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Incentive Design and Mis-Allocated Effort

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Incentive Design and Mis-Allocated Effort

Preliminary Version

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Wendelin Schnedler*

August 30, 2013

Abstract

In their seminal work on multi-tasking, Holmström and Milgrom (1991) argue that apart from incentive and insurance issues, incentive design is also about the allocation of time, effort or attention. Despite its popularity this argument has never been formalized. The present paper offers such a formalization. Some perceived wisdoms surrounding the multi-tasking model are confirmed others challenged. In particular, it is pointed out that the reason to ignore informative signals is a mis-allocation of effort. Multi-tasking is neither necessary nor sufficient for this mis-allocation to occur. The results also challenge the equal compensation principle. With the proposed concepts, it becomes possible to generalize the sufficient statistic result to the situation, where the agent's action is from an arbitrary compact set. It also becomes apparent that the LEN model is not capable of illustrating why mis-allocated effort can render it optimal to ignore informative signals.

Keywords: incentive scheme, multi-tasking, alignment, gaming, LEN model

JEL-Codes: D86, M52, J33, D82, M41

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Introduction

Sometimes it is optimal to discard information rather than employing it to provide incentives. A legendary case is Lincoln Electric's attempt to boost their typists' productivity by paying for the number of key strokes; a scheme that was abolished when a typist was found hitting the 'most convenient key' in her lunch break.¹ In their seminal paper on multi-tasking, Holmström and Milgrom (1991) provide a formal example (the home-contractor model), where a builder decides on quality and speed and it is optimal not to use the only information available, which is the time of completion. In explaining the result, they appeal to the intuition that incentives are not only about getting the builder (agent) to work and insuring him against income fluctuations but also about the allocation of time or attention between multiple tasks (or multiple dimensions of the agent's action choice). Probably because this explanation is very natural and appealing, it has never been formalized.

Not surprisingly, the practical implications drawn from Holmström and Milgrom's argument are often vague. As an example, consider Prendergast's claim (1999) that 'more holistic measures of performance suffer less from the multi-tasking dilemma.' Other implications based on the argument seem to be at odds with reality. The Equal-Compensation Principle from the textbook by (Milgrom and Roberts, 1992) states that marginal returns to two different tasks need to be equal, otherwise the agent spends his entire time on the better remunerated task. Consider an academic who faces stronger rewards for spending the next minute to think about research rather than the upcoming lesson (as many academics do). According to the principle, the academic should not engage in preparing the lesson at all. The comparison of the two rates of return seem to miss the fact that the academic does not like standing unprepared in front of the class. The example raises the question which fundamental forces an incentive designer needs to take into account and which trade-offs she faces.

The burgeoning literature following Holmström and Milgrom (1991) spans fields as diverse as accounting and labor economics (see e.g. Feltham and Xie, 1994; Datar et al., 2001; Feltham and Wu, 2000; Baker, 2000, 2002; Schnedler, 2008) but does not formalize Holmström and Milgrom's explanation. Instead, it focuses on performance measures and their properties. Of particular interest in this literature is whether the performance measure is congruent with the principal's objective. Using the linear-exponential normal model (LEN model), it then directly stipulates a trade-off between the congruity of a performance measure and its precision, for example, its variance, signal-to-noise ratio, etc. This trade-off seems

¹The example has been popularized by Norman A. Berg's Harvard Business School Case "Lincoln Electric Co," 1975.

to be motivated by the idea that congruent performance measures induce a well-balanced activity by the agent. It also implicitly assumes that a more congruent performance measure is ‘better’ than an otherwise identical less congruent one. However, unless the agent has special preferences, optimally using a congruent signal generates a lower surplus than optimally using another less congruent but equally noisy signal (Schnedler, 2008). In addition, it will become clear later that it is never optimal to ignore a dis-congruent performance measure in the LEN model—however large the induced mis-allocation may be.

Two decades after Holmström and Milgrom’s insightful explanation, it still awaits a formal foundation. Lacking such a foundation, it is not possible to answer the following questions. What fundamental forces are involved in incentive design? How are they traded-off? Why and when are informative signals ignored? Are aligned incentives desirable and if so in what sense? The present paper provides answers to these questions. At the heart is a definition of mis-allocated effort and its costs, where the term ‘effort’ is used differently from the common practice in moral-hazard models. Using this somewhat unusual definition, it becomes possible to confirm various conceived wisdoms prevailing in the multi-tasking literature, while other such wisdoms are refuted.

The moral-hazard literature tends not to distinguish between the agent’s action choice and ‘effort.’ In contrast, I propose to carefully separate the agent’s action choice a from the effort it requires. Effort here is some real number e that captures all effects of the agent’s action on his well-being. Let the action choice be from some compact set \mathcal{A} . This set could, for example, be a list of choices or a single or multi-dimensional interval. Then, effort maps from this set into the real numbers. Formally, $e : \mathcal{A} \rightarrow \mathbb{R}, a \mapsto e(a)$. Two very different action choices may require the same effort. In other words, they can have the same opportunity costs for the agent. Suppose the an academic decides on how many hours to spend on working on a research project, a_1 , and preparing his lecture, a_2 . Then the academic could be indifferent between 7h on the project and 1h for lecture preparation and 2h on the project and 3h on the lecture: $e((7, 1)) = e((2, 3))$. Both action choices lead to the same utility for the academic, or, alternatively require the same effort, say \tilde{e} . (This academic is clearly concerned about more than just the opportunity costs of time.) Separating the action choice from effort enables us to speak of different ways to use or allocate effort. The effort \tilde{e} can be used for action $a = (7, 1)$ or $\tilde{a} = (2, 3)$ (and possibly even other effort choices). Put differently, the notion of allocating effort becomes meaningful.

In addition to effort, the agent also cares about rewards. Absent rewards, the agent is by definition indifferent between all action choices that require the same effort. The benefit b that the principal derives from these action choices, however, may be different. For example, the university employing the agent may derive a larger benefit b from the more balanced action choice $\tilde{a} = (2, 3)$ rather

than the extreme $a = (7, 1) : b(\bar{a}) > b(a)$. Denote the set of all Pareto-optimal allocations of \tilde{e} by $A^*(\tilde{e})$. Since the agent is indifferent between the different uses of a given effort level \tilde{e} , any Pareto-optimal allocation of this effort level, say $a^*(\tilde{e}) \in A^*(\tilde{e})$, maximizes the principal's benefit. If incentives induce the agent to choose activity a^I which requires effort $e^I := e(a^I)$ but this effort is not used Pareto-optimally, $a^I \notin A^*(e^I)$, they lead to *mis-allocated effort*. The loss of benefit to the principal from mis-allocated effort, $b(a^*(e^I)) - b(a^I)$, are the *mis-allocation costs* of incentive scheme I .

With these concepts, it can be shown that the fundamental forces involved in designing incentives are effort allocation and elicitation (Proposition 1). Formally, the costs from running incentives I rather than being able to stipulate the action choice directly, *the agency costs (of incentives I)*, are composed of effort allocation and elicitation costs, where the latter comprise the loss due to sub-optimal effort and an information premium. The proof borrows from the Slutsky decomposition. Agency costs are decomposed using the Pareto-optimal allocation of the effort induced by incentives e^I . Although the proof is mathematically trivial, the result almost directly implies the following insights.

Since the mechanism designer wants to minimize agency costs, it follows immediately from the proposition that mis-allocation costs are reduced whenever this is possible without increasing elicitation costs. Eventually these opportunities are exhausted and the mechanism designer has to trade-off mis-allocation and elicitation costs (Corollary 1). In contrast to the trade-offs proposed in the literature, which only work in the LEN model under specific restriction on the agent's preferences, this trade-off is more widely applicable. Moreover, it can be regarded as an extension of the traditional idea that incentive design involves a trade-off between incentives to exert effort and insurance (because elicitation costs are composed of the loss due to sub-optimal effort and the information premium). Any decomposition of agency costs in terms of eliciting and using time (or attention) would not relate to this traditional idea.

Using an informative signal typically reduces elicitation costs. Accordingly, the only reason to dismiss such a signal is an increase in mis-allocation costs (Corollary 5). Mis-allocation is hence the reason to ignore informative signals. Multi-tasking (or a multi-dimensional choice set) is neither necessary nor sufficient for mis-allocation. Re-consider the typist and assume (for simplicity's sake) that he can only choose between { business as usual, type important letter, hit space bar in lunch break }. Although this choice set is clearly not multi-dimensional, the introduction of piece rates will induce the typist to exert effort, which is directed towards the less beneficial hitting of the space bar rather than the typing of the important letter. The argument is taken up in more depth, later.

Suppose the mechanism designer knows little about the agent's preference. Then, effort is allocated Pareto-optimally if and only if incentives are aligned in the

sense that a larger benefit is associated with stochastically larger rewards for the agent (Proposition 2). This contradicts the spirit of Milgrom and Roberts' Equal Compensation Principle Milgrom and Roberts (1992) because marginal returns to different dimensions may be very different even if incentives are aligned.

The results can be used to generalize famous insights into the value of information, such as the sufficient statistic result by Holmström (1979, 1982), from the original one-dimensional choice set of the agent to an arbitrary compact set \mathcal{A} . The sufficient statistic result and a related result by Kim (1995) hold in the more general setting considered here with respect to elicitation rather than agency cost (Corollary 3 and 4). The failure of these results in the multi-tasking setting is thus due to effort mis-allocation and not multi-tasking.

The results also generate insights into the literature on the congruity-precision trade-off. It can be shown that congruent performance measures are desirable in the sense that using them leads to no mis-allocation (Corollary 6). This justifies what seems to be an implicit assumption in this literature. A congruent performance measure thus leads to lower surplus than an otherwise identical dis-congruent one because the latter has lower elicitation costs (Corollary 7). Finally, performance measures that introduce a mis-allocation are never optimally dismissed in the LEN model, however large this mis-allocation may be (Corollary 8). This shows that not being able to measure the performance on one task does not necessarily mean that incentives have to be renounced (which contradicts an implication of the Equal Compensation Principle). It also indicates that the LEN model cannot be used to illustrate the notion that mis-allocation prevents signals from being used.

The remainder of the paper is structured as follows. Section 1 introduces a fairly general moral hazard framework that allows, for example, for single and multi-dimensional action choices. Section 2 formally defines effort mis-allocation and elicitation costs. Section 3 provides the central results. Section 4 returns to the literature on the value of information, adapts respective results to the more general framework, and singles out effort mis-allocation (as defined here) as the reason why these results fail in multi-tasking models. Section 5 re-visits the congruence-precision trade-off and substantiates the claim that the LEN model cannot explain why it is optimal to discard performance signals. Section 6 concludes.

1 A General Moral Hazard Framework

The following framework tries to capture the essentials of most moral-hazard models in which a principal engages an agent to chose an action on her behalf.

Agent's action choice, effort and utility The agent (he) chooses an action a , which affects a principal (she). This action is from some (compact) set \mathcal{A} . Suppose that each choice $a \in \mathcal{A}$ requires effort e , where effort captures all consequences

of the action choice a on the agent's well-being in a real number e . Formally, e is a continuous function mapping from \mathcal{A} to the real line. While the agent dislikes effort e , he enjoys rewards r . Put differently, the agent's utility, u , strictly increases in r and strictly falls in e , where r and e are real numbers and u is assumed to be a continuous function. Notice that the agent's utility does not directly depend on the action choice but only indirectly via effort.

Principal's utility. In order to reflect that the agent's effort may be employed in unhelpful ways, the principal does not care about effort in itself, but draws a real-valued benefit b from the agent's action, a . Formally, $b : \mathcal{A} \rightarrow \mathbb{R}, a \mapsto b(a)$. Apart from increasing in the benefit b , the principal's utility falls in rewards r to the agent. For simplicity, let her utility be $b - r$. Restricting attention to this simple function is without loss of generality. Results remain valid if the principal's utility is some continuous function that strictly increases in b and strictly falls in r ; this function may also depend on influences beyond the agent's control.

Incentives. Let \mathcal{J} be the set of all incentive schemes in a given contractual environment. Any $I \in \mathcal{J}$ assigns some distribution of rewards to each action choice a by the agent. This reward distribution is denoted here by a real-valued random variable $R^I(a)$, with the action choice a as a parameter. Designing incentives can be regarded as selecting an incentive scheme $I \in \mathcal{J}$. The action choice that is induced by incentive scheme I is denoted by a^I and the respective effort by $e^I := e(a^I)$.² Given I , the principal expects to pay the agent a reward of $r^I := E [R^I(a^I)]$.

Scope of the framework. The assumption that the agent's utility does not directly depend on the action is equivalent to supposing that the agent's preferences are separable in effort and rewards. This is not innocuous because it excludes the possibility that the relative effect of different dimensions of the action choice on the agent's well-being changes with the certainty equivalent of rewards. For example, an academics preference of research over teaching is not allowed to when he get 'richer.' This and all other assumptions, however, are implicitly or explicitly assumed in all well-known moral hazard models, which are hence covered by the proposed framework. These include, among others, the traditional moral-hazard models, such as Gjesdal (1976), Holmström (1979, 1982) and Kim (1995), or Grossman and Hart (1983) but also multi-tasking models in the tradition of Holmström and Milgrom (1991) and Baker (1992), such as Feltham and Xie (1994), Datar et al. (2001) Feltham and Wu (2000), Baker (2000, 2002), or Schnedler (2008). The considerable generality of the framework even allows to embed incentive problems, such as the Averch-Johnson-Effect that were originally not placed in a principal-agent setting (Averch and Johnson, 1962).³

²If incentives leave the agent indifferent between multiple action choices, a^I is assumed to be the choice suggested by the principal.

³The regulator in their model is the principal here; the monopolist is the agent; the choice of

Level and Use of Effort. The introduction of effort allows us to separate out two aspects of the agent's choice. Formally, we can (disjointly) decompose the agent's choice space into subsets that require the same effort: $\mathcal{A} = \dot{\bigcup}_{\tilde{e}} \{a | e(a) = \tilde{e}\}$. On the one hand, the agent's choice thus determines how much effort \tilde{e} is required. On the other hand, the agent determines how effort is used. In absence of incentives, the agent only cares about the level of effort and not its use. Each effort level \tilde{e} is hence associated with an indifference set $\{a | e(a) = \tilde{e}\}$

2 Effort Mis-Allocation and Elicitation Costs

This section provides definitions of mis-allocation costs and other more traditional types of costs related to incentives.

In contrast to how the term 'effort' is used in the literature, different action choices may require the same effort, here: $e(a) = e(\tilde{a}) = \tilde{e}$ for $a \neq \tilde{a}$. This renders it meaningful to speak of different allocations, say a and \tilde{a} , of the same effort \tilde{e} . Observe that the agent's utility is constant when effort is fixed and that the principal's utility increases in the generated benefit. Consequently, all Pareto-optimal ways of employing effort e can be obtained by maximizing the principal's benefit over all action choices that require effort \tilde{e} :⁴

$$A^*(\tilde{e}) := \arg \max_a b(a) \text{ such that } e(a) = \tilde{e}.$$

In order to refer to a specific action choice from $A^*(e)$ let us use $a^*(e)$. The effort e^I elicited by incentives I is *mis-allocated* if it is not employed Pareto-optimally: $a^I \notin A^*(e^I)$. Whenever there is only one way to use effort, as in the traditional moral-hazard models with a one-dimensional action choice, effort is never mis-allocated.

The negative consequences of mis-allocation can be measured in terms of a benefit loss to the principal; again, the agent is irrelevant because he is indifferent between all action choices that require the same effort. Given incentives I , the principal's benefit from the incentive I amount to $b(a^I)$. The following definition compares this benefit with the benefit, the principal would have obtained with effort e^I if the behavior had not been distorted, $b(a^*(e^I))$.

Definition 1 (Effort Mis-allocation Costs). *The (effort) mis-allocation costs of incentives I are*

$$MC(I) := b(a^*(e^I)) - b(a^I).$$

labor and capital intensity is the action choice; production costs are the effort; and the social gains from production are the benefit.

⁴The set of maximizers is not empty because $b(\cdot)$ is a continuous function and the set $\{a | e(a) = \tilde{e}\}$ is compact (as $e(\cdot)$ is a continuous function).

Mis-allocation costs are uniquely defined even if there are multiple Pareto-optimal ways to use effort e^I , ($|A^*(e^I)| > 1$), because all these ways entail the same benefit. Mis-allocation costs are zero if effort is not mis-allocated, i.e., when $a^I \in A^*(e^I)$; otherwise they are positive.

Effort Elicitation costs. Let us now come to some more traditional costs related to incentives. Denote the first-best action, i.e., the action that the principal would stipulate if actions could be stipulated, by a^* and the respective effort by $e^* := e(a^*)$. Let the compensation required by the agent to work for the principal in the first-best be r^* . Then, the *information premium*, $r^I - r^*$, describes how much more the principal expects to pay given incentives I and in relation to the first-best. The effort elicited by incentives, e^I , may differ from that in the first-best, e^* . The pure effect of this difference on the principal can be separated out from that of mis-allocated effort by supposing that effort e^* and e^I are both used Pareto-optimally; the *benefit loss due to sub-optimal effort* is $b(a^*(e^*)) - b(a^*(e^I))$. Observe that in the traditional ‘one-dimensional’ moral-hazard model there is only one way of employing effort, which necessarily coincides with $a^*(e^*)$ for e^* and $a^*(e^I)$ for e^I . The benefit loss due to sub-optimal effort and the information premium thus stand for well-known issues of incentive design: getting the agent to exert effort and ensuring him against income fluctuations. Both are related to eliciting rather than allocating effort.

Definition 2 (Effort Elicitation Costs). *The costs of eliciting effort e^I with incentives I , or short, elicitation costs are composed of the benefit loss due to sub-optimal effort and the information premium:*

$$EC(I) := b(a^*(e^*)) - b(a^*(e^I)) + (r^I - r^*).$$

If the expected payment under the incentive scheme, r^I , is below that in the first-best world, r^* , the information premium is negative. Likewise, the benefit loss due to sub-optimal effort can be negative if the incentive scheme elicits more than the first-best effort. Effort elicitation costs, however, are always larger than or equal to zero. The reason is that eliciting effort is at least as costly as directly stipulating it in a contract.⁵

For completeness, let us define the loss from operating incentives I in relation to what would be possible if actions were contractible, the *agency costs (of incentive scheme I)*, as

$$AC(I) := b(a^*) - r^* - (b(a^I) - r^I).$$

The definitions from this section are used in the next section to identify effort mis-allocation and elicitation as the fundamental forces in incentive design.

⁵Strictly speaking, it is not the effort, which is stipulated in the contract, but the set of action choices that lead to this effort.

3 Forces and Trade-Offs in Incentive Design

The following proposition answers which forces have to be taken into account when designing incentives.

Proposition 1 (Fundamental Forces in Incentive Design). *The costs from running an incentive scheme (agency costs) are composed of elicitation and mis-allocation costs:*

$$AC(I) = EC(I) + MC(I).$$

Proof. The proof follows directly from the definitions of elicitation and mis-allocation costs: $EC(I) + MC(I) = r^I - r^* + b(a^*(e^*)) - b(a^*(e^I)) + (b(a^*(e^I)) - b(a^I)) = AC(I)$. \square

Similar to the famous Slutsky decomposition, the proof is mathematically trivial, while the economic content of the proposition is important. Since the aim of incentive design is to minimize agency costs, the proposition clarifies the two issues faced by a mechanism designer: (i) keeping the costs of eliciting effort low and (ii) reducing the benefit loss from mis-allocation effort. It thus renders precise the very intuitive but vague insight that incentives are about getting the agent to work and ensuring that this work is useful, which has so far successfully resisted any attempts to be captured in a theoretical model.

Let us return to the mechanism designer's aim to minimize agency costs. From the decomposition, we can conclude that she will reduce mis-allocation costs as long as this is possible without increasing elicitation costs. Eventually, all respective opportunities are exhausted and she faces a trade-off: mis-allocation costs can only be reduced further if elicitation costs are increased. This argument directly leads to the following corollary.⁶

Corollary 1 (Central Trade-Off of Incentive Design). *Suppose all feasible incentives entail agency costs, $AC(I) > 0$ for all $I \in \mathcal{J}$. Then, optimal incentive design has to trade off mis-allocation and elicitation costs.*

This corollary offers an alternative to the congruity-precision trade-off from the literature. The trade-off suggested here is not confined to a specific model setting (like the LEN model). Moreover, it embeds the traditional idea that incentives trade-off the benefit loss due to sub-optimal effort and the information premium because these make up elicitation costs. While it is probably possible to decompose

⁶Formally, the proof works by contradiction. Take optimal incentives I and suppose there is no trade-off. Then, it would be possible to reduce mis-allocation costs in relation to incentives I without increasing elicitation costs. Consequently, there must exist another incentive scheme, say \tilde{I} , with agency costs, which are strictly lower than those of I . This, however, contradicts the fact that incentives I were optimal.

agency costs into the costs of eliciting and using time (or attention), the resulting trade-off would not relate to this traditional idea.

The decomposition of agency costs can also be used to state a first result about the use of information. Later, we will re-visit to two famous results on the value of information in their respective specific settings (see Section 4). For the moment, however, we remain in the relatively general moral hazard framework. Consider an optimal incentive scheme I_0 , i.e., an incentive scheme that minimizes agency costs given some specific contractual framework. Now suppose that a new signal arrives which is *informative about effort* in the sense that it can be incorporated in some new incentive scheme, I_1 , which either induces the same effort as the old incentive scheme, $e^{I_1} = e^{I_0}$ but has a lower information premium: $r^{I_1} < r^{I_0}$, or, alternatively, has the same information premium $r^{I_1} = r^{I_0}$, but yields more benefit from optimally used effort, $b(a^*(e^{I_1})) > b(a^*(e^{I_0}))$. Under the assumptions used by Holmström (1979, 1982) for his sufficient statistic result, this notion of informativeness is equivalent to his definition of informative signals.

If a signal is informative about effort, the elicitation costs of I_0 are below those of I_1 . Using the signal by moving from I_0 to I_1 thus reduces agency costs by $EC(I_0) - EC(I_1)$. The signal is hence only discarded if mis-allocation costs increase by more than $EC(I_0) - EC(I_1)$ for all incentives I_1 .

Corollary 2 (Discarding Informative Signals). *A signal that is informative about effort is ignored if and only if the reduction in elicitation costs is more than outweighed by the increase in mis-allocation costs*

$$EC(I_0) - EC(I_1) < MC(I_1) - MC(I_0),$$

for all incentives I_1 that use this signal.

If some signal is informative about effort but using it in some incentive scheme does not increase mis-allocation costs, it would not be dismissed. The increase in mis-allocation is thus necessary for discarding informative signals.

While mis-allocation is necessary for discarding informative signals, multi-tasking is not. Incorporating an informative signal may well increase mis-allocation costs in a model where the agent's choice is not multi-dimensional. As a toy model, consider a (risk-neutral) typist who can choose between three actions { business as usual, type important letter, hit space bar in lunch break }. Clearly these choices are not multi-dimensional. Now suppose, that the typist prefers business as usual but is indifferent between the other two actions: $e(\text{business as usual}) < e(\text{type important letter}) = e(\text{hit space bar in lunch break})$. The employer benefits most from the important letter being typed and least from space bar hits: $b(\text{letter}) > b(\text{business as usual}) > b(\text{space bar})$. Next assume that a new technology allows the mechanism designer to record the number of key strokes; further, hitting the space bar leads

to more key strokes than typing an important letter, which in turn results in more keystrokes than ‘business as usual.’ Now suppose that the mechanism designer only considers the use of piece rates based on the number of key-strokes to provide incentives. If the piece rate is set sufficiently high, the typist hits the space bar. If the number of key strokes is not used the typist does ‘business as usual.’ The former leads to considerably more effort for the same information premium as the latter (because the base payment can always be reduced by the average gains from the piece rate). Key strokes are hence informative about effort. Moreover, using them increases effort. This effort, however, is grossly mis-allocated, while the initial effort is not. (It can only be used in one way). We thus observe an increase in mis-allocation costs that prevents the use of a signal even though the model involves no multi-dimensional action choice by the agent. Of course, the same result could also be obtained by modeling the agent’s choice as being multi-dimensional. But if it does not matter for the result whether the choice is modeled multi-dimensional or not, multi-dimensionality cannot be the reason why information is ignored. If we eliminate the possibility of mis-allocating effort, for example, by assuming a one-to-one mapping, informative signals are no longer optimally ignored.

Next, we address the question why aligned incentives may be considered attractive. To do so, we first need a general notion of alignment.

Definition 3 (Aligned Incentives). *Incentives I are aligned (with the principal’s benefit) if a change in the agent’s action choice that increases benefits also entails stochastically larger rewards: for all a, \tilde{a} with $b(a) > b(\tilde{a})$: $R^I(a) \succ_{FOSD} R^I(\tilde{a})$.*⁷

Essentially alignment means that the agent’s rewards are increasing (albeit stochastically) in the principal’s benefit, or, equivalently, that the principal’s benefit iso-quants coincide with the iso-quants induced by rewards for the agent. This notion can be applied to a much wider range of problems than the existing notions, which are formulated for linear benefit and rewards within the LEN model. In the LEN model, aligned incentives are closely related to congruity and we will examine this relationship in detail in Section 5.

It is relatively clear that aligned incentives eliminate mis-allocations. The reason is simple: if incentives are aligned, the principal’s benefit-isoquants are identical with those induced by rewards for the agent. Effort is thus always used in the most beneficial way. Conversely, if effort is allocated Pareto-optimally, benefit and rewards have to be aligned near the action induced by incentives (otherwise the agent has a reason to deviate). The principal can, of course, only infer the induced action choice in advance if she is well informed about the agent’s preferences. If

⁷Let X and Y be random variables, then $X \succ_{FOSD} Y$ means that X first-order stochastically dominates Y

she lacks this information, mis-allocation can only be avoided if rewards are locally aligned with benefits for any possible action choice, which means that incentives have to be aligned. This intuition is summarized in the following proposition (the formal proof is in the appendix).

Proposition 2 (Aligned Incentives and Mis-allocation). *(i) If incentives are aligned, effort is allocated Pareto-optimally. (ii) If mis-allocation should be avoided irrespective of the agent's preferences, incentives have to be aligned.*

This result formalizes Baker, Gibbons and Murphy's claim (Baker et al., 1994) that 'Basing pay on an employee's contribution to firm value would have prevented [...] seemingly dysfunctional behaviors' if 'seemingly dysfunctional behavior' is interpreted as effort mis-allocation.

The proposition offers an alternative to Milgrom and Robert's equal compensation principle (1992). According to this principle, a mis-allocation (albeit of time or attention) can only be avoided if the marginal of return to the employee is equal for any two tasks. In contrast, the proposition suggests that it suffices if the marginal rate of substitution between any two tasks in terms of agent's return is equal to the marginal rate of substitution between the same two tasks in terms of the principal's benefit. Marginal rates of return may well be different while these substitution rates are the same and incentives are aligned; effort may thus be Pareto-optimally allocated although Milgrom and Roberts' crucial condition does not hold.

4 Results on the Value of Information

Probably one of the most important insights from the early moral hazard literature is that freely available independent information about performance, which is not yet reflected in incentives, should be incorporated because doing so reduces agency costs (Gjesdal, 1976; Holmström, 1979). According to Holmström (1979), a signal X is informative about effort e in relation to information already used, say Y , if Y is no sufficient statistic for e . Re-phrased in terms of agency costs, Holmström's famous sufficient statistic result states that using signal X optimally reduces agency costs if and only if X is informative (Proposition 3 in Holmström 1979).

With their home contractor model, Holmström and Milgrom (1991) provide a multi-task example in which it is optimal to ignore informative signals. The sufficient statistic in its original form is thus not valid for the general framework considered here. In the following, I show how it can be adapted to hold for any choice set \mathcal{A} .

In Holmström’s pioneering model (1979), the agent decides on a (one-dimensional) choice variable. Since each choice a requires a different effort e in the model, the question of how to allocate effort does not arise, and the optimal way of using effort $e(a)$ is the only way: $a^*(e(a)) = a$. This means also that there are no mis-allocation costs. Put differently, agency costs in his model are equivalent to elicitation costs in our more general framework. Accordingly, what is true for agency costs in his model remains valid for elicitation costs in the more general framework here.

Corollary 3 (Generalized Sufficient Statistic Result). *Given Holmström’s assumptions (1979) while allowing the agent’s choice set to be \mathcal{A} , incorporating a signal that is informative about effort reduces elicitation costs.*

The corollary clarifies that it is not multi-tasking, which invalidates the sufficient statistic result, but effort mis-allocation. Suppose that effort is never mis-allocated, for example, because all feasible incentives are aligned. Then, agency costs amount to elicitation costs and the sufficient statistic result holds with respect to agency costs—irrespective of whether the agent’s action choice is multi-dimensional or not. The necessary condition for the sufficient statistic result to fail is not the multi-dimensional action choice. It is the fact that multiple action choices require the same effort.

Implicitly, the sufficient statistic result compares two information systems, one which includes and one which excludes signal X . Kim (1995) extends such comparisons to information systems that are not necessarily nested. Kim’s Proposition 1 states that based on some information system Y it is possible to obtain lower agency costs than with a different system Y whenever the likelihood ratio distribution of X is a mean-preserving spread of that of Y .⁸ Like Holmstrom’s models, Kim’s model also features a one-to-one relationship between action choice and effort. In perfect analogy, his agency costs are thus equivalent to elicitation costs in the present setting and we obtain the following more general version of his result.

Corollary 4 (Information System Comparison with Multiple Tasks). *Given Kim’s assumptions (1995) while allowing the agent’s choice set to be \mathcal{A} , relying on information system X rather than information system Y , one can obtain lower elicitation costs if the likelihood ratio distribution of X is a mean-preserving spread of that of Y .*

Both corollaries identify the same reason why it is optimal to discard ‘more’ information, where ‘more’ refers to adding an informative signal in case of Holmström’s model and to a lower mean-preserving spread in Kim’s model. The only reason to discard ‘more’ information is that using it would lead to higher agency costs.

⁸The likelihood ratio distribution of X , which measures the responsiveness of X to changes in effort, thus has the same mean as that of Y but varies more.

In both cases, using ‘more’ information reduces elicitation costs. Hence, it only makes sense to discard ‘more’ information if doing so increases mis-allocation costs more than it lowers elicitation costs.

Corollary 5. *Given Holmström’s (1979) or Kim’s (1995) assumptions, ‘more’ information is optimally discarded because using it would result in mis-allocated effort.*

Holmström and Milgrom’s finding that it is optimal to discard information (1991) can hence formally be traced back to mis-allocated effort—as defined here. Observe that a similar argument cannot be made in terms of mis-allocated time or attention—unless these happen to coincide with effort because the agent does not care about how he uses his time or attention.

5 Re-Visiting Congruence-Precision-Trade-Off

In search for an alternative to the traditional trade-off, the literature turned to performance measure properties. Particular attention has been devoted to the congruence between performance measure and the principal’s benefit. Congruence is then traded-off against the measure’s precision as captured—for example—by its signal-to-noise ratio.

The focus in this section is on three aspects. First, by focusing on congruence, the literature implicitly assumes rather than proves that measuring performance congruently avoids some kind of mis-allocation. In the following, this implicit assumption will be justified: whenever incentives are based on congruent performance measures, they are aligned and effort is allocated Pareto-optimally (Corollary 6). Second, more congruence does not imply lower agency costs even if precision is held constant—see Proposition 3 by Schnedler (2008). Schnedler claims that ‘insurance plays a crucial role’ for explaining this observation. Here, this claim is substantiated (Corollary 7). Third, performance measures that lead to mis-allocation are never discarded in the LEN model, however large this mis-allocation may be (Corollary 8). The LEN model thus cannot explain why key strokes are discarded as a performance measure in the typist example.

Implicit in any incentives is a performance measure, i.e., a mapping from action choices to observable outcomes that are rewarded or punished. The literature has heavily focused on how action choices affect performance measures in relation to how they affect the principal’s benefit. For tractability reasons, all these models rely on the LEN model, which is based on the following assumptions: performance measure, Y , and benefit, B , are assumed to be linear in action choices and noise: $Y(a, \varepsilon) = \mu_1 a_1 + \dots + \mu_n a_n + \varepsilon$, and $B(a, \eta) = b_1 a_1 + \dots + b_n a_n + \eta$, where ε and η are normally distributed error terms. Moreover, rewards are assumed to be linear in

the performance measure. Finally, the effect of action choices on effort is assumed to be quadratic: $e(a) = a' E a$, with E being a positive-definite matrix.

The performance measure with marginal effects μ is said to be *congruent* if the relative effect of tasks is the same as on the benefit. Formally, there is some $\lambda \neq 0$ such that the measure's weight vector $\mu = (\mu_1, \dots, \mu_n)$ is a multiple of that of the benefit $b = (b_1, \dots, b_n)$: $\mu = \lambda b$, for some $\lambda > 0$.⁹ A performance measure is *dis-congruent* if the weight vector for performance measure and benefit are not aligned: there is no $\lambda > 0$ such that $\mu = \lambda b$.

The notion of a trade-off between congruence and precision only makes sense if congruence is desirable. This implicit assumption in the literature can be justified by appealing to mis-allocation costs. Rewarding the realization of a performance measure that is congruent with the benefit means that it is not possible to (stochastically) increase measured performance without (stochastically) increasing benefit. In other words, incentives based on rewards for congruent performance measures are aligned and thus entail no mis-allocation costs. Moreover, the converse also holds. Since effort is quadratic and the benefit linear in actions, there is a unique action that maximizes the principal's benefit for a given effort level. A dis-congruent performance measure thus induces a different and non-optimal use of effort in contrast to a congruent one. The following corollary summarizes these considerations.

Corollary 6. *In the LEN model, using incentives I leads to Pareto-optimally used effort, $MC(I) = 0$, if and only if performance is measured congruently with the benefit, $\mu = \lambda b$, with $\lambda > 0$.*

The corollary establishes that from a mis-allocation perspective, congruent performance measures are clearly superior to dis-congruent ones. It thus justifies in a specific sense the implicit assumption from the literature that congruent performance measures are desirable.

The trade-off between congruence and precision only makes sense if congruent performance measures yield no higher agency costs than any dis-congruent one with the same precision. As Schnedler (2008) points out, this is indeed true *but* only in the arguably unusual case that the effect of different dimensions of the action choice on effort is identical and independent (see his Proposition 3), i.e., $E = \gamma I$ with $\gamma > 0$, or equivalently, $e(a) = \sum_{i=1}^n \gamma a_i^2$. He argues that 'insurance plays a crucial role' why some dis-congruent performance measures lead to lower agency costs than congruent ones (p. 610). The next result supports this claim. Since agency costs are composed of elicitation costs and mis-allocation costs

⁹A measure Y with $\lambda < 0$ measures 'bad performance' and can be turned into a congruent performance measure \tilde{Y} with some positive λ by flipping the scale: $\tilde{Y} = -1 \cdot Y$. Without loss of generality we can thus assume that if a measure is congruent $\lambda > 0$.

(Proposition 1), either elicitation or mis-allocation costs must be smaller for the dis-congruent performance measure. Congruent performance measures, however, do not entail mis-allocation costs by Corollary 6. It can thus be excluded that mis-allocation costs are lower. Thus, agency costs have to be smaller because of lower elicitation costs.

Corollary 7. *The reason why some dis-congruent performance measures entail lower agency costs than congruent ones with the same precision are lower elicitation costs.*

Since elicitation costs result from the noisiness of performance measures in the LEN model, this confirms Schnedler’s argument that insurance issues are the reason why dis-congruent measures are superior to congruent ones. At the same time, the result shows that some distorted behavior may well be preferable to no mis-allocation costs (as this may allow the reduction of elicitation costs). The elicitation-mis-allocation trade-off here leads to an inner solution.

Finally, let us examine whether it is possible that performance information is optimally discarded to avoid mis-allocation in the LEN model. Let us begin with an incentive scheme that uses a performance measure Y , which is congruent, $\mu = \lambda b$ for some $\lambda > 0$, so that effort is used Pareto-optimally, initially. Next, we want to examine whether some additional measure $X = \mu^x a + \varepsilon^x$ should be discarded.

In analogy to Holmström’s sufficient statistic result, Feltham and Xie characterize when adding a signal X helps reducing agency costs in the LEN model (see Proposition 3 Feltham and Xie, 1994). A direct implication of their proposition for the situation at hand is the following. Measure X is only useless in reducing agency cost if it is aligned with Y : $\mu^x = \delta\mu$ for some δ . Using that Y is congruent, the signal should be discarded only if $\mu^x = \delta\lambda b$. Suppose now that using X leads to a mis-allocation, then it must be dis-congruent. Put differently, there is no α such that $\mu^x = \alpha b$. This, however, means that condition $\mu^x = \delta\lambda b$ is never fulfilled and that X should never be discarded—irrespective of how strong the mis-allocation is and how noisy the performance measure is.

Corollary 8. *In the LEN model, dis-congruent performance information is never discarded —irrespective of how strong a mis-allocation it induces.*

The intuition is that with a linear benefit function, marginal losses from mis-allocation are small in relation to the marginal gains from lower elicitation costs.

The result yields a contradiction to Milgrom and Robert’s Equal Compensation Principle. They claim that setting marginal rates of return differently has drastic consequences: the agent completely ignores the task with the lower marginal rate. As they point out, this ‘imposes a serious constraint on incentive-compensation formulas used in practice’: as soon as one dimension cannot be measured, incentives

cannot be used. This statement, however, is not generally true. Corollary 8 holds in particular if the performance information concerns only one dimension while the agent's choice is two-dimensional. So, even if only one task can be measured, the gains from using this measurement in reduced elicitation costs can outweigh the losses in mis-allocation costs. This contradicts Milgrom and Roberts' conclusion that incentives cannot be used if only one dimension is measurable.

The corollary also demonstrates that the LEN model is not capable to represent the notion from the typist example that mis-allocated effort prevents the use of performance measures.¹⁰ This notion can be immediately obtained by dropping the assumption that benefit is linear in action choices.

6 Conclusion

More than two decades ago, Holmström and Milgrom's influential work on multi-tasking (1991) challenged the traditional view that incentives result from a trade-off between insurance and incentives to exert effort. They have argued that this view misses an important element, namely, that the agent can allocate attention, effort or time across tasks. While this intuitive argument has become very popular, it has never been formalized. Not surprisingly, the practical implications of this argument have remained rather foggy and the issues have been vaguely attributed to 'multi-tasking' or more formally to the fact that the agent's action choice is multi-dimensional.

The present paper has provided a specific definition of mis-allocated effort and its respective costs. The results which follow from this definition confirm some perceived wisdoms and can be regarded as a formalization of these. The fundamental forces that need to be taken into account when designing incentives are effort mis-allocation and elicitation (Proposition 1). The mechanism designer of any incentive scheme has to trade-off lower keep mis-allocation and effort elicitation costs (Corollary 1). The suggested trade-off offers a viable alternative to those, which have been suggested in the literature so far and which only apply under restrictive conditions in a specific setting that does not allow for performance information to be optimally discarded (the LEN model). In contrast to these trade-offs, the one suggested here can be regarded as an extension of the traditional incentive-insurance trade-off because elicitation costs are composed of the information premium and benefit losses due to sub-optimal effort. The paper also proves that signals that are informative about effort are ignored only if they

¹⁰In fact, there is generally very little reason to ignore signals in the LEN model. Any generic signal X should be employed in this model. Formally, the set of valuable signals characterized in Feltham and Xie's Proposition 3 (1994) is dense in the set of all possible signals.

increase mis-allocation costs (Corollary 2). Finally, aligning incentives eliminates mis-allocation (Proposition 2).

The results from this paper do not only confirm perceived wisdom, they also challenge it. The fact that informative signals may be optimally discarded is often attributed to multi-tasking. The reason is probably that the dimensionality of the agent's choice set is the glaring difference between the early moral-hazard models (where informative signals are never discarded) and the later models (where they are). The results here identify mis-allocation of effort as the reason. Moreover, it has been demonstrated that multi-tasking is neither necessary nor sufficient for effort to be mis-allocated. Multi-tasking is thus not essential for optimally discarding informative signals. Using the term 'multi-tasking problem' for the issues involved in incentive design in multi-tasking models is thus largely a red herring.

The results also challenge the Equal Compensation Principle. Marginal rates of return for different tasks do not need to be equal for effort to avoid mis-allocation of effort (as the principle suggests). Moreover, renouncing incentives can generally not be justified by the observation that only the performance of one of two tasks can be measured (as implied by the principle).

The proposed formalization has hopefully helped to clarify some claims from the multi-tasking literature and dismantle some myths.

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Appendix

Proof of Proposition 2 . Let us begin with part (i) and suppose that incentives are aligned. The proof works by contradiction: suppose that the agent’s behavior is distorted, i.e., he chooses an action \tilde{a} rather than a , although $b(a) > b(\tilde{a})$ and both require the same $e(a) = e(\tilde{a})$. Then, $R^I(a)$ is stochastically larger than $R^I(\tilde{a})$, because incentives are aligned. Consequently, $E[u_A(R^I(a), e^I)] \geq E[u_A(R^I(\tilde{a}), e^I)]$.

Then, however, the agent can only gain from deviating to action choice a . This, however, contradicts the assumption that the agent chooses \tilde{a} rather than a .¹¹ Hence, the assumption that the agent's behavior is distorted cannot be true.

Now turn to (ii). Suppose there are two arbitrary action choices, a and \tilde{a} , leading to the same effort $e(a) = e(\tilde{a})$. For the agent's behavior to be undistorted it must hold that $E [u_A(R^I(a), e^I)] \geq E [u_A(R^I(\tilde{a}), e^I)]$ if $b(a) > b(\tilde{a})$. In other words, $R^I(a)$ has to be stochastically larger than $R^I(\tilde{a})$, for all a and \tilde{a} with $b(a) > b(\tilde{a})$. This, however, is the definition of alignment. \square

¹¹Recall that in case of indifference the agent selects the action preferred by the principal.