$. In view of (23) and (28), we can then rewrite χ and $a(\tau)$ in the following way:

$$\chi = \left(\frac{f}{f_x} (\rho(\tau) - 1) \right)^{\frac{\nu}{\xi}} \quad a(\tau) = \frac{\rho(\tau)^{\frac{\gamma-1}{\gamma}} - 1}{\rho(\tau) - 1}. \quad (66)$$

Totally differentiating u with respect to τ , therefore gives

$$\begin{aligned} \frac{du}{d\tau} &= u^a \left\{ \frac{\chi f_x/f}{1 + \chi f_x/f} \chi^{\frac{\xi}{\gamma\nu}} \frac{da(\cdot)}{d\rho} \right. \\ &\quad \left. + \frac{f_x/f}{(1 + \chi f_x/f)^2} \left[\frac{\xi}{\gamma\nu} \chi^{\frac{\xi}{\gamma\nu}} a(\tau) \left(1 + \chi \frac{f_x}{f} \right) + \chi^{\frac{\xi}{\gamma\nu}} a(\tau) - 1 \right] \frac{d\chi}{d\rho} \right\} \rho'(\tau), \end{aligned} \quad (67)$$

according to (28). Substituting

$$\frac{da(\cdot)}{d\rho} = -\frac{1}{\rho(\tau) - 1} \left(\frac{a(\tau)}{\gamma} - \frac{\gamma - 1}{\gamma} \frac{1 - \rho(\tau)^{-\frac{1}{\gamma}}}{\rho(\tau) - 1} \right), \quad \frac{d\chi}{d\rho} = \frac{\nu}{\xi} \frac{\chi}{\rho(\tau) - 1}, \quad (68)$$

we can calculate

$$\frac{du}{d\tau} = \frac{\Omega}{\rho(\tau) - 1} \frac{\chi f_x/f}{(1 + \chi f_x/f)^2} \frac{d\rho(\tau)}{d\tau}, \quad (69)$$

with

$$\Omega \equiv \chi^{\frac{\xi}{\gamma\nu}} \left(1 + \chi \frac{f_x}{f} \right) \frac{\gamma - 1}{\gamma} \frac{1 - \rho(\tau)^{-\frac{1}{\gamma}}}{\rho(\tau) - 1} + \frac{\nu}{\xi} \left(\chi^{\frac{\xi}{\gamma\nu}} a(\cdot) - 1 \right). \quad (70)$$

Noting that $1 + \chi f_x/f = 1 + \chi^{1-\xi/\nu}(\rho(\tau) - 1)$ holds, according to (66), it is easily confirmed that $1 + \chi f_x/f < \rho(\tau)$ for any $\chi < 1$. This implies

$$\Omega < \chi^{\frac{\xi}{\gamma\nu}} \frac{\gamma-1}{\gamma} \frac{\rho(\tau) - \rho(\tau)^{\frac{\gamma-1}{\gamma}}}{\rho(\tau) - 1} + \frac{\nu}{\xi} \left(\chi^{\frac{\xi}{\gamma\nu}} a(\cdot) - 1 \right) = - \left(\frac{\nu}{\xi} - \frac{\gamma-1}{\gamma} \right) \chi^{\frac{\xi}{\gamma\nu}} [1 - a(\cdot)] - \frac{\nu}{\xi} \left(1 - \chi^{\frac{\xi}{\nu\gamma}} \right).$$

Since the right-hand side of this inequality is negative, we can conclude that $\Omega < 0$ and, in view of $d\rho(\tau)d\tau < 0$, $du/d\tau > 0$ must hold. This confirms that a marginal decline in τ unambiguously lowers underemployment u in our setting and thus completes the proof. *QED*