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Wage Bargaining when Workers Have Fairness Concerns*

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Abstract

We analyze optimal labor contracts when the worker is inequity averse towards the employer. Welfare is maximized for an equal sharing rule of surplus between the worker and the firm. That is, profit sharing is optimal even if effort is contractible. If the firm can make a take-it-or-leave-it offer, the optimal contract is also state-dependent but always suboptimal with respect to welfare. The reason is that the firm will always pay the worker less than half of the surplus, thereby leading to agency costs due inequity aversion. If the parties bargain over the optimal contract, the optimal division of surplus is more equitable compared to the case with a purely selfish worker. Moreover, the optimal contract with bargaining approaches the welfare-optimal contract as the parties’ bargaining power converges. Our results help explain why workers are willing to accept lower wages in times of crisis but demand higher wages in times of economic rise. Moreover, our findings imply that raising the bargaining power of the less powerful party may increase welfare.

JEL Classification: M52, M55, D03, D86

Keywords: principal-agent, Nash bargaining, inequity aversion, performance pay, profit sharing, labor contracts, wage

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1 Introduction

Empirical evidence suggests that workers not only care for absolute but also for relative payoff and often have a preference for equitable payoff distributions.\(^1\) Not only dislike workers income inequity among peers but also in regard to firm profit when it comes to the distribution of productive surplus between labor and firm owners. In wage bargaining situations, workers often call for fairness, thereby demanding a wage level that awards them a reasonable part of the productive rent.

The existent literature has extensively examined the impact of social preferences on optimal labor contracts in standard principal-agent models in which the whole bargaining power is assigned to the principal. In such a situation, the principal makes the worker a take-it-or-leave-it offer that is accepted by the worker if it makes him just as well off as in his second-best opportunity. This approach is generally useful when we are mainly interested in the efficiency properties of the resulting optimal contract. Yet, the results are not always suitable when we are concerned with distributionary characteristics of optimal labor contracts. In particular, real-world wage setting is often more appropriately reflected by assuming that the firm and the worker bargain over the wage contract. Typically, firm owners as well as workers possess some power in such bargaining scenarios. For example, unions and employers’ associations frequently engage in extensive bargaining procedures in the process of which workers demand higher or ‘fairer’ wages.

In the current paper, we present a model that introduces social preferences in an environment in which a firm bargains over the optimal labor contract with its worker in the absence of moral hazard. We assume that the worker exhibits inequity aversion in the sense of Fehr and Schmidt (1999) towards the firm.\(^2\) More precisely, the worker compares his own net income under the contract with firm profit and dislikes inequitable surplus distributions in any productive state. Formally, we employ the Generalized Nash-bargaining framework (Nash (1950)) so that the firm and the worker may possess different levels of bargaining power. The present analysis highlights an interesting novelty regarding the impact of inequity aversion on the optimal labor contract. In particular, the optimal wage contract endogenously determines the level of surplus to be shared in the process of bargaining over the optimal contract. Our analysis proceeds in three steps. As a benchmark, we first determine the first-best welfare-maximizing contract. Second, we derive the profit-maximizing contract which the firm offers the worker if it has all the bargaining power. Then we solve the Generalized Nash-bargaining problem for the optimal wage contract and compare the results to the foregoing contracts. Moreover, we investigate the impact of bargaining power and the strength of the worker’s inequity aversion on the parties’ respective income and the efficiency of the contract.

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\(^1\)See, e.g., Goranson and Berkowitz (1966), Berg, Dickhaut, and McCabe (1995), Fehr, Kirchsteiger, and Riedl (1998), Fehr and Schmidt (1999), or Charness and Rabin (2002). For an overview of the extensive literature on the effects of relative pay comparisons see Akerlof and Yellen (1990). An overview of the experimental literature on other-regarding preferences is provided by, e.g., Camerer (2003) or Fehr and Schmidt (2006).

\(^2\)Regarding the formalization of the worker’s utility function, we follow Englmaier and Wambach (2010) so that the worker’s disutility due to inequity aversion is convex in income inequity. Moreover, in line with Fehr and Schmidt (1999), we assume that the worker’s feelings of envy exceed his feelings of empathy.
We find that the welfare-maximizing contract stipulates an equal sharing rule in each productive state. Notably, this result contradicts the Pareto-efficient contract with purely self-regarding workers where a fixed wage suffices to implement the first-best solution. Intuitively, a contract that awards the worker half of the surplus in each state avoids income inequity altogether and thus avoids any welfare loss due to inequity aversion. By contrast, the profit-maximizing contract if the worker possesses no bargaining power, never implements the first-best solution. In particular, the firm sets the wage so that it earns more than the worker in any state. Interestingly, this implies that agency costs arise due to the worker’s inequity aversion even though effort is contractible. Moreover, the optimal contract depends on the realized output level although moral hazard is absent and incentives are not needed.

The optimal contract with bargaining is also state-dependent, involves profit sharing, and entails a welfare loss due to inequity aversion whenever bargaining power is not equally distributed among the parties. Depending on the parties’ relative bargaining power, the firm may earn more or less than the worker. Compared to the case with purely selfish workers, the optimal contract, however, implements a more egalitarian distribution of surplus. The reason is that such a contract lowers the welfare loss due to inequity aversion. Interestingly, the worker’s optimal wage may be increasing or decreasing in the worker’s concern for inequity. Intuitively, depending on the relative bargaining power of labor, and, hence, relative earnings, increasing the fixed wage may entail more or less income inequity. The optimal fixed wage reduces the possible welfare loss due to both advantageous and disadvantageous income inequity from the worker’s perspective. Notably, the optimal contract approaches the welfare-optimal contract as the parties’ bargaining weights converge.

Our results help explain why workers are willing to accept lower wages in times of crisis but demand higher wages in times of economic rise. Moreover, we can conclude that an increase in the worker’s concern for equity leads to a more equitable income distribution in the bargaining process and thus, a more even overall distribution of the productive surplus from the employment relationship. Our findings also imply that raising the bargaining power of the less powerful party may increase overall welfare. It may thus be socially efficient to design institutions on the labor market in a way to achieve a well-balanced distribution of bargaining power among firm owners and productive workers.

The present paper brings together the literature on wage bargaining and that on social preferences. Optimal labor contracts in Nash-bargaining settings have been extensively discussed (see, e.g., Pissarides (2000), Demougin and Helm (2006)). Typically, the optimal sharing rule awards each party its outside option and divides the additional surplus of the relationship proportional to the players’ bargaining power. Yet we are aware of only one paper that analyzes the consequences of social preferences in such an environment: Dur and Tichem (2012) study the effects of altruism and spite on relational incentives in a repeated bargaining-setting.3 In contrast to our work, they emphasize the relevance of social relations between the principal and the agent and focus on the provision of implicit incentives in a moral-hazard setting. Moreover,

3 The impact of envy on relational incentive contracts without bargaining has been investigated by, e.g., Kragl and Schmid (2009).
both parties have social preferences in their model and are concerned with relative utilities levels rather than relative net income. We contribute to that literature by focusing on the impact of workers’ preferences for equitable surplus distributions in regard to the firm on the optimal labor contract with bargaining.\footnote{That is, in line with most papers on vertical social preferences, we consider the firm as an unemotional entity.}

Following a multiplicity of field and laboratory evidence on the relevance of social preferences, during recent years, there is a steadily evolving literature that investigates the effects of other-regarding preferences in theoretical models of optimal decision-making and labor contracts. Alternative approaches regarding the formalization of social preferences have been presented (Rabin (1993), Fehr and Schmidt (1999), Bolton and Ockenfels (2000), Falk and Fischbacher (2006)). The majority of subsequent studies is either concerned with inequity aversion as proposed by Fehr and Schmidt (1999), that is, workers feel envy and empathy, or focus on envy alone. As already noted above, almost all these studies focus on optimal incentive contracting in principal-agent models in which the principal possesses all bargaining power. Moreover, the majority of papers focus on fairness concerns among workers, i.e., horizontal social preferences.\footnote{For example, Demougin, Fluet, and Helm (2006), Bartling and von Siemens (2010), and Neilson and Stowe (2010) analyze the impact of envy or inequity aversion on optimal independent incentive contracts. Inequity aversion in rank-order tournaments is analyzed by, e.g., Demougin and Fluet (2003) and Grund and Sliwka (2005). Other contributions compare the efficiency of different peer-dependent incentive regimes with social preferences (e.g., Itoh (2004), Demougin and Fluet (2006), Goel and Thakor (2006), Rey-Biel (2008), Bartling (2011)).}

More closely related to our work are the papers by Dur and Glazer (2008) and Englmaier and Wambach (2010) which examine optimal labor contracts when workers care about inequality relative to the principal. While Englmaier and Wambach (2010) analyze inequity aversion, Dur and Glazer (2008) focus on envy towards the firm and, in line with our work, consider an environment with contractible effort. By contrast, Englmaier and Wambach (2010) also analyze optimal contracts under moral hazard. Both papers moreover consider the impact of risk aversion but assume that the principal has the whole bargaining power. In line with our results, both studies find that the firm benefits from profit sharing due to the worker’s social preference. We contribute to that literature by extending the discussion to optimal contracts under bargaining. In particular, we highlight that the first-best solution is never implemented under the profit-maximizing contract while it may be achieved with bargaining by converging the bargaining power of the worker and the firm. We moreover show that inequity aversion not only reduces social welfare but also affects the optimal distribution of surplus with bargaining. In particular, a higher concern for equity increases (lowers) the worker’s share when he has less (more) bargaining power than the firm.

The remainder of the paper proceeds as follows. The next section introduces the model. In Sections 3 and 4, we derive the welfare-maximizing contract and the profit-maximizing if the worker has no bargaining power, respectively. In Section 5, we derive our main results concerning the optimal labor contract with bargaining and analyze the impact of the worker’s bargaining weight and inequity aversion on the results. Finally, Section 6 offers some concluding remarks.
2 The Model

We model the interaction between a profit-maximizing firm (principal) and a utility-maximizing worker (agent). The worker exerts effort \( e \) in order to produce verifiable output \( Q \) and incurs effort costs \( c(e) \), where \( c(0) = c'(0) = 0 \), \( c'(e) > 0 \) if \( e > 0 \), \( c''(e) \geq 0 \), and \( \lim_{e \to 1} c'(e) = \infty \). Output can take a high or low value, i.e., \( Q \in \{L, H\} \) with \( H > L \geq 0 \). The level of output is low if the worker exerts ‘normal’ effort, i.e., \( e = 0 \), and high output is realized with probability \( \Pr[Q = H | e] = \epsilon \).

The worker is risk neutral but inequity averse towards the firm in the sense of Fehr and Schmidt (1999). Formally, we employ the utility function proposed by Englmaier and Wambach (2010) but depart by assuming that the agent is concerned with relative net income, i.e., in any state the worker compares his wage net of effort costs to firm profit.\(^6\) Accordingly, the agent’s utility of wage \( W \) is given by:

\[
U(Q, W, e) = W - c(e) - \gamma G(D),
\]

where \( D := [Q - W] - [W - c(e)] \),
\[
G(0) = G'(0) = 0,
\]
\[
G'(D) < 0 \text{ if } D < 0, \quad G'(D) > 0 \text{ if } D > 0,
\]
\[
G''(D) > 0,
\]
\[
G(-D) < G(D) \text{ for } D > 0
\]

Denoting by \( D \) the difference in firm profit \( (Q - W) \) and the worker’s net payoff \( (W - c(e)) \), the function \( G(D) \) captures the worker’s preference for equitable income distributions in any state. Figure 1 illustrates the basic properties of the function as stated above. Accordingly, the worker’s disutility from income inequity is represented by a strictly convex function which is positive whenever \( D \neq 0 \) and not symmetric around the equitable allocation \( (D = 0) \). In the reminder, we speak of income inequity whenever \( D \neq 0 \). The asymmetry of the function \( G(D) \) implies that the worker suffers more from disadvantageous inequity \( (D > 0) \) than he dislikes advantageous inequity \( (D < 0) \) for some given absolute difference in net payoffs. Intuitively, in line with Fehr and Schmidt (1999), the worker’s propensity for envy exceeds his feelings of compassion. Moreover, the worker’s disutility increases overproportionally as income differences rise in either direction. Finally, the parameter \( \gamma > 0 \) denotes the individual weight the worker puts on achieving equitable outcomes.

We focus on the case where the worker’s effort is contractible, i.e., there is no moral hazard and thus no need for incentives. Yet, in contrast to the case with a purely self-regarding worker, the distribution of surplus across states plays a role not only for the worker’s willingness to accept a contract but also for the size of the overall surplus. Accordingly, the wage \( W(e, Q) \) is

\(^6\)Former papers on vertical inequity aversion (Englmaier and Wambach (2010), Dur and Glazer (2008)) assume that the agent compares his gross wage to firm profit, thereby presuming that income inequality coincides with income inequity. Yet we believe that effort costs play an important role in relative income comparisons given that the considered parties are not symmetric. Intuitively, an agent who perceives his wage as unfair may reduce effort to reestablish equity. By contrast, an equal split of surplus may be perceived as unfair if the agent’s associated productive effort is huge.
paid if the worker exerts the stipulated effort level and depends on the realized output level:\(^7\)

\[
W = \begin{cases} 
0 & \text{if } e < e^* \\
w & \text{if } e \geq e^* \text{ and } Q = L \\
w + \Delta & \text{if } e \geq e^* \text{ and } Q = H
\end{cases}
\]

The worker is paid a fixed wage \(w\) in any state and obtains an additional amount of \(\Delta\) if output is high. The size of the latter payment hence determines the distribution of surplus across states. In the remainder, we will refer to \(\Delta\) as the wage spread. Under such a contract, the parties’ respective expected payoffs become:

\[
\pi_W = w + e\Delta - c(e), \quad (2)
\]
\[
\pi_F = e(H - w - \Delta) + (1 - e)(L - w), \quad (3)
\]

where \(\pi_W\) denotes the worker’s expected net payoff and \(\pi_F\) the expected firm profit. The worker’s expected utility depends on \(\pi_W\) and the expected level of income inequity \(D = Q - 2W + c(e)\) in the two states. Denote the (ex-post) difference in net payoffs in state \(Q = H\) by \(D_H := H - 2w - 2\Delta + c(e)\) and in state \(Q = L\) by \(D_L := L - 2w + c(e)\). The worker’s expected utility is thus given by:

\[
EU = \pi_W - e\gamma G(D_H) - (1 - e)\gamma G(D_L) \quad (4)
\]

The timing is as follows. First, the worker and the firm bargain over the labor contract \(\{e, w, \Delta\}\). Formally, we apply the Generalized Nash-bargaining framework and denote with \(\alpha \in [0, 1]\) the bargaining power of labor (Nash (1950)). If bargaining fails, both parties receive their respective outside options which we, for simplicity, set to zero.\(^8\) If bargaining is successful, the worker undertakes effort. Afterwards output \(Q\) is realized and the worker is paid. As a

\(^7\)It is well known that, in standard principal-agent models with purely self-regarding workers and contractible effort, the first-best solution is implemented by means of a state-independent fixed-wage contract. As we will show in the following sections, this does not hold true for the case of inequity averse workers (see also Englmaier and Wambach (2010), Dur and Glazer (2008)). In particular, we find that \(\Delta\) is always non-zero in the optimal contract.

\(^8\)This may appear like a strong assumption. Doing so, however, greatly simplifies the exposition of the paper. Allowing for non-zero outside options does not change our main results but adds complexity to the model. In particular, as long as both parties’ outside options do not exceed half of the surplus, our results reestablish.
benchmark, we also analyze the case where the worker has no bargaining power \( \alpha = 0 \) so that the firm makes a take-it-or-leave-it offer to the worker.

3 Welfare-maximizing Contract

In this section, we determine the welfare-maximizing (Pareto-efficient) contract \( \{ \hat{e}, \hat{w}, \hat{\Delta} \} \). The overall expected welfare is given by the sum of expected firm profit \( \pi_F \) as given in equation (2) and the worker’s expected utility \( EU \) as given in equation (4). Thus, the welfare-maximizing contract solves:

\[
\max_{e, w, \Delta} S(e, w, \Delta) = L + e(H - L) - c(e) - e\gamma G(D_H) - (1 - e)\gamma G(D_L)
\]

s.t. \( \pi_F, EU \geq 0 \) (I)

By inspection of the objective function, overall welfare \( S \) is decreasing in the worker’s disutility due to inequity aversion. The following proposition states the solution to problem (I) and shows that the first-best wage payments \( \hat{w} \) and \( \hat{\Delta} \) are such that \( D_H = D_L = 0 \). That is, in the welfare-maximizing solution, parties split the ex-post surplus \( S_Q = Q - c(e) \) equally in any state.

**Proposition 1** The welfare-maximizing contract \( \{ \hat{e}, \hat{w}, \hat{\Delta} \} \) stipulates an equal sharing rule in any state. In particular, the optimal contract elements are:

\[
\hat{e} = \arg \max_e S(e, \hat{w}, \hat{\Delta}) \iff H - L = c'(\hat{e})
\]

\[
\hat{w} = \frac{S_L}{2} + c(\hat{e}),
\]

\[
\hat{\Delta} = \frac{H - L}{2}.
\]

The associated first-best expected surplus is given by:

\[
\hat{S} = S(\hat{e}, \hat{w}, \hat{\Delta}) = L + \hat{e}(H - L) - c(\hat{e})
\]

**Proof.** See the Appendix. ■

The proposition shows that the worker’s wage is \( \hat{w} = S_L/2 + c(\hat{e}) \) if output is low and \( \hat{w} + \hat{\Delta} = S_H/2 + c(\hat{e}) \) if it is high. Accordingly, firm profit amounts to \( S_L/2 \) and \( S_H/2 \), respectively. Hence, in contrast to the case with a purely self-regarding worker, the first-best contract is not a fixed-wage contract but awards the worker half of the surplus in each state in addition to his cost of effort. This implies that not only the firm earns more when output is high but also the worker (\( \hat{\Delta} > 0 \)). Obviously, the first-best contract maximizes overall welfare by avoiding the occurrence of inequity aversion altogether. As a result, the contract implements first-best effort \( \hat{e} \) for which marginal productivity equals marginal costs of effort:

\[
H - L = c'(\hat{e})
\]
4 Profit-maximizing Contract

In this section, we analyze the benchmark case where the firm possesses all the bargaining power, i.e., $\alpha = 0$. The case coincides with the principal-agent game in which the firm makes the worker a take-it-or-leave-it offer. We thus replicate and extend some of the findings from the existent literature on vertical inequity aversion without bargaining (Englmaier and Wambach (2010), Dur and Glazer (2008)).

When the firm makes a take-it-or-leave-it offer, its optimization problem is given by:

$$\max_{e,w,\Delta} \pi_F = L + e(H - L) - (w + e\Delta) \quad \text{(II)}$$

subject to

$$EU = w + e\Delta - c(e) - e\gamma G(D_H) - (1 - e)\gamma G(D_L) \geq 0 \quad \text{(PC)}$$

Condition (PC) denotes the worker’s participation constraint. In the proof of the following proposition, we verify that condition (PC) is binding in the optimal contract. This implies that the firm’s expected wage costs amount to the worker’s cost of effort plus the so-called inequity premium. The latter is the worker’s expected disutility due to income inequity across states and is represented by the absolute value of the last two terms in $EU$. Notably, the level of wage payments $w, \Delta$ affects the size of the inequity premium. For any effort level, the firm seeks to minimize the expected inequity premium. We show that this is achieved by setting $w, \Delta$ such that income inequity $D$ is equal across states. The following proposition characterizes the optimal contract.

**Proposition 2** Suppose that the firm possesses all the bargaining power. Then under the optimal contract $(e^*, w^*, \Delta^*)$:

(i) effort is first-best, i.e., $e^* = \hat{e}$,

(ii) income inequity is equal across states; $D_H = D_L =: D^*$,

(iii) the optimal wage spread is $\Delta^* = \hat{\Delta} = (H - L)/2$,

(iv) the firm earns more than half of the surplus in each state; $D^* > 0$,

(v) the optimal fixed wage $w^*$ is uniquely defined by the solution to

$$w = c(e^*) - e^*\Delta^* + \gamma G(D(w)),$$

(vi) the worker’s expected wage is given by

$$EW^* = w^* + e^*\Delta^* = c(e^*) + \gamma G(D^*).$$

**Proof.** For the proof of (i)-(iii), see the Appendix. There we show that the participation constraint (PC) is binding, and thus, the optimal fixed wage $w^*$ is implicitly defined by equation (10), where $D$ is a function of $w$. The left-hand side of the latter equation is the identity function, thus strictly increasing in $w$ with a slope of 1, as shown in Figure 2. Define by $g(w)$ the function on the right-hand side of equation (10). Since $c(e^*) - e^*\Delta^*$ is a constant, it implies that $\frac{\partial g(w)}{\partial w} = -2G'(D)$. Therefore, for $w < \frac{L + c(e)}{2}$, i.e., for $D > 0$, $g(w)$ is decreasing, while
for \( w > \frac{L+c(e)}{2} \), \( g(w) \) is increasing and moreover, since in the second case the feasible solutions are for \( G'(D) > -1/2 \), the slope of \( g'(w) < 1 \). In addition, one can easily verify that \( g(w) \) is a convex function with a minimum value \( g\left(\frac{L+c(e)}{2}\right) = c(e^*) - e^*\Delta^* \). In Figure 2, we have plotted two examples of the function depending on whether minimum value of \( g(w) \) is negative or positive. Comparing the values of both functions for \( w = \frac{L+c(e)}{2} \) (the left and the right hand sides of equation (10)), we can see that \( w = \frac{L+c(e)}{2} > c(e^*) - e^*\Delta^* = g\left(\frac{L+c(e)}{2}\right) \), since \( \frac{S}{2} > 0 \)

Thus, for \( w < \frac{L+c(e)}{2} \), the function at the left hand side is negative, for negative \( w \), it is strictly increasing and for \( w = \frac{L+c(e)}{2} \) is greater than \( g(w) \), while the function at the right hand side is positive for relatively small \( w \), it is strictly decreasing and for \( w = \frac{L+c(e)}{2} \) has a smaller value than the one of the identity function. Therefore, there is a solution of equation (10) such that \( w^* < \frac{L+c(e)}{2} \) (Figure 2 depicts two examples of the intersection of both functions, where the optimal fixed wage is negative or positive). In addition, for \( w > \frac{L+c(e)}{2} \), \( g(w) \) has a smaller slope than the identity function, and thus, both functions cannot intersect each other implying that the found solution is indeed a unique one. Moreover, \( w^* < \frac{L+c(e)}{2} \), i.e., the solution is such that the payoff of the firm exceeds the one of the worker implying that \( D^* > 0 \), for any \( 0 < \gamma < \infty \).

Note, however, that if \( \gamma \) converges towards infinity (the slope of \( g(w) \) for \( w < \frac{L+c(e)}{2} \) will be almost a vertical line), the optimal fixed wage \( w^* \rightarrow \frac{L+c(e)}{2} \), and thus, the optimal to the firm contract converges towards the welfare-maximizing one.\(^9\) Since (PC) is binding at the optimum, the expected wage equals the costs of effort and the minimized inequity premium, as shown in (11).

\(^9\)To examine this result in another way, let us assume that there is still some \( \gamma \), sufficiently large, such that \( D^* = 0 \) is optimal for the firm, i.e., \( EU^* = \frac{S}{2} > 0 \). Since between any two different real numbers, one can find another real one, there exists a positive number between \( \frac{S}{2} \) and 0, such that it equals \( \frac{S}{2} - 2\varepsilon \) for some \( \varepsilon > 0 \). Thus, there should be a small number \( \tilde{\varepsilon} \), such that \( \varepsilon / \gamma = G(2\tilde{\varepsilon}) \), and since \( \gamma \) is sufficiently large, \( G(2\tilde{\varepsilon}) \) will be close to zero, or \( \tilde{\varepsilon} \) will be closed to zero and thus, smaller than \( \varepsilon \). If the firm pays instead the optimal fixed wage a wage \( \bar{w} = w^* - \tilde{\varepsilon} \), the profit of the firm will be \( \pi_F = \frac{S}{2} + \bar{\varepsilon} \), thus, increased, while the expected utility of the worker will be \( EU = \frac{S}{2} - \varepsilon - \gamma G(2\tilde{\varepsilon}) \), or equivalently, \( EU = \frac{S}{2} - 2\tilde{\varepsilon} - \varepsilon > \frac{S}{2} - 2\varepsilon \), where the later is positive by definition of \( \varepsilon \). Hence, the firm profit is increased, while the participation constraint is still satisfied, which contradicts to the optimality of the contract.
As under the first-best contract, the firm implements the first-best effort level $\hat{c}$. Moreover, the firm sets the wage spread $\Delta$ so that the worker obtains half of the increase in output due to work effort. Notably, this result is in sharp contrast to the case of self-regarding agents in which a fixed-wage contract ($\Delta = 0$) implements the first-best solution. With an inequity averse worker, the optimal contract is state-dependent ($\Delta > 0$) and does generally not implement the first-best solution. To see this, observe that the firm faces an interesting trade-off regarding expected wage costs (sum of effort costs and inequity premium). Increasing the worker’s wage towards half of the surplus in any state ($SQ/2$) reduces the inequity premium and thus expected wage costs. At the same time, however, doing so directly increases wage costs and hence reduces the firm’s profit. The contract $(c^*, w^*, \Delta^*)$ optimally trades off these two counteracting effects on firm profit. As in the case with a self-regarding worker, the firm extracts all the surplus from the relationship on expectation. The associated firm profit hence coincides with surplus and becomes:

$$\pi_F = L + e^*(H - L) - c(e^*) - \gamma G(D^*) = S(e^*, w^*, \Delta^*) =: S^* \quad (12)$$

The above equation shows that surplus, and consequently, firm profit are reduced by the worker’s disutility due to inequity aversion, since $D^* > 0$, i.e., there is some income inequity between firm and worker. The reason is that the worker suffers from envy and must be compensated accordingly. Note that, from Proposition 2 (iv), we know that the optimal contract, never leads to $D^* < 0$. Intuitively, this is never optimal for the firm because it then not only earns less than the worker but also needs to pay inequity-premium costs because the worker would suffer from empathy.

By inspection of the foregoing proposition, the optimal level of effort, $e^*$, and the wage spread, $\Delta^*$ are unaffected by the worker’s concern for income equity, $\gamma$. By contrast, equation (10) implies that the optimal fixed wage $w^*$ implicitly depends on $\gamma$, as stated in the following proposition.

**Proposition 3** The optimal fixed wage $w^*$ is increasing in the individual weight the worker puts on achieving equitable outcomes $\gamma$.

**Proof.** Applying the Implicit-Function Theorem to equation (10) yields:

$$\frac{\partial w^*}{\partial \gamma} = -\frac{-G(D)}{1 + 2\gamma G'(D)} \quad (13)$$

As shown in the proof of Proposition 2 in the Appendix, feasible are solutions such that $\gamma G'(D) > -1/2$, and thus, the denominator of (13) is positive. Together with $G(D) > 0$ if $D \neq 0$, this implies that the derivative of $w$ with respect to $\gamma$ is positive, which concludes the proof. ■

Intuitively, the more inequity averse the worker, the more he suffers from the fact that
the firm earns more. The firm counteracts the increase in inequity-premium costs by reducing income inequity $D$. That is, it increases the worker’s wage by the same amount in both states, hence raises the fixed wage. Clearly, doing so is useful only as long as the worker’s net income falls below firm profit. The reason is that, if $D = 0$, a further increase of the fixed wage leads to income inequity as well - in this case advantageous inequity - and hence raises inequity-premium costs due to the worker’s feelings of empathy. The consequences of an increase in $\gamma$ on firm profit, income and utility of the worker under the optimal contract are summarized in the next proposition.

**Proposition 4** The expected profit of the firm decreases in the worker’s concern for income inequity, $\gamma$, while the expected payoff of the worker increases. The worker’s expected utility is, however, unaffected by his concern for income inequity.

**Proof.** By Proposition 2, the payoffs of both parties (2) and (3) can be rewritten as follows:

\[ \pi^*_W = c(e^*) + \gamma G(D(w^*(\gamma))) \]
\[ \pi^*_F = L + e^*(H - L) - c(e^*) - \gamma G(D(w^*(\gamma))) \]

Taking the derivatives with respect to $\gamma$, and applying Lemma 3, yields

\[ \frac{\partial \pi^*_W}{\partial \gamma} = \frac{1}{1+2\gamma G'(D)} G(D) \]
\[ \frac{\partial \pi^*_F}{\partial \gamma} = -\frac{1}{1+2\gamma G'(D)} G(D) \]

As stated above, $\gamma G'(D) > -1/2$ and $G(D) > 0$ for $D \neq 0$, therefore, we have $\frac{\partial \pi^*_W}{\partial \gamma} > 0$ and $\frac{\partial \pi^*_F}{\partial \gamma} < 0$. Finally, the firm chooses the contract so that condition (PC) is binding for any $\gamma$, yielding $EU^* = 0$. ■

The intuition is straightforward. As the worker’s concern for inequity rises, the firm needs to pay him a larger inequity premium and hence raise his wage, thereby reducing its own profit. However, if the worker is extremely inequity averse, i.e., $\gamma \rightarrow \infty$, the firm optimally chooses $w^*$ so that the worker earns almost half of the surplus in any state ($G(D^*) \rightarrow 0$). Intuitively, when the individual weight the worker puts on achieving equitable outcomes $\gamma$ is so large, the firm increases the fixed wage, which yields to a decrease in the value of $G(D)$. Yet, the forgoing findings imply that the net difference of the firm profit and the worker’s net payoff is always positive ($D^* > 0$), and thus, the profit-maximizing contract leads to reduced welfare. Notably, this implies that, in contrast to the case with a self-regarding worker, agency costs then arise although effort is contractible. These observations lead to the following conclusion.

**Corollary 1** With an inequity averse worker and contractible effort, the profit-maximizing wage contract $(e^*, w^*, \Delta^*)$ does not implement the first-best solution. In particular, the optimal contract then leads to income inequity in all states ($D^* > 0$). As a result, firm profit, and thus, overall welfare are reduced by the amount of the inequity premium; $S^* = \hat{S} - \gamma G(D^*) < \hat{S}$.

The foregoing results are related to the findings by Englmaier and Wambach (2010) and Dur and Glazer (2008). In particular, both studies also find that surplus sharing is optimal with inequity aversion and envy, respectively, so that firm profit is decreasing in the worker’s social preference.
5 Optimal Contract with Nash Bargaining

In this section, we turn to the case where workers and the firm engage in Generalized Nash bargaining. Denoting with $\alpha \in (0,1)$ the bargaining power of labor, the optimal contract is then defined as:

$$e^{**}, w^{**}, \Delta^{**} = \arg \max_{e, w, \Delta} (EU)^\alpha (\pi_F)^{1-\alpha}$$  \hspace{1cm} (III)

Notably, in contrast to models with purely selfish workers, in our model, the surplus to be shared in the process of bargaining, $S(e, w, \Delta) = EU + \pi_F$, is (weakly) decreasing in the level of income inequity among the parties. This implies that the surplus endogenously depends on the result of bargaining, that is, the parties’ negotiated shares as determined by the contract $(e, w, \Delta)$. This endogeneity makes solving the above problem non-trivial.

As a benchmark, initially consider the case of a purely self-regarding worker; $\gamma = 0$. In such a case, we have $EU = \pi_W$ and the above problem (III) corresponds to the standard Nash bargaining scenario. In this case, surplus becomes $S(e, w, \Delta) = \hat{S}$, as given in equation (8), and the optimal contract assigns both parties a share of this surplus that corresponds to their bargaining power, respectively. More specifically, the worker obtains $\alpha \hat{S}$, while the firm earns $(1-\alpha)\hat{S}$. As we will show in the following, the surplus with an inequity averse agent depends on the worker’s concern for inequity, $\gamma$, and generally falls below $\hat{S}$. Moreover, the parties’ optimal shares of surplus depend not only on $\alpha$ but also on $\gamma$ and are in general different from those with a purely selfish worker. The following proposition presents the solution to problem (III), and thus, characterizes the optimal contract with Nash bargaining and an inequity averse worker.

Proposition 5 Suppose that the firm and the worker engage in Nash bargaining and the worker’s bargaining power is given by $\alpha \in (0,1)$. Then under the optimal contract $(e^{**}, w^{**}, \Delta^{**})$:

(i) effort is first-best, i.e., $e^{**} = \hat{e}$;
(ii) income inequity is equal across states; $D_H = D_L =: D^{**}$,
(iii) the optimal wage spread is $\Delta^{**} = \hat{\Delta} = (H - L)/2$,
(iv) overall surplus is given by

$$S^{**} := S(e^{**}, w^{**}, \Delta^{**}) = \hat{S} - \gamma G(D^{**}),$$  \hspace{1cm} (16)

(v) the expected utility of the worker and expected firm profit are given by:

$$EU^{**} := EU (e^{**}, w^{**}, \Delta^{**}) = \alpha \left[ \hat{S} - \gamma G(D^{**}) \right] \frac{1 + 2\gamma G'(D^{**})}{1 + 2\alpha \gamma G'(D^{**})};$$  \hspace{1cm} (17)

$$\pi_F^{**} := \pi_F (e^{**}, w^{**}, \Delta^{**}) = (1-\alpha) \left[ \hat{S} - \gamma G(D^{**}) \right] \frac{1}{1 + 2\alpha \gamma G'(D^{**})}$$  \hspace{1cm} (18)

Proof. See the Appendix. ■

10For the Nash-bargaining solution in economic modelling see, e.g., Nash (1950), Binmore, Rubinstein, and Wolinsky (1986), or Muthoo (1999).
The implemented effort level is first-best also with Nash bargaining. In line with the profit-maximizing contract, the optimal contract with bargaining is state-dependent and equalizes the difference in net payoffs across states. Comparing the surplus with bargaining, \( S^{**} \), to the one under the profit-maximizing contract, \( S^* \), as given in equation (12), shows that expressions coincide if \( D^* = D^{**} \). In the foregoing section, we have verified that income inequity, \( D^* \), is always positive, i.e., the firm earns more, and surplus always falls below first-best. In the following, we investigate the optimal level of income inequity under bargaining, \( D^{**} \), depending on the distribution of bargaining power between the parties. Closer inspection of equations (16), (17) and (18) leads to the following conclusions for \( D^{**} \) and regarding the parties’ relative shares of surplus under the optimal contract.

**Corollary 2** Under the optimal contract \( (e^{**}, w^{**}, \Delta^{**}) \):

(i) if \( \alpha \in (0, 1/2) \), firm profit exceeds the worker’s net income in each state; \( D^{**} > 0 \),

(ii) if \( \alpha \in (1/2, 1) \), firm profit falls below the worker’s net income in each state; \( D^{**} < 0 \), and

(iii) if \( \alpha = 1/2 \), firm profit coincides with the worker’s net income in each state; \( D^{**} = 0 \).

(iv) Accordingly, the following holds for the worker’s expected utility and firm profit:

\[
EU^{**} > \alpha S^{**} \quad \text{and} \quad \pi_F^{**} < (1 - \alpha) S^{**} \quad \text{if} \quad \alpha \in (0, 1/2)
\]
\[
EU^{**} = \bar{S}/2 \quad \text{and} \quad \pi_F^{**} = \bar{S}/2 \quad \text{if} \quad \alpha = 1/2
\]
\[
EU^{**} < \alpha S^{**} \quad \text{and} \quad \pi_F^{**} > (1 - \alpha) S^{**} \quad \text{if} \quad \alpha \in (1/2, 1)
\]

**Proof.** The optimal contract with Nash bargaining implements wage spread \( \Delta^{**}, e^{**} \) disregarding the value of the bargaining power. Therefore, the shares of both parties for different values of \( \alpha \), or respectively the differences in the net income, are adjusted through the wage \( w^{**} \). Moreover, by definition \( D^{**} \) is strictly decreasing in the fixed wage, while \( w^{**} \) is strictly increasing in \( \alpha \), for \( \alpha \neq 1/2 \) (consider Proposition 6 (i)).

For every \( \gamma \), the optimal income inequity \( D^{**} = 0 \), if and only if the optimal fixed wage \( w^{**} = \frac{L + c(e^{**})}{2} \) satisfies the equation (44), and thus, it follows:

\[
\frac{L + c(e^{**})}{2} = c(e^{**}) + \alpha(L + e^{**}(H - L) - c(e^{**})) - e^{**}\Delta^{**}
\]

or equivalently, \( \frac{\bar{S}}{2} = \alpha \bar{S} \). The later yields \( \alpha = 1/2 \). To conclude, for \( \alpha = 0 \), \( D^{**} > 0 \) as shown in Section 4, \( D^{**} = 0 \), for \( \alpha = 1/2 \) and \( D^{**} \) is strictly decreasing in \( \alpha \) (for \( \alpha \neq 1/2 \)).

By equation (17) the worker still receives \( \alpha \)-share of the accumulated surplus, but multiplied by a factor \( \frac{1+2\gamma G'(D^{**})}{1+2\alpha \gamma G'(D^{**})} \), depending on the weight, the worker puts on achieving equitable outcome, \( \gamma \). One can easily verify that this factor is greater than 1, if and only if \( (1 - \alpha)G'(D^{**}) > 0 \), or equivalently, \( G'(D^{**}) > 0 \), i.e., the profit of the firm exceeds the one of the worker, and the factor is less than one otherwise. Thus, for \( \alpha \in (0, 1/2) \) an inequity-averse worker obtains a larger share than \( \alpha \)-share of the accumulated surplus, and for \( \alpha \in (1/2, 1) \) actually a smaller one. In addition, if \( \alpha = 1/2 \), as shown above, \( D^{**} = 0 \) and equation (17) implies that both parties share equally half of the first-best surplus. The profit of the firm follows similar logic. If the bargaining power of labor is less than a half, the firm will obtain less than \( (1 - \alpha) \)-share of the
surplus and for a bigger $\alpha$, vice versa.

Parts (i)-(iii) of the above corollary show that there will always be some income inequity in the optimal contract unless both parties have the same bargaining power. In the course of bargaining, the net production, $Q - c(e)$, is distributed among the parties in each state. Recall that $D = (Q - W) - (W - c(e))$. Hence, similar to the case with purely selfish agents, the parties earn the same net income, $(Q - c(e))/2$, if $\alpha = 1/2$ while the firm earns a larger (smaller) share of $Q - c(e)$ than the worker if $\alpha < 1/2$ ($\alpha > 1/2$). It is worth noting the difference to the optimal contract with a take-or-leave-it offer in the foregoing section. With bargaining, it is not always the firm who earns more but also the worker may earn more or the same.

Part (iv) of the corollary summarizes important observations regarding the relative utility levels of the parties under the optimal contract. These results allow for a comparison with the standard case with purely selfish workers where the firm and the worker obtain $(1 - \alpha)\hat{S}$ and $\alpha\hat{S}$, respectively. The results in (iv) imply that, with an inequity averse worker, the party with the smaller bargaining power obtains a larger share of the surplus compared to the case with a purely selfish worker, respectively. Obviously, the opposite is true for the party with the higher bargaining power. That is, inequity aversion leads to a more egalitarian distribution of surplus. The reason is that the associated optimal shares lead to an increase in the overall surplus. To grasp the intuition, suppose that one party - say party 1 - has a smaller bargaining power than the other party - say party 2. Now consider the impact of an increase in the two party’s bargaining power on both parties’ shares and overall surplus, respectively: Since party 1’s bargaining power is below $1/2$ initially, by Corollary 2 (i),(ii), party 1 earns less than party 2. An increase in party 1’s bargaining weight then not only increases party 1’s share but at the same time reduces the level of income inequity and thus raises overall surplus. Obviously, the latter has a favorable effect on both parties’ income. Consequently, the optimal share of party 1 rises compared to the case without equity concerns. By contrast, party 2’s bargaining power is above $1/2$ so that a further increase in the bargaining weight raises income inequity and thus reduces overall surplus. As a result, the optimal share of party 2 is lower than in the case without equity concerns.

The foregoing results directly lead to the following conclusion regarding the efficiency of the optimal contract with Nash bargaining.

**Corollary 3** With an inequity averse worker and contractible effort, the optimal wage contract with Nash-bargaining $(e^{**}, w^{**}, \Delta^{**})$:

(i) does not implement the first-best solution if bargaining power is not equally distributed among the parties; $\alpha \neq 1/2$. Then welfare falls below the first-best level; $S^{**} < \hat{S}$.

(ii) implements the first-best solution if both parties possess the same bargaining power; $\alpha = 1/2$. The optimal contract then stipulates an equal sharing rule and welfare is at the first-best level; $S^{**} = \hat{S}$.

**Proof.** By Corollary 2 (i)-(iii), the optimal contract implements $D^{**} \neq 0$ whenever $\alpha \neq 1/2$. The claims then directly follow from equations (8) and (16) by noting that $G(D^{**}) > 0$ whenever $D^{**} \neq 0$ and $G(D^{**}) = 0$ if $D^{**} = 0$. ■
Notably, while the profit-maximizing contract in which the firm has the whole bargaining power, never leads to the first-best welfare, efficiency is reestablished with Nash bargaining if the firm’s bargaining power is sufficiently reduced so that both parties are equally powerful. Whenever one party possesses a higher bargaining power, it obtains a larger part of the surplus. It is worth noting the following two observations. First, the worker always earns a rent, that is, his expected utility is positive, if $\alpha > 0$. Second, high bargaining power above $1/2$ is not necessarily in the worker’s interest. The surplus is maximized for a bargaining power of $1/2$ since there is no income inequity, and the worker receives half of it. Increasing $\alpha$ above $1/2$ not only reduces the overall surplus (Corollary 3 (i)), but also yields the worker an expected utility is less than $\alpha S^{**}$ , ceteris paribus (Corollary 2 (iv)). Even though $\alpha$ is increased, and thus, he obtains a relatively larger share of the surplus, it is still possible that the worker’s utility is reduced for an increased value of $a$.

By Proposition (5), the optimal level of effort, $e^{**}$, and the wage spread, $\Delta^{**}$, are unaffected by the worker’s concern for income equity, $\gamma$, and the worker’s bargaining power, $\alpha$. Yet expected utility, firm profit, and surplus vary in both parameters. The reason is that the worker’s optimal fixed wage, and hence his overall payment in both states, reacts to changes in inequity aversion as well as bargaining power. The following proposition states the associated comparative statics.

**Proposition 6** The worker’s optimal fixed wage with Nash bargaining $w^{**}$

(i) is increasing in the worker’s bargaining power; $\alpha$,

(ii) is increasing in the worker’s concern for inequity, $\gamma$, if $\alpha \in (0, 1/2)$,

(iii) may be increasing or decreasing in the worker’s concern for inequity, $\gamma$, if $\alpha \in (1/2, 1)$.

**Proof.** Applying the Implicit-Function Theorem on equation (44) with respect to $\gamma$ and $\alpha$, and rearranging we obtain:

$$\frac{\partial w^{**}}{\partial \alpha} = \frac{(\hat{S} - \gamma G(D)) + 2\gamma G'(D)\pi_F}{1 + 2\gamma G''(D) + 4\alpha \gamma G'''(D)\pi_F} \quad (20)$$

$$\frac{\partial w^{**}}{\partial \gamma} = \frac{(1 - \alpha)G(D) + 2\alpha G'(D)\pi_F}{1 + 2\gamma G''(D) + 4\alpha \gamma G'''(D)\pi_F} \quad (21)$$

As shown in the proof of Proposition 5 in the Appendix, for a solution to exist, it must hold that $\gamma G'(D) > -1/2$. Since $\pi_F \geq 0$, $G''(D) > 0$, the denominator in both derivatives is strictly positive. Moreover, the first terms in both numerators are non-negative, since the accumulated surplus with inequity-averse agents, $\hat{S} - G(D)$, must be positive and the inequity premium is non-negative, $G(D) \geq 0$. The second term in both numerators may be negative if $D < 0$, respectively.

First, consider the numerator of equation (20) and assume that it is negative. Since $\gamma G'(D) > -1/2$, it follows that $-\pi_F < 2\gamma G'(D)\pi_F < -(\hat{S} - \gamma G(D))$, or equivalently, $\pi_F > \hat{S} - \gamma G(D)$. Hence, there is a contradiction, since the profit of the firm cannot exceed the accumulated by the relationship surplus, and consequently, the derivative of $w^{**}$ with respect to $\alpha$ is strictly positive.

Second, consider the numerator of equation (21). If $\alpha \in (0, 1/2)$, we have $\gamma G'(D) > 0$, and the numerator as well as the overall derivative of $w^{**}$ with respect to $\gamma$ is positive. If $\alpha \in (1/2, 1)$,
it follows that $\gamma G'(D) < 0$, and the numerator is positive iff $(1 - \alpha)G(D) > 2\alpha G'(D)\pi_F$ where $\pi_F = \pi_F(\gamma, \alpha, D)$ is given by equation (18). Simplifying the expression verifies that the sign of the numerator and hence the overall derivative in (21) depends on the relative levels of $\alpha, \gamma G(D)$, and $\hat{S}$.

The intuition of the first claim of the proposition is straightforward. The more powerful the worker is in the process of bargaining, the higher is also his wage in both states for given $\alpha$. Since the wage spread is optimally set so that the loss due to inequity aversion is minimized across states, the fixed wage rises accordingly.

With respect to claims $(ii)$ and $(iii)$, note that an increase in the fixed wage entails a higher inequity premium if $D > 0$, hence if $\alpha < 1/2$. The opposite is true if $\alpha > 1/2$ and $D < 0$. In the first case, an increase in the fixed wage lowers the worker’s disutility due to envy while, in the second case, the disutility due to empathy is raised. Accordingly, the optimal fixed wage rises in the former case in the worker’s inequity aversion as this not only raises the worker’s share but also yields a higher overall surplus. By contrast, if the worker is more powerful than the firm, a higher fixed wage has two counteracting effects. First, it raises the worker’s share but, second, it reduces the overall surplus due to an increased level of income inequity at the same time. The optimal fixed wage balances these two effects for given $\alpha, \gamma$ and $\hat{S}$. If the first effect outweighs the second, the optimal fixed wage is increasing in $\gamma$ and vice versa.

6 Conclusion

In the current paper, we introduce social preferences in a bargaining setting. More specifically, we consider the productive relationship between a firm and an inequity averse worker. The latter compares his own net income under the contract with firm profit and dislikes inequitable surplus distributions in any productive state. We show that the welfare-maximizing contract stipulates an equal sharing rule in each productive state. If the worker possesses no bargaining power, the profit-maximizing contract never implements the first-best solution while converging the bargaining power of the parties may reestablish the efficient contract.

Altogether, our findings show that an increase in the worker’s concern for equity leads to a more equitable income distribution in the bargaining process and thus, a more even overall distribution of the productive surplus from the employment relationship. Moreover, our results imply that it may be socially efficient to design institutions on the labor market in a way to achieve a well-balanced distribution of bargaining power among firm owners and productive workers.

Appendix

Proof of Proposition 1. Considering the maximization problem as shown in (I), we ignore the constraints at first, compute the optimal solution, and then show that it is indeed feasible. The
first-order conditions with respect to $e, w, \Delta$, respectively, are:

\begin{align*}
H - L - c'(e)[1 + e\gamma G'(D_H) + (1 - e)\gamma G'(D_L)] - \gamma G(D_H) + \gamma G(D_L) &= 0 \\
2e\gamma G'(D_H) + 2(1 - e)\gamma G'(D_L) &= 0 \\
2e\gamma G'(D_H) &= 0
\end{align*}

Equation (24) implies that, for a positive level of effort, if high output is realized optimal is that both parties split equally, i.e., $G(D_H) = 0$, or equivalently, $D_H = 0$. It follows that the second term in (23) is zero as well. With other words, if low output is observed the firm and the worker share again equally the accumulated surplus, i.e., $D_L = L - 2w + c(e) = 0$, or equivalently, $w = \frac{L - c(e)}{2} + c(e)$. Equal differences in the payoffs in both states implies $\Delta = \frac{H - L}{2}$. Substituting the result that at the optimum $D_L = D_H = 0$ in equation (22), we observe that the optimal effort is such that the marginal productivity equals the marginal costs $H = L = c'(e)$. Obviously, at the optimum the profit of the firm and the utility of the worker are strictly positive, and furthermore, since both parties split equally, there are no additional cost due to envy or empathy and the surplus of the relationship is as given in (8).

**Proof of Proposition 2 (i)-(iii).** The Lagrangian of the maximization problem (II) can be defined as follows:

\[
L = L + e(H - L) - (w + e\Delta) + \lambda[w + e\Delta - c(e) - e\gamma G(D_H) - (1 - e)\gamma G(D_L)]
\]

Thus, the first-order conditions with respect to $e, w, \Delta$ and the Kuhn-Tucker conditions are:

\begin{align*}
L_e &= H - L - \Delta + \lambda[\Delta - c'(e)[1 + e\gamma G'(D_H) + (1 - e)\gamma G'(D_L)] - \gamma G(D_H) + \gamma G(D_L)] = 0 \\
L_w &= -1 + \lambda[1 + 2e\gamma G'(D_H) + 2(1 - e)\gamma G'(D_L)] = 0 \\
L_\Delta &= -e + \lambda[e + 2e\gamma G'(D_H)] = 0 \\
\lambda &\geq 0 \text{ and } \lambda[w + e\Delta - c(e) - e\gamma G(D_H) - (1 - e)\gamma G(D_L)] = 0 \\
w + e\Delta - c(e) - e\gamma G(D_H) - (1 - e)\gamma G(D_L) &\geq 0
\end{align*}

Since the firm would implement effort level $e > 0$, rearranging (28) yields:

\[
\lambda[1 + 2\gamma G'(D_H)] = 1
\]

If $G'(D_H) = -1/2$ the equation has no solution, and moreover, if $G'(D_H) < -1/2$, $\lambda < 0$ and (29) is violated. Thus, $G'(D_H) > -1/2$, and

\[
\lambda = \frac{1}{1 + 2\gamma G'(D_H)}
\]

A positive $\lambda$ suggests that the participation constraint (PC) is binding. Rearranging (27) we
obtain:
\[
\lambda[1 + 2e\gamma G'(D_H) + 2(1 - e)\gamma G'(D_L)] = 1
\]  
(32)

Similarly to the above, we see that \(e\gamma G'(D_H) + (1 - e)\gamma G'(D_L) > -1/2\) implies that \(\gamma G'(D_L) > -1/2.\) Combining (31) and (32) we obtain
\[
1 + 2e\gamma G'(D_H) + 2(1 - e)\gamma G'(D_L) = 1 + 2\gamma G'(D_H),
\]  
(33)
or equivalently,
\[
G'(D_H) = G'(D_H).
\]  
(34)

Thus, the payoffs of both parties in each of the state are equal, and consequently \(\Delta^* = \frac{H - L}{2}.\)

We denote \(D^* := D_H = D_L,\) the difference between the payoffs at the optimum, substitute \(\lambda^*\) and \(\Delta^*\) in (26), and after canceling common terms, we obtain:
\[
\frac{1 + \gamma G'(D^*)}{1 + 2\gamma G'(D^*)}(H - L - c'(e)) = 0
\]  
(35)

Since \(\gamma G'(D^*) > -1/2,\) it follows that \(H - L = c'(e).\) \(\blacksquare\)

**Proof of Proposition 5.** The Nash Product from (III) can be rewritten as follows:
\[
N(e, w, \Delta) = [w + e\Delta - c(e) - e\gamma G(D_H) - (1 - e)\gamma G(D_L)]^\alpha[L + e(H - L) - (w + e\Delta)]^{1-\alpha}
\]  
(36)

Denoting \(C := EU/\pi_F,\) the first-order conditions with respect to \(e, w, \Delta\) become:
\[
\alpha C^{\alpha-1}[\Delta - c'(e)](1 + e\gamma G'(D_H) + (1 - e)\gamma G'(D_L) - \gamma(G(D_H) + \gamma G(D_L))] + (1 - \alpha)C^\alpha[H - L - \Delta] = 0
\]  
(37)
\[
\alpha C^{\alpha-1}[1 + 2e\gamma G'(D_H) + 2(1 - e)\gamma G'(D_L)] - (1 - \alpha)C^\alpha = 0
\]  
(38)
\[
\alpha C^{\alpha-1}[e + 2e\gamma G'(D_H)] - (1 - \alpha)eC^\alpha = 0
\]  
(39)

Rearranging (39) and assuming that \(e > 0,\) we obtain:
\[
C = \frac{\alpha}{1 - \alpha}(1 + 2\gamma G'(D_H))
\]  
(40)

Note, however, that the above equation is satisfied only for \(\gamma G(D_H) > -1/2.\) By using (38), we can rewrite \(C\) as:
\[
C = \frac{\alpha}{1 - \alpha}(1 + 2e\gamma G'(D_H) + 2(1 - e)\gamma G'(D_L))
\]  
(41)

Equating this to (40) and cancelling common terms, we obtain \(G'(D_H) = G'(D_L),\) i.e. \(D_H = D_L = D^{**},\) or equivalently, \(\Delta^{**} = (H - L)/2.\) If we further use this result in (37) and rearrange terms, we obtain:
\[
(1 + \gamma G'(D^{**}))(H - L - c'(e)) = 0
\]  
(42)

\(^{11}\)For a solution to exist, it must be hold that \((1 - e)\gamma G'(D_L) > -1/2 - e\gamma G'(D_H).\) However, as discussed above, \(\gamma G'(D_H) > -1/2,\) and thus, \((1 - e)\gamma G'(D_L) > -1/2(1 - e),\) or equivalently, \(\gamma G'(D_L) > -1/2.\)
Since $\gamma G(D^{**}) > -1/2$, it must be the case that $H - L = c'(e)$, i.e., $e^{**} = \dot{e}$. Considering (40), or equivalently, $EU^{**} = \frac{\alpha}{\alpha} (1 + 2\gamma G'(D^{**})) \pi_F^{**}$, we substitute the expected utility of the worker (4) and the profit of the firm (2) at the optimum, and solve it for $w$:

$$w(1 + 2\alpha \gamma G'(D(w))) = (1 - \alpha)(c(e^{**}) + \gamma G(D(w))) + \alpha(1 + 2\gamma G'(D(w)))(L + e^{**}(H - L)) - e^{**}\Delta^{**}(1 + 2\alpha \gamma G'(D(w)))$$

(43)

Since $\alpha \in (0, 1)$, $\gamma G(D^{**}) > -1/2$ implies that $1 + 2\alpha \gamma G'(D^{**}) > 0$, and thus, we can define $w^{**}$ as the solution of the following equation:

$$w = \frac{1}{1 + 2\alpha \gamma G'(D(w))} \{(1 - \alpha)(c(e^{**}) + \gamma G(D(w))) + \alpha(1 + 2\gamma G'(D(w)))(L + e^{**}(H - L))\} - e^{**}\Delta^{**}$$

(44)

Solving for $\Delta^{**}$ and substituting it in (2), we obtain:

$$\pi_F = L + e^{**}(H - L) - \frac{1}{1 + 2\alpha \gamma G'(D^{**})} \{(1 - \alpha)(c(e^{**}) + \gamma G(D^{**})) + \alpha(1 + 2\gamma G'(D^{**})))(L + e^{**}(H - L))\}$$

(45)

or equivalently,

$$\pi_F = \frac{1 - \alpha}{1 + 2\alpha \gamma G'(D^{**})} \left[L + e^{**}(H - L) - c(e^{**}) - \gamma G(D^{**})\right].$$

(46)

Considering equation (40), we can conclude that the expected utility of the worker and firm profit can be presented as given in equations (17) and (18). ■

References


