

Groll, Dominik

Conference Paper

When do Countries Benefit from Forming a Monetary Union?

Beiträge zur Jahrestagung des Vereins für Socialpolitik 2013: Wettbewerbspolitik und Regulierung in einer globalen Wirtschaftsordnung - Session: Monetary Policy II, No. B01-V2

Provided in Cooperation with:

Verein für Socialpolitik / German Economic Association

Suggested Citation: Groll, Dominik (2013) : When do Countries Benefit from Forming a Monetary Union?, Beiträge zur Jahrestagung des Vereins für Socialpolitik 2013: Wettbewerbspolitik und Regulierung in einer globalen Wirtschaftsordnung - Session: Monetary Policy II, No. B01-V2, ZBW - Deutsche Zentralbibliothek für Wirtschaftswissenschaften, Leibniz-Informationszentrum Wirtschaft, Kiel und Hamburg

This Version is available at:

<https://hdl.handle.net/10419/79787>

Standard-Nutzungsbedingungen:

Die Dokumente auf EconStor dürfen zu eigenen wissenschaftlichen Zwecken und zum Privatgebrauch gespeichert und kopiert werden.

Sie dürfen die Dokumente nicht für öffentliche oder kommerzielle Zwecke vervielfältigen, öffentlich ausstellen, öffentlich zugänglich machen, vertreiben oder anderweitig nutzen.

Sofern die Verfasser die Dokumente unter Open-Content-Lizenzen (insbesondere CC-Lizenzen) zur Verfügung gestellt haben sollten, gelten abweichend von diesen Nutzungsbedingungen die in der dort genannten Lizenz gewährten Nutzungsrechte.

Terms of use:

Documents in EconStor may be saved and copied for your personal and scholarly purposes.

You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.

If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.

WHEN DO COUNTRIES BENEFIT FROM FORMING A MONETARY UNION?

Dominik Groll*

Kiel Institute for the World Economy and University of Kiel, Germany

Preliminary version:
August 30, 2013

The New Keynesian DSGE literature has come to the consensus that, from the perspective of business cycle stabilization, countries are worse off in terms of welfare by forming a monetary union compared to a flexible exchange rate regime. This consensus, however, is based on the assumption of monetary policy being optimal. This paper shows that in the basic two-country New Keynesian DSGE model, in which monetary policy is conducted via Taylor-type interest rate rules, forming a monetary union is welfare-enhancing if prices are relatively sticky. In that case, the cost of greater output gap and terms of trade gap instability is outweighed by the benefit of greater inflation stability. Two endogenous effects drive this result: First, the stabilizing property of a flexible nominal exchange rate declines as prices become stickier. Second, fixing the exchange rate entails the inherent benefit of stabilizing inflation expectations by inducing inertia into the economy. Importantly, whether countries benefit from forming a monetary union depends crucially on the way monetary policy is conducted: If monetary policy responds to inflation aggressively or if it implements a high degree of interest rate smoothing, countries are better off maintaining a flexible nominal exchange rate. In contrast, asymmetries across countries in the degree of price stickiness and in economic size are not of any importance for a monetary union to be beneficial.

Email: dominik.groll@ifw-kiel.de

Contents

1. Introduction	3
2. Literature	4
3. Model	6
3.1. Model equations	6
3.1.1. Flexible prices	7
3.1.2. Sticky prices under the MU regime	7
3.1.3. Sticky prices under the FX regime	7
3.1.4. Variables and parameters	7
3.2. Model description	8
3.3. Welfare-based loss function	9
3.4. Calibration	9
4. Results under symmetry	10
4.1. Analytical insights	11
4.2. Price stickiness	13
4.2.1. Inherited stickiness of the nominal exchange rate	16
4.2.2. Inherent benefit of monetary unions	17
4.2.3. Summary of baseline results	19
4.3. Monetary policy	21
4.3.1. Inflation coefficient	21
4.3.2. Output coefficient	22
4.3.3. Interest rate smoothing coefficient	23
5. Results under asymmetry	25
5.1. Country size	25
5.2. Price stickiness	25
6. Conclusion	26
A. Monetary union regime (MU)	29
A.1. Consumer problem	29
A.2. Producer problem	32
A.3. Terms of trade	34
A.4. Log-linearization	34
A.4.1. Flexible prices	34
A.4.2. Sticky prices	35
B. Flexible exchange rate regime (FX)	36
C. Welfare-based loss function	38
C.1. The term $U(C_t)$	38
C.2. The term $\frac{1}{n} \int_0^n V(y_t(h), z_t^H) dh$	39
C.3. Expanding Y_t^H	42
C.4. Combining the results	44
C.5. Special case: $\alpha^H = \alpha^F$	51
C.6. Version containing country-specific output gaps	52
D. Variances	54
D.1. MU regime	54
D.2. FX regime	55

1. Introduction

When do countries benefit from forming a monetary union? This question is at least as old as the Optimum Currency Area (OCA) literature initiated by Mundell (1961). One of the key insights of the traditional OCA literature is the following: For asymmetric countries to benefit from forming a monetary union, prices and wages have to be flexible and production factors have to be mobile.

More recently, the New Keynesian literature using dynamic stochastic general equilibrium (DSGE) models has come to the consensus that, from the perspective of business cycle stabilization, forming a monetary union makes countries generally worse off in terms of welfare.¹ The rationale is simple: Countries forming a monetary union relinquish one important policy instrument to stabilize the business cycle, namely the short-term nominal interest rate controlled by national central banks. Thus, forming a monetary union induces a cost to participating countries.

This consensus, however, is based on the assumption that monetary policy is conducted optimally. Clearly, optimal monetary policy constitutes an important theoretical benchmark from a normative perspective. Yet, the practical limitations of the textbook prescription of optimal monetary policy are widely acknowledged.² More importantly, the application of optimal policy in the OCA context has led to the conclusion that if both monetary policy and fiscal policy are conducted optimally, the exchange rate regime is *irrelevant* for stabilization policy (Adao, Correia, and Teles, 2009). If there are as many policy instruments as distortions in the economy, the efficient equilibrium can be obtained and welfare losses can be reduced to zero under any international monetary regime.

Against this background, I take a different approach in this paper: Monetary policy follows Taylor-type interest rate rules, according to which it responds only to macroeconomic variables that are observable, such as the inflation rate or output. The central result: In the standard two-country New Keynesian DSGE model, in which labor is immobile and in which monetary policy follows interest rate rules, countries may gain in welfare by forming a monetary union. The welfare gain comes from a higher stability of inflation rates, which overcompensates the cost of greater output gap and terms of trade gap instability. Moreover, whether countries benefit depends strongly on the degree of price stickiness. If prices are relatively sticky, forming a monetary union is welfare-enhancing; if prices are relatively flexible, countries are better off maintaining a flexible exchange rate.

Two economic mechanisms are responsible for this result: First, the benefit of maintaining flexible exchange rates diminishes as prices become stickier. In particular, the nominal exchange rate inherits the stickiness of prices. As a result, an increasing degree of price stickiness reduces the effectiveness of the nominal exchange rate as a stabilization mechanism. Second, forming a monetary union entails an inherent benefit (in this context, "inherent" refers to the fact that the benefit is not modeled explicitly, like a reduction in transaction costs, but emerges from within the model). Since the nominal exchange rate is fixed, the terms of trade and, therefore, the inflation rates display an inertial or history-dependent behavior. The history-dependence has the advantage of affecting inflation expectations of price setters in such a way as to lower the responsiveness of inflation to changing economic conditions. As a result, inflation rates are more stable under the monetary union regime.

The first mechanism is due to the presence of the uncovered interest parity condition on the one hand and monetary policy following Taylor-type interest rate rules on the other hand. Importantly, for this mechanism to be effective, the uncovered

¹See surveys by Corsetti (2008), Dellas and Tavlás (2009), and Beetsma and Giuliodori (2010).

²E.g. the monetary authority needs to be able to observe the flexible-price equilibrium. For more practical shortcomings of optimal monetary policy, see Galí (2008, Ch. 4.3.2).

interest parity condition need not hold exactly. The link between interest rates and the nominal exchange rate is sufficient. The second mechanism is due to the fact that price setters are forward-looking in the presence of nominal price rigidities. Those features are at the core of new open economy macroeconomic (NOEM) models and, therefore, are present also in many medium-to-large-scale models that have been built around this core.³

The benefit of inertia is a well-known phenomenon from optimal monetary policy analysis in a closed-economy environment: Optimal monetary policy *under discretion* is inferior to optimal policy *under commitment*, because the former does not influence inflation expectations of firms in a favorable way. It suffers from the *stabilization bias*.⁴ In contrast, optimal monetary policy under commitment induces history-dependence into the economy and, therefore, exploits the fact that price setters are forward-looking. The intuition in this paper is completely analogous: Forming a monetary union may be superior to maintaining a flexible exchange rate under certain conditions, because fixing the nominal exchange rate induces history-dependence.

The result that forming a monetary union is beneficial if prices are relatively sticky is very sensitive to the way monetary policy is conducted: If monetary policy responds to inflation aggressively or if it implements a high degree of interest rate smoothing, then maintaining a flexible nominal exchange rate is superior. On the one hand, by reacting more strongly to inflation, monetary policy induces a greater response in the nominal exchange rate, thereby counteracting the stickiness the nominal exchange rate inherits from goods prices. On the other hand, by smoothing interest rates, monetary policy induces history-dependence into the economy and therefore manipulates inflation expectations in its favor. The stabilizing effect on inflation expectations is exactly the same as under the monetary union regime, only the source differs. With flexible exchange rates, monetary policy has to engage in interest rate smoothing to induce history-dependence. In a monetary union, history-dependence is induced automatically by the fact that the nominal exchange rate is fixed.

The plan of the paper is the following: Section 2 relates the paper to the relevant literature. Section 3 outlines briefly the structure of the model. Section 4 first provides important analytical insights in the case of symmetric countries. Then, welfare losses in the presence of asymmetric productivity shocks are computed for both the monetary union regime and the flexible exchange rate regime. Results are presented graphically. The economic explanation for the results are given on the basis of the analytical insights and with the aid of impulse response functions. Section 5 presents the results in the case of asymmetric countries. Finally, Section 6 concludes.

2. Literature

This paper is related along several dimensions to the New Keynesian literature that analyzes under which conditions countries benefit from forming a monetary union. Few authors considered an environment with suboptimal monetary policy. In a small open economy version of the basic New Keynesian model, Galí and Monacelli (2005) find that fixing the exchange rate induces higher welfare losses compared to flexible exchange rate regimes with different Taylor-type interest rate rules. In a two-country setting, Devereux (2004) concludes that whether fixing the exchange

³The introduction of nominal price rigidities à la Calvo (1983) in NOEM models goes back to Kollmann (2001), Galí and Monacelli (2005), and Clarida, Galí, and Gertler (2002).

⁴For details on the stabilization bias, see e.g. Woodford (2003, Ch. 7), Galí (2008, Ch. 5), or Walsh (2010, Ch. 8).

rate involves welfare losses depends critically on how monetary policy is conducted. If monetary policy targets the employment gap, flexible exchange rate regimes fare better. Under constant money growth rules, fixed exchange rate regimes fare better.

In a three-country and two-country setting respectively, Dellas and Tavlas (2005) and Dellas (2006) find that under certain conditions countries may be better off in terms of welfare by forming a monetary union compared to a flexible exchange rate regime. One of those conditions seems to be that nominal rigidities are present in wages instead of prices. Their model differs from the basic New Keynesian model in several dimensions. Finally, Ferreira-Lopes (2010) calibrates a two-country model specifically to the UK and Sweden and finds that they would be worse off by forming a monetary union. None of the aforementioned studies highlights the inherent benefit of monetary unions, the role of the degree of price stickiness nor the closely related issue of the inherited stickiness of the exchange rate, which are crucial for the welfare ranking between the monetary union and the flexible exchange rate regime.

Several authors have introduced explicit benefits to monetary unions to create a counterpart to the cost of giving up national monetary policy as a stabilization device. Such explicit benefits of monetary unions include the elimination of shocks to the uncovered interest parity condition (Kollmann, 2004), the gain in potential output (Ca'Zorzi, De Santis, and Zampolli, 2005), the gain in central bank credibility (Clerc, Dellas, and Loisel, 2011), and the possibility of higher consumption risk sharing across countries (Ching and Devereux, 2003). In contrast, in this paper no explicit benefits are introduced into the model. The benefit of stabilizing inflation expectations is inherent to monetary unions as a result of a fixed nominal exchange rate.

This paper is also related to Monacelli (2004).⁵ Similarly, he finds that a fixed exchange rate regime induces inertia into the economy. On the one hand, I show that this benefit carries over to a two-country environment and is inherent to monetary union regimes as well.⁶ On the other hand, I show that it does not hinge upon the stationarity of the price level, as stressed by Monacelli (2004). Stationarity of the price level is a special feature of the small open economy environment and does not carry over to a two-country setting employed here. Furthermore, Monacelli (2004) does not analyze the role of the degree of price stickiness and the related issue of the inherited stickiness of the exchange rate for the welfare ranking of flexible versus fixed exchange rate regimes.

With the exception of Monacelli (2004), none of the aforementioned studies recognizes the benefit of more stable inflation rates that monetary unions may entail. This benefit, which is related to the *stabilization bias*, adds to the benefit of eliminating a potential *inflation bias*, which was stressed by Alesina and Barro (2002) and Cooley and Quadrini (2003).⁷ Whereas the latter benefit has been acknowledged in the literature (e.g. Dellas and Tavlas, 2009; Beetsma and Giuliodori, 2010),

⁵In a small open economy setting, he finds that a fixed exchange rate regime may dominate a flexible exchange rate regime with optimal monetary policy under discretion. A flexible exchange rate regime with optimal monetary policy under commitment, however, always dominates the other two regimes. Soffritti and Zanetti (2008) conduct virtually the same exercise, but come to the conclusion that a fixed exchange rate regime fares worst compared to the flexible exchange rate regimes. One possible explanation could be the different weight attached to the output gap variance relative to the weight attached to the inflation variance in the welfare loss function. In both Monacelli (2004) and Soffritti and Zanetti (2008), the welfare loss function is ad-hoc in the sense that the weights do not depend on the deep parameters of the model, but must be chosen exogenously. Another explanation could be the different assumption on whether the rest of the world is also subject to shocks or not.

⁶In general, fixed exchange rate regimes and monetary union regimes do not coincide. This depends on how the fixed exchange rate regime is implemented by the monetary authorities.

⁷Giavazzi and Pagano (1988)'s "advantage of tying one's hands" by joining the European Monetary System (1979-1999) followed the same logic.

the first still seems to be unknown.

3. Model

In this section, I outline the structure of the model. Since it is a completely standard two-country New Keynesian DSGE model, the description is kept very brief. Two international monetary regimes will be considered:

1. Monetary union (MU) regime: Both countries share the same currency. The common monetary policy governs the common nominal interest.
2. Flexible exchange rate (FX) regime: Each country maintains its national currency and independent monetary policy. Nominal interest rates are country-specific. The nominal exchange rate between the two currencies is flexible.

In particular, I use the model developed by Benigno (2004) and by Benigno and Benigno (2008) including the microfounded welfare measure. The model economy features two countries with trade in consumption goods (as opposed to trade in intermediate goods). Consumption preferences are of the Cobb-Douglas type and, in addition, identical across countries, i.e. there is no home bias in consumption. Both assumptions lead to the equalization of consumption across countries at all times. The only factor of production is labor, which is immobile between countries. The only rigidity in the economy is the nominal price rigidity à la Calvo (1983).

Under the FX regime, prices are set in the currency of the producer's country, i.e. the producer does not discriminate the price between countries. The nominal exchange rate converts the price into foreign currency, i.e. the law of one price holds and exchange rate pass-through is complete. Given consumption preferences, purchasing power parity holds as well, i.e. the real exchange rate is constant. The nominal exchange rate is determined by the uncovered interest parity.

In both the MU and the FX regime, monetary policy is conducted via Taylor-type interest rate rules. Importantly, I assume that monetary policy is not able to observe the flexible-price equilibrium of the economy, in particular the flexible-price interest rate and flexible-price output. Thus, monetary policy reacts to inflation and to output (deviation from steady state), not to the output gap (deviation from flexible-price output). The only shocks considered are country-specific productivity shocks.

3.1. Model equations

In the following, the equations of the final log-linearized model are displayed. Appendix A and Appendix B contain the full derivation of the model. Deviations of the logarithm of a variable X_t from its steady state are denoted by \tilde{X}_t under flexible prices and by \hat{X}_t under sticky prices.

3.1.1. Flexible prices

Under flexible prices, the system of model equations is identical in both the FX and MU regime and given by

$$\tilde{C}_t = \frac{\eta}{\rho + \eta} \bar{Y}_t^W \quad (3.1)$$

$$\tilde{T}_t = -\frac{\eta}{1 + \eta} \bar{Y}_t^R \quad (3.2)$$

$$\tilde{Y}_t^W = \frac{\eta}{\rho + \eta} \bar{Y}_t^W \quad (3.3)$$

$$\bar{Y}_t^i = \rho_i \bar{Y}_{t-1}^i + \nu_t^i. \quad (3.4)$$

3.1.2. Sticky prices under the MU regime

Under sticky prices, the system of model equations in the MU regime is given by

$$E_t \hat{C}_{t+1} = \hat{C}_t + \frac{1}{\rho} (\hat{R}_t - E_t \pi_{t+1}^W) \quad (3.5)$$

$$\hat{Y}_t^H = (1 - n) \hat{T}_t + \hat{C}_t \quad (3.6)$$

$$\hat{Y}_t^F = -n \hat{T}_t + \hat{C}_t \quad (3.7)$$

$$\pi_t^H = (1 - n) k_T^H (\hat{T}_t - \tilde{T}_t) + k_C^H (\hat{C}_t - \tilde{C}_t) + \beta E_t \pi_{t+1}^H \quad (3.8)$$

$$\pi_t^F = -n k_T^F (\hat{T}_t - \tilde{T}_t) + k_C^F (\hat{C}_t - \tilde{C}_t) + \beta E_t \pi_{t+1}^F \quad (3.9)$$

$$\hat{T}_t = \hat{T}_{t-1} + \pi_t^F - \pi_t^H \quad (3.10)$$

$$\hat{R}_t = \phi_R \hat{R}_{t-1} + (1 - \phi_R) (\phi_\pi \pi_t^W + \phi_Y \hat{Y}_t^W) \quad (3.11)$$

$$\pi_t^W = n \pi_t^H + (1 - n) \pi_t^F. \quad (3.12)$$

3.1.3. Sticky prices under the FX regime

The system of model equations in the FX regime is given by

$$E_t \hat{C}_{t+1} = \hat{C}_t + \frac{1}{\rho} \left(n (\hat{R}_t^H - E_t \pi_{t+1}^H) + (1 - n) (\hat{R}_t^F - E_t \pi_{t+1}^F) \right) \quad (3.13)$$

$$\hat{Y}_t^H = (1 - n) \hat{T}_t + \hat{C}_t \quad (3.14)$$

$$\hat{Y}_t^F = -n \hat{T}_t + \hat{C}_t \quad (3.15)$$

$$\pi_t^H = (1 - n) k_T^H (\hat{T}_t - \tilde{T}_t) + k_C^H (\hat{C}_t - \tilde{C}_t) + \beta E_t \pi_{t+1}^H \quad (3.16)$$

$$\pi_t^F = -n k_T^F (\hat{T}_t - \tilde{T}_t) + k_C^F (\hat{C}_t - \tilde{C}_t) + \beta E_t \pi_{t+1}^F \quad (3.17)$$

$$\hat{T}_t = \hat{T}_{t-1} + \pi_t^F - \pi_t^H + \Delta \hat{S}_t \quad (3.18)$$

$$E_t \Delta \hat{S}_{t+1} = \hat{R}_t^H - \hat{R}_t^F \quad (3.19)$$

$$\hat{R}_t^H = \phi_R \hat{R}_{t-1}^H + (1 - \phi_R) (\phi_\pi \pi_t^H + \phi_Y \hat{Y}_t^H) \quad (3.20)$$

$$\hat{R}_t^F = \phi_R \hat{R}_{t-1}^F + (1 - \phi_R) (\phi_\pi \pi_t^F + \phi_Y \hat{Y}_t^F). \quad (3.21)$$

3.1.4. Variables and parameters

Variables:

C_t	Consumption (identical across countries)
Y_t^i	Output of country $i = H, F$
Y_t^W	World output (weighted average of country-specific output)
π_t^i	Producer price inflation in country $i = H, F$
π_t^W	World inflation (weighted average of country-specific inflation)
π_t^R	Inflation differential between the two countries $\pi_t^F - \pi_t^H$
R_t^i	Nominal interest rate in country $i = H, F$
R_t	Nominal interest rate in monetary union
T_t	Terms of trade
S_t	Nominal exchange rate
\bar{Y}_t^i	Productivity shock in country $i = H, F$
ν_t^i	White noise process in country $i = H, F$

Parameters:

ρ	Inverse of elasticity of intertemporal substitution in consumption
n	Country size measured by population
β	Discount factor
η	Inverse of elasticity of producing the differentiated good
σ	Elasticity of substitution between differentiated goods
α^i	Probability of not being able to reset the price in country $i = H, F$
ϕ_π	Inflation coefficient in interest rate rule
ϕ_Y	Output coefficient in interest rate rule
ϕ_R	Interest rate smoothing coefficient in interest rate rule
ρ_i	Persistence of productivity shock in country $i = H, F$
k_C^i	$k_C^i = \frac{(1-\alpha^i\beta)(1-\alpha^i)}{\alpha^i} \frac{\rho+\eta}{1+\sigma\eta}$
k_T^i	$k_T^i = \frac{(1-\alpha^i\beta)(1-\alpha^i)}{\alpha^i} \frac{1+\eta}{1+\sigma\eta}$

3.2. Model description

Since risk sharing is perfect due to Cobb-Douglas preferences in consumption in both the MU and FX regime, consumption is equal across countries at all times and is described by only one Euler equation, equation (3.5) in the MU regime and equation (3.13) in the FX regime. The only difference is that under the MU regime the nominal interest rate is common to both countries. The country-specific national account identities are the same in both regimes and given by equations (3.6), (3.7), (3.14), and (3.15). Also the country-specific New Keynesian Phillips curves are the same in both regimes and given by (3.8), (3.9), (3.16), and (3.17). In contrast to the closed-economy framework, not only the consumption gap but also the terms of trade gap (difference between sticky price and flexible price terms of trade) matters for producer price inflation.⁸ The terms of trade identity is given by (3.10) under the MU regime and by (3.18) under the FX regime, the difference being the presence of the nominal exchange rate in the latter. Equation (3.19) is the uncovered interest parity condition: The expected change in the nominal exchange rate corresponds to the interest rate differential across countries. Finally, monetary policy is conducted via Taylor-type interest rate rules, given by equation (3.11) under the MU regime and by equations (3.20) and (3.21) under the FX regime.

Under flexible prices, prices are set as a markup over marginal costs, monetary policy is neutral, and consumption, output, and the terms of trade are driven by productivity shocks only, given by equations (3.1), (3.2), and (3.3). Since money is

⁸Note that the consumption gap is equal to the world output gap: $\hat{C}_t - \bar{C}_t = \hat{Y}_t^W - \bar{Y}_t^W$. Accordingly, the New Keynesian Phillips curves can be expressed in terms of the world output gap as well.

neutral under flexible prices, the international monetary regime does not affect real variables, which therefore behave identically in both monetary regimes.

3.3. Welfare-based loss function

The world welfare loss function is the discounted value of a weighted average across countries of the average utility flow of agents using a second-order Taylor series expansion in the spirit of Woodford (2003).⁹ Throughout the paper, it is assumed that the distortion induced by monopolistic competition is completely offset by an appropriate subsidy (Appendix C contains the full derivation). Thus,

$$\begin{aligned}
W_t = & -\frac{1}{2} \left((\rho + \eta) \text{var}(\hat{C}_t - \tilde{C}_t) + (1 + \eta)n(1 - n) \text{var}(\hat{T}_t - \tilde{T}_t) \right. \\
& + \sigma(1 + \sigma\eta)n \frac{\alpha^H}{(1 - \alpha^H)(1 - \alpha^H\beta)} \text{var} \pi_t^H \\
& \left. + \sigma(1 + \sigma\eta)(1 - n) \frac{\alpha^F}{(1 - \alpha^F)(1 - \alpha^F\beta)} \text{var} \pi_t^F \right) \\
& + t.i.p. + O(\|\xi\|^3). \tag{3.22}
\end{aligned}$$

As in the closed-economy framework, the welfare loss depends on inflation and the consumption gap.¹⁰ In the open economy, the welfare loss depends additionally on the terms of trade gap. Intuitively, when the terms of trade deviate from the terms of trade that would prevail under flexible prices, the resulting allocation of production across countries is inefficient due to the presence of prices stickiness.

In the special case where prices are equally rigid in both countries ($\alpha^H = \alpha^F = \alpha$), the welfare loss function simplifies to

$$\begin{aligned}
W_t = & -\frac{1}{2} \left((\rho + \eta) \text{var}(\hat{C}_t - \tilde{C}_t) + (1 + \eta)n(1 - n) \text{var}(\hat{T}_t - \tilde{T}_t) \right. \\
& \left. + \sigma(1 + \sigma\eta) \frac{\alpha}{(1 - \alpha)(1 - \alpha\beta)} [\text{var} \pi_t^W + n(1 - n) \text{var} \pi_t^R] \right) \\
& + t.i.p. + O(\|\xi\|^3). \tag{3.23}
\end{aligned}$$

3.4. Calibration

The values for the baseline calibration are taken from Benigno (2004), except for the interest rate rule coefficients (Table 1). A value of 0.99 for the discount factor β implies a steady state real interest rate of around 4.1 percent on an annual basis. A value of 7.66 for the elasticity of substitution between differentiated goods σ implies a steady state markup of prices over marginal costs of 15 percent. A value of 0.75 for the probability of not being able to reset the price α^i implies an average duration of price contracts of 4 quarters.

In the baseline calibration, monetary policy responds to inflation with $\phi_\pi = 1.5$, it does not react to output ($\phi_Y = 0$) and does not engage in interest rate smoothing ($\phi_R = 0$). Finally, it is assumed throughout the paper that the interest rate rule

⁹Computing country-specific welfare would complicate the calculations significantly in the sense that more accurate approximations of the non-linear model equations would be necessary (Benigno and Woodford, 2005). This is beyond the scope of this paper.

¹⁰In the basic closed-economy framework, consumption usually equals output. Note that the welfare loss function (3.22) could alternatively be expressed in terms of the world output gap or the country-specific output gaps. The specification in terms of the consumption gap was chosen for analytical convenience.

coefficients ϕ_π , ϕ_Y , and ϕ_R are identical across countries and regimes.

ρ	1/6	Inv. of elasticity of intertemporal substitution in consumption
n	0.5	Country size measured by population
β	0.99	Discount factor
η	0.67	Inv. of elasticity of producing the differentiated good
ϵ_{wy}	0.5	Production elasticity of average real wage
γ	0.75	Labor income share
σ	7.66	Elasticity of substitution between differentiated goods
α^i	0.75	Probability of not being able to reset the price
ϕ_π	1.5	Inflation coefficient in interest rate rule
ϕ_Y	0	Output coefficient in interest rate rule
ϕ_R	0	Interest rate smoothing coefficient in interest rate rule
ρ_i	0.9	Persistence of productivity shock
$\text{var } \nu_t^i$	1	Variance of white noise process
$\text{corr}(\nu_t^H, \nu_t^F)$	0	Correlation between country-specific white noise processes

Table 1: Baseline calibration

Following Rotemberg and Woodford (1998) and Benigno (2004), the inverse of the elasticity of producing the differentiated good η is calculated as

$$\eta = \epsilon_{wy} - \rho + \frac{1 - \gamma}{\gamma}, \quad (3.24)$$

where ϵ_{wy} denotes the elasticity of the average real wage with respect to production and γ denotes the labor income share.

The paper considers a broad range of values for the parameters of the model to check for the validity of the results (Table 2).

ρ	[0.1,1.1]	Inv. of elasticity of intertemporal substitution in consumption
n	[0.05,0.95]	Country size measured by population
β	[0.9,1.0]	Discount factor
η	[0.2,3.0]	Inv. of elasticity of producing the differentiated good
ϵ_{wy}	[0.2,1.2]	Production elasticity of average real wage
γ	[0.5,0.9]	Labor income share
σ	[5,25]	Elasticity of substitution between differentiated goods
α^i	[0.05,0.95]	Probability of not being able to reset the price
ϕ_π	[1.1,3.5]	Inflation coefficient in interest rate rule
ϕ_Y	[0,3]	Output coefficient in interest rate rule
ϕ_R	[0,0.95]	Interest rate smoothing coefficient in interest rate rule

Table 2: Parameter range

4. Results under symmetry

First, I will conduct the analysis under the assumption that the two countries are symmetric (except for country size n). In particular, the degree of price stickiness is equal across countries. The resulting analytical insights will be crucial in understanding the main results of the paper.

The world welfare loss function under symmetry, equation (3.23), is repeated here for convenience:

$$\begin{aligned}
W_t = & -\frac{1}{2} \left((\rho + \eta) \text{var}(\hat{C}_t - \tilde{C}_t) + (1 + \eta)n(1 - n) \text{var}(\hat{T}_t - \tilde{T}_t) \right. \\
& \left. + \sigma(1 + \sigma\eta) \frac{\alpha}{(1 - \alpha)(1 - \alpha\beta)} [\text{var} \pi_t^W + n(1 - n) \text{var} \pi_t^R] \right) \\
& + t.i.p. + O(\|\xi\|^3). \tag{4.1}
\end{aligned}$$

It contains four components: The variance of the consumption gap ($\hat{C}_t - \tilde{C}_t$), the variance of the terms of trade gap ($\hat{T}_t - \tilde{T}_t$), the variance of world inflation (π_t^W), and the variance of the inflation differential (π_t^R).

4.1. Analytical insights

The analytical results in this Section are based on the assumption that the degree of price stickiness and the persistence of productivity shocks are identical across countries ($\alpha^H = \alpha^F$ and $\rho_H = \rho_F$) and that monetary policy does not engage in interest rate smoothing ($\phi_R = 0$). In order to obtain analytical expressions for the variances contained in the loss function, I derive the recursive law of motion (RLOM) of the model for each monetary regime using the method of undetermined coefficients.

Fortunately, it is not necessary to derive the RLOM for the variables consumption and world inflation, since they both behave identically across monetary regimes. For the MU regime, substituting out the nominal interest rate \hat{R}_t in the Euler equation (3.5) by inserting the interest rate rule (3.11) and the national account identities (3.6) and (3.7) yields

$$\rho E_t \hat{C}_{t+1} = (\rho + \phi_Y) \hat{C}_t + \phi_\pi \pi_t^W - E_t \pi_{t+1}^W. \tag{4.2}$$

The same equation is obtained completely analogously for the FX regime.

Inserting the New Keynesian Phillips curves, which are identical across regimes, into the definition of world inflation $\pi_t^W = n\pi_t^H + (1 - n)\pi_t^F$, where due to symmetry $\alpha^H = \alpha^F = \alpha$ and therefore $k_T^H = k_T^F = k_T$ and $k_C^H = k_C^F = k_C$, yields

$$\pi_t^W = k_C(\hat{C}_t - \tilde{C}_t) + \beta E_t \pi_{t+1}^W. \tag{4.3}$$

The reason why world inflation is the same under both the MU and the FX regime is that the terms of trade vanish from the equation when the degree of price stickiness is equal across countries. The fact that both consumption and world inflation behave identically across monetary regimes implies that the variance of consumption and the variance of world inflation are identical as well. As a result, these two components do not produce differences in welfare across the two regimes.

To obtain analytical expressions for the variance of the terms of trade gap and of the inflation differential under each monetary regime, I first reduce the number of equations as much as possible. Second, I derive the RLOM. Finally, the variance-covariance matrix of the corresponding system of equations contains the variances of interest. Appendix D lays out the entire derivation.

The reduced system of equations under the MU regime is given by

$$\pi_t^R = -k_T(\hat{T}_t - \tilde{T}_t) + \beta E_t \pi_{t+1}^R \tag{4.4}$$

$$\hat{T}_t = \hat{T}_{t-1} + \pi_t^R. \tag{4.5}$$

The reduced system of equations under the FX regime is given by

$$\pi_t^R = -k_T(\hat{T}_t - \tilde{T}_t) + \beta E_t \pi_{t+1}^R \quad (4.6)$$

$$\hat{T}_t = \hat{T}_{t-1} + \pi_t^R + \Delta \hat{S}_t \quad (4.7)$$

$$E_t \Delta \hat{S}_{t+1} = -\phi_\pi \pi_t^R + \phi_Y \hat{T}_t. \quad (4.8)$$

Equations (4.4) and (4.6) are obtained by subtracting the New Keynesian Phillips curve of country H from that of country F . Equations (4.5) and (4.7) are the terms of trade identities. Equation (4.8) is obtained by inserting the interest rate rules (3.20) and (3.21) as well as the national account identities (3.14) and (3.15) into the uncovered interest parity condition (3.19).

The RLOM under the MU regime is given by

$$\hat{T}_t = b_1 \hat{T}_{t-1} + c_1 \tilde{T}_t \quad (4.9)$$

$$\pi_t^R = b_2 \hat{T}_{t-1} + c_2 \tilde{T}_t, \quad (4.10)$$

with coefficients

$$b_1 = \frac{1 + k_T + \beta - \sqrt{(1 + k_T + \beta)^2 - 4\beta}}{2\beta}$$

$$b_2 = \frac{1 + k_T - \beta - \sqrt{(1 + k_T + \beta)^2 - 4\beta}}{2\beta}$$

$$c_1 = c_2 = c = \frac{k_T}{1 + k_T + \beta(1 - \rho_H - b_1)}.$$

The RLOM under the FX regime is given by

$$\hat{T}_t = b_1 \hat{T}_{t-1} + c_1 \tilde{T}_t \quad (4.11)$$

$$\pi_t^R = b_2 \hat{T}_{t-1} + c_2 \tilde{T}_t \quad (4.12)$$

$$\Delta \hat{S}_t = b_3 \hat{T}_{t-1} + c_3 \tilde{T}_t, \quad (4.13)$$

with coefficients

$$b_1 = 0$$

$$b_2 = 0$$

$$b_3 = -1$$

$$c_1 = \frac{(\phi_\pi - \rho_H)k_T}{(\phi_\pi - \rho_H)k_T + (1 - \rho_H + \phi_Y)(1 - \beta\rho_H)}$$

$$c_2 = \frac{(1 - \rho_H + \phi_Y)k_T}{(\phi_\pi - \rho_H)k_T + (1 - \rho_H + \phi_Y)(1 - \beta\rho_H)}$$

$$c_3 = \frac{(\phi_\pi - 1 - \phi_Y)k_T}{(\phi_\pi - \rho_H)k_T + (1 - \rho_H + \phi_Y)(1 - \beta\rho_H)}.$$

Consequently, the variances of the terms of trade gap and the variances of the

inflation differential under both regimes are given by

$$\text{var}_{MU}(\hat{T}_t - \tilde{T}_t) = \left[\frac{(1 + \rho_H b_1)c^2}{(1 - b_1^2)(1 - \rho_H b_1)} - \frac{2c}{1 - \rho_H b_1} + 1 \right] \text{var} \tilde{T}_t \quad (4.14)$$

$$\text{var}_{FX}(\hat{T}_t - \tilde{T}_t) = (c_1 - 1)^2 \text{var} \tilde{T}_t \quad (4.15)$$

$$\text{var}_{MU} \pi_t^R = \frac{2c^2(1 - \rho_H)}{(1 + b_1)(1 - \rho_H b_1)} \text{var} \tilde{T}_t \quad (4.16)$$

$$\text{var}_{FX} \pi_t^R = c_2^2 \text{var} \tilde{T}_t \quad (4.17)$$

$$\text{var} \tilde{T}_t = \frac{1}{1 - \rho_H^2} \left(\frac{\eta}{1 + \eta} \right)^2 [\text{var} \nu_t^H + \text{var} \nu_t^F - 2 \text{cov}(\nu_t^H, \nu_t^F)]. \quad (4.18)$$

Two important differences exist between the MU and FX regime:

1. In contrast to the MU regime, there is no persistence in the terms of trade nor in the inflation differential under the FX regime ($b_1 = b_2 = 0$). Hence, once the shock has vanished, both variables return immediately to steady state. This is due to the nominal exchange rate. Intuitively, the coefficient $b_3 = -1$ implies that, if the terms of trade were e.g. one percent below the steady state in the previous period, the nominal exchange rate will increase by one percent in the current period, so that the terms of trade are at steady state. Naturally, this mechanism is absent in the MU regime, since the nominal exchange rate is fixed. Both the terms of trade and the inflation differential are inertial or history-dependent in the sense that they depend on the realization of the terms of trade in the previous period.
2. In contrast to the MU regime, monetary policy is able to influence the terms of trade gap and the inflation differential under the FX regime. Technically, the variance of the terms of trade gap and of the inflation differential depend on the interest rate rule coefficients ϕ_π and ϕ_Y . Moreover, if monetary policy is extremely aggressive towards inflation under the FX regime ($\phi_\pi \rightarrow \infty$), the variance of the terms of trade gap and of the inflation differential converge towards zero (since $c_1 \rightarrow 1$ and $c_2 \rightarrow 0$). Thus, the efficient equilibrium can be approximated arbitrarily well, reducing the welfare loss to zero. In contrast, the variance of the terms of trade gap and of the inflation differential under the MU regime cannot be zero, and therefore the efficient equilibrium is not feasible.¹¹

The analytical expressions for the variances can be used to derive the condition under which world welfare is larger in one or the other monetary regime. Unfortunately, the resulting condition is a complex inequality that provides hardly any intuition of the economics behind the result. Therefore, I will compute the welfare losses numerically and display the results graphically. In the following, the deep parameters are calibrated according to the baseline calibration (Table 1), except for the parameters of interest, which take on a broad range of values (Table 2).

4.2. Price stickiness

Whether the world welfare loss is higher in one than in the other monetary regime, depends crucially on the Calvo parameter α , i.e. the degree of price stickiness in both economies (Figure 1). In both regimes, the world welfare loss increases with the

¹¹Mathematically, the variance of the inflation differential cannot be zero since $c > 0$. The variance of the terms of trade gap cannot be zero since the quadratic equation in square brackets does not have real roots.

degree of price stickiness.¹² If the degree of price stickiness is rather low, the world welfare loss is higher under the MU regime than under the FX regime. The countries are better off in terms of world welfare with their own currency and independent monetary policy. However, beyond a certain threshold ($\alpha \approx 0.5$), where the degree of price stickiness is rather high, the world welfare loss is higher under the FX regime than under the MU regime. The countries are better off forming a monetary union with one currency and one common monetary authority. Quantitatively, the difference in welfare between the two monetary regimes can be substantial. Under the baseline calibration, the welfare loss under the MU regime is roughly 40 percent lower than under the FX regime (0.8/1.3).

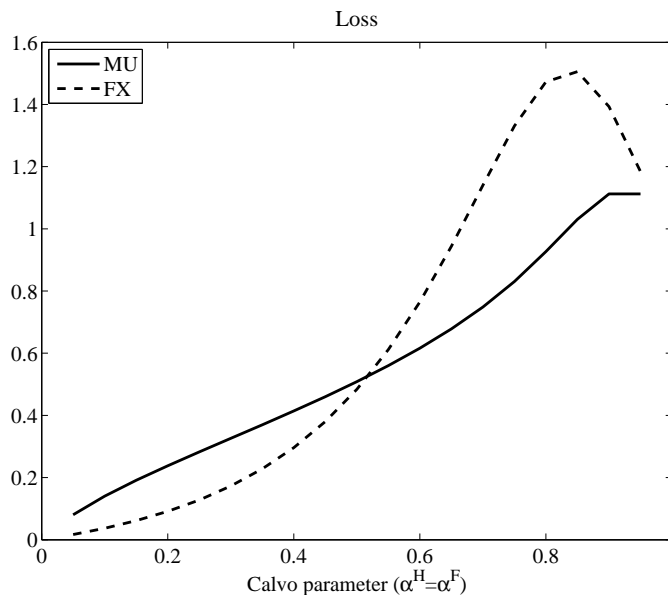


Figure 1: World welfare loss under different degrees of price stickiness ($\alpha^H = \alpha^F$)

As described above, two components of the world welfare loss function (4.1) behave identically across monetary regimes and, therefore, cannot create welfare differences across regimes: consumption gap and world inflation (Figure 2, upper and lower left panel). This does not hold for the terms of trade gap and the inflation differential (Figure 2, upper and lower right panel). The contribution of the terms of trade gap is higher under the MU regime than under the FX regime regardless of the degree of price stickiness. This points to a cost of the MU regime. However, the contribution of the terms of trade gap is rather small compared to the contribution of the inflation differential. The result that the MU regime yields higher world welfare if prices are relatively sticky is overwhelmingly driven by the inflation differential. In fact, the pattern in the lower right panel of Figure 2 resembles closely the pattern in Figure 1, with a similar threshold value of $\alpha \approx 0.5$. This points to a benefit of the MU regime.

The contribution of a component of the world welfare loss function is the product of the weight and the variance of that component. The weight and variance of the inflation differential show opposite patterns with respect to price stickiness. Whereas the variance decreases with a rising degree of price stickiness (Figure 3), the weight increases (Figure 4). Thus, although the variance decreases with the degree of price stickiness, which per se enhances the agent's welfare, the agent

¹²This feature is common to the closed-economy setup of the basic New Keynesian model, as in Galí (2008).

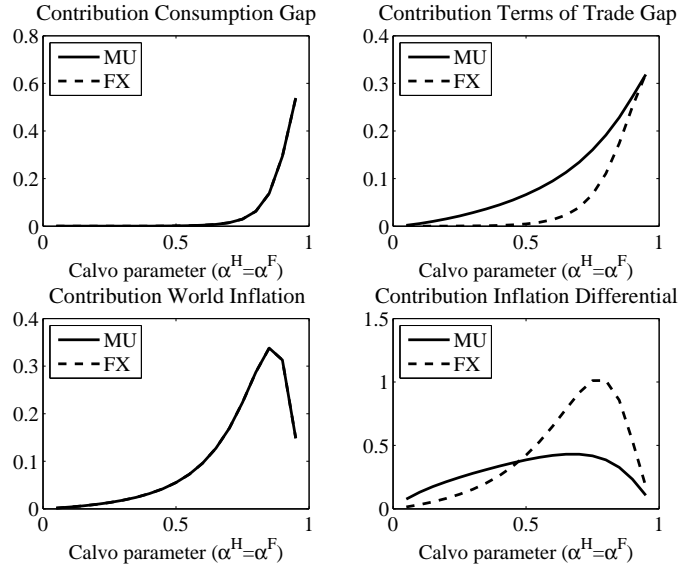


Figure 2: Contributions to world welfare loss in Figure 1

attaches a higher weight to inflation as prices become stickier.¹³

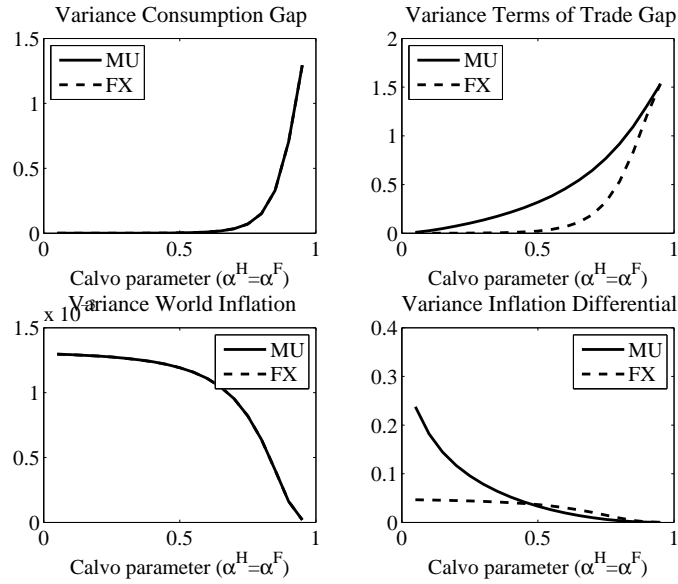


Figure 3: Variances underlying the contributions in Figure 2

Since the weight of the inflation differential is identical across monetary regimes (this holds for all components), the variance of the inflation differential is the driving force behind the result that the MU regime yields higher world welfare beyond a certain degree of price stickiness. In both regimes, the variance of the inflation differential decreases with the degree of price stickiness. More importantly, whereas

¹³The agent attaches a higher weight to inflation as prices become stickier, because the degree of inefficient price dispersion between differentiated goods increases with the degree of price stickiness for a given level of aggregate inflation. This feature is common to the closed-economy setup of the basic New Keynesian model, as in Galí (2008).

the variance is higher in the MU regime for lower degrees of price stickiness, it is higher in the FX regime for higher degrees of price stickiness.

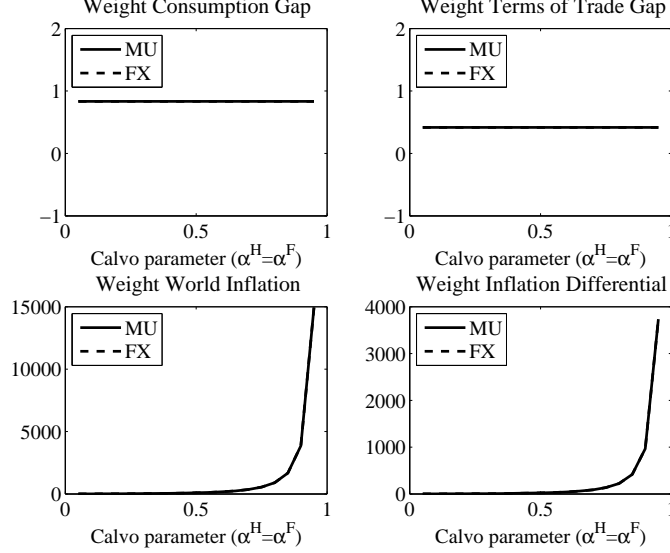


Figure 4: Weights underlying the contributions in Figure 2

The inflation differential in period t can be expressed as the sum of current and discounted expected future terms of trade gaps. Solving equation (4.4) forward, which holds under both regimes, gives

$$\pi_t^R = -k_T E_t \sum_{k=0}^{\infty} \beta^k (\hat{T}_{t+k} - \tilde{T}_{t+k}). \quad (4.19)$$

Accordingly, current and expected future terms of trade gaps are the only determinant for the current inflation differential. As shown below, two mechanisms that both affect the terms of trade gap explain why the variance of the inflation differential is lower under the MU regime for relatively sticky prices: First, the stabilizing property of a flexible nominal exchange rate declines as prices become stickier ("inherited stickiness of the nominal exchange rate"). Second, fixing the exchange rate entails an inherent benefit of affecting inflation expectations in a favorable way by inducing history-dependence into the economy ("inherent benefit of monetary unions").

4.2.1. Inherited stickiness of the nominal exchange rate

Whether the nominal exchange rate stabilizes or destabilizes the terms of trade gap, thereby facilitating an efficient allocation across countries, depends on the way monetary policy is conducted. Recall the recursive law of motion for the nominal exchange rate, (4.13):

$$\Delta \hat{S}_t = b_3 \hat{T}_{t-1} + c_3 \tilde{T}_t, \quad (4.20)$$

with

$$c_3 = \frac{(\phi_\pi - 1 - \phi_Y)k_T}{(\phi_\pi - \rho_H)k_T + (1 - \rho_H + \phi_Y)(1 - \beta\rho_H)}. \quad (4.21)$$

Under the baseline calibration, where monetary policy reacts to inflation, but not to output ($\phi_Y = 0$), the coefficient c_3 is unambiguously positive (since $\phi_\pi > 1$) and smaller than one. Accordingly, in response to a shock that leads to an increase in

the flexible-price terms of trade, the nominal exchange rate will increase as well, pushing up the sticky-price terms of trade closer to the flexible-price terms of trade. Thus, the nominal exchange rate stabilizes the terms of trade gap.

However, the stabilizing effect of the nominal exchange rate weakens as prices become stickier. The size of the response of the nominal exchange rate to a productivity shock declines with the degree of price stickiness. Analytically, as the degree of price stickiness α increases, k_T decreases and c_3 decreases. In the limit, when prices become fixed ($\alpha \rightarrow 1$), the nominal exchange rate is fixed as well ($k_T \rightarrow 0, c_3 \rightarrow 0$).

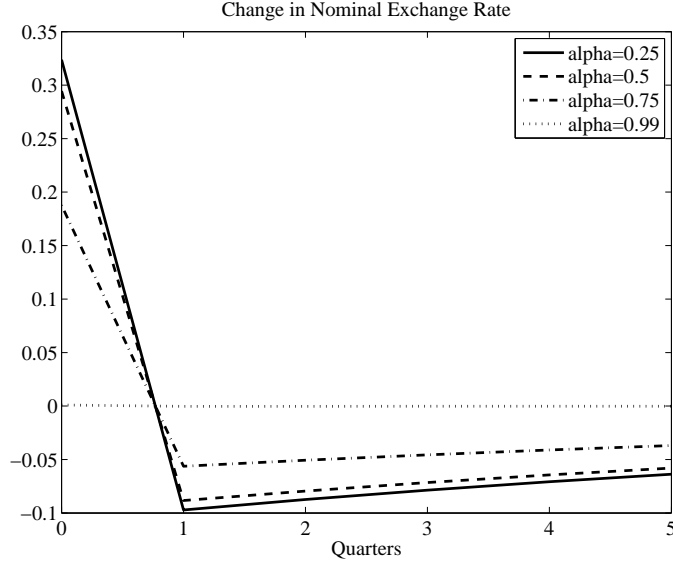


Figure 5: Impulse response of the change in the nominal exchange rate to a positive productivity shock in country H under different degrees of price stickiness (α)

The reason is the following: The expected change in the nominal exchange rate depends on the interest rate differential across countries, recall the uncovered interest parity condition:

$$E_t \Delta \hat{S}_{t+1} = \hat{R}_t^H - \hat{R}_t^F. \quad (4.22)$$

Interest rates, in turn, are set by monetary policy in response to inflation according to the interest rate rules. Therefore, an increase in price stickiness, which reduces inflation variability, reduces interest rate variability and, ultimately, reduces the variability of the nominal exchange rate. Figure 5 shows how the variability of the nominal exchange rate declines with the degree of price stickiness. When prices are virtually fixed ($\alpha = 0.99$), the nominal exchange rate is virtually fixed as well.

Thus, the nominal exchange rate inherits the stickiness of goods prices. The increased stickiness of the nominal exchange rate, in turn, hampers the stabilization of the terms of trade gap. The stabilizing property of the nominal exchange rate, therefore, declines with the degree of price stickiness.

4.2.2. Inherent benefit of monetary unions

The fact that the benefit of a flexible nominal exchange rate declines with the degree of price stickiness cannot explain alone the result that the MU regime is welfare-improving over the FX regime. For even under relatively sticky prices, the nominal exchange rate stabilizes the terms of trade gap at least to some extent compared to a situation with a completely fixed nominal exchange rate as under the MU regime. That is also the reason why the variance of the terms of trade gap is lower under

the FX regime regardless of the degree of price stickiness (Figure 3, upper right panel). Thus, there must be a benefit of the MU regime as well.

As the analytical insights from Section 4.1 have shown, there is one important difference between the two monetary regimes: In contrast to the FX regime, the economy under the MU regime is intrinsically inertial. So, even in the presence of a one-off shock, the inflation differential and the terms of trade gap display persistence. To elaborate on this qualitative difference between the two monetary regimes, consider the impulse response of the terms of trade gap to a positive one-off productivity shock in country H (Figure 6).¹⁴

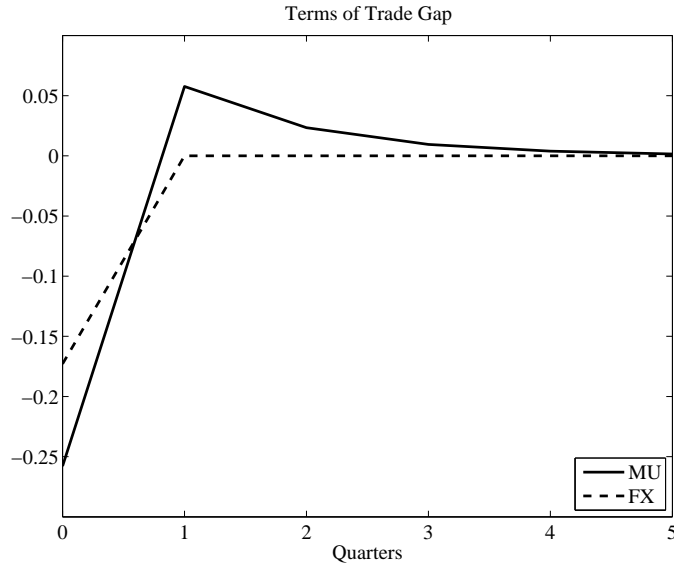


Figure 6: Impulse response of the terms of trade gap to a positive one-off productivity shock in country H with $\rho_H = 0$ and $\alpha = 0.2$

On impact, the terms of trade gap declines under both regimes, because the sticky-price terms of trade does not increase as much as the flexible-price terms of trade due to the stickiness of prices. In the following period, when the shock has vanished, the terms of trade gap is back at steady state under the FX regime, but not under the MU regime. Under the FX regime, it is the nominal exchange rate that brings the terms of trade gap automatically back to steady state in the absence of shocks.¹⁵ Under the MU regime, this mechanism is absent. As a result, the terms of trade gap is intrinsically inertial or history-dependent.

Importantly, the history-dependence of the terms of trade gap manifests itself in an overshooting pattern. The terms of trade gap overshoots, because the sticky-price terms of trade are still elevated above steady state after the shock has vanished, whereas the flexible-price terms of trade are back at steady state. In subsequent periods, the terms of trade gap converges back to steady state.

The qualitative difference in the dynamics between the two monetary regimes prevails in situations in which the productivity shock itself is persistent. Whereas the terms of trade gap converges monotonically back to steady state under the FX regime, it overshoots under the MU regime (Figure 7, left panel).¹⁶ Recall that

¹⁴The degree of price stickiness was chosen to be low ($\alpha = 0.2$) as to make the differences in the impulse responses clearly visible. The differences are much smaller for higher degrees of price stickiness, but qualitatively the same.

¹⁵Interestingly, this mechanism is independent of the interest rate rule coefficients ϕ_π and ϕ_Y (recall that the RLOM coefficient $b_3 = -1$).

¹⁶The calibration underlying the impulse responses in Figure 7 is identical to the calibration under-

the current terms of trade gap and its expected future development are the only determinants of the inflation differential:

$$\pi_t^R = -k_T E_t \sum_{k=0}^{\infty} \beta^k (\hat{T}_{t+k} - \tilde{T}_{t+k}). \quad (4.23)$$

Accordingly, under the FX regime the inflation differential increases on impact and converges monotonically back to steady state (Figure 7, right panel), because the terms of trade gap declines, remains negative and converges monotonically back to steady state, which is anticipated by price setters. In contrast, under the MU regime, price setters adjust their prices less in the initial period despite the stronger initial decline in the terms of trade gap, because they anticipate the future overshooting of the terms of trade gap. Therefore, in subsequent periods, inflation returns to steady state regions faster than under the FX regime and eventually overshoots.¹⁷

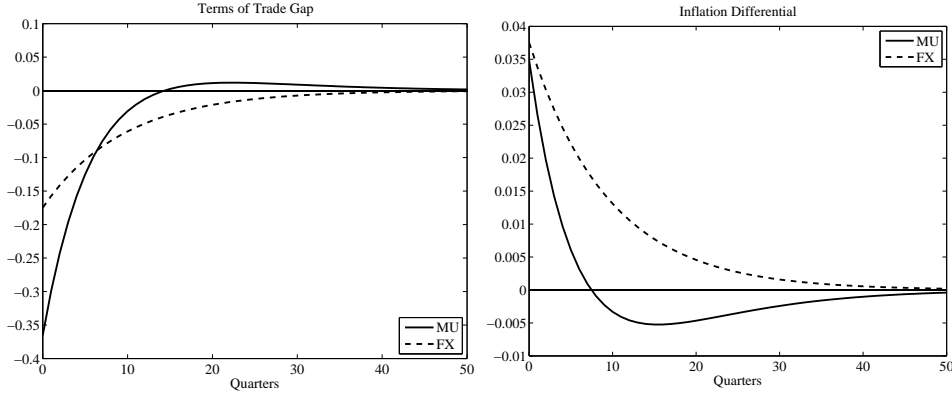


Figure 7: Impulse responses to a positive productivity shock in country H with $\rho_H = 0.9$ and $\alpha = 0.75$

To sum up: Since price setters are forward-looking, not only present, but also expected future terms of trade gaps matter for current inflation. Since the nominal exchange rate is fixed under the MU regime, the terms of trade gap overshoots in response to a shock at some point in time, which would then call for the opposite price adjustments as in the present. In anticipation of this, current price responses are smaller in magnitude than under the FX regime. As a result, the variance of the inflation differential (i.e. the sum of squared deviations of the inflation differential from zero) is lower under the MU regime than under the FX regime. Thus, the inherent benefit of monetary union consists of affecting inflation expectations in such a way as to lower the welfare-relevant variance of inflation by inducing history-dependence into the economy.

4.2.3. Summary of baseline results

In the case of two identical countries, world welfare is higher under the monetary union regime if prices are relatively sticky, and higher under the flexible exchange rate regime if prices are relatively flexible. If prices are relatively sticky, the benefit of greater inflation stability outweighs the cost of greater output gap and terms of

lying the welfare results in Figures 1 through 4.

¹⁷Throughout the paper, I use the term "overshooting" to describe both "overshooting" and "undershooting".

trade gap instability.¹⁸ For the economic agent puts the highest weight on inflation stability. This is traditionally the case in microfounded welfare measures derived from New Keynesian models.

Two effects drive the result:

1. **Inherited stickiness of the nominal exchange rate:** The nominal exchange rate inherits the stickiness of goods prices through the uncovered interest parity condition in combination with monetary policy setting the nominal interest rate in reaction to inflation. As a result, the property of the nominal exchange rate of facilitating an efficient allocation across countries by stabilizing the terms of trade gap and therefore inflation declines as prices become stickier.
2. **Inherent benefit of monetary unions:** Fixing the nominal exchange rate induces history-dependence into the economy in the form of terms of trade overshooting. Terms of trade overshooting, in turn, implies that future price adjustments will have the opposite sign of current price adjustments. Since price setters are forward-looking and anticipate this pattern, current price adjustments are smaller in the first place. As a result, inflation is more stable.

For the first mechanism to be effective, the uncovered interest parity condition need not hold exactly. A link between interest rates and the nominal exchange rate is sufficient.

The second effect is closely related to a well-known phenomenon from optimal monetary policy analysis in a closed-economy environment. Optimal monetary policy *under discretion* is inferior from a welfare perspective to optimal policy *under commitment*, because the former does not influence inflation expectations of price setters in a favorable way. It suffers from the so-called stabilization bias.¹⁹ In contrast, optimal monetary policy under commitment induces history-dependence into the economy, therefore taking advantage of the fact that price setters are forward-looking. This results in a higher stability of inflation. In exactly the same sense, forming a monetary union may be superior to maintaining flexible exchange rates under certain conditions, because fixing the nominal exchange rate affects inflation expectations in a favorable way by inducing history-dependence into the economy.

Moreover, this effect is in place despite non-stationary price levels.²⁰ Thus, and in contrast to Monacelli (2004), the benefit does not hinge upon stationarity of the price level. This is a particular feature of the small open economy assumption and does not generally carry over to a two-country environment. Instead, the benefit hinges upon the overshooting pattern of the terms of trade, the anticipation of which reduces the magnitude of price changes, rendering the inflation rates more stable.

The inherent benefit of stabilizing inflation, which is related to the *stabilization bias*, adds to the benefit of eliminating a potential *inflation bias*, which was stressed by Alesina and Barro (2002) and Cooley and Quadrini (2003). Whereas the latter benefit has been acknowledged in the literature (e.g. Dellas and Tavlas, 2009; Beetsma and Giuliodori, 2010), the first still seems to be unknown.

¹⁸As mentioned earlier, the welfare loss function can alternatively be expressed in terms of country-specific output gaps instead of the consumption gap, see equation (C.72). The output gap variance is larger under the MU regime than under the FX regime regardless of the degree of price stickiness. This was not shown to save space, but the corresponding graphs are available from the author upon request.

¹⁹For details on the stabilization bias, see e.g. Woodford (2003, Ch. 7), Galí (2008, Ch. 5), or Walsh (2010, Ch. 8).

²⁰This was not shown to save space, but the corresponding impulse responses for price levels are available from the author upon request.

4.3. Monetary policy

The result that forming a monetary union is beneficial if prices are relatively sticky is very robust to the range of parameter values considered in Table 2.²¹ The important exception are the parameters that govern the behavior of monetary policy, i.e. the coefficients of the interest rate rules. Those will be considered in the following subsections.

4.3.1. Inflation coefficient

Whether forming a monetary union turns out to be beneficial depends crucially on the inflation coefficient ϕ_π in the interest rate rules, i.e. the aggressiveness of monetary policy towards inflation (Figure 8).²² Starting out at a very low response of monetary policy to inflation (ϕ_π above, but close to one), the MU regime yields a lower world welfare loss for every degree of price stickiness. Increasing the aggressiveness of monetary policy a little bit results in the FX regime being superior for very low degrees of price stickiness, but inferior for higher degrees of price stickiness. As the aggressiveness of monetary policy increases further, the threshold value for α increases, beyond which the MU regime yields a lower world welfare loss. Eventually, beyond a certain aggressiveness of monetary policy towards inflation ($\phi_\pi \approx 2.5$), the MU regime is inferior to the FX regime regardless of the degree of price stickiness.

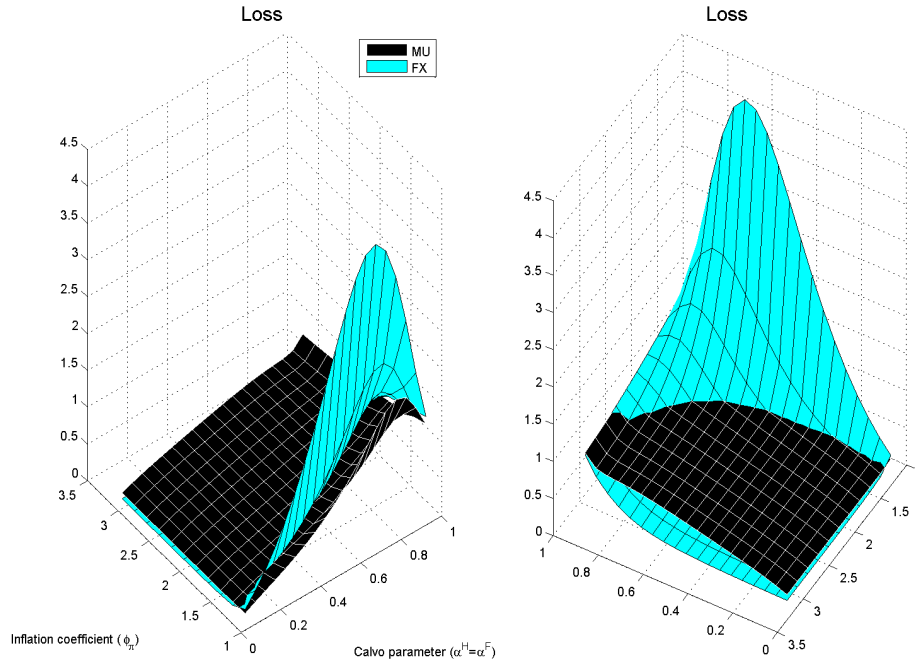


Figure 8: World welfare loss under different degrees of price stickiness ($\alpha^H = \alpha^F$) and different values for the inflation coefficient (ϕ_π) from two different angles

²¹The corresponding graphs are available from the author upon request.

²²The graph in the right panel of Figure 8 is a rotation of the graph in the left panel, in order to be able to see behind the steep surface area. Cutting through the two surface areas along $\phi_\pi = 1.5$ produces Figure 1.

The intuition is the following: Under the FX regime, when monetary policy reacts to inflation only, the nominal exchange rate stabilizes the terms of trade gap in response to shocks (recall Section 4.2.1). The strength of this stabilizing property increases with the aggressiveness of monetary policy towards inflation, since monetary policy influences directly the nominal exchange rate via the uncovered interest parity condition (recall equation (4.8) and the coefficient c_3 of the RLOM). As a result, even when prices are relatively sticky, monetary policy can counteract by being more aggressive towards inflation. This is not possible under the MU regime, where the common monetary policy has no influence on the terms of trade whatsoever when prices are equally sticky across countries.

4.3.2. Output coefficient

The welfare ranking between the two monetary regimes depends on the output coefficient in the interest rate rules ϕ_Y as well, i.e. the aggressiveness of monetary policy towards output (Figure 9). For almost all combinations of ϕ_Y and ϕ_π considered, the FX regime yields a higher world welfare loss than the MU regime, although the degree of price stickiness was deliberately chosen to favor the FX regime ($\alpha = 0.2$). Increasing the degree of price stickiness would favor the MU regime further. In general, the stronger monetary policy reacts to output, the stronger it needs to react to inflation for the FX regime to remain superior. This relationship is very steep; a small increase in ϕ_Y (e.g. from 0 to 0.25) requires a strong increase in ϕ_π (from roughly 1.5 to 2.3).

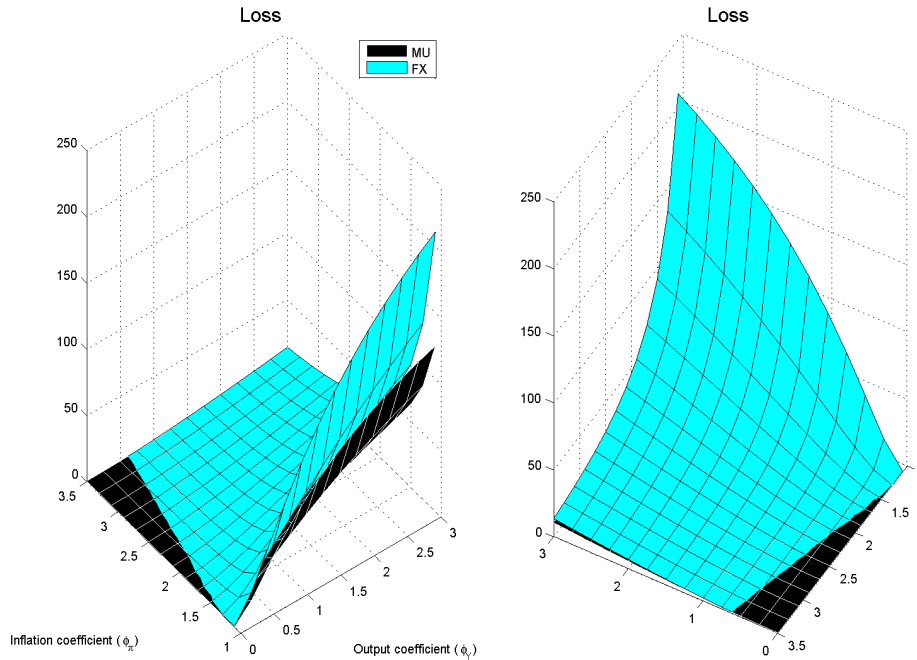


Figure 9: World welfare loss under different values for the output coefficient (ϕ_Y) and for the inflation coefficient (ϕ_π) with $\alpha = 0.2$, from two different angles

Unlike in the case of the response to inflation, the more aggressive monetary policy reacts to output, the smaller becomes the impact response of the nominal

exchange rate to shocks (recall coefficient c_3 of the RLOM). When the aggressiveness of monetary policy towards output relative to inflation exceeds a certain degree ($\phi_Y > \phi_\pi - 1$), the nominal exchange rate destabilizes the terms of trade gap in response to shocks ($c_3 < 0$).

The reason is that from a welfare perspective, a response to output by monetary policy is counterproductive. Welfare-relevant is not the deviation of output from steady state, but the deviation from the flexible-price counterpart (output gap).²³ For example, a positive productivity shock in country H induces a rise in output, but a fall in the output gap, since the increase of output is lower than the increase of flexible-price output. A welfare-oriented reaction of monetary policy would require a reduction in the interest rate due to the negative output gap. Instead, monetary policy raises the interest rate due to the rise in output. As a result, the variance of the inflation differential is higher, when monetary policy reacts to output ($\phi_Y > 0$) than when it does not react to output at all ($\phi_Y = 0$).²⁴

A reaction to output is counterproductive under both regimes (in Figure 9, the welfare loss increases under both regimes with an increasing output coefficient ϕ_Y). However, recall that under the MU regime monetary policy only affects the consumption gap and world inflation, but not the inflation differential and the terms of trade. Under the FX regime, monetary policy affects all four components of the welfare loss function. Thus, conducting counterproductive monetary policy is more harmful under the FX regime, because monetary policy is more powerful under the FX regime. The nominal exchange rate does not compensate for bad monetary policy, it rather inherits the quality of monetary policy.

4.3.3. Interest rate smoothing coefficient

The welfare ranking between the two monetary regimes depends on the degree of interest rate smoothing ϕ_R as well (Figure 10). No interest rate smoothing ($\phi_R = 0$) produces Figure 1: The MU regime yields a lower welfare loss for relatively sticky prices. As the degree of interest rate smoothing increases, the threshold value for α , beyond which the MU regime is superior, also increases. For very high degrees of interest rate smoothing, the MU regime is welfare-improving only for extremely high degrees of price stickiness. Thus, interest rate smoothing makes a beneficial monetary union less likely.

In the following, instead of deriving the recursive laws of motion, the consequences of interest rate smoothing for the economy under the FX regime are presented graphically. Consider the impulse response of the terms of trade gap to a positive one-off productivity shock in country H , but now with a relatively high degree of interest rate smoothing (Figure 11). The impulse response under the MU regime is identical to the situation without interest rate smoothing (Figure 6), because monetary policy continues to exert no influence on the terms of trade when prices are equally sticky across countries. In contrast, the impulse response under the FX regime now resembles the response under the MU regime. Although the productivity shock is one-off, the terms of trade gap displays inertia in the form of overshooting.

As a result, inflation expectations are affected in the same favorable way as under the MU regime, namely by inducing history-dependence into the economy. Only the source of history-dependence is different. Under the FX regime, monetary policy has to engage in interest rate smoothing to induce history-dependence. Under the MU regime, history-dependence is induced automatically by the fact that the nominal exchange rate is fixed. For the FX regime to be welfare-improving over

²³Recall that the welfare loss function can alternatively be expressed in terms of country-specific output gaps instead of the consumption gap, see equation (C.72).

²⁴This feature is common to the closed-economy setup of the basic New Keynesian model, as in Galí (2008).

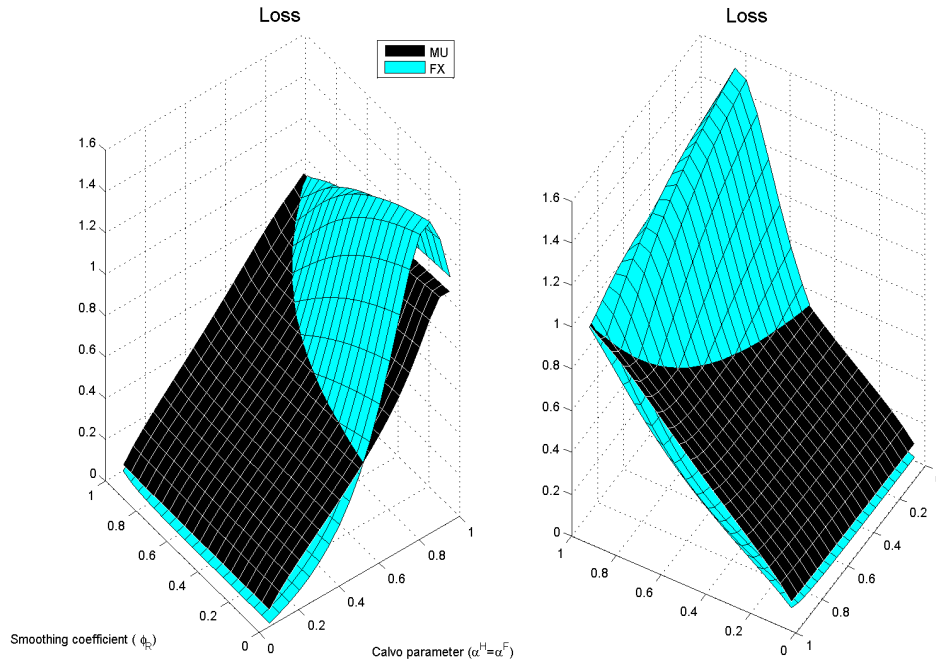


Figure 10: World welfare loss under different degrees of price stickiness ($\alpha^H = \alpha^F$) and different values for the interest rate smoothing coefficient (ϕ_R) from two different angles

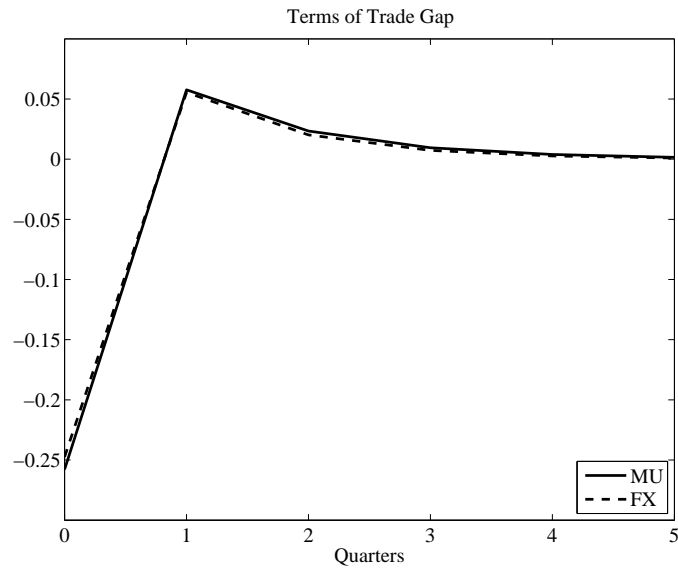


Figure 11: Impulse response of the terms of trade gap to a positive one-off productivity shock in country H with $\phi_R = 0.9$, $\rho_H = 0$ and $\alpha = 0.2$

the MU regime under relatively sticky prices, monetary policy has to implement a sufficiently high degree of interest rate smoothing. This will render the inflation

differential more stable under the FX regime.

The fact that interest rate smoothing under the FX regime reduces the welfare loss by stabilizing inflation comes as no surprise. As shown by Woodford (1999), one way for monetary policy to implement the kind of history-dependence that is desirable from the perspective of optimal monetary policy is to engage in interest rate smoothing by including a feedback of the current nominal interest rate to past realizations of the nominal interest rate, as in the interest rate rules given by equations (3.20) and (3.21).

5. Results under asymmetry

5.1. Country size

If the two countries only differ in population size, the analytical insights from Section 4.1 carry over. Thus, the systems of RLOM of both monetary regimes, equations (4.4) through (4.8), are valid in this case. As one can see, the RLOM are independent of the country size n . Therefore, the welfare-relevant inflation differential and the terms of trade gap are independent of n . As a consequence, the threshold value for α , beyond which the MU regime yields higher world welfare, is completely insensitive with respect to n . The welfare ranking between the MU and FX regime does not depend on country size.

5.2. Price stickiness

The world welfare loss function under different degrees of price stickiness across the two countries is given by equation (3.22) and repeated here for convenience:

$$\begin{aligned}
W_t = & -\frac{1}{2} \left((\rho + \eta) \text{var}(\hat{C}_t - \tilde{C}_t) + (1 + \eta)n(1 - n) \text{var}(\hat{T}_t - \tilde{T}_t) \right. \\
& + \sigma(1 + \sigma\eta)n \frac{\alpha^H}{(1 - \alpha^H)(1 - \alpha^H\beta)} \text{var} \pi_t^H \\
& \left. + \sigma(1 + \sigma\eta)(1 - n) \frac{\alpha^F}{(1 - \alpha^F)(1 - \alpha^F\beta)} \text{var} \pi_t^F \right) \\
& + t.i.p. + O(\|\xi\|^3). \tag{5.1}
\end{aligned}$$

It contains four components: The variance of the consumption gap ($\hat{C}_t - \tilde{C}_t$), the variance of the terms of trade gap ($\hat{T}_t - \tilde{T}_t$), the variance of inflation in country H (π_t^H), and the variance of inflation in country F (π_t^F).

Unless the degree of price stickiness is extremely high ($\alpha \geq 0.9$, which corresponds to an average duration of price contracts of 10 quarters), asymmetry in the degree of price stickiness does not matter for the welfare ranking between the MU and FX regime (Figure 12). Again, this result is predominantly driven by the variances of the inflation rates. Drawing from the analysis in the previous section, the intuition is the following: First, the inherent benefit of monetary unions of inducing history-dependence is insensitive to country characteristics. It depends only on the fact that the nominal exchange rate is fixed and that price setters are forward-looking. Second, the nominal exchange rate inherits the stickiness of goods prices from both countries. It does not matter if the stickiness is equally present in both countries or if the stickiness comes primarily from one country. Thus, as long as the aggregate degree of price stickiness in the world as a whole is sufficiently high, the MU regime continues to be beneficial.

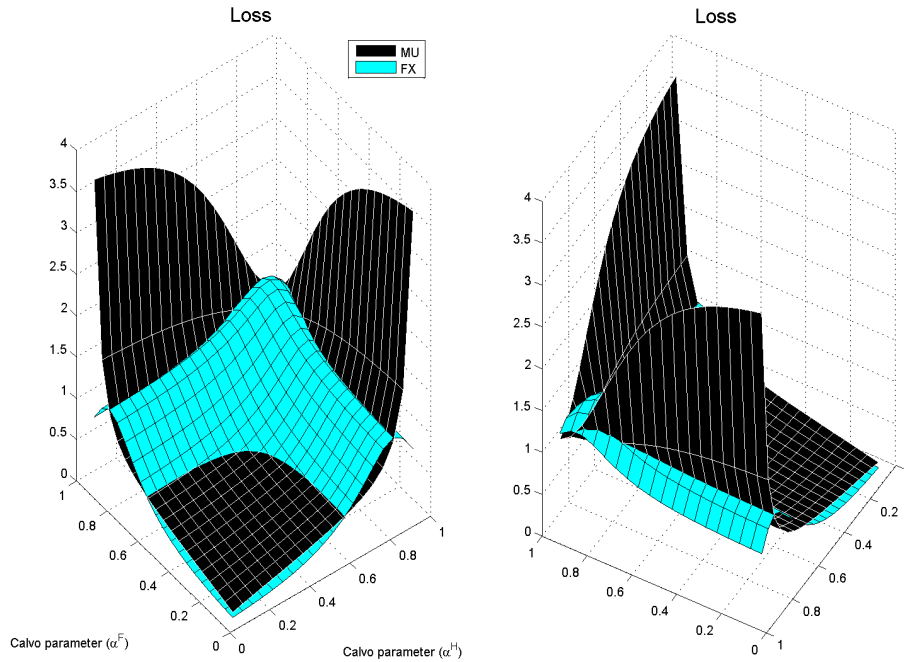


Figure 12: World welfare loss under different degrees of price stickiness across countries ($\alpha^H \neq \alpha^F$), from two different angles

6. Conclusion

The results of the analysis are clear-cut: In the basic two-country New Keynesian model, in which monetary policy is conducted via Taylor-type interest rate rules, forming a monetary union is welfare-improving if prices are relatively sticky. In this case, the cost of relinquishing national monetary policy as a stabilization device is outweighed by the inherent benefit of stabilizing inflation expectations by fixing the nominal exchange rate. This result is very sensitive to the way monetary policy is conducted: If monetary policy responds to inflation aggressively or if it implements a high degree of interest rate smoothing, maintaining a flexible nominal exchange rate is superior. In contrast, asymmetries in the degree of price stickiness and country size are not critical.

Clearly, the results stand in contrast to the predictions of the traditional OCA theory. Probably the most important reason for this discrepancy is the absence of expectations in the theoretical models of the traditional OCA literature. For the inherent benefit of monetary unions works through inflation expectations.

References

- Adao, B., I. Correia, and P. Teles (2009). On the relevance of exchange rate regimes for stabilization policy. *Journal of Economic Theory* 144, 1468–1488.
- Alesina, A. and R. J. Barro (2002). Currency unions. *Quarterly Journal of Economics* 117, 409–436.
- Beetsma, R. and M. Giuliodori (2010). The macroeconomic costs and benefits of the EMU and other monetary unions: An overview of recent research. *Journal of Economic Literature* 48, 603–641.
- Benigno, G. and P. Benigno (2006). Designing targeting rules for international monetary policy cooperation. *Journal of Monetary Economics* 53, 473–506.
- Benigno, G. and P. Benigno (2008). Exchange rate determination under interest rate rules. *Journal of International Money and Finance* 27, 971–993.
- Benigno, P. (2003). Appendix to "Optimal monetary policy in a currency area". <http://docenti.luiss.it/benigno/files/2007/02/appendix1.pdf>.
- Benigno, P. (2004). Optimal monetary policy in a currency area. *Journal of International Economics* 63, 293–320.
- Benigno, P. and M. Woodford (2005). Inflation stabilization and welfare: The cost of a distorted steady state. *Journal of the European Economic Association* 3, 1185–1236.
- Calvo, G. A. (1983). Staggered prices in a utility-maximizing framework. *Journal of Monetary Economics* 12, 383–398.
- Ca'Zorzi, M., R. A. De Santis, and F. Zampolli (2005). Welfare implications of joining a common currency. ECB Working Paper 445.
- Ching, S. and M. B. Devereux (2003). Mundell revisited: A simple approach to the costs and benefits of a single currency area. *Review of International Economics* 11, 674–691.
- Clarida, R., J. Galí, and M. Gertler (2002). A simple framework for international monetary policy analysis. *Journal of Monetary Economics* 49, 879–904.
- Clerc, L., H. Dellas, and O. Loisel (2011). To be or not to be in monetary union: A synthesis. *Journal of International Economics* 83, 154–167.
- Cole, H. L. and M. Obstfeld (1991). Commodity trade and international risk sharing. *Journal of Monetary Economics* 28, 3–24.
- Cooley, T. F. and V. Quadrini (2003). Common currencies vs. monetary independence. *Review of Economic Studies* 70, 785–806.
- Corsetti, G. (2008). A modern reconsideration of the theory of optimal currency areas. CEPR Discussion Paper 6712.
- Corsetti, G., L. Dedola, and S. Leduc (2011). Optimal monetary policy in open economies. In B. M. Friedman and M. Woodford (Eds.), *Handbook of Monetary Economics* 3B. North-Holland, Amsterdam.
- Corsetti, G. and P. Pesenti (2001). Welfare and macroeconomic interdependence. *Quarterly Journal of Economics* 116, 421–4415.

- Dellas, H. (2006). Monetary policy in open economies. *European Economic Review* 50, 1471–1486.
- Dellas, H. and G. S. Tavlas (2005). Wage rigidity and monetary union. *Economic Journal* 115, 907–927.
- Dellas, H. and G. S. Tavlas (2009). An optimum-currency-area odyssey. *Journal of International Money and Finance* 28, 1117–1137.
- Devereux, M. B. (2004). Monetary policy rules and exchange rate flexibility in a simple dynamic general equilibrium model. *Journal of Macroeconomics* 26, 287–308.
- Dixit, A. K. and J. E. Stiglitz (1977). Monopolistic competition and optimum product diversity. *American Economic Review* 67, 297–308.
- Ferreira-Lopes, A. (2010). In or out? The welfare costs of EMU membership. *Economic Modelling* 27, 585–594.
- Galí, J. (2008). *Monetary policy, inflation, and the business cycle*. Princeton University Press, Princeton and Oxford.
- Galí, J. and T. Monacelli (2005). Monetary policy and exchange rate volatility in a small open economy. *Review of Economic Studies* 72, 707–734.
- Giavazzi, F. and M. Pagano (1988). The advantage of tying one’s hands. *European Economic Review* 32, 1055–1082.
- Hamilton, J. D. (1994). *Time series analysis*. Princeton University Press, Princeton.
- Kollmann, R. (2001). The exchange rate in a dynamic-optimizing business cycle model with nominal rigidities: A quantitative investigation. *Journal of International Economics* 55, 243–262.
- Kollmann, R. (2004). Welfare effects of a monetary union: The role of trade openness. *Journal of the European Economic Association* 2, 289–301.
- Monacelli, T. (2004). Commitment, discretion and exchange rates in an open economy. In J.-O. Hairault and T. Sopraseuth (Eds.), *Exchange rate dynamics: A new open economy macroeconomics perspective*. Routledge, London and New York.
- Mundell, R. A. (1961). A theory of optimum currency areas. *American Economic Review* 51, 657–665.
- Obstfeld, M. and K. Rogoff (2001). Risk and exchange rates. Prepared for the conference in honor of Assaf Razin, Tel-Aviv University, March 2001.
- Rotemberg, J. J. and M. Woodford (1998). An optimization-based econometric framework for the evaluation of monetary policy: Expanded version. NBER Technical Working Paper 233.
- Soffritti, M. and F. Zanetti (2008). The advantage of tying one’s hands: Revisited. *International Journal of Finance and Economics* 13, 135–149.
- Walsh, C. E. (2010). *Monetary theory and policy*. MIT Press, Cambridge.
- Woodford, M. (1999). Optimal monetary policy inertia. *Manchester School Supplement* 67, 1–35.
- Woodford, M. (2003). *Interest and prices: Foundations of a theory of monetary policy*. Princeton University Press, Princeton.

A. Monetary union regime (MU)

The world, which consists of two countries labeled H and F , is populated by a continuum of agents on the interval $[0, 1]$. The population on the segment $[0, n]$ lives in country H , the population on the segment $[n, 1]$ lives in country F . Thus, n measures the population size as a fraction of world population. An agent is both consumer and producer. He produces a single differentiated good and consumes all the goods produced in both countries.

A.1. Consumer problem

Agent j in country $i = H, F$ derives positive utility from consumption C^j and negative utility from producing the differentiated good y^j . The present discounted value of lifetime utility U^j is thus given by²⁵

$$U^j = E_0 \sum_{t=0}^{\infty} \beta^t \left[U(C_t^j) - V(y_t^j, z_t^i) \right], \quad (\text{A.1})$$

where E_t denotes the expectations operator and β the discount factor.

V is an increasing, convex function of agent j 's supply of his product y_t^j and a decreasing convex function of productivity z_t^i , which is common to all agents in country i . One can think of V as the combination of the agent's disutility of working and the production function. If the disutility of working is given by $g(N_t^j)$, where N_t^j is the number of hours worked, and the production function is given by $y_t^j = f(N_t^j, z_t^i)$, then $V = g(f^{-1}(y_t^j, z_t^i))$.

U is an increasing, concave function of consumption C_t^j . The agent consumes both a bundle of differentiated goods from country H and from country F with a preference structure of the Cobb-Douglas type, so that

$$C_t^j = \left(\frac{C_{H,t}^j}{n} \right)^n \left(\frac{C_{F,t}^j}{1-n} \right)^{1-n}, \quad (\text{A.2})$$

where the bundles of differentiated goods are given by aggregators according to Dixit and Stiglitz (1977):

$$C_{H,t}^j = \left[\left(\frac{1}{n} \right)^{\frac{1}{\sigma}} \int_0^n c_t^j(h)^{\frac{\sigma-1}{\sigma}} dh \right]^{\frac{\sigma}{\sigma-1}} \quad (\text{A.3})$$

$$C_{F,t}^j = \left[\left(\frac{1}{1-n} \right)^{\frac{1}{\sigma}} \int_n^1 c_t^j(f)^{\frac{\sigma-1}{\sigma}} df \right]^{\frac{\sigma}{\sigma-1}}.$$

These preferences imply (1) that the elasticity of substitution between differentiated goods c_t^j from one country is σ , which is assumed to be greater than one and equal across countries, (2) that the elasticity of substitution between the bundle of goods from the two countries $C_{H,t}$ and $C_{F,t}$ is one and equal across countries, and (3) that the share of a bundle of goods from one country in the overall consumption expenditures of an agent coincides with the country's share in world population, i.e. there is no home bias in consumption.

Accordingly, the aggregate price index in country i is given by

$$P_t^i = (P_{H,t}^i)^n (P_{F,t}^i)^{1-n}, \quad (\text{A.4})$$

²⁵In Benigno (2004), the agent derives utility also from holding money. However, money in the utility function is not necessary if monetary policy is conducted via the interest rate.

where the price indices for the bundles of differentiated goods in each country are defined by

$$P_{H,t}^i = \left[\frac{1}{n} \int_0^n p_t^i(h)^{1-\sigma} dh \right]^{\frac{1}{1-\sigma}} \quad (\text{A.5})$$

$$P_{F,t}^i = \left[\frac{1}{1-n} \int_n^1 p_t^i(f)^{1-\sigma} df \right]^{\frac{1}{1-\sigma}}.$$

In their role as producers, agents charge only one price for their good irrespective of whether the good remains in the country or is exported (no price discrimination). Furthermore, exporting does not entail transportation costs. These assumptions imply that a single good has the same price in both countries, i.e. $p_t^H(h) = p_t^F(h)$ and $p_t^H(f) = p_t^F(f)$. Given the Cobb-Douglas type structure of preferences, this immediately leads purchasing power parity to hold, so $P_t^H = P_t^F = P_t$. Consequently, the superscript i can be dropped from all the price indices.²⁶

Agent j takes three decisions with respect to his consumption choices. First, he decides on the overall level of consumption C_t^j .²⁷ Second, given C_t^j he optimally allocates expenditures between the bundles of differentiated goods from the two countries $C_{H,t}^j$ and $C_{F,t}^j$ by minimizing total expenditure $P_t C_t^j$ with respect to (A.2). As a result, demand for these bundles is given by

$$C_{H,t}^j = n \left(\frac{P_{H,t}}{P_t} \right)^{-1} C_t^j, \quad C_{F,t}^j = (1-n) \left(\frac{P_{F,t}}{P_t} \right)^{-1} C_t^j. \quad (\text{A.6})$$

Third, given $C_{H,t}$ and $C_{F,t}$ the agent optimally allocates expenditures between the differentiated goods by minimizing $P_{H,t} C_{H,t}$ and $P_{F,t} C_{F,t}$ with respect to equations (A.3). This yields

$$c_t^j(h) = \frac{1}{n} \left(\frac{p_t(h)}{P_{H,t}} \right)^{-\sigma} C_{H,t}^j, \quad c_t^j(f) = \frac{1}{1-n} \left(\frac{p_t(f)}{P_{F,t}} \right)^{-\sigma} C_{F,t}^j. \quad (\text{A.7})$$

Combining (A.6) and (A.7) yields

$$c_t^j(h) = \left(\frac{p_t(h)}{P_{H,t}} \right)^{-\sigma} \left(\frac{P_{H,t}}{P_t} \right)^{-1} C_t^j, \quad c_t^j(f) = \left(\frac{p_t(f)}{P_{F,t}} \right)^{-\sigma} \left(\frac{P_{F,t}}{P_t} \right)^{-1} C_t^j. \quad (\text{A.8})$$

The terms of trade are defined from the perspective of country F , i.e. the ratio of the price of the bundle of goods produced in country F to the price of the bundle of goods imported from country H :

$$T_t = \frac{P_{F,t}}{P_{H,t}}. \quad (\text{A.9})$$

Equations (A.8) can then be expressed in terms of the terms of trade as follows:

$$c_t^j(h) = \left(\frac{p_t(h)}{P_{H,t}} \right)^{-\sigma} T_t^{1-n} C_t^j, \quad c_t^j(f) = \left(\frac{p_t(f)}{P_{F,t}} \right)^{-\sigma} T_t^{-n} C_t^j. \quad (\text{A.10})$$

where the terms of trade were substituted by rearranging the aggregate price equation (A.4) and using the definition of the terms of trade (A.9).

Aggregating over all agents living in both countries, global demand for the dif-

²⁶Note that P_t can be interpreted as a consumer price index, $P_{H,t}$ and $P_{F,t}$ as producer price indices.

²⁷As shown below, C_t^j is determined by the usual Euler consumption equation (A.14).

ferentiated goods h and f can be written as

$$y_t(h) = \left(\frac{p_t(h)}{P_{H,t}} \right)^{-\sigma} T_t^{1-n} C_t^W, \quad y_t(f) = \left(\frac{p_t(f)}{P_{F,t}} \right)^{-\sigma} T_t^{-n} C_t^W, \quad (\text{A.11})$$

where world consumption is given by

$$C_t^W = \int_0^1 C_t^j dj. \quad (\text{A.12})$$

There are two types of assets agents can trade in. Within each country, agents can insure against all possible states of nature by holding a portfolio of contingent, one-period securities whose real value (denominated in units of the consumption-based price index) is denoted by $B_t^{i,j}$ and whose vector of prices is denoted by q_t^i . Across countries, agents can trade in a non-contingent, one-period bond whose nominal value (denominated in the currency of the union) is denoted by B_t^j and whose nominal interest rate is denoted by R_t . Thus, asset markets are incomplete across countries, but complete within countries. The intertemporal budget constraint of agent j in country i is then given by

$$C_t^j + q_t^i B_t^{i,j} + \frac{B_t^j}{P_t(1+R_t)} = B_{t-1}^{i,j} + \frac{B_{t-1}^j}{P_t} + (1-\tau^i) \frac{p_t(j)y_t(j)}{P_t}, \quad (\text{A.13})$$

where the left-hand side represents the agent's expenditures and the right-hand side his income. The latter stems also from sales revenues $p_t(j)y_t(j)$ net of a proportional, country-specific tax τ^i .²⁸

All contingent securities and non-contingent bonds are assumed to be in zero supply in the initial period, so $B_0^{i,j} = B_0^j = 0$ for all i and j . Together with the facts that agents have identical preferences and that asset markets are complete within countries, this assumption implies perfect risk sharing of consumption within each country. Therefore, it is possible to analyze the consumer problem from the viewpoint of the representative agent of country H and country F .

The representative agent in country i maximizes his lifetime utility (A.1) with respect to the budget constraint (A.13). By combining the resulting first order conditions with respect to consumption and bond holdings, the usual Euler consumption equation is then given by

$$U_C(C_t^i) = (1+R_t)\beta E_t \left\{ U_C(C_{t+1}^i) \frac{P_t}{P_{t+1}} \right\}. \quad (\text{A.14})$$

One important implication of the Cobb-Douglas type consumption preferences given by (A.2) together with the initial condition $B_0^H = B_0^F = 0$ is that risk sharing is perfect across countries as well despite incomplete asset markets at the international level, in the sense that²⁹

$$C_t^H = C_t^F = C_t. \quad (\text{A.15})$$

To gain intuition, first note that, similar to (A.3), aggregate demand for the bundles

²⁸The tax will turn out to be a subsidy to exactly offset the distortion caused by monopolistic competition.

²⁹For a proof, see Benigno (2003), Appendix A.

of goods in the two countries can be expressed as

$$\begin{aligned} Y_t^H &= \left[\left(\frac{1}{n} \right) \int_0^n y_t(h)^{\frac{\sigma-1}{\sigma}} dh \right]^{\frac{\sigma}{\sigma-1}} \\ Y_t^F &= \left[\left(\frac{1}{1-n} \right) \int_n^1 y_t(f)^{\frac{\sigma-1}{\sigma}} df \right]^{\frac{\sigma}{\sigma-1}}. \end{aligned} \quad (\text{A.16})$$

Then, applying (A.16) to (A.11) and using (A.15) yields the national account identities

$$Y_t^H = T_t^{1-n} C_t, \quad Y_t^F = T_t^{-n} C_t. \quad (\text{A.17})$$

Making use of the definition of the terms of trade (A.9) and the aggregate price equation (A.4), this can be rearranged to

$$P_{H,t} Y_t^H = P_t C_t, \quad P_{F,t} Y_t^F = P_t C_t. \quad (\text{A.18})$$

Finally, the ratio of the two equations is given by

$$\frac{P_{F,t} Y_t^F}{P_{H,t} Y_t^H} = T_t \frac{Y_t^F}{Y_t^H} = 1. \quad (\text{A.19})$$

Nominal output equals nominal consumption in both countries at all times, as can be seen from (A.18). Thus, current accounts are always balanced. The reason is that any variation in the terms of trade is accompanied by an exact proportional variation in relative output across countries, as shown by (A.19). Agents shift consumption from the good that has become relatively expensive to the good that has become relatively cheap (expenditure switching effect) in such a way that a one percent increase in the relative price (terms of trade) leads to one percent decrease in relative quantities. This is ultimately due to Cobb-Douglas preferences.

As a result, relative nominal output and therefore relative income between the two countries are constant at all times. Thus, there are no gains from asset trade across countries, and the internationally traded bond becomes redundant ($B_t^H = B_t^F = 0 \ \forall \ t$). The terms of trade fully stabilize output variations.³⁰

A.2. Producer problem

In their role as producers, agents act in an environment of monopolistic competition, in which they dispose of some degree of market power. Furthermore, prices are sticky in the sense that the agent is able to change his price in a given period with a fixed probability, as in Calvo (1983). The probability of being able to change the price may differ across countries and is given by $1 - \alpha^i$.

Agent j in country i maximizes expected, discounted profits by choosing the price $\tilde{p}_t(j)$ taking into account that demand for his good depends on the chosen price and that the price may remain unchanged for some periods. Formally, the

³⁰The result that, under Cobb-Douglas preferences, the terms of trade provide perfect insurance against output variations was already shown by Cole and Obstfeld (1991). Note that the result does not hinge upon the specification in which the expenditure share in the Cobb-Douglas function coincides with the population size n (a feature also common to the Obstfeld and Rogoff (2001) model). If the expenditure share does not coincide with the population size, as in Corsetti and Pesenti (2001), relative consumption across countries as well as relative income across countries are still constant over time. However, they are not equal to one, as in (A.15) and (A.19) respectively. Consumption and nominal output, then, differ across countries.

agent maximizes

$$E_t \sum_{k=0}^{\infty} (\alpha^i \beta)^k [\lambda_{t+k}(1 - \tau^i) \tilde{p}_t(j) \tilde{y}_{t,t+k}(j) - V(\tilde{y}_{t,t+k}(j), z_{t+k}^i)] \quad (\text{A.20})$$

subject to the demand function

$$\tilde{y}_{t,t+k}(h) = \left(\frac{\tilde{p}_t(h)}{P_{H,t+k}} \right)^{-\sigma} T_{t+k}^{1-n} C_{t+k} \quad (\text{A.21})$$

if the agent lives in country H or

$$\tilde{y}_{t,t+k}(f) = \left(\frac{\tilde{p}_t(f)}{P_{F,t+k}} \right)^{-\sigma} T_{t+k}^{-n} C_{t+k}, \quad (\text{A.22})$$

if the agent lives in country F , where $\tilde{y}_{t,t+k}(j)$ denotes total demand of good j at time $t+k$ if the price $\tilde{p}_t(j)$ prevails. Profits are expressed in utility units. Therefore, nominal sales revenues net of taxes $(1 - \tau^i) \tilde{p}_t(j) \tilde{y}_{t,t+k}(j)$ are converted into utility units using the marginal utility of nominal revenues $\lambda_{t+k} = \frac{U_C(C_{t+k})}{P_{t+k}}$, which is the same for all agents in both countries due to perfect risk sharing within and across countries and due to purchasing power parity. The cost of production expressed in utility units is given by the function V .

The first order condition yields the optimal price

$$\tilde{p}_t(j) = \frac{\sigma}{(\sigma - 1)(1 - \tau^i)} \frac{E_t \sum_{k=0}^{\infty} (\alpha^i \beta)^k V_y(\tilde{y}_{t,t+k}(j), z_{t+k}^i) \tilde{y}_{t,t+k}(j)}{E_t \sum_{k=0}^{\infty} (\alpha^i \beta)^k \lambda_{t+k} \tilde{y}_{t,t+k}(j)}, \quad (\text{A.23})$$

where V_y denotes the derivative of function V with respect to output $\tilde{y}(j)$. All agents that live in the same country and are able to reset their price in a certain period will set the same price since they share identical preferences (function V) and face the same demand curves, which depend only on aggregate variables such as P_H , P_F , T , and C , and the common elasticity of substitution σ . Hence, in a given period, a fraction $1 - \alpha^i$ of agents will set the same optimal price, while for a fraction α^i of agents the price from the previous period remains effective:

$$\begin{aligned} P_{H,t} &= [\alpha^H P_{H,t-1}^{1-\sigma} + (1 - \alpha^H) \tilde{p}_t(h)^{1-\sigma}]^{\frac{1}{1-\sigma}} \\ P_{F,t} &= [\alpha^F P_{F,t-1}^{1-\sigma} + (1 - \alpha^F) \tilde{p}_t(f)^{1-\sigma}]^{\frac{1}{1-\sigma}}. \end{aligned} \quad (\text{A.24})$$

When prices are flexible, the optimal price equation (A.23) for country H simplifies to

$$T_t^{n-1} = \frac{\sigma}{(\sigma - 1)(1 - \tau^H)} \frac{V_y(y_t^H, z_t^H)}{U_C(C_t)}, \quad (\text{A.25})$$

and for country F to

$$T_t^n = \frac{\sigma}{(\sigma - 1)(1 - \tau^F)} \frac{V_y(y_t^F, z_t^F)}{U_C(C_t)}. \quad (\text{A.26})$$

Note that the closed-economy counterpart is given by

$$1 = \frac{\sigma}{(\sigma - 1)(1 - \tau)} \frac{V_y(y_t, z_t)}{U_C(C_t)}. \quad (\text{A.27})$$

Moreover, variations in the marginal disutility of production of one country relative to the other country are reflected in variations in the terms of trade. Dividing

(A.26) by (A.25) yields

$$T_t = \frac{1 - \tau^H}{1 - \tau^F} \frac{V_y(y_t^F, z_t^F)}{V_y(y_t^H, z_t^H)}. \quad (\text{A.28})$$

A.3. Terms of trade

It is necessary to express the terms of trade equation (A.9) in changes, since the model will only contain price changes (i.e. inflation) rather than price levels. Thus

$$\frac{T_t}{T_{t-1}} = \frac{P_{F,t}}{P_{F,t-1}} \frac{P_{H,t-1}}{P_{H,t}}. \quad (\text{A.29})$$

A.4. Log-linearization

Since the model is not solvable in non-linear form, the equations need to be log-linearized. Given a variable X_t , the following definitions will be used:

$$X_t^W = nX_t^H + (1 - n)X_t^F \quad (\text{A.30})$$

$$X_t^R = X_t^F - X_t^H \quad (\text{A.31})$$

Furthermore, deviations of the logarithm of a variable X_t from its steady state are denoted by \tilde{X}_t under flexible prices and by \hat{X}_t under sticky prices.

A.4.1. Flexible prices

Under flexible prices, prices are set as a markup over marginal costs, monetary policy is neutral, and consumption, output, and the terms of trade are driven by productivity shocks only. Accordingly, consumption, world output, and the terms of trade evolve as follows:³¹

$$\tilde{C}_t = \frac{\eta}{\rho + \eta} \bar{Y}_t^W \quad (\text{A.32})$$

$$\tilde{T}_t = -\frac{\eta}{1 + \eta} \bar{Y}_t^R \quad (\text{A.33})$$

$$\tilde{Y}_t^W = \frac{\eta}{\rho + \eta} \bar{Y}_t^W. \quad (\text{A.34})$$

The first equation is derived by log-linearizing (A.25) and (A.26) and taking the weighted average with weight n . The second equation is derived by subtracting the log-linear approximation of (A.25) from the log-linear approximation of (A.26). The third equation is derived by inserting the first two equations into the weighted average of the log-linear approximations of equations (A.17).

The following definitions were used:

$$\rho = -\frac{U_{CC}\bar{C}}{U_C} \quad (\text{A.35})$$

denotes the inverse of the elasticity of intertemporal substitution in consumption,

$$\eta = \frac{V_{yy}\bar{C}}{V_y} \quad (\text{A.36})$$

³¹In contrast to Benigno (2004), I abstract from fiscal policy shocks.

denotes the inverse of the elasticity of producing the differentiated good, and finally

$$\bar{Y}_t^i = -\frac{V_{yz}}{V_{yy}\bar{C}}\hat{z}_t^i \quad (\text{A.37})$$

reparameterizes the productivity shock in country i .

The productivity shock in country i follows an AR(1) process of the form

$$\bar{Y}_t^i = \rho_i \bar{Y}_{t-1}^i + \nu_t^i, \quad (\text{A.38})$$

where ν_t^i is a white noise process with $\text{var } \nu_t^i = 1$.

A.4.2. Sticky prices

Under sticky prices, the system of equations is given by

$$E_t \hat{C}_{t+1} = \hat{C}_t + \frac{1}{\rho} (\hat{R}_t - E_t \pi_{t+1}^W) \quad (\text{A.39})$$

$$\hat{Y}_t^H = (1-n)\hat{T}_t + \hat{C}_t \quad (\text{A.40})$$

$$\hat{Y}_t^F = -n\hat{T}_t + \hat{C}_t \quad (\text{A.41})$$

$$\pi_t^H = (1-n)k_T^H(\hat{T}_t - \tilde{T}_t) + k_C^H(\hat{C}_t - \tilde{C}_t) + \beta E_t \pi_{t+1}^H \quad (\text{A.42})$$

$$\pi_t^F = -nk_T^F(\hat{T}_t - \tilde{T}_t) + k_C^F(\hat{C}_t - \tilde{C}_t) + \beta E_t \pi_{t+1}^F \quad (\text{A.43})$$

$$\hat{T}_t = \hat{T}_{t-1} + \pi_t^F - \pi_t^H \quad (\text{A.44})$$

$$\hat{R}_t = \phi_\pi \pi_t^W + \phi_Y \hat{Y}_t^W. \quad (\text{A.45})$$

Equation (A.39) is the log-linear approximation of the Euler consumption equation (A.14), where $C_t^i = C_t$ and $\pi_t = \ln(P_t/P_{t-1})$. Recall that, due to perfect risk sharing, consumption is the same across countries, which implies that there is only one Euler equation. Equations (A.40) and (A.41) are log-linear approximations of the national account identities (A.17).

Equations (A.42) and (A.43) represent the New Keynesian Phillips curves for country H and country F respectively, where $\pi_t^H = \ln(P_{H,t}/P_{H,t-1})$ and $\pi_t^F = \ln(P_{F,t}/P_{F,t-1})$. They are derived by combining the log-linear approximation of the optimal price (A.23) with the log-linear approximation of (A.24) for each country separately. The parameters in front of the terms of trade gap ($\hat{T}_t - \tilde{T}_t$) and the consumption gap ($\hat{C}_t - \tilde{C}_t$) are defined as follows (for $i = H, F$):

$$k_C^i = \frac{(1-\alpha^i\beta)(1-\alpha^i)}{\alpha^i} \frac{\rho + \eta}{1 + \sigma\eta} \quad (\text{A.46})$$

$$k_T^i = \frac{(1-\alpha^i\beta)(1-\alpha^i)}{\alpha^i} \frac{1 + \eta}{1 + \sigma\eta} \quad (\text{A.47})$$

Equation (A.44) is the log-linear approximation of the terms of trade equation (A.29). Finally, equation (A.45) represents the Taylor-type interest rate rule, according to which the common monetary policy reacts to union-wide inflation and to union-wide output (measured as the weighted average of country-specific inflation and output respectively) with coefficients ϕ_π and ϕ_Y .

B. Flexible exchange rate regime (FX)

The main difference to the MU regime, of course, is that both countries possess their own currency and independent monetary policy. Notwithstanding, the model structure is to a large extent identical. The behavior of output, consumption, and the terms of trade under flexible prices is given by equations (A.32) through (A.34).

Under sticky prices, the system of equations is given by

$$E_t \hat{C}_{t+1} = \hat{C}_t + \frac{1}{\rho} \left(n(\hat{R}_t^H - E_t \pi_{t+1}^H) + (1-n)(\hat{R}_t^F - E_t \pi_{t+1}^F) \right) \quad (\text{B.1})$$

$$\hat{Y}_t^H = (1-n)\hat{T}_t + \hat{C}_t \quad (\text{B.2})$$

$$\hat{Y}_t^F = -n\hat{T}_t + \hat{C}_t \quad (\text{B.3})$$

$$\pi_t^H = (1-n)k_T^H(\hat{T}_t - \tilde{T}_t) + k_C^H(\hat{C}_t - \tilde{C}_t) + \beta E_t \pi_{t+1}^H \quad (\text{B.4})$$

$$\pi_t^F = -nk_T^F(\hat{T}_t - \tilde{T}_t) + k_C^F(\hat{C}_t - \tilde{C}_t) + \beta E_t \pi_{t+1}^F \quad (\text{B.5})$$

$$\hat{T}_t = \hat{T}_{t-1} + \pi_t^F - \pi_t^H + \Delta \hat{S}_t \quad (\text{B.6})$$

$$E_t \Delta \hat{S}_{t+1} = \hat{R}_t^H - \hat{R}_t^F \quad (\text{B.7})$$

$$\hat{R}_t^H = \phi_\pi \pi_t^H + \phi_Y \hat{Y}_t^H \quad (\text{B.8})$$

$$\hat{R}_t^F = \phi_\pi \pi_t^F + \phi_Y \hat{Y}_t^F. \quad (\text{B.9})$$

Given the same assumptions as in the MU regime on the set of assets agents can trade in, on the agents' preferences, and on the law of one price, the result of perfect risk sharing carries over to the FX regime.³² Therefore, consumption is described by one Euler equation (B.1). In contrast to the MU regime, the Euler equation contains two interest rates, since monetary policy is country-specific.

The national account identities (B.2) and (B.3) as well as the New Keynesian Phillips curves (B.4) and (B.5) are the same as in the MU regime. Agents are assumed to set their price in the currency of their country (producer currency pricing). The assumption of no price discrimination and no transportation costs implies that the law of one price holds, which in turn implies that exchange rate pass-through is complete. The law of one price together with Cobb-Douglas preferences implies that purchasing power parity holds as well.

The terms of trade are now given by

$$T_t = \frac{S_t P_{F,t}}{P_{H,t}}, \quad (\text{B.10})$$

where $P_{H,t}$ denotes the price of the bundle of differentiated goods produced in country H denominated in country H 's currency, $P_{F,t}$ denotes the price of the bundle of differentiated goods produced in country F denominated in country F 's currency, and S_t is the nominal exchange rate defined as the price of country F 's currency in terms of country H 's currency. First-differencing and log-linearizing the definition of the terms of trade yields (B.6).

Equation (B.7) represents the uncovered interest parity condition, which can be obtained by subtracting the log-linearized Euler equation of country F from the one of country H , using the fact that purchasing power parity holds. Thus, the expected change in the nominal exchange rate corresponds to the interest rate differential across countries. Finally, equations (B.8) and (B.9) represent the Taylor-

³²See Obstfeld and Rogoff (2001). By adopting the assumption from the MU regime that asset markets are incomplete across countries I deviate from Benigno and Benigno (2008), who assume asset markets across countries to be complete in the FX regime. Under Cobb-Douglas preferences, however, both assumptions imply perfect risk sharing and the same Euler equation.

type interest rate rules, according to which monetary policy reacts to country-specific inflation and output with coefficients ϕ_π and ϕ_Y .

C. Welfare-based loss function

The world welfare loss function is the discounted value of a weighted average across countries of the average utility flow of agents using a second-order Taylor series expansion in the spirit of Woodford (2003).³³

The average utility among agents in country H is given by

$$w_t^H = U(C_t) - \frac{1}{n} \int_0^n V(y_t(h), z_t^H) dh, \quad (\text{C.1})$$

and average utility among agents in country F is given by

$$w_t^F = U(C_t) - \frac{1}{1-n} \int_{1-n}^1 V(y_t(f), z_t^F) df. \quad (\text{C.2})$$

The discounted value of the weighted average of the two flows is then given by

$$\tilde{W}_t = E_t \sum_{k=0}^{\infty} \beta^k (n w_{t+k}^H + (1-n) w_{t+k}^F). \quad (\text{C.3})$$

Each term of the utility function is treated separately.

C.1. The term $U(C_t)$

Taking a second-order linear expansion of $U(C_t)$ around the steady state value \bar{C} yields

$$U(C_t) = U(\bar{C}) + U_C(C_t - \bar{C}) + \frac{1}{2} U_{CC}(C_t - \bar{C})^2 + O(\|\xi\|^3), \quad (\text{C.4})$$

where the term $O(\|\xi\|^3)$ groups all the terms that are of third or higher order in the deviations of the various variables from their steady state.

Furthermore, a second-order Taylor expansion to C_t yields

$$\frac{C_t - \bar{C}}{\bar{C}} = \hat{C}_t + \frac{1}{2} \hat{C}_t^2 + O(\|\xi\|^3) \Leftrightarrow C_t - \bar{C} = \bar{C} \hat{C}_t + \frac{1}{2} \bar{C} \hat{C}_t^2 + O(\|\xi\|^3), \quad (\text{C.5})$$

where $\hat{C}_t = \ln(C_t) - \ln(\bar{C})$.

³³The derivation follows Benigno (2003), Appendix D. Here, I do not abstract from exogenous government expenditures. The loss function without government expenditure shocks is identical.

Inserting (C.5) into (C.4) yields

$$\begin{aligned}
U(C_t) &= \underbrace{U(\bar{C})}_{=t.i.p.} + U_C(\bar{C}\hat{C}_t + \frac{1}{2}\bar{C}\hat{C}_t^2) + \frac{1}{2}U_{CC}(\bar{C}\hat{C}_t + \frac{1}{2}\bar{C}\hat{C}_t^2)^2 + O(\|\xi\|^3) \\
&= U_C\bar{C}\hat{C}_t + \frac{1}{2}U_C\bar{C}\hat{C}_t^2 + \frac{1}{2}U_{CC}(\bar{C}^2\hat{C}_t^2 + \underbrace{\bar{C}^2\hat{C}_t^3 + \frac{1}{4}\bar{C}^2\hat{C}_t^4}_{=O(\|\xi\|^3)}) + t.i.p. + O(\|\xi\|^3) \\
&= U_C\bar{C}\hat{C}_t + \frac{1}{2}U_C\bar{C}\hat{C}_t^2 + \frac{1}{2}U_{CC}\bar{C}^2\hat{C}_t^2 + t.i.p. + O(\|\xi\|^3) \\
&= U_C\bar{C} \left[\hat{C}_t + \frac{1}{2}\hat{C}_t^2 + \frac{1}{2}\underbrace{\frac{U_{CC}\bar{C}}{U_C}}_{=-\rho}\hat{C}_t^2 \right] + t.i.p. + O(\|\xi\|^3) \\
&= U_C\bar{C} \left[\hat{C}_t + \frac{1}{2}(1-\rho)\hat{C}_t^2 \right] + t.i.p. + O(\|\xi\|^3), \tag{C.6}
\end{aligned}$$

where the term *t.i.p.* collects all the terms that are independent of monetary policy and independent of whether the two countries form a monetary union or not.

C.2. The term $\frac{1}{n} \int_0^n V(y_t(h), z_t^H) dh$

A second-order Taylor expansion of the second term in (C.1) around a steady state, where $y_t(h) = \bar{Y}^H$ for all h and t , and where $z_t^H = 0$ for all t yields

$$\begin{aligned}
V(y_t(h), z_t^H) &= V(\bar{Y}^H, 0) + V_y(y_t(h) - \bar{Y}^H) + V_z z_t^H + \frac{1}{2}V_{yy}(y_t(h) - \bar{Y}^H)^2 \\
&\quad + V_{yz}(y_t(h) - \bar{Y}^H) z_t^H + \frac{1}{2}V_{zz} z_t^{H2} + O(\|\xi\|^3). \tag{C.7}
\end{aligned}$$

Global demand for a differentiated good produced in country H (including demand from government expenditures G^H) can be expressed by

$$\begin{aligned}
y(h) &= \left(\frac{p(h)}{P_H} \right)^{-\sigma} [T^{1-n} C^W + G^H] \\
&= \underbrace{\left(\frac{p(h)}{P_H} \right)^{-\sigma} T^{1-n} C^W}_{=y^d(h)} + \underbrace{\left(\frac{p(h)}{P_H} \right)^{-\sigma} G^H}_{=y^g(h)} \\
&= y^d(h) + y^g(h). \tag{C.8}
\end{aligned}$$

A second-order Taylor expansion to $y_t^d(h)$ yields

$$y_t^d(h) - \bar{Y}^H = \bar{Y}^H \hat{y}_t^d(h) + \frac{1}{2}\bar{Y}^H \hat{y}_t^d(h)^2 + O(\|\xi\|^3), \tag{C.9}$$

where $\hat{y}_t^d(h) = \ln(y_t^d(h)) - \ln(\bar{Y}^H)$.

A second-order Taylor expansion to $y_t^g(h)$ yields

$$y_t^g(h) = \bar{Y}^H \hat{y}_t^g(h) + \frac{1}{2}\bar{Y}^H \hat{y}_t^g(h)^2 + O(\|\xi\|^3). \tag{C.10}$$

which can be rearranged to

$$(1 - \Phi^H)U_C(\bar{C}) = \bar{T}^{1-n}V_y\left(\bar{T}^{1-n}\bar{C}, 0\right) \quad (\text{C.15})$$

$$(1 - \Phi^F)U_C(\bar{C}) = \bar{T}^{-n}V_y\left(\bar{T}^{-n}\bar{C}, 0\right) \quad (\text{C.16})$$

with

$$(1 - \Phi^H) = (1 - \tau^H)\frac{\sigma - 1}{\sigma} \quad (\text{C.17})$$

$$(1 - \Phi^F) = (1 - \tau^F)\frac{\sigma - 1}{\sigma}. \quad (\text{C.18})$$

The analysis must be restricted to the case in which distortions from the efficient steady state are small, i.e. the deviations of Φ^H and Φ^F are at least of order $O(\|\xi\|)$. Furthermore, for reasons of tractability, it is assumed that $\Phi^H = \Phi^F$. If $\tau^H = \tau^F$, it follows that $\bar{T} = 1$ and $\bar{Y}^H = \bar{Y}^F = \bar{C}$. Then, equation (C.15) yields

$$\begin{aligned} (1 - \Phi^H)U_C(\bar{C}) &= \bar{T}^{1-n}V_y\left(\bar{T}^{1-n}\bar{C}, 0\right) \\ &= V_y\left(\bar{Y}^H, 0\right) \\ &= V_y. \end{aligned} \quad (\text{C.19})$$

Plugging into (C.12) yields

$$\begin{aligned} V(y_t(h), z_t^H) &= (1 - \Phi^H)U_C(\bar{C})\bar{Y}^H\left(\hat{y}_t^d(h) + \frac{1}{2}\hat{y}_t^d(h)^2 + \frac{\eta}{2}\hat{y}_t(h)^2\right. \\ &\quad \left. - \eta\hat{y}_t^d(h)\bar{Y}_t^H\right) + t.i.p. + O(\|\xi\|^3). \end{aligned} \quad (\text{C.20})$$

With $\bar{Y}^H = \bar{C}$ and $U_C(\bar{C}) = U_C$ together with the small distortion assumption, i.e. the product of Φ^H with second-order terms can be neglected, the last equation can be written as

$$\begin{aligned} V(y_t(h), z_t^H) &= U_C\bar{C}\left((1 - \Phi^H)\hat{y}_t^d(h) + \frac{1}{2}\hat{y}_t^d(h)^2 + \frac{\eta}{2}\hat{y}_t(h)^2\right. \\ &\quad \left. - \eta\hat{y}_t^d(h)\bar{Y}_t^H\right) + t.i.p. + O(\|\xi\|^3). \end{aligned} \quad (\text{C.21})$$

Integrating across agents belonging to country H yields

$$\begin{aligned} \frac{1}{n}\int_0^n V(y_t(h), z_t^H)dh &= U_C\bar{C}\left((1 - \Phi^H)\frac{1}{n}\int_0^n \hat{y}_t^d(h)dh + \frac{1}{2}\frac{1}{n}\int_0^n \hat{y}_t^d(h)^2dh\right. \\ &\quad \left.+ \frac{\eta}{2}\frac{1}{n}\int_0^n \hat{y}_t(h)^2dh - \eta\frac{1}{n}\int_0^n \hat{y}_t^d(h)dh\bar{Y}_t^H\right) \\ &\quad + t.i.p. + O(\|\xi\|^3) \\ &= U_C\bar{C}\left((1 - \Phi^H)E_h\hat{y}_t^d(h) + \frac{1}{2}E_h\hat{y}_t^d(h)^2 + \frac{\eta}{2}E_h\hat{y}_t(h)^2\right. \\ &\quad \left.- \eta E_h\hat{y}_t^d(h)\bar{Y}_t^H\right) + t.i.p. + O(\|\xi\|^3). \end{aligned} \quad (\text{C.22})$$

Recall the basic relationship

$$\text{var}(X) = E(X^2) - (E(X))^2 \Leftrightarrow E(X^2) = \text{var}(X) + (E(X))^2. \quad (\text{C.23})$$

Thus,

$$\begin{aligned} \frac{1}{n} \int_0^n V(y_t(h), z_t^H) dh &= U_C \bar{C} \left((1 - \Phi^H) E_h \hat{y}_t^d(h) + \frac{1}{2} (\text{var}_h \hat{y}_t^d(h) + [E_h \hat{y}_t^d(h)]^2) \right. \\ &\quad \left. + \frac{\eta}{2} (\text{var}_h \hat{y}_t(h) + [E_h \hat{y}_t(h)]^2) - \eta E_h \hat{y}_t^d(h) \bar{Y}_t^H \right) \\ &\quad + t.i.p. + O(\|\xi\|^3). \end{aligned} \quad (\text{C.24})$$

C.3. Expanding Y_t^H

Recall the aggregator

$$Y_t^H = \left\{ \frac{1}{n} \int_0^n y_t(h)^{\frac{\sigma-1}{\sigma}} dh \right\}^{\frac{\sigma}{\sigma-1}}. \quad (\text{C.25})$$

I conduct a second-order Taylor series expansion of both sides of the equation. Note that the more general case of (C.5) is given by

$$\frac{C_t^a - \bar{C}^a}{\bar{C}^a} = a \hat{C}_t + \frac{1}{2} a^2 \hat{C}_t^2 + O(\|\xi\|^3). \quad (\text{C.26})$$

Thus, approximating $y_t(h)^{\frac{\sigma-1}{\sigma}}$ up to second-order yields

$$y_t(h)^{\frac{\sigma-1}{\sigma}} = \bar{Y}^H \frac{\sigma-1}{\sigma} \left[1 + \frac{\sigma-1}{\sigma} \hat{y}_t(h) + \frac{1}{2} \left(\frac{\sigma-1}{\sigma} \right)^2 \hat{y}_t(h)^2 \right] + O(\|\xi\|^3). \quad (\text{C.27})$$

Inserting into (C.25) yields

$$\begin{aligned} Y_t^H \frac{\sigma-1}{\sigma} &= \frac{1}{n} \int_0^n \bar{Y}^H \frac{\sigma-1}{\sigma} \left[1 + \frac{\sigma-1}{\sigma} \hat{y}_t(h) + \frac{1}{2} \left(\frac{\sigma-1}{\sigma} \right)^2 \hat{y}_t(h)^2 \right] dh + O(\|\xi\|^3) \\ &= \bar{Y}^H \frac{\sigma-1}{\sigma} \left\{ \frac{1}{n} \int_0^n 1 dh + \frac{\sigma-1}{\sigma} \frac{1}{n} \int_0^n \hat{y}_t(h) dh + \frac{1}{2} \left(\frac{\sigma-1}{\sigma} \right)^2 \frac{1}{n} \int_0^n \hat{y}_t(h)^2 dh \right\} \\ &\quad + O(\|\xi\|^3) \\ &= \bar{Y}^H \frac{\sigma-1}{\sigma} \left\{ 1 + \frac{\sigma-1}{\sigma} E_h \hat{y}_t(h) + \frac{1}{2} \left(\frac{\sigma-1}{\sigma} \right)^2 E_h \hat{y}_t(h)^2 \right\} + O(\|\xi\|^3). \end{aligned} \quad (\text{C.28})$$

A second-order Taylor expansion to $Y_t^H \frac{\sigma-1}{\sigma}$ yields

$$Y_t^H \frac{\sigma-1}{\sigma} = \bar{Y}^H \frac{\sigma-1}{\sigma} \left\{ 1 + \frac{\sigma-1}{\sigma} \hat{Y}_t^H + \frac{1}{2} \left(\frac{\sigma-1}{\sigma} \right)^2 \hat{Y}_t^{H^2} \right\} + O(\|\xi\|^3). \quad (\text{C.29})$$

Equating the previous two equations yields

$$\hat{Y}_t^H + \frac{1}{2} \frac{\sigma-1}{\sigma} \hat{Y}_t^{H^2} = E_h \hat{y}_t(h) + \frac{1}{2} \frac{\sigma-1}{\sigma} E_h \hat{y}_t(h)^2 + O(\|\xi\|^3). \quad (\text{C.30})$$

This expression raised to the power of two gives

$$\hat{Y}_t^{H^2} = (E_h \hat{y}_t(h))^2 + O(\|\xi\|^3). \quad (\text{C.31})$$

Inserting back into (C.30) and simplifying yields

$$\begin{aligned} \hat{Y}_t^H + \frac{1}{2} \frac{\sigma - 1}{\sigma} (E_h \hat{y}_t(h))^2 &= E_h \hat{y}_t(h) + \frac{1}{2} \frac{\sigma - 1}{\sigma} E_h \hat{y}_t(h)^2 + O(\|\xi\|^3) \\ \hat{Y}_t^H &= E_h \hat{y}_t(h) + \frac{1}{2} \frac{\sigma - 1}{\sigma} [E_h \hat{y}_t(h)^2 - (E_h \hat{y}_t(h))^2] + O(\|\xi\|^3) \\ \hat{Y}_t^H &= E_h \hat{y}_t(h) + \frac{1}{2} \frac{\sigma - 1}{\sigma} \text{var}_h \hat{y}_t(h) + O(\|\xi\|^3). \end{aligned} \quad (\text{C.32})$$

Analogously,

$$\hat{Y}_t^{H,d} = E_h \hat{y}_t^d(h) + \frac{1}{2} \frac{\sigma - 1}{\sigma} \text{var}_h \hat{y}_t^d(h) + O(\|\xi\|^3). \quad (\text{C.33})$$

Using the previous two equations to substitute out $E_h \hat{y}_t(h)$ and $E_h \hat{y}_t^d(h)$ in (C.24)

gives

$$\begin{aligned}
& \frac{1}{n} \int_0^n V(y_t(h), z_t^H) dh \\
&= U_C \bar{C} \left((1 - \Phi^H) \left[\hat{Y}_t^{H,d} - \frac{1}{2} \frac{\sigma - 1}{\sigma} \text{var}_h \hat{y}_t^d(h) \right] \right. \\
&+ \frac{1}{2} \left(\text{var}_h \hat{y}_t^d(h) + \left[\hat{Y}_t^{H,d} - \frac{1}{2} \frac{\sigma - 1}{\sigma} \text{var}_h \hat{y}_t^d(h) \right]^2 \right) \\
&+ \frac{\eta}{2} \left(\text{var}_h \hat{y}_t(h) + \left[\hat{Y}_t^H - \frac{1}{2} \frac{\sigma - 1}{\sigma} \text{var}_h \hat{y}_t(h) \right]^2 \right) \\
&- \eta \left[\hat{Y}_t^{H,d} - \frac{1}{2} \frac{\sigma - 1}{\sigma} \text{var}_h \hat{y}_t^d(h) \right] \bar{Y}_t^H \Big) + t.i.p. + O(\|\xi\|^3) \\
&= U_C \bar{C} \left((1 - \Phi^H) \hat{Y}_t^{H,d} - \frac{1}{2} \frac{\sigma - 1}{\sigma} \text{var}_h \hat{y}_t^d(h) + \underbrace{\Phi^H \frac{1}{2} \frac{\sigma - 1}{\sigma} \text{var}_h \hat{y}_t^d(h)}_{=O(\|\xi\|^3)} \right. \\
&+ \frac{1}{2} \left(\text{var}_h \hat{y}_t^d(h) + \hat{Y}_t^{H,d^2} - \underbrace{\hat{Y}_t^{H,d} \frac{\sigma - 1}{\sigma} \text{var}_h \hat{y}_t^d(h) + \frac{1}{4} \left(\frac{\sigma - 1}{\sigma} \right)^2 (\text{var}_h \hat{y}_t^d(h))^2}_{=O(\|\xi\|^3)} \right) \\
&+ \frac{\eta}{2} \left(\text{var}_h \hat{y}_t(h) + \hat{Y}_t^{H^2} - \underbrace{\hat{Y}_t^H \frac{\sigma - 1}{\sigma} \text{var}_h \hat{y}_t(h) + \frac{1}{4} \left(\frac{\sigma - 1}{\sigma} \right)^2 (\text{var}_h \hat{y}_t(h))^2}_{=O(\|\xi\|^3)} \right) \\
&- \eta \hat{Y}_t^{H,d} \bar{Y}_t^H + \underbrace{\frac{\eta}{2} \frac{\sigma - 1}{\sigma} \text{var}_h \hat{y}_t^d(h) \bar{Y}_t^H}_{=O(\|\xi\|^3)} \Big) + t.i.p. + O(\|\xi\|^3) \\
&= U_C \bar{C} \left((1 - \Phi^H) \hat{Y}_t^{H,d} + \frac{1}{2} \hat{Y}_t^{H,d^2} + \frac{\eta}{2} \hat{Y}_t^{H^2} - \eta \hat{Y}_t^{H,d} \bar{Y}_t^H \right. \\
&- \left. \frac{1}{2} \frac{\sigma - 1}{\sigma} \text{var}_h \hat{y}_t^d(h) + \frac{1}{2} \text{var}_h \hat{y}_t^d(h) + \frac{\eta}{2} \text{var}_h \hat{y}_t(h) \right) + t.i.p. + O(\|\xi\|^3). \quad (\text{C.34})
\end{aligned}$$

Note that since $\hat{y}_t(h) = \hat{y}_t^d(h)$, $\text{var}_h \hat{y}_t(h) = \text{var}_h \hat{y}_t^d(h)$. Therefore, the previous expression can be simplified to

$$\begin{aligned}
\frac{1}{n} \int_0^n V(y_t(h), z_t^H) dh &= U_C \bar{C} \left((1 - \Phi^H) \hat{Y}_t^{H,d} + \frac{1}{2} \hat{Y}_t^{H,d^2} + \frac{\eta}{2} \hat{Y}_t^{H^2} - \eta \hat{Y}_t^{H,d} \bar{Y}_t^H \right. \\
&+ \left. \frac{1}{2} (\sigma^{-1} + \eta) \text{var}_h \hat{y}_t(h) \right) + t.i.p. + O(\|\xi\|^3). \quad (\text{C.35})
\end{aligned}$$

C.4. Combining the results

Inserting (C.35) and (C.6) into (C.1) yields

$$\begin{aligned}
w_t^H &= U_C \bar{C} \left(\hat{C}_t + \frac{1}{2} (1 - \rho) \hat{C}_t^2 - (1 - \Phi^H) \hat{Y}_t^{H,d} - \frac{1}{2} \hat{Y}_t^{H,d^2} - \frac{\eta}{2} \hat{Y}_t^{H^2} + \eta \hat{Y}_t^{H,d} \bar{Y}_t^H \right. \\
&- \left. \frac{1}{2} (\sigma^{-1} + \eta) \text{var}_h \hat{y}_t(h) \right) + t.i.p. + O(\|\xi\|^3). \quad (\text{C.36})
\end{aligned}$$

Average utility among agents living in country F is derived completely analogously. Thus,

$$w_t^F = U_C \bar{C} \left(\hat{C}_t + \frac{1}{2}(1-\rho)\hat{C}_t^2 - (1-\Phi^F)\hat{Y}_t^{F,d} - \frac{1}{2}\hat{Y}_t^{F,d^2} - \frac{\eta}{2}\hat{Y}_t^{F^2} + \eta\hat{Y}_t^{F,d}\bar{Y}_t^F \right. \\ \left. - \frac{1}{2}(\sigma^{-1} + \eta) \text{var}_f \hat{y}_t(f) \right) + t.i.p. + O(\|\xi\|^3). \quad (\text{C.37})$$

World welfare consists of the linear combination of country H 's and country F 's welfare with weight n and $1-n$:

$$w_t = nw_t^H + (1-n)w_t^F \\ = U_C \bar{C} \left(\underbrace{n\hat{C}_t + (1-n)\hat{C}_t}_{=\hat{C}_t} + \frac{1}{2}(1-\rho)\underbrace{(n\hat{C}_t^2 + (1-n)\hat{C}_t^2)}_{=\hat{C}_t^2} \right. \\ \left. - n(1-\Phi^H)\hat{Y}_t^{H,d} - (1-n)(1-\Phi^F)\hat{Y}_t^{F,d} - \frac{1}{2}\left(n\hat{Y}_t^{H,d^2} + (1-n)\hat{Y}_t^{F,d^2}\right) \right. \\ \left. - \frac{\eta}{2}\left(n\hat{Y}_t^{H^2} + (1-n)\hat{Y}_t^{F^2}\right) + \eta\left(n\hat{Y}_t^{H,d}\bar{Y}_t^H + (1-n)\hat{Y}_t^{F,d}\bar{Y}_t^F\right) \right. \\ \left. - \frac{1}{2}(\sigma^{-1} + \eta)[n \text{var}_h \hat{y}_t(h) + (1-n) \text{var}_f \hat{y}_t(f)] \right) + t.i.p. + O(\|\xi\|^3). \quad (\text{C.38})$$

Inserting the expressions

$$\hat{Y}_t^H = (1-n)\hat{T}_t + \hat{C}_t + g_t^H \quad (\text{C.39})$$

$$\hat{Y}_t^F = -n\hat{T}_t + \hat{C}_t + g_t^F \quad (\text{C.40})$$

$$\hat{Y}_t^{H,d} = (1-n)\hat{T}_t + \hat{C}_t \quad (\text{C.41})$$

$$\hat{Y}_t^{F,d} = -n\hat{T}_t + \hat{C}_t \quad (\text{C.42})$$

and simplifying yields

$$\begin{aligned}
w_t &= U_C \bar{C} \left(\hat{C}_t + \frac{1}{2}(1-\rho)\hat{C}_t^2 \right. \\
&\quad - n(1-\Phi^H)[(1-n)\hat{T}_t + \hat{C}_t] - (1-n)(1-\Phi^F)[-n\hat{T}_t + \hat{C}_t] \\
&\quad - \frac{1}{2} \left(n[(1-n)\hat{T}_t + \hat{C}_t]^2 + (1-n)[-n\hat{T}_t + \hat{C}_t]^2 \right) \\
&\quad - \frac{\eta}{2} \left(n[(1-n)\hat{T}_t + \hat{C}_t + g_t^H]^2 + (1-n)[-n\hat{T}_t + \hat{C}_t + g_t^F]^2 \right) \\
&\quad + \eta \left(n[(1-n)\hat{T}_t + \hat{C}_t]\bar{Y}_t^H + (1-n)[-n\hat{T}_t + \hat{C}_t]\bar{Y}_t^F \right) \\
&\quad - \frac{1}{2}(\sigma^{-1} + \eta)[n \text{var}_h \hat{y}_t(h) + (1-n) \text{var}_f \hat{y}_t(f)] \\
&\quad + t.i.p. + O(\|\xi\|^3) \tag{C.43} \\
&= U_C \bar{C} \left(\hat{C}_t + \frac{1}{2}(1-\rho)\hat{C}_t^2 \right. \\
&\quad - (1-\Phi^H)n(1-n)\hat{T}_t - n\hat{C}_t + n\Phi^H\hat{C}_t + (1-\Phi^F)n(1-n)\hat{T}_t - (1-n)\hat{C}_t + (1-n)\Phi^F\hat{C}_t \\
&\quad - \frac{1}{2} \left(n(1-n)^2\hat{T}_t^2 + 2n(1-n)\hat{T}_t\hat{C}_t + n\hat{C}_t^2 + (1-n)n^2\hat{T}_t^2 - 2n(1-n)\hat{T}_t\hat{C}_t + (1-n)\hat{C}_t^2 \right) \\
&\quad - \frac{\eta}{2} \left(n(1-n)^2\hat{T}_t^2 + n\hat{C}_t^2 + \underbrace{ng_t^{H^2}}_{=t.i.p.} + 2n(1-n)\hat{T}_t\hat{C}_t + 2n(1-n)\hat{T}_t g_t^H + 2\hat{C}_t g_t^H \right. \\
&\quad \left. + (1-n)n^2\hat{T}_t^2 + (1-n)\hat{C}_t^2 + \underbrace{(1-n)g_t^{F^2}}_{=t.i.p.} - 2n(1-n)\hat{T}_t\hat{C}_t - 2n(1-n)\hat{T}_t g_t^H + 2\hat{C}_t g_t^F \right) \\
&\quad + \eta \left(n(1-n)\hat{T}_t\bar{Y}_t^H + n\hat{C}_t\bar{Y}_t^H - (1-n)n\hat{T}_t\bar{Y}_t^F + (1-n)\hat{C}_t\bar{Y}_t^F \right) \\
&\quad - \frac{1}{2}(\sigma^{-1} + \eta)[n \text{var}_h \hat{y}_t(h) + (1-n) \text{var}_f \hat{y}_t(f)] \\
&\quad + t.i.p. + O(\|\xi\|^3). \tag{C.44}
\end{aligned}$$

Further simplification gives

$$\begin{aligned}
w_t = & U_C \bar{C} \left(\hat{C}_t + \frac{1}{2}(1-\rho)\hat{C}_t^2 \right. \\
& - \underbrace{\left((1-\Phi^H) - (1-\Phi^F) \right)}_{=0 \text{ if } \Phi^H = \Phi^F} n(1-n)\hat{T}_t \underbrace{\left(-n\hat{C}_t - (1-n)\hat{C}_t \right)}_{=-\hat{C}_t} + \hat{C}_t [n\Phi^H + (1-n)\Phi^F] \\
& - \frac{1}{2} \left(\hat{C}_t^2 + \underbrace{[n(1-n)^2 + (1-n)n^2]}_{=n(1-n)} \hat{T}_t^2 \right) \\
& - \frac{\eta}{2} \left(\hat{C}_t^2 + n(1-n)\hat{T}_t^2 + 2n(1-n)\hat{T}_t \underbrace{[g_t^H - g_t^F]}_{=-g_t^R} + 2\hat{C}_t \underbrace{[ng_t^H + (1-n)g_t^F]}_{=g_t^W} \right) \\
& + \eta \left(\hat{C}_t \underbrace{[n\bar{Y}_t^H + (1-n)\bar{Y}_t^F]}_{=\bar{Y}_t^W} + n(1-n)\hat{T}_t \underbrace{[\bar{Y}_t^H - \bar{Y}_t^F]}_{=-\bar{Y}_t^R} \right) \\
& - \frac{1}{2}(\sigma^{-1} + \eta)[n \text{var}_h \hat{y}_t(h) + (1-n) \text{var}_f \hat{y}_t(f)] \\
& + t.i.p. + O(\|\xi\|^3). \tag{C.45}
\end{aligned}$$

This yields³⁴

$$\begin{aligned}
w_t = & U_C \bar{C} \left(\hat{C}_t [n\Phi^H + (1-n)\Phi^F] + \frac{1}{2}(1-\rho)\hat{C}_t^2 \right. \\
& + \eta \left(\hat{C}_t \bar{Y}_t^W - n(1-n)\hat{T}_t \bar{Y}_t^R \right) \\
& - \frac{1}{2} \left(\hat{C}_t^2 + n(1-n)\hat{T}_t^2 \right) \\
& - \frac{\eta}{2} \left(\hat{C}_t^2 + n(1-n)\hat{T}_t^2 + 2\hat{C}_t g_t^W - 2n(1-n)\hat{T}_t g_t^R \right) \\
& - \frac{1}{2}(\sigma^{-1} + \eta)[n \text{var}_h \hat{y}_t(h) + (1-n) \text{var}_f \hat{y}_t(f)] \\
& + t.i.p. + O(\|\xi\|^3). \tag{C.46}
\end{aligned}$$

³⁴This equation corresponds to equation (E.21) in Benigno (2003), Appendix D, except for a typo: There must be a minus sign in front of $n(1-n)\hat{T}_t \bar{Y}_t^R$.

Factoring out a minus sign yields

$$\begin{aligned}
w_t = & -U_C \bar{C} \left(-\hat{C}_t [n\Phi^H + (1-n)\Phi^F] - \frac{1}{2}(1-\rho)\hat{C}_t^2 \right. \\
& - \eta \left(\hat{C}_t \bar{Y}_t^W - n(1-n)\hat{T}_t \bar{Y}_t^R \right) \\
& + \frac{1}{2} \left(\hat{C}_t^2 + n(1-n)\hat{T}_t^2 \right) \\
& + \frac{\eta}{2} \left(\hat{C}_t^2 + n(1-n)\hat{T}_t^2 + 2\hat{C}_t g_t^W - 2n(1-n)\hat{T}_t g_t^R \right) \\
& + \frac{1}{2}(\sigma^{-1} + \eta)[n \text{var}_h \hat{y}_t(h) + (1-n) \text{var}_f \hat{y}_t(f)] \left. \right) \\
& + t.i.p. + O(\|\xi\|^3). \tag{C.47}
\end{aligned}$$

Expanding gives

$$\begin{aligned}
w_t = & -U_C \bar{C} \left(-\hat{C}_t [n\Phi^H + (1-n)\Phi^F] - \frac{1}{2}(1-\rho)\hat{C}_t^2 \right. \\
& - \eta \hat{C}_t \bar{Y}_t^W + \eta n(1-n)\hat{T}_t \bar{Y}_t^R \\
& + \frac{1}{2}\hat{C}_t^2 + \frac{1}{2}n(1-n)\hat{T}_t^2 \\
& + \frac{\eta}{2}\hat{C}_t^2 + \frac{\eta}{2}n(1-n)\hat{T}_t^2 + \eta \hat{C}_t g_t^W - \eta n(1-n)\hat{T}_t g_t^R \\
& + \frac{1}{2}(\sigma^{-1} + \eta)[n \text{var}_h \hat{y}_t(h) + (1-n) \text{var}_f \hat{y}_t(f)] \left. \right) \\
& + t.i.p. + O(\|\xi\|^3). \tag{C.48}
\end{aligned}$$

Rearranging yields

$$\begin{aligned}
w_t &= -U_C \bar{C} \left(-\hat{C}_t [n\Phi^H + (1-n)\Phi^F] \right. \\
&\quad + \frac{1}{2}(\rho + \eta) \hat{C}_t^2 - \underbrace{\eta [\bar{Y}_t^W - g_t^W]}_{=(\rho+\eta)\tilde{C}_t} \hat{C}_t \\
&\quad + \frac{1}{2}(1 + \eta)n(1 - n) \hat{T}_t^2 - n(1 - n) \underbrace{\eta [g_t^R - \bar{Y}_t^R]}_{=(1+\eta)\tilde{T}_t} \hat{T}_t \\
&\quad \left. + \frac{1}{2}(\sigma^{-1} + \eta)[n \text{var}_h \hat{y}_t(h) + (1 - n) \text{var}_f \hat{y}_t(f)] \right) \\
&\quad + t.i.p. + O(\|\xi\|^3) \tag{C.49}
\end{aligned}$$

$$\begin{aligned}
&= -U_C \bar{C} \left(-\hat{C}_t [n\Phi^H + (1 - n)\Phi^F] \right. \\
&\quad + \frac{1}{2}(\rho + \eta) [\hat{C}_t^2 - 2\tilde{C}_t \hat{C}_t] \\
&\quad + \frac{1}{2}(1 + \eta)n(1 - n) [\hat{T}_t^2 - 2\tilde{T}_t \hat{T}_t] \\
&\quad \left. + \frac{1}{2}(\sigma^{-1} + \eta)[n \text{var}_h \hat{y}_t(h) + (1 - n) \text{var}_f \hat{y}_t(f)] \right) \\
&\quad + t.i.p. + O(\|\xi\|^3) \tag{C.50}
\end{aligned}$$

$$\begin{aligned}
&= -U_C \bar{C} \left(-\hat{C}_t [n\Phi^H + (1 - n)\Phi^F] \right. \\
&\quad + \frac{1}{2}(\rho + \eta) \underbrace{[\hat{C}_t^2 - 2\tilde{C}_t \hat{C}_t + \tilde{C}_t^2]}_{=[\hat{C}_t - \tilde{C}_t]^2} - \underbrace{\frac{1}{2}(\rho + \eta)\tilde{C}_t^{W^2}}_{=t.i.p.} \\
&\quad + \frac{1}{2}(1 + \eta)n(1 - n) \underbrace{[\hat{T}_t^2 - 2\tilde{T}_t \hat{T}_t + \tilde{T}_t^2]}_{=[\hat{T}_t - \tilde{T}_t]^2} - \underbrace{\frac{1}{2}(1 + \eta)n(1 - n)\tilde{T}_t^2}_{=t.i.p.} \\
&\quad \left. + \frac{1}{2}(\sigma^{-1} + \eta)[n \text{var}_h \hat{y}_t(h) + (1 - n) \text{var}_f \hat{y}_t(f)] \right) \\
&\quad + t.i.p. + O(\|\xi\|^3) \tag{C.51}
\end{aligned}$$

$$\begin{aligned}
&= -U_C \bar{C} \left(-\hat{C}_t [n\Phi^H + (1 - n)\Phi^F] \right. \\
&\quad + \frac{1}{2}(\rho + \eta) [\hat{C}_t - \tilde{C}_t]^2 + \frac{1}{2}(1 + \eta)n(1 - n) [\hat{T}_t - \tilde{T}_t]^2 \\
&\quad \left. + \frac{1}{2}(\sigma^{-1} + \eta)[n \text{var}_h \hat{y}_t(h) + (1 - n) \text{var}_f \hat{y}_t(f)] \right) \\
&\quad + t.i.p. + O(\|\xi\|^3). \tag{C.52}
\end{aligned}$$

The difference between steady state consumption under the presence of the monopolistic distortion \bar{C} and the efficient level of consumption C^* (situation without the distortion) is given by

$$\bar{c} = -\ln \left(\frac{\bar{C}}{C^*} \right) = \frac{n\Phi^H + (1 - n)\Phi^F}{\rho + \eta}. \tag{C.53}$$

Inserting yields

$$\begin{aligned}
w_t &= -U_C \bar{C} \left(-\hat{C}_t(\rho + \eta)\bar{c} \right. \\
&\quad + \frac{1}{2}(\rho + \eta) \left[\hat{C}_t - \tilde{C}_t \right]^2 + \frac{1}{2}(1 + \eta)n(1 - n) \left[\hat{T}_t - \tilde{T}_t \right]^2 \\
&\quad \left. + \frac{1}{2}(\sigma^{-1} + \eta)[n \text{var}_h \hat{y}_t(h) + (1 - n) \text{var}_f \hat{y}_t(f)] \right) \\
&\quad + t.i.p. + O(\|\xi\|^3) \tag{C.54}
\end{aligned}$$

$$\begin{aligned}
&= -U_C \bar{C} \left(\frac{1}{2}(\rho + \eta) \left(\left[\hat{C}_t - \tilde{C}_t \right]^2 - 2\hat{C}_t\bar{c} \right) + \frac{1}{2}(1 + \eta)n(1 - n) \left[\hat{T}_t - \tilde{T}_t \right]^2 \right. \\
&\quad \left. + \frac{1}{2}(\sigma^{-1} + \eta)[n \text{var}_h \hat{y}_t(h) + (1 - n) \text{var}_f \hat{y}_t(f)] \right) \\
&\quad + t.i.p. + O(\|\xi\|^3) \tag{C.55}
\end{aligned}$$

$$\begin{aligned}
&= -U_C \bar{C} \left(\frac{1}{2}(\rho + \eta) \left(\underbrace{\left[\hat{C}_t - \tilde{C}_t \right]}_{\equiv c_t} - \bar{c} \right)^2 + \frac{1}{2}(1 + \eta)n(1 - n) \left[\hat{T}_t - \tilde{T}_t \right]^2 \right. \\
&\quad \left. + \frac{1}{2}(\sigma^{-1} + \eta)[n \text{var}_h \hat{y}_t(h) + (1 - n) \text{var}_f \hat{y}_t(f)] \right) \\
&\quad + t.i.p. + O(\|\xi\|^3). \tag{C.56}
\end{aligned}$$

Thus,

$$\begin{aligned}
w_t &= -U_C \bar{C} \left(\frac{1}{2}(\rho + \eta) [c_t - \bar{c}]^2 + \frac{1}{2}(1 + \eta)n(1 - n) \left[\hat{T}_t - \tilde{T}_t \right]^2 \right. \\
&\quad \left. + \frac{1}{2}(\sigma^{-1} + \eta)[n \text{var}_h \hat{y}_t(h) + (1 - n) \text{var}_f \hat{y}_t(f)] \right) \\
&\quad + t.i.p. + O(\|\xi\|^3). \tag{C.57}
\end{aligned}$$

Next, $\text{var}_h \hat{y}_t(h)$ can be linked to inflation π_t^H and $\text{var}_f \hat{y}_t(f)$ to π_t^F . Note that

$$\text{var}_h \hat{y}_t(h) = \text{var}_h y_t(h) = \sigma^2 \text{var}_h \hat{p}_t(h) = \sigma^2 \text{var}_h p_t(h). \tag{C.58}$$

Then, the following relationship is derived in a completely analogous way as in Woodford (2003):

$$\sum_{t=0}^{\infty} \beta^t \text{var}_h p_t(h) = \frac{\alpha^H}{(1 - \alpha^H)(1 - \alpha^H \beta)} \sum_{t=0}^{\infty} \beta^t \pi_t^{H^2} + t.i.p. + O(\|\xi\|^3). \tag{C.59}$$

Finally, calculating the discounted value of all future utility flows yields

$$\begin{aligned}
\tilde{W}_t &= E_t \sum_{k=0}^{\infty} \beta^k w_{t+k} \\
&= E_t \sum_{k=0}^{\infty} \beta^k (-U_C \bar{C}) \left(\frac{1}{2} (\rho + \eta) [c_{t+k} - \bar{c}]^2 + \frac{1}{2} (1 + \eta) n (1 - n) [\hat{T}_{t+k} - \tilde{T}_{t+k}]^2 \right. \\
&\quad \left. + \frac{1}{2} (\sigma^{-1} + \eta) [n \text{var}_h \hat{y}_{t+k}(h) + (1 - n) \text{var}_f \hat{y}_{t+k}(f)] \right) \\
&\quad + t.i.p. + O(\|\xi\|^3). \tag{C.60}
\end{aligned}$$

Thus,

$$\begin{aligned}
\tilde{W}_t &= -\frac{1}{2} U_C \bar{C} E_t \sum_{k=0}^{\infty} \beta^k \left((\rho + \eta) [c_{t+k} - \bar{c}]^2 + (1 + \eta) n (1 - n) [\hat{T}_{t+k} - \tilde{T}_{t+k}]^2 \right. \\
&\quad \left. + \sigma (1 + \sigma \eta) n \frac{\alpha^H}{(1 - \alpha^H)(1 - \alpha^H \beta)} \pi_{t+k}^{H^2} \right. \\
&\quad \left. + \sigma (1 + \sigma \eta) (1 - n) \frac{\alpha^F}{(1 - \alpha^F)(1 - \alpha^F \beta)} \pi_{t+k}^{F^2} \right) \\
&\quad + t.i.p. + O(\|\xi\|^3). \tag{C.61}
\end{aligned}$$

This expression is equivalent to equation (26) in Benigno (2004) with $\bar{c} = 0$, i.e. the monopolistic distortion is perfectly neutralized by an appropriate subsidy, and with $c_t = y_t^W$.

Dividing both sides by $U_C \bar{C}$, letting $\beta \rightarrow 1$, and with $\bar{c} = 0$, the loss function can be written as

$$\begin{aligned}
W_t &= -\frac{1}{2} \left((\rho + \eta) \text{var}(\hat{C}_t - \tilde{C}_t) + (1 + \eta) n (1 - n) \text{var}(\hat{T}_t - \tilde{T}_t) \right. \\
&\quad \left. + \sigma (1 + \sigma \eta) n \frac{\alpha^H}{(1 - \alpha^H)(1 - \alpha^H \beta)} \text{var} \pi_t^H \right. \\
&\quad \left. + \sigma (1 + \sigma \eta) (1 - n) \frac{\alpha^F}{(1 - \alpha^F)(1 - \alpha^F \beta)} \text{var} \pi_t^F \right) \\
&\quad + t.i.p. + O(\|\xi\|^3). \tag{C.62}
\end{aligned}$$

This equation corresponds to equation (3.22) in the main text.

C.5. Special case: $\alpha^H = \alpha^F$

When prices are equally rigid in the two countries ($\alpha^H = \alpha^F$), the world welfare loss function can be simplified further in a useful way.

When $\alpha^H = \alpha^F = \alpha$, it immediately follows that

$$\begin{aligned}
\tilde{W}_t &= -\frac{1}{2} U_C \bar{C} E_t \sum_{k=0}^{\infty} \beta^k \left((\rho + \eta) [c_{t+k} - \bar{c}]^2 + (1 + \eta) n (1 - n) [\hat{T}_{t+k} - \tilde{T}_{t+k}]^2 \right. \\
&\quad \left. + \sigma (1 + \sigma \eta) \frac{\alpha}{(1 - \alpha)(1 - \alpha \beta)} [n \pi_{t+k}^{H^2} + (1 - n) \pi_{t+k}^{F^2}] \right) \\
&\quad + t.i.p. + O(\|\xi\|^3). \tag{C.63}
\end{aligned}$$

The last term in square brackets can be modified in the following way:

$$\begin{aligned}
& n\pi_t^{H^2} + (1-n)\pi_t^{F^2} \\
&= n \underbrace{\left(n\pi_t^{H^2} + (1-n)\pi_t^{H^2} \right)}_{=\pi_t^{H^2}} + (1-n) \underbrace{\left(n\pi_t^{F^2} + (1-n)\pi_t^{F^2} \right)}_{=\pi_t^{F^2}} \\
&= n^2\pi_t^{H^2} + n(1-n)\pi_t^{H^2} + n(1-n)\pi_t^{F^2} + (1-n)^2\pi_t^{F^2}. \tag{C.64}
\end{aligned}$$

Adding $2n(1-n)\pi_t^H\pi_t^F - 2n(1-n)\pi_t^H\pi_t^F$ and simplifying yields

$$\begin{aligned}
n\pi_t^{H^2} + (1-n)\pi_t^{F^2} &= n^2\pi_t^{H^2} + 2n(1-n)\pi_t^H\pi_t^F + (1-n)^2\pi_t^{F^2} \\
&\quad + n(1-n)\pi_t^{F^2} - 2n(1-n)\pi_t^H\pi_t^F + n(1-n)\pi_t^{H^2} \\
&= \underbrace{n^2\pi_t^{H^2} + 2n(1-n)\pi_t^H\pi_t^F + (1-n)^2\pi_t^{F^2}}_{(n\pi_t^H + (1-n)\pi_t^F)^2} \\
&\quad + n(1-n) \underbrace{\left(\pi_t^{F^2} - 2\pi_t^H\pi_t^F + \pi_t^{H^2} \right)}_{(\pi_t^F - \pi_t^H)^2} \\
&= \underbrace{\left(n\pi_t^H + (1-n)\pi_t^F \right)^2}_{\pi_t^{W^2}} + n(1-n) \underbrace{\left(\pi_t^F - \pi_t^H \right)^2}_{\pi_t^{R^2}} \\
&= \pi_t^{W^2} + n(1-n)\pi_t^{R^2}. \tag{C.65}
\end{aligned}$$

The world welfare loss function is, then, given by

$$\begin{aligned}
\tilde{W}_t &= -\frac{1}{2}U_C\bar{C}E_t \sum_{k=0}^{\infty} \beta^k \left((\rho + \eta) [c_{t+k} - \bar{c}]^2 + (1 + \eta)n(1-n) [\hat{T}_{t+k} - \tilde{T}_{t+k}]^2 \right. \\
&\quad \left. + \sigma(1 + \sigma\eta) \frac{\alpha}{(1-\alpha)(1-\alpha\beta)} \left[\pi_{t+k}^{W^2} + n(1-n)\pi_{t+k}^{R^2} \right] \right) \\
&\quad + t.i.p. + O(\|\xi\|^3). \tag{C.66}
\end{aligned}$$

Dividing both sides by $U_C\bar{C}$, letting $\beta \rightarrow 1$, and with $\bar{c} = 0$, the loss function can be written as

$$\begin{aligned}
W_t &= -\frac{1}{2} \left((\rho + \eta) \text{var}(\hat{C}_t - \tilde{C}_t) + (1 + \eta)n(1-n) \text{var}(\hat{T}_t - \tilde{T}_t) \right. \\
&\quad \left. + \sigma(1 + \sigma\eta) \frac{\alpha}{(1-\alpha)(1-\alpha\beta)} \left[\text{var} \pi_t^W + n(1-n) \text{var} \pi_t^R \right] \right) \\
&\quad + t.i.p. + O(\|\xi\|^3). \tag{C.67}
\end{aligned}$$

This equation corresponds to equation (3.23) in the main text.

C.6. Version containing country-specific output gaps

The welfare loss function can alternatively be expressed in terms of the country-specific output gaps instead of the consumption gap. This makes the analogy to the closed-economy counterpart, which is expressed in terms of the output gap as well, more obvious.

Inserting the gap-version of the national account identities

$$\hat{Y}_t^H - \tilde{Y}_t^H = (1-n)(\hat{T}_t - \tilde{T}_t) + \hat{C}_t - \tilde{C}_t \quad (\text{C.68})$$

$$\hat{Y}_t^F - \tilde{Y}_t^F = -n(\hat{T}_t - \tilde{T}_t) + \hat{C}_t - \tilde{C}_t \quad (\text{C.69})$$

into the weighted average of the squared output gaps yields

$$n(\hat{Y}_t^H - \tilde{Y}_t^H)^2 + (1-n)(\hat{Y}_t^F - \tilde{Y}_t^F)^2 = n(1-n)(\hat{T}_t - \tilde{T}_t)^2 + (\hat{C}_t - \tilde{C}_t)^2. \quad (\text{C.70})$$

Solving this equation for $(\hat{C}_t - \tilde{C}_t)^2$ and inserting the resulting expression into equation (C.61) with $\bar{c} = 0$ yields

$$\begin{aligned} \tilde{W}_t = & -\frac{1}{2}U_C\bar{C}E_t \sum_{k=0}^{\infty} \beta^k \left((\rho + \eta) \left[n(\hat{Y}_{t+k}^H - \tilde{Y}_{t+k}^H)^2 + (1-n)(\hat{Y}_{t+k}^F - \tilde{Y}_{t+k}^F)^2 \right] \right. \\ & + (1-\rho)n(1-n) \left[\hat{T}_{t+k} - \tilde{T}_{t+k} \right]^2 \\ & + \sigma(1+\sigma\eta)n \frac{\alpha^H}{(1-\alpha^H)(1-\alpha^H\beta)} \pi_{t+k}^{H^2} \\ & \left. + \sigma(1+\sigma\eta)(1-n) \frac{\alpha^F}{(1-\alpha^F)(1-\alpha^F\beta)} \pi_{t+k}^{F^2} \right) \\ & + t.i.p. + O(\|\xi\|^3). \end{aligned} \quad (\text{C.71})$$

Expressed in variances, the welfare loss function is then given by

$$\begin{aligned} W_t = & -\frac{1}{2} \left((\rho + \eta) \left[n \text{var}(\hat{Y}_t^H - \tilde{Y}_t^H) + (1-n) \text{var}(\hat{Y}_t^F - \tilde{Y}_t^F) \right] \right. \\ & + (1-\rho)n(1-n) \text{var}(\hat{T}_t - \tilde{T}_t) \\ & + \sigma(1+\sigma\eta)n \frac{\alpha^H}{(1-\alpha^H)(1-\alpha^H\beta)} \text{var} \pi_t^H \\ & \left. + \sigma(1+\sigma\eta)(1-n) \frac{\alpha^F}{(1-\alpha^F)(1-\alpha^F\beta)} \text{var} \pi_t^F \right) \\ & + t.i.p. + O(\|\xi\|^3). \end{aligned} \quad (\text{C.72})$$

This welfare loss function resembles closely Benigno and Benigno (2006, eq. 21) as well as Corsetti, Dedola, and Leduc (2011, eq. 40).

D. Variances

The derivations in this Appendix are only valid if the degree of price stickiness and the persistence of productivity shocks are identical across countries ($\alpha^H = \alpha^F$ and $\rho_H = \rho_F$) and if monetary policy does not engage in interest rate smoothing ($\phi_R = 0$). To obtain analytical expressions for the variance of the inflation differential under each monetary regime, I first derive the recursive laws of motion (RLOM). Then I set up the corresponding vector autoregressive (VAR) model of the system of equations. Finally, since the matrix algebra is very extensive, I use MATLAB Symbolic Math Toolbox to obtain the expressions of interest from the variance-covariance matrix. As shown in section 4, the variables consumption and world inflation need not be considered as they behave identically across monetary regimes and independently of the variables terms of trade, nominal exchange rate and inflation differential.

D.1. MU regime

The number of equations can be reduced by subtracting the New Keynesian Phillips curve of country H (A.42) from the one of country F (A.43). As a result, the consumption gap vanishes due to $k_C^H = k_C^F$. The second equation is given by the terms of trade identity (A.44). The resulting system of equation is, then, given by

$$\pi_t^R = -k_T(\hat{T}_t - \tilde{T}_t) + \beta E_t \pi_{t+1}^R \quad (\text{D.1})$$

$$\hat{T}_t = \hat{T}_{t-1} + \pi_t^R. \quad (\text{D.2})$$

The general form of the corresponding RLOM is given by

$$\hat{T}_t = b_1 \hat{T}_{t-1} + c_1 \tilde{T}_t \quad (\text{D.3})$$

$$\pi_t^R = b_2 \hat{T}_{t-1} + c_2 \tilde{T}_t \quad (\text{D.4})$$

$$\tilde{T}_t = \rho_H \tilde{T}_{t-1} - \frac{\eta}{1 + \eta} \nu_t^R, \quad (\text{D.5})$$

where $\nu_t^R = \nu_t^F - \nu_t^H$. Equation (D.5) is obtained by inserting the country-specific shock processes (A.38) into the equation of the flexible-price terms of trade (A.33).

To obtain the unknown coefficients as functions of the deep parameters of the model, I use the method of undetermined coefficients. First, inserting equations (D.3) through (D.5) into equations (D.1) and (D.2) and rearranging yields

$$b_2 \hat{T}_{t-1} + c_2 \tilde{T}_t = [-k_T b_1 + \beta b_2 b_1] \hat{T}_{t-1} \quad (\text{D.6})$$

$$+ [-k_T(c_1 - 1) + \beta b_2 c_1 + \beta c_2 \rho_H] \tilde{T}_t$$

$$b_1 \hat{T}_{t-1} + c_1 \tilde{T}_t = [1 + b_2] \hat{T}_{t-1} + c_2 \tilde{T}_t. \quad (\text{D.7})$$

Setting $\hat{T}_{t-1} = 1, \tilde{T}_t = 0$ and $\hat{T}_{t-1} = 0, \tilde{T}_t = 1$ respectively gives the following four conditions for the four unknown coefficients:

$$b_2 = -k_T b_1 + \beta b_2 b_1 \quad (\text{D.8})$$

$$c_2 = -k_T(c_1 - 1) + \beta b_2 c_1 + \beta c_2 \rho_H \quad (\text{D.9})$$

$$b_1 = 1 + b_2 \quad (\text{D.10})$$

$$c_1 = c_2. \quad (\text{D.11})$$

Straightforward manipulation yields the quadratic equation

$$0 = \beta b_1^2 - (1 + k_T + \beta) b_1 + 1 \quad (\text{D.12})$$

and therefore two solutions for b_1 . Only one solution fulfills the requirement for a stable equilibrium, i.e., $|b_1| < 1$. Using b_1 immediately yields the other coefficients. Thus, the coefficients of the RLOM take the following form:

$$b_1 = \frac{1 + k_T + \beta - \sqrt{(1 + k_T + \beta)^2 - 4\beta}}{2\beta} \quad (\text{D.13})$$

$$b_2 = \frac{1 + k_T - \beta - \sqrt{(1 + k_T + \beta)^2 - 4\beta}}{2\beta} \quad (\text{D.14})$$

$$c_1 = c_2 = c = \frac{k_T}{1 + k_T + \beta(1 - \rho_H - b_1)}. \quad (\text{D.15})$$

The corresponding VAR model can be written as follows³⁵:

$$\begin{pmatrix} \pi_t^R \\ \hat{T}_t \\ \tilde{T}_t \end{pmatrix} = \underbrace{\begin{pmatrix} 0 & b_2 & \rho_H c \\ 0 & b_1 & \rho_H c \\ 0 & 0 & \rho_H \end{pmatrix}}_{\equiv A} \begin{pmatrix} \pi_{t-1}^R \\ \hat{T}_{t-1} \\ \tilde{T}_{t-1} \end{pmatrix} - \underbrace{\begin{pmatrix} c \frac{\eta}{1+\eta} \\ c \frac{\eta}{1+\eta} \\ \frac{\eta}{1+\eta} \end{pmatrix}}_{\equiv B} \nu_t.$$

A closed-form solution of the variance-covariance matrix Σ can be obtained in terms of the *vec* operator as follows:³⁶

$$\text{vec}(\Sigma) = (I - A \otimes A)^{-1} \text{vec}(B), \quad (\text{D.16})$$

where I denotes the identity matrix.

Since matrix A is of dimension 3×3 , matrix $A \otimes A$ is of dimension 9×9 . Although matrix $A \otimes A$ is triangular, calculating the inverse of that matrix is very cumbersome. Therefore, I resort to MATLAB Symbolic Math Toolbox. Further simplification of the resulting expressions finally yields

$$\text{var}_{MU}(\hat{T}_t - \tilde{T}_t) = \left[\frac{(1 + \rho_H b_1)c^2}{(1 - b_1^2)(1 - \rho_H b_1)} - \frac{2c}{1 - \rho_H b_1} + 1 \right] \text{var} \tilde{T}_t \quad (\text{D.17})$$

$$\text{var}_{MU} \pi_t^R = \frac{2c^2}{(1 + b_1)(1 - \rho_H b_1)(1 + \rho_H)} \text{var} \tilde{T}_t \quad (\text{D.18})$$

$$\text{var} \tilde{T}_t = \frac{1}{1 - \rho_H^2} \left(\frac{\eta}{1 + \eta} \right)^2 [\text{var} \nu_t^H + \text{var} \nu_t^F - 2 \text{cov}(\nu_t^H, \nu_t^F)]. \quad (\text{D.19})$$

D.2. FX regime

The derivation of the variances under the FX regime follows the exact same steps as under the MU regime. The number of equations can be reduced by subtracting the New Keynesian Phillips curves from each other. Furthermore, the expected change in the nominal exchange rate can be expressed as a function of the inflation differential and the terms of trade by inserting the interest rate rules (B.8) and (B.9) as well as the national account identities (B.2) and (B.3) into the uncovered

³⁵The order of variables was chosen as to render matrix A and therefore matrix $A \otimes A$ triangular. This facilitates the calculation of the determinant considerably since, in that case, the determinant is simply given by the product of the diagonal elements.

³⁶See e.g. Hamilton (1994).

interest parity condition (B.7). The resulting system of equations is, then, given by

$$\pi_t^R = -k_T(\hat{T}_t - \tilde{T}_t) + \beta E_t \pi_{t+1}^R \quad (\text{D.20})$$

$$\hat{T}_t = \hat{T}_{t-1} + \pi_t^R + \Delta \hat{S}_t \quad (\text{D.21})$$

$$E_t \Delta \hat{S}_{t+1} = -\phi_\pi \pi_t^R + \phi_Y \hat{T}_t. \quad (\text{D.22})$$

The general form of the corresponding RLOM is given by

$$\hat{T}_t = b_1 \hat{T}_{t-1} + c_1 \tilde{T}_t \quad (\text{D.23})$$

$$\pi_t^R = b_2 \hat{T}_{t-1} + c_2 \tilde{T}_t \quad (\text{D.24})$$

$$\Delta \hat{S}_t = b_3 \hat{T}_{t-1} + c_3 \tilde{T}_t \quad (\text{D.25})$$

$$\tilde{T}_t = \rho_H \tilde{T}_{t-1} - \frac{\eta}{1+\eta} \nu_t^R. \quad (\text{D.26})$$

Inserting equations (D.23) through (D.26) into equations (D.20) through (D.21) and rearranging yields

$$b_2 \hat{T}_{t-1} + c_2 \tilde{T}_t = [-k_T b_1 + \beta b_2 b_1] \hat{T}_{t-1} + [-k_T(c_1 - 1) + \beta b_2 c_1 + \beta c_2 \rho_H] \tilde{T}_t \quad (\text{D.27})$$

$$b_1 \hat{T}_{t-1} + c_1 \tilde{T}_t = [1 + b_2 + b_3] \hat{T}_{t-1} + [c_2 + c_3] \tilde{T}_t \quad (\text{D.28})$$

$$b_1 b_3 \hat{T}_{t-1} + [b_3 c_1 + c_3 \rho_H] \tilde{T}_t = [-\phi_\pi b_2 + \phi_Y b_1] \hat{T}_{t-1} + [-\phi_\pi c_2 + \phi_Y c_1] \tilde{T}_t. \quad (\text{D.29})$$

Setting $\hat{T}_{t-1} = 1, \tilde{T}_t = 0$ and $\hat{T}_{t-1} = 0, \tilde{T}_t = 1$ respectively gives the following six conditions for the six unknown coefficients:

$$b_2 = -k_T b_1 + \beta b_2 b_1 \quad (\text{D.30})$$

$$c_2 = -k_T(c_1 - 1) + \beta b_2 c_1 + \beta c_2 \rho_H \quad (\text{D.31})$$

$$b_1 = 1 + b_2 + b_3 \quad (\text{D.32})$$

$$c_1 = c_2 + c_3 \quad (\text{D.33})$$

$$b_1 b_3 = -\phi_\pi b_2 + \phi_Y b_1 \quad (\text{D.34})$$

$$b_3 c_1 + c_3 \rho_H = -\phi_\pi c_2 + \phi_Y c_1. \quad (\text{D.35})$$

Straightforward manipulation yields the quadratic equation

$$0 = \beta b_1^2 - [1 + k_T + (1 + \phi_Y)\beta] b_1 + (1 + \phi_\pi k_T + \phi_Y). \quad (\text{D.36})$$

In this case, there are either two real or two imaginary solutions for b_1 , depending on the realizations of the deep parameters. However, neither solution fulfills the requirement for a stable equilibrium. Yet, $b_1 = 0$ is another solution to the above system of equations, and it implies a stable equilibrium since $|b_1| < 1$. Given $b_1 = 0$,

the coefficients of the RLOM take the following form:

$$b_1 = 0 \quad (\text{D.37})$$

$$b_2 = 0 \quad (\text{D.38})$$

$$b_3 = -1 \quad (\text{D.39})$$

$$c_1 = \frac{(\phi_\pi - \rho_H)k_T}{(\phi_\pi - \rho_H)k_T + (1 - \rho_H + \phi_Y)(1 - \beta\rho_H)} \quad (\text{D.40})$$

$$c_2 = \frac{(1 - \rho_H + \phi_Y)k_T}{(\phi_\pi - \rho_H)k_T + (1 - \rho_H + \phi_Y)(1 - \beta\rho_H)} \quad (\text{D.41})$$

$$c_3 = \frac{(\phi_\pi - 1 - \phi_Y)k_T}{(\phi_\pi - \rho_H)k_T + (1 - \rho_H + \phi_Y)(1 - \beta\rho_H)}. \quad (\text{D.42})$$

The corresponding VAR model can be written as follows³⁷:

$$\begin{pmatrix} \Delta \hat{S}_t \\ \hat{T}_t \\ \pi_t^R \\ \tilde{T}_t \end{pmatrix} = \underbrace{\begin{pmatrix} 0 & -1 & 0 & \rho_H c_3 \\ 0 & 0 & 0 & \rho_H c_1 \\ 0 & 0 & 0 & \rho_H c_2 \\ 0 & 0 & 0 & \rho_H \end{pmatrix}}_{\equiv A} \begin{pmatrix} \Delta \hat{S}_{t-1} \\ \hat{T}_{t-1} \\ \pi_{t-1}^R \\ \tilde{T}_{t-1} \end{pmatrix} - \underbrace{\begin{pmatrix} c_3 \frac{\eta}{1+\eta} \\ c_1 \frac{\eta}{1+\eta} \\ c_2 \frac{\eta}{1+\eta} \\ \frac{\eta}{1+\eta} \end{pmatrix}}_{\equiv B} \nu_t.$$

A closed-form solution of the variance-covariance matrix Σ can be obtained in terms of the *vec* operator as follows:³⁸

$$\text{vec}(\Sigma) = (I - A \otimes A)^{-1} \text{vec}(B), \quad (\text{D.43})$$

where I denotes the identity matrix.

Since matrix A is of dimension 4×4 , matrix $A \otimes A$ is of dimension 16×16 . Although matrix $A \otimes A$ is triangular, calculating the inverse of that matrix is very cumbersome. Therefore, I resort to MATLAB Symbolic Math Toolbox. Further simplification of the resulting expressions finally yields

$$\text{var}_{FX}(\hat{T}_t - \tilde{T}_t) = (c_1 - 1)^2 \text{var} \tilde{T}_t \quad (\text{D.44})$$

$$\text{var}_{FX} \pi_t^R = c_2^2 \text{var} \tilde{T}_t \quad (\text{D.45})$$

$$\text{var} \tilde{T}_t = \frac{1}{1 - \rho_H^2} \left(\frac{\eta}{1 + \eta} \right)^2 [\text{var} \nu_t^H + \text{var} \nu_t^F - 2 \text{cov}(\nu_t^H, \nu_t^F)]. \quad (\text{D.46})$$

³⁷The order of variables was chosen as to render matrix A and therefore matrix $A \otimes A$ triangular. This facilitates the calculation of the determinant considerably since, in that case, the determinant is simply given by the product of the diagonal elements.

³⁸See e.g. Hamilton (1994).