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Taxation, Innovation and Entrepreneurship

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We examine how basic research should be financed. While basic research is a public good benefiting innovating entrepreneurs it also affects the entire economy: occupational choices of potential entrepreneurs, wages of workers, dividends to shareholders, and aggregate output. We show that the general economy impact of basic research rationalizes a pecking order of taxation to finance basic research. In particular, in a society with desirable dense entrepreneurial activity, a large share of funds for basic research should be financed by labor taxation and a minor share is left to profit taxation. Such tax schemes induce a significant share of agents to become entrepreneurs, thereby rationalizing substantial investments in basic research. These entrepreneurial economies, however, may make a majority of citizens worse off if those individuals do not possess shares of final good producers in the economy. In such circumstances, stagnation may prevail.

**Keywords**  Basic research · Economic growth · Entrepreneurship · Income taxation · Political economy

**JEL Classification**  D72 · H20 · H40 · O31 · O38

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1 Introduction

Motivation: Financing basic research

Basic research is a public good and has to be provided by the government. The main beneficiaries are innovating entrepreneurs as basic research improves their chances to develop new varieties or new, less cost-intensive production technologies. However, basic research impacts on the entire economy. Specifically, it impacts on:

- the occupational choice of individuals to become entrepreneurs
- wages earned by workers
- dividends paid to shareholders of final good producers
- aggregate output

Financing the public good basic research is therefore an intricate task as taxation will affect the four channels described above and thus it will interact with the incentives to provide basic research. In this paper we examine financing of optimal basic research investments in a general equilibrium framework.

Motivation: Entrepreneurship

The role of entrepreneurship for the well-being of societies has been a constant concern for policy-makers and is at the center of policy debates on how to induce growth in the Eurozone (Economist, 2012). With basic research and taxation, we will examine key drivers that shape entrepreneurial activities in societies. We will analyze in which circumstances societies are in favor of entrepreneurial economies or are prone to remain stagnant.

Model

We develop a simple model of creative destruction where a final consumption good is produced using labor and a continuum of indivisible intermediate goods as inputs. Entrepreneurs can take up basic research provided by the government and invest in applied research in order to develop improved production technologies for intermediates, allowing successful entrepreneurs to earn monopoly profits. In addition, entrepreneurship has immaterial cost (such as entrepreneurial effort cost) and benefits (such as social status). Potential entrepreneurs weigh these benefits against the labor income lost when deciding on whether or not to become entrepreneurs. The government fi-
nances its basic research investments using a combination of labor income, profit, and potentially lump-sum taxes. Among others, this financing decision affects the occupational choice by potential entrepreneurs and hence it impacts on the effectiveness of basic research investments.

**Main results**

Our analyses show that the general economy implications of basic research rationalize a pecking order of taxation to finance basic research. In particular, in an innovative economy with desirable dense entrepreneurial activity, a large share of funds for basic research should be financed by labor taxation and a minor share is left to profit taxation. This pecking order of taxation with labor income tax first exploits the complementarity between basic research and tax policies: The resulting tax stimulus fosters entrepreneurship, thereby increasing benefits from investments in basic research. In the absence of (constitutional) bounds on taxation, however, such aggregate consumption optimal entrepreneurial economies may make a majority of citizens worse off if those individuals do not possess shares of final good producers in the economy. Depending on whether or not these individuals can set the political agenda, stagnation may prevail in such circumstances or it may be politically viable to implement constraint optimal policy changes that make the economy closer in nature to an entrepreneurial economy when compared to a benchmark economy with zero investments in basic research and neutral tax policies.

The main insights are detailed and qualified in a series of formal results: First, existence and uniqueness of an equilibrium given basic research investments and given tax policies is established. Second, if the government is solely interested in maximizing aggregate consumption, the main insights occur as described, both for a scenario with and without lump-sum taxes. Third, the optimality of a pecking order of taxation survives the use of a broader welfare measure which includes in addition immaterial cost and benefits associated with entrepreneurial activity.

**Organization of the paper**

The paper is organized as follows: Section 2 embeds our paper in the literature. Sections 3 and 4 outline the model and derive the equilibrium for given tax policies and basic research investments. In section 5 we analyze aggregate consumption optimal policies for the scenario with and without lump-sum taxes. Section 6 presents an
analysis of the political economy of financing basic research. Section 7 concludes.

2 Literature

Our paper is related to several important strands of the literature.

Rationale for public funding of basic research

The case for public funding of basic research is well established in the literature, in particular since the seminal paper of Nelson (1959). He identifies fundamental conflicts between providing basic research and the interests of profit making firms in a competitive economy. First, the provision of basic research has significant positive external effects that cannot be internalized by private firms. Basic research should not be directed towards particular technologies and the resulting scientific knowledge has typically practical value in many fields. As a consequence, technological specialization and a lack of patentability frequently prevent private firms from exploiting all the potential benefits from undirected basic research. Even more, Nelson argues that full and free dissemination of scientific knowledge would be socially desirable due to its non-rivalry. Second, Nelson argues that the long lag between basic research and the reflection thereof in marketable products might prevent short-sighted firms from investing. And third, he points out that the high uncertainty involved in the process might induce a private provision of basic research that is below the socially optimal level. These three problems are the more severe, the more basic the research is and they therefore motivate public provision of basic research in particular.

The case for publicly funded basic research has further been substantiated by several authors. Arrow (1962), for example, points out that invention which he defines as the production of knowledge is prone to three classical reasons for market failures: indivisibility, inappropriability, and uncertainty. Similar to Nelson (1959), he argues that these problems result in an underinvestment in research on the free market and that this problem is the more severe, the more basic the research is. Kay and Smith (1985) stress the enormous benefits from basic research and argue that public provision is necessary due to the public good nature of basic research. They also make a case for domestic provision of basic research rather than free-riding on basic research performed by other countries.

Beginning in the late 1980s, some authors have questioned the public goods nature
of scientific knowledge. In particular, the view that existing knowledge is non-rival has been criticized. It has been argued that the utilization of specialized knowledge requires significant investments in complementary research capabilities. This might motivate private provision of basic research (see, for example, Cohen and Levinthal (1989), Rosenberg (1990), or Callon (1994)). Notwithstanding, these authors do not question public provision of basic research. Callon points out that public engagement in the field of science is needed in order to preserve variety and flexibility in the economy.

In summary, there is a strong case for publicly funded research, in particular basic research. This rationale is matched by empirical evidence. Gersbach et al. (2012) reported data showing that for a selection of 15 countries the average share of basic research that was performed in the government and higher education sector was approximately 75% in 2009. From the OECD main science and technology indicators we find that across OECD member countries around 80% of total research performed in the government or higher education sector is also funded by the government.\(^1\) Taken together, these findings suggest that indeed a major share of basic research investments are publicly funded.

*Financing of basic research*

In this paper, we start from the rationale why basic research has to be funded publicly, in particular because pure private provision will result in underinvestment when compared to the social optimum. Our main question is then how optimally chosen basic research expenditures should be financed. Our paper is thus related to the literature on financing productive government expenditures. In the seminal paper, Barro (1990) examines the case of productive government expenditures as a flow variable. Futagami et al. (1993) develop the case of productive government expenditures as a stock variable. In both cases, the public service provided is not subject to congestion effects, as for publicly provided basic research. These authors develop investment-based endogenous growth models where the individual firm faces constant returns to scale with respect to both, private capital and the public services provided by the government. According to the comprehensive survey by Irmen and Kuehnel (2009), this applies more general to

\(^1\)Data have been downloaded from OECD (2012) in May 2012. As far as available, 2008 data has been used. For each country, the share of public funding in the government and higher education sector has been computed as follows: 
\[
\text{average of these shares across all OECD member countries was found to be slightly below 80%}.
\]
the main body of the literature on productive government expenditures and economic growth. By contrast, our model is rooted in the tradition of R&D based endogenous growth models, and particularly those that explicitly take into account the hierarchical order of basic and applied research (see, for example, Arnold (1997), Morales (2004), or Gersbach et al. (2009)). In these kind of models, basic research has no productive use in itself, but rather fuels into the productivity of the applied research sector, where knowledge is transformed into blueprints for new or improved products. In our case, basic research affects the innovation probability of entrepreneurs that engage in applied research. Using more public funds for basic research improves the chances of success of private entrepreneurs at the cost of diverting resources away from intermediate and final good production.

Moreover, a second important role of financing basic research will be addressed in this paper. Basic research may be financed via a combination of labor income, profit, and lump-sum taxes. The relative size of labor to profit taxes affects the trade-off faced by potential entrepreneurs between being employed in the labor market and becoming an entrepreneur and hence influences the number of innovating entrepreneurs in our economy.

Tax structures and entrepreneurial activity

Empirical evidence in the literature suggests that the tax structure does indeed influence the level of entrepreneurial activities in an economy. Using cross-sectional data of US personal income tax returns, Cullen and Gordon (2007) estimate the impact of various tax measures on entrepreneurial risk-taking as proxied by an indicator variable for whether or not an individual reports business losses greater than 10% of reported wage income. They find that a cut in personal income tax rates significantly reduces entrepreneurial risk taking. The evidence for a cut in corporate tax rates is less clear: depending on the model specification used, such a cut is predicted to either rise or not to significantly affect entrepreneurial risk taking. Cullen and Gordon interpret their results to be in line with their theory, as risk-sharing of non-diversifiable entrepreneurial risks with the government is positively related to the corporate income tax rate. Djankov et al. (2010) analyze cross-country data for 85 countries. They find that higher effective tax rates paid by a hypothetical new company have a significant adverse effect on aggregate investment and entrepreneurship. Gentry and Hubbard (2000) analyze 1979 to 1992 data from the Panel Study on Income Dynamics to find
that less progressive tax rates significantly increase entrepreneurship.

**Optimal taxation in an economy with entrepreneurship**

We analyze the optimal mix of basic research and tax policies. Hence, our paper is also related to the literature on optimal income taxation standing in the tradition of the seminal paper by Mirrlees (1971). Initial work in this area analyzed tax distortions of the labor leisure choice by households and derived optimal tax policies balancing the efficiency losses from more progressive taxes against welfare gains associated with more egalitarian income distributions (see also Sheshinski (1972) or Stern (1976), for example).

At the heart of our model is the occupational choice by (potential) entrepreneurs. Hence, our paper is closer related to papers with endogenous wages and occupational choice. Feldstein (1973), Allen (1982), and Stiglitz (1982) develop models with two types of workers, skilled and unskilled, with endogenous wages and endogenous labor supply by both types of labor but no occupational choice. Boadway et al. (1991) present a model with heterogeneous agents who can chose between becoming entrepreneurs or workers. These papers have in common that they analyze optimal taxes, where tax rates are the same for both types of labor or income. By contrast, in our model the government can discriminate between taxes on profits and on labor income. Kanbur (1981) considers a model with endogenous occupational choice of homogeneous agents between becoming a worker earning a safe wage or an entrepreneur earning risky profits. Among others, he considers entrepreneurial risk-taking given occupational dependent taxation, but he does not derive optimal tax policies. In this regard his work is close in nature to recent work on calibrated dynamic general equilibrium models that are used to assess the effects of stylized tax reforms (see Meh (2005) or Cagetti and De Nardi (2009), for example).

Moresi (1998) and Scheuer (2011), for example, analyze optimal tax policies in models of asymmetric information with occupational choice, where the government faces a trade-off between efficiency and equity. The distinctive feature of our model is that we analyze optimal tax policies in the presence of basic research that allows the government to use tax revenues in order to directly foster innovativeness of entrepreneurs. We

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2In the paper by Allen (1982), workers belong to either of two skill groups and they can chose among two types of labor, but workers perfectly select into these types of work on the basis of their skill-group.
show that efficient policies make use of a pecking order of taxation. In particular, in our model investments in basic research that allow for efficiency gains in aggregate should be accompanied by low profit taxes and high labor income taxes. We consider distributional effects when analyzing the political economics of our results.

3 The Model

3.1 Production

The economy is populated by a continuum of measure \( \bar{L} > 1 \) of households who derive utility \( u(c) = c \) from a final consumption good. Agents are indexed by \( k \) (\( k \in [0, \bar{L}] \)). The output of the final good, denoted by \( y \), is produced with a continuum of intermediate goods \( x(i) \) (\( i \in [0, 1] \)). The production technology is given by

\[
y = L_y^{1-\alpha} \int_0^1 x(i)^\alpha \, di,
\]

where \( L_y \) denotes labor employed in final good production and where \( 0 < \alpha < 1 \). The final good is only used for consumption, hence in equilibrium output of the final good equals aggregate consumption (\( C = y \)).

We assume that intermediate goods \( x \) are indivisible, i.e. \( x(i) \) is either 1 or 0. The price of the final consumption good is normalized to 1 and thus it is the numéraire. Firms in the final good sector operate under perfect competition. They take the price \( p(i) \) of intermediate goods as given. In the following we work with a representative final good firm maximizing

\[
\pi_y = y - \int_0^1 p(i)x(i) \, di - wL_y
\]

by choosing the quantities \( x(i) \in \{0, 1\} \) and the amount of labor \( L_y \). If the final good producer choose \( x(i) = 1 \) for all \( i \), the demand for labor in final good production is

\[
L_y = \left( \frac{1 - \alpha}{w} \right)^{\frac{1}{\alpha}}.
\]

3.2 Behavior of intermediate good producers

Each intermediate good can be produced by a given technology using \( m > 0 \) units of labor. Hence the marginal production costs are \( mw \) and we assume that the standard
technology is freely available. This implies perfect competition and a price equal to the marginal costs. If an entrepreneur engages in research and development and successfully innovates, the production costs decline by a factor $\gamma$ ($\gamma < 1$) leading to marginal production costs of $\gamma mw$. The innovating entrepreneur obtains a monopoly and it will turn out that he still offers his product at the price equal to the marginal cost of potential competitors, $mw$, thereby gaining profit $\pi_{xm} = (1 - \gamma)mw$.

3.3 Innovation

There is a measure 1 of individuals $[0, 1] \subset [0, \bar{L}]$ who are potential entrepreneurs. Individuals face different costs and benefits when deciding to become an entrepreneur. Specifically, we assume that agents are ordered in $[0, 1]$ according to their immaterial utilities from entrepreneurial activities: In particular, individual $k$ faces the utility factor $\lambda_k = (1 - k)b$ ($k \in [0, 1]$, $b$ being a positive parameter). This factor rescales the profit earned from entrepreneurial activities in order to take into account immaterial cost (such as cost from exerting efforts as an entrepreneur or utility cost from entrepreneurial risk taking that are not reflected in the utility from consumption) and immaterial benefits (such as excitement, initiative taking, or social status) associated with entrepreneurial activity.\(^3\) Agents with a higher index $k$ have lower utility factors. A utility factor $\lambda_k < 1$ represents net utility cost from being an entrepreneur while a factor $\lambda_k > 1$ represents net immaterial benefits. For individuals $k$ with $\lambda_k = 1$, and thus $k^{crit} = 1 - \frac{1}{b}$, immaterial cost and benefits associated with entrepreneurial activities cancel out. If $b$ is small and thus $k^{crit}$ is small or even zero, the society is characterized by a population of potential entrepreneurs for whom effort cost matter most. If $b$ is large and thus $k^{crit}$ is large, the potential entrepreneurs enjoy being one compared to a worker. We assume that $\lambda_k$ is private information and thus only observed by agent $k$.\(^4\)

The chances of entrepreneurs to successfully innovate can be fostered by basic research. Basic research generates knowledge that is taken up by entrepreneurs and transformed into innovations applied in the production process. Suppose that the government em-

\(^3\)Cf. footnote 11 for a discussion on how differences in risk-attitudes might give rise to occupational choice effects similar to the ones arising from our immaterial benefit factor $\lambda_k$. In our model, there is no aggregate risk and entrepreneurship is not prone to moral hazard. Hence, with complete markets entrepreneurs can perfectly insure against entrepreneurial risk. In this case, $\lambda_k$ does not capture utility cost from entrepreneurial risk taking.

\(^4\)This does preclude to condition taxation on $\lambda_k$. We note that our results remain unaffected if $\lambda_k$ is common knowledge but tax policies do not condition thereon.
ploys $L_B$ ($0 \leq L_B \leq \bar{L}$) researchers in basic research. Then the probability that an entrepreneur successfully innovates is given by $\eta(L_B)$ where $\eta(L_B)$ fulfills $\eta(0) \geq 0$, $\eta'(\cdot) > 0$, $\eta''(\cdot) < 0$ and $\eta(\bar{L}) \leq 1$.\(^5\) Depending on whether $\eta(0) = 0$ or $\eta(0) > 0$, basic research is a necessary condition for innovation or not.

Accordingly, if a measure $L_E$ of the population decided to become entrepreneurs and each has the success probability $\eta(L_B)$, the share of intermediate sectors with successful innovation is equal to $\eta(L_B)L_E$.\(^6\) We note that the property $L_E \leq 1$ allows that entrepreneurs perform research on a variety different from others.\(^7\)

### 3.4 Financing scheme

Expenditures for basic research have to be financed by taxes. The government can levy taxes on labor income or profits. Additionally, we assume that the government can levy lump sum taxes or make lump sum transfers.\(^8\) Later we examine the case when this is not possible. A tax scheme is a vector $(\tau_L, \tau_P, \tau_H)$ where $\tau_L$ and $\tau_P$ are the tax rates on labor income and on profits, respectively, and $\tau_H$ denotes a lump sum tax or transfer. We assume that there are upper bounds (and potentially lower bounds) of labor income and profit taxes given by the constitution. For example in Germany, average taxes on income cannot exceed 50%. We denote the upper and lower bounds by $\tau_j$ and $\tau_{j-l}$, $j \in \{L, P\}$, respectively.

Throughout our paper, we assume that the government needs to run a balanced budget, i.e. the government budget constraint is given by

\[ wL_B = \tau_L(\bar{L} - L_E)w + \tau_P(\pi_y + \eta(L_B)L_E\pi_{x_m}) + \tau_H\bar{L}, \]  

where $\tau_H = 0$ in the scenario without lump-sum taxes.

### 3.5 Sequence of events

We summarize the sequence of events as follows.

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\(^5\) $\eta'(\cdot)$ and $\eta''(\cdot)$ denote the first and second derivative, respectively, of $\eta(\cdot)$ with respect to $L_B$.

\(^6\) We use a suitable version of the law of large numbers for a continuum of random variables.

\(^7\) Strictly speaking we assume that there is no duplication of research efforts. It is straightforward to incorporate formulations in which several researchers compete for innovation on one variety. This would decrease the benefits from basic research for entrepreneurs and for the society.

\(^8\) Our model allows for unsuccessful entrepreneurs which earn zero profits. Consequently, in case that their share of the profits of the final good firm are not too high, they may not be able to pay the lump sum tax. Here we assume that all individuals have a certain endowment, which could be drawn on by the government in this case.
(1) The government hires a number $L_B$ of researchers to provide public basic research and chooses a financing scheme.

(2) A share $L_E$ of the population decide to become entrepreneurs. With probability $\eta(L_B)$ they successfully innovate, which enables them to capture monopoly rents. A share $(1 - \eta(L_B))L_E$ will not be successful and will earn zero profits.

(3) Each intermediate good firm $i$ hires a number $L_x(i)$ of workers in order to produce the intermediate good $x(i)$.

(4) The representative final good firm buys the intermediate goods $x(i)$ at a price $p(i)$ and produces the homogeneous final good $y$.

4 Equilibrium

In this section we derive the equilibrium for a given amount of basic research and a given financing scheme.

4.1 Occupational choice by potential entrepreneurs

We first address the choice of occupation. Potential entrepreneurs, i.e., agents in the interval $[0, 1]$, can choose between being employed as workers and trying to develop an innovation to be used in the production of intermediate goods. We are left with two cases: all agents choose to be workers or both occupations are chosen in equilibrium.\(^9\) If both occupations are chosen in equilibrium, the marginal entrepreneur has to be indifferent between being employed as a worker and becoming an entrepreneur. The expected net profit of an entrepreneur is

$$\pi^E = (1 - \tau_p)\eta(L_B)\pi_{xm}.$$ 

Hence, the expected utility for an individual $k$ with (dis-)utility factor $\lambda_k = (1 - k)b$ from being an entrepreneur is:

$$EU^E(k) = (1 - \tau_p)\eta(L_B)m(1 - \gamma)w(1 - k)b$$

\(^9\)More precisely, in the first case only a set of individuals of measure 0 decides to become an entrepreneur.
We note that we have chosen a multiplicative functional form. The individual is indifferent between being employed as a worker and becoming an entrepreneur if 

\[ EU^E(k) = (1 - \tau_L)w. \]

Solving for the equilibrium amount of entrepreneurs yields

\[ L_E^e = \max \left\{ 0, 1 - \frac{1 - \tau_L}{1 - \tau_P} \frac{1}{\eta(L_B)m(1 - \gamma)b} \right\}. \] (4)

In the following we use \( \tau \) as an abbreviation for \( \frac{1 - \tau_L}{1 - \tau_P} \), with the upper and lower bounds of \( \tau \) denoted by \( \tau \) and \( \bar{\tau} \) being defined by the respective bounds of \( \tau_L \) and \( \tau_P \). \( \tau \) is a measure of tax incentives given to (potential) entrepreneurs.

Knowing \( L_E \), we obtain the amount of labor employed in the production of intermediates as

\[ L_x^e = \int_0^1 L_x(i)di = m - \eta(L_B)m(1 - \gamma)L_E^e \] (5)

if \( x(i) = 1 \ \forall \ i \). Using the market clearing condition in the labor market

\[ \bar{L} = L_E + L_B + L_y + L_x, \] (6)

as well as (2), we derive the equilibrium wage rate as

\[ w^e = (1 - \alpha)(\bar{L} - L_E^e - L_B - L_x^e)^{-\alpha}. \] (7)

\(^{10}\)An alternative approach is to use an additive functional form by deducting the cost. The multiplicative approach is more convenient and analytically much easier. In addition, it implies that the net immaterial benefit is scaled by entrepreneurial profits. The multiplicative approach may therefore be more appropriate to reflect effort costs and social status benefits, in particular, as these would typically be related to profits. For \( \lambda_k > 1 \) the effort cost dominate and for \( \lambda_k < 1 \) the social status benefits. Qualitatively, however, the additive and the multiplicative approach involve the same trade-offs and pecking order considerations.

\(^{11}\)In our model, potential entrepreneurs differ in their immaterial cost and benefits from being an entrepreneur. Agents whose expected utility from being an entrepreneur exceeds the utility from working in the labor market opt to become an entrepreneur, thus giving rise to continuous occupational choice effects. We note that a similar result for the occupational choice would arise if agents differed in the risk attitude rather than in an extra (dis-)utility term. Suppose for example that potential entrepreneurs differ only in their degree of constant relative risk-aversion with \( \gamma \) being defined by the respective bounds of \( \tau_L \) and \( \tau_P \). \( \gamma \) is a measure of tax incentives given to (potential) entrepreneurs.
4.2 Equilibrium for given basic research and financing scheme

The derivation of the equilibrium involves some subtleties. We start with the following observation:

**Lemma 1**

(i) In any equilibrium with positive production in the final good sector, intermediate good producers supplying their product will charge \( p_i = mw \). \(^{12}\)

(ii) In any equilibrium with positive production in the final good sector, the final good producer uses all varieties of intermediate goods and chooses \( L_y = (\frac{1-\alpha}{m})^\frac{1}{\alpha} \).

The proof of Lemma 1 can be found in appendix A.1. We next introduce the following condition:

**Definition 1 (Positive Profit Condition)**

\[
\frac{m}{\alpha} \leq \begin{cases} 
\bar{L} - L_B & \text{if } \frac{1}{\tau \eta(L_B)m(1-\gamma)b} \geq 1 \\
\bar{L} - L_B + \left[ 1 - \frac{1}{\tau \eta(L_B)m(1-\gamma)b} \right] \eta(L_B)m(1 - \gamma) - 1 & \text{if } \frac{1}{\tau \eta(L_B)m(1-\gamma)b} < 1 
\end{cases},
\]

which guarantees that profits of the representative final good firm are non-negative in equilibrium. Indeed, the following Lemma proven in appendix A.2 holds:

**Lemma 2**

If and only if condition (PPC) is satisfied, the final good producer is operating and he uses all varieties in production.

We are now in a position to characterize the allocation and prices in the equilibrium of the economy for given basic research investments \( L_B \) and a given financing scheme \( \tau \).

**Proposition 1**

(i) If \( L_B \) and \( \tau \) satisfy condition (PPC), there is a unique equilibrium with \( x(i) = 1 \) for all \( i \) and

\[
(1) \ L_E^i = \max \left\{ 0; 1 - \frac{1}{1 - \tau \eta(L_B)m(1-\gamma)b} \frac{1}{\eta(L_B)m(1 - \gamma)} \right\} \\
(2) \ L_x^e = m - mL_E^i \eta(L_B)(1 - \gamma)
\]

\(^{12}\)To avoid the need to discretize the strategy-space in order to obtain existence of equilibria in the price-setting game in the intermediate good industry \( i \), we assume as a tie-breaking rule that the final good producer demands the product from the innovating entrepreneur if he offers the same price as non-innovating competitors.
(3) \( L_y = \bar{L} - L_B - m + L_E^e \left[ \eta(L_B)m(1 - \gamma) - 1 \right] \)

(4) \( w^e = (1 - \alpha) \left( L_y^e \right)^{-\alpha} \)

(5) \( p(i)^c = m(1 - \alpha) \left( L_y^e \right)^{-\alpha} \forall i \)

(6) \( y^c = (L_y^e)^{1-\alpha} \)

(7) \( \pi_y^e = (L_y^e)^{-\alpha} \left( \alpha L_y^e - m(1 - \alpha) \right) \)

(8) \( \pi_{xm}^e = (1 - \gamma)m(1 - \alpha)(L_y^e)^{-\alpha} \).

(ii) If \( L_B \) and \( \tau \) do not satisfy condition (PPC), there is a unique equilibrium with \( x(i) = 0 \) for all \( i \), \( L_E^e = L_x^e = L_y^e = 0 \), and zero profits.

The proof of Proposition 1 can be found in appendix A.3.

5 Optimal Policies

The government can manipulate the previously established equilibrium outcomes by investing in basic research and via the tax scheme. We consider the case of a government seeking to maximize aggregate consumption and analyze optimal policies with and without lump-sum taxes and transfers.\(^{13}\) Throughout this analysis we assume:

Assumption 1

\[ \bar{L} + \frac{L_E}{\tau = 1} \left[ \rho_0 m(1 - \gamma) - 1 \right] \geq \frac{m}{\alpha}, \]

where \( L_E \left| \tau = 1 \right. \) denotes the equilibrium number of entrepreneurs given \( L_B = 0 \) and \( \tau = 1 \). Either \( L_E \left| \tau = 1 \right. = 0 \) in which case Assumption 1 reduces to \( \bar{L} \geq \frac{m}{\alpha} \). Or \( L_E \left| \tau = 1 \right. = 1 - \frac{1}{\rho_0 m(1 - \gamma)b} \) yielding:

\[ \bar{L} + \left[ 1 - \frac{1}{\rho_0 m(1 - \gamma)b} \right] \left[ \rho_0 m(1 - \gamma) - 1 \right] \geq \frac{m}{\alpha}. \]

As we verify in Appendix A.4, Assumption 1 defines values of \( \bar{L} \) such that any choice of \( L_B \) and financing scheme characterized by \( \tau \) that maximizes aggregate consumption will yield an equilibrium according to item (i) in Proposition 1. Economically this assumption is not very restrictive as it merely states that in an economy without basic research and with neutral tax policy an equilibrium with positive production prevails.

\(^{13}\)In an extended version of this paper we also consider the case of a government that maximizes aggregate welfare, which in addition to utility derived from consumption accounts for the utility costs and benefits from becoming an entrepreneur. There, we show that our main insight regarding the pecking order of taxation prevails and may be reinforced with a broader welfare measure.
Lemma 3

Suppose Assumption 1 is satisfied. Then, any aggregate consumption maximizing policy choice \( (\tau_1^*, \tau_p^*, L_B) \) will satisfy condition (PPC).

We first consider the case where lump-sum taxes and transfers are available to the government. As the number of entrepreneurs only depends on the relation between profit and labor income taxes as captured by \( \tau \), the assumption of lump-sum transfers allows us to separate the choice of \( L_B \) from the choice of the government’s optimal relation between profit and labor income taxes. This scenario will yield the major insights.\(^{14}\) If no lump sum taxes and transfers are available, the choices of \( \tau \) and \( L_B \) cannot in all cases be separated. We discuss these issues in section 5.2 and abstract from such problems in the next section.

5.1 Optimal policy with lump-sum taxes and transfers

The government’s problem boils down to maximizing aggregate consumption, \( C \), by choosing the amount of basic research \( L_B \) and the optimal ratio between profit and labor taxes, \( \tau \), while either levying an additional lump sum tax if labor and profit taxes satisfying the optimal \( \tau \) do not suffice to finance the desired amount of \( L_B \) or making a lump sum transfer in case that the revenue generated by \( \tau \) is larger than needed for the basic research expenditures.

\[
\max_{\{\tau_L, \tau_P, \tau_H, L_B\}} C = \pi_y + \eta(L_B) \pi_x \mu + wL_y + wL_x + wL_B - (\bar{L} - L_E) w \tau_L - \tau_P \left[ \pi_y + \eta(L_B) \pi_x \mu \right] - \tau_H \bar{L}
\]

s.t. \( wL_B = (\bar{L} - L_E) w \tau_L + \tau_P \left[ \pi_y + \eta(L_B) \pi_x \mu \right] + \tau_H \bar{L} \)

where \( \tau_H \bar{L} \) denotes the lump-sum taxes or transfers and we use \( C \) to denote aggregate consumption.

Inserting the constraint into the objective function, using \( \tau = \frac{1 - \tau_P}{1 - \pi_L} \) and the aggregate income identity \( y = \pi_y + \eta(L_B) \pi_x \mu + wL_y + wL_x \) reduces the problem to

\[
\max_{\{L_B, \tau\}} C(L_B, \tau) = y(L_B, \tau) = \left[ \bar{L} - L_E(L_B, \tau) - L_B - L_x(L_B, \tau) \right]^{1-\alpha}.
\]

\(^{14}\)Given that basic research investments account for a share of government expenditures only, the scenario with lump-sum taxes might also be interpreted as one where any excess funds are used to finance other government expenditures that benefit all members of the population equally. For a broad range of parameter values, lump-sum taxes are negative in optimum, i.e. we have lump-sum transfers. Then, our analysis is equivalent to an analysis with no lump-sum taxes but investments in an additional public good \( g \) which directly impacts on households’ utilities and where \( u(c, g) = c + \frac{g}{L} \).
Hence, the objective of the government is to maximize the amount of productive labor in final good production. By inserting $L_x$, the objective function can be written as $\overline{L} - L_B - m + L_E[\eta(L_B)m(1 - \gamma) - 1]$. From the Kuhn-Tucker conditions we obtain:

$$\frac{\partial L_E}{\partial L_B}[\eta(L_B)m(1 - \gamma) - 1] + L_E\eta'(L_B)m(1 - \gamma) - 1 \leq 0$$

$$\frac{\partial L_y}{\partial L_B}L_B = 0$$

$$\frac{\partial L_E}{\partial \tau}[\eta(L_B)m(1 - \gamma) - 1] \geq 0$$

$$\frac{\partial L_y}{\partial \tau}(\tau - \underline{\tau})(\tau - \overline{\tau}) = 0.$$  

The first-order conditions for interior solutions ($L_B > 0$, $\underline{\tau} < \tau < \overline{\tau}$) are:

$$\frac{\partial L_E}{\partial L_B}[\eta(L_B)m(1 - \gamma) - 1] + L_E\eta'(L_B)m(1 - \gamma) = 1,$$  

(9)  

$$\frac{\partial L_E}{\partial \tau}[\eta(L_B)m(1 - \gamma) - 1] = 0.$$  

(10)

The term in brackets on the left-hand sides of (9) and (10) expresses how much labor is saved in expectation for final good production by having another entrepreneur. The first term in (10) reflects the increase in the number of individuals becoming entrepreneurs when the labor income tax rate marginally increases relative to the profit tax rate. $\frac{\partial L_y}{\partial \tau}$ is clearly non-negative and strictly positive for $\tau > \frac{1}{\eta(L_B)m(1 - \gamma)\eta}$. Consequently, if the term in brackets is positive, the government will increase $\tau$ to its maximum to make entrepreneurship most attractive. The opposite is the case if the term in brackets is negative. Then the government aims at reducing the number of entrepreneurs to a minimum by setting $\tau$ to its lowest level. In the first situation where $\tau$ is maximized we speak of an entrepreneurial economy and refer to the situation with a minimal level of $\tau$ as a stagnant economy.

We also observe in (10) that the expected benefit of another entrepreneur (reflected by the term in brackets in (10)) depends on the level of basic research expenditures. For example, if $\rho_0(= \eta(0)) \approx 0$, an entrepreneur is not as productive in innovating than when working in final good production. Only if the amount of basic research is larger than $L_{B,\min} := \eta^{-1}\left(1/(m(1 - \gamma))\right)$, where $\eta^{-1}(\cdot)$ denotes the inverse of $\eta(\cdot)$, will an increase in the relative labor income tax be favorable for aggregate consumption and lead to an entrepreneurial economy. This reveals that the government’s choices of basic research and the financing scheme interact in an important way. Given a taxation scheme the optimal amount of basic research is determined by (9) where $L_E$
and \( \frac{\partial L_E}{\partial B} \) depend on \( \tau \). We use \( \tilde{L}_B(\tau) \) to denote the candidate for a maximum derived by (9) for a given \( \tau \). From the reasoning given above, we conclude that \( \tau^* \in \{ \overline{\tau}, \underbar{\tau} \} \).

Hence, \( (\tau, \tilde{L}_B(\tau)) \) is optimal if and only if \( c(\tau, \tilde{L}_B(\tau)) \geq c(\overline{\tau}, \tilde{L}_B(\overline{\tau})) \). A necessary condition for this to happen is that \( \tilde{L}_B(\tau) > L_{B,\min} \).

In the following, we assume that it is feasible for the government to fully discourage entrepreneurship in the economy by not providing any basic research and at the same time penalizing entrepreneurship via tax policies.

**Assumption 2**

\[ \tau \rho_0 m(1 - \gamma)b \leq 1 \]

To verify that Assumption 2 is indeed sufficient for \( L_E = 0 \) to be feasible, use \( \tau \rho_0 m(1 - \gamma)b \) in the expression for \( L_E \) stated in Proposition 1.

We next introduce the following condition:

**Definition 2 (Positive Labor Saving R&D)**

When the maximal amount of entrepreneurs is induced by tax policies, \( \tau = \overline{\tau} \), and the optimal amount of basic research given \( \overline{\tau} \) is provided, \( L_B = \tilde{L}_B(\overline{\tau}) \), then labor savings are positive, i.e.

\[
-L_B(\overline{\tau}) + \left[ 1 - \frac{1}{\tau \eta(\tilde{L}_B(\overline{\tau})) m(1 - \gamma)b} \right] \left[ \eta(L_B(\overline{\tau})) m(1 - \gamma) - 1 \right] \geq 0. \quad \text{(PLS)}
\]

Using condition PLS, we can characterize the optimal policy schemes as follows:

**Proposition 2**

Suppose the government maximizes aggregate consumption using \( (\tau_L, \tau_P, \tau_H, L_B) \) as policy instruments and let Assumptions 1 and 2 both be satisfied. Then:

(i) If and only if condition (PLS) is satisfied, there will be an entrepreneurial economy with \( \tau^* = \overline{\tau}, L_B^* = \tilde{L}_B(\overline{\tau}) \) and \( L_E = 1 - \frac{1}{\tau \eta(\tilde{L}_B(\overline{\tau})) m(1 - \gamma)b} \).

(ii) Else, there will be a stagnant economy with \( \tau^* = \underbar{\tau}, L_B^* = 0 \) and \( L_E = 0 \).

The proof of Proposition 2 is given in appendix A.5. Proposition 2 implies that an entrepreneurial economy will be established if and only if the optimal amount of basic research given maximal tax incentives to entrepreneurs yields higher final good production than would obtain in an economy without entrepreneurship.
We next analyze condition (PLS) more closely in order to deduce when an entrepreneurial economy is likely to prevail.

**Corollary 1**

*Suppose the government maximizes aggregate consumption using* $(\tau_L, \tau_P, \tau_H, L_B)$ *as policy instruments and let Assumptions 1 and 2 both be satisfied. Then, the higher* $m$, $b$, *and the lower* $\gamma$, *the more likely it is that an entrepreneurial economy prevails.*

The proof of Corollary 1 is given in appendix A.6. Corollary 1 implies that the more valuable innovations are, i.e. the higher is $m$ and the lower is $\gamma$, the more likely it is that we will observe an entrepreneurial economy. Further, an entrepreneurial economy is the more likely the higher is the maximum admissible level of $\tau$, $\bar{\tau}$, and the higher are the utility benefits (the lower are the utility cost) derived from becoming an entrepreneur, i.e. the higher is $b$.

### 5.2 Optimal policy without lump-sum taxes and transfers

In the previous section, separating the choice of $L_B$ from that of the ratio between labor and profit taxes as captured in $\tau$ was feasible. We now ask whether this is always possible even when lump-sum taxes or transfers are not available. This means that given optimal values of $L_B$ and $\tau$ we can always find values of $\tau_L$ and $\tau_P$ resulting in the desired value of $\tau$ and satisfying the budget constraint

$$wL_B = w\tau_L \left[ \bar{L} - L_E \right] + \tau_P \left[ \pi_y + \eta(L_B) L_E \pi_{xm} \right].$$

(11)

Using the equilibrium values of $\pi_y$ and $\pi_{xm}$, the budget constraint can be rewritten as

$$L_B = \tau_L \left[ \bar{L} - L_E \right] + \tau_P \left[ \frac{\alpha}{1 - \alpha} L_y - m + L_E \eta(L_B) m(1 - \gamma) \right].$$

(12)

The right hand side of equation (12) corresponds to the tax revenue in working hour equivalents. It will subsequently be denoted by $TR$.

The definition of $\tau$ yields $\tau_L = 1 - \frac{1 - \tau_P}{\tau}$. Inserting into equation (12) and solving for $\tau_P$, we obtain that the choice of $L_B$ and $\tau$ can be separated only if this value of $\tau_P$, which we denote by $\bar{\tau}_P$, is in the feasible range $[\underline{\tau}_P, \bar{\tau}_P]$ and $\bar{\tau}_L = 1 - \frac{1 - \bar{\tau}_P}{\tau}$ is in $[\underline{\tau}_L, \bar{\tau}_L]$.

\[\bar{\tau}_P = \left( L_B + \frac{1 - \tau}{\tau} (\bar{L} - L_E) \right) \left/ \left( \frac{\alpha}{1 - \alpha} L_y - m + L_E \eta(L_B) m(1 - \gamma) + \frac{\bar{L} - L_E}{\tau} \right) \right. \].

The exact formula for $\bar{\tau}_P$ is

\[\bar{\tau}_P = \left( L_B + \frac{1 - \tau}{\tau} (\bar{L} - L_E) \right) \left/ \left( \frac{\alpha}{1 - \alpha} L_y - m + L_E \eta(L_B) m(1 - \gamma) + \frac{\bar{L} - L_E}{\tau} \right) \right. \].
In the previous section we have seen that in the setting with lump-sum taxes either 
\((\tau, \tilde{L}_B(\tau))\) or \((\tilde{\tau}, \tilde{L}_B(\tilde{\tau}))\) is optimal. These optimal policies are not feasible in general in the setting without lump-sum taxes as they would require that \((\tau_L, \tilde{\tau}_P, \tilde{L}_B(\tau))\), \((\tilde{\tau}_L, \tau_P, \hat{L}_B(\tau))\), respectively, exactly satisfy equation (12).

We now analyze optimal policies when the choice of \(L_B\) and \(\tau\) are not fully separable. To sharpen our results, we assume \(\tau_L = \tilde{\tau}_P = 0\) in this section which is necessary for \(L_B = 0\) to be feasible in general.

In the case that no lump-sum taxes and transfers are available and the choices of \(L_B\) and \(\tau\) cannot be separated fully, we obtain the first-order condition with respect to the optimal level of basic research as

\[
\frac{\partial L_y}{\partial L_B} + \frac{\partial L_y}{\partial L_E} \frac{\partial \tau}{\partial \tau} \frac{\partial \tau}{\partial L_B} = 0.
\]

The first partial derivative of the objective \(L_y\) with respect to \(L_B\) corresponds to the necessary condition for maximization of aggregate consumption when lump sum taxes and transfers are feasible (9). The second summand captures the effect of \(L_B\) on \(\tau\) implying that a marginal increase of basic research additionally influences the amount of entrepreneurs making use of it via the tax scheme. The sign of \(\frac{\partial L_y}{\partial L_E}\) corresponds to the sign of \(\eta(L_B)m(1-\gamma) - 1\) and hence depends on the level of basic research, as discussed earlier in connection with (10). For \(L_E > 0\), the term \(\frac{\partial L_E}{\partial \tau}\) is clearly positive as indicated by the equilibrium value of \(L_E\) given in Section 4.2. Finally, the last expression represents the marginal effect of basic research on \(\tau\) as implied by the government budget constraint. The sign of this effect depends on three interdependent factors: First, it depends on whether or not an increase in \(L_B\) requires additional funding. An increase in \(L_B\) might in principle generate additional tax returns in working hour equivalents that exceed the increase in \(L_B\). Second, it depends on how exactly the basic research is financed: via a change in labor income or via a change in profit taxes. And third, it is driven by the side of the Laffer curve at which the respective tax rate is located. Suppose for example that both tax measures are located at the increasing part of the Laffer curve and that an increase in basic research requires additional funding. Then, \(\frac{\partial \tau}{\partial L_B} > 0\) if basic research is financed via labor income taxes, while if basic research is financed via profit taxes the last expression transforms to \(\frac{\partial \tau}{\partial L_B} < 0\).

The insight we gain from (13) is as follows: If \(\frac{\partial L_y}{\partial L_E} > 0\), the marginal productivity of basic research is higher if financed in a way that increases \(\tau\) and vice versa if \(\frac{\partial L_y}{\partial L_E} < 0\).
The reason is that the first way of financing basic research increases the degree to which it is used while the second way of financing reduces the degree of usage. As we will show below, this leads to a pecking order of taxation to finance basic research. We refer to a *pecking order of taxation* if one tax is used, e.g. the labor income tax, and the other tax (e.g. profit tax) is levied only if an increase of the former cannot be used further to finance the public good. The latter case may occur if the prioritized tax reached its upper constitutional limit or is located at the decreasing part of the Laffer curve. In our model, there are two pecking orders of taxation: use labor income tax first and profit tax only if an increase in $\tau_L$ cannot be used to finance additional basic research and vice versa.

As discussed earlier, the sign of $\frac{\partial L}{\partial L_E}$ depends on $L_B$. In particular, $\frac{\partial L}{\partial L_E} > 0$ if and only if $L_B > L_{B,\text{min}}$. Hence, the labor income tax first pecking order should be used to finance $L_B > L_{B,\text{min}}$ and vice versa for $L_B < L_{B,\text{min}}$. We formalize these insights in Proposition 3.

**Proposition 3 (Pecking Order of Taxation)**

Consider a government that maximizes aggregate consumption and finances a given amount of basic research $L_B$ using $(\tau_L, \tau_P)$ as tax measures. Suppose that financing $L_B$ is feasible in a sense that $\exists$ a tax policy satisfying the government budget constraint and the positive profit condition for the final-good producer, condition (PPC). Then:

(i) If $L_B > L_{B,\text{min}}$, basic research should be financed using a pecking order with labor income tax first. In particular, $\tau_P > 0$ only if $TR$ cannot be increased further by a ceteris paribus increase of $\tau_L$.

(ii) If $L_B < L_{B,\text{min}}$, basic research should be financed using a pecking order with profit tax first. In particular, $\tau_L > 0$ only if $TR$ cannot be increased further by a ceteris paribus increase of $\tau_P$.

A proof of Proposition 3 is given in appendix A.7.

Proposition 3 characterizes the optimal tax policies to finance a given amount of basic research $L_B$. We now analyze when it is optimal to use the pecking order of taxation with labor income tax first if both, basic research and tax policy are simultaneously optimized.

Each pecking order implies an optimal amount of basic research given the order. We denote the optimal investment level when using the pecking order with labor income tax...
first and with profit tax first by $L_{B,\tau_L}$ and $L_{B,\tau_P}$, respectively, and the corresponding levels of $\tau$ by $\tau_{\tau_L}$ and $\tau_{\tau_P}$. Proposition 3 implies that the pecking order with labor income tax first is optimal only if $L_{B,\tau_L} > L_{B,min}$. In addition, optimality of this pecking order requires that it is preferable to the alternative pecking order, i.e. that $c(\tau_{\tau_L}, L_{B,\tau_L}) \geq c(\tau_{\tau_P}, L_{B,\tau_P})$. Otherwise, the pecking order with profit taxes first would be aggregate consumption maximizing.

In the following, we strengthen Assumption 2 to read:

**Assumption 3**

\[ \rho_0 m(1 - \gamma)b \leq 1. \]

Assumption 3 implies that $L_E = 0$ is also possible if no lump-sum taxes are available. It is for example satisfied if $\rho_0 = 0$, i.e. when basic research is a necessary condition for innovation.

We next adapt condition (PLS) as follows:

**Definition 3 (Positive Labor Saving R&D 2)**

When the pecking order of taxation with labor income tax first is used and the optimal amount of basic research given the order is provided, $L_B = L_{B,\tau_L}$, then labor savings are positive, i.e.

\[ -L_{B,\tau_L} + \left[ 1 - \frac{1}{\tau_{\tau_L} \eta(L_{B,\tau_L}) m(1 - \gamma)b} \right] \left[ \eta(L_{B,\tau_L}) m(1 - \gamma) - 1 \right] \geq 0. \]  

(PLS2)

Proposition 4 shows that condition (PLS2) is necessary and sufficient for the labor income tax first pecking order to be optimal.

**Proposition 4**

Suppose the government maximizes aggregate consumption using $(\tau_L, \tau_P, L_B)$ as policy instruments and let Assumptions 1 and 3 both be satisfied. Then:

(i) If and only if condition (PLS2) is satisfied, there will be an entrepreneurial economy with $\tau^* = \tau_{\tau_L}$, $L_B^* = L_{B,\tau_L}$ and $L_E = 1 - \frac{1}{\tau_{\tau_L} \eta(L_{B,\tau_L}) m(1 - \gamma)b}$.

(ii) Else, there will be a stagnant economy with $\tau_L^* = \tau_P^* = 0$, $L_B^* = 0$ and $L_E = 0$. 

21
6 The political economy of financing basic research

6.1 The set-up

So far, we have taken on the viewpoint of a government seeking to maximize aggregate consumption, without caring about distributional effects. Our analyses of the previous sections suggested that innovation stimulating investments in basic research should be complemented by a pecking order of taxation. Obviously, such innovation policies might have substantial distributional effects. In this section we explore such distributional effects and whether the optimal innovation policies are politically viable.

In our model, the government has two main policy areas at its discretion to foster entrepreneurship and innovation in the economy: basic research and tax policy. The latter policy instruments have direct distributional effects: labor income and profit taxes allow for redistribution of wealth between workers on the one hand and entrepreneurs and shareholders of the final good producer on the other hand.

Basic research investments have a direct effect on entrepreneurs by improving their chances of success. However, these direct effects are accompanied by important general equilibrium feedback effects. In our model, basic research investments support labor-saving technological progress in the intermediate good sectors. As a consequence of innovations, labor is set free in the intermediate good sectors and additionally supplied to final good production. This increases output and the profits of the representative final good producer but lowers wages.\footnote{These implications are consistent with the common trend across industrialized economies that labor income - in particular labor income of low skilled workers - as a share of total value added is decreasing over time. Timmer et al. (2010), for example, show that for the European Union worker’s share in total value added decreased from 72.1% in 1980 to 66.2% in 2005. In the US, this share decreased from 66.8% to 63.2%. At the same time, the share of high-skilled workers’ income in total value added increases rapidly over time: In the EU, this share increased from 8.3% in 1980 to 16.0% in 2005, whereas in the US it increased from 18.5% to 30.4%.}

Hence, while ownership in the final good producer is irrelevant for the consumption maximizing policies, it is crucial for the distributional effects of such policies.

In what follows, we assume that a fraction \( \frac{1}{2} < \mu < 1 \) of the population are workers who do not own shares in the final good producer.\footnote{The situation with a majority of the population being workers who are not engaged in the stock market is in line with empirical evidence on stock market participation rates. For example, Guiso et al. (2008) show for a selection of 12 OECD member states percentages of households that are engaged in the stock market. Even if indirect stockholdings are also considered, Sweden is the only country where a majority of households is engaged in the stock market with most countries having a share of} We will refer to these workers as \textit{pure}...
workers and ask how a majority of pure workers might affect the viability of policy changes that intend to move the economy towards an entrepreneurial economy. In doing so, we compare outcomes to a benchmark case with tax-neutrality and without basic research, i.e. with $\tau = 1$ and $L_B = 0$. We focus on circumstances for which Assumption 1 is satisfied, implying that the final good producer makes non-negative profits in our benchmark economy.

6.2 The political economy without lump-sum taxation

It is instructive to first consider the case when lump-sum taxation is not feasible. Without lump-sum taxes, the total compensation of a pure worker, $I_w$, is simply given by his after-tax wage:

$$I_w = (1 - \tau_L)w.$$  \hspace{1cm} (14)

From Proposition 1 we observe that wages are inversely related to labor used in final good production and hence to output of the final good. It follows that if pure workers were able to design policies to maximize their own well being, they would seek to minimize $\tau_L$ and aggregate consumption, potentially facing some trade-offs between the two endeavors. Hence, they would not opt for aggregate consumption stimulating policy changes that make the economy closer in nature to an entrepreneurial economy, i.e. with $\tau > 1$ and $\eta(L_B)m(1 - \gamma) > 1$.

An analogous result can be derived for a setting where these workers are confronted with a policy choice between an aggregate consumption increasing entrepreneurial economy and the benchmark economy. To see this, note that in the setting without lump-sum taxes, pure workers are clearly worse off upon introduction of an entrepreneurial economy which is beneficial for aggregate consumption. First, as noted above any aggregate consumption stimulating policy change decreases pre-tax wages earned by workers. Second, in an entrepreneurial economy investments in basic research are financed using the pecking order with labor income taxes first. Hence, labor income taxes increase vis-à-vis the benchmark case.\footnote{Note that in the benchmark economy without lump-sum taxes we have $\tau_L = \tau_P = 0$ as this is the only combination of tax rates satisfying $\tau = 1$ and yielding $L_B = 0$.} More generally, in the absence of lump-sum taxes and starting from the benchmark economy with $L_B = \tau_L = \tau_P = 0$ pure workers would not support any aggregate consumption stimulating investment in basic research. This follows immediately from observing that these workers would not even households that is engaged in the stock market of less than one third.
support such investments if they were financed purely via profit taxes as they would still decrease the pre-tax wages. We summarize our insights in the following Proposition:

**Proposition 5**

*In an economy without lump-sum taxes, pure workers would never prefer aggregate consumption stimulating investments in basic research over the benchmark economy with \( \tau_L = \tau_P = L_B = 0 \).*

### 6.3 The political economy with lump-sum taxation

#### 6.3.1 Workers’ compensation

With lump-sum taxes, a worker’s total compensation is composed of his after-tax wage plus his share in the lump-sum tax:

\[
I_{w,ls} = (1 - \tau_L)w - (1 - \tau_L)w \frac{L_B}{L} + \tau_L w \frac{\bar{L} - L_E - L_B}{L} + \tau_P \frac{\pi_y + \eta(L_B)L_E \pi_{xm}}{L} 
\]

Intuitively, the worker receives his wage plus the share in the final good producer’s profits that is allocated to him via the lump-sum taxes. Similarly, for every entrepreneur in the economy he receives a share in the difference between the expected tax returns from entrepreneurs and from workers. These incomes are reduced by the worker’s share in the cost of providing basic research.

From equation (15) we observe that the net effect of introducing an entrepreneurial economy on pure workers depends on several factors. Any policy change that is beneficial for aggregate consumption unambiguously increases \( \pi_y \) and decreases \( w \). In an entrepreneurial economy, entrepreneurship stimulating basic research is provided, hence \( L_B \) and \( L_E \) are larger than in the benchmark case. The net effect on aggregate entrepreneurial profits, \( \eta(L_B)L_E \pi_{xm} \) is undetermined. While the profit per successful entrepreneur, \( \pi_{xm} \), is decreasing, the number of entrepreneurs and the probability of successful innovation of each entrepreneur is increasing. Finally, as has been shown in section 5.1, in an entrepreneurial economy basic research is complemented by entrepreneurship stimulating tax policies, i.e. \( \tau_L = \overline{\tau}_L \) and \( \tau_P = \overline{\tau}_P \). The worker is adversely affected by the decrease in his wage income and by tax policies, in particular. Conversely, he participates in the increase in \( \pi_y \) via the lump-sum tax. This latter benefit is however reduced by any reduction in \( \tau_P \). Finally, the net effect of introducing
an entrepreneurial economy on $I_{w,ls}$ depends also on the benchmark income. In the setting with lump-sum taxes, this is not uniquely determined by $\tau$ and $L_B$. To see this, it is convenient to use the aggregate income identity in the expression for $I_{w,ls}$ given above and to substitute $\tau P \pi y + \eta(L_B)m(1-\gamma) \geq 1$, on $I_{w,ls}$ is undetermined. Among others, the bounds of the tax measures are crucial. We will therefore consider the cases with upper bounds on taxation only and with upper and lower bounds on taxation separately.\footnote{With no tax constraints, i.e. with any $\tau_L \in [0,1]$, $\tau_P \in [0,1]$ feasible, any $\tau$ can be implemented by choosing both, $\tau_L$ and $\tau_P$ arbitrarily close to 1 with $\tau_L > \tau_P$ if $\tau > 1$ and vice versa if $\tau < 1$. This implies that an entrepreneurial economy can be established with both $\tau_L$ and $\tau_P$ arbitrarily close to one, i.e. with aggregate income being effectively socialized and redistributed in equal shares to the population via the lump-sum taxes. If such an entrepreneurial economy is aggregate consumption maximizing, then it is also $I_{w,ls}$-optimal, unless workers who are not engaged in the stock market can earn a more than proportionate share from a smaller aggregate output. This requires positive entrepreneurship with entrepreneurs earning a less than proportionate share. Consider for example a benchmark economy with $\tau_L = \tau_P = L_B = 0$ and $\eta(L_B)m(1-\gamma) \leq 1$. Furthermore, let $b$ be large such that $L_E > 0$. Then, each entrepreneur earns in expectation $\eta(L_B)m(1-\gamma) < w$, the after-tax income of each worker. If finally profits of the final good producer are small in the benchmark economy and final good production is only slightly larger in the entrepreneurial economy, then workers who are not engaged in the stock market might prefer the described benchmark economy over the entrepreneurial economy.}

Now, all combinations of $\tau_L$ and $\tau_P$ satisfying $\tau_L = 1 - \frac{1-\tau_P}{\tau}$ yield the same $\tau$. Such $\tau$-preserving tax policy changes do not affect any variables other than the tax-measures themselves. Substituting $\tau_L$ by $1 - \frac{1-\tau_P}{\tau}$ and differentiating with respect to $\tau_P$ we get:

$$\frac{\partial I_{w,ls}}{\partial \tau_P} \bigg|_{\tau=\tau^*} = \frac{L_E}{\tau^* L} - \frac{L_x}{L} + \frac{\alpha}{1-\alpha} \frac{L_y}{L}. \quad (17)$$

This expression is not equal to 0 in general.

Taken together, in the setting with lump-sum taxes the net effect of introducing an entrepreneurial economy, i.e. an aggregate consumption optimal policy choice with $\tau_L = \tau_L$, $\tau_P = \tau_P$ and $L_B > 0$, s.t. $\eta(L_B)m(1-\gamma) > 1$, on $I_{w,ls}$ is undetermined.
the occupational choice by potential entrepreneurs. In addition, labor income taxes might affect the labor/leisure choice of workers and might hence be effectively bound from above.\textsuperscript{20} Lower bounds on profit taxes, in particular, might be demanded by the international community. The European Council of Economics and Finance Ministers, for example, has agreed upon a code of conduct for business taxation which is intended to tackle harmful competition in the field of business taxation (European Union, 1998). Although this code of conduct does not define explicit lower bounds on taxation and is not legally binding, it still represents a considerable political commitment not to have extremely low tax rates on profits.

6.3.2 No lower bounds on taxation

Suppose that tax rates are bound from above, but not from below, i.e. any $\tau_L \in [0, \bar{\tau}_L]$, $\tau_P \in [0, \bar{\tau}_P]$ with $\bar{\tau}_L, \bar{\tau}_P < 1$ is admissible and let an entrepreneurial economy be aggregate consumption maximizing. From our discussions in section 5.1 we know that $\tau_P = 0$ and $\tau_L = \bar{\tau}_L$ in the entrepreneurial economy. It follows that pure workers cannot participate in the increase in profits earned by final good producers nor can they benefit from entrepreneurial profits. Furthermore, pre-tax wages and hence after-tax wages are lower than in the benchmark economy with $\tau = 1$ and $L_B = 0$, as discussed above. We conclude that workers who are not engaged in the stock market are clearly worse-off in the entrepreneurial economy than in the benchmark economy and summarize these insights in the following Proposition:

**Proposition 6**

Consider an economy with lump-sum taxes and with upper but no lower bounds on taxation, i.e. where any $\tau_L \in [0, \bar{\tau}_L]$, $\tau_P \in [0, \bar{\tau}_P]$ with $\bar{\tau}_L, \bar{\tau}_P < 1$ is admissible. Then an aggregate consumption optimal policy change from a benchmark economy with $\tau = 1$ and $L_B = 0$ to an entrepreneurial economy with $\tau_L = \bar{\tau}_L$, $\tau_P = \bar{\tau}_P$ and $L_B$ s.t. $\eta (L_B) m (1 - \gamma) > 1$ is never supported by the majority of pure workers.

Proposition 6 states that in the absence of lower bounds on taxation, pure workers will prevent the introduction of an entrepreneurial economy when confronted with a choice between an entrepreneurial and the benchmark economy. Obviously, this also implies that pure workers would never opt for an aggregate consumption optimal en-

\textsuperscript{20}In an open economy, the government might also be confronted with harmful supply effects associated with high profit taxes that are not considered here and that may give rise to effective upper bounds on profit taxes.
entrepreneurial economy if they were the political agenda setter. We next analyze whether these results also prevail in the presence of upper and lower bounds on taxation. Alternatively, we might ask whether in the case of an aggregate consumption maximizing entrepreneurial economy not being politically viable it might be possible to implement (constrained optimal second best) policy changes that make the economy closer in nature to an entrepreneurial economy when compared to the benchmark economy, i.e. to set policies such that $\tau \geq 1$ and $\eta(L_B)m(1-\gamma) > 1$.

### 6.3.3 Upper and lower bounds on taxation

Suppose that tax rates are bound from below and from above, i.e. any $\tau_L \in [\tau_L, \tau_L]$, $\tau_P \in [\tau_P, \tau_P]$ with $\tau_L, \tau_P > 0$, $\tau_L, \tau_P < 1$ is admissible. Then, in the scenario with lump-sum taxes, an entrepreneurial economy according to section 5.1 is politically viable if bounds on taxation are sufficiently restrictive. To illustrate this, we consider the polar case where $\tau_L = \tau_P \approx 1$ is the only admissible tax policy, i.e. an economy where effectively all income is socialized and redistributed with equal shares to the population. In such an economy, the society unanimously agrees on maximizing aggregate consumption and hence supports output stimulating investments in basic research.

By continuity of $I_{w,ls}$ in $\tau_P$ it follows then that in an economy with lump-sum taxes a movement in direction of an entrepreneurial economy, i.e. a policy choice of $L_B$ s.t. $\eta(L_B)m(1-\gamma) > 1$ and of $\tau > 1$, might be politically viable in a sense that it might be designed in a way to satisfy the constraint that pure workers are no worse-off than in the benchmark economy.

We next analyze in more detail when the entrepreneurial economy may be politically viable. Lemma 4 below states that pure workers prefer the benchmark over the entrepreneurial economy if both yield the same level of final good production.

**Lemma 4**

Suppose that in the entrepreneurial economy with $\tau \geq 1$ and $\eta(L_B)m(1-\gamma) > 1$ final good production is just as large as in the benchmark economy. Then pure workers strictly prefer the benchmark economy over the entrepreneurial economy.

A formal proof of Lemma 4 is given in appendix A.9. Intuitively, if final good production is just as large in the entrepreneurial economy as in the benchmark economy, so are wages and profits of the final good producer. However, due to changes in tax policies after-tax wages are lower and after tax profits of the final good producer are
higher than in the benchmark economy. Furthermore, for \( \eta(L_B)m(1-\gamma) > 1 \) and \( \tau_P \leq \tau_L \) expected after-tax profits per entrepreneur are larger than the after-tax wage income he could earn on the labor market. It follows that workers are also harmed by the increase in entrepreneurship and hence they would prefer the benchmark economy.

Lemma 4 implies that in order for the entrepreneurial economy to be politically viable it needs to be sufficiently beneficial for final good production. We note that this requires that basic research is sufficiently productive. From our discussions in section 5.1 we know that for any level \( L_B \) such that \( \eta(L_B)m(1-\gamma) > 1 \), final good production increases in \( \tau \), so final good production in the entrepreneurial economy would increase as \( \tau_P \) decreases and hence \( \tau \) increases. However, from Proposition 6 we know that at some point this would undermine political support for the entrepreneurial economy.

Lemma 5 re-considers \( I_{w,s} \) for \( \tau \) and \( L_B \) given. It states that for any policy with \( \eta(L_B)m(1-\gamma) > 1 \) and \( \tau \geq 1 \), i.e. in any entrepreneurial economy, pure workers get the highest income if \( \tau \) is implemented in the \( \tau_P \)-maximizing way.

**Lemma 5**

*In an economy with lump-sum taxes and where increasing the number of entrepreneurs adds to aggregate consumption, i.e. where \( \eta(L_B)m(1-\gamma) > 1 \), total income given \( \tau \geq 1 \) of a pure worker is maximized if this \( \tau \) is implemented in the \( \tau_P \)-maximizing way.*

A formal proof of Lemma 5 can be found from appendix A.10. Intuitively, a \( \tau \)-preserving increase of \( \tau_L \) and \( \tau_P \) makes the economy closer in nature to the polar-case discussed above and in particular it provides a higher share of the final good producer’s profits to the workers. This effect is beneficial for the worker. However, the change in tax rates also affects the difference between the expected tax returns from entrepreneurs and from workers, a share of which the worker receives via the lump-sum taxes. If \( \eta(L_B)m(1-\gamma) > 1 \) and if \( \tau \geq 1 \), then this difference increases as both tax variables are increased in a \( \tau \)-preserving manner.

In the entrepreneurial economy \( \tau \) is at its upper bound, \( \bar{\tau} \), i.e. \( \tau_L = \bar{\tau} \) and \( \tau_P = \bar{\tau}_P \). Lemma 5 implies that an entrepreneurial economy with a given \( \bar{\tau} \) is the more likely to be politically viable, the higher are the underlying tax bounds for labor income and profit taxes. We summarize our insights from the preceding discussion in the following Proposition:

\footnote{For the case of \( \bar{\tau}_P = \bar{\tau}_L \) tax policies are just the same in the entrepreneurial economy as in the benchmark economy. In this case, so are after tax labor income and final good producer’s profits.}
Proposition 7

Consider an economy with lump-sum taxes and with upper and lower bounds on taxation, i.e. where any \( \tau_L \in [\underline{\tau}_L, \overline{\tau}_L] \), \( \tau_P \in [\underline{\tau}_P, \overline{\tau}_P] \) with \( \underline{\tau}_L, \overline{\tau}_L, \underline{\tau}_P, \overline{\tau}_P > 0 \), \( \overline{\tau}_L, \overline{\tau}_P < 1 \) is admissible. An aggregate consumption optimal policy change from a benchmark economy with \( \tau = 1 \) and \( L_B = 0 \) to an entrepreneurial economy with \( \tau_L = \overline{\tau}_L, \tau_P = \overline{\tau}_P \) and \( L_B \) s.t. \( \eta(L_B)m(1 - \gamma) > 1 \)

(i) may be supported by the majority of pure workers if tax constraints are sufficiently restrictive and the entrepreneurial economy is sufficiently beneficial for final good production when compared to the benchmark economy.

(ii) For any given \( \overline{\tau} \), such a policy change is the more likely to be supported by pure workers, the higher are the supporting \( \overline{\tau}_L \) and \( \overline{\tau}_P \).

So far, we have analyzed conditions such that an entrepreneurial economy would be accepted by the majority of pure workers if they were confronted with the alternative of this policy choice and a benchmark economy with \( L_B = 0 \) and \( \tau = 1 \). We now consider what happens if these workers can themselves design policies. Clearly, these workers would choose tax policies and basic research investments to maximize their income given by equation (15) for the scenario with lump-sum taxes.

Proposition 6 immediately implies that in the absence of lower bounds on taxation these workers would never opt for an entrepreneurial economy. Trivially, if they do not prefer the entrepreneurial economy over the benchmark, it cannot be \( I_{w,ls} \)-optimal. Proposition 8 also allows for lower bounds on taxation. It shows that even in this case pure workers would not opt for an aggregate consumption optimal entrepreneurial economy if they were the political agenda setter.

Proposition 8

An aggregate consumption optimal entrepreneurial economy with positive investment in basic research is never \( I_{w,ls} \)-optimal.

A formal proof of Proposition 8 is given in Appendix A.11.

6.3.4 Discussion

In this section we have analyzed the political economy of financing basic research investments, mainly for the case with lump-sum taxes. From Proposition 6 we know that in the absence of lower bounds on taxation, the introduction of an entrepreneurial
economy that is optimal in aggregate does harm workers who are not engaged in the stock market. It follows that such policies may not be politically viable if these pure workers form a majority of the population. This is not true in general for a situation with upper and lower bounds on taxation as shown in Proposition 7. Here, pure workers may prefer an entrepreneurial economy over a benchmark economy with $\tau = 1$ and $L_B = 0$ if the entrepreneurial economy is sufficiently beneficial for final good production and tax bounds are sufficiently restrictive. However, from Proposition 8 we learn that such policy choices are no longer politically viable if the majority of pure workers are able to set policies themselves. Hence, for the feasibility of an aggregate consumption optimal entrepreneurial economy it matters who is the political agenda setter.

If pure workers would not support an entrepreneurial economy when confronted with a choice between this economy and a benchmark economy with $\tau = 1$ and $L_B = 0$, then it may be constrained optimal to realize policy changes that make the economy closer in nature to an entrepreneurial economy when compared to the benchmark economy, i.e. to implement a policy with $\tau \geq 1$ and $\eta(L_B)m(1 - \gamma) > 1$. From Proposition 7 we learn that in the scenario with lump-sum taxes such an aggregate consumption increasing policy choice may in principle be supported by pure workers. Again, Lemma 5 implies that in order to increase political support for policy changes that bring the economy closer in nature to an entrepreneurial economy, $\tau$ should be implemented in the $\tau_P$-maximizing way, i.e. at least one tax measure should be located at its upper bound. Hence, the larger $\tau_L$ and $\tau_P$, the larger is $\tau_P$ in the $\tau_P$-maximizing implementation of $\tau$. It follows that an aggregate consumption increasing policy change towards an entrepreneurial economy is the more likely to be supported by the majority of pure workers the higher are $\tau_L$ and $\tau_P$. Furthermore, if such a policy change can be implemented, then it can be designed the more pronounced, i.e. with a higher $\tau$, the higher are $\tau_L$ and $\tau_P$.

In our analysis, we have assumed that profit taxes are the same in the intermediate and in the final good sector. Of course, within our model, if distributional reasons prevent the existence of an entrepreneurial economy, it might be optimal to tax profits in the final good sector differently from those in the intermediate good sector. Such tax discrimination might be interpreted as different tax treatment of corporate versus non-corporate income found in the US, for example. It would allow compensating workers with the tax revenues from the beneficiaries of the final good producer’s profits.
Clearly, such compensations would also affect the occupational choice by potential entrepreneurs and hence they would have a deeper impact on optimal policy choices. For any choice of $L_B$ such that $\eta(L_B)m(1 - \gamma) > 1$, it would be optimal to tax profits of the final good producer at the highest possible rate, make workers just indifferent between the status quo and the new policy choice (in order for the policy choice to be politically viable) and use the remainder of the output as incentives to entrepreneurs.

If tax differentiation between entrepreneurs and final good producers is not possible due to asymmetric information, for example, then progressive taxes might also be used to support the implementation of an entrepreneurial economy. Of course, this would only be a viable option if profits of the representative final good producer exceeded those of the successful entrepreneur. From Proposition 1 we conclude that this is the case if and only if

$$L_y > (2 - \gamma) \frac{1 - \alpha}{\alpha} m.$$

7 Conclusions

We have outlined a rationale for a pecking order of taxation to finance basic research investments, thus presenting an important new perspective on the theory of optimal income taxation. Moreover, we have characterized the conditions under which the optimal taxation scheme is politically viable. In particular, our political economy analysis suggests that optimal policies might harm workers, if they are not engaged in the stock-market. We have shown that an entrepreneurship and innovation stimulating policy might therefore not be politically viable if these workers formed a majority of the population.

Given its importance for future economic growth and prosperity, further analyses might take on different perspectives on the political economy of financing basic research. For example, it may be interesting to analyze optimal policies from the point of view of entrepreneurs or shareholders. Also, possible ways of compensating workers for resulting welfare losses deserve further scrutiny. On a similar note, our analysis of optimal financing of basic research investments might also be further linked to the theory on optimal taxation in the tradition of Mirrlees (1971). With concave utilities and traditional supply side effects of labor income taxation, optimal policies would account for losses in aggregate utility from income inequality and for potential adverse effects on labor supply. These additional equity- / efficiency trade-offs might push optimal...
tax policies towards a more egalitarian economy, thus stimulating political support for welfare optimal policies. In the presence of incomplete markets, concave utilities might also allow for additional beneficial effects of basic research on entrepreneurship and thus innovation in the economy: next to fostering expected profits from being entrepreneur, basic research affects associated idiosyncratic risks.
A Proofs

A.1 Proof of Lemma 1

We prove each part of Lemma 1 in turn.

(i) We consider innovative and non-innovative intermediate good producer separately. Intermediate-goods in non-innovative industries are produced using the freely available standard technology. Perfect competition implies that these intermediate goods are sold at cost in equilibrium, i.e. non-innovative intermediate good producer will offer their goods at price \( p(i) = mw \).

The cost of production of innovative intermediate good producer are reduced to \( \gamma mw \). These firms are still confronted with competition from non-innovative intermediate good producers in their industry. Taken together, this implies that an innovative intermediate good producer will charge a price \( p(i) = \delta mw \) with \( \delta_i \in [\gamma, 1] \). We now show by contradiction that \( \delta_i \in [\gamma, 1) \) cannot be optimal. We show that there do not exist symmetric equilibria such that all innovative intermediate good producer charge the common price \( p(i) = \delta mw \), with \( \delta \in [\gamma, 1) \) and leave it to the reader to verify that no non-symmetric equilibrium exists with \( \delta_i < 1 \) for some \( i \).

Let us define \( \tilde{X} := \int_{|p(i) = \delta mw|} x(i)^{\alpha} di \) and \( \hat{X} := \int_{|p(i) = mw|} x(i)^{\alpha} di \). This allows us to write the maximization problem of the final good producer as

\[
\max_{L_y, X} \pi_y = L_1^{1-\alpha}(\hat{X} + \tilde{X}) - wL_y - \delta mw\hat{X} - mw\tilde{X} = \tilde{X}(L_y^{1-\alpha} - \delta mw) + \hat{X}(L_y^{1-\alpha} - mw) - wL_y. \quad (18)
\]

\( \delta < 1 \) implies that \( L_y^{1-\alpha} - \delta mw > 0 \) is a necessary condition for the final goods producer to operate making non-negative profits. \( L_y^{1-\alpha} - \delta mw \) is the net marginal benefit of the final good producer from using intermediate good \( x(i) \) offered at price \( p(i) = \delta mw \) in production. Hence, \( L_y^{1-\alpha} - \delta mw > 0 \) implies first, that if the final good producer is operating he always demands \( x(i) = 1 \) of every intermediate offered at price \( p(i) = \delta mw \). And second, that the innovative intermediate good producer \( i \) would want to set a price \( \tilde{p}(i) = \delta mw + \epsilon \), \( \epsilon > 0 \) but small, such that \( L_y^{1-\alpha} - \tilde{p}(i) > 0 \). Then the net marginal benefit of the final good producer from using intermediate good \( x(i) \) in production remains positive. Furthermore, given that each intermediate good producer has measure 0, it would not affect the profitability of the representative final
good firm. Hence, the final good firm would still demand \( x(i) = 1 \), a contradiction to \( p(i) = \delta mw \) being profit maximizing for intermediate good producer \( i \).

The contradiction establishes the result.

(ii) Let us define \( X := \int_0^1 x(i)^{\alpha} di \). \( X \) assumes the value 0 if \( x(i) = 0 \forall i \), 1 if \( x(i) = 1 \forall i \), and values between 0 and 1 only if a subset of the varieties is used. If \( p_i = mw \forall i \), the maximization problem of the final good producer can be written as

\[
\max_{L_y, X} \pi_y = L_y^{1-\alpha} X - wL_y - mwX = X(L_y^{1-\alpha} - mw) - wL_y. 
\]  

(19)

Hence, the profit function is linear in \( X \). A necessary condition for non-negative profits is \( L_y^{1-\alpha} - mw > 0 \). As a consequence, if it is optimal for the final goods producer to operate, i.e. to demand \( X > 0 \) then it must hold that \( L_y^{1-\alpha} - mw > 0 \) and hence profits are maximized by setting \( X = 1 \).

Finally, if \( x(i) = 1 \forall i \), then profit maximizing labor demand of the final good producer is given by

\[
(1 - \alpha)L_y^{-\alpha} - w = 0 \quad \text{or} \quad L_y = \left( \frac{1 - \alpha}{w} \right)^{\frac{1}{\alpha}}. 
\]

A.2 Proof of Lemma 2

The final good producer operates in equilibrium if and only if he can earn non-negative profits from doing so. From Lemma 1 we know, that if the final good producer operates, he uses all intermediate goods in production which are offered at a price \( p = mw \). Hence, if the final good producer operates, he earns profit

\[
\pi_y = L_y^{1-\alpha} - wL_y - wm. 
\]

Inserting the equilibrium wage rate (7) yields:

\[
\pi_y = L_y^{-\alpha} (\alpha L_y - (1 - \alpha)m). 
\]

Hence, the profits of the final good producer are non-negative if and only if

\[
L_y \geq m \frac{1 - \alpha}{\alpha}. 
\]  

(20)

Using \( L_y = \bar{L} - L_B - m + L_E[\eta(L_B)m(1 - \gamma) - 1] \) and replacing \( L_E \) with its equilibrium value given in (4), we obtain condition (PPC).
A.3 Proof of Proposition 1

From Lemmas 1 and 2 we know that if condition (PPC) is satisfied, the final good producer is operating and he uses all varieties in production. Conversely, if condition (PPC) is not satisfied, he is not operating and \( L_E^c = L_x^c = L_y^c = 0 \) and zero profits follow immediately. It remains to show that in case (i) the other variables take on the unique equilibrium values stated in the Proposition.

(i) Conditions (1), (2), and (4) have been derived in the main text. Condition (3) follows from using \( L_E^c \) and \( L_x^c \) in the labor market clearing condition. Combining \( w^c \) with the observation that \( p(i) = mw \forall i \) yields condition (5). Condition (6) follows from \( x(i) = 1 \forall i \) and the production technology in the final goods sector. Using \( y^c \), \( p(i)^c \), \( x(i)^c \), and \( w^c \) in the profit function of the final good producer yields condition (7). Finally, condition (8) follows from using \( w^c \) in the expression for profits of a monopolistic intermediate good producer.

A.4 Proof of Lemma 3

According to condition (PPC), profits of the representative final good producer are non-negative if

\[
\hat{L} - L_B + L_E[\eta(L_B)m(1 - \gamma) - 1] \geq \frac{m}{\alpha}. \tag{21}
\]

The left-hand side of condition (21) represents \( L_y + m \). As we will show at the onset of section 5.1, the government effectively maximizes labor available for final good production. Hence, if condition (21) is satisfied for some values \( L_B \) and \( \tau \), then it must also be satisfied for the aggregate consumption maximizing choices \( L_B^* \) and \( \tau^* \). If Assumption 1 holds, then condition (21) is satisfied for a choice of \( L_B = 0 \) and \( \tau = 1 \). This completes the proof.

A.5 Proof of Proposition 2

As shown in the main text, the government either chooses \( (\tau, \tilde{L}_B(\tau)) \) or \( (\tau, \tilde{L}_B(\tau)) \). We consider two separate cases: (i) \( \tilde{L}_B(\tau) > 0 \) and (ii) \( \tilde{L}_B(\tau) = 0 \). We show that if \( \tilde{L}_B(\tau) > 0 \) condition (PLS) is always satisfied and that the government opts for \( (\tau, \tilde{L}_B(\tau)) \). We then show that for \( \tilde{L}_B(\tau) = 0 \) the government prefers \( (\tau, \tilde{L}_B(\tau)) \) over \( (\tau, \tilde{L}_B(\tau)) \) if and only if condition (PLS) is satisfied. The value for \( L_E \) follows from
the considerations in the main text.

(i) Suppose \( \tilde{L}_B(\tau) > 0 \). Then
\[
-\tilde{L}_B(\tau) + \left[ 1 - \frac{1}{\tau \eta(\tilde{L}_B(\tau)) m(1 - \gamma) b} \right] \left[ \eta(\tilde{L}_B(\tau)) m(1 - \gamma) - 1 \right] \geq 0.
\]
We know that \( \tilde{L}_B(\tau) > 0 \) can only be optimal if \( L_E > 0 \), allowing us to conclude that
\[
\eta(\tilde{L}_B(\tau)) m(1 - \gamma) - 1 > 0.
\]
Hence, it follows that
\[
c(\tau, \tilde{L}_B(\tau)) > c(\tau, 0)
\]
and from the optimality of \( \tilde{L}_B(\tau) \) we know
\[
c(\tau, \tilde{L}_B(\tau)) \geq c(\tilde{L}_B(\tau), \tilde{L}_B(\tau)) > c(\tilde{L}_B(\tau), \tau) \geq c(\tau, 0).
\]
\[
c(\tau, \tilde{L}_B(\tau)) > c(\tau, \tilde{L}_B(\tau)) \text{ implies that } (\tau, \tilde{L}_B(\tau)) \text{ is optimal. } c(\tau, \tilde{L}_B(\tau)) > c(\tau, 0)
\]
implies that condition (PLS) is satisfied.

(ii) We now turn to the case \( \tilde{L}_B(\tau) = 0 \). Then by Assumption 2 \( L_E(\tau, \tilde{L}_B(\tau)) = 0 \) and final good production is given by
\[
c(\tau, \tilde{L}_B(\tau)) = (\bar{L} - m)^{1-\alpha}.
\]
Hence, the government prefers \((\tau, \tilde{L}_B(\tau))\) over \((\tau, \tilde{L}_B(\tau))\) if and only if
\[
\bar{L} - \tilde{L}_B(\tau) - m + \left[ 1 - \frac{1}{\tau \eta(\tilde{L}_B(\tau)) m(1 - \gamma) b} \right] \left[ \eta(\tilde{L}_B(\tau)) m(1 - \gamma) - 1 \right] \geq \bar{L} - m.
\]
Simplifying terms yields condition (PLS).

A.6 Proof of Corollary 1

By Proposition 2 there will be an entrepreneurial economy if and only if condition (PLS) is satisfied. Now, in response to a change in \( m, b, \tau, \) or \( \gamma \), the government could
leave $\tilde{L}_B(\tau)$ unaffected. Hence, if it opts for a $\hat{L}_B(\tau) \neq \tilde{L}_B(\tau)$, then we must have $c(\tau, \hat{L}_B(\tau)) \geq c(\tau, \tilde{L}_B(\tau))$, which implies

$$-\hat{L}_B(\tau) + \left[1 - \frac{1}{\tau \eta(\tilde{L}_B(\tau)) m(1 - \gamma)b} \right] \left[\eta(\hat{L}_B(\tau)) m(1 - \gamma) - 1\right] \geq$$

$$-\tilde{L}_B(\tau) + \left[1 - \frac{1}{\tau \eta(\tilde{L}_B(\tau)) m(1 - \gamma)b} \right] \left[\eta(\tilde{L}_B(\tau)) m(1 - \gamma) - 1\right].$$

A proof then follows from the fact that for a constant $\tilde{L}_B(\tau)$

$$\left[1 - \frac{1}{\tau \eta(\tilde{L}_B(\tau)) m(1 - \gamma)b} \right] \left[\eta(\tilde{L}_B(\tau)) m(1 - \gamma) - 1\right]$$

is increasing in $m$, $b$, and $\tau$ and decreasing in $\gamma$.

**A.7 Proof of Proposition 3**

We proof Proposition 3 (i) by contradiction. Part (ii) can be shown using a similar argument.

(i) We first note that $L_B > L_{B,\text{min}}$ implies that if $(\hat{\tau}_L, \hat{\tau}_P, \hat{L}_B)$ satisfies condition (PPC), then so does any $(\tau'_L, \tau'_P, \hat{L}_B)$ satisfying $\frac{1 - \tau'_P}{1 - \hat{\tau}_L} \geq \frac{1 - \hat{\tau}_P}{1 - \tau'_L}$.

Let $TR(\tau_L, \tau_P, L_B)$ denote tax revenues in working hour equivalents given $\tau_L$, $\tau_P$, and $L_B$. Consider a policy choice $(\hat{\tau}_L, \hat{\tau}_P, \hat{L}_B)$, such that $\hat{\tau}_P > 0$, $\hat{L}_B > L_{B,\text{min}}$, and $\exists \tilde{\tau}_L > \hat{\tau}_L$ such that $TR(\hat{\tau}_L, \hat{\tau}_P, \hat{L}_B) > TR(\hat{\tau}_L, \hat{\tau}_P, \hat{L}_B)$. Furthermore, let $(\hat{\tau}_L, \hat{\tau}_P, \hat{L}_B)$ satisfy condition (PPC). Then, by continuity of $TR$ in $\tau_L$ and $\tau_P$, it is possible to finance $\hat{L}_B$ using some alternative financing scheme $(\tau'_L, \tau'_P)$ satisfying:

$$\tau'_L = \hat{\tau}_L + \Delta_1, \quad \Delta_1 \geq 0,$$

$$\tau'_P = \hat{\tau}_P - \Delta_2, \quad \Delta_2 \geq 0,$$

$$\frac{1 - \tau'_P}{1 - \tau'_L} > \frac{1 - \hat{\tau}_P}{1 - \hat{\tau}_L}.$$

In particular, depending on whether $\frac{\partial TR}{\partial \tau_L} \leq 0$ and $\frac{\partial TR}{\partial \tau_P} \leq 0$, the following alternative financing schemes satisfy the conditions above:
1. Suppose \( \frac{\partial TR}{\partial \tau_L} \bigg|_{\tau_L=\hat{\tau}_L, \tau_P=\hat{\tau}_P} < 0 \) or \( \left( \frac{\partial TR}{\partial \tau_P} \bigg|_{\tau_P=\hat{\tau}_P} = 0 \right) \) and \( \frac{\partial^2 TR}{(\partial \tau_L)^2} \bigg|_{\tau_L=\hat{\tau}_L} < 0 < 0 \). Then by the existence of \( \hat{\tau}_L > \hat{\tau}_P \) such that \( TR(\hat{\tau}_L, \hat{\tau}_P) > TR(\hat{\tau}_L, \hat{\tau}_P, \hat{L}_B) \). We conclude that \( \exists \Delta_1 > 0 \) and \( \Delta_2 = 0 \) satisfying the conditions stated above.

2. Suppose \( \frac{\partial TR}{\partial \tau_P} \bigg|_{\tau_L=\hat{\tau}_L, \tau_P=\hat{\tau}_P} < 0 \) or \( \left( \frac{\partial TR}{\partial \tau_L} \bigg|_{\tau_L=\hat{\tau}_L} = 0 \right) \) and \( \frac{\partial^2 TR}{(\partial \tau_P)^2} \bigg|_{\tau_P=\hat{\tau}_P} > 0 \). We show that given \( \hat{\tau}_L \) and \( \hat{L}_B \), \( TR \) is minimized at \( \tau_P = 0 \). Then it follows from continuity of \( TR \) in \( \tau_P \) that \( \exists \tau'_P < \hat{\tau}_P \) satisfying \( TR(\hat{\tau}_L, \tau'_P, \hat{L}_B) = TR(\hat{\tau}_L, \hat{\tau}_P, \hat{L}_B) \). Hence, \( \exists \Delta_1 = 0 \) and \( \Delta_2 > 0 \) satisfying the conditions stated above.

To show that given \( \hat{\tau}_L \) and \( \hat{L}_B \), \( TR \) is minimized at \( \tau_P = 0 \), note first that \( L_E \) is non-increasing in \( \tau_P \). Hence, the term \((\hat{L} - L_E)\hat{\tau}_L \) is non-decreasing in \( \tau_P \). Furthermore, all \( \tau_P < \hat{\tau}_P \) satisfy condition (PPC) and hence we have \( \tau_P \left[ \frac{\partial}{\partial \tau} L_B - m + L_E \eta(L_B) m(1 - \gamma) \right] \geq 0 \). We conclude that \( TR \) is minimized at \( \tau_P = 0 \).

3. Finally, suppose \( \frac{\partial TR}{\partial \tau_L} \bigg|_{\tau_L=\hat{\tau}_L, \tau_P=\hat{\tau}_P} > 0 \) or \( \left( \frac{\partial TR}{\partial \tau_P} \bigg|_{\tau_P=\hat{\tau}_P} = 0 \right) \) and \( \frac{\partial^2 TR}{(\partial \tau_L)^2} \bigg|_{\tau_L=\hat{\tau}_L} > 0 \). Then by continuity of \( TR \) in \( \tau_L \) and \( \tau_P \exists \tau_L > \hat{\tau}_L \) and \( \tau_P < \hat{\tau}_P \) satisfying \( TR(\tau'_L, \tau'_P, \hat{L}_B) = TR(\hat{\tau}_L, \hat{\tau}_P, \hat{L}_B) \). We conclude that \( \exists \Delta_1 > 0 \) and \( \Delta_2 > 0 \) satisfying the conditions stated above.

\( \frac{1-\hat{\tau}_L}{1-\hat{\tau}_P} > \frac{1-\hat{\tau}_P}{1-\hat{\tau}_L} \) implies \( L'_E > \hat{L}_E \). Since \( \hat{L}_B > L_B, \min \) and hence \( \eta(\hat{L}_B) m(1 - \gamma) > 1 \) it follows \( L'_y > \hat{L}_y \), a contradiction to \( (\hat{\tau}_L, \hat{\tau}_P, \hat{L}_B) \) being optimal.

The contradiction establishes the result.

### A.8 Proof of Proposition 4

Analogous to proof of Proposition 2.

\(^{22}\)We note that it cannot ever be optimal to finance \( L_B > 0 \) in a way yielding \( L_E = 0 \).
A.9 Proof of Lemma 4

We start by rewriting the expression for $I_{w,ls}$ given in equation (15) as follows:

$$I_{w,ls} = \frac{w}{L} \left[ \bar{L} - L_B + \tau_L \bar{L}_E + \tau_P \eta(L_B) m(1 - \gamma) L_E \right] + \frac{\bar{\tau}_P \pi_y}{L}$$

$$= \frac{w}{L} \left\{ \bar{L} - L_B + L_E [\eta(L_B) m(1 - \gamma) - 1] - (1 - \tau_P) L_E [\eta(L_B) m(1 - \gamma) - 1] \right\}$$

$$+ (\tau_P - \tau_L) L_E \} + \frac{\tau_P \pi_y}{L}$$

We subsequently denote variable values in the entrepreneurial and the benchmark economy by a superscript “ee” and “be”, respectively. Pure workers (strictly) prefer the entrepreneurial economy if and only if:

$$I^{be}_{w,ls} - I^{ee}_{w,ls} > 0$$

$L^{be}_y = L^{ee}_y$ implies that $w^{be} = w^{ee}$ and $\pi_y^{be} = \pi_y^{ee}$. Furthermore from $L_y = \bar{L} - L_B - m + L_E [\eta(L_B) m(1 - \gamma) - 1]$ we know that $-L^{be}_B + L^{be}_E [\eta(L^{be}_B) m(1 - \gamma) - 1] = -L^{ee}_B + L^{ee}_E [\eta(L^{ee}_B) m(1 - \gamma) - 1]$. Finally, using $\tau^{ee}_L = \bar{\tau}_L$, $\tau^{ee}_P = \bar{\tau}_P$, $\tau^{be}_L = \tau^{be}_P$, and $L^{be}_B = 0$ we get:

$$I^{be}_{w,ls} - I^{ee}_{w,ls} = \frac{w^{be}}{L} \left\{ (1 - \bar{\tau}_P) L^{ee}_E [\eta(L^{ee}_B) m(1 - \gamma) - 1] - (1 - \tau^{be}_P) L^{be}_E [\eta(L^{be}_B) m(1 - \gamma) - 1] \right\}$$

$$- (\bar{\tau}_P - \tau_L) L^{ee}_E \} + \frac{(\tau^{be}_P - \tau_L) \pi_y^{be}}{L}$$

Now, $(1 - \bar{\tau}_P) L^{ee}_E [\eta(L^{ee}_B) m(1 - \gamma) - 1] > (1 - \tau^{be}_P) L^{be}_E [\eta(L^{be}_B) m(1 - \gamma) - 1]$, $(\bar{\tau}_P - \tau_L) L^{ee}_E \leq 0$ with the inequality being strict as long as $\bar{\tau}_P < \tau_L$, and $(\tau^{be}_P - \tau_L) \pi_y^{be} \geq 0$ with the inequality being strict as long as $\tau^{be}_P \neq \tau_P$ and $\pi_y^{be} > 0$. It follows that $I^{be}_{w,ls} - I^{ee}_{w,ls} > 0.$

This completes the proof.

A.10 Proof of Lemma 5

From equation (17) we know:

$$\frac{\partial I_{w,ls}}{\partial \tau_P} \bigg|_{\tau = \tau^*} = \frac{L_E}{\tau^* L} - \frac{L_x}{L} + \frac{\alpha}{1 - \alpha} \frac{L_y}{L} .$$

Using the equilibrium value for $L_x$ given in Proposition 1 yields:

$$\frac{\partial I_{w,ls}}{\partial \tau_P} \bigg|_{\tau = \tau^*} = \frac{L_E}{\tau^* L} - \frac{m - L_E \eta(L_B) m(1 - \gamma)}{L} + \frac{\alpha}{1 - \alpha} \frac{L_y}{L} .$$
Now, \( \frac{\alpha L_y}{1 - \alpha} - \frac{w}{E} \geq 0 \) is the positive profit condition for the final good producer. By Assumption 1, this condition is always satisfied by an aggregate consumption maximizing policy maker. Furthermore, for \( \eta(L_B) m(1 - \gamma) > 1 \) and \( \tau^* \geq 1 \), \( L_E \left[ \eta(L_B) m(1 - \gamma) - \frac{1}{\tau} \right] > 0 \). We conclude that \( \frac{\partial I}{\partial \tau} \Bigg|_{\tau = \tau^*} > 0 \). This completes the proof.

### A.11 Proof of Proposition 8

We show the result by contradiction.

Let \( L_B^* > 0 \) be the aggregate consumption maximizing investment in basic research for a given tax policy \( \tau_L^* \) and \( \tau_p^* \). Then we know from our discussions in section 5.1 that if \( L_B^* \) satisfies the following first-order condition for an interior optimum:

\[
\frac{\partial L_E}{\partial L_B} \Bigg|_{\tau_L = \tau_L^* \atop \tau_p = \tau_p^* \atop L_B = L_B^*} = 0
\]

\[
\Rightarrow \frac{\partial L_E}{\partial L_B} \Bigg|_{\tau_L = \tau_L^* \atop \tau_p = \tau_p^* \atop L_B = L_B^*} [\eta(L_B^*) m(1 - \gamma) - 1] + L_E \bigg|_{\tau_L = \tau_L^* \atop \tau_p = \tau_p^* \atop L_B = L_B^*} \eta'(L_B^*) m(1 - \gamma) - 1 = 0
\]

Suppose \( L_B^* \) was also \( I_{w,ls} \)-optimal. Then, \( L_B^* \) would satisfy the following first-order condition for an interior optimum:

\[
\frac{\partial I_{w,ls}}{\partial L_B} \bigg|_{\tau_L = \tau_L^* \atop \tau_p = \tau_p^* \atop L_B = L_B^*} = 0
\]

Differentiating the expression for \( I_{w,ls} \) given in equation (16) with respect to \( L_B \) yields:

\[
\frac{\partial w}{\partial L_B} \bigg|_{\tau_L = \tau_L^* \atop \tau_p = \tau_p^* \atop L_B = L_B^*} \left[ L - L_B^* - \tau_L^* \right] \left[ L - L_B^* - \tau_L^* \right] L_E \bigg|_{\tau_L = \tau_L^* \atop \tau_p = \tau_p^* \atop L_B = L_B^*} - \tau_p^* L_x \bigg|_{\tau_L = \tau_L^* \atop \tau_p = \tau_p^* \atop L_B = L_B^*} + \frac{\alpha}{1 - \alpha} \left[ \frac{L_y}{L_B^*} \bigg|_{\tau_L = \tau_L^* \atop \tau_p = \tau_p^* \atop L_B = L_B^*} \right]
\]

\[
+ \frac{1}{L} \left[ \eta(L_B^*) \right] - \tau_p^* \left[ \frac{\partial L_E}{\partial L_B} \Bigg|_{\tau_L = \tau_L^* \atop \tau_p = \tau_p^* \atop L_B = L_B^*} \right] - \tau_p^* \left[ \frac{\partial L_x}{\partial L_B} \Bigg|_{\tau_L = \tau_L^* \atop \tau_p = \tau_p^* \atop L_B = L_B^*} \right] + \frac{\alpha}{1 - \alpha} \left[ \frac{\partial L_y}{\partial L_B} \Bigg|_{\tau_L = \tau_L^* \atop \tau_p = \tau_p^* \atop L_B = L_B^*} \right] = 0
\]

Using \( \frac{\partial I_{w,ls}}{\partial L_B} = -\eta(L_B^*) m(1 - \gamma) \frac{\partial L_E}{\partial L_B} - \eta'(L_B^*) m(1 - \gamma) L_E \), \( \frac{\partial w}{\partial L_B} = -\alpha(1 - \alpha) L_y \frac{\partial L_E}{\partial L_B} \).
and \( \frac{\partial L_y}{\partial L_B} \bigg|_{t_L=t_L^*, t_P=t_P^*, L_B=L_B^*} = 0 \), this simplifies to:

\[
\begin{align*}
\left. \frac{1}{L} \left\{ -1 + \tau^*_P \left[ \eta(L_B^*) (1 - \gamma) - 1 \right] \frac{\partial E}{\partial L_B} \bigg|_{t_L=t_L^*, t_P=t_P^*, L_B=L_B^*} \right. \right. \\
&\left. \left. + \tau^*_P \eta'(L_B^*) m (1 - \gamma) L_E \right|_{t_L=t_L^*, t_P=t_P^*, L_B=L_B^*} \right. \\
&\left. \left. + [\tau^*_P - \tau^*_L] \frac{\partial E}{\partial L_B} \bigg|_{t_L=t_L^*, t_P=t_P^*, L_B=L_B^*} \right. \right. \\
&\left. \left. - 1 + \tau^*_P \left[ \eta(L_B^*) (1 - \gamma) - 1 \right] \frac{\partial E}{\partial L_B} \bigg|_{t_L=t_L^*, t_P=t_P^*, L_B=L_B^*} \right.
\end{align*}
\]

Now, \( \frac{\partial L_y}{\partial L_B} \bigg|_{t_L=t_L^*, t_P=t_P^*, L_B=L_B^*} = 0 \) and \( \tau^*_P < 1 \) imply that

\[
-1 + \tau^*_P \left[ \eta(L_B^*) (1 - \gamma) - 1 \right] \frac{\partial E}{\partial L_B} \bigg|_{t_L=t_L^*, t_P=t_P^*, L_B=L_B^*} + \tau^*_P \eta'(L_B^*) m (1 - \gamma) L_E \bigg|_{t_L=t_L^*, t_P=t_P^*, L_B=L_B^*} < 0.
\]

\( \tau \geq 1 \) implies that \( \tau^*_P - \tau^*_L \leq 0 \). Furthermore, \( \frac{\partial L_E}{\partial L_B} \bigg|_{t_L=t_L^*, t_P=t_P^*, L_B=L_B^*} > 0 \). We conclude that

\[
\frac{\partial w_{ls}}{\partial L_B} \bigg|_{t_L=t_L^*, t_P=t_P^*, L_B=L_B^*} < 0,
\]

a contradiction to \( (\tau^*_L, \tau^*_P, L_B^*) \) being \( I_{w_{ls}} \)-optimal.

The contradiction establishes the result.
References


