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# Bidding in common value fair division games\*

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## Abstract

In a fair division game an indivisible object with an unknown common value is owned by a group of individuals and should be allocated to one of them while the others are compensated monetarily. Implementing fair division games in the lab, we find many occurrences of the winner's curse under the first-price rule but only few occurrences under the second-price rule. Moreover, bidding behavior is very heterogeneous across subjects. A considerable share of our subjects anticipates that other bidders overbid and respond by bidding lower than in equilibrium. We find that the level- $k$  model performs well in explaining our results.

JEL Classification: C73, C91, D44

Key words: common value auction, winner's curse, level- $k$

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# 1 Introduction

Auctions and fair division games are used to allocate indivisible goods among a group of bidders. Whereas in an auction this indivisible good is owned by an external party and bidders seek to buy it from that party, in a fair division game the good is collectively owned by the group *ex ante*. Each bidder has the same legal right to obtain the good, therefore, if one bidder gains acceptance, the price she pays is equally shared among the bidders. Fair division games are proposed as simple mechanisms that dissolve equal partnerships efficiently (Cramton, Gibbons, and Klemperer, 1987, McAfee, 1992).<sup>1</sup> They can be used in conflict settlements, e.g., in case of inheritance, divorce, or the dissolution of a joint venture, where the owner after the bidding has to compensate the others.

A typical example that is due to Engelbrecht-Wiggans (1994) is from Amish inheritance law: When an Amish farmer dies, the farm estate is handed down as a whole. *Ex ante* all heirs have the same legal right to receive the estate. Which of the heirs inherits the farm is determined by a fair division game. When the heirs make their bids the exact return is unknown but each heir has a private estimate of it. The heir with the highest bid receives the farm land and the price the winning bidder pays is split among all the heirs.

The present study is the first to analyze fair division games in a common value environment. The inherited object, e.g., the formerly mutually owned possessions in a marriage or the joint venture, has the same value to all bidders *ex post* and this value is unknown when bidding takes place. Instead, each bidder has private information on what the future value might be.

Studying common value fair division games in the lab is important for two reasons. Firstly, understanding the bidding behavior of real subjects in fair division games

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<sup>1</sup>In the partnership dissolution literature, the terms winner's bid auction and loser's bid auction are used for the first- and second-price fair division game, respectively (see, e.g., McAfee, 1992).

is interesting in its own right. Since fair division games are recommended as a way to dissolve partnerships in a wide variety of contexts one might want to know more about how people respond to fair division games. Do they play the equilibrium strategy straight away? If not, do they learn to play the equilibrium? How costly are deviations from optimal bidding?

Secondly, studying common value fair division games provides a new angle on recent attempts to explain the winner's curse in common value auctions. Despite the similarity to auctions, fair division games have some unique properties. The fact that in fair division games the price is not paid to an outside party but divided among the bidders raises the possibility for a sophisticated bidder to bid deliberately low in expectation of other bidders falling prey to the winner's curse with this bidder receiving an excessive price for her share of the object.

Previous experimental studies on auctions with common values have shown that actual behavior differs substantially from what theory predicts. Winning bidders systematically overbid the (unknown) true value of the item and end up earning negative payoffs (see, e.g., Bazerman and Samuelson, 1983; Kagel and Levin, 1986; Kagel, Levin, and Harstad, 1987 and Kagel and Richard, 2001). The winner's curse phenomenon has been studied extensively in the lab. It is especially distinct with inexperienced bidders. Moreover, it is pervasive under first- and second-price rule.<sup>2</sup> Two prominent explanations of the winner's curse are cursed equilibrium (Eyster and Rabin, 2005) and level- $k$  bidding (Crawford and Iriberri, 2007a). More recently, two studies analyze whether belief-based models such as level- $k$  reasoning and cursed equilibrium can explain the winner's curse. Charness and Levin (2009) study an individual decision making version of the acquiring-a-company game. Ivanov, Levin, and Niederle (2010) analyze the maximal game. Both studies find that the win-

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<sup>2</sup>Some studies claim to find the winner's curse in the field: Capen, Clapp, and Campbell (1971), Lorenz and Dougherty (1983), Dessauer (1981), Cassing and Douglas (1980) and van Damme (2002). See also Harrison and List (2008) for a more skeptical view on the winner's curse in the field.

ner's curse persists in situations in which beliefs do not matter. They conclude that belief-based models fail to explain the essence of the winner's curse.

In this paper we take the opposite direction. By studying fair division games we *increase* the strategic interaction component between bidders relative to auctions. By doing this we can not only analyze whether a bidder falls prey to the winner's curse but also study how other bidders respond.

The theory of fair division games is closely related to the auctions considered in Engelbrecht-Wiggans (1994) and charity auctions Goeree, Maasland, Onderstal, and Turner (2005). Experimentally, fair division games have so far only been studied with private values. Güth, Ivanova-Stenzel, Königstein, and Strobel (2002, 2005) compare repeated first- and second-price auctions and fair division games, and analyze to what extent learning is influenced by the structural differences between the two games. They find for both games that learning does not drive bidding towards the benchmark solution. More recently, there are also a few experimental studies about dissolution of partnership (Kittsteiner, Ockenfels, and Trhal, 2009 and Brooks, Lando, and Spier, 2009). These studies, however, mainly focus on efficiency which is not an issue in a common value environment.

We provide the symmetric risk neutral Nash equilibrium strategies for both first- and second-price fair division games. Based on this theoretical investigation of the fair division game with common values and independently and identically distributed private signals, we experimentally study the extent of the winner's curse and bidding behavior over time. We find a high incidence of the winner's curse in first-price fair division games; but in second-price fair division games there are fewer occurrences of the winner's curse than one would expect in equilibrium. Subjects learn to bid more cautiously over time under the first-price rule but not under the second-price rule. Moreover, bidding behavior in fair division games is very heterogeneous across subjects and noisier than in auctions.

In order to explain our findings we use two alternative solution concepts: the  $\chi$ -cursed equilibrium introduced by Eyster and Rabin (2005) and level- $k$  reasoning as

applied to auctions by Crawford and Iriberri (2007a). We find that level- $k$  reasoning is better suited to explain the heterogeneity in our data: While a considerable part of the subjects overbids, there is also a large fraction of subjects that anticipates this and underbids. These subjects prefer not to win the object in order to exploit the excessive bidding behavior of other bidders. Level- $k$  reasoning provides a good explanation for how deviations from equilibrium bidding can lead to fewer incidences of the winner's curse than under equilibrium bidding.

The remainder of this paper is organized as follows. Section 2 provides the symmetric risk neutral Nash equilibrium strategies for the fair division games. Section 3 presents the experimental design and section 4 provides the experimental results for the fair division games. Section 5 compares the results obtained in fair division games and auctions. In section 6 we discuss the  $\chi$ -cursed equilibrium and the level- $k$  bidding strategies and study which model performs better at explaining our experimental results. Section 7 concludes. The detailed derivation of the equilibrium,  $\chi$ -cursed equilibrium and level- $k$  bidding strategies of the fair division game is deferred to the Appendix.

## 2 Fair division games

In a fair division setting, all bidders have ex ante the same legal rights concerning the object. The highest bidder therefore earns the value of the item, but has to compensate the losers monetarily at the same time. The highest bidder pays the  $n$ -th share of the price to each of the other group members and thus earns the value of the object less  $\frac{n-1}{n}$  times its price.

This logic results in the following payoff function for bidder  $i = 1, \dots, n$ , which is

common knowledge,

$$\Pi_i^{FDG} = \begin{cases} v - \frac{n-1}{n}p & \text{if } i = w \\ \frac{p}{n} & \text{otherwise,} \end{cases} \quad (1)$$

where index  $w$  denotes the highest bidder and  $p$  equals the highest bid under the first-price rule and second-highest bid under the second-price rule. Since we are studying a common value setting the true value of the object is unknown to the bidders, who receive an imprecise private information signal instead.

## Theoretical solution

For our derivation we assume that the true value  $v$  is uniformly distributed on  $[\underline{v}, \bar{v}]$ . The signals  $x_i$ , for  $i = 1, \dots, n$ , are drawn independently from a uniform distribution on the interval  $[v - \frac{a}{2}, v + \frac{a}{2}]$ . The parameter values  $\underline{v}, \bar{v}$  and  $a$  are common knowledge. For signals in the region of  $x_i \in [\underline{v} + \frac{a}{2}, \bar{v} - \frac{a}{2}]$ , i.e., without corner cases, the symmetric risk neutral Nash equilibrium (SRNNE) strategy is given by <sup>3</sup>

$$b_1^*(x_i) = x_i - \frac{a}{2} + \frac{a}{n^2} + \frac{a(n^3 - n^2 - n + 1)}{n^2(n^2 + n - 1)} \exp\left\{-\frac{n^2}{a(n-1)}\left(x_i - \left(\underline{v} + \frac{a}{2}\right)\right)\right\} \quad (2)$$

for the first-price fair division game and

$$b_2^*(x_i) = x_i - \frac{a}{2} + \frac{a(n+1)}{n^2} + C_0 \exp\left\{\frac{n^2}{a} x_i\right\} \quad (3)$$

for the second-price fair division game.<sup>4</sup> The derivation of the equilibrium bidding functions is transferred to the Appendix.<sup>5</sup> Table 2 shows the SRNNE bidding strategies for the fair division game (FDG) with respect to the parameters in our experiment.

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<sup>3</sup>Following most of the experimental literature on common value auctions, we focus only on the interior region of signals. Note that signals in the corner regions  $x_i < \underline{v} + \frac{a}{2}$  and  $x_i > \bar{v} - \frac{a}{2}$  contain

<b>First-price FDG</b>	$b_1^*(x) = x - 13.125 + \frac{675}{152} \exp\{-\frac{8}{45}(x - 65)\}$
<b>Second-price FDG</b>	$b_2^*(x) = x - 5.625 - 8.070 \times 10^{-34} \exp\{\frac{8}{15}x\}$

Table 1: SRNNE bidding strategies

Note:  $n = 4$ ,  $v \sim U[50, 150]$ ,  $x_i \sim U[v - 15, v + 15]$ ,  $x \in [65, 135]$

### 3 Experimental design and procedures

This study focuses on sealed bid common value fair division games and auctions in which a single indivisible object is awarded to the highest among  $n$  bidders. The true value of the object  $v$  is not known at the time bids are placed and is uniformly distributed on  $[\underline{v}, \bar{v}]$ . Each bidder receives a private information signal  $x$  about the true value. Four different games are investigated: the first- and second-price auction and the first- and second-price fair division game. In our experiment subjects bid in groups of  $n = 4$ . The true values were randomly drawn from the uniform interval  $v \in [50, 150]$  and the private signals from the interval  $x_i \in [v - 15, v + 15]$ . Subjects were asked to place a single bid  $b_i \in [0, 200]$  in each round. All values are denoted in a fictitious currency termed ECU (Experimental Currency Unit). In order to keep monetary incentives in both games approximately constant, we varied the exchange rate (100 ECU = 14 Euro in the Auction, 100 ECU = 1 Euro in the fair division game).<sup>6</sup>

We conducted 4 sessions, 2 with fair division games and 2 with auctions. In each additional information about the true value.

<sup>4</sup>Note that there is no analytical solution for the constant  $C_0$ .  $C_0$  is found numerically for the parameter values in the experiment.

<sup>5</sup>Solutions to a similar problem are derived in Engelbrecht-Wiggans (1994).

<sup>6</sup>Güth, Ivanova-Stenzel, Königstein, and Strobel (2002, 2005) face the same problem of unequal incentives for both games. Due to the lack of previous data on fair division games with common values, we accommodated our adjustment of the exchange rate to the one used in Güth et al. (2002, 2005) for private values.



<b>Treatm.</b>	<b>Period 1-20</b>	<b>Period 21-40</b>	<b>No. of subjects</b>
<b>1</b>	FDG1st	FDG2nd	28
<b>2</b>	FDG2nd	FDG1st	28
<b>3</b>	Auc1st	Auc2nd	32
<b>4</b>	Auc2nd	Auc1st	32

Table 2: Experimental design

Note: FDG1st/2nd: Fair division game under first/second-price rule, Auc1st/2nd:

Auction under first/second-price rule

session each subject played 40 rounds: 20 rounds under first-price rule and 20 rounds under second-price rule. We reversed the order of the price rules for the same game to check for order effects. Bidding groups were rematched after each round within fixed matching groups.<sup>7</sup> See Table 2 for a summary of the experimental design.

In the main text we only report the results of periods 1-20. The games in periods 21-40 are slightly contaminated in the first few rounds after the rule change but the qualitative results are the same as for periods 1-20. An analysis of periods 21-40 can be found in the Appendix.

Subjects took part in only one of the sessions and therefore either played the auction or the fair division game. In the invitation to this experiment, they were informed that it was possible to make losses during the experiment. When they entered the laboratory, the possibility of losses was announced once more, together with the information that a loss would not be charged in monetary terms but in the form of a simple task that the subjects concerned would have to perform after the experiment. The length of this task would depend on how much loss they made.<sup>8</sup> Furthermore,

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<sup>7</sup>In the auctions we had four matching groups of 8 subjects and in the fair division games two matching groups of 8 and one of 12 subjects.

<sup>8</sup>The task consisted of searching for the letter 'a' in a document produced by a random words generator. This was, however, only revealed to the subjects that were actually concerned. 25 subjects had to do this chore in the auction treatments. No subject made losses in the fair division

subjects were told that it was still possible to leave in case they did not agree. However, only one out of 121 subjects left.

The computerized experiment was conducted in October/November 2007 at the laboratory of the Max-Planck-Institute of Economics in Jena using z-Tree (Fischbacher, 2007). We recruited 120 undergraduate students from various fields such as economics, biology, law and informatics from Jena University, using the ORSEE software (Greiner, 2004). After reading aloud the instructions several control questions had to be solved to make sure that the participants understood the game. The experiment lasted for approximately 2 hours. Average earnings in the fair division games were 11.93 Euro plus a show up fee of 2.50 Euro.

## 4 Results

The winner's curse is usually described as bidders' systematic failure to "condition upon the critical future event (e.g. winning the auction) and correctly infer and incorporate the relevant posterior in their current bidding decisions." (Charness and Levin, 2009, p. 207) This description of the winner's curse, however, is not very helpful when analyzing data, because it is difficult to tell whether a deviation from equilibrium bidding is due to a systematic failure of correct Bayesian updating or something else. As a result, the following definition of winner's curse is often applied in the common value auction literature (see, e.g., Bazerman and Samuelson, 1983): The winner of an auction suffers from the winner's curse if her payoff is negative. Ex post she prefers to lose the auction and get zero payoff. We apply the same logic to the fair division game. A winning bidder compares her payoff from winning to the payoff she would have received if she had lost. We assume that a bidder can make sure to lose the fair division game by placing the lowest possible bid, a zero

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games.

bid. More formally, the definition of the winner's curse applied here looks as follows:

**Definition** *The winner of the fair division game suffers the winner's curse, if her payoff is lower than the payoff resulting from a zero bid, i.e.*

- (i) *in a first-price fair division game, winning bidder  $i$  suffers the winner's curse if  $v - \frac{n-1}{n}b_i < \frac{q}{n}$ , where  $b_i$  is bidder  $i$ 's bid and  $q$  is the highest bid among the remaining  $n - 1$  bidders and*
- (ii) *in a second-price fair division game, winning bidder  $i$  suffers the winner's curse if  $v - \frac{n-1}{n}q < \frac{r}{n}$ , where  $q$  and  $r$  are the highest bid and the second-highest bid among the remaining  $n - 1$  bidders, respectively.*

Note that this is a conservative definition of the winner's curse for the second-price rule. Alternatively, a winning bidder might compare her winning bid with the strategy of undercutting the highest bid of the remaining bidders by a small amount. The bidder would lose but receive a high price for her share. Since the decisions of the other bidders are unknown when placing the bid, this strategy entails the risk of winning the fair division game and is therefore not an appropriate benchmark for comparing payoffs from winning and losing the fair division game.

For auctions we will say that the winner's curse occurs whenever the ex post payoff of the winner is negative. Consistent with the theoretical benchmark in the previous section, we will throughout the analysis only consider bids that are based on signals within the inner region of  $v$ , i.e.,  $x \in [\underline{v} + \frac{a}{2}, \bar{v} - \frac{a}{2}]$ . Signals outside of this region were not per se excluded from the experiment; however, they contained additional information regarding the true value which could possibly have changed the bidding behavior for those signals. We also frequently refer to the shading rate which is defined as the amount a bid falls short of the signal relative to the dispersion of the signals. Formally, the shading rate is defined by  $\frac{x-b}{a/2}$ .

	FDG 1st	FDG 2nd	Auc 1st	Auc 2nd
<b>All bidders</b> (obs.)	385	360	433	472
Signal overbidding	15.8%	51.4%	10.2%	39.6%
Mean shading rate	0.7786	-0.018	0.5354	0.1542
Mean equil. shading rate	0.8566	0.375	0.9581	0.5
<b>Only winners</b> (obs.)	97	92	101	114
Winner's curse	54.6%	7.6%	70.3%	56.1%
Winner's curse (in SRNNE)	9.3%	26.1%	3.0%	21.1%
Winners with highest signal	48.5%	59.8%	71.3%	55.3%
Signal overbidding	40.2%	78.3%	21.8%	71.1%
Mean shading rate	0.1445	-0.5281	0.2916	-0.2413
Mean profit margin	-3.084	-4.050	-2.888	-1.113
Mean equil. profit margin	7.662	4.882	7.179	3.429
<b>Only losers</b> (obs.)	288	268	332	358
Signal overbidding	7.6%	42.2%	6.6%	29.6%
Mean shading rate	0.985	0.1627	0.6014	0.2816

Table 3: Descriptive statistics for inexperienced bidders (periods 1–20)

Note: FDG/Auc: Fair division game/Auction; 1st/2nd: first/second-price rule

The first two columns of Table 3 present a general picture of the data obtained in the fair division game treatments. It provides mean shading rates, equilibrium shading rates and the fraction of bids that exceed the signal (signal overbidding). Equilibrium shading rates, i.e.,  $\frac{x-b^*(x)}{a/2}$ , with  $b^*(x)$  as the SRNNE bidding functions, are around 0.85 for the first-price fair division game and 0.375 for the second-price fair division game.

Considering all bidders, we find that for both price rules subjects overbid their signals to a substantial amount and average shading rates are below their equilibrium predictions. Looking at winners and losers separately suggests that under both price rules, losers bid relatively close to equilibrium, whereas winners' bids lie dramati-

cally below the equilibrium shading rate. As a consequence, more than 50% of all winners suffer from the winner’s curse in the first-price fair division game, while with SRNNE bidding less than 10% of the fair division games would lead to the winner’s curse.

Surprisingly, we find the opposite picture for the second-price rule. Here, SRNNE bidding would lead to the winner’s curse in 26% of times. Actual winners, however, suffer from the winner’s curse in only 7.6% of the cases. According to our definition of the winner’s curse in fair division games, a bidder suffers the winner’s curse whenever  $v < \frac{(n-1)q+r}{n}$ . Thus, the higher the second- and third-highest bid the more likely it is that the winner suffers the winner’s curse. Since there are less occurrences of the winner’s curse compared to SRNNE the second- and third-highest bids must be below their SRNNE values for some of the periods.

Profit margin is defined as the difference between the realized value of the object and the price, i.e., it measures the profit the winner makes per unit of the object exchanged. In an auction one unit of the object is exchanged; in the fair division games only  $\frac{n-1}{n}$  units are exchanged because the winner already possesses  $\frac{1}{n}$ -th of the object. If all bidders bid according to SRNNE the average price the winner pays would be well below the realized value of the object. Average profit margins in equilibrium are 7.7 and 4.9 for the first- and second-price rule, respectively. Average profit margins resulting from actual bidding behavior are negative; winning bidders overpay on average.

Finally, Table 3 reveals that bidding behavior in our experiment is very heterogeneous. This is evident in the large differences in shading rates between winners and losers. Moreover, the probability that the holder of the highest signal wins the fair division game is only around 50% for the first-price rule and around 60% for the second-price rule. This suggests that the subjects employ very different bidding strategies.

Table 4 explores learning behavior for both price rules separately. The endogenous variables in these regressions are the shading rate and the winner’s profit margin.

Dep. variable	FDG 1st		FDG 2nd	
	shading rate	profit margin	shading rate	profit margin
	Coef.	Coef.	Coef.	Coef.
<i>(Intercept)</i>	0.9443***	-7.251***	0.0152	-5.2024***
<i>Period</i>	-0.013*	0.4016**	-0.0033	0.0649
Obs	385	97	360	92

\*\*\*  $p < 0.01$  \*\*  $p < 0.05$  \*  $p < 0.1$

Table 4: Shading rates and winner profit margins in the fair division game Note: Mixed effects linear regression model with nested random effects on the individual and the matching group level

Looking at shading rates first, we find that they are on average significantly positive for first-price fair division games. Under the second-price rule we find that average shading rates at the outset of the game are not significantly different from zero. Moreover, shading rates do not increase in the number periods played. With respect to the profit margins, we find that under both first- and second-price rule winners make on average considerable losses. Under the first-price rule this is reduced over time, since we find a positive effect of period. Overall, the influence of time suggests that a learning process is taking place under the first-price rule but not the second-price rule.

## 5 Comparing fair division games and auctions

The theoretical solutions for the first- and second-price auction with common values for the same distributions of the random variables are available in the literature and can be directly adapted (see, e.g., Milgrom and Weber, 1982, Kagel and Levin, 2002). For signals in the region of  $x_i \in [65, 135]$ , i.e., without corner cases, the

symmetric risk neutral Nash equilibrium strategies for the auctions are given by

$$b_{A1}^*(x_i) = x_i - 15 + 6 \exp\left\{-\frac{2}{15}(x_i - 65)\right\} \quad (4)$$

for the first-price rule and

$$b_{A2}^*(x_i) = x_i - \frac{15}{2} \quad (5)$$

for the second-price rule.

Figure 1 represents the SRNNE solutions for both auctions and fair division games graphically for the parameter values employed in the experiment. For a given price rule and a given signal the equilibrium bid in the fair division game is slightly higher than in the corresponding auction. Furthermore, Figure 1 shows that the nonlinearities at both ends of the range of signals we consider are rather small.<sup>9</sup> In contrast to the rather similar SRNNE bidding strategies, we find behavioral differences between fair division games and auctions. The fair division games display a higher proportion of bids that exceed the signal (see columns three and four of Table 3). 15.8% of all bids in the first-price fair division game and 51.4% in the second-price fair division game exceed the signal, whereas this is true for only 10.2% in first-price auctions and 39.6% in second-price auctions (pairwise comparisons are statistically significant at  $p=0.0059$  for the first-price rule and  $p=7.24e-05$  for the second-price rule).

There are more occurrences of the winner's curse for auctions than for the fair division games than in the auctions. 70.3% of all winners in the first-price auction and 56.1% in the second-price auction make losses and consequently suffer the winner's curse.

Figure 2 graphically shows the development of shading rates (averaged over the matching groups). The upper horizontal line in each of the four graphs indicates the shading rate implied by the SRNNE bidding strategy. The lower horizontal

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<sup>9</sup>This is because the exponential term in the equilibrium bidding functions is negligible in that region.

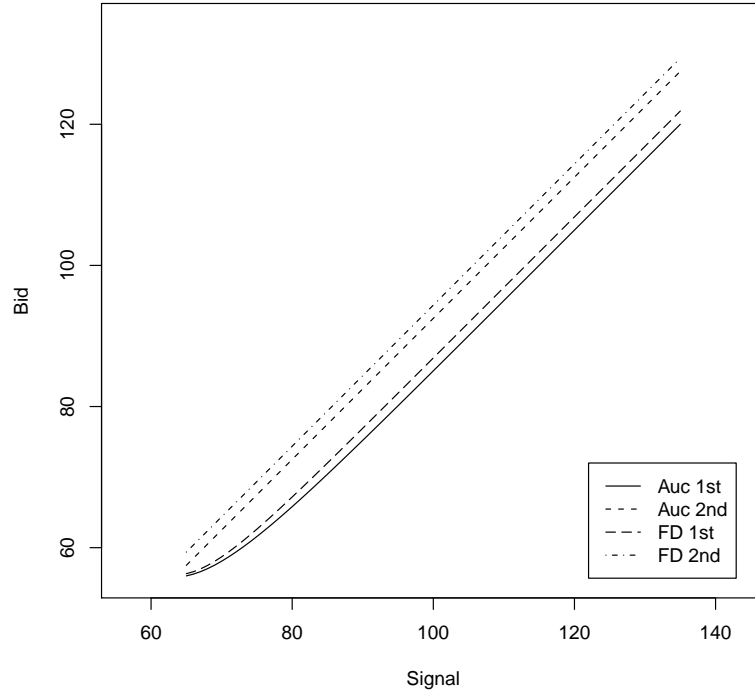


Figure 1: SRNNE bidding functions for the auction and fair division games  
Note:  $n = 4$ ,  $v \sim U[50, 150]$ ,  $x_i \sim U[v - 15, v + 15]$ ,  $x \in [65, 135]$ , FD 1st/2nd: Fair division game under first/second-price rule, Auc 1st/2nd: Auction under first/second-price rule



Dep. variable	Auc 1st		Auc 2nd	
	shading rate	profit margin	shading rate	profit margin
	Coef.	Coef.	Coef.	Coef.
<i>(Intercept)</i>	0.3096***	-7.5595***	0.1629**	-0.6652
<i>Period</i>	0.0197***	0.4393***	-0.0013	-0.0460
Obs	433	101	472	114

\*\*\*  $p < 0.01$  \*\*  $p < 0.05$  \*  $p < 0.1$

Table 5: Shading rates and winner profit margins in the auction games Note: Mixed effects linear regression model with nested random effects on the individual and the matching group level

line displays the shading rate leading to an expected profit margin of exactly zero, the break-even shading rate. Figure 2 suggests that bidding behavior in auctions converges on average to the region around the break even shading rate and, thus, subjects learn to avoid the winner’s curse. In the fair division games, however, bidding behavior is rather dispersed and does not converge.

The observations from Figure 2 are confirmed in Table 5. For both price rules average shading rates are significantly positive at the outset of the game. Profit margins are significantly negative in the starting periods of the first-price auction but shading rates increase significantly and losses are avoided in the final periods. In the second-price auction profit margins are not significantly different from zero in the first periods and no learning takes place.

## 6 Explaining the observed bidding behavior

The previous sections have shown that the bidding behavior in the experiment differs significantly from SRNNE bidding. Moreover, bidding behavior is very heterogeneous. This leads to more occurrences of the winner’s curse for the first-price rule

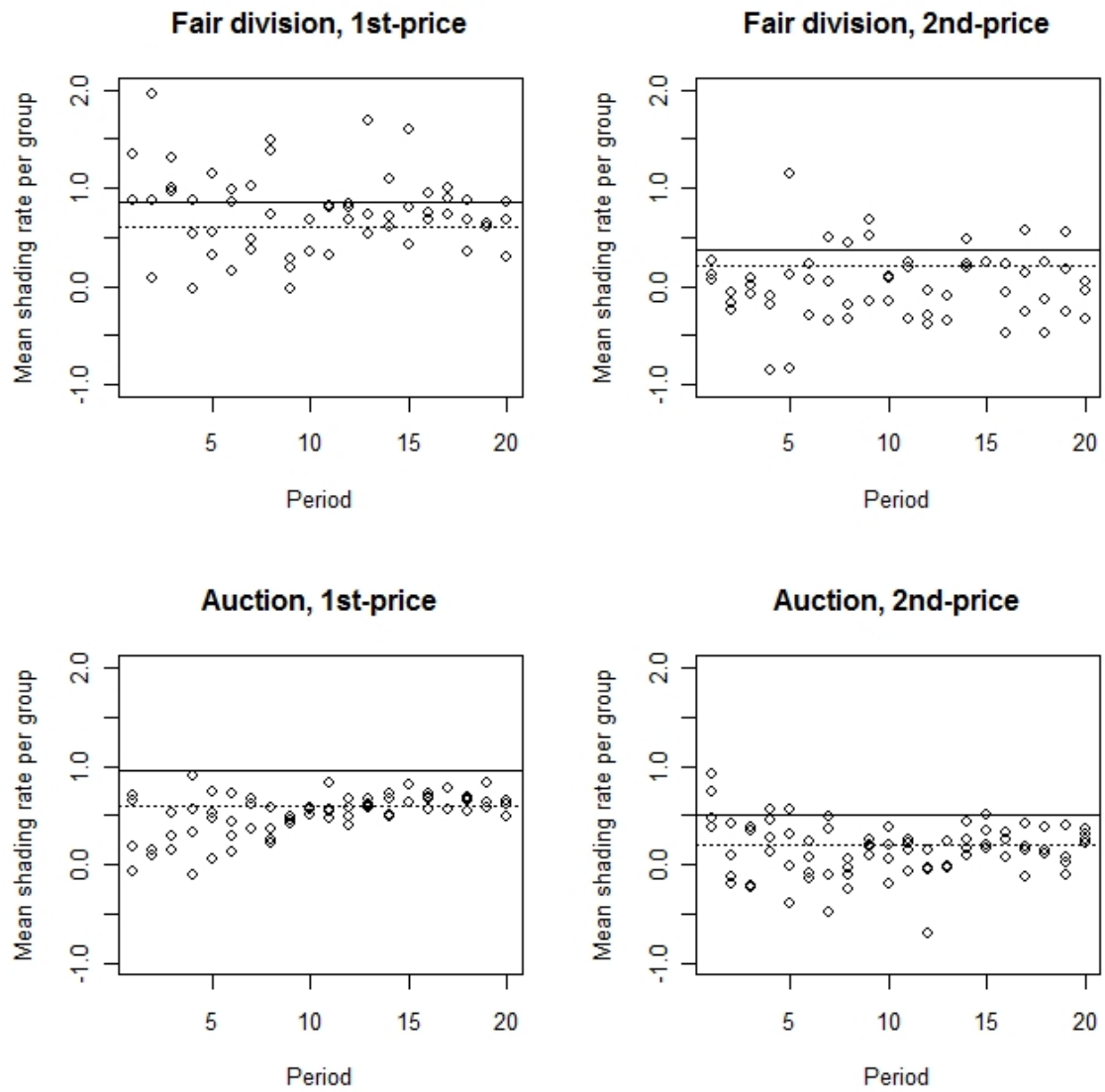


Figure 2: Mean shading rates over time (matching group level)

Note: '—' equilibrium behavior, '- - -' break even strategy

and less occurrences of winner’s curse for the second-price rule relative to SRNNE bidding. In order to better understand bidding behavior in our experimental data, we therefore turn to alternative solution concepts. The solution concepts that we will apply are the  $\chi$ -cursed equilibrium and level- $k$  reasoning.

## 6.1 Cursed equilibrium

Eyster and Rabin (2005) account for the winner’s curse in their  $\chi$ -cursed equilibrium by assuming that each bidder “correctly predicts the probability distribution over her opponents actions, but she underappreciates the connection between her opponents’ equilibrium action profile and their types.” (Eyster and Rabin, 2005, p. 1629) If in the first- and second-price fair division game, bidders are fully cursed, i.e.  $\chi = 1$ , they do not see any correlation between the other bidders’ bids and the true value and act as if in a private value environment. Their expected value of the item, conditional on winning, is perceived to be  $E(v|x_i)$ . If  $\chi = 0$ , bidders are perfectly rational, and their expectation of the true value conditional on winning is  $E(v|x_i, x_i \geq x_j, \forall j)$ . Consequently, they play the SRNNE strategies. For the  $\chi$ -cursed equilibrium it is typically assumed that all bidders have the same degree of cursedness  $\chi \in [0, 1]$  and that this is common knowledge.

The  $\chi$ -cursed equilibrium bidding functions for the first- and second-price fair division game are<sup>10</sup>

$$b_1^\chi(x) = x - 13.125 + 7.5 \chi$$

and

$$b_2^\chi(x) = x - 5.625 + 7.5 \chi.$$

As discussed above, for a bidder with  $\chi = 0$  the bidding strategies collapse to the Nash equilibrium strategies. A fully cursed bidder ( $\chi = 1$ ) overbids by 7.5 under

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<sup>10</sup>See Appendix A.1 for the derivation. The exponential terms are neglected.

both price rules. For a degree of cursedness strictly between 0 and 1 the bidding function is between these two extremes. However, as Fudenberg (2006) points out, equilibria with  $\chi \in (0, 1)$  are difficult to justify from a learning-theoretic perspective. Moreover, evidence from experiments with experienced bidders, such as Kagel and Richard (2001), suggest that the winner's curse is a transitory phenomenon that vanishes with experience (Fudenberg, 2006). In the  $\chi$ -cursed equilibrium, however, the winner's curse is a feature of the stable outcome.

## 6.2 Level- $k$ reasoning

Crawford and Iriberri (2007a) propose level- $k$  reasoning as an alternative explanation for the winner's curse. Level- $k$  reasoning was introduced in Stahl and Wilson (1994, 1995) and Nagel (1995) and it relaxes the assumption that players' beliefs about other players' behavior coincide with the Nash equilibrium strategies.<sup>11</sup> Instead, players are classified according to different types: Level-0 type players are assumed to choose a non-strategic action. In many applications this is a random action from a uniform distribution over the action space. Level-1 types assume that all other players are level-0 types and they play the best response, level-2 types best respond to the level-1 type players, and so on. In Crawford and Iriberri (2007a) and other applications of level- $k$  reasoning it is considered sufficient to concentrate on levels 0 to 2.

Level- $k$  reasoning is best suited to explain initial play. When players play a game repeatedly, they learn about other players' behavior, increase their level of reasoning and may ultimately play the Nash equilibrium. Following Crawford and Iriberri (2007a) we, therefore, restrict our analysis to the first five rounds. Note that the level-0 type is not meant to explain a significant fraction of players' behavior. Its

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<sup>11</sup>See Crawford, Costa-Gomes, and Iriberri (2012) for a recent survey on level- $k$  reasoning and alternative models of strategic thinking.

	<b>first-price FDG</b>	<b>second-price FDG</b>
R.L1	$b_1^{R.L1}(x) = x - 6$	$b_2^{R.L1}(x) = x + 6$
R.L2	$b_1^{R.L2}(x) = x - 13.6$	$b_2^{R.L2}(x) = x - 6.4$
T.L1	$b_1^{T.L1}(x) = x - 14$	$b_2^{T.L1}(x) = x - 6$
T.L2	$b_1^{T.L2}(x) = x - 12.84$	$b_2^{T.L2}(x) = x + 1.69$

Table 6: Level- $k$  strategies for the first- and second-price fair division games

Note: The derivation of the level- $k$  strategies is given in the Appendix A.2.

main purpose is to anchor the beliefs of the level-1 types.<sup>12</sup> In an auction setting, apart from random play, there is a second candidate for level-0 behavior, namely naive or truthful bidding: A truthful level-0 bidder (T.L0) bids her private signal. A truthful level-1 bidder (T.L1) chooses her optimal bidding function assuming that all other bidders bid their own private signals, and so on. As in Crawford and Iriberri (2007a), we consider the following types: random level-0 (R.L0), random level-1 (R.L1), random level-2 (R.L2), truthful level-0 (T.L0), truthful level-1 (T.L1) and truthful level-2 (T.L2).

Table 6 presents the level- $k$  strategies for the parameter values used in our experiment. The derivation of the different level- $k$  bidding strategies for fair division games can be found in Appendix A.2. Comparing the level- $k$  bidding strategies in Table 6 with the SRNNE bidding strategies, we observe the following general pattern: R.L1 types overbid relative to SRNNE bidding. The R.L2 types best respond by bidding slightly lower than in equilibrium. Similarly, since the T.L0 type overbids by construction, the T.L1 underbids relative to equilibrium. The T.L2 type responds by overbidding, but for the first-price fair division game the T.L2 type is quite close to the Nash equilibrium both in terms of her bidding strategy as well as her beliefs about her opponents' strategies.

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<sup>12</sup>A random level-1 bidder with signal  $x$  believes that her opponents (random level-0 bidders) bid uniformly on the range of possible signals  $[x - a, x + a]$ .

### 6.3 Estimation of $\chi$ - and $k$ -types in our population

In order to categorize the subjects in our experiment into the different level- $k$ ,  $\chi$ -cursed or equilibrium types, we closely follow the mixture model approach pioneered in Stahl and Wilson (1994, 1995) which was later adapted to the auction setting by Crawford and Iriberri (2007a). In this specification all bidders make logistic errors. Let  $k$  denote the type of a bidder (e.g. random level-1, but  $k$  can also represent the degree of cursedness or equilibrium beliefs) and  $g$  denote the game (i.e. first-price or second-price fair division game). Bidder  $i$ 's observed bid in game  $g$  in the  $t$ -th round is  $c_{it}^g$  and her corresponding expected utility given her belief type  $k$  and signal  $x_{it}$  is  $S_k^g(c_{it}^g|x_{it})$ . The probability of bid  $c_{it}^g$  if subject  $i$  is of type  $k$  is then given by

$$Pr(c_{it}^g|k, x_{it}, g, \lambda_i) = \frac{\exp(\lambda_i S_k^g(c_{it}^g|x_{it}))}{\int_{\underline{c}}^{\bar{c}} \exp(\lambda_i S_k^g(e|x_{it}))de}, \quad (6)$$

where  $\lambda_i$  denotes the precision of subject  $i$  and  $\underline{c}$  and  $\bar{c}$  are the lower and upper bound of the relevant action space, respectively.<sup>13</sup> As  $\lambda_i \rightarrow 0$ , subject  $i$ 's bids are uniformly distributed on the interval  $[\underline{c}, \bar{c}]$ ; as  $\lambda_i \rightarrow \infty$  bidder  $i$  always plays exactly the best response.

Let  $\pi_{ik}$  denote the probability that bidder  $i$  is of type  $k$ , with  $\sum_{k=1}^K \pi_{ik} = 1$ . For each individual  $i = 1, \dots, N$  we find the values  $(\pi_{i1}, \dots, \pi_{iK}, \lambda_i)$  that maximize the likelihood

$$\sum_{k=1}^K \pi_{ik} \prod_{t=1}^5 Pr(c_{it}^g|k, x_{it}, g, \lambda_i). \quad (7)$$

Using the estimates of the  $\pi_{ik}$ s, we can classify the subjects into different types and finally obtain the proportion  $\pi_k$  of each type  $k$  in our population of subjects.

The results of the classification are presented in Table 7.<sup>14</sup> For each price rule, i.e. first-price fair division game and second-price fair division game, we estimate two

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<sup>13</sup>Following Crawford and Iriberri (2007a) we assume that the relevant action space is equal to the possible values of  $v$  given signal  $x_i$ , i.e.,  $\underline{c} = \max\{v, x_i - \frac{a}{2}\}$  and  $\bar{c} = \min\{\bar{v}, x_i + \frac{a}{2}\}$ .

<sup>14</sup>Tables that present the classification of types on the subject-level are provided in Appendix B.

Level- $k$ plus Equilibrium			Cursed Equilibrium		
	first-price	second-price		first-price	second-price
Type ( $k$ )	$\pi_k$	$\pi_k$	$\chi$	$\pi_k$	$\pi_k$
R.L0	0.25	0.39	R.L0	0.25	0.61
R.L1	0.11	0.14	0	0.52	0.07
R.L2	0.14	0.21	0.1	0.02	0.07
T.L1	0.50	0.09	0.2	0.00	0.00
T.L2	$\sim$ EQ	0.05	0.3	0.00	0.07
EQ	0.00	0.11	0.4	0.04	0.00
			0.5	0.00	0.04
			0.6	0.04	0.04
			0.7	0.04	0.00
			0.8	0.04	0.00
			0.9	0.00	0.00
			1	0.07	0.11
Log-likelihood	-998.97	-1039.84		-1034.20	-1049.52

Table 7: Models and estimates for the fair division games

models: In the first model, called level- $k$  plus equilibrium, we estimate the fractions of  $R.L1$ ,  $R.L2$ ,  $T.L1$  and  $T.L2$  types and, in addition, equilibrium bidders. Subjects that cannot be assigned to any of these types (i.e.,  $\pi_{ik} = 0$  for all  $k$ ) are labeled  $R.L0$  types. The second model, called cursed equilibrium, assumes that the subjects are  $\chi$ -cursed (and believe that other subjects are equally cursed), where we vary the degree of cursedness from 0 to 1 in steps of 0.1. Note that this model also includes equilibrium bidders, since  $\chi = 0$  implies equilibrium behavior. Again, subjects that cannot be assigned to any level of cursedness, are labeled  $R.L0$  bidders. Comparing the likelihoods of the two models, we see that, for both price rules, the level- $k$  plus equilibrium model explains the experimental data better than the cursed equilibrium model. Given that the level- $k$  plus equilibrium model uses fewer parameters,

this is a considerable advantage.

Table 7 also shows that subjects are very heterogeneous: In the first-price fair division game 11% overbid (*R.L1* types) and 64% underbid (*R.L2* and *T.L1* types). In the second-price fair division game 19% overbid (*R.L1* and *T.L2* types) and 30% underbid (*R.L2* and *T.L1* types). This heterogeneity explains the low probability that the bidder with the highest signal wins and the difference in shading rates between winners and losers, documented in section 5.

Comparing the  $\chi$ -cursed equilibrium and the level- $k$  model (both plus equilibrium), we find that the level- $k$  model fares better in explaining our results. Note, that the  $\chi$ -cursed bidding functions describe a symmetric equilibrium. Thus, it seems obvious that a non-equilibrium model that allows for heterogeneous types performs better. For our econometric specification, however, we allow for different degrees of cursedness that can coexist within the population. In fact, the cursed equilibrium model has more parameters than the level- $k$  plus equilibrium model. Nonetheless, the level- $k$  model has a clear likelihood advantage.

Compared to the results of Crawford and Iriberri (2007a) there is a greater fraction of *R.L0* types in our data. Recall that level-0 types are usually not thought to describe the behavior of real subjects, but are merely present in the minds of level-1 bidders. Thus, Crawford, Costa-Gomes, and Iriberri (2012) interpret a low estimated fraction of level-0 types as soundness of the results. This suggests that behavior is more noisy in our fair division game experiments. Our results for the first-price and second-price auction (reported in Appendix B) are similar to the results of Crawford and Iriberri (2007a) and show a lower fraction of *R.L0* types. This suggests that the higher fraction of *R.L0* types is indeed attributable to the fair division game and not to our experimental procedures or to the subject pool. How can the level- $k$  model help to explain the finding that there are many occurrences of the winner's curse in the first-price fair division game and few in the second-price fair division game? Suppose two overbidders (say *R.L1* types) and two underbidders (*R.L2* or *T.L1* types) play a fair division game together. One of the



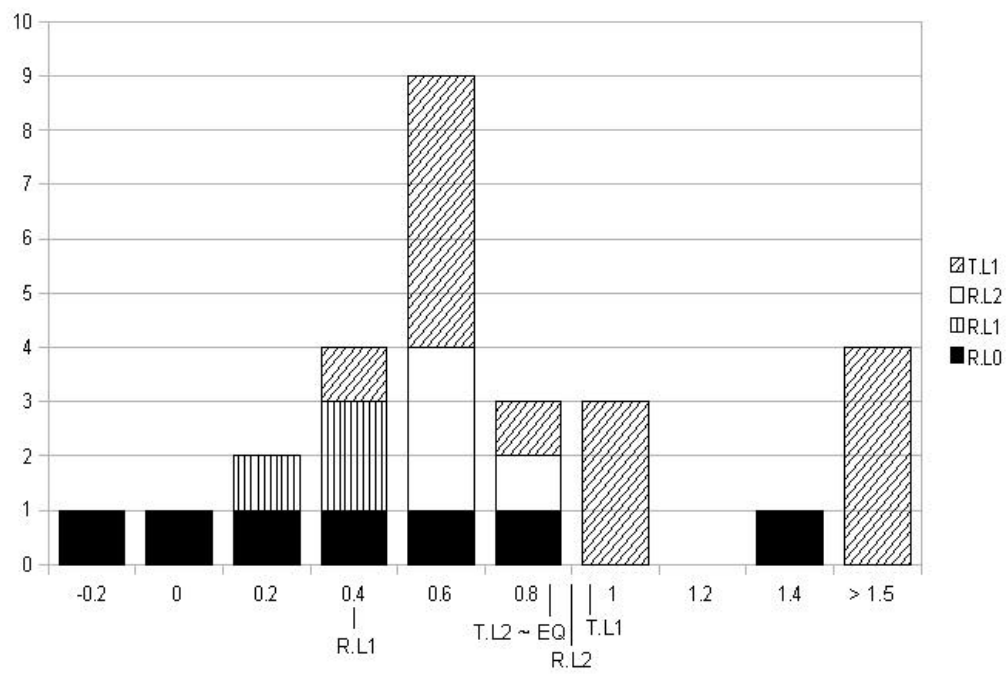


Figure 3: Histogram of average shading rates per subject for the first-price fair division game.

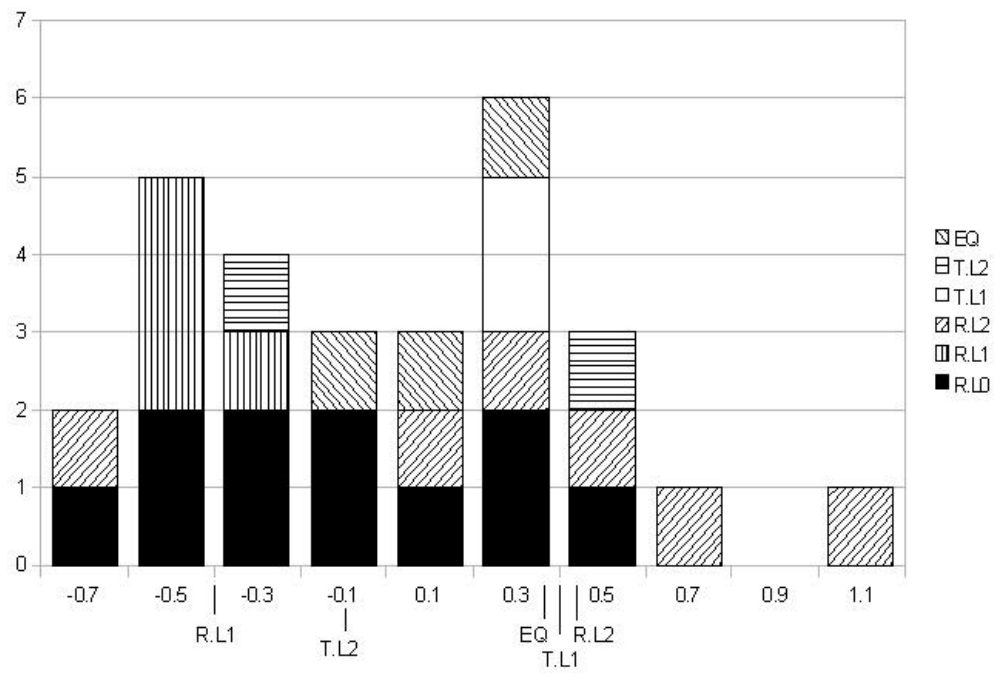


Figure 4: Histogram of average shading rates per subject for the second-price fair division game.

two overbidders is likely to win and pays her bid under the first-price rule. If this bidder had made a zero bid she would receive a share of the second-highest bid which is likely to be high, because it is either from the second overbidder or from an underbidder with a very high signal. Thus, it is quite likely that the winner's payoff is less than what she would have received as a loser. The winner suffers from the winner's curse. In a second-price fair division game, however, the winner's payoff from making a zero bid is a share of the third-highest bid which is likely to be from an underbidder and thus is low. This makes losing the game unattractive and the winner may prefer her payoff even if she pays more than in equilibrium for the object.

This argument assumes that our subjects can be classified in overbidders who might fall prey to the winner's curse and bidders that best respond to this behavior by bidding low. However, there are other explanations that can lead to the same result. The first alternative explanation is that subjects are not consistent. They sometimes shade their signal a lot, sometimes they shade too little. In this case subjects cannot be classified into over- or underbidders. If this explanation were true, average shading rates for the 20 rounds would be very similar across subjects and centered around a moderate value. Figures 3 and 4 show the histograms of average shading rates for the first-price and second-price fair division game, respectively. The histogram for the first-price fair division game is roughly centered around an average shading rate of 0.6. However, there are extreme values well above 1 that are hard to obtain as average shading rates from random over- and underbidding. The histogram for the second-price rule is bimodal with one mode at a negative average shading rate of -0.5 and the other mode at 0.3.

Moreover, the classification according to the level- $k$  model (although only based on the first five rounds) works rather well: The R.L0 bidders are evenly spread across the range of average shading rates. The highest average shading rates stem from T.L1 bidders for the first-price rule and from R.L2 bidders for the second-price rule. R.L1 bidders are consistently among the bidders with the lowest shading rate. Based

on this evidence we reject the alternative explanation that all subjects choose their shading rates randomly each round.

A second alternative explanation is that each subject has an inborn shading rate and that these inborn shading rates stem from a fixed distribution over some interval. On average the bidder with the highest shading rate wins and the losers have lower shading rates. The difference to the level- $k$  explanation is that this behavior is not driven by different beliefs and the corresponding best responses. If this explanation were true we would observe similar histograms for both price rules. Figures 3 and 4, however, show that subjects react differently when faced with different price rules. Finally, if subjects do not best respond to beliefs, the level- $k$  model and the  $\chi$ -cursed model are equally wrong. Since the cursed model allows for more types and hence covers the interval more densely, one would expect that the  $\chi$ -cursed model fits the data better than the level- $k$  model. The fact that the level- $k$  model fares better for both price rules (and for first- and second-price auctions) suggests that best responding to lower types' actions does play a role at least for some of our subjects.

## 7 Discussion and conclusions

The level- $k$  analysis highlights that in fair division games some bidders exploit the excessive bidding of other bidders. Note that this is easier under the first-price rule. In the second-price fair division game, two overbidding players are necessary to push the price above the value of the object. If three players bid low and attempt to exploit the winner, the winner pays only a rather low price. Moreover, by placing a high bid, it is even possible for a bidder to increase the price she receives for her share. This might explain why we observe a smaller fraction of overbidding types and more underbidding types in the first-price fair division game as compared to the second-price rule.

Since fair division games are advocated in situations of conflict about who should

get an object and how other parties involved should be compensated, one could ask whether they are a useful mechanism to solve such conflicts. The results of this experiment suggest that fair division games are not a very appropriate mechanism in a common value environment. The winners systematically overpay and there is little learning at least for the second-price rule. Bidding behavior is very heterogeneous and largely driven by the attempt to outguess the other bidders. This behavior leads to bids that do not reflect the private signals very well. Consequently, the price will not be an unbiased estimator of the common value, and the fair division game will not accurately aggregate the privately owned signals.

Our results support the finding in Crawford and Iriberri (2007a) that the level- $k$  model can explain bidding behavior and occurrences of the winner’s curse in common value environments better than the cursed equilibrium model. Moreover, level- $k$  reasoning explains the surprising finding that subjects in our experiment generate fewer occurrences of the winner’s curse than SRNNE bidding predicts for the second price rule. Charness and Levin (2009) and Ivanov, Levin, and Niederle (2010), however, find that belief based models such as cursed equilibrium and level- $k$  reasoning fail to explain the winner’s curse. How can we reconcile this seemingly contradictory evidence?

One argument is provided by Camerer, Nunnari, and Palfrey (2012). In our econometric specification we allow for logistic errors in subjects’ best responses. Thus, as for the majority of empirical studies that use non-equilibrium beliefs, our explanation of bidding behavior is not purely belief-based but a combination of non-equilibrium beliefs and imperfect best responses.

But there is a second important difference. Charness and Levin (2009) and Ivanov, Levin, and Niederle (2010) concentrate on “correct Bayesian updating”, i.e., overbidding as a result of ignoring the information contained by the event of winning. In the level- $k$  specification that we use to classify our subjects, only random L1-types do not condition on the event of winning the object. Since they assume that all other bidders bid randomly, winning the object is not informative from their perspective.

Random L1-types form only a relatively small fraction of subjects in our experiment (11% and 14% in the first- and second-price fair division game, respectively). All other types may overbid or underbid relative SRNNE but these deviations are driven by strategic considerations (or noise in case of random L0 types) and are not a result of incorrect updating. Thus, we believe that there is no contradiction between our results and those of Charness and Levin (2009) and Ivanov, Levin, and Niederle (2010): While the level- $k$  model provides a good description of the bidding behavior, it does not necessarily lead to the winner's curse. In fact, in the second-price fair division game level- $k$  bidding even helps to overcome the winner's curse.

## Appendix

### A Derivation of bidding strategies

We use the framework introduced by Milgrom and Weber (1982) and extended by Eyster and Rabin (2005) to incorporate the  $\chi$ -cursed equilibrium to find the equilibrium bidding strategies for the fair division game both under the first- and second-price rule.

An indivisible object is auctioned off to  $n \geq 3$  risk neutral bidders.<sup>15</sup> At the outset of the fair division game each bidder owns one  $n$ -th of the object. The vector  $(x_1, \dots, x_n) \in [\underline{x}, \bar{x}]^n \subset \mathbb{R}^n$  is a profile of private signals held by the individual bidders and  $v \in \mathbb{R}$  is an additional possibly payoff relevant random variable with density  $h(v)$ . We assume that for every  $i$ ,  $g(x_i|v)$  satisfies the monotone-likelihood property. In our common value environment the signals held by different bidders

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<sup>15</sup>For the first-price fair division game the minimum number of bidders is 2. For our presentation of the second-price fair division game, however, we require that  $n \geq 3$ .

are uncorrelated given  $v$ . Thus, the joint density of  $x_1, \dots, x_n, v$  is  $f(x_1, \dots, x_n, v) = \prod_{i=1}^n g(x_i|v)h(v)$ . The value of the object to a bidder is  $u(x_1, \dots, x_n, v)$  which is continuous and increasing in the signals  $x_i$  and  $v$ . We further assume that bidders are symmetric, i.e.,  $u(x_1, \dots, x_n, v)$  is symmetric in the private signals  $x_i$ .

Let  $Y_{-i}$  and  $Z_{-i}$  be the highest and second-highest signals among all bidders except  $i$ . Following Milgrom and Weber (1982) we define the following two functions:  $r(x_i) = E[u(x_1, \dots, x_n, v)|x_i]$  is the expectation of the object's value given the private signal  $x_i$  and  $\nu(x_i, y) = E[u(x_1, \dots, x_n, v)|x_i, Y_{-i} = y]$  is the expectation of the object's value given the private signal  $x_i$  and given that the highest signal of the other bidders is  $y$ .

We derive the general bidding functions and provide the bidding functions for the following specification:  $u(x_1, \dots, x_n, v) = v$ ,  $v \sim U[\underline{v}, \bar{v}]$  and  $x_i \sim U[v - \frac{a}{2}, v + \frac{a}{2}]$ , for  $i = 1, \dots, n$ .

But before we derive the different bidding strategies, this section presents some important results that will be used in the next sections. Firstly, we need to know bidder  $i$ 's expectation of the common value given that she observes signal  $x$  and that the highest bid of the remaining  $n-1$  bidders is  $y$ . Secondly, we need the probability density function (pdf) and cumulative density function (cdf) of the highest signal among all bidders except bidder  $i$ , given that bidder  $i$  observes signal  $x$ . Thirdly, for the second-price fair division game, we need an expression for the joint pdf and cdf of the highest and second-highest bids among all bidders except bidder  $i$ , given that bidder  $i$  observes signal  $x$ . The first two are already derived in Crawford and

Iriberri (2007b):

$$\begin{aligned}
\nu(x, y) &= \begin{cases} x - \frac{a}{2} + \frac{a}{n} - \frac{x-y}{n}, & x - a < y \leq x, \\ y - \frac{a}{2} + \frac{a}{n} - \frac{(\frac{y-x}{a})^{n-1}}{1 - (\frac{y-x}{a})^{n-1}} \left(\frac{n-1}{n}\right)(x + a - y), & x < y \leq x + a, \end{cases} \\
f_{Y|X}(y|x) &= \begin{cases} \frac{1}{a} \left(1 - \frac{x-y}{a}\right)^{n-1}, & x - a < y \leq x, \\ \frac{1}{a} \left[1 - \left(\frac{y-x}{a}\right)^{n-1}\right], & x < y \leq x + a, \end{cases} \\
F_{Y|X}(y|x) &= \begin{cases} \frac{1}{n} \left(1 - \frac{x-y}{a}\right)^n, & x - a < y \leq x, \\ \frac{1}{a} \left[y - x - \frac{a}{n} \left(\frac{y-x}{a}\right)^n\right] + \frac{1}{n}, & x < y \leq x + a, \end{cases}
\end{aligned}$$

The joint density of the highest ( $y$ ) and second-highest ( $z$ ) valuation of the other  $n - 1$  bidders given bidder  $i$ 's own valuation  $x$  is

$$f_{Y,Z|X}(y, z|x) = \left[ -\frac{n-1}{a^2} \left(\frac{z-v}{a} + \frac{1}{2}\right)^{n-2} \right]_{\text{lowerbound}}^{\text{upperbound}}, \quad (8)$$

for  $y \geq z$  and zero otherwise. We have to distinguish the following cases:

case		lowerbound	upperbound
A	$X \geq Y \geq Z$	$x - a/2$	$z + a/2$
B	$Y \geq X \geq Z$	$y - a/2$	$z + a/2$
C	$Y \geq Z \geq X$	$y - a/2$	$x + a/2$

**Case A:**

$$f_{Y,Z|X}(y, z|x) = \frac{n-1}{a^2} \left(1 - \frac{x-z}{a}\right)^{n-2}, \text{ for } a \geq y - z \geq 0 \text{ and } 0 \text{ otherwise.} \quad (9)$$

$$\begin{aligned}
F_{Y,Z|X}(y, z|x) &= \int_{x-a}^z \int_{\tilde{z}}^y \frac{n-1}{a^2} \left(1 - \frac{x-\tilde{z}}{a}\right)^{n-2} d\tilde{y} d\tilde{z} \\
&= \frac{1}{n} \left(1 - \frac{x-z}{a}\right)^n + \left(1 - \frac{x-z}{a}\right)^{n-1} \frac{y-z}{a}
\end{aligned}$$

**Case B:**

$$f_{Y,Z|X}(y, z|x) = \frac{n-1}{a^2} \left(1 - \frac{y-z}{a}\right)^{n-2}, \text{ for } a \geq y - z \geq 0 \text{ and } 0 \text{ otherwise.} \quad (10)$$



$$\begin{aligned}
F_{Y,Z|X}(y, z|x) &= \int_{x-a}^z \int_{\tilde{z}}^x \frac{n-1}{a^2} \left(1 - \frac{x-\tilde{z}}{a}\right)^{n-2} d\tilde{y} d\tilde{z} \\
&\quad + \int_x^y \int_{\tilde{y}-a}^z \frac{n-1}{a^2} \left(1 - \frac{y-z}{a}\right)^{n-2} d\tilde{z} d\tilde{y} \\
&= \frac{2}{n} \left(1 - \frac{x-z}{a}\right)^n - \frac{1}{n} \left(1 - \frac{y-z}{a}\right)^n + \left(1 - \frac{x-z}{a}\right)^{n-1} \frac{x-z}{a}
\end{aligned}$$

**Case C:**

$$f_{Y,Z|X}(y, z|x) = \frac{n-1}{a^2} \left[ \left(1 - \frac{y-z}{a}\right)^{n-2} - \left(\frac{z-x}{a}\right)^{n-2} \right], \quad (11)$$

for  $a \geq y - z \geq 0$  and 0 otherwise.

$$\begin{aligned}
F_{Y,Z|X}(y, z|x) &= F_{Y,Z|X}(y, x|x) + \int_x^z \int_{\tilde{z}}^y \frac{n-1}{a^2} \left[ \left(1 - \frac{\tilde{y}-\tilde{z}}{a}\right)^{n-2} - \left(\frac{\tilde{z}-x}{a}\right)^{n-2} \right] d\tilde{y} d\tilde{z} \\
&= \frac{2}{n} - \frac{1}{n} \left(1 - \frac{y-x}{a}\right)^n + \frac{z-x}{a} - \frac{1}{n} \left(1 - \frac{y-z}{a}\right)^n \\
&\quad + \frac{1}{n} \left(1 - \frac{y-x}{a}\right)^n - \frac{1}{n} \left(\frac{z-x}{a}\right)^n - \left(\frac{z-x}{a}\right)^{n-1} \frac{y-z}{a} \\
&= \frac{2}{n} + \frac{z-x}{a} - \frac{1}{n} \left(1 - \frac{y-z}{a}\right)^n - \frac{1}{n} \left(\frac{z-x}{a}\right)^n - \left(\frac{z-x}{a}\right)^{n-1} \frac{y-z}{a}
\end{aligned}$$

For the derivation of the (cursed) equilibrium strategies we also need the expressions for the conditional valuation, pdf and cdf for the corner regions of the signal range:

	$x \in [\underline{v} - \frac{a}{2}, \underline{v} + \frac{a}{2}]$	$x \in [\underline{v} + \frac{a}{2}, \bar{v} - \frac{a}{2}]$	$x \in [\bar{v} - \frac{a}{2}, \bar{v} + \frac{a}{2}]$
$r(x)$	$\frac{x + \frac{a}{2} + \underline{v}}{2}$	$x$	$\frac{x - \frac{a}{2} + \bar{v}}{2}$
$\nu(x, x)$	$\underline{v} + \frac{x + \frac{a}{2} - \underline{v}}{n}$	$x - \frac{a}{2} + \frac{a}{n}$	$\bar{v} + \frac{(x - \frac{a}{2} - \bar{v})a^{n-1} + (a^n - (x - \bar{v} + \frac{a}{2})^n)/n}{a^{n-1} - (x - \bar{v} + \frac{a}{2})^{n-1}}$
$f_{Y X}(x x)$	$\frac{1}{a} \left(\frac{x - \underline{v} + \frac{a}{2}}{a}\right)^{n-2}$	$\frac{1}{a}$	$\frac{1}{\bar{v} - x + \frac{a}{2}} \left(1 - \left(\frac{x - \bar{v} + \frac{a}{2}}{a}\right)^{n-1}\right)$
$F_{Y X}(x x)$	$\frac{1}{n} \left(\frac{x - \underline{v} + \frac{a}{2}}{a}\right)^{n-1}$	$\frac{1}{n}$	$\frac{1}{n} \frac{a}{\bar{v} - x + \frac{a}{2}} \left(1 - \left(\frac{x - \bar{v} + \frac{a}{2}}{a}\right)^n\right)$

## A.1 Derivation of SRNN and $\chi$ -cursed equilibrium bidding strategies

### A.1.1 First-price fair division game

In the first-price fair division game the bidder with the highest bid wins and pays her own bid. In contrast to the auction, however, she does not pay the price to an auctioneer but to all bidders, including herself, in equal parts. Therefore, a bidder receives some positive payoff even if she is not the highest bidder. A  $\chi$ -cursed bidder with private signal  $x$  maximizes the expected payoff

$$\begin{aligned} \max_b \int_{\underline{x}}^{b^{*-1}(b)} & \left( (1 - \chi)\nu(x, y) + \chi r(x) - \frac{n-1}{n} b \right) f_{Y|X}(y|x) dy \\ & + \int_{b^{*-1}(b)}^{\tilde{x}} \frac{b^*(y)}{n} f_{Y|X}(y|x) dy, \end{aligned} \quad (12)$$

where  $\underline{x} = \max\{\underline{v} - \frac{a}{2}, x - a\}$  and  $\tilde{x} = \min\{\bar{v} + \frac{a}{2}, x + a\}$  are the lowest and highest signal one of the other bidders can have received from the perspective of a bidder with signal  $x$ . Setting the first derivative of (12) equal to zero and applying the symmetry condition  $b = b^*(x)$ , we obtain the differential equation

$$b^{*'}(x) = \frac{n}{n-1} \left( (1 - \chi)\nu(x, x) + \chi r(x) - b^*(x) \right) \frac{f_{Y|X}(x|x)}{F_{Y|X}(x|x)}. \quad (13)$$

Let  $\lambda(x) = \frac{n}{n-1} \frac{f_{Y|X}(x|x)}{F_{Y|X}(x|x)}$  and  $v_\chi(x) = (1 - \chi)\nu(x, x) + \chi r(x)$ . Then we can rewrite equation (13) as

$$b^{*'}(x) + b^*(x)\lambda(x) = v_\chi(x)\lambda(x). \quad (14)$$

Let  $\underline{x} = \underline{v} - \frac{a}{2}$  and  $\bar{x} = \bar{v} + \frac{a}{2}$  be the lowest and highest signal a bidder can receive, respectively. Using the integrating factor  $P(x) = \exp\{-\int_x^{\bar{x}} \lambda(u) du\}$  we obtain

$$(b^*(x)P(x))' = v_\chi(x)\lambda(x)P(x).$$

Integrating both sides yields

$$\begin{aligned}
b^*(x)P(x) - b^*(\underline{x})P(\underline{x}) &= \int_{\underline{x}}^x v_\chi(t)\lambda(t)P(t)dt \\
&= \int_{\underline{x}}^x v_\chi(t)P'(t)dt \\
&= v_\chi(x)P(x) - v_\chi(\underline{x})P(\underline{x}) + \int_{\underline{x}}^x v_\chi(t)P'(t)dt.
\end{aligned}$$

Using the initial condition  $b^*(\underline{x}) = v_\chi(\underline{x})$  this yields

$$b^*(x) = v_\chi(x) - P(x)^{-1} \int_{\underline{x}}^x P(t) \frac{dv_\chi(t)}{dt} dt$$

and plugging in the expression for the integrating factor, we finally obtain

$$b^*(x) = v_\chi(x) - \int_{\underline{x}}^x \exp\left\{-\int_t^x \lambda(u)du\right\} \frac{dv_\chi(t)}{dt} dt.$$

$$b^*(x) = (1 - \chi)\nu(x, x) + \chi r(x) - \int_{\underline{x}}^x \exp\left\{-\int_t^x \frac{n}{n-1} \frac{f_{Y|X}(u|u)}{F_{Y|X}(u|u)} du\right\} \frac{d(1 - \chi)\nu(t, t) + \chi r(t)}{dt} dt.$$

Plugging in the expressions for  $r(x)$ ,  $\nu(x, x)$ ,  $f_{Y|X}(x|x)$  and  $F_{Y|X}(x|x)$  for the three different regions of  $x$ , we obtain the following solution

$$b^*(x) = \begin{cases} \frac{n(x+\frac{a}{2})+n^2\underline{v}-\underline{v}}{n^2+n-1} + \chi \frac{(x-\underline{v}+\frac{a}{2})n(n-2)}{2(n^2+n-1)} & , x \in [\underline{v} - \frac{a}{2}, \underline{v} + \frac{a}{2}], \\ x - \frac{a}{2} + \frac{a}{n^2} + \chi \frac{a}{2} \frac{n-2}{n} + \left[\frac{1}{n^2} - \left(\frac{1-\chi}{n} + \frac{\chi}{2}\right) \frac{1}{n^2+n-1}\right] \\ \quad \times (n-1) a \exp\left\{-\frac{n^2}{n-1} \frac{x-\underline{v}-\frac{a}{2}}{a}\right\} & , x \in [\underline{v} + \frac{a}{2}, \bar{v} - \frac{a}{2}], \\ v_\chi(x) + \left(\left[\frac{1}{n^2} - \left(\frac{1-\chi}{n} + \frac{\chi}{2}\right) \frac{1}{n^2+n-1}\right] \exp\left\{-\frac{n^2}{n-1} \frac{\bar{v}-\underline{v}-a}{a}\right\} - \frac{1}{n^2}\right) \\ \quad \times (n-1) a P(\bar{v} - \frac{a}{2}; x) - \int_{\bar{v}-\frac{a}{2}}^x P(t; x) \frac{dv_\chi(t)}{dt} dt & , x \in [\bar{v} - \frac{a}{2}, \bar{v} + \frac{a}{2}], \end{cases} \quad (15)$$

where  $P(t; x) = \exp\left\{-\int_t^x \lambda(u)du\right\}$ .

### A.1.2 Second-price fair division game

In the second-price fair division game the highest bidder receives the object and has to pay the second-highest bid to all bidders in equal parts. A bidder who does

not win the object might therefore receive one  $n$ -th of her own bid, if she is the second-highest bidder or she receives one  $n$ -th of the second-highest bid of the other  $n - 1$  bidders, if her bid is below this bid. Thus, the  $\chi$ -cursed equilibrium strategy solves

$$\begin{aligned} \max_b \int_{\underline{x}}^{b^{*-1}(b)} & ((1 - \chi)\nu(x, y) + \chi r(x) - \frac{n-1}{n} b^*(y)) f_{Y|X}(y|x) dy \\ & + \int_{b^{*-1}(b)}^{\tilde{x}} \int_{\underline{x}}^{b^{*-1}(b)} \frac{b}{n} f_{Y,Z|X}(y, z|x) dz dy \\ & + \int_{b^{*-1}(b)}^{\tilde{x}} \int_z^{\tilde{x}} \frac{b^*(z)}{n} f_{Y,Z|X}(y, z|x) dy dz. \end{aligned} \quad (16)$$

Setting the first derivative of (16) equal to zero and applying the symmetry condition  $b = b^*(x)$ , we obtain the differential equation

$$b^{*'}(x) = -n((1 - \chi)\nu(x, x) + \chi r(x) - b^*(x)) \frac{f_{Y|X}(x|x)}{F_{Y,Z|X}(\tilde{x}, x|x) - F_{Y,Z|X}(x, x|x)}. \quad (17)$$

We define  $\sigma(x) = n \frac{f_{Y|X}(x|x)}{F_{Y,Z|X}(\tilde{x}, x|x) - F_{Y,Z|X}(x, x|x)}$ . Together with the definition  $v_\chi(x) = (1 - \chi)\nu(x, x) + \chi r(x)$  we can simplify the first-order condition to

$$b^{*'}(x) - b^*(x)\sigma(x) = -v_\chi(x)\sigma(x). \quad (18)$$

Using the integrating factor  $Q(x) = \exp\{\int_x^{\tilde{x}} \sigma(u) du\}$  we obtain

$$(b^*(x)Q(x))' = -v_\chi(x)\sigma(x)Q(x).$$

Integrating both sides yields

$$\begin{aligned} b^*(\bar{x})Q(\bar{x}) - b^*(x)Q(x) &= \int_x^{\bar{x}} -v_\chi(t)\sigma(t)Q(t)dt \\ &= \int_x^{\bar{x}} v_\chi(t)Q'(t)dt \\ b^*(x)Q(x) &= b^*(\bar{x})Q(\bar{x}) - \int_x^{\bar{x}} v_\chi(t)Q'(t)dt \\ &= b^*(\bar{x})Q(\bar{x}) - [v_\chi(t)Q(t)]_x^{\bar{x}} + \int_x^{\bar{x}} v_\chi'(t)Q(t)dt \\ &= b^*(\bar{x})Q(\bar{x}) - v_\chi(\bar{x})Q(\bar{x}) + v_\chi(x)Q(x) + \int_x^{\bar{x}} v_\chi'(t)Q(t)dt \end{aligned}$$

The boundary condition that we used for the first-price fair division game,  $b^*(\underline{x}) = (1 - \chi)\nu(\underline{x}, \underline{x}) + \chi r(\underline{x})$ , is not applicable for the second-price fair division game. Since equilibrium bidding strategies are strictly increasing in signals, a bidder who observes  $x = \underline{x}$  knows that she has the lowest bid and can increase her expected payoff by raising her bid slightly above  $(1 - \chi)\nu(\underline{x}, \underline{x}) + \chi r(\underline{x})$ . By doing this she will still not win the object but she increases the probability of being the second-highest bidder and, thus, increase her payment in case of losing. Instead, we use the condition  $b^*(\bar{x}) = (1 - \chi)\nu(\bar{x}, \bar{x}) + \chi r(\bar{x})$ . A bidder with the highest possible signal knows that she is the highest bidder. Increasing her bid further does not change her expected payoff and reducing her bid increases the probability of not winning the object and receiving a smaller payment.<sup>16</sup> With this boundary condition  $b^*(\bar{x}) = v_\chi(\bar{x})$  the first two terms on the right-hand side cancel out. Thus, we obtain

$$b^*(x) = v_\chi(x) + Q(x)^{-1} \int_x^{\bar{x}} v'_\chi(t) Q(t) dt \quad (19)$$

Plugging in the expression for the integrating factor yields

$$b^*(x) = v_\chi(x) + \int_x^{\bar{x}} \exp\left\{-\int_x^t \sigma(u) du\right\} \frac{dv_\chi(t)}{dt} dt. \quad (20)$$

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<sup>16</sup>For our specific example we obtain that a bidder with the lowest possible signal  $\underline{x} = \underline{v} - \frac{a}{2}$ , and who therefore knows that  $v = \underline{v}$ , bids  $\underline{v}$  in the first-price fair division game and above  $\underline{v}$  in the second-price fair division game. Conversely, a bidder with the highest possible signal  $\bar{x} = \bar{v} + \frac{a}{2}$  knows that  $v = \bar{v}$  and bids  $\bar{v}$  in the second-price fair division game and below  $\bar{v}$  in the first-price fair division game. This parallels the result in Güth and van Damme (1986) that the first-price rule guarantees overbidding proofness and the second-price rule guarantees underbidding proofness in fair division games in a private value environment.

Plugging in the expressions for  $v_\chi(x)$  and  $\sigma(x)$  for the three different regions of  $x$  in equation (20) for our setting yields

$$b^*(x) = \begin{cases} (1 - \chi)(\underline{v} + \frac{x - \underline{v} + a/2}{n}) + \chi \frac{x + \underline{v} + a/2}{2} + (\frac{1 - \chi}{n} + \frac{\chi}{2}) \int_x^{\underline{v} + \frac{a}{2}} Q(x; t) dt \\ \quad + \int_{\underline{v} + \frac{a}{2}}^{\bar{v} - \frac{a}{2}} Q(x; t) dt + \int_{\bar{v} - \frac{a}{2}}^{\bar{v} + \frac{a}{2}} Q(x; t) \frac{dv_\chi(t)}{dt} dt & , x \in [\underline{v} - \frac{a}{2}, \underline{v} + \frac{a}{2}] , \\ x - \frac{a}{2} + \frac{a(n+1)}{n^2} + \chi \frac{a}{2} \frac{n-2}{n} + \exp\{-\frac{n^2}{a}(\bar{v} - \frac{a}{2} - x)\} \\ \quad \times (\int_{\bar{v} - \frac{a}{2}}^{\bar{v} + \frac{a}{2}} Q(\bar{v} - \frac{a}{2}; t) \frac{dv_\chi(t)}{dt} dt - \frac{a}{n^2}) & , x \in [\underline{v} + \frac{a}{2}, \bar{v} - \frac{a}{2}] , \\ v_\chi(x) + \int_x^{\bar{v} + \frac{a}{2}} Q(x; t) \frac{dv_\chi(t)}{dt} dt & , x \in [\bar{v} - \frac{a}{2}, \bar{v} + \frac{a}{2}] , \end{cases} \quad (21)$$

where  $Q(x; t) = \exp\{-\int_x^t \sigma(u) du\}$ . For  $x \in [\underline{v} + \frac{a}{2}, \bar{v} - \frac{a}{2}]$  we can simplify this to

$$b^*(x) = x - \frac{a}{2} + \frac{a(n+1)}{n^2} + \chi \frac{a}{2} \frac{n-2}{n} + C_0 \exp\{\frac{n^2 x}{a}\}. \quad (22)$$

From the numerical solution for the specific values  $\chi = 0, n = 4, a = 30$  and  $\bar{v} = 150$  we obtain  $C_0 = -8.070 \times 10^{-34}$ . Plotting the bidding function in equation (22) with these parameters on the range  $x \in [65, 135]$  gives an almost straight line, which suggest that the exponential term is negligible.

## A.2 Derivation of the level- $k$ bidding strategies

### A.2.1 First-price fair division game

If bidder  $i$  believes all other bidders employ the bidding strategy  $b(x)$ , she solves the maximization problem:

$$\begin{aligned} \max_{b_i} \int_{x_i - a}^{b^{-1}(b_i)} (\nu(x_i, y) - \frac{n-1}{n} b_i) f_{Y|X}(y|x_i) dy \\ + \int_{b^{-1}(b_i)}^{x_i + a} \frac{b(y)}{n} f_{Y|X}(y|x_i) dy. \end{aligned} \quad (23)$$

The first-order condition is

$$(\nu(x_i, b^{-1}(b_i)) - b_i) f_{Y|X}(b^{-1}(b_i)|x_i) = \frac{n-1}{n} F_{Y|X}(b^{-1}(b_i)|x_i) \frac{\partial b^{-1}(b_i)}{\partial b_i}. \quad (24)$$

*Truthful L0-players:*

A truthful level-0 bidder bids  $b_1^{T.L0}(x) = x$ . A truthful L1 bidder best responds to the truthful L0 bidder. Thus, the first-order condition becomes

$$(\nu(x_i, b_i) - b_i) f_{Y|X}(b_i|x_i) = \frac{n-1}{n} F_{Y|X}(b_i|x_i). \quad (25)$$

Using the conditional expectation and the conditional pdf and cdf for the case where  $y \leq x$  and solving for  $b_i$  we obtain

$$b_1^{T.L1}(x) = x - \frac{n^2 - 2}{2(n^2 - 1)} a. \quad (26)$$

A truthful L2 bidder best responds to the truthful L1 bidder. Plugging the truthful L1 bidder's strategy in the first-order condition we obtain

$$(\nu(x_i, (b_1^{T.L1})^{-1}(b_i)) - b_i) f_{Y|X}((b_1^{T.L1})^{-1}(b_i)|x_i) = \frac{n-1}{n} F_{Y|X}((b_1^{T.L1})^{-1}(b_i)|x_i). \quad (27)$$

We conjecture and verify later that  $b_1^{T.L2}(x) \geq b_1^{T.L1}(x)$ . Thus, we plug in the expected value, pdf and cdf for the case  $y \geq x$ . There is no closed form solution for the general case. For our parameter values we obtain

$$b_1^{T.L2}(x) = x - 12.8355. \quad (28)$$

*Random L0-players:*

Random L0-types bid uniform on  $[x-a, x+a]$ . Thus, for random L1-bidders winning the fair division game is not informative. Their maximization problem is:

$$\max_{b_i} \int_{x_i-a}^{b_i} (r(x_i) - \frac{n-1}{n} b_i) f_Y(y) dy + \int_{b_i}^{x_i+a} \frac{y}{n} f_Y(y) dy, \quad (29)$$

where  $r(x) = x$  and

$$\begin{aligned} f_Y(y) &= \frac{n-1}{2a} \left( \frac{y - x_i + a}{2a} \right)^{n-2}, \\ F_Y(y) &= \left( \frac{y - x_i + a}{2a} \right)^{n-1}. \end{aligned}$$

The first-order condition is

$$x_i - b_i - \frac{n-1}{n} \frac{b_i - x_i + a}{n-1} = 0 \quad (30)$$

which yields the bidding strategy

$$b_1^{R.L1}(x) = x - \frac{a}{n+1}. \quad (31)$$

For random L2-bidders the first-order condition is the same as in equation (24) with  $b^{-1}(b_i) = b_i + \frac{a}{n+1}$ . For  $b_1^{R.L2}(x) \leq b_1^{R.L1}(x)$ , we have  $y = (b_1^{R.L1})^{-1}(b) \leq x$ . Using the corresponding pdf and cdf, we can solve the first-order condition and obtain the bidding strategy

$$b_1^{R.L2}(x) = x - \frac{n^3 + n^2 - 2n - 4}{2(n+1)(n^2-1)} a. \quad (32)$$

The objective function for the random L1 decision rule is

$$S_{R.L1}^1(b|x) = \begin{cases} \frac{x}{n} + \frac{n-2}{n^2} a & \text{if } b \leq x - a, \\ \frac{x}{n} + \frac{n-2}{n^2} a + \left(\frac{b-x+a}{2a}\right)^{n-1} \left(x - b + \frac{b-x+a}{n^2}\right) & \text{if } x - a < b < x + a, \\ x - \frac{n-1}{n} b & \text{if } x + a \leq b. \end{cases} \quad (33)$$

For  $k = T.L1, T.L2, R.L2$  the objective functions are

$$S_k^1(b|x) = \begin{cases} \frac{x}{n} + \frac{(n-2)a}{2n^2} - \frac{d}{n} & \text{if } b_{k-1}^{-1}(b) < x - a, \\ \frac{x}{n} + \frac{(n-2)a}{2n^2} - \frac{d}{n} + \left(1 - \frac{x-b-d}{a}\right)^n \frac{2(n-1)(x-b)-(n-2)a+2d}{2n^2} & \text{if } x - a < b_{k-1}^{-1}(b) < x, \\ \frac{x}{n} - \frac{na+2(n-1)(x-b)+2d}{2n^2} \left(\frac{b+d-x}{a}\right)^n \\ + \frac{n(b+d-x)-n^2(b-d-x)-a(n^2-2)}{2n^2} \frac{b+d-x}{a} & \text{if } x < b_{k-1}^{-1}(b) < x + a, \\ x - \frac{n-1}{n} b & \text{if } x + a < b_{k-1}^{-1}(b), \end{cases} \quad (34)$$

where  $d$  is the amount a bid falls short of the signal of a bidder of type  $k-1$ , so that  $(b_1^{k-1})^{-1}(b) = b + d$ .



### A.2.2 Second-price fair division game

If bidder  $i$  believes all other bidders employ the bidding strategy  $b(x)$ , she solves the maximization problem:

$$\begin{aligned}
\max_{b_i} & \int_{x_i-a}^{b^{-1}(b_i)} \left( \nu(x_i, y) - \frac{n-1}{n} b(y) \right) f_{Y|X}(y|x_i) dy \\
& + \int_{b^{-1}(b_i)}^{x_i+a} \int_{x_i-a}^{b^{-1}(b_i)} \frac{b_i}{n} f_{Y,Z|X}(y, z|x_i) dz dy \\
& + \int_{b^{-1}(b_i)}^{x_i+a} \int_z^{x_i+a} \frac{b(z)}{n} f_{Y,Z|X}(y, z|x_i) dy dz.
\end{aligned} \tag{35}$$

The first-order condition for the second-price fair division game is:

$$\begin{aligned}
& (\nu(x_i, b^{-1}(b_i)) - b_i) f_Y(b^{-1}(b_i)|x_i) \frac{\partial b^{-1}(b_i)}{\partial b_i} \\
& + \frac{1}{n} (F_{Y,Z|X}(\bar{y}, b^{-1}(b_i)|x_i) - F_{Y,Z|X}(b^{-1}(b_i), b^{-1}(b_i)|x_i)) = 0.
\end{aligned}$$

*Truthful L0-players:*

A truthful level-1 bidder best responds to the bidding function  $b_2^{T.L0}(x) = x$ . Plugging this in the first-order condition and assuming that  $b^{-1}(b_i) \leq x$  we get

$$b_2^{T.L1}(x) = x - \frac{n^2 - 2n - 2}{2(n^2 - 1)} a. \tag{36}$$

A truthful level-2 bidder best responds to the bidding function  $b_2^{T.L1}(x)$ . We conjecture and verify later that  $b^{-1}(b_i) \geq x$ . There is no general closed form solution, but for our specific parameter values ( $a = 30$  and  $n = 4$ ) we get

$$b_2^{T.L2}(x) = x + 1.687.$$

*Random L0-players:*

Since a random L0 bidder's bid is uniform on  $[x - a, x + a]$ , having the highest bid

is not informative about the value of the object for a R.L1 bidder. Thus, she solves

$$\begin{aligned} \max_{b_i} \int_{x_i-a}^{b_i} (r(x_i) - \frac{n-1}{n} y) f_Y(y) dy + \int_{b_i}^{x_i+a} \int_{x_i-a}^{b_i} \frac{b}{n} f_{Y,Z}(y, z) dz dy \\ + \int_{b_i}^{x_i+a} \int_z^{x_i+a} \frac{z}{n} f_{Y,Z}(y, z) dy dz, \end{aligned}$$

where  $r(x_i) = x_i$  and for  $y \geq z$

$$\begin{aligned} f_{Y,Z}(y, z) &= (n-1)(n-2) \left(\frac{1}{2a}\right)^2 \left(\frac{z-x_i+a}{2a}\right)^{n-3} \\ F_{Y,Z}(y, z) &= (n-1) \frac{y-x_i+a}{2a} \left(\frac{z-x_i+a}{2a}\right)^{n-2} - (n-2) \left(\frac{z-x_i+a}{2a}\right)^{n-1}. \end{aligned}$$

This yields the following bidding function for random L1-bidders:

$$b_2^{R.L1}(x) = x + \frac{a}{n+1}. \quad (37)$$

For random L2-bidders the same first-order condition as for truthful L1- and L2-bidders applies, with  $b^{-1}(b_i) = b_i - \frac{a}{n+1}$ . This yields the bidding function

$$b_2^{R.L2}(x) = x - \frac{n^3 - n^2 - 4n}{2(n+1)(n^2-1)} a. \quad (38)$$

The objective function for the Random L1 decision rule is

$$S_{R.L1}^2(b|x) = \begin{cases} \frac{x}{n} + \frac{n-4}{n^2} a & \text{if } b \leq x-a, \\ \frac{x}{n} + \frac{n-4}{n^2} a + \left(\frac{b-x+a}{2a}\right)^{n-1} \left(x-b + \frac{2a}{n} \left(1 + \frac{1}{n} \frac{b-x+a}{2a}\right)\right) & \text{if } x-a < b < x+a, \\ \frac{x}{n} - \frac{(n-2)(n-1)}{n^2} a & \text{if } x+a \leq b. \end{cases} \quad (39)$$

For  $k = T.L1, T.L2, R.L2$  the objective functions are

$$S_k^2(b|x) = \begin{cases} \frac{x}{n} + \frac{(n-4)a}{2n^2} - \frac{d}{n} & \text{if } b_{k-1}^{-1}(b) < x-a, \\ \frac{x}{n} + \frac{(n-4)a}{2n^2} - \frac{d}{n} + \left(1 - \frac{x-b-d}{a}\right)^n \frac{2(n-1)(x-b) - (n-4)a + 2d}{2n^2} & \text{if } x-a < b_{k-1}^{-1}(b) < x, \\ \frac{x}{n} + \frac{(2d-a)n^2 + 2a(n+1)}{2n^2} \frac{b+d-x}{a} - \frac{2(b+d-x) + 2n(x-b) + (n+2)a}{2n^2} \left(\frac{b+d-x}{a}\right)^n & \text{if } x < b_{k-1}^{-1}(b) < x+a, \\ \frac{x}{n} - \frac{(n-2)(n-1)}{2n^2} a + \frac{n-1}{n} d & \text{if } x+a < b_{k-1}^{-1}(b), \end{cases} \quad (40)$$

where  $d$  is the amount a bid falls short of the signal of a bidder of type  $k-1$ , so that  $(b_2^{k-1})^{-1}(b) = b+d$ .

## B Additional tables

	FDG 1st	FDG 2nd	Auc 1st	Auc 2nd
<b>All bidders</b> (obs.)	428	392	496	468
Signal overbidding	26.2%	36.5%	5.04%	37.6%
Mean shading rate	0.4476	0.0813	0.6248	0.1853
Mean equil. shading rate	0.8494	0.375	0.9517	0.5
<b>Only winners</b> (obs.)	104	92	126	111
Winner's curse	67.3%	19.6%	61.11%	50.54%
Winner's curse (in SRNNE)	11.5%	29.3%	6.25%	20.63%
Winners with highest signal	50.6%	48.9%	69.84%	66.66%
Signal overbidding	56.7%	70.7%	13.49%	57.65%
Mean shading rate	-0.0201	-0.6521	0.4508	-0.1006
Mean profit margin	-5.885	-2.284	-1.363	0.098
Mean equil. profit margin	7.191	3.943	6.277	2.959
<b>Only losers</b> (obs.)	324	300	370	357
Signal overbidding	16.4%	26.0%	2.16%	31.37%
Mean shading rate	0.5906	0.301	0.6898	0.2808

Table 8: Descriptive statistics for experienced bidders (periods 21–40)

Note: FDG 1st/2nd: Fair division game under first/second-price rule; Auc 1st/2nd:

Auction under first/second-price rule

Dep. variable	FDG 1st		FDG 2nd	
	shading rate	profit margin	shading rate	profit margin
	Coef.	Coef.	Coef.	Coef.
<i>(Intercept)</i>	0.2692**	-8.6685***	0.1147	-2.5788
<i>Period</i>	0.0162**	0.2773*	-0.0040	-0.0203
Obs	428	104	392	92

\*\*\*  $p < 0.01$  \*\*  $p < 0.05$  \*  $p < 0.1$

Table 9: Shading rates and winner profit margins in the fair division games with experienced subjects (mixed effects linear regression model with nested random effects on the individual and the matching group level)

Dep. variable	Auc 1st		Auc 2nd	
	shading rate	profit margin	shading rate	profit margin
	Coef.	Coef.	Coef.	Coef.
<i>(Intercept)</i>	0.5389***	-4.0632***	0.2411***	1.2485
<i>Period</i>	0.0086***	0.2556**	-0.0088***	-0.1416
Obs	496	126	468	111

\*\*\*  $p < 0.01$  \*\*  $p < 0.05$  \*  $p < 0.1$

Table 10: Shading rates and winner profit margins in the auction games with experienced subjects (mixed effects linear regression model with nested random effects on the individual and the matching group level)

<u>Level-<math>k</math> plus Equilibrium</u>			<u>Cursed Equilibrium</u>		
	first-price	second-price		first-price	second-price
Type ( $k$ )	$\pi_k$	$\pi_k$	$\chi$	$\pi_k$	$\pi_k$
R.L0	0.13	0.09	R.L0	0.13	0.25
R.L1	0.59	0.31	0	0.28	0.06
R.L2	0.00	0.28	0.1	0.00	0.06
T.L1	0.19	$\sim$ R.L2	0.2	0.00	0.06
T.L2	$\sim$ EQ	0.19	0.3	0.00	0.03
EQ	0.09	0.13	0.4	0.00	0.06
			0.5	0.00	0.00
			0.6	0.00	0.00
			0.7	0.03	0.03
			0.8	0.00	0.09
			0.9	0.06	0.00
			1	0.50	0.34
Log-likelihood	-1112.65	-1178.66		-1114.73	-1201.62

Table 11: Models and estimates for the auction games

Subject	<u>Level-<math>k</math> plus Equilibrium</u>		<u>Cursed Equilibrium</u>	
	Type	Subject-Specific	$\chi$	Subject-Specific
		Precision ( $\lambda_i$ )		Precision ( $\lambda_i$ )
1	T.L1	15.74	0	1.37
2	R.L1	0.77	0.7	0.27
3	T.L1	10.91	0	4.24
4	T.L1	0.37	0	0.27
5	R.L1	0.52	1	0.17
6	R.L0	0	R.L0	0
7	T.L1	6.29	0	0.45
8	T.L1	7.75	0	0.33
9	T.L1	0.81	0.8	0.63
10	R.L0	0	R.L0	0
11	T.L1	9.00	0	0.63
12	R.L2	3.39	0.4	40.58
13	R.L1	2.32	1	0.76
14	T.L1	0.67	0	0.63
15	T.L1	0.95	0	0.61
16	T.L1	184.38	0	8.32
17	R.L2	0.18	0	0.17
18	T.L1	5.30	0	0.34
19	T.L1	5.30	0	0.26
20	R.L0	0	R.L0	0
21	R.L0	0	R.L0	0
22	R.L2	0.08	0	0.07
23	R.L2	0.55	0.6	0.66
24	R.L0	0	R.L0	0
25	R.L0	0	R.L0	0
26	T.L1	6.85	0	1.28
27	R.L0	0	R.L0	0
28	T.L1	0.02	0/0.1	0.01

Table 12: Models and estimates for the first-price fair division game

Subject	<u>Level-<math>k</math> plus Equilibrium</u>		<u>Cursed Equilibrium</u>	
	Type	Subject-Specific	$\chi$	Subject-Specific
		Precision ( $\lambda_i$ )		Precision ( $\lambda_i$ )
1	T.L1	3.40	0.3	3.61
2	R.L2	2.10	0.1	6.24
3	R.L0	0	R.L0	0
4	R.L2	4.51	R.L0	0
5	R.L0	0	R.L0	0
6	R.L1	0.91	R.L0	0
7	R.L1	0.43	R.L0	0
8	R.L0	0	R.L0	0
9	R.L0	0	R.L0	0
10	R.L0	0	R.L0	0
11	T.L1	0.16	R.L0	0
12	T.L1/T.L2	8.54	1	1.99
13	R.L0	0	R.L0	0
14	R.L0	0	R.L0	0
15	R.L2	1.86	R.L0	0
16	R.L2	2.83	0.5	1.65
17	R.L0	0	R.L0	0
18	R.L1	1.12	0	2.23
19	EQ	75.34	0.6	45.89
20	EQ	16.96	0	16.96
21	T.L2	90.45	1	13.93
22	R.L2	2.54	R.L0	0
23	R.L0	0	R.L0	0
24	R.L0	0	R.L0	0
25	R.L2	0.59	0.3	1.07
26	R.L0	0	R.L0	0
27	R.L1	0.89	1	0.05
28	EQ	87.82	0.1	27.21

Table 13: Models and estimates for the second-price fair division game

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