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Copula Dynamics in CDOs

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Copula Dynamics in CDOs. *

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Abstract

Values of tranche spreads of collateralized debt obligations (CDOs) are driven by the joint default performance of the assets in the collateral pool. The dependence between the names in the portfolio mainly depends on current economic conditions. Therefore, a correlation implied from tranches can be seen as a measure of the general health of the credit market. We analyse the European market of standardized CDOs using tranches of iTraxx index in the periods before and during the global financial crisis. We investigate the evolution of the correlations using different copula models: the standard Gaussian, the NIG, the double- t , and the Gumbel copula model. After calibration of these models one obtains a time varying vector of parameters. We analyse the dynamic pattern of these coefficients. That enables us to forecast future parameters and consequently calculate Value-at-Risk measures for iTraxx Europe tranches.

Keywords: CDO, multivariate distributions, copula, implied correlations, Value-at-Risk.

JEL classification: C13, C22, C53, G32

1 Introduction

Financial institutions have been facing difficulties over the years for a wide variety of reasons, however, the last financial crisis has shown that one of the major source of problems was the credit risk management. The credit derivatives market was the most innovative and fastest growing derivative market during the past ten years. The rapid development

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was due to new possibilities that were offered by credit derivatives. Credit instruments are flexible financial products that enable the efficient repackaging and transfer of credit risk. Credit derivatives are attractive for yield seeking investors and banks that need to hedge their investments and fulfil the capital requirements. The most popular securities traded on open markets are credit default swaps (CDS), default baskets, and collateralized debt obligations (CDO). In this paper we consider tranches of the iTraxx Europe index because of the availability of market quotes. iTraxx tranches have a structure of a synthetic CDO. They are written on the portfolio of the 125 most liquid CDS on European companies.

The market standard tool for pricing CDO tranches is the one factor Gaussian copula model. The core assumption of this model is that one value of the correlation is sufficient to model the correlation of every pair of assets. The one factor model for the CDO valuation is an analogy to the Black-Scholes option pricing model where the implied correlation plays the role of the implied volatility parameter. The correlations implied from the different tranches of the same CDO are not equal and the observed phenomenon is called a correlation smile.

Modelling the risk of CDOs involves determining the loss distribution of the underlying portfolio. If we have a portfolio of several assets, we have to quantify the default risk of each obligor and also take into account the synergy of these risks. Because of the high dimensionality, the valuation of a CDO is usually achieved by applying a factor model. For a guide to credit risk models and credit derivatives we refer to Bluhm & Overbeck (2006) and Bluhm, Overbeck & Wagner (2010).

There has been a multitude of CDO pricing methods proposed. The most popular models are based on copula functions. The market standard is the Gaussian copula model proposed by Li (2000). Burtshell, Gregory & Laurent (2008) compare selected copula approaches. The alternative valuations methods are the random factor loading model (Andersen & Sidenius 2004), the intensity based models (Duffie & Gârleanu 2001), the multivariate asset value models (Zhou 2001, Overbeck & Schmidt 2005). Another stream of CDO pricing models come from a top-down framework. Representatives of this approach are Schönbucher (2005), Bennani (2005), Sidenius, Piterbarg & Andersen (2008), Filipovic, Overbeck & Schmidt (2011).

Most of the above cited models are fully parametric and static. The focus of this research is on the dynamics of CDO parameters. We compare different copula models and investigate the dynamic evolution of the calibrated base parameters, which we explain in the next section. We study the dynamic pattern in data, check the stability of coefficients over time, and forecast the models' parameters. We calculate Value-at-Risk (VaR) measures for iTraxx Europe tranches with the aim to improve the understanding of the risk associated with trading credit derivatives.

An issue of the VaR estimation has appeared already in CDO studies. O'Kane & Schlögl (2005) calculate VaR for credit portfolios using the Gaussian, Student- t , Clayton, and Gumbel copula models. Fender, Tarashev & Zhu (2008) compare VaR of corporate bonds and CDO tranches of the same ratings. However, both works do not use empirical data and do not carry any investigations over time. In consequence, they do not analyse the

dynamics in CDO models, neither conduct any backtesting for VaR which are the main contributions of this research.

The paper is structured as follows. Section 2 introduces the concept of CDOs. Section 3 presents the models. Section 4 shows results. Section 5 concludes.

2 CDOs

2.1 Market for credit risk transfer

The international standards for measuring and recognizing risk are provided in the Basel I and II Accords developed by the Basel Committee on Banking Supervision. The Basel capital requirements rules state that credit institutions must at all times maintain a minimum amount of financial capital, in order to cover the risks to which they are exposed. In relation to credit risk, Basel II permits two approaches. Banks can assess risk using the standardised approach, which involves external credit assessments, or they can use their own internal systems for rating credit risk. The latter possibility encouraged banks to develop more sophisticated risk management techniques. The strong capital requirements motivated banks to transfer risk from their balance sheets directly to investors leading to the development of new risk dispersal instruments like CDS, default baskets, CDOs. We judge the magnitude of losses incurred by the CDO tranches by calculating VaR. VaR is the most widely used risk measure, mostly because of Basel II requirements for financial services. The effectiveness of VaR models is commonly assessed by a backtesting procedure. For an overview of VaR we refer to Jorion (2006).

2.2 CDO Valuation

Assume the existence of a risk neutral pricing measure P , under which all discounted price processes are martingales. All expectations are taken with respect to this measure. Consider a CDO with a maturity T , J tranches and a pool of d entities at the valuation day t_0 . Every obligor i is represented by a default indicator

$$\Gamma_i(t) = \mathbf{1}(\tau_i \leq t), \quad i = 1, \dots, d, \quad (1)$$

such that the obligor defaults at time t within a period $[t_0, T]$ if the time of default variable $\tau_i \leq t$. The portfolio loss at time t is defined as

$$L(t) = \frac{\text{LGD}}{d} \sum_{i=1}^d \Gamma_i(t), \quad t \in [t_0, T], \quad (2)$$

where LGD is a common loss given default. Each tranche $j = 1, \dots, J$ is defined by the detachment l_j and attachment u_j point which are the percentages of the portfolio losses and $l_j < u_j$. The loss L_j at time t is expressed as

$$L_j(t) = L^u(t, u_j) - L^u(t, l_j),$$

where L^u is specified from (2)

$$L^u(t, x) = \min\{L(t), x\} \quad \text{for } x \in [0, 1].$$

The outstanding notional $F_j(t)$ of the tranche j is written as

$$F_j(t) = F^u(t, u_j) - F^u(t, l_j),$$

where F^u is

$$F^u(t, x) = x - L^u(t, x) \quad \text{for } x \in [0, 1].$$

The fair spread of a CDO tranche is defined by the equivalence of the protection (called also default) and premium leg. The protection leg DL_j is calculated as the expected value of the discounted stream of payments made upon defaults

$$DL_j(t_0) = \sum_{t=t_1}^T \beta(t_0, t) \mathbf{E}\{L_j(t) - L_j(t - \Delta t)\}, \quad j = 1, \dots, J, \quad (3)$$

where β is a discount factor and Δt is a time between t and the previous payment day. The premium leg PL_j is expressed as the expectation of the present value of all premium payments

$$PL_j(t_0) = \sum_{t=t_1}^T \beta(t_0, t) s_j(t_0) \Delta t \mathbf{E}\{F_j(t)\}, \quad j = 2, \dots, J, \quad (4)$$

where s_j denotes the spread of tranche j . For the equity tranche, the premium leg (4) turns into

$$PL_1(t_0) = \alpha(t_0)(u_1 - l_1) + \sum_{t=t_1}^T \beta(t_0, t) \cdot 500 \cdot \Delta t \mathbf{E}\{F_1(t)\},$$

with an upfront payment α and a fixed spread of 500 bp. The tranche spread is found by solving $PL_j(t_0) = DL_j(t_0)$ for $s_j(t_0)$

$$s_j(t_0) = \frac{\sum_{t=t_1}^T \beta(t_0, t) \mathbf{E}\{L_j(t) - L_j(t - \Delta t)\}}{\sum_{t=t_1}^T \beta(t_0, t) \Delta t \mathbf{E}\{F_j(t)\}}, \quad \text{for } j = 2, \dots, J. \quad (5)$$

By denoting the denominator of (5) by $PL_j^*(t_0)$ we get

$$s_j(t_0) = \frac{DL_j(t_0)}{PL_j^*(t_0)}, \quad \text{for } j = 2, \dots, J.$$

For the equity tranche the upfront payment is

$$\begin{aligned} \alpha(t_0) &= \frac{100}{u_1 - l_1} \sum_{t=t_0}^T (\beta(t, t_0) [\mathbf{E}\{L_1(t) - L_1(t - \Delta t)\} - 0.05 \Delta t \mathbf{E}\{F_1(t)\}]) \\ &= \frac{100}{u_1 - l_1} \{DL_1(t_0) - 0.05 PL_1^*(t_0)\}. \end{aligned}$$

For more details we refer to Chapter 3 in Bluhm & Overbeck (2006).

2.3 Types of Dependence Parameters

The standard CDO pricing methods are based on the Gaussian distribution. Therefore, it is common to equate the problem of modelling CDOs with modelling correlations. However, there are numerous approaches where the dependency is represented by a parameter (or parameters) that is not a linear correlation.

Compound correlation and compound parameter

In a given copula model, a compound dependency parameter $\rho(l_j, u_j)$, $j = 1, \dots, J$, is a parameter that prices the tranche j so that it fits the market value. A present value PV_j of a tranche j is given by

$$PV_j(t_0) = \sum_{t=t_1}^T \beta(t_0, t) [s_j(t_0) \Delta t \mathbb{E}_{\rho(l_j, u_j)} \{F_j(t)\} - \mathbb{E}_{\rho(l_j, u_j)} \{L_j(t) - L_j(t - \Delta t)\}], \quad j = 2, \dots, J,$$

where the expected value is calculated with respect to the distribution determined by the compound parameter $\rho(l_j, u_j)$.

In this work we investigate implied correlations and implied dependency parameters. An implied dependency parameter is a parameter calculated out of a market spread by inverting the pricing model. The standard Gaussian model uses only one correlation to specify the loss distribution and price all the tranches. However, the implied correlations are not the same across the tranches. The phenomenon observed is called a correlation smile and has been widely studied in the literature, see Amato & Gyntelberg (2005).

The main disadvantage of the compound parameters is that the mezzanine tranches are not monotonic in correlation. In consequence, there could be two parameters that yield the same market spread. Moreover, there is no guarantee that the implied parameter exists. These shortcomings motivate us to turn to base parameters.

Base correlation and base parameter

The base correlations were introduced by McGinty & Ahluwalia (2004) from JP Morgan in the framework of the Homogeneous Large Pool Gaussian copula model, see Section 3. The main idea behind the concept of the base correlation is that every tranche can be decomposed into two tranches that have lower attachment point zero. Being long the mezzanine tranche with the attachment points l_j and u_j can be viewed as being simultaneously long the equity tranche with upper attachment point u_j and short the equity tranche with upper attachment point l_j . The base correlations are computed using a bootstrapping technique, i.e. we use the base correlation of the first tranche to calculate the second tranche, and so on. The expected losses of successive tranches are calculated recursively

$$\begin{aligned} \mathbb{E}\{L_{(3\%,6\%)}\} &= \mathbb{E}_{\rho(0,6\%)}\{L_{(0,6\%)}\} - \mathbb{E}_{\rho(0,3\%)}\{L_{(0,3\%)}\}, \\ \mathbb{E}\{L_{(6\%,9\%)}\} &= \mathbb{E}_{\rho(0,9\%)}\{L_{(0,9\%)}\} - \mathbb{E}_{\rho(0,6\%)}\{L_{(0,6\%)}\}, \dots \end{aligned}$$

A present value PV_j of a tranche $j = 2, \dots, J$ in this approach is given by

$$PV_j(t_0) = \sum_{t=t_1}^T \beta(t_0, t) \left(s_j(t_0) \Delta t \left[\mathbb{E}_{\rho(0, u_j)} \{F_j^u(t)\} - \mathbb{E}_{\rho(0, l_j)} \{F_j^l(t)\} \right] - \right. \\ \left. \mathbb{E}_{\rho(0, u_j)} \{L_j^u(t, u_j) - L_j^u(t - \Delta t, u_j)\} + \mathbb{E}_{\rho(0, l_j)} \{L_j^l(t, l_j) - L_j^l(t - \Delta t, l_j)\} \right). \quad (6)$$

Although the base correlations overcome some limitations of the compound correlations, they also have drawbacks. The analysis of the Gaussian base correlations can be found in Willemann (2005). Willemann (2005) lists problems with the use of base correlations. He shows that even if the true default correlation increases, base correlations might decrease. Moreover, the expected losses for mezzanine tranches can be negative.

The concept of the base correlation can be applied to non-Gaussian copula. Then the expectations in the above formulae are taken with respect to a given distribution of the portfolio loss and the dependence parameters calculated using the above approach we call the base parameters.

3 CDO Dynamics

The most popular CDO pricing models are based on a factor approach combined with various copula functions. In this study we apply and compare the following one-factor models: the Gaussian copula model, the Normal Inverse Gaussian (NIG) copula model, the double- t copula model, the Gumbel Archimedean copula model. The copulae determine the dependency structure between entities in the pool. The industry standard methodology assumes that the values of assets are driven by one unobserved factor such that the individual defaults are conditionally independent given the realization of the factor. The factor reflects a state of economy and is common to all assets. We assume, according to the market practice, that the portfolio is homogeneous. For all models we use the factor representation and the large portfolio approximation technique.

3.1 Copula models

Let $(\tau_1, \dots, \tau_d)^\top$ be a vector of default times with a (risk-neutral) joint cumulative distribution function

$$F(t_1, \dots, t_d) = \mathbb{P}(\tau_1 \leq t_1, \dots, \tau_d \leq t_d) \quad \text{for all } (t_1, \dots, t_d)^\top \in \mathbb{R}_+^d.$$

We denote by F_1, \dots, F_d the marginal distribution functions. From the Sklar theorem we know that there exists a copula $C : [0, 1]^d \rightarrow [0, 1]$, such that

$$F(t_1, \dots, t_d) = C\{F_1(t_1), \dots, F_d(t_d)\}.$$

For a survey over the mathematical foundations of copulae we refer to Nelsen (2006). A default time τ_i of the asset $i = 1, \dots, d$ is taken to be the first jump time of a Poisson process with an intensity $\lambda_i(t)$ and with exponentially distributed jumps

$$F_i(t) = P(\tau_i \leq t) = 1 - \exp \left\{ - \int_{t_0}^t \lambda_i(u) du \right\}, \quad t \geq t_0$$

An intensity function $\lambda_i(t)$ represents the instantaneous default probability of the obligor at time t . We assume that every individual name has a constant intensity function $\lambda_i(t) = \lambda_i$. Then the default probability is calculated as

$$p_i(t) = F_i(t) = 1 - \exp\{-\lambda_i(t - t_0)\}. \quad (7)$$

Large portfolio approximation

In the one-factor model, introduced by Vasicek (1987), default times are calculated from a vector of latent variables $(X_1, \dots, X_d)^\top$ and each variable X_i , $i = 1, \dots, d$ is represented as

$$X_i = \sqrt{\rho}Y + \sqrt{1 - \rho}Z_i, \quad (8)$$

where Y is a systematic risk factor, $\{Z_i\}_{i=1}^d$ are idiosyncratic risk factors and all are independent.

The analytical tractability of the factor model is reached by assuming that the portfolio is homogeneous i.e. all the assets have the same exposure, share the same pairwise correlation ρ , default probability p , and recovery rate R and all these values are constant for all time horizons. The number of obligors in the reference portfolio is large so that one may apply asymptotic techniques. A default occurs when the value of the variable X_i drops below the default threshold $C = F_X^{-1}(p)$, where F_X is a distribution function of X_i .

From the representation (8) we get that conditionally on the realization of the systematic factor Y , the variables X_i , $i = 1, \dots, d$, are independent. Therefore, the individual probability that $X_i < C$ given that $Y = y$ is derived from (8) and equals

$$p(y) = P(X_i < C | Y = y) = F_Z \left(\frac{C - \sqrt{\rho}y}{\sqrt{1 - \rho}} \right), \quad (9)$$

where $Z_i \sim F_Z$. As a single default event is a binary variable, the conditional distribution of the loss L of the portfolio of d assets follows a binomial distribution

$$P \left(L = \frac{k}{d} \right) = \binom{d}{k} p(y)^k \{1 - p(y)\}^{d-k}. \quad (10)$$

For portfolios of a sufficiently large size d the fraction of defaulted obligors for a given state of economy $Y = y$ is approximately equal the conditional default probability (9). By the law of large numbers the percentage loss given Y tends in probability to $p(Y)$

$$\begin{aligned} P(L \leq x) &= P\{p(Y) \leq x\} \\ &= 1 - F_Y \left\{ \frac{C - \sqrt{\rho}F_Z^{-1}(x)}{\sqrt{1 - \rho}} \right\}. \end{aligned} \quad (11)$$

The expected tranche loss in (3) and (4) is calculated as an integral with respect to the distribution (11) and with $C(t) = F_X^{-1}\{p(t)\}$ for $t_0 < t \leq T$.

Gaussian copula model

In the Gaussian copula model the variables (8) are decomposed by the factors Y and $\{Z_i\}_{i=1}^d$ that are i.i.d. $N(0, 1)$. In this framework the default times are given by $\tau_i = F_i^{-1}\{\Phi(X_i)\}$ with Φ denoting the cdf of a standard normal. The Gaussian copula was introduced in the CDO valuation by Li (1999). The large portfolio approximation for the Gaussian one-factor model, which is often referred to as Homogeneous Large Pool Gaussian Copula model, provides a simple and a fast solution, therefore, it quickly became an industry standard for pricing CDOs. However, the Gaussian copula has fundamental drawbacks. The main problem is that the Gaussian copula is not able to model properly the joint extreme events. The following copula models incorporate an effect of tail dependence.

Normal Inverse Gaussian (NIG) copula model

The default times are modelled from a latent vector $(X_1, \dots, X_d)^\top$ as in (8), where now $Y, \{Z_i\}_{i=1}^d$ are independent NIG distributed variables

$$Y \sim \text{NIG}\left(\alpha, \beta, -\frac{\beta\gamma^2}{\alpha^2}, \frac{\gamma^3}{\alpha^2}\right), \quad (12)$$

$$Z_i \sim \text{NIG}\left(\frac{\sqrt{1-\rho}}{\sqrt{\rho}}\alpha, \frac{\sqrt{1-\rho}}{\sqrt{\rho}}\beta, -\frac{\sqrt{1-\rho}}{\sqrt{\rho}}\frac{\beta\gamma^2}{\alpha^2}, \frac{\sqrt{1-\rho}}{\sqrt{\rho}}\frac{\gamma^3}{\alpha^2}\right) \quad (13)$$

with $0 \leq |\beta| < \alpha$ and $\gamma = \sqrt{\alpha^2 - \beta^2}$. For more detailed explanation of the NIG copula model refer to Kalemanova, Schmid & Werner (2007). In order to simplify notation, denote $\text{NIG}\left(s\alpha, s\beta, -s\frac{\beta\gamma^2}{\alpha^2}, s\frac{\gamma^3}{\alpha^2}\right)$ as $\text{NIG}_{(s)}$. Hence, (12) and (13) can be rewritten as $Y \sim \text{NIG}_{(1)}$ and $Z_i \sim \text{NIG}_{\left(\frac{\sqrt{1-\rho}}{\sqrt{\rho}}\right)}$. The NIG distribution is stable under convolution

$$X_i \sim \text{NIG}\left(\frac{\alpha}{\sqrt{\rho}}, \frac{\beta}{\sqrt{\rho}}, -\frac{1}{\sqrt{\rho}}\frac{\beta\gamma^2}{\alpha^2}, \frac{1}{\sqrt{\rho}}\frac{\gamma^3}{\alpha^2}\right) = \text{NIG}_{(1/\sqrt{\rho})},$$

however, the vector (X_1, \dots, X_d) is not multivariate NIG distributed. The default times are computed as $\tau_i = F_i^{-1}\{\text{NIG}_{(1/\sqrt{\rho})}(X_i)\}$.

Double- t model

In this model default times are created from

$$X_i = \sqrt{\rho}\sqrt{\frac{\nu_Y - 2}{\nu_Y}}Y + \sqrt{1-\rho}\sqrt{\frac{\nu_Z - 2}{\nu_Z}}Z_i, \quad i = 1, \dots, d, \quad (14)$$

where Y and Z_i are t distributed with ν_Y and ν_Z degrees of freedom respectively. The double- t one-factor model was introduced by Hull & White (2004). Since the Student t distribution is not stable under convolution, X_i are not t distributed and the copula is not a Student t copula. The default times are such that

$$\tau_i = F_i^{-1}\{F_X(X_i)\},$$

where the distribution F_X of X_i needs to be approximated numerically.

Gumbel Archimedean copula model

An Archimedean copula function $C : [0, 1]^d \rightarrow [0, 1]$ is a copula that can be represented in the following form

$$C(u_1, \dots, u_d) = \phi\{\phi^{-1}(u_1) + \dots + \phi^{-1}(u_d)\}, \quad u_1, \dots, u_d \in [0, 1], \quad (15)$$

where $\phi \in \{\phi : [0; \infty) \rightarrow [0, 1] \mid \phi(0) = 1, \phi(\infty) = 0; (-1)^j \phi^{(j)} \geq 0; j = 1, \dots, \infty\}$ is called a generator of the copula and usually incorporates a parameter θ . Each generator is a Laplace transform of a cumulative distribution function F_Y of a positive random variable Y , i.e. $\phi(t) = \int_0^\infty e^{-tw} dF_Y(w)$.

The random variables of Archimedean copula possess a factor structure that allows us to derive a large portfolio approximation similar to the one obtained in the classical approach (11). The factor representation emerges from the sampling algorithm proposed by Marshall & Olkin (1988). If we generate X_1, \dots, X_d i.i.d. uniformly distributed on $[0, 1]$ and a variable Y that is independent of X_1, \dots, X_d and whose Laplace transform is ϕ , then the variables $U_i = \phi\left(-\frac{\log X_i}{Y}\right)$, $i = 1, \dots, d$, have the Archimedean copula function (15) as the joint distribution function.

In this algorithm the dependence between the variables U_i , $i = 1, \dots, d$, is generated by the mixing variable Y . Therefore, conditional on the realisation of Y , the random variables U_i are independent.

Let

$$U_i = \bar{p}_i(\tau_i) \sim U[0, 1], \quad i = 1, \dots, d,$$

where \bar{p}_i is a survival probability. Recall that the i th obligor survives until $t < T$ if and only if

$$\begin{aligned} \tau_i &\geq t \\ \text{or } U_i &\leq \bar{p}_i(t). \end{aligned} \quad (16)$$

So instead of determining the joint default probability of τ_i one can specify a joint distribution of U_i by a copula

$$C\{\bar{p}_1(t), \dots, \bar{p}_d(t)\} = P\{U_1 \leq \bar{p}_1(t), \dots, U_d \leq \bar{p}_d(t)\},$$

where the margins satisfy $P\{U_i \leq \bar{p}_i(t)\} = \bar{p}_i(t)$. Hence, the default times are calculated as $\tau_i = \bar{p}_i^{-1}(U_i)$, where U_i have a joint distribution of the Archimedean copula.

If U_i , $i = 1, \dots, d$, have the same unconditional survival probability \bar{p} and the number of obligors d is very large, then the limiting loss distribution is

$$P(L \leq x) = F_Y \left\{ -\frac{\log(1-x)}{\phi^{-1}(\bar{p})} \right\},$$

where F_Y is a distribution of the mixing variable Y . For more details we refer to Schönbucher (2003), Chapter 10.

From (16) we see that a default occurs when U_i is large. Since for the credit portfolios we are mostly interested in modeling the joint defaults, in the applications we will use

a Gumbel copula as it exhibits an upper tail dependence. Namely, it assigns a positive probability to a simultaneous occurrence of positive extreme values. The Gumbel copula is given by

$$C(u_1, \dots, u_d; \theta) = \exp \left[- \left\{ \sum_{j=1}^d (-\log u_j)^\theta \right\}^{\theta^{-1}} \right],$$

where the generator

$$\phi(x; \theta) = \exp \{-x^{1/\theta}\}, \quad 1 \leq \theta < \infty, \quad x \in [0, \infty).$$

is a Laplace transform of an α -stable distribution with $\alpha = 1/\theta$.

A generalization of the one-parameter Archimedean copulae are hierarchical Archimedean copulae (HAC), see Okhrin, Okhrin & Schmid (2010). HAC are flexible copulas that define the dependency structure in a recursive way using multiple parameters. Choroś-Tomczyk, Härdle & Okhrin (2010) apply HAC to CDO pricing.

3.2 Time dynamics

The copula structures of the above presented models contain only one parameter. This parameter reflects the strength of the dependence between the entities. As the market conditions change over time, the relation between the companies also change. In this study we investigate the evolution of the parameters over time and calculate their forecasts. The parameters' forecasts are used to compute the Value-at-Risk (VaR) measures for spreads. VaR states the maximum expected loss of a particular investment for a defined time horizon and for a given confidence level. It is especially useful if we are interested in assessing the tail risk. Big portfolios of financial assets are characterised by a high risk of joint extreme outcomes. The ability of a pricing model to describe the joint downward and upward movements is crucial for assigning the correct CDO prices.

In this paper we calculate a one-day VaR for the CDO tranches. The forecast of the next day tranche spreads are computed from the forecasted models' parameters.

Let $\theta_j(t)$ denote a copula parameter implied from a tranche $j = 1, \dots, J$ at time t using a copula-based CDO model. By calibrating the models to data day by day we construct time series of parameters. The econometric analysis is further conducted on the first difference $X_j(t) = \theta_j(t) - \theta_j(t-1)$ or on the first difference of logarithms of the parameters $X_j(t) = \log \theta_j(t) - \log \theta_j(t-1)$. The stationarity is checked using the augmented Dickey-Fuller test and the heteroskedasticity is detected by the Engle test for residual heteroscedasticity. Afterwards, we choose a time series model to describe the dynamic behavior of $X_j(t)$. The models considered are ARMA(R, M)-GARCH(P, Q)

$$\begin{aligned} X_j(t) &= \mu_j(t) + \varepsilon_j(t), \\ X_j(t) &= C_j + \sum_{i=1}^R \phi_{j,i} X_j(t-i) + \sum_{l=1}^M \psi_{j,l} \varepsilon_j(t-l) + \varepsilon_j(t), \end{aligned} \quad (17)$$

where $\varepsilon_j(t) = \sigma_j(t)Z_j(t)$ are innovations, $Z_j(t)$ are standardised innovations that follow the standard normal or t distribution, and

$$\sigma_j^2(t) = K_j + \sum_{i=1}^P G_{j,i} \sigma_j^2(t-i) + \sum_{l=1}^Q A_{j,l} \varepsilon_j^2(t-l) \quad (18)$$

with the following constraints $\sum_{i=1}^P G_{j,i} + \sum_{l=1}^Q A_{j,l} < 1$, $K_j > 0$, $G_{j,i} \geq 0$, and $A_{j,i} \geq 0$.

The following investigation is carried out in moving windows. A moving window procedure is used when only the most recent data are considered to be relevant for estimation. Here, a static window of $h = 250$ elements is applied. Then for every time t_0 between h and \tilde{T} , the end of the period considered, we look at $\{X_j(t)\}_{t=t_0-h}^{t_0-1}$. In our work the size of the interval is fixed, however, more advance methods estimate the size of the window adaptively, see Giacomini, Härdle & Spokoiny (2009).

We restrict the orders in (17) and (18) to be $R, M, P, Q \in \{0, 1, 2\}$. The selection is done in the first window $\{X_j(t)\}_{t=1}^h$ using AIC and BIC criteria. Then the orders of the time series models are fixed but in each following window we re-estimate the parameters. The normality of the standardised residuals is also checked in the first window. If the normality is rejected, we impose a t distribution.

After selecting a model and fitting it in the moving window we forecast the conditional mean $\{\hat{\mu}_j(t)\}_{t=h+1}^{\tilde{T}}$ and the conditional standard deviations $\{\hat{\sigma}_j(t)\}_{t=h+1}^{\tilde{T}}$ of the residuals of the process $X_j(t)$. Using these results we calculate predictions of $\hat{X}_j(t)$ and then transform them into $\hat{\theta}_j(t)$ which are needed for calculating VaR measures for tranche spreads.

For a given level α and a time horizon, the VaR is an α -quantile of a profit-loss distribution. A profit-loss process is the first difference of the spread process $\Delta s_j(t) = s_j(t) - s_j(t-1)$ and the probability that $\Delta s_j(t)$ exceeds the value of $\text{VaR}_{s_j}^{1-\alpha}(t)$ or is smaller than $\text{VaR}_{s_j}^\alpha(t)$ is equal α :

$$\begin{aligned} P\{\Delta s_j(t) > \text{VaR}_{s_j}^{1-\alpha}(t)\} &= \alpha, \\ P\{\Delta s_j(t) < \text{VaR}_{s_j}^\alpha(t)\} &= \alpha. \end{aligned}$$

The calculation of $\text{VaR}_{s_j}^\alpha(t)$ requires prediction of $\hat{s}_j(t)$ as the value of $s_j(t-1)$ is known. The spread is a function of correlations and default probabilities and both have to be forecasted in order to get the next day spread. As the predictor of the intensity $\hat{\lambda}(t)$ (7) one applies a forecast of the conditional mean of an ARMA model.

The spread of a tranche $j = 2, \dots, J$ depends on two dependence parameters $\hat{\theta}_j(t)$ and $\hat{\theta}_{j-1}(t)$. Therefore, we investigate the dependence of $X_j(t)$ and $X_{j-1}(t)$. Because of the representation (17-18) we determine the joint distribution of the innovations. A pair $[\varepsilon_j(t), \varepsilon_{j-1}(t)]$ follows a certain distribution G_j . As the marginals are known $\varepsilon_j \sim F_{\varepsilon_j}$, the joint distribution can be modeled with a copula function

$$C_{G_j}(u_1, u_2) = G_j\{F_{\varepsilon_j}^{-1}(u_1), F_{\varepsilon_{j-1}}^{-1}(u_2)\}.$$

The copula chosen in this instance are a bivariate Gaussian, Gumbel and Clayton.

Afterwards, we generate N random bivariate vectors from a copula C_{G_j} and then we transform them into bivariate vectors $[\hat{\theta}_j(t), \hat{\theta}_{j-1}(t)]$. The set of theta parameters gives us a vector of possible next day spreads. Finally, $\widehat{\text{VaR}}_{s_j}^\alpha(t)$ is calculated as a sample quantile from N values.

The adequacy of VaR is examined by performing a backtesting on historical spreads. The basic technique is to calculate an exceedances ratio of the number of exceedances to the number of observations

$$\hat{\alpha}_j^u = \frac{1}{\tilde{T} - h} \sum_{t=h+1}^{\tilde{T}} \mathbf{1}\{\Delta s_j(t) > \widehat{\text{VaR}}_{s_j}^{1-\alpha}(t)\}, \quad (19)$$

$$\hat{\alpha}_j^l = \frac{1}{\tilde{T} - h} \sum_{t=h+1}^{\tilde{T}} \mathbf{1}\{\Delta s_j(t) < \widehat{\text{VaR}}_{s_j}^\alpha(t)\}. \quad (20)$$

If a model works well, the exceedances ratio is close to the confidence level. The test that checks the frequency of exceedances is Kupiec's likelihood ratio test with the statistics

$$LR_j = -2 \log\{(1 - \alpha_j)^{\tilde{T}-h-n} \alpha_j^n\} + 2 \log\{(1 - \hat{\alpha}_j)^{\tilde{T}-h-n} \hat{\alpha}_j^n\} \sim \chi^2(1), \quad (21)$$

where α_j is either α_j^u or α_j^l and n is a number of corresponding exceedances.

The model applied to compute VaR is correctly specified if the exceedances happen only in the effect of unpredictable events. Moreover, the exceedances should not cluster over time. However, this dependence in time is not taken into account by the Kupiec test. Therefore, we use a dynamic quantile (DQ) test proposed by Engle & Manganelli (2004). Define a hit H_j as $\{H_j^u(t)\}_{t=h+1}^{\tilde{T}}$ for the upper VaR or $\{H_j^l(t)\}_{t=h+1}^{\tilde{T}}$ for the lower VaR such that

$$\begin{aligned} H_j^u(t) &= \mathbf{1}\{\Delta s_j(t) > \widehat{\text{VaR}}_{s_j}^{1-\alpha}(t)\} - \alpha, \\ H_j^l(t) &= \mathbf{1}\{\Delta s_j(t) < \widehat{\text{VaR}}_{s_j}^\alpha(t)\} - \alpha. \end{aligned} \quad (22)$$

The test regresses the hits on their lags and the other variables. The statistics is given by

$$DQ_j = H_j V_j^\top (V_j^\top V_j)^{-1} V_j^\top H_j / \{\alpha(1 - \alpha)(\tilde{T} - h - n)\}, \quad (23)$$

where V_j is a vector of explanatory variables. Following Engle & Manganelli (2004) we include in V_j a constant, H_j , $\widehat{\text{VaR}}_{s_j}$, defined for the upper VaR as $\{\widehat{\text{VaR}}_{s_j}^{1-\alpha}(t)\}_{t=t_0-h}^{t_0-1}$ and for the lower VaR as $\{\widehat{\text{VaR}}_{s_j}^\alpha(t)\}_{t=t_0-h}^{t_0-1}$, and also their four lagged values. The test's null hypothesis states that H_j and V_j are orthogonal. Under the null hypothesis the statistics $DQ_j \sim \chi^2(q_j)$, where $q_j = \text{rank}(V_j)$.

4 Empirical Results

The empirical research of this study was performed using iTraxx Euro indices with a maturity of 5 years for a time period between 20 September 2006 and 2 February 2009.

This time interval is especially interesting as it covers time before and during the global financial crisis. In the 4th quarter of 2008 the European market of credit derivatives suffered from lack of demand. In the first quarter of 2009 the iTraxx tranches became highly illiquid. Many missing data made the analysis for the year 2009 impossible.

The time series are constructed from the on-the-run indices of series 6, 7, 8 and 9 consecutively plus observations of series 9 till the end of the term considered. The construction of our data set is motivated by the fact that the latest series of the index are the most liquid. However, we observed that at the end of 2008 the tranches on Series 9 of iTraxx, as well as of the American equivalent index CDX, were more liquid than the tranches of the on-the-run Series 10. For that reason Series 10 was not included in this study.

The following series and time periods are concatenated as follows:

1. Series 6: 20060920-20070322
2. Series 7: 20070323-20070919
3. Series 8: 20070920-20080320
4. Series 9: 20080321-20090202

In total we have 619 days. We assume a flat correlation structure, deterministic LGD of 60%, and constant intensity parameters derived from iTraxx indices. The discount curve is calculated from rates of Euribor and Euro Swaps.

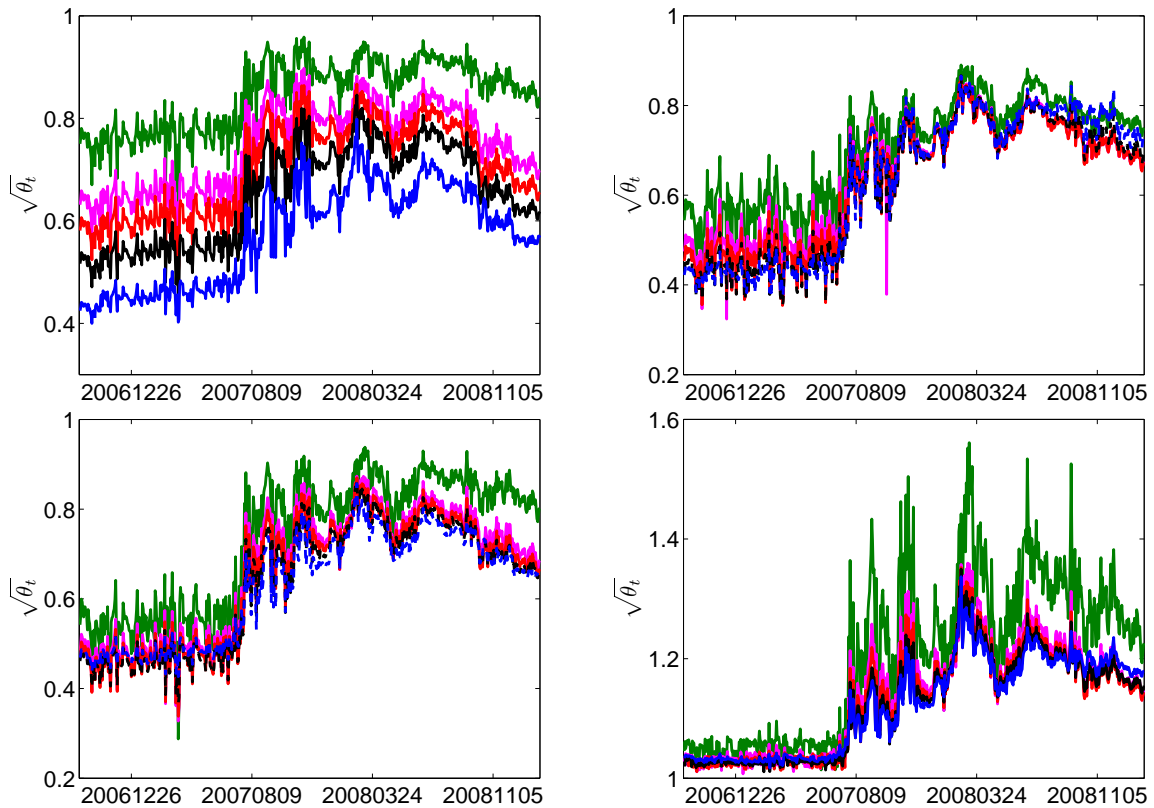


Figure 1: Square root of the base correlation from the Gaussian (upper panel, left), the NIG (upper panel, right), the double- t (lower panel, left) model. Gumbel base parameter (lower panel, right). Tranches: 1 (blue), 2 (black), 3 (red), 4 (pink), 5 (green).

Model	Tranche	Mean	Std. Dev	Maximum	Minimum
Gaussian	1	0.573	0.097	0.798	0.400
	2	0.652	0.096	0.845	0.453
	3	0.704	0.088	0.867	0.494
	4	0.745	0.082	0.898	0.529
	5	0.846	0.067	0.959	0.610
NIG	1	0.635	0.158	0.866	0.375
	2	0.629	0.148	0.867	0.354
	3	0.634	0.135	0.864	0.354
	4	0.645	0.127	0.864	0.324
	5	0.703	0.116	0.890	0.371
Double- t	1	0.630	0.119	0.857	0.427
	2	0.636	0.135	0.866	0.366
	3	0.647	0.138	0.870	0.338
	4	0.664	0.140	0.873	0.328
	5	0.733	0.146	0.938	0.288
Gumbel	1	1.124	0.078	1.337	1.017
	2	1.124	0.082	1.350	1.011
	3	1.129	0.085	1.352	1.011
	4	1.138	0.090	1.360	1.007
	5	1.202	0.132	1.561	1.014

Table 1: Summary statistics for the implied square root of the base parameter for the Gaussian, NIG, double- t , and Gumbel model.

Figure 1 shows the implied square root of the base parameter from the Gaussian, the NIG, the double- t , and the Gumbel model. Table 1 presents their summary statistics. We see that the parameters for mezzanine and senior tranches are much lower for the NIG and the double- t model than for the Gaussian copula. In addition, the tranches of the benchmark model are the least volatile.

In general, the NIG distribution has four free parameters. In the model (12) and (13) two parameters are chosen in such a way that the NIG distributions have zero mean and unit variance. In consequence, the copula has two free parameters α and β . As it was shown by Kalemanova et al. (2007), setting β to zero does not significantly affect the results. However, α has to be calibrated to data. Since from one market value of a tranche spread we can imply only one parameter, and this is in our case the correlation, other parameters have to be determined through a preliminary investigation. Therefore, we calibrate the NIG model to data of all five tranches simultaneously by minimizing the sum of the relative difference between the calculated and historical spread. Hence, at any time $t > 0$ two parameters $\alpha(t)$ and $\rho(t)$ price all tranches, see Figure 2. $\{\alpha(t)\}_{t=0}^T$ obtained with this procedure are afterwards used in the individual calibration of the tranches.

In the double- t model (14) one has to choose the number of degrees of freedom for the common and the idiosyncratic factor. We follow Hull & White (2004) and set both to be equal four.

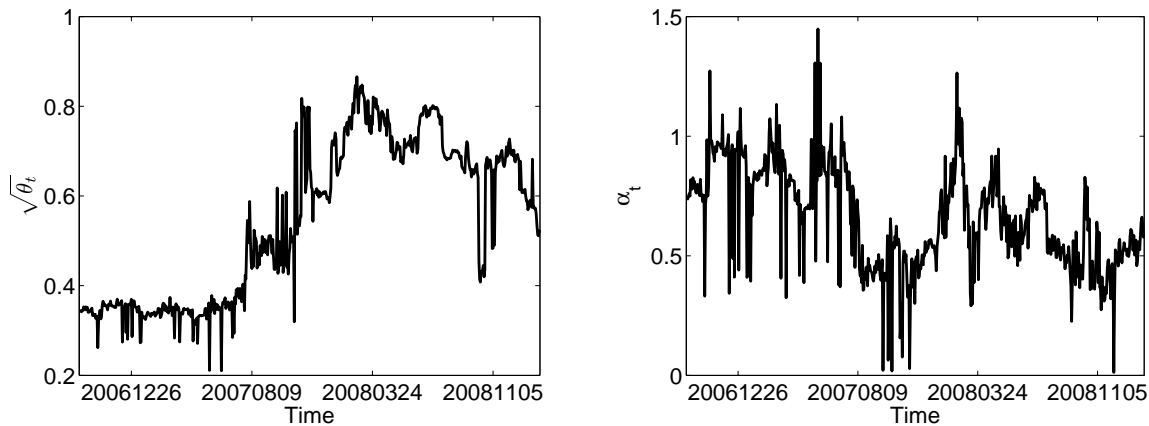


Figure 2: Square root of the base correlation (left) and α parameter (right) calibrated for all tranches using the NIG model.

All integrals in (6) are computed using the Legendre-Gauss quadrature.

The ARMA-GARCH models (17)-(18) are estimated for the first difference and for the first difference of logarithms of the parameters. The model type is chosen in the first window using AIC and BIC criteria and then estimated for the whole period. The final model selection for parameters is also based on the Kupiec test (21) and the dynamic quantile test (23). The parameters' forecasts that are incorporated in the calculations of the spreads' forecasts are computed from the ARMA-GARCH models that provide the best quantiles predictions. We do not present these intermediate results as we find them of minor interest to the reader.

Table 2 depicts the exceedance ratios and Tables 3-10 present the p -values of the Kupiec test (21) and the dynamic quantile test (23) for the VaR at the level α equal 0.05 and 0.95. Each table shows the results of the CDO valuation approaches: the Gaussian, NIG, double- t , and Gumbel copula model. In Tables 2-10 the columns refer to the bivariate copulae (Gaussian, Gumbel and Clayton) that are applied to model the parameters of the neighbor tranches. A sample from which the VaR is calculated has a size of 1000 elements. Figure 3 illustrates selected results.

The results of the Kupiec and the DQ tests are different in nearly 38% of the cases which confirms that it is important to check the time series dependence of the exceedances. The number of statistically significant results for every CDO pricing model for both tests are shown in Table 11. The results detect that the bivariate Gumbel copula was most often the optimal choice for modelling the dependence between the parameters according to the Kupiec test and the bivariate Gaussian copula according to the DQ test.

Tables 12 and 13 present the highest computed p -values for every tranche and every pricing method. Here each column refers to a different CDO valuation approach but each item is a largest p -value out of three obtained from the models where the bivariate Gaussian, Gumbel and Clayton copulae were used to model the parameters. Using the Kupiec test for the lower VaR the Gumbel copula was the optimal model for the equity tranche and NIG for other tranches from 2 to 5. For the tranche 3 also the Gaussian model can be selected. According to the Kupiec test for the upper VaR the Gumbel model is the best

choice for the equity tranche, the double- t for the tranches 2 and 4, the Gaussian model for the tranches 3 and 5. Regarding the DQ test, which is more reliable, both lower and upper VaR is best calculated by the double- t model for the tranches 2 and 4, and by the NIG model for the tranches 3 and 5. The optimal result for the equity tranche for the lower VaR was achieved by the Gaussian model and for the upper VaR by the Gumbel model. As we see from both tests, against all the odds, the simple Gaussian copula is not entirely outperformed by the more sophisticated models. However, the Gaussian and the Gumbel model perform better for the equity tranche, and the NIG and the double- t are rather better for more senior tranches.

Tranche	VaR(5%)			VaR(95%)			
	Gauss	Gumbel	Clayton	Gauss	Gumbel	Clayton	
Gaussian	1	0.035		0.054			
	2	0.038	0.038	0.014	0.065	0.054	0.046
	3	0.033	0.038	0.016	0.068	0.052	0.052
	4	0.016	0.008	0.003	0.016	0.008	0.003
	5	0.019	0.027	0.005	0.060	0.071	0.052
NIG	1	0.019		0.030			
	2	0.054	0.087	0.011	0.052	0.087	0.046
	3	0.068	0.120	0.038	0.082	0.120	0.035
	4	0.030	0.038	0.016	0.043	0.057	0.016
	5	0.038	0.087	0.008	0.038	0.057	0.016
Double- t	1	0.033		0.057			
	2	0.030	0.033	0.016	0.038	0.049	0.041
	3	0.033	0.033	0.011	0.060	0.065	0.043
	4	0.022	0.024	0.008	0.054	0.046	0.030
	5	0.008	0.022	0.000	0.035	0.038	0.030
Gumbel	1	0.044		0.046			
	2	0.024	0.030	0.014	0.027	0.030	0.033
	3	0.019	0.014	0.003	0.011	0.008	0.008
	4	0.011	0.005	0.003	0.014	0.008	0.005
	5	0.011	0.024	0.003	0.005	0.008	0.005

Table 2: Backtesting results. Exceedance ratios for all VaR models.

Tranche	VaR(5%)			VaR(95%)		
	Gauss	Gumbel	Clayton	Gauss	Gumbel	Clayton
1	0.174			0.706		
2	0.273	0.273	0.000	0.200	0.706	0.735
3	0.103	0.273	0.001	0.133	0.886	0.886
4	0.001	0.000	0.000	0.001	0.000	0.000
5	0.002	0.028	0.000	0.403	0.086	0.886

Table 3: Backtesting results for the Gaussian copula model. Kupiec test's p -values.

Tranche	VaR(5%)			VaR(95%)		
	Gauss	Gumbel	Clayton	Gauss	Gumbel	Clayton
1		0.755			0.011	
2	0.735	0.723	0.111	0.000	0.008	0.005
3	0.513	0.425	0.184	0.000	0.003	0.000
4	0.188	0.037	0.009	0.063	0.037	0.009
5	0.244	0.474	0.019	0.000	0.000	0.000

Table 4: Backtesting results for the Gaussian copula model. DQ test's p -values.

Tranche	VaR(5%)			VaR(95%)		
	Gauss	Gumbel	Clayton	Gauss	Gumbel	Clayton
1		0.002			0.056	
2	0.706	0.003	0.000	0.886	0.003	0.735
3	0.133	0.000	0.273	0.011	0.000	0.174
4	0.056	0.273	0.001	0.557	0.543	0.001
5	0.273	0.003	0.000	0.273	0.543	0.001

Table 5: Backtesting results for the NIG copula model. Kupiec test's p -values.

Tranche	VaR(5%)			VaR(95%)		
	Gauss	Gumbel	Clayton	Gauss	Gumbel	Clayton
1		0.137			0.068	
2	0.119	0.000	0.068	0.009	0.000	0.000
3	0.126	0.000	0.780	0.000	0.000	0.067
4	0.014	0.165	0.026	0.001	0.000	0.122
5	0.484	0.000	0.037	0.335	0.052	0.165

Table 6: Backtesting results for the NIG copula model. DQ test's p -values.

Tranche	VaR(5%)			VaR(95%)		
	Gauss	Gumbel	Clayton	Gauss	Gumbel	Clayton
1		0.103			0.543	
2	0.056	0.103	0.001	0.273	0.924	0.401
3	0.103	0.103	0.000	0.403	0.200	0.557
4	0.005	0.013	0.000	0.706	0.735	0.056
5	0.000	0.005	0.000	0.174	0.273	0.056

Table 7: Backtesting results for the double- t copula model. Kupiec test's p -values.

Tranche	VaR(5%)			VaR(95%)		
	Gauss	Gumbel	Clayton	Gauss	Gumbel	Clayton
1		0.653			0.020	
2	0.574	0.741	0.182	0.223	0.001	0.240
3	0.740	0.738	0.054	0.000	0.000	0.005
4	0.382	0.221	0.033	0.001	0.089	0.510
5	0.036	0.019	-	0.269	0.031	0.114

Table 8: Backtesting results for the double- t copula model. DQ test's p -values.

Tranche	VaR(5%)			VaR(95%)		
	Gauss	Gumbel	Clayton	Gauss	Gumbel	Clayton
1		0.558			0.735	
2	0.013	0.056	0.000	0.028	0.056	0.103
3	0.002	0.000	0.000	0.000	0.000	0.000
4	0.000	0.000	0.000	0.000	0.000	0.000
5	0.000	0.013	0.000	0.000	0.000	0.000

Table 9: Backtesting results for the Gumbel copula model. Kupiec test's p -values.

Tranche	VaR(5%)			VaR(95%)		
	Gauss	Gumbel	Clayton	Gauss	Gumbel	Clayton
1		0.605			0.085	
2	0.482	0.596	0.110	0.001	0.132	0.003
3	0.137	0.118	0.009	0.009	0.035	0.033
4	0.055	0.018	0.009	0.026	0.034	0.018
5	0.050	0.251	0.009	0.018	0.033	0.017

Table 10: Backtesting results for the Gumbel copula model. DQ test's p -values.

Test (Significance Level)	VaR(5%)				VaR(95%)			
	Gauss	NIG	double- t	Gumbel	Gauss	NIG	double- t	Gumbel
Kupiec (0.01)	7	7	7	9	3	4	0	9
Kupiec (0.05)	8	7	8	11	3	5	0	10
DQ (0.01)	1	3	1	3	10	7	5	3
DQ (0.05)	3	6	4	5	12	7	7	11

Table 11: Number of statistically significant results.

Tranche	VaR(5%)				VaR(95%)			
	Gauss	NIG	double- t	Gumbel	Gauss	NIG	double- t	Gumbel
1	0.174	0.002	0.103	0.557	0.706	0.056	0.543	0.735
2	0.273	0.706	0.103	0.056	0.735	0.886	0.924	0.103
3	0.273	0.273	0.103	0.002	0.886	0.174	0.557	0.000
4	0.001	0.273	0.013	0.000	0.001	0.557	0.735	0.000
5	0.028	0.273	0.005	0.013	0.886	0.543	0.273	0.000

Table 12: Backtesting results for all VaR models. Kupiec test's p -values.

Tranche	VaR(5%)				VaR(95%)			
	Gauss	NIG	double- t	Gumbel	Gauss	NIG	double- t	Gumbel
1	0.755	0.137	0.653	0.605	0.011	0.068	0.020	0.085
2	0.735	0.119	0.741	0.596	0.008	0.009	0.240	0.132
3	0.513	0.780	0.740	0.137	0.003	0.067	0.005	0.035
4	0.188	0.165	0.382	0.055	0.063	0.122	0.510	0.034
5	0.474	0.484	0.036	0.251	0.000	0.335	0.269	0.033

Table 13: Backtesting results for all VaR models. DQ test's p -values.

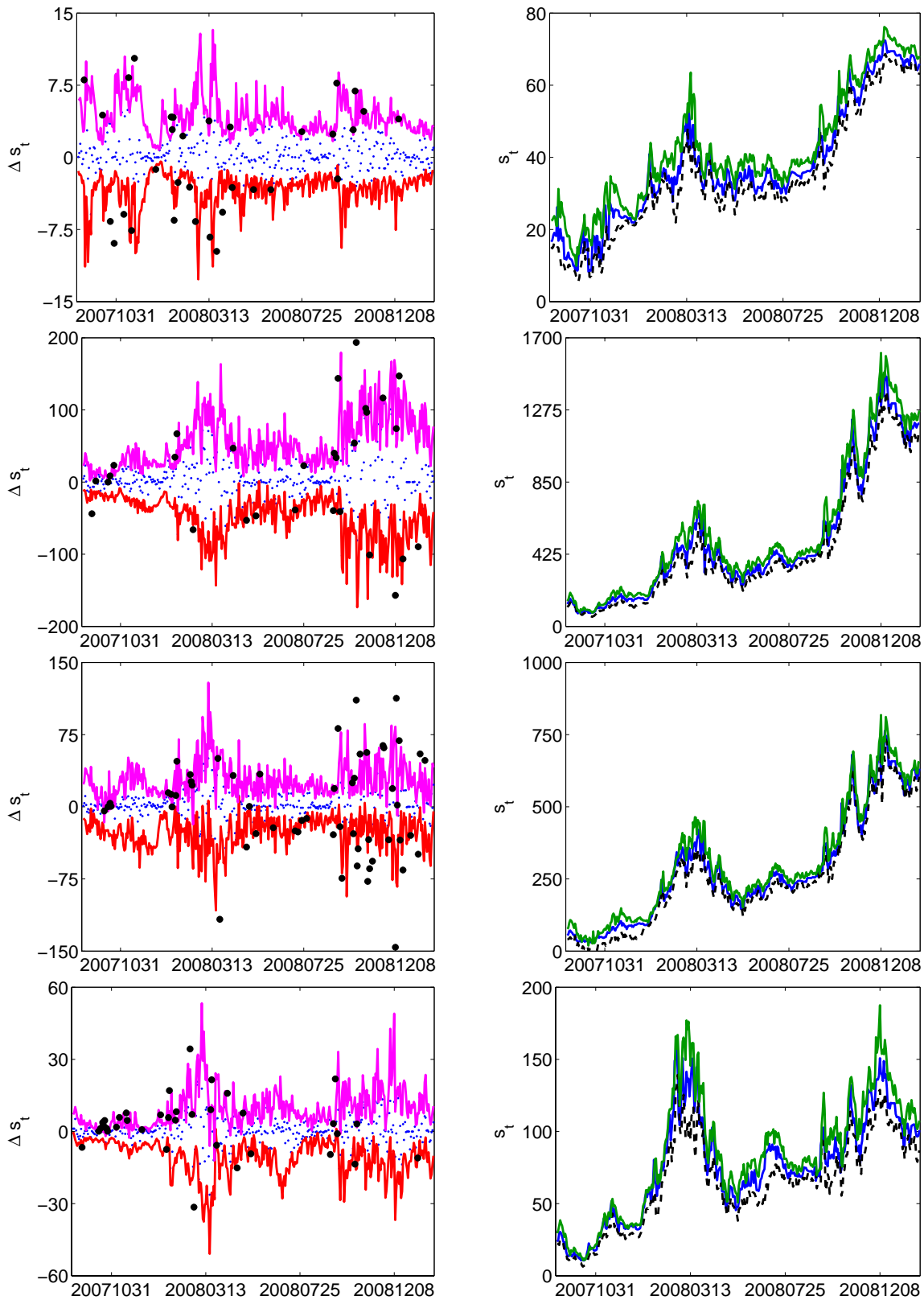


Figure 3: VaR for the tranche 1 calculated with the Gumbel model (first row), for the tranche 2 calculated with the double- t model with the inner Gumbel copula (second row), for the tranche 3 calculated with the NIG model with the inner Gaussian copula (third row), and for the tranche 5 calculated with the Gaussian model with the inner Gumbel copula (fourth row). Left: spread difference (blue), $\text{VaR}_{s_j}^\alpha$ (red), $\text{VaR}_{s_j}^{1-\alpha}$ (pink), exceedances (black). Right: market spreads (blue), spread predictions (green, dashed black)

Tranche	Gauss	NIG	double- t	Gumbel
1	5.474	5.485	5.179	5.379
2	1524.232	1510.394	1474.177	1532.652
3	510.229	563.453	508.706	646.390
4	219.462	274.981	211.025	495.953
5	33.687	39.228	33.199	46.246
Total	2293.083	2393.541	2232.286	2726.620

Table 14: Mean squared error of the spread predictions.

In addition to the VaR, Table 14 shows the mean squared error of the spread predictions. The best next day spread was forecasted by the double- t model for all the tranches.

5 Conclusions

This paper investigates the dynamic changes in the dependence structure in Collateralized Debt Obligations (CDOs). The CDO valuation procedure and four risk models, Gaussian, NIG, double- t , Gumbel, are presented. The empirical study is conducted using iTraxx Europe tranches for the time period between 20 September 2006 and 2 February 2009. We imply base correlations and analyse their evolution in time. By applying time series models we forecast the implied parameters. Afterwards, the predictions are used to compute the spread forecasts. The forecasting ability of the CDO models is exploited by calculating Value-at-Risk measures for spreads and carrying out a backtesting using the Kupiec test and the dynamic quantile test. The performance of the Gaussian and the more advance models are comparable. The empirical results do not confirm that the benchmark approach is entirely inefficient in assessing risk. Therefore, the simple Gaussian model should not be excluded from the analysis of the iTraxx spreads.

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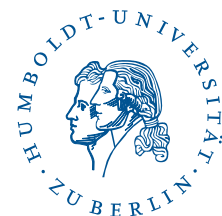
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