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## Risk Sensitivity of Banks, Interbank Markets and the Effects of Liquidity Regulation

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# Risk Sensitivity of Banks, Interbank Markets and the Effects of Liquidity Regulation

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December 2012

First Draft. Please do not quote without permission.

## Abstract

The industrial organization approach to banking is extended to analyze the effects of interbank market activity and regulatory liquidity requirements on bank behavior. A multi-stage decision situation allows for considering the interaction between credit risk and liquidity risk of banks. This interaction is found to make a risk neutral bank behave as if it were risk averse in an environment where there is no interbank market and liquidity regulation. Introducing a buoyant interbank money market destroys endogenous risk aversion and allows banks to manage credit risk and liquidity risk independently. The paper shows that a liquidity regulation just like the one proposed in BCBS (2010) is not generally able to offset the separating effect of interbank money markets and recreate endogenous risk aversion of banks.

*Keywords: credit risk, liquidity risk, liquidity regulation, bank behavior*

*JEL classification: G21, G28, G32*

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The industrial organization approach to banking is extended to analyze the effects of interbank market activity and regulatory liquidity requirements on bank behavior. A multi-stage decision situation allows for considering the interaction between credit risk and liquidity risk of banks. This interaction is found to make a risk neutral bank behave as if it were risk averse in an environment where there is no interbank market and liquidity regulation. Introducing a buoyant interbank money market destroys endogenous risk aversion and allows banks to manage credit risk and liquidity risk independently. The paper shows that a liquidity regulation just like the one proposed in BCBS (2010) is not generally able to offset the separating effect of interbank money markets and recreate endogenous risk aversion of banks.

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## 1 Introduction

Risk sensitivity of banks appears to be closely related to the availability of buoyant financial markets. In the late 1980's and early 1990's a number of financial instruments established which were designed to support banks' risk management. The new instruments were usually traded in financial markets with trading volumes growing nearly exponentially from year to year and in which banks played a dominant role as actors on both sides of these markets. The majority of the new financial instruments were conceived to mitigate risks that are associated with risks that arise from maturity transformation and the riskiness of banks' assets. For instance, the emergence of loan securitization using asset backed securities not only allowed for transferring credit risk off the banks' balance sheet. These instruments gave banks

the opportunity to dispose on liquid funds immediately without being reliant on returns from maturing loans. Moreover, the new instruments were also widely accepted as collateral in interbank money markets and, in this way, helped banks to gain funds at short notice. As a consequence, credit risk and liquidity risk were increasingly managed independently which seemingly fostered the expansion of risky lending and the treatment of funding requirements at short notice using interbank money markets. That this proceeding made banks' business models more vulnerable to shocks in the financial system became obvious when the global financial crisis started in 2007. When interbank money markets dried up and loan securitization became unavailable a large number of banks were to collapse and had to be rescued by the states.

As a response, the Basel Committee on Banking Supervision (BCBS) not only revised its capital adequacy framework. Rather by end 2009 the BCBS for the first time published internationally standardized liquidity requirements for banks. These standards build on a number of principles for sound liquidity risk management and supervision<sup>1</sup> and develop basically two measures: the Liquidity Coverage Ratio (LCR) and the Net Stable Funding Ratio (NSFR). While the LCR aims at ensuring that banks are able to meet liquidity needs within a 30 calendar day time horizon, in particular the NSFR is designed to provide incentives for banks to consider liquidity risk over a longer period of time.<sup>2</sup> The two main objectives of the NSFR are, first, the limitation of an over-reliance on short-term wholesale funding during times of buoyant market liquidity and, second, to encourage a better assessment of liquidity risk across all on- and off-balance sheet items.<sup>3</sup>

This paper extends the industrial organization approach to banking to analyze the effect of the availability of buoyant interbank money markets on lending and deposit-taking decisions of a risk neutral bank and asks whether a NSFR-type regulatory liquidity requirement is able to support the BCBS's intention of encouraging a better assessment of liquidity risk and reduce the over-reliance on short-term wholesale funding. The paper develops a two-period version of the industrial organization approach to banking that allows for including a kind of endogenous liquidity risk into the analysis: Granting risky loans in the first period of the model causes uncertainty regarding profits in the following period. In this way the bank faces funding liquidity risk in the latter period since the proceeds from uncertain first-period loans may be used to prolongate first-period lending or grant new loans without additional costs. Therefore low earnings from first-period loans may require a bank to use a higher volume of costly external funds to finance a certain volume of lending in the second period.

The main results may be summarized as follows: an unregulated risk neutral

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<sup>1</sup>See BCBS (2008).

<sup>2</sup>See BCBS (2010).

<sup>3</sup>See BCBS (2010, p. 25).

bank behaves as if it were risk averse when there is no short-term interbank money market available. Today's credit risk exposure affects future profits which, in turn, affects future funding costs since internally generated funds (profits) are cheaper than external funds (deposits). This finding is closely related to Froot et al. (1993) and Froot and Stein (1998). However, in the present model it appears to be a consequence of the interaction of credit risk and funding liquidity risk on the cost level. The introduction of a buoyant interbank money market destroys endogenous risk aversion of risk neutral banks. It is shown that banks find it more beneficial to lend out internally generated funds (pre-stage profits) in the interbank market at a riskless interest rate than using internally generated profits to reduce the amount of deposits taken. However, NSFR-style liquidity requirements do revitalize risk averse behavior only in a situation when internally generated pre-stage profits are low and the bank needs funds from the interbank market (then NSFR creates a strict link between a bank's assets and liabilities). Otherwise the bank behaves risk neutral. The reason for this is that although the liquidity requirement directly affects lending and deposit taking decisions, the effect of NSFR is linear. The NSFR requirement only changes costs of lending and deposit taking relative to interbank market activities. As a result the regulation reduces the optimal volume of deposits. The regulatory effect on loans, however, is ambiguous. The results, therefore suggest that NSFR-style liquidity requirements only partially generate correct steering mechanisms. Borrowing and lending in the interbank market becomes less valuable to banks which is in line with the intention of the BCBS. However, there may be situations when banks reduce the total volume of deposits and grant more risky loans, ie increase credit risk exposure, compared to the unregulated case (this is the opposite of risk averse behavior!) in response to the regulation because the NSFR-type liquidity requirement creates incentives for integrated risk management (ie considering credit and liquidity risk as well as interactions) only in some states of the world.

The present paper is closely related to the literature on endogenously-forming risk attitudes (cf Froot et al. (1993) and Froot and Stein (1998)) as well as the literature on banks' decision making in the lending and deposit-taking business in situations of risk (cf, for instance, Wong (1997), Dermine (1986), Wahl and Broll (2000)). The model of the present paper is also related to the seminal papers of Diamond and Dybvig (1983) and Bryant (1980) by using a similar time structure. However, in contrast to these models, liquidity risk in the present paper is not a consequence of uncertain liquidity needs of depositors (liquidity shocks). Rather, liquidity risk arises from uncertain proceeds from loans (ie credit risk) which were granted at an earlier point in time. In other words the model of the present paper is, to the author's best knowledge, the first which combines the industrial organization approach of banking of Klein (1971) and Monti (1972) with the timely dynamic of Diamond and Dybvig (1983) and Bryant (1980). In this way the present model is able to capture the interaction between credit risk and liquidity risk and allows for

the explicit calculation of an evaluation function that may show the same features as a risk averse utility function.

The paper proceeds as follows: Section two sets out the model. Section three analyzes the benchmark situation: optimal behavior of a risk neutral bank that is not subject to liquidity regulation and that does not have access to an interbank market. Section four examines the effect of the introduction of a buoyant interbank money market. Section five considers the effect of a NSFR-type liquidity requirement on bank behavior. Section six concludes. The proofs of the propositions can be found in the appendix.

## 2 The base model

The analysis of the present paper builds on a modified and extended version of the industrial organization approach of banking. The seminal model of Klein (1971) and Monti (1972) with a risk neutral bank that enjoys market power in the deposit business and the loan business is augmented by uncertainty of the credit risk type. Furthermore, by considering an intermediate point of time where decisions need to be made the standard one-period setting is transferred to a two-period version of the model. This allows for explicitly considering the aspect of maturity transformation which is an inherent feature of the banking business and explains a major part of the economic value of banking.<sup>4</sup>

The bank faces an exogenously given (inverse) deposit supply function  $r_D(D)$  which is increasing and convex in the volume of deposits, ie  $r'_D(D) > 0$  and  $r''_D(D) > 0$ .<sup>5</sup> Deposits are contracted for one period only. Funding loans with a longer maturity, therefore, requires to roll over deposits after one period. Loans are basically contracted with a maturity of two periods. The exogenously given (inverse) loan demand function  $r_L(L)$  is decreasing and concave in the volume of loans, ie  $r'_L(L) < 0$  and  $r''_L(L) < 0$ .

Taking deposits and granting loans, moreover, causes management costs  $C(D, L)$  which are increasing and convex in deposits and loans as well, ie  $\partial C(D, L)/\partial i \equiv C_i(D, L) > 0$  and  $\partial^2 C(D, L)/(\partial i)^2 \equiv C_{ii}(D, L) > 0 \forall i = D, L$ . In addition, I do not

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<sup>4</sup>See the seminal papers of Diamond and Dybvig (1983) and Bryant (1980) for details.

<sup>5</sup>The deposit rate in the present model should be interpreted as an expected interest payment to depositors. Due to credit risk the bank faces a risk of bankruptcy which, in the absence of deposit insurance, needs to be assumed (at least partially) by depositors. In a seminal paper Dermine (1986) argues that in this situation a bank is not able to decide on deposit and loan rates independently. However, Dermine (1986) shows that the decisions on expected deposit rates and loan rates can be separated which also allows for a separation of a bank's decisions on the optimal volumes of deposits and loans.

consider any economies or diseconomies of scope by assuming  $\partial^2 C(D, L)/\partial D \partial L = 0$ .<sup>6</sup>

The deposit supply, the loan demand as well as the management costs do not depend on the point of time considered. In other words: these functions do not change over time.

There are three dates of decision making: at the initial date  $t = 0$  the bank, which owns no initial funds or capital, chooses the optimal volumes of deposits  $d$  and loans  $l$ . Since loans are long-term contracts they will mature in  $t = 2$  and the corresponding loan rate  $r_L(l)$  prevails until that time. Loans are, furthermore, subject to credit risk. To model credit risk I follow Wong (1997) and let  $\tilde{\mu} \in (0, 1)$  denote the random share of loans  $l$  that fail on paying interest at the interim date  $t = 1$ .

At the interim date  $t = 1$  the bank observes the realization of initial credit risk  $\tilde{\mu}$ , earns interest on performing loans, pays back deposits, which are short-term, and makes corresponding interest payments to depositors. As a result, the bank realizes an interim profit  $\pi$  which is entirely retained and may be used to prolongate the funding of loans granted in  $t = 0$  or to fund new loans at an interest rate  $r_L(l)$ . In other words: it is assumed that the bank does not make dividend payments to shareholders etc. and does not adjust the interest rate on loans which was fixed in  $t = 0$ .<sup>7</sup> The bank may, however, decide to adjust (ie reduce, increase or leave

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<sup>6</sup>This assumption is less restrictive than it may appear at a first glance. The results of the present paper might only change in case of very strong economies of scope. Although some parts of the literature argue that the joint production of deposits and loans provides the foundation for the economic value of banks. (See for instance Diamond (1984), Boyd and Prescott (1986), and Calomiris and Kahn (1991).) There is, however, another part of the literature that argues for the existence of a business volume when other cost-driving effects more than outweigh potential economies of scope between the deposit business and the loan business of the bank. For example Krasa and Villamil (1992) show in an agency-theoretic setting that economies of scale at banks arise when the number of risky investment projects (loans) increases as this creates a diversification of credit risk and reduces the bank's risk of insolvency. At the same time depositors' costs of monitoring a bank in the case of insolvency increase. As a result they find an optimal size of a bank, which is bounded and where benefits from diversification just outweigh monitoring costs. Cerasi and Daltung (2000) also provide theoretical arguments for the existence of an optimal size of a bank, which is bounded. They explain that as banks grow larger internal agency costs and communication costs increase and absorb potential benefits arising from a larger and better diversified loan portfolio.

<sup>7</sup>One may argue that assuming  $r_L(l)$  to be also the valid interest rate for new loans is not realistic. However, it is easily verified that relaxing this assumption does not change the qualitative results of the analysis but complicates the formal representations. In the interest of formal clarity, therefore, it seems to be acceptable to assume  $r_L(l)$  to remain unchanged.

unchanged) the total volume of loans. Let therefore  $L$  denote the bank's risky volume of loans to be held until  $t = 2$  and to be chosen optimally. Reducing the volume of loans in  $t = 1$  is costly to the bank because of foregone  $t = 2$ -profits. I refrain from imposing additional adjustment costs on the bank. Furthermore, let  $D$  denote the bank's volume of deposits to be taken at  $t = 1$  in order to fund lending until  $t = 2$ .

At the final date  $t = 2$  the credit risk parameter  $\tilde{\theta} \in (0, 1)$  materializes and determines the share of loans that fail on paying interest between  $t = 1$  and  $t = 2$ . The bank realizes a final profit which is to be maximized by decisions at earlier dates. Since both sources of credit risk  $\tilde{\mu}$  and  $\tilde{\theta}$  need not be stochastically independent, let  $F(\tilde{\mu}, \tilde{\theta}) \in (0, 1)$  denote the joint probability distribution function. In this context it appears to be reasonable to assume a non-negative stochastic relationship between both credit risk parameters. To include this in the model, I follow Wong (1996) and assume that the conditional probability distribution function of any  $\tilde{\theta}$  conditional on  $\tilde{\mu}$  deteriorates in the sense of first-order stochastic dominance, formally:

$$\frac{dF(\tilde{\theta}|\tilde{\mu})}{d\tilde{\mu}} \leq 0 \quad \forall \tilde{\theta}.$$

Before starting to analyze the model two aspects should be noted. First, the present model considers a going-concern situation: it is assumed that there does not appear any kind of crisis which requires liquidation of the bank under consideration. Therefore, the bank's decisions do not take into account future crisis events. Second, in the model endogenously arises liquidity risk in the sense that credit risk, which the bank assumes in  $t = 0$ , affects the costs of funding the loan volume  $L$  at  $t = 1$ . In other words: the present model considers liquidity risk in terms of uncertain funding costs in the future rather than a bank's ability to meet payment obligations (which is the traditional view in the literature)<sup>8</sup>.

### 3 Endogenous risk aversion

The analysis of a bank's optimal decisions on deposits and loans in the previously described setting will provide the benchmark for the examination of the effects arising from the availability of a buoyant interbank money market and liquidity regulation as currently proposed by the Basel Committee on Banking Supervision. The main focus of the analysis is on the way a risk neutral bank evaluates a risky decision making situation in which long-term loans are funded by short-term deposits and therefore requires rolling over deposits at the intermediate date. Internally

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<sup>8</sup>Consider for instance Diamond and Dybvig (1983) for details of the traditional notion of liquidity risk.



generated interim profits may help to mitigate the problem of rolling over deposits and reduce funding costs. Alternatively the bank may adjust the size of the loan portfolio in order to reduce funding requirements at the interim date. From an ex-ante point of view credit risk causes uncertainty of future profits and, hence, of funds available for lending at the interim date. The resulting decision-making problem by no means is trivial and is, therefore, examined carefully in the following.

### 3.1 The endogenously derived utility function

As it is common in the literature I start the analysis of the benchmark model by examining the bank's decisions at time  $t = 1$  taking the realizations of  $\tilde{\mu}$  and hence  $\tilde{\pi}$  as given (ie the bank observes the realizations of  $\tilde{\mu}$  and  $\tilde{\pi}$  before deciding on  $D$  and  $L$ ).

It is important to note that the bank's balance sheet constraint at  $t = 1$  is

$$L = \pi + D. \quad (1)$$

Funds available to prolongate loans granted in  $t = 0$  (and for new lending)  $L$  come from (new) deposits  $D$  and retained profits  $\pi$ . In other words: c.p. the volume of retained profits  $\pi$  directly affects the volume of funds available for lending. The balance sheet constraint is considered to be binding because it is not profitable for a bank to leave parts of available funds uninvested in loans as long as  $E(1 - \tilde{\theta})r_L(l) > 0$ . Note, funds that are not used for lending do not generate positive returns.

The bank's expected final profit at  $t = 2$  for any given realization of  $\tilde{\mu}$  is

$$E(\tilde{\Pi}(\tilde{\theta})|\tilde{\mu}) = (1 - E(\tilde{\theta}|\tilde{\mu}))r_L(l)L - r_D(L - \pi)(L - \pi) - C(L, L - \pi) \quad (2)$$

which is to be maximized by deciding on the optimal volume of loans  $L$  and deposits  $D$ . Since at the current date of decision making  $\pi$  is given, the bank only needs to optimally choose the volume of one of both businesses. When, eg, the volume of loans  $L$  is set to maximize expected profit in  $t = 1$ , the corresponding optimal volume of deposits  $D$  is determined by the balance sheet constraint (1). Using this finding, the bank's maximization problem can be rewritten as

$$\max_L E(\tilde{\Pi}(\tilde{\theta})|\tilde{\mu})$$

where  $E(\tilde{\Pi}(\tilde{\theta})|\tilde{\mu})$  is given in (2) above.

The optimal volume of loans  $L$  is defined by the first-order condition

$$\begin{aligned} \frac{dE(\tilde{\Pi}(\tilde{\theta})|\tilde{\mu})}{dL} &= (1 - E(\tilde{\theta}|\tilde{\mu}))r_L(l) - r_D(L - \pi) - r'_D(L - \pi)(L - \pi) - \\ &\quad - C_L(L, L - \pi) - C_D(L, L - \pi) = 0. \end{aligned} \quad (3)$$

From (3) it can be easily seen that the bank earns a strictly positive interest margin at the optimal volume of  $L$ . The expected net return from the loan business  $(1 - E(\tilde{\theta}|\tilde{\mu}))r_L(l)$  exceeds the payment obligations in the deposit business  $r_D(L - \pi)$ :

$$(1 - E(\tilde{\theta}|\tilde{\mu}))r_L(l) - r_D(L - \pi) = C_L(L, L - \pi) + C_D(L, L - \pi) + r'_D(L - \pi)(L - \pi) > 0$$

where the inequality follows from  $C_L(\cdot), C_D(\cdot) > 0$ ,  $r'_D(L - \pi) > 0$ , and  $L - \pi > 0$ .

The balance-sheet effect of  $\pi$  on the optimal volume of  $L$  can be figured out by applying the implicit function theorem to the first-order condition (3):

$$\frac{dL}{d\pi} = -\frac{\frac{\partial}{\partial \pi} \left( \frac{dE(\tilde{\Pi}(\tilde{\theta})|\tilde{\mu})}{dL} \right)}{\frac{\partial}{\partial L} \left( \frac{dE(\tilde{\Pi}(\tilde{\theta})|\tilde{\mu})}{dL} \right)}.$$

Partially differentiating (3) with respect to  $\pi$  yields for the numerator:

$$\frac{\partial}{\partial \pi} \left( \frac{dE(\tilde{\Pi}(\tilde{\theta})|\tilde{\mu})}{dL} \right) = 2r'_D + r''_D(L - \pi)(L - \pi) + C_{DD}(L, L - \pi) > 0$$

where the inequality follows from assumptions  $C_{DL}(\cdot) = C_{LD}(\cdot) = 0$ ,  $r'_D(\cdot), r''_D(\cdot) > 0$ ,  $L - \pi > 0$ , and  $C_{DD}(\cdot) > 0$ .

For the denominator partially differentiating (3) with respect to  $L$  yields:

$$\begin{aligned} \frac{\partial}{\partial L} \left( \frac{dE(\tilde{\Pi}(\tilde{\theta})|\tilde{\mu})}{dL} \right) &= -2r'_D(L - \pi) - r''_D(L - \pi)(L - \pi) \\ &\quad - C_{LL}(L, L - \pi) - C_{DD}(L, L - \pi) < 0 \end{aligned}$$

given that  $C_{DL}(\cdot) = C_{LD}(\cdot) = 0$ . The inequality is a result of  $r'_D(\cdot), r''_D(\cdot) > 0$  and  $C_{LL}(L, L - \pi), C_{DD}(L, L - \pi) > 0$ .

One, therefore, finds a positive relationship between the optimal volume of loans  $L$  and the interim profit  $\pi$ :

$$\frac{dL}{d\pi} > 0. \tag{4}$$

A closer look at the formal derivation gives an intuition for this result: when  $\pi$  increases, a bank can reduce the volume of deposits which are necessary to fund a given amount of loans. Since interest to be payed on deposits is increasing and convex in the volume of deposits, the bank's funding costs decrease disproportionately when  $\pi$  grows. The positive relationship between the optimal volume of loans  $L$  and  $\pi$  is, therefore, a consequence of a bank's funding costs which are directly affected by  $\pi$ : the higher  $\pi$  the lower the bank's cost to fund a given volume of loans.

Taking into account this relationship between  $L$  and  $\pi$  the bank's expected profit  $E(\tilde{\Pi}(\tilde{\theta})|\tilde{\mu})$ , which represents the bank's evaluation of the risky situation at date  $t = 1$  conditional on the realization of  $\tilde{\mu}$ , can be shown to be increasing and concave in  $\pi$ :

$$\frac{dE(\tilde{\Pi}(\tilde{\theta})|\tilde{\mu})}{d\pi}\Big|_{opt} = r_D(L - \pi) + r'_D(L - \pi)(L - \pi) + C_D(L, L - \pi) > 0 \quad (5)$$

and

$$\begin{aligned} \frac{d^2E(\tilde{\Pi}(\tilde{\theta})|\tilde{\mu})}{(d\pi)^2}\Big|_{opt} &= (2r'_D(L - \pi) + r''_D(L - \pi)(L - \pi) + C_{DD}(L, L - \pi)) \left( \frac{dL}{d\pi} - 1 \right) \\ &= -\frac{dL}{d\pi} (C_{LL}(L, L - \pi) + C_{DD}(L, L - \pi)) < 0. \end{aligned} \quad (6)$$

**Proof:** Differentiating the bank's expected conditional profit (2) with respect to  $\pi$  yields

$$\begin{aligned} \frac{dE(\tilde{\Pi}(\tilde{\theta})|\tilde{\mu})}{d\pi}\Big|_{opt} &= (1 - E(\tilde{\theta}|\tilde{\mu}))r_L(l) \frac{dL}{d\pi} - \\ &\quad - (r_D(L - \pi) + r'_D(L - \pi)(L - \pi)) \frac{dL}{d\pi} - \\ &\quad - (C_L(L, L - \pi) - C_D(L, L - \pi)) \frac{dL}{d\pi} + \\ &\quad + r_D(L - \pi) + r'_D(L - \pi)(L - \pi) + C_D(L, L - \pi). \end{aligned}$$

Note that the first three lines on the right hand of the latter equation are equal to

$$\left[ (1 - E(\tilde{\theta}|\tilde{\mu}))r_L(l) - r_D(L - \pi) - r'_D(L - \pi)(L - \pi) - C_L(\cdot) - C_D(\cdot) \right] \frac{dL}{d\pi}$$

and that the term in squared brackets is exactly the first-order condition for the optimal loan rate which is zero in the optimum (see (3)). Therefore the formal representation of the relationship between the bank's expected profit and  $\pi$  simplifies to (5) and the inequality comes from  $r_D(\cdot), r'_D(\cdot) > 0$ ,  $C_D(\cdot) > 0$  and  $L - \pi > 0$ .

The first line of (6) is the direct result of differentiating (5) with respect to  $\pi$ . While the first term of this line is unambiguously positive due to  $r'_D(\cdot), r''_D(\cdot), C_{DD}(\cdot), L - \pi > 0$ , the sign of the second term  $dL/d\pi - 1$  is not clear. That is, the previous analysis showed that there is a positive relationship between  $L$  and  $\pi$  in the optimum (see equation (4)). But the analysis so far did not provide any conclusions regarding the strength of this relationship. Therefore, the first-order condition for the optimal volume of loans (3) was used to replace  $r_D(\cdot) + r'_D(\cdot)(L - \pi) + C_{DD}(\cdot)$  in (5) by  $(1 - E(\tilde{\theta}|\tilde{\mu}))r_L(l) - C_L(L)$ . Differentiating the latter term with respect to  $\pi$  yields the second line of (6). The inequality follows from  $C_{LL}(\cdot), C_{DD}(\cdot) > 0$  and  $dL/d\pi > 0$ .  $\square$

Equation (5) shows that  $\pi$  positively affects the bank's optimal expected profit. The same equation also shows that this is a result of the previously mentioned

funding-costs effect: the higher  $\pi$  the less deposits a bank needs to attract in order fund a given amount of loans. Since the costs of taking deposits are increasing and convex in  $D$ , a higher amount of  $\pi$  makes it beneficial for the bank to increase lending (see equation (4)). In other words: leaving funding costs (and hence the volume of deposits  $D$ ) unchanged, the bank is able to grant a higher volume of loans which generate additional (expected) returns which, in turn, increases the bank's expected profit.

The results derived so far can be summarized as follows: the bank's expected  $t = 2$ -profit is an increasing and concave function of the profit  $\pi$  earned from business activities in previous times. In other words: due to the random share  $\tilde{\theta}$  of non-performing loans in  $t = 1$ , the per se risk neutral bank evaluates the final profits that will be earned in  $t = 2$  for a given realization of  $\pi$  by taking expectations over all potential realizations of  $\tilde{\theta}$ . Let  $U$  represent the bank's function to evaluate  $t = 2$ -profits for a given realization of  $\pi$  in general, then one can write more explicitly:

$$U(\pi) \equiv E(\tilde{\Pi}(\tilde{\theta})|\tilde{\mu}). \quad (7)$$

Furthermore, the evaluation of (random)  $t = 2$ -profits at  $t = 1$  was found to be increasing and concave in the volume of  $\pi$  (see equations (5) and (6), respectively). This implies for the evaluation function  $U(\cdot)$ :

$$U'(\pi) > 0 \text{ and } U''(\pi) < 0. \quad (8)$$

If one recalls that, basically,  $\pi$  is also a random variable (at least at  $t = 0$  when the banks makes decisions that determine the level of  $\pi$  earned in  $t = 1$ ),  $U(\cdot)$  may be interpreted as a utility function which exhibits risk aversion as it is widely used in the literature to analyze decision making in situations of risk. In contrast to the literature, in the setting of the present paper this kind of utility functions arises endogenously from the impact of  $\pi$  on the opportunities to earn profits in the future (ie  $t = 2$ ) rather than being assumed to be given exogenously. I can thus state

**Proposition 1** *The interaction of credit risk at date  $t = 0$  and its effect on funding costs at date  $t = 1$  make the per se risk neutral bank to evaluate the risky situation at  $t = 1$  in a risk averse way.*

Moreover, from (6) I can conclude that a higher profit from previous business activities makes the bank's optimal volume of loans  $L$  increase disproportionately. The result of equation (4) can, hence, be extended to

**Corollary 1**

$$1 > \frac{dL}{d\pi} > 0. \quad (9)$$

Furhtermore, using the balance sheet constraint (1) this result implies

**Corollary 2**

$$\frac{dD}{d\pi} = \frac{dL}{d\pi} - 1 < 0. \quad (10)$$

An increase of the profit  $\pi$  is used for both extending the loan business and reducing funding via taking deposits and, therefore, affects both sides of a risk neutral bank's balance sheet.

### 3.2 Risk averse decisions

The current subsection will show that the endogenously risk averse evaluation of the risky situation at date  $t = 1$  will affect bank behavior at date  $t = 0$ . A major driving force in this context is that the bank's choice of deposits  $d$  and loans  $l$  at  $t = 0$ , on the one hand, directly determine the interim profit  $\tilde{\pi}$ . On the other hand, the bank is long-term oriented and is to set  $d$  and  $l$  in  $t = 0$  in a way that maximizes the expected profit in  $t = 2$ . The interim profit  $\tilde{\pi}$  is only of indirect relevance because it affects funds available in  $t = 1$ . The bank's  $t = 2$ -profit, therefore, includes the profit  $\tilde{\pi}$  in  $t = 1$ :

$$\begin{aligned} \tilde{\Pi}(\tilde{\theta}, \tilde{\mu}) &= (1 - \tilde{\theta})r_L(l)L(\tilde{\pi}) - r_D(L(\tilde{\pi}) - \tilde{\pi})(L(\tilde{\pi}) - \tilde{\pi}) - C(L(\tilde{\pi}), L(\tilde{\pi}) - \tilde{\pi}) \\ &\text{with} \\ \tilde{\pi} &= (1 - \tilde{\mu})r_L(l) - r_D(d)d - C(d, l). \end{aligned} \quad (11)$$

Since the bank is not endowed with deposits and loans when entering  $t = 0$ , the initial balance sheet constraint can be written as

$$l = d. \quad (12)$$

Lending out an amount of  $l$  loans in  $t = 0$  requires to take deposits (at least) of the same amount. But because deposit-taking is costly to the bank it is not profit maximizing to acquire more deposits than are necessary for lending. The balance sheet constraint (12) is, hence, considered to be binding in the optimum.

For formulating the maximization problem of the bank in  $t = 0$  note that the calculation of the expected profit in  $t = 2$  comprises both sources of credit risk,  $\tilde{\theta}$  as well as  $\tilde{\mu}$ :

$$\max_{d, l} E(\tilde{\Pi}(\tilde{\theta}, \tilde{\mu})) = \int_{\tilde{\mu}} \int_{\tilde{\theta}} \tilde{\Pi}(\tilde{\theta}, \tilde{\mu}) dF(\tilde{\theta}|\tilde{\mu}) dF(\tilde{\mu}) \quad (13)$$

where  $\tilde{\Pi}$  is defined in (11) and  $L(\tilde{\pi})$  is the result of the bank's optimization at time  $t = 1$  which was found to depend on the realization of  $\tilde{\pi}$ .

Given the binding balance sheet constraint (12) it is sufficient to choose either the optimal volume of loans or the optimal volume of deposits to find the bank's profit maximizing business volume. Therefore, in the profit function (11)  $d$  is replaced by  $l$  which, in turn, is chosen such that the bank's expected profit is maximized. The corresponding first-order necessary condition can then be written as follows<sup>9</sup>:

$$\frac{dE(\tilde{\Pi})}{dl} = \int_{\tilde{\mu}} \int_{\tilde{\theta}} (1 - \tilde{\theta}) r'_L(l) L(\tilde{\pi}) dF(\tilde{\theta}|\tilde{\mu}) dF(\tilde{\mu}) + E \left( U'(\tilde{\pi}) \frac{\partial \tilde{\pi}}{\partial l} \right) = 0. \quad (14)$$

The first-order condition (14) is composed of two parts: first, the integral term in (14) represents the expected marginal long-term income from fixed-rate loans. This part appears due to the assumption that the loan rate set in  $t = 0$  does not change when making decisions in  $t = 1$ . If one, alternatively, assumes that loans are granted using floating interest rates – ie giving the bank the opportunity to adjust loan rates in  $t = 1$  – the integral term disappears. Second, the expected marginal utility of the banks lending decision in  $t = 0$  is, actually, a long-term oriented evaluation of risky lending activities. In particular, risky lending in  $t = 0$  makes the interim profit  $\tilde{\pi}$  in  $t = 1$  uncertain from an ex-ante perspective and creates a certain kind of funding liquidity risk for the bank in  $t = 1$ . That is,  $\tilde{\pi}$  affects funds available for lending in  $t = 1$  as well as funding costs. In order to fund a certain amount of loans  $L$  in  $t = 1$  the bank, depending on the realization of  $\tilde{\pi}$  needs to take a higher or lower amount of deposits  $D$ . This affects the interest rate to be payed on deposits where the concavity of the deposit supply function leads to convex funding costs with respect to deposits.

The joint effect of both parts of the first-order condition (14) leads to

**Proposition 2** *The per se risk neutral long-term oriented bank behaves endogenously risk averse in  $t = 0$  and chooses lower volumes of deposits  $d$  and loans  $l$  than a risk neutral short-term oriented bank.*

**Proof:** See the Appendix.  $\square$

The endogenously risk averse behavior, in the end, reflects the fact that the bank applies an integrated approach to manage assets, liabilities and risks including longer-term risk effects. In particular risk management comprises the interaction between credit and liquidity risk (in the sense of risky funding costs) over time.

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<sup>9</sup>For a detailed derivation see the Appendix.

## 4 The effect of an interbank market

Before the onset of the 2007-2009 financial crisis banks intensively used financial markets to lend and borrow funds. In particular (unsecured) interbank money markets played an important role to gain funds which were needed to meet funding requirements at short notice. One of the main features of this kind of interbank markets was that they were buoyant, ie in the short run interest rates that had to be paid were stable and bid-ask spreads were low and stable, too.

In the following I will focus on this kind of interbank market environment. It was one of the major lessons learned from the 2007-2009 financial crisis that banks tend to oversee potential threats when their business activities heavily rely on well-functioning interbank markets. Not surprisingly, this observation was also one of the main justifications for the creation of liquidity standards which were recently published by the Basel Committee on Banking Supervision.<sup>10</sup>

But before analyzing the effects of the new liquidity standards on bank behavior - I will do this in the next section - it needs to be figured out whether and in which way a buoyant interbank money market affects banks' optimal behavior in the loan and deposit business. For this purpose the model is extended by an interbank market which is available in  $t = 0$  and  $t = 1$  as well. Banks can lend or borrow any desired amount of funds at an interest rate  $r$  which they consider exogenously given. The interest rate is time invariant and applies in  $t = 0$  as well as  $t = 1$ . It is easy to see that this set of assumption quite well represents the previously explained environment of buoyant interbank markets before the onset of the 2007-2009 financial crisis.

Let, furthermore,  $m$  and  $M$  denote the bank's position in the interbank market in  $t = 0$  and  $t = 1$ , respectively. A bank's position in the interbank market may be positive or negative as well. A positive position means that the bank is a lender in the interbank market. If a bank acts as a borrower in the interbank market  $m$  or  $M$  will be negative.

The analysis, again, runs down in two steps: first determine a bank's optimal decisions at time  $t = 1$  for given  $t = 0$  decisions. Second determine optimal decisions in  $t = 0$  by taking into account decisions in  $t = 1$ .

### 4.1 Separation of assets and liabilities management

The bank's decisions on deposits  $D$  and loans  $L$  at  $t = 1$  are, basically, made in perfect analogy to the benchmark situation without interbank market in section 3.1 above. The conditional expected profit to be maximized needs to be extended,

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<sup>10</sup>See BCBS (2010).

however, by profits earned or costs paid in the interbank market:

$$\mathbb{E} \left( \tilde{\Pi}(\tilde{\theta}) | \tilde{\mu} \right) = \left( 1 - \mathbb{E}(\tilde{\theta} | \tilde{\mu}) \right) r_L(l)L + rM - r_D(D)D - C(D, L). \quad (15)$$

The availability of the interbank market also affects the bank's balance sheet constraint:

$$L + M = \pi + D. \quad (16)$$

Using the balance sheet constraint (16) to replace M in (15) yields

$$\mathbb{E} \left( \tilde{\Pi}(\tilde{\theta}) | \tilde{\mu} \right) = \left( (1 - \mathbb{E}(\tilde{\theta} | \tilde{\mu}))r_L(l) - r \right) L + (r - r_D(D)) D - C(D, L) + r\pi \quad (17)$$

which the bank maximizes by deciding on  $D$  and  $L$ .

The corresponding first-order necessary conditions are

$$\frac{\partial \mathbb{E}(\tilde{\Pi}(\tilde{\theta}) | \tilde{\mu})}{\partial D} = r - r_D(D) - r'_D(D)D - C_D(D, L) = 0 \quad (18)$$

$$\frac{\partial \mathbb{E}(\tilde{\Pi}(\tilde{\theta}) | \tilde{\mu})}{\partial L} = (1 - \mathbb{E}(\tilde{\theta} | \tilde{\mu}))r_L(l) - r - C_L(D, L) = 0. \quad (19)$$

For an analysis of the impact of  $\pi$  on the optimal volumes of  $D$  and  $L$  I apply, just like in section 3.1, the implicit function theorem to the first-order necessary conditions (18) and (19). In the present situation, however, one observes immediately

$$\frac{dD}{d\pi} = \frac{dL}{d\pi} = 0$$

due to

$$\frac{\partial^2 \mathbb{E}(\tilde{\Pi}(\tilde{\theta}) | \tilde{\mu})}{\partial D \partial \pi} = \frac{\partial^2 \mathbb{E}(\tilde{\Pi}(\tilde{\theta}) | \tilde{\mu})}{\partial L \partial \pi} = 0.$$

That is, in contrast to the benchmark case the profit  $\pi$  which arises from the bank's decisions in  $t = 0$  does not affect the optimal volumes of deposits and loans in  $t = 1$ . As a consequence, the expected profit in  $t = 1$  is linearly affected by the level of  $\pi$ :

$$\frac{d\mathbb{E}(\tilde{\Pi}(\tilde{\theta}) | \tilde{\mu})}{d\pi} = r ; \quad \frac{d^2 \mathbb{E}(\tilde{\Pi}(\tilde{\theta}) | \tilde{\mu})}{(d\pi)^2} = 0.$$

Therefore, if one again defines the utility function to evaluate  $\pi$  in the same way as in section 3.1, ie

$$U(\pi) \equiv \mathbb{E}(\tilde{\Pi}(\tilde{\theta}) | \tilde{\mu}),$$

one observes that the availability of an interbank market in  $t = 1$  prevents an endogenously risk averse evaluation of early profits  $\pi$ . The risk neutrality of the bank remains intact.



**Proposition 3** *The availability of a buoyant interbank market in  $t = 1$  separates the bank's decisions on  $D$  and  $L$  at that time. Moreover, the interbank market separates  $D$  and  $L$  from earlier business decisions and makes the bank to evaluate risky prospects in  $t = 1$  in a risk neutral way.*

To give an intuition for Proposition 3 consider the first-order conditions (18) and (19) above. The interbank market separates the liabilities, ie deposit, side from the assets, ie loan, side of the bank's balance sheet. In this respect the interbank market acts as a kind of benchmark in the determination of the optimal volumes of  $D$  and  $L$ : In the deposit business from (18) one observes a strictly positive interest rate spread between the interbank rate  $r$  and the deposit rate  $r_D$  in the optimum:

$$r - r_D(D) = r'_D(D)D + C_D(D, L) > 0.$$

The inequality results from the assumptions  $r'_D(D) >$  and  $C_D(\cdot) > 0$ . For the bank it is, therefore, beneficial to increase the volume of deposits as long as doing so is cheaper than borrowing funds in the interbank market. Moreover, since increasing the amount of  $D$  also requires to increase the deposit rate  $r_D$  and causes higher operative costs  $C$  it is not valuable for the bank to increase  $D$  until  $r = r_D$ . Given this logic behind the bank's deposit decision it is not surprising that a potential profit  $\pi$  is irrelevant in this context. Note that using  $\pi$  to reduce the volume of deposits taken while leaving constant the volume of loans may increase the bank's expected profit by approximately

$$\pi \frac{\text{E}(\tilde{\Pi}(\tilde{\theta})|\tilde{\mu})}{\partial D} = r\pi - (r_D(D) + r'_D(D)D + C_D(D, L))\pi > 0$$

for any  $D$  smaller than the one that satisfies the first-order condition (18) due to the concavity of the expected profit in  $D$ .<sup>11</sup> Alternatively, lending out  $\pi$  in the interbank market at the certain interest rate  $r$  and leaving  $D$  and  $L$  unchanged increases the bank's expected profit by

$$r\pi > r\pi - (r_D(D) + r'_D(D)D + C_D(D, L))\pi$$

because of  $r_D(D), r'_D(D), C_D(D, L) > 0$ . In other words: the bank finds it more profitable to completely invest profits earned on earlier business decisions in the interbank market than reducing the volume of deposits taken.

A similar line of argument may be applied to show that using  $\pi$  to extend its loan business is also not beneficial for the bank. From (19) one observes that in

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<sup>11</sup>It is easily verified that

$$\frac{\partial^2 \text{E}(\tilde{\Pi}(\tilde{\theta})|\tilde{\mu})}{(\partial D)^2} = -2r'_D(D) - r''_D(D)D - C_{DD}(D, L) < 0.$$

the optimum  $L$  ensures that the bank earns a strictly positive expected interest rate spread in the loan business

$$(1 - E(\tilde{\theta}|\tilde{\mu}))r_L(l) - r = C_L(D, L) > 0.$$

The expected interest rate spread in the loan business, hence, just covers the marginal costs of loans in the optimum which are assumed to be strictly positive. Furthermore, in  $t = 1$   $L$  does, by assumption, not affect the loan rate. Nevertheless, using profit from earlier business decisions  $\pi$  to further increase the volume of granted loans leaving the volume of deposits unchanged reduces the bank's expected profit in  $t = 1$  by approximately

$$\pi \frac{\partial E(\tilde{\Pi}(\tilde{\theta})|\tilde{\mu})}{\partial L} = \pi \left( (1 - E(\tilde{\theta}|\tilde{\mu}))r_L(l) - r - C_L(D, L) \right) < 0$$

for any  $L$  larger than the one that satisfies the first-order condition (19) due to the concavity of the expected profit in  $L$ .<sup>12</sup> That is, due to the convexity of the cost function in  $L$  increasing the volume of loans beyond the level that is defined by the first-order condition (19) is not beneficial. In contrast, lending out  $\pi$  at the certain interest rate  $r$  in the interbank market immediately increases the bank's expected profit by

$$r\pi.$$

In sum, the bank will not adjust the optimal volumes of deposits  $D$  and loans  $L$  when the profit from  $\pi$  earned from business decisions in  $t = 0$  changes. Regardless of  $\pi$  the optimal decisions on  $D$  and  $L$  are constant and defined by the first-order conditions (18) and (19).

**Corollary 3** *Any additional unit of  $\pi$  is used to increase the bank's interbank market position by the same amount:*

$$\frac{dM}{d\pi} = \frac{dD}{d\pi} - \frac{dL}{d\pi} + 1 = 1.$$

The optimal volumes of deposits  $D$  and loans  $L$  are set in a way that all costs and benefits – including using the interbank market – are balanced optimally. A reduction of  $D$  or / and an increase of  $L$  which is funded by an additional unit of  $\pi$  distorts this balance and is, hence, not optimal. Consequently, the maximum additional profit is earned when any additional unit of  $\pi$  is completely lent out in the interbank market.

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<sup>12</sup>Standard calculations show:

$$\frac{\partial^2 E(\tilde{\Pi}(\tilde{\theta})|\tilde{\mu})}{(\partial L)^2} = -C_{LL}(D, L) < 0.$$

## 4.2 Separation of risk management over time

The analysis of optimal bank behavior in  $t = 0$  can now be carried out based on the findings in the previous section. Just like in the benchmark situation in section 3.2, in  $t = 0$  the bank chooses  $d$  and  $l$  in a way that the expected final profit  $\tilde{\Pi}$  in  $t = 2$  becomes maximal. However,  $d$  and  $l$  directly determine only  $\tilde{\pi}$ . The final profit  $\tilde{\Pi}$  is affected by  $d$  and  $l$  only indirectly via  $\tilde{\pi}$ .

Given that the bank's decisions  $D$  and  $L$  in  $t = 1$  have been found to be independent of  $\pi$ , the profit of the bank in  $t = 2$  can be written as

$$\tilde{\Pi}(\tilde{\theta}, \tilde{\mu}) = \left( (1 - \tilde{\theta})r_L(l) - r \right) L + (r - r_D(D)) D - C(D, L) + r\tilde{\pi} \quad (20)$$

with

$$\tilde{\pi} = (1 - \tilde{\mu})r_L(l)l + rm - r_D(d)d - C(d, l) \quad (21)$$

and

$$l + m = d \quad (22)$$

because of the availability of a buoyant interbank market in  $t = 0$ .

The bank's maximization problem can, hence, be written as

$$\max_{d, l} E(\tilde{\Pi}(\tilde{\theta}, \tilde{\mu})) = \int_{\tilde{\mu}} \int_{\tilde{\theta}} \tilde{\Pi}(\tilde{\theta}, \tilde{\mu}) dF(\tilde{\theta}|\tilde{\mu}) dF(\tilde{\mu})$$

where  $\tilde{\Pi}(\tilde{\theta}, \tilde{\mu})$  is defined in (20) above, which includes  $\tilde{\pi}$  as defined in (21) as well as the balance sheet constraint (22).

The first-order necessary conditions which determine the optimal volumes of deposits  $d$  and loans  $l$  in  $t = 0$  are:

$$\frac{\partial E(\tilde{\Pi})}{\partial d} = \int_{\tilde{\mu}} \int_{\tilde{\theta}} r(r - r_D(d) - r'_D(d)d - C_D(d, l)) dF(\tilde{\theta}|\tilde{\mu}) dF(\tilde{\mu}) = 0 \quad (23)$$

$$\begin{aligned} \frac{\partial E(\tilde{\Pi})}{\partial l} &= \int_{\tilde{\mu}} \int_{\tilde{\theta}} (1 - \tilde{\theta})r'_L(l)L dF(\tilde{\theta}|\tilde{\mu}) dF(\tilde{\mu}) + \\ &+ \int_{\tilde{\mu}} \int_{\tilde{\theta}} r((1 - \tilde{\mu})(r_L(l) + r'_L(l)l) - r - C_L(d, l)) dF(\tilde{\theta}|\tilde{\mu}) dF(\tilde{\mu}) = 0. \end{aligned} \quad (24)$$

Inspection of (23) shows that the terms are not affected by random variables. Taking additionally into account that  $r$  was assumed to be strictly positive, the first-order condition for the optimal volume of  $d$  may be rewritten as

$$r - r_D(d) - r'_D(d)d - C_D(d, l) = 0.$$

It is easily verified that this modified first-order condition for the optimal  $d$  is equivalent to the first-order condition of a bank that aims at choosing  $d$  in a way to

maximize  $E(\tilde{\pi})$ . That is, future business decisions do not affect the optimal volume of deposits of the risk neutral bank in the present model in  $t = 0$ .

Using standard calculations, the first-order condition for the optimal loan volume  $l$  (24) can also be rewritten:

$$\int_{\tilde{\mu}} \int_{\tilde{\theta}} (1 - \tilde{\theta}) r'_L(l) L dF(\tilde{\theta}|\tilde{\mu}) dF(\tilde{\mu}) + r E \left( \frac{\partial \tilde{\pi}}{\partial l} \right) = 0 \quad (25)$$

where

$$E \left( \frac{\partial \tilde{\pi}}{\partial l} \right) = \frac{\partial E(\tilde{\pi})}{\partial l} = (1 - E(\tilde{\mu})) (r_L(l) - r'_L(l)l) - r - C_L(d, l).$$

From (25) one can, first, observe that it significantly differs from the first-order condition of a bank choosing  $l$  such that  $E(\tilde{\pi})$  is maximized. The reason for this is that the  $l$ -decision in  $t = 0$  fixes the interest rate earned on loans not only for this period. Rather  $r_L(l)$  is, by assumption, still relevant for loan decisions to be made in  $t = 1$ . The effect of this long-term decision on bank behavior is captured in the first term, ie the integral term, in (24). Also note that this term is the same as was derived in the first-order condition for the optimal volume of  $l$  in the benchmark situation without interbank market (14). The major difference between the first-order condition in the benchmark situation and the present situation with interbank market availability comes from the second term of (25). Instead of

$$E \left( U'(\tilde{\pi}) \frac{d\tilde{\pi}}{dl} \right)$$

which was found to be the second term in the first-order condition in the benchmark case, in the present situation the second term in the first-order condition for the optimal volume of  $l$  is

$$r E \left( \frac{\partial \tilde{\pi}}{\partial l} \right).$$

That is, the bank evaluates the impact of the loan decision  $l$  based on the constant interbank rate  $r$ . This has two effects: First, the risk neutral bank does not behave as if it were risk averse. In contrast to the benchmark case without interbank market there arises no endogenous risk aversion and decisions are completely driven by risk neutrality. Second, the interconnection between the time periods due to the impact of  $\pi$  on a bank's business decisions in  $t = 1$  does no longer exist when a buoyant interbank market is available. This is also the reason why I do not observe an endogenous risk aversion in the present situation.

Nevertheless, because of the assumption of fixed interest rates on loans – ie  $r_L(l)$ , which is (implicitly) chosen in  $t = 0$ , also prevails in  $t = 1$  – there is a link between the time periods which affects the optimal volume of loans in the benchmark situation as well in the present situation with interbank market. This effect, which is

represented by the term

$$\int_{\tilde{\theta}} \int_{\tilde{\mu}} (1 - \tilde{\theta}) r'_L(l) L dF(\tilde{\theta}|\tilde{\mu}) dF(\tilde{\mu})$$

in both first-order conditions (14) and (25), however, enters the decision on  $l$  additively and does not affect the mode of decision making. If one were to assume that the bank can adjust the loan rate in  $t = 1$ , the latter term and hence its linear impact on the decision on  $l$  would vanish.

The next Proposition summarizes the findings of the present section:

**Proposition 4** *The availability of a buoyant interbank money market enables a risk neutral bank to separate decisions on loans  $l$  from decisions on deposits  $d$ . This is not only true at a certain point in time, this is also true over time. In this way the interbank money market also separates the management of credit risk and liquidity risk and prevents endogenous risk averse behavior of the considered bank.*

## 5 Liquidity regulation

The results derived in the previous section may be interpreted as arguments that support the implementation of regulatory liquidity requirements for banks. Indeed, observations made during the 2007-2009 financial crisis suggest that banks increasingly used buoyant interbank markets to manage liquidity risk at short notice before the onset of the crisis. Moreover, observations suggest that banks behaved less risk sensitive and became more and more vulnerable to external shocks at that time. When interbank markets dried up after the crisis had started, funding became a major problem for a number of banks.

As a response to this adverse development during the crisis the Basel Committee on Banking Supervision (BCBS) formulated internationally harmonized liquidity standards. In particular the Net Stable Funding Ratio (NSFR)

[...] aims to limit over-reliance on short-term wholesale funding during times of buoyant market liquidity and encourage better assessment of liquidity risk across all on- and off-balance sheet items. (BCBS, 2010, p. 25)

Therefore, in the following I analyze a bank's behavior in the deposit and loan business that faces a NSFR-type liquidity regulation.

## 5.1 Modeling liquidity regulation

The general idea behind the NSFR is that all illiquid assets and securities held by a bank require a certain amount of stable funding. The NSFR considers a time period of one year and is defined as the ratio of the available amount of stable funding (ASF) over the required amount of stable funding (RSF). This ratio is required to be at least one:<sup>13</sup>

$$\text{NSFR} = \frac{\text{ASF}}{\text{RSF}} \geq 1.$$

The ASF contains equity capital, other liabilities with a maturity of at least one year, deposits with a maturity of less than one year but which are expected to stay with the bank even in a crisis situation, and wholesale funding with a maturity of less than one year but which is available also in a crisis. For the calculation of ASF the BCBS defines a number of categories of capital and liabilities. Each category has assigned a so called ASF factor to weight corresponding capital and liabilities positions. Banks categorize their capital and liabilities by this taxonomy and multiply the respective amount with the corresponding ASF factor. The total ASF is then calculated as the sum over all weighted categories.<sup>14</sup>

To calculate the ASF in the context of the model of the present paper it should, first, be noted that all the ASF factors are exogenously given by the BCBS and constant. Second, the profit  $\pi$  which arises from the bank's deposit and loan decisions in  $t = 0$  is assumed to be retained in the present model. Regarding ASF  $\pi$ , therefore, belongs to the equity category of the NSFR and is assigned an ASF factor of one – ie  $\pi$  is considered as an Available Stable Funding to the full amount. Third, deposits  $d$  and  $D$  basically represent demand deposits within the present model framework. They are considered in the calculation of ASF with an ASF factor of 0.8. Fourth, funds borrowed in the interbank market get an ASF factor of zero. That is,  $M < 0$  in the present model does not add to the ASF.<sup>15</sup> For including the ASF in the model, I define the following function to be considered in  $t = 1$ :

$$\begin{aligned} ASF = ASF(D, \pi) &= ASF_D(D, \pi) \cdot D + ASF_\pi(D, \pi) \cdot \pi \\ &\text{with} \end{aligned} \tag{26}$$

$$ASF_D(D, \pi) = \frac{\partial ASF}{\partial D} = 0.8 > 0 \quad \text{and} \quad ASF_\pi(D, \pi) = \frac{\partial ASF}{\partial \pi} = 1 > 0.$$

The ASF function to be considered in  $t = 0$  is defined analogously with the

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<sup>13</sup>See BCBS (2010), p. 25.

<sup>14</sup>See BCBS (2010), p. 26.

<sup>15</sup>See BCBS (2010), p. 27.

exception that there is no  $\pi$  at that point in time. I, therefore, write

$$\begin{aligned} ASF = ASF(d) &= ASF_d(d) \cdot d \\ &\text{with} \\ ASF_d(d) &= \frac{\partial ASF}{\partial d} = 0.8 > 0. \end{aligned} \tag{27}$$

In addition note that in both cases second-order derivatives and cross derivatives of are all zero because of constant exogenous ASF factors.

The RSF is defined and calculated in a similar way. BCBS (2010, p. 29f.) defines a number asset categories and assigns RSF factors to each. The RSF factors depend on the liquidity risk profile of the respective assets. Loans to retail costumers as they are considered in the model framework of the present paper are assigned a RSF factor of 0.85. Funds lend out in the interbank market even are considered to their full amount in the RSF. That is, if the bank in the model chooses  $M > 0$  this enters RSF with a RSF factor of one.

To model the RSF, I define the following RSF function which the bank needs to consider in  $t = 1$ :

$$\begin{aligned} RSF = RSF(L, M) &= RSF_L(L, M) \cdot L + RSF_M(L, M) \cdot M \\ &\text{with} \\ RSF_L(L, M) &= \frac{\partial RSF}{\partial L} = 0.85 > 0 \\ RSF_M(L, M)|_{M \geq 0} &= \frac{\partial RSF}{\partial M}|_{M \geq 0} = 1 > 0 \\ RSF_M(L, M)|_{M < 0} &= \frac{\partial RSF}{\partial M}|_{M < 0} = 0. \end{aligned} \tag{28}$$

The RSF function to be considered in  $t = 0$  looks similar:

$$\begin{aligned} RSF = RSF(l, m) &= RSF_l(l, m) \cdot l + RSF_m(l, m) \cdot m \\ &\text{with} \\ RSF_l(l, m) &= \frac{\partial RSF}{\partial l} = 0.85 > 0 \\ RSF_m(l, m)|_{m \geq 0} &= \frac{\partial RSF}{\partial m}|_{m \geq 0} = 1 > 0 \\ RSF_m(l, m)|_{m < 0} &= \frac{\partial RSF}{\partial m}|_{m < 0} = 0. \end{aligned} \tag{29}$$

Note that the definitions and calculation rules of the BCBS imply the  $RSF(\cdot)$  to be right-continuous at  $M = 0$  and  $m = 0$  with respect to the interbank market exposure  $M$  or  $m$ . Furthermore, just like with ASF all second-order derivatives and cross derivatives of RSF are zero because of the constant and exogenously given RSF factors.

## 5.2 Liquidity regulation and interbank markets

The analysis of the impact of the previously described kind of liquidity regulation starts – just like the analysis in the previous sections – with the bank’s decision making at time  $t = 1$ . I examine the optimal volumes of deposits  $D$  and loans  $L$  as well as the optimal position in the interbank market  $M$  which are set by a regulated bank given a certain realization of profit  $\pi$ . The optimal decisions solve the following maximization problem:

$$\max_{D,L,M} E \left( \tilde{\Pi}(\tilde{\theta}) | \tilde{\mu} \right) = \left( 1 - E(\tilde{\theta} | \tilde{\mu}) \right) r_L(l)L + rM - r_D(D)D - C(D, L) \quad (30)$$

$$\text{s.t. } L + M \leq D + \pi \quad (31)$$

$$RSF(L, M) \leq ASF(D, \pi). \quad (32)$$

The bank chooses deposits  $D$ , loans  $L$ , and interbank market exposure  $M$  such that the expected profit conditional on the realization of  $\tilde{\pi}$ , and hence  $\tilde{\mu}$ , is maximized. These decisions are constrained by balance sheet limitations (31) and the NSFR-type liquidity regulation (32). Both constraints are formulated as inequalities, because it is not clear per se which one (or whether even both) will bind in the optimum. Consequently, the solution to the optimization problem will be derived using the Kuhn-Tucker Theorem.

Based on the corresponding Lagrangian

$$\mathcal{L} = E \left( \tilde{\Pi}(\tilde{\theta}), \tilde{\mu} \right) + \Lambda (D + \pi - L - M) + \Phi (ASF(D, \pi) - RSF(L, M))$$

the first-order necessary conditions for the optimal decisions  $D$ ,  $L$ ,  $M$ ,  $\Lambda$ , and  $\Phi$  can be derived where  $\Lambda$  and  $\Phi$  represent the Lagrangian Multipliers for the constraints (31) and (32), respectively:<sup>16</sup>

$$\frac{\partial \mathcal{L}}{\partial D} = -r_D(D) - r'_D(D)D - C_D(D, L) + \Lambda + \Phi ASF_D(D, \pi) = 0 \quad (33)$$

$$\frac{\partial \mathcal{L}}{\partial L} = \left( 1 - E(\tilde{\theta} | \tilde{\mu}) \right) r_L(l) - C_L(D, L) - \Lambda - \Phi RSF_L(L, M) = 0 \quad (34)$$

$$\frac{\partial \mathcal{L}}{\partial M} = r - \Lambda - \Phi RSF_M(L, M) = 0 \quad (35)$$

$$\frac{\partial \mathcal{L}}{\partial \Lambda} = D + \pi - L - M \geq 0 ; \Lambda \geq 0 ; \frac{\partial \mathcal{L}}{\partial \Lambda} \Lambda = 0 \quad (36)$$

$$\frac{\partial \mathcal{L}}{\partial \Phi} = ASF(D, \pi) - RSF(L, M) \geq 0 ; \Phi \geq 0 ; \frac{\partial \mathcal{L}}{\partial \Phi} \Phi = 0. \quad (37)$$

Inspection of first-order conditions (35), (36), and (37) shows that the optimum comprises three cases. In the first case (a) the bank chooses  $M \geq 0$  which implies

<sup>16</sup>First-order partial derivatives of  $ASF(\cdot)$  and  $RSF(\cdot)$  will be denoted by  $ASF_i(\cdot)$ ;  $i = D, \pi, d$  and  $RSF_j(\cdot)$ ;  $j = L, M, l, m$ , respectively, in the following.



that in this case the regulatory requirement (37) is the only binding constraint, ie  $\Phi > 0$  and  $\Lambda = 0$  holds. In the second case (b) the bank selects  $M < 0$  which directly implies the balance sheet constraint to be binding, ie  $\Lambda > 0$ . In this case, however, the first-period profit  $\pi$  is sufficiently high to leave the regulatory liquidity requirement non-binding, ie  $\Phi = 0$ . In the third case (c), again, the bank prefers  $M < 0$  causing the balance sheet constraint to bind in the optimum. In this case, however, the profit from earlier business decisions  $\pi$ , however, is quite low which makes the liquidity requirement to bind in the optimum, too – ie  $\Phi > 0$  in addition to  $\Lambda > 0$ . These three cases will be analyzed in detail subsequently.

### 5.2.1 Case (a): $M \geq 0$

In this situation it is easy to see that the balance sheet constraint (36) cannot be binding in the optimum. To see this consider alternatively  $\Lambda > 0$  which means that  $L + M = D + \pi$ , holds. The liquidity regulation (37) at the same time requires

$$ASF_D(D, \pi)D + ASF_\pi(D, \pi)\pi \geq RSF_L(L, M)L + RSF_M(L, M)|_{M \geq 0}M.$$

Rearrange terms to rewrite the liquidity requirement to

$$ASF_D(D, \pi)(D - L) - (RSF_L(L, M) - ASF_D(D, \pi))L \geq M - \pi$$

with  $ASF_\pi(D, \pi) = RSF_M(L, M)|_{M \geq 0} = 1$  and the binding balance sheet constraint to

$$M - \pi = D - L.$$

Combining both conditions yields

$$ASF_D(D, \pi)(D - L) \geq D - L + (RSF_L(L, M) + ASF_D(D, \pi))L$$

which is incorrect due to  $ASF_D(D, L) < 1$  and  $RSF_L(L, M) - ASF_D(D, \pi) > 0$ . That is, when  $M \geq 0$  a binding balance sheet constraint implies that the liquidity requirement is violated. Therefore, when  $M \geq 0$  in the optimum  $\Lambda = 0$  needs to be met. Moreover, (35) in this situation implies

$$\Phi = \frac{r}{RSF_M(L, M)|_{M \geq 0}} > 0$$

since  $r, RSF_M(\cdot)|_{M \geq 0} > 0$ . In other words, when the bank chooses  $M \geq 0$  the liquidity regulation is binding to the bank's deposit and loan decisions whereas the balance sheet constraint is not.

Given the previous findings the first-order conditions (33) to (37) can be rewritten. Using  $\Phi = r/RSF_M(L, M)|_{M \geq 0} > 0$  yields:

$$\frac{\partial \mathcal{L}}{\partial D} = -r_D(D) - r'_D(D)D - C_D(D, L) + r \frac{ASF_D(D, \pi)}{RSF_M(L, M)|_{M \geq 0}} = 0 \quad (38)$$

$$\frac{\partial \mathcal{L}}{\partial L} = \left(1 - E(\tilde{\theta}|\tilde{\mu})\right) r_L(l) - C_L(D, L) - r \frac{RSF_L(L, M)}{RSF_M(L, M)|_{M \geq 0}} = 0 \quad (39)$$

$$\frac{\partial \mathcal{L}}{\partial \Phi} = ASF(D, \pi) - RSF(L, M) = 0. \quad (40)$$

In this regard it should be noted that  $\Phi$ , which is calculated using the first-order condition (35) above, is constant due to the definitions underlying  $RSF(L, M)$ . As a consequence,  $M$  is implicitly determined by the binding liquidity requirement constraint (40) when  $D$ ,  $L$ , and  $\pi$  are fixed.

To get deeper insights regarding the effects of liquidity regulation I next analyze the impact of  $\pi$  on the bank's optimal decisions  $D$ ,  $L$ , and  $M$ . For the purpose of this comparative static analysis I apply the implicit function theorem and Cramer's rule to the first-order conditions (38), (39) and (40) to find:<sup>17</sup>

$$\frac{dD}{d\pi} = \frac{dL}{d\pi} = 0 \quad (41)$$

and

$$\frac{dM}{d\pi} = 1. \quad (42)$$

This result immediately yields

**Proposition 5** *In case of  $M \geq 0$  a NSFR-type liquidity regulation does not neutralize the separating property of an buoyant interbank market. Optimal decisions  $D$  and  $L$  are not affected by the realization of  $\pi$  and the bank continues to evaluate risky prospects in  $t = 1$  in a risk neutral way.*

**Proof:** Define as in previous sections

$$U(\pi) \equiv E\left(\tilde{\Pi}(\tilde{\theta})|\tilde{\mu}\right)$$

and interpret  $U(\pi)$  as an endogenously derived utility function. The analysis of the

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<sup>17</sup>See the Appendix for an exposition of the calculation.

shape of this utility function shows:

$$\begin{aligned} \frac{dU(\pi)}{d\pi} &= \frac{dE(\tilde{\Pi}(\tilde{\theta})|\tilde{\mu})}{d\pi} = \left( (1 - E(\tilde{\theta}|\tilde{\mu}))r_L(l) - C_L(D, L) \right) \frac{dL}{d\pi} + r \frac{dM}{d\pi} - \\ &\quad - (r_D(D) + r'_D(D)D - C_D(D, L)) \frac{dD}{d\pi} \\ &= r \frac{dM}{d\pi} \\ &= r > 0 \end{aligned}$$

due to (41) and (42). Moreover,

$$\frac{d^2U(\pi)}{(d\pi)^2} = \frac{d^2E(\tilde{\Pi}(\tilde{\theta})|\tilde{\mu})}{(d\pi)^2} = 0$$

since  $dM/d\pi = 1$  is constant.  $\square$

The proof of Proposition (5) shows that in the case of  $M \geq 0$  liquidity regulation does not affect a bank's endogenous attitudes towards the risky profit  $\pi$ . The bank still acts risk neutral. Compared to an unregulated bank, a regulated one even puts the same (linear) weight on  $\pi$  which is just the interbank market rate  $r$ .

Although Proposition (5) shows that the implementation of a NSFR-type liquidity requirement does not neutralize the separating property of a buoyant interbank market the optimal levels of  $D$  and  $L$  are affected in a certain way. A comparison of the respective first-order conditions of an unregulated bank (18) and (19) with the corresponding first-order conditions with regulation (38) and (39) shows two important aspects: first, regardless of regulation being implemented or not the bank uses the interbank rate  $r$  as a benchmark to decide on the optimal volumes of deposits and loans. Second, the regulatory constraint creates constant weights which are multiplied with  $r$ . In the first-order condition for the optimal volume of  $D$  (38) one observes

$$r \frac{ASF_D(D, \pi)}{RSF_M(L, M)|_{M \geq 0}} = r \frac{0.8}{1} < r \quad (43)$$

and the first-order condition (39) for the optimal loan volume  $L$  shows

$$r \frac{RSF_L(L, M)}{RSF_M(L, M)|_{M \geq 0}} = r \frac{0.85}{1} < r. \quad (44)$$

**Corollary 4** *The NSFR-type regulation reduces the impact of the interbank market rate  $r$  on the optimal deposit and loan decisions. This effect constant and does not depend on the amount of  $D$ ,  $L$ , or  $\pi$ . Compared to a situation without liquidity regulation the regulated bank will reduce the optimal volume of deposits  $D$  and increase the loan volume  $L$ .*

**Proof:** The second part of Corollary (4) is immediately derived from comparing first-order conditions. In the unregulated situation the optimal decisions on  $D$  and  $L$  satisfy the conditions

$$-r_D(D) - r'_D(D)D - C_D(D, L) + r = 0$$

and

$$(1 - E(\tilde{\theta}|\tilde{\mu}))r_L(l) - C_L(D, L) - r = 0,$$

respectively. For a regulated bank that sets the same volumes of  $D$  and  $L$ , one would observe

$$-r_D(D) - r'_D(D)D - C_D(D, L) + r \frac{ASF_D(D, \pi)}{RSF_M(L, M)|_{M \geq 0}} < 0$$

and

$$(1 - E(\tilde{\theta}|\tilde{\mu}))r_L(l) - C_L(D, L) - r \frac{RSF_L(L, M)}{RSF_M(L, M)|_{M \geq 0}} > 0$$

due to (43) and (44), respectively. Since I found for the second-order derivatives

$$\frac{\partial^2 \mathcal{L}}{(\partial D)^2} < 0, \quad \frac{\partial^2 \mathcal{L}}{(\partial L)^2} < 0 \text{ and } \frac{\partial^2 \mathcal{L}}{\partial D \partial L} = 0$$

in the calculation of the Jacobian matrix in the Appendix, it is straightforward that optimality requires the regulated bank to reduce  $D$  and increase  $L$  compared to a situation without liquidity regulation.  $\square$

Therefore, I can summarize: With  $M \geq 0$  being optimal, liquidity regulation of the NSFR-type does not create endogenous risk aversion of a risk neutral bank because this kind of regulation does not have the ability to offset the separating property of the interbank market on deposit and loan decisions. The optimal volumes of deposits and loans, however, change because the regulation adds constant weights to the interbank rate  $r$  which serves as a benchmark in the decision-making process.

### 5.2.2 Case (b): $M < 0$ and $\Phi = 0$

Consider now the situation where the bank finds it optimal to choose  $M < 0$ . This implies

$$RSF_M(L, M)|_{M < 0} = 0,$$

see (26) above, and therefore

$$\Lambda = r > 0$$

regardless of  $\Phi$  being strictly positive or zero – see the first-order condition (37). Moreover, the liquidity requirement may be rewritten in this case as

$$ASF_D(D, \pi)D + ASF_\pi(D, \pi) \geq RSF_L(L, M)$$

because a negative interbank market position  $M$  does not add to  $ASF$  or  $RSF$ . Rearranging terms and using  $ASF_{\pi}(D, \pi) = 1$  shows that the liquidity requirement is met as long as

$$\pi \geq RSF_L(L, M) - ASF_D(D, \pi)D$$

without violating the (rewritten) balance sheet constraint

$$\pi = L - D + M$$

and vice versa. That is, in the current case the binding balance sheet constraint determines the optimal volume of  $M < 0$  when  $D$  and  $L$  are chosen optimally and  $\pi$  is given. Moreover,  $\pi \geq RSF_L(L, M) - ASF_D(D, \pi)$  ensures that the liquidity requirement is satisfied anyway and is, hence, not binding in the optimum, ie  $\Phi = 0$ .

As a result, the current case of  $M < 0$  and  $\Phi = 0$  is equivalent to the decision-making situation of an unregulated bank which has the opportunity to use an unsecured interbank money market as analyzed in section 4. The finding derived in section 4, therefore, still apply to the current case:

$$\begin{aligned} \frac{dD}{d\pi} &= 0 \\ \frac{dL}{d\pi} &= 0 \\ \frac{dM}{d\pi} &= 1. \end{aligned}$$

In addition, the bank sets the same volumes of deposits  $D$  and loans  $L$  compared to the unregulated situation and variations in  $\pi$  completely transfer to the interbank market position  $M$ . Liquidity regulation is also not able to alter the bank's risk neutrality.

### 5.2.3 Case (c): $M < 0$ and $\Phi > 0$

In the situation where  $M < 0$  is optimal a very low realization of  $\pi$  may cause the liquidity requirement constraint (37) to be binding in addition to the balance sheet constraint. More precisely, if it comes out that

$$\pi < RSF_L(L, M)L - ASF_D(D, \pi)D$$

the liquidity requirement needs to bind in the optimum, ie  $\Phi > 0$ . Otherwise the bank's decisions on deposit  $D$  and loan volumes  $L$  would violate the regulatory liquidity requirement. The reason is that the binding balance sheet constraint in this case only determines the bank's position  $M$  in the interbank market for the optimal decisions on  $D$  and  $L$  and the given realization of  $\pi$ . When  $\pi$ , however, is very low these volumes of  $D$  and  $L$  may be not in line with the regulatory liquidity

requirement. The latter, in this case, needs to be binding in the optimum meaning that the liquidity regulation now determines the relation between  $D$  and  $L$ , formally:

$$D = \frac{1}{ASF_D(D, \pi)} (RSF_L(L, M)L - ASF_\pi(D, \pi)\pi).$$

For a more detailed analysis of this case I insert  $\lambda = r$  and the binding balance sheet constraint in the first-order conditions (33) and (34), solve (33) for  $\Phi$  which is, then, inserted in (34). As a result, the following two first-order conditions define the bank's optimal behavior in the current case:

$$\begin{aligned} \left(1 - E(\tilde{\theta}|\tilde{\mu})\right) r_L(l) - \left(1 - \frac{RSF_L(L, M)}{ASF_D(D, \pi)}\right) r - C_L(D, L) - \\ - \frac{RSF_L(L, M)}{ASF_D(D, \pi)} (r_D(D) - r'_D(D)D - C_D(D, L)) = 0 \end{aligned} \quad (45)$$

$$ASF(D, \pi) - RSF(L, M) = 0. \quad (46)$$

Applying the implicit function theorem to (45) and (46) shows that in the current situation there exists an effect of  $\pi$  on the optimal volumes of deposits and loans:<sup>18</sup>

$$\frac{dD}{d\pi} < 0; \quad \frac{dL}{d\pi} > 0.$$

That is, in contrast to the previous cases (a) and (b) the realization of  $\pi$  affects the optimal volumes of deposits and loans when the situation on hand is characterized by  $M < 0$  and  $\Phi > 0$ .

This raises the question whether the bank still evaluates the early profit  $\pi$  in a risk neutral way or whether one observes a kind of endogenous risk aversion in the current situation. Defining, again, the endogenous utility function of the bank as

$$U(\pi) \equiv E(\tilde{\Pi}(\tilde{\theta})|\tilde{\mu})$$

I can state and prove

**Proposition 6** *When the bank finds it optimal to choose  $M < 0$  and the realization of  $\tilde{\pi}$  is below  $\frac{1}{ASF_\pi} (RSF_L L - ASF_D D)$  the per se risk neutral bank evaluates the realizations of the early profit  $\pi$  in a risk averse way. As a consequence the optimal volume of deposits  $D$  increases and the optimal volume of loans decreases compared to an unregulated bank with access to a buoyant interbank market.*

**Proof** See the Appendix.  $\square$

<sup>18</sup>See the Appendix for an formal derivation.

The reason for this result is that in the current case the regulatory liquidity requirement imposes a strict link between both sides of the bank's balance sheet:

$$D = \frac{1}{ASF_D} (RSF_L L - ASF_\pi \pi).$$

This means that as soon as the bank has fixed the optimal loan volume  $L$ , the liquidity requirement strictly determines the volume of deposits that need to be taken. In this way in the current situation I find a mechanism which is basically the same as the mechanism that endogenously created risk averse profit evaluation in the benchmark model of section 3. The only difference is, that the regulatory liquidity requirement uses additional factors to weight the respective balance sheet position. In the benchmark model, in contrast, the balance sheet constraint created the link between the bank's assets and liabilities without weighting the several positions.

To sum up, the previous arguments show two important findings: first, the implementation of a NSFR-type liquidity regulation replaces the balance sheet constraint as binding for the bank's decision making only in case of  $M \geq 0$ . When the bank prefers  $M < 0$  the liquidity regulation is either ineffective or complements the balance sheet constraint as binding in the decision making process. Second, the the NSFR-type liquidity regulation is effective in creating an endogenously risk averse evaluation of the random profit  $\pi$  – and is thus effective in offsetting the separating effect of the interbank market – only if  $M < 0$  and  $\pi < \frac{1}{ASF_\pi} (RSF_L L - ASF_D D)$ . In all other cases the interbank market still separates the banks optimal decision on deposits from the optimal decision on loans which maintains the bank's risk neutrality in evaluating early profits  $\pi$ .

### 5.3 Regulation and assessment of liquidity risk

The analysis of the impact of interbank market availability on an unregulated bank above found a separating effect of the interbank market not only between a bank's assets and liabilities at a certain point in time. Moreover, separation was also observed over time in the sense that future decisions did not affect early ones and vice versa.

To examine whether this result is also valid when the bank under consideration is subject to liquidity regulation of the NSFR type, in the following I determine optimal bank behavior in  $t = 0$  taking into account the results of the previous section. The bank chooses deposits  $d$ , loans  $l$  and its position in the interbank market  $m$  in  $t = 0$

in the way to maximize the expected final profit  $E(\tilde{\Pi}(\tilde{\theta}, \tilde{\mu}))$  in  $t = 2$ , formally:

$$\max_{d,l,m} E \left( \tilde{\Pi}(\tilde{\theta}, \tilde{\mu}) \right) = \int_{\tilde{\mu}} \int_{\tilde{\theta}} \tilde{\Pi}(\tilde{\theta}|\tilde{\mu}) dF(\tilde{\theta}|\tilde{\mu}) dF(\tilde{\mu}) \quad (47)$$

with

$$\tilde{\Pi}(\tilde{\theta}|\tilde{\mu}) = (1 - \tilde{\theta})r_L(l)L + rM(\tilde{\pi}) - r_D(D)D - C(D, L) \quad (48)$$

and

$$\tilde{\pi} = (1 - \tilde{\mu})r_L(l)l + rm - r_D(d)d - C(d, l) \quad (49)$$

s.t.

$$l + m \leq d \quad (50)$$

$$RSF(l, m) \leq ASF(d). \quad (51)$$

Just like in the previous section the balance sheet constraint (50) and the liquidity requirement constraint (51) have been formulated as inequalities because of uncertainty regarding the question which condition will be binding in the optimum.

Moreover, the results of the analysis of decisions in  $t = 1$  require to rewrite the expected profit that will materialize in  $t = 2$  as follows:

$$\begin{aligned} E \left( \tilde{\Pi}(\tilde{\theta}, \tilde{\mu}) \right) &= \int_{\{\tilde{\mu}: M \geq 0\}} E \left( \tilde{\Pi}(\tilde{\theta})|\tilde{\mu} \right) dF(\tilde{\mu}) + \\ &+ \int_{\{\tilde{\mu}: M < 0, \Phi = 0\}} E \left( \tilde{\Pi}(\tilde{\theta})|\tilde{\mu} \right) dF(\tilde{\mu}) + \\ &+ \int_{\{\tilde{\mu}: M < 0, \Phi > 0\}} E \left( \tilde{\Pi}(\tilde{\theta})|\tilde{\mu} \right) dF(\tilde{\mu}). \end{aligned} \quad (52)$$

This representation accounts for the fact that depending on the realization of  $\tilde{\mu}$  and hence  $\tilde{\pi}$  the bank was found to set  $M \geq 0$  or  $M < 0$  and, in particular in the last case, the regulatory constraint may be non-binding ( $\Phi = 0$ ) or binding ( $\Phi > 0$ ). The bank's optimal behavior in the respective cases has been analyzed in the previous section and needs to be considered in the following.

The relevant first-order conditions for the optimal volumes of deposits  $d$  and loans  $l$  in the current situation are:<sup>19</sup>

$$\frac{ASF_d(d)}{RSF_m(l, m)|_{m \geq 0}} r - r_D(d) - r'_D(d)d - C_D(d, l) = 0 \quad (53)$$

<sup>19</sup>For a formal derivation see the Appendix.



and

$$\begin{aligned}
& \int_{\{\tilde{\mu}: M \geq 0\}} \frac{dU(\tilde{\pi})}{d\pi} \left( (1 - \tilde{\mu})(r_L(l) + r'_L(l)l) - \frac{RSF_l(l, m)}{RSF_m(l, m)|_{m \geq 0}} r - C_L(d, l) \right) dF(\tilde{\mu}) + \\
& \int_{\{\tilde{\mu}: M < 0, \Phi = 0\}} \frac{dU(\tilde{\pi})}{d\pi} \left( (1 - \tilde{\mu})(r_L(l) + r'_L(l)l) - \frac{RSF_l(l, m)}{RSF_m(l, m)|_{m \geq 0}} r - C_L(d, l) \right) dF(\tilde{\mu}) + \\
& \int_{\{\tilde{\mu}: M < 0, \Phi > 0\}} \frac{dU(\tilde{\pi})}{d\pi} \left( (1 - \tilde{\mu})(r_L(l) + r'_L(l)l) - \frac{RSF_l(l, m)}{RSF_m(l, m)|_{m \geq 0}} r - C_L(d, l) \right) dF(\tilde{\mu}) = 0.
\end{aligned} \tag{54}$$

Examining these first-order conditions yields

**Proposition 7** *Under a NSFR-type liquidity regulation the bank continues to be risk neutral in the deposit business. The regulation, however, imposes a linear effect on the bank which leads to a reduction of the optimal volume of deposits compared to an unregulated bank. Regarding the loan business the bank's risk attitudes arise endogenously by averaging over risk neutral and risk averse situations. The regulatory effect on the lending decision is ambiguous and depends on which situation - risk neutral or risk averse - are more likely.*

Comparing the current first-order condition (53) to the corresponding one (23) which defines the optimal volumes of deposits  $d$  of an unregulated bank with interbank market participation shows that the only difference is the factor

$$\frac{ASF_d(d)}{RSF_m(l, m)|_{m \geq 0}} < 1$$

which is multiplied with the interbank rate  $r$  in the case of regulation. The inequalities follow immediately from the definitions of  $ASF(d)$  (27) and  $RSF(l, m)$  (29).

Regarding deposits the mode of decision making of the bank, therefore, does not change due to the implementation of NSFR-type liquidity regulation. The bank still behaves risk neutral. Nevertheless, the liquidity regulation imposes a linear effect on the bank's first-order condition which determine the optimal volumes of  $d$ . Multiplying a factor which is below one to the interbank rate  $r$  in the first-order condition (53) causes a reduction of the optimal volume of deposits  $d$  compared to

the unregulated situation.<sup>20</sup>

Regarding the optimal volume of loans  $l$ , however, the effect of a NSFR-type liquidity regulation is ambiguous. Comparing the relevant first-order necessary conditions in the current case (54) with the one in the unregulated situation (24) shows that only the term

$$\int_{\tilde{\mu}} (1 - E(\tilde{\theta}|\tilde{\mu}))r'_L(l)LdF(\tilde{\mu}) = \int_{\tilde{\mu}} \int_{\tilde{\theta}} (1 - \tilde{\theta})r'_L(l)LdF(\tilde{\theta}|\tilde{\mu})dF(\tilde{\mu})$$

is the same in both conditions. The most interesting difference between the unregulated and the regulated situation is that in the latter the bank averages over several ex-ante possible situations. Depending on  $M$  being positive or negative and the liquidity requirement being binding or not, the bank behaves either risk neutral – when  $M \geq 0$  or  $M < 0$  and  $\Phi = 0$  – or risk averse – when  $M < 0$  and  $\Phi > 0$ . Since in section 5.2 it was found that in  $t = 1$  the bank will increase, leave unchanged, decrease the optimal volume of loans  $L$  compared to an unregulated bank depending on  $M \geq 0$ ,  $M < 0$  and  $\Phi = 0$ ,  $M < 0$  and  $\Phi > 0$ , respectively, the total effect depends on which effect is dominant. Moreover, the effects of averaging over the situations interacts with an additional effect which arises from the liquidity regulation at  $t = 0$ . Note, that in the first-order condition (54) the term

$$\frac{RSF_l(l, m)}{RSF_m(l, m)|_{m \geq 0}} < 1$$

is multiplied with the interbank rate  $r$  which works towards increasing the optimal volume of loans  $l$  compared to the situation without liquidity regulation. As a result, the total effect of a NSFR-type liquidity requirement on the lending decision of a risk neutral bank arises from the complex interaction of risk evaluation effects – which arise endogenously – and the direct impact of the regulation due to adding weights to the interbank market rate  $r$ .

With respect to policy implications these results provide a somewhat mixed picture. Although, in particular, the finding that the bank will not borrow in the interbank market in  $t = 0$  is perfectly in line the intentions of the BCBS to implement the NSFR standard, it is not clear whether the reaction regarding deposit and loan decisions is also intended. Note that on the one hand, the bank reduces the optimal volume of deposits in  $t = 0$  which are considered to be a more stable source of funding compared to interbank money by the BCBS. On the other hand depending on the probability distributions of the credit risk parameters and the market environment the bank may find it optimal to increase the volume of loans

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<sup>20</sup>The proof is actually the same as the one shown in section 5.2. A regulated bank that chooses the same volume of  $d$  as an unregulated one does violates the first order condition:  $\partial Z/\partial d < 0$ . Since it is easily verified that  $\partial^2 Z/(\partial d)^2 < 0$  and, by assumption,  $\partial^2 Z/(\partial d \partial l) = 0$ , the predicted reaction of optimal  $d$  follows immediately.

as a response to the liquidity regulation. Although this is just one of the possible reactions on the implementation of a NSFR-type liquidity regulation, in particular this behavior may increase the bank's exposure to credit risk. This may, in turn, increase liquidity risk in the future – note that the choice of  $l$  in  $t = 0$  affects the uncertain intermediate profit  $\tilde{\pi}$  in  $t = 1$  which represents internally generated funds to the bank. Furthermore, the NSFR-type liquidity regulation does not generate effects that provide incentives for banks to take an integrated view on risk management. Long-term effects of a bank's decisions and the interaction of credit risk and liquidity risk only enter the decision making process when the amount of internally generated funds, ie early profits  $\pi$ , is low and the bank is in need of interbank money to fund lending. Otherwise the liquidity requirement only affects the extent of interbank market usage in a linear (risk neutral) way. From a risk management perspective the NSFR-type regulation, hence, does not effectively address liquidity risk.

## 6 Conclusion

The effects arising from the availability of buoyant interbank markets may be considered as a main justification for the introduction of regulatory liquidity requirements in the aftermath of the financial crisis 2007-2009. Observations made during this crisis suggest that buoyant interbank markets reduced banks' risk sensitivity by allowing banks to treat credit risk and liquidity risk independently and on short notice. The current liquidity requirements proposed in BCBS (2010) aim at mitigating the effects of buoyant interbank markets and at recreating risk sensitivity of banks.

Against this background the present paper analyzes the effects of the availability of buoyant interbank money markets and liquidity requirements just like the ones currently proposed by the Basel Committee on Banking Supervision on risk sensitivity and behavior of banks. For this purpose the industrial organization approach to banking is extended by allowing for multiple stages of bank decisions. In this way the model is able to account for an interaction between credit risk and liquidity risk over time.

In the absence of interbank money markets and liquidity regulation the interaction between credit risk and liquidity risk is found to make risk neutral banks behave as if they were risk averse. They evaluate future risky prospects in a risk averse fashion which induces risk sensitivity regarding risky lending decisions.

The risk sensitivity, however, disappears if the bank has the opportunity to use buoyant interbank money markets to manage liquidity risk. When interbank money markets allow for lending or borrowing any desired amount of funds at a (nearly) risk free rate – which is characteristic for buoyant markets – the interaction between credit risk and liquidity risk does not longer drive a bank's decisions. Credit risk

and liquidity risk are treated separately at short notice.

A regulatory liquidity requirement just like the one proposed in BCBS (2010) is, however, found to be not generally able to recreate endogenous risk sensitive behavior of banks. Only in the case of a bank being a net borrower in the interbank market and earning low profits on granted loans the regulation makes a per se risk neutral bank to behave in a risk averse way. In all other situations this is not the case, and a bank may even prefer to engage in more risky lending which increases the bank's exposure to liquidity risk in the future.

The model and the results of the present paper provide a first set of insights into the effects of the interaction of credit risk and liquidity risk, the impact of interbank markets and the potential role of liquidity regulations. An important issue that is beyond the scope of this paper but which will be very relevant in practice is the interaction between several regulatory requirements. That is, the liquidity requirements are added to an environment where there is already capital adequacy regulation in force. The question, therefore, is whether the effect of the liquidity regulation may work against the effect of capital adequacy regulation.<sup>21</sup> This question, however, is left for future research.

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<sup>21</sup>Capital adequacy regulation of the kind formulated by the BCBS requires equity capital to be held to cover losses from risky loans. When holding and extending equity capital is costly, capital adequacy regulation creates an incentive to a risk neutral bank to reduce risky lending. See Pausch and Welzel (2002) for details.

## Appendix

### Derivation of the first-order condition (14)

Differentiating 13 with respect to  $l$  yields

$$\begin{aligned}
\frac{dE(\tilde{\Pi})}{dl} &= \int_{\tilde{\theta}} \int_{\tilde{\mu}} \left[ (1 - \tilde{\theta}) \left( r'_L(l)(\tilde{\pi}) + r_L(l) \frac{dL}{d\tilde{\pi}} \frac{d\tilde{\pi}}{dl} \right) \right] dF(\tilde{\theta}|\tilde{\mu}) dF(\tilde{\mu}) - \\
&\quad - \int_{\tilde{\theta}} \int_{\tilde{\mu}} [r_D(L - \tilde{\pi}) + r'_D(L - \tilde{\pi})(L - \tilde{\pi})] \left[ \frac{dL}{d\tilde{\pi}} - 1 \right] \frac{d\tilde{\pi}}{dl} dF(\tilde{\theta}|\tilde{\mu}) dF(\tilde{\mu}) - \\
&\quad - \int_{\tilde{\theta}} \int_{\tilde{\mu}} C_D(L(\tilde{\pi}), L(\tilde{\pi}) - \tilde{\pi}) \left[ \frac{dL}{d\tilde{\pi}} - 1 \right] \frac{d\tilde{\pi}}{dl} dF(\tilde{\theta}|\tilde{\mu}) dF(\tilde{\mu}) - \\
&\quad + \int_{\tilde{\mu}} \int_{\tilde{\theta}} C_L(L(\tilde{\pi}), L(\tilde{\pi}) - \tilde{\pi}) \frac{dL}{d\tilde{\pi}} \frac{d\tilde{\pi}}{dl} dF(\tilde{\theta}|\tilde{\mu}) dF(\tilde{\mu}) = 0. \tag{55}
\end{aligned}$$

Taking a closer look at (55) one observes that it contains the first-order necessary condition (3) for the optimal volume of loans  $L$  in  $t = 1$ :

$$\begin{aligned}
\frac{dE(\tilde{\Pi}(\tilde{\theta})|\tilde{\mu})}{dL} &= (1 - E(\tilde{\theta}|\tilde{\mu}))r_L(l) - r_D(L - \tilde{\pi}) - r'_D(L - \tilde{\pi})(L - \tilde{\pi}) - \\
&\quad - C_L(L, L - \tilde{\pi}) - C_D(L, L - \tilde{\pi}) = 0
\end{aligned}$$

where  $E(\tilde{\theta}|\tilde{\mu}) = \int_{\tilde{\theta}} \tilde{\theta} dF(\tilde{\theta}|\tilde{\mu})$ . It should be also noted that  $dL/d\tilde{\pi}$  as well as  $\partial\tilde{\pi}/\partial d$  are both not affected by the random variable  $\tilde{\theta}$  and, hence, need not be considered in the calculation of the conditional expected profit above.

The first-order condition (55), therefore, reduces to

$$\begin{aligned}
\frac{dE(\tilde{\Pi})}{dl} &= \int_{\tilde{\mu}} \int_{\tilde{\theta}} (1 - \tilde{\theta}) r'_L(l) L(\tilde{\pi}) dF(\tilde{\theta}|\tilde{\mu}) dF(\tilde{\mu}) + \int_{\tilde{\mu}} U'(\tilde{\pi}) \frac{d\tilde{\pi}}{dl} dF(\tilde{\mu}) \\
&= \int_{\tilde{\mu}} \int_{\tilde{\theta}} (1 - \tilde{\theta}) r'_L(l) L(\tilde{\pi}) dF(\tilde{\theta}|\tilde{\mu}) dF(\tilde{\mu}) + E \left( U'(\tilde{\pi}) \frac{\partial\tilde{\pi}}{\partial l} \right) = 0 \tag{56}
\end{aligned}$$

where equation (5) and the definition of  $U(\tilde{\pi})$  in (7) have been used:

$$\begin{aligned}
U'(\tilde{\pi}) &= \frac{dE(\tilde{\Pi}(\tilde{\theta})|\tilde{\mu})}{d\tilde{\pi}} = \\
&= \int_{\tilde{\theta}} (r_D(L - \tilde{\pi}) + r'_D(L - \tilde{\pi})(L - \tilde{\pi}) + C_D(L, L - \pi)) dF(\tilde{\theta}|\tilde{\mu}).
\end{aligned}$$

### Proof of Proposition 2

For the Proof of Proposition 2 compare the first-order necessary condition of a long-term oriented bank with the corresponding one of a short-term oriented bank.

First, note that the following first-order condition determines the optimal volume of loans (and due to balance-sheet equivalence also the optimal volume of deposits) of a short-term oriented bank acting in the current setting:

$$\begin{aligned} \frac{dE(\tilde{\pi})}{dl} &= E \left[ \frac{d\tilde{\pi}}{dl} \right] = \\ &= E [(1 - \tilde{\mu}) (r_L(l) + r'_L(l)l) - r_D(l) - r'_D(l)l - C_L(l, l) - C_D(l, l)] = 0. \end{aligned} \quad (57)$$

To facilitate the comparison of the standard short-term oriented behavior with endogenous risk averse behavior of a long-term oriented bank, I rewrite the first-order condition (14):

$$\frac{dE(\tilde{\Pi})}{dl} = r'_L(l)E \left( L(\tilde{\pi})(1 - E(\tilde{\theta}|\tilde{\mu})) \right) + E(U'(\tilde{\pi}))E \left( \frac{d\tilde{\pi}}{dl} \right) + \text{Cov} \left( U'(\tilde{\pi}), \frac{d\tilde{\pi}}{dl} \right) = 0 \quad (58)$$

where

$$E \left( U'(\tilde{\pi}) \frac{d\tilde{\pi}}{dl} \right) = E(U'(\tilde{\pi})) \cdot E \left( \frac{d\tilde{\pi}}{dl} \right) + \text{Cov} \left( U'(\tilde{\pi}), \frac{d\tilde{\pi}}{dl} \right).$$

Taking into account

$$\frac{\partial \tilde{\pi}}{\partial l} = (1 - \tilde{\mu}) (r_L(l) + r'_L(l)l) r_D(l) - r'_D(l)l - C_L(l, l) - C_L(l, l)$$

one finds

$$\text{Cov} \left( U'(\tilde{\pi}), \frac{d\tilde{\pi}}{dl} \right) = -(r_L(l) + r'_L(l)l) \text{Cov} (U'(\tilde{\pi}), \tilde{\mu}) < 0.$$

Moreover, it should be noted that

$$r'_L(l)E \left( L(\tilde{\pi})(1 - E(\tilde{\theta}|\tilde{\mu})) \right) = \int_{\tilde{\mu}} \int_{\tilde{\theta}} (1 - \tilde{\theta}) r'_L(l) L(\tilde{\pi}) dF(\tilde{\theta}|\tilde{\mu}) dF(\tilde{\mu})$$

because

$$\int_{\tilde{\theta}} (1 - \tilde{\theta}) dF(\tilde{\theta}) = 1 - E(\tilde{\theta}|\tilde{\mu})$$

and  $r'_L(l)$  as well as  $L(\tilde{\pi})$  are not affected by the random variable  $\tilde{\theta}$ . Using a similar manipulation technique as in the case of the second term of (58) yields

$$\begin{aligned} &r'_L(l)E \left( L(\tilde{\pi})(1 - E(\tilde{\theta}|\tilde{\mu})) \right) \\ &= r'_L(l) \left[ E(L(\tilde{\pi})) \cdot E(1 - E(\tilde{\theta}|\tilde{\mu})) - \text{Cov} \left( L(\tilde{\pi}), E(\tilde{\theta}|\tilde{\mu}) \right) \right] < 0 \end{aligned}$$

given the assumption of non-negative stochastic dependence between the random variables  $\tilde{\theta}$  and  $\tilde{\mu}$ . Using the non-negative regression dependence to formalize this stochastic dependence one finds

$$\frac{d}{d\mu} E(\tilde{\theta}|\mu) = - \int_{\tilde{\theta}} \frac{d}{d\mu} F(\tilde{\theta}|\mu) d\tilde{\theta} \geq 0.$$

Moreover, due to  $dL/d\tilde{\pi} > 0$  and  $d\tilde{\pi}/d\tilde{\mu} < 0$ , the covariance between  $L(\tilde{\pi})$  and  $E(\tilde{\theta}|\tilde{\mu})$  appears to be non-positive. The negative sign of the first term of (58), then follows because of  $r'_L(l) < 0$ ,  $E(U'(\tilde{\pi})) > 0$  and  $E(1 - E(\tilde{\theta}|\tilde{\mu})) > 0$ .

Rearranging terms of (58) one observes that for a long-term oriented, endogenously risk averse bank the optimal volume of loans implies

$$E\left(\frac{d\tilde{\Pi}}{dl}\right) > 0$$

since  $E(U'(\tilde{\pi})) > 0$  and  $\text{Cov}(U'(\tilde{\pi}), \tilde{\mu}) > 0$ , which are both implications of  $U(\tilde{\pi})$  being increasing and concave in the optimum (see (8) above), as well as  $r_L(l) + r'_L(l)l > 0$ , which represents the bank's marginal revenue in the loan business and which needs to be positive in the optimum because of the bank's market power.

This, in turn, implies that the long-term oriented bank sets a lower volume of loans  $l$  in the optimum compared to a short-term oriented bank due to  $d^2E(\tilde{\pi})/(dl)^2 < 0$  in the optimum.<sup>22</sup>

Due to the balance sheet identity (12) the optimal volume of deposits  $d$  of a long-term oriented bank is also lower than the optimal volume of deposits of a short-term oriented bank.

## Comparative static analysis of section 5.2.1

The impact of  $\pi$  on the optimal decisions  $D$ ,  $L$ , and  $M$  of a regulated bank that sets  $M \geq 0$  is analyzed by applying the implicit function theorem to first-order conditions (38), (39) and (40). For this purpose I calculate the Jacobian Matrix

$$J = \begin{pmatrix} \frac{\partial^2 \mathcal{L}}{(\partial D)^2} & \frac{\partial^2 \mathcal{L}}{\partial D \partial L} & \frac{\partial^2 \mathcal{L}}{\partial D \partial M} \\ \frac{\partial^2 \mathcal{L}}{\partial L \partial D} & \frac{\partial^2 \mathcal{L}}{(\partial L)^2} & \frac{\partial^2 \mathcal{L}}{\partial L \partial M} \\ \frac{\partial^2 \mathcal{L}}{\partial \Phi \partial D} & \frac{\partial^2 \mathcal{L}}{\partial \Phi \partial L} & \frac{\partial^2 \mathcal{L}}{\partial \Phi \partial M} \end{pmatrix}$$

<sup>22</sup>Using the earlier presented technique the second-order sufficient condition may be written as

$$\begin{aligned} E\left(\frac{d^2E(\tilde{\Pi})}{(dl)^2}\right) &= r''_L(l)E\left(L(\tilde{\pi})(1 - E(\tilde{\theta}|\tilde{\mu}))\right) + \\ &+ r'_L(l)\left[E\left(\frac{d\tilde{\pi}}{dl}\right) \cdot E\left(\frac{dL}{d\tilde{\pi}}(1 - E(\tilde{\theta}|\tilde{\mu}))\right) + \text{Cov}\left(\frac{d\tilde{\pi}}{dl}, \frac{dL}{d\tilde{\pi}}(1 - E(\tilde{\theta}|\tilde{\mu}))\right)\right] + \\ &+ E\left(U''(\tilde{\pi})\left(\frac{d\tilde{\pi}}{dl}\right)^2 + U'(\tilde{\pi})\frac{d^2\tilde{\pi}}{(dl)^2}\right) < 0. \end{aligned}$$

with

$$\begin{aligned}
\frac{\partial^2 \mathcal{L}}{(\partial D)^2} &= -2r'_D(D) - r''_D(D)D - C_{DD}(D, L) < 0 \\
\frac{\partial^2 \mathcal{L}}{\partial L \partial D} &= \frac{\partial^2 \mathcal{L}}{\partial D \partial L} = -C_{DL}(D, L) = 0 \\
\frac{\partial^2 \mathcal{L}}{\partial \Phi \partial D} &= ASF_D(D, \pi) = 0.8 > 0 \\
\frac{\partial^2 \mathcal{L}}{(\partial L)^2} &= -C_{LL}(D, L) < 0 \\
\frac{\partial^2 \mathcal{L}}{\partial \Phi \partial L} &= RSF_L(L, M) = 0.85 > 0 \\
\frac{\partial^2 \mathcal{L}}{\partial D \partial M} &= \frac{\partial^2 \mathcal{L}}{\partial L \partial M} = 0 \\
\frac{\partial^2 \mathcal{L}}{\partial \Phi \partial M} &= -RSF_M(L, M)|_{M \geq 0} = 1 > 0
\end{aligned}$$

and the vector

$$V = \begin{pmatrix} -\frac{\partial^2 \mathcal{L}}{\partial D \partial \pi} \\ -\frac{\partial^2 \mathcal{L}}{\partial L \partial \pi} \\ -\frac{\partial^2 \mathcal{L}}{\partial \phi \partial \pi} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ -ASF_\pi(D, \pi) \end{pmatrix}.$$

Using Cramer's rule one can now calculate

$$\frac{dD}{d\pi} = \frac{|J_1|}{|J|}; \quad \frac{dL}{d\pi} = \frac{|J_2|}{|J|}; \quad \frac{dM}{d\pi} = \frac{|J_3|}{|J|}$$

where  $|J|$  denotes the determinant of the Jacobian matrix and  $|J_i|$ ;  $i = 1, 2, 3$  denotes the determinant of the Jacobian in which the  $i$ -th column was replaced by the vector  $V$  above. This yields

$$\frac{dD}{d\pi} = \frac{0}{(-2r'_D(D) - r''_D(D)D - C_{DD}(D, L))(-C_{LL}(D, L))(-RSF_M(L, M)|_{M \geq 0})} = 0,$$

$$\frac{dL}{d\pi} = \frac{0}{(-2r'_D(D) - r''_D(D)D - C_{DD}(D, L))(-C_{LL}(D, L))(-RSF_M(L, M)|_{M \geq 0})} = 0$$

and

$$\frac{dM}{d\pi} = \frac{(-2r'_D(D) - r''_D(D)D - C_{DD}(D, L))(-C_{LL}(D, L))(-ASF_\pi(D, \pi))}{(-2r'_D(D) - r''_D(D)D - C_{DD}(D, L))(-C_{LL}(D, L))(-RSF_M(L, M)|_{M \geq 0})} = 1$$

because of  $ASF_\pi(D, \pi) = RSF_M(L, M)|_{M \geq 0} = 1$ , see (26) and (28).



### Comparative static analysis of section 5.2.3

Applying the implicit function theorem to (45) and (46) and using Cramer's Rule yields

$$\frac{dD}{d\pi} = \frac{|J_1|}{|J|}; \quad \frac{dL}{d\pi} = \frac{|J_2|}{|J|}$$

where  $J$  denotes the Jacobian Matrix and  $J_i$   $i = 1, 2$  denotes the Jacobian where column  $i$  has been replaced by the vector

$$V = \begin{pmatrix} 0 \\ -ASF_{\pi}(D, \pi) \end{pmatrix}.$$

The Jacobian in the current case is

$$J = \begin{pmatrix} \frac{\partial(45)}{\partial D} & \frac{\partial(45)}{\partial L} \\ \frac{\partial(46)}{\partial D} & \frac{\partial(46)}{\partial L} \end{pmatrix}$$

where

$$\begin{aligned} \frac{\partial(45)}{\partial D} &= -\frac{RSF_L(L, M)}{ASF_D(D, \pi)} (2r'_D(D) + r''_D(D)D + C_{DD}(D, L)) < 0 \\ \frac{\partial(46)}{\partial D} &= ASF_D(D, \pi) > 0 \\ \frac{\partial(45)}{\partial L} &= -C_{LL}(D, L) < 0 \\ \frac{\partial(46)}{\partial L} &= -RSF_L(L, M) < 0 \end{aligned}$$

due to  $C_{DL}(\cdot) = 0$ .

Standard calculations, then, yield

$$\begin{aligned} \frac{dD}{d\pi} &= \frac{-C_{LL}(D, L)ASF_{\pi}}{\frac{RSF_L}{ASF_D} (2r'_D(D) + r''_D(D)D + C_{DD}(D, L)) RSF_L + C_{LL}(D, L)ASF_D} < 0 \\ \frac{dL}{d\pi} &= \frac{\frac{RSF_L}{ASF_D} (2r'_D(D) + r''_D(D)D + C_{DD}(D, L)) ASF_{\pi}}{\frac{RSF_L}{ASF_D} (2r'_D(D) + r''_D(D)D + C_{DD}(D, L)) RSF_L + C_{LL}(D, L)ASF_D} > 0. \end{aligned}$$

### Proof of Proposition 6

First note that the initial first-order necessary conditions (33) and (34) can be rewritten considering  $M < 0$ ,  $\Lambda = r$  and  $\Phi > 0$  as

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial D} &= r - r_D(D) - r'_D(D)D - C_D(D, L) + \Phi ASF_D(D, \pi) = 0 \\ \frac{\partial \mathcal{L}}{\partial L} &= (1 - E(\tilde{\theta}|\tilde{\mu}))r_L(l) - r - C_L(D, L) - \Phi RSF_L(L, M) = 0. \end{aligned}$$

To prove Proposition 6 I now rewrite the bank's expected profit in  $t = 1$  using the binding balance sheet constraint:

$$\mathbb{E}(\tilde{\Pi}(\tilde{\theta})|\tilde{\mu}) = \left( (1 - \mathbb{E}(\tilde{\theta}|\tilde{\mu}))r_L(l) - r \right) L + (r - r_D(D))D + r\pi - C(D, L).$$

Differentiation with respect to  $\pi$  yields

$$\begin{aligned} \frac{d\mathbb{E}(\tilde{\Pi}(\tilde{\theta})|\tilde{\mu})}{d\pi} &= \left( (1 - \mathbb{E}(\tilde{\theta}|\tilde{\mu}))r_L(l) - r - C_L(D, L) \right) \frac{dL}{d\pi} + \\ &\quad + (r - r_D(D) - r'_D(D)D - C_D(D, L)) \frac{dD}{d\pi} + r \\ &= \Phi ASF_D \frac{ASF_\pi}{ASF_D} + r > 0 \end{aligned}$$

where the second line is a result of using first-order necessary conditions (33) and (34) to replace

$$(1 - \mathbb{E}(\tilde{\theta}|\tilde{\mu}))r_L(l) - r - C_L(D, L) = \Phi RSF_L$$

and

$$r - r_D(D) - r'_D(D)D - C_D(D, L) = -\Phi ASF_D.$$

Furthermore, due to the binding liquidity requirement, ie  $\Phi > 0$ , in the present situation one can replace

$$\frac{dD}{d\pi} = \frac{RSF_L}{ASF_D} \frac{dL}{d\pi} - \frac{ASF_\pi}{ASF_D}.$$

Moreover, again differentiating  $\mathbb{E}(\tilde{\Pi}(\tilde{\theta})|\tilde{\mu})$  with respect to  $\pi$  yields

$$\frac{d^2\mathbb{E}(\tilde{\Pi}(\tilde{\theta})|\tilde{\mu})}{(d\pi)^2} = (2r'_D(D) + r''_D(D)D + C_{DD}(D, L)) \frac{dD}{d\pi} < 0.$$

Using the definition

$$U(\pi) \equiv \mathbb{E}(\tilde{\Pi}(\tilde{\theta})|\tilde{\mu})$$

the previously derived results imply

$$U'(\pi) > 0 \text{ and } U''(\pi) < 0$$

which may be interpreted as risk averse evaluation of the risky situation in  $t = 1$ .

Moreover, it is easy to show that the bank's optimal volumes of deposits  $D$  and loans  $L$  in the present situation increase and decrease, respectively, compared to the same decisions of an unregulated bank that is active in the interbank market. The first-order conditions (18) and (19) for the optimal volumes of deposits and loans of an unregulated bank which is active in the interbank market only slightly differ from the the relevant conditions in the present situation. In the condition determining  $D$  there appears an additional term

$$\Phi ASF_D(D, \pi) > 0.$$

In the optimality condition for  $L$  there has also been added a term which is

$$-\Phi RSF_L(L, M) < 0.$$

As a result, would the regulated bank in the current situation choose the same amounts of  $D$  and  $L$  that an unregulated bank sets, first-order conditions were violated:

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial D} &> 0 \\ \frac{\partial \mathcal{L}}{\partial L} &< 0. \end{aligned}$$

Because of

$$\frac{\partial^2 \mathcal{L}}{(\partial D)^2} < 0 \text{ and } \frac{\partial^2 \mathcal{L}}{(\partial L)^2} < 0,$$

which has already been shown in the analysis of case (a), optimality of the regulated bank requires to increase the volume of  $D$  and decrease the volume of  $L$  compared to an unregulated bank.

### Deriving first-order conditions in 5.3

To solve the current optimization problem (47) to (51) in 5.3 the Kuhn-Tucker Theorem is applied to the Lagrangian

$$\mathcal{Z} = \mathbb{E} \left( \tilde{\Pi}(\tilde{\theta}, \tilde{\mu}) \right) + \lambda(d - l - m) + \phi(ASF(d) - RSF(l, m)).$$

The corresponding first-order necessary conditions are:

$$\begin{aligned}
\frac{\partial \mathcal{Z}}{\partial d} &= \int_{\{\tilde{\mu}: M \geq 0\}} \frac{dE(\tilde{\Pi}(\tilde{\theta})|\tilde{\mu})}{d\pi} (-r_D(d) - r'_D(d)d - C_D(d, l)) dF(\tilde{\mu}) + \\
&+ \int_{\{\tilde{\mu}: M < 0, \Phi = 0\}} \frac{dE(\tilde{\Pi}(\tilde{\theta})|\tilde{\mu})}{d\pi} (-r_D(d) - r'_D(d)d - C_D(d, l)) dF(\tilde{\mu}) \\
&+ \int_{\{\tilde{\mu}: M < 0, \Phi > 0\}} \frac{dE(\tilde{\Pi}(\tilde{\theta})|\tilde{\mu})}{d\pi} (-r_D(d) - r'_D(d)d - C_D(d, l)) dF(\tilde{\mu}) + \\
&+ \lambda + \phi ASF_d(d) = 0
\end{aligned} \tag{59}$$

$$\begin{aligned}
\frac{\partial \mathcal{Z}}{\partial l} &= \int_{\tilde{\mu}} (1 - E(\tilde{\theta}|\tilde{\mu})) r'_L(l) L dF(\tilde{\mu}) + \\
&+ \int_{\{\tilde{\mu}: M \geq 0\}} \frac{dE(\tilde{\Pi}(\tilde{\theta})|\tilde{\mu})}{d\pi} ((1 - \tilde{\mu})(r_L(l) + r'_L(l)l) - C_L(d, l)) dF(\tilde{\mu}) + \\
&+ \int_{\{\tilde{\mu}: M < 0, \Phi = 0\}} \frac{dE(\tilde{\Pi}(\tilde{\theta})|\tilde{\mu})}{d\pi} ((1 - \tilde{\mu})(r_L(l) + r'_L(l)l) - C_L(d, l)) dF(\tilde{\mu}) + \\
&+ \int_{\{\tilde{\mu}: M < 0, \Phi > 0\}} \frac{dE(\tilde{\Pi}(\tilde{\theta})|\tilde{\mu})}{d\pi} ((1 - \tilde{\mu})(r_L(l) + r'_L(l)l) - C_L(d, l)) dF(\tilde{\mu}) - \\
&- \lambda - \phi RSF_l(l, m) = 0
\end{aligned} \tag{60}$$

$$\begin{aligned}
\frac{\partial \mathcal{Z}}{\partial m} &= \int_{\{\tilde{\mu}: M \geq 0\}} \frac{dE(\tilde{\Pi}(\tilde{\theta})|\tilde{\mu})}{d\pi} r dF(\tilde{\mu}) + \int_{\{\tilde{\mu}: M < 0, \Phi = 0\}} \frac{dE(\tilde{\Pi}(\tilde{\theta})|\tilde{\mu})}{d\pi} r dF(\tilde{\mu}) + \\
&+ \int_{\{\tilde{\mu}: M < 0, \Phi > 0\}} \frac{dE(\tilde{\Pi}(\tilde{\theta})|\tilde{\mu})}{d\pi} r dF(\tilde{\mu}) - \lambda - \phi RSF_m(l, m) = 0
\end{aligned} \tag{61}$$

$$\frac{\partial \mathcal{Z}}{\partial \lambda} = d - l - m \geq 0 ; \lambda \geq 0 ; \frac{\partial \mathcal{Z}}{\partial \lambda} \lambda = 0 \tag{62}$$

$$\frac{\partial \mathcal{Z}}{\partial \phi} = ASF(d) - RSF(l, m) \geq 0 ; \phi \geq 0 ; \frac{\partial \mathcal{Z}}{\partial \phi} \phi = 0. \tag{63}$$

Inspection of the first-order necessary conditions (61), (62), and (63) shows that in the optimum

$$\lambda = 0 ; \phi > 0 ; m \geq 0$$

hold. It is easily verified that for any combination of  $d$ ,  $l$ , and  $m$  the liquidity requirement in (63) would be violated when the balance sheet constraint (62) were binding, ie  $\lambda > 0$ : for  $m \geq \lambda > 0$  implies  $d = l + m$  which leads to  $ASF_d d < RSF_l l + RSF_m|_{m \geq 0} m$  due to  $ASF_d < RSF_l < RSF_m|_{m \geq 0}$  which, in turn, contradicts the requirement of the first-order condition (63). In case of  $m < 0$   $\lambda > 0$  implies

$d < l$  and  $ASF_d d < RSF_l l$  due to  $ASF_d < RSF_l$  and  $RSF_m|_{m<0} = 0$ , which also contradicts the first-order condition (63). As a result, in the optimum it must be true that  $\lambda = 0$  and  $\phi > 0$ . Furthermore, choosing  $m < 0$  implies  $RSF_m(l, M)|_{m<0} = 0$  which would, in combination with  $\lambda = 0$ , violate condition (61):

$$\begin{aligned} \frac{\partial \mathcal{Z}}{\partial m} = & \int_{\{\tilde{\mu}: M \geq 0\}} \frac{dE(\tilde{\Pi}(\tilde{\theta})|\tilde{\mu})}{d\pi} r dF(\tilde{\mu}) + \int_{\{\tilde{\mu}: M < 0, \Phi = 0\}} \frac{dE(\tilde{\Pi}(\tilde{\theta})|\tilde{\mu})}{d\pi} r dF(\tilde{\mu}) + \\ & + \int_{\{\tilde{\mu}: M < 0, \Phi > 0\}} \frac{dE(\tilde{\Pi}(\tilde{\theta})|\tilde{\mu})}{d\pi} r dF(\tilde{\mu}) \neq 0 \end{aligned}$$

because  $dE(\tilde{\Pi}(\tilde{\theta})|\tilde{\mu})/d\pi$  has been shown to be strictly positive regardless of the respective choice of  $M$  in section 5.2. Hence, optimality requires  $m \geq 0$ . That is, in the optimum the liquidity regulation constraint is the only binding constraint for the bank's decisions. Furthermore, it is not optimal for the bank to borrow funds in the interbank market in  $t = 0$ .

Using  $\lambda = 0$ ,  $m \geq 0$  and the earlier definition  $U(\pi) \equiv E(\tilde{\Pi}(\tilde{\theta})|\tilde{\mu})$  in the optimum condition (61) can be used to find

$$\begin{aligned} \phi = & \frac{r}{RSF_m|_{m \geq 0}} \left( \int_{\{\tilde{\mu}: M \geq 0\}} \frac{dU(\tilde{\pi})}{d\pi} dF(\tilde{\mu}) + \right. \\ & \left. + \int_{\{\tilde{\mu}: M < 0, \Phi = 0\}} \frac{dU(\tilde{\pi})}{d\pi} dF(\tilde{\mu}) + \int_{\{\tilde{\mu}: M < 0, \Phi > 0\}} \frac{dU(\tilde{\pi})}{d\pi} dF(\tilde{\mu}) \right) \end{aligned}$$

which is then used to restate the first-order necessary condition for the optimal deposit volume in  $t = 0$

$$\begin{aligned} & \left( \int_{\{\tilde{\mu}: M \geq 0\}} \frac{dU(\tilde{\pi})}{d\pi} dF(\tilde{\mu}) + \int_{\{\tilde{\mu}: M < 0, \Phi = 0\}} \frac{dU(\tilde{\pi})}{d\pi} dF(\tilde{\mu}) + \int_{\{\tilde{\mu}: M < 0, \Phi > 0\}} \frac{dU(\tilde{\pi})}{d\pi} dF(\tilde{\mu}) \right) \cdot \\ & \cdot \left( \frac{ASF_d(d)}{RSF_m(l, m)|_{m \geq 0}} r - r_D(d) - r'_D(d)d - C_D(d, l) \right) = 0 \end{aligned}$$

The modified first-order condition (53) follows because  $dU(\tilde{\pi})/d\pi > 0$  regardless of the optimal choice of  $M$  and the modified first-order condition (54) is a direct consequence of inserting  $\lambda = 0$  and  $\phi$ .

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