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Working Paper

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cemmap working paper, No. CWP28/13

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Suggested Citation: Linton, Oliver; Whang, Yoon-Jae; Yen, Yu-Min (2013) : A nonparametric test of a strong leverage hypothesis, cemmap working paper, No. CWP28/13, Centre for Microdata Methods and Practice (cemmap), London,
<https://doi.org/10.1920/wp.cem.2013.2813>

This Version is available at:

<https://hdl.handle.net/10419/79535>

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A nonparametric test of a strong leverage hypothesis

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A Nonparametric Test of a Strong Leverage Hypothesis*

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June 25, 2013

Abstract

The so-called leverage hypothesis is that negative shocks to prices/returns affect volatility more than equal positive shocks. Whether this is attributable to changing financial leverage is still subject to dispute but the terminology is in wide use. There are many tests of the leverage hypothesis using discrete time data. These typically involve fitting of a general parametric or semiparametric model to conditional volatility and then testing the implied restrictions on parameters or curves. We propose an alternative way of testing this hypothesis using realized volatility as an alternative direct nonparametric measure. Our null hypothesis is of conditional distributional dominance and so is much stronger than the usual hypotheses considered previously. We implement our test on a number of stock return datasets using intraday data over a long span. We find powerful evidence in favour of our hypothesis.

Key words: Distribution function; Leverage Effect; Gaussian Process;

Journal of Economic Literature Classification: C14, C15.

*We thank Valentina Corradi, Jean-Marie Dufour, and Haim Levy for helpful comments. R software for carrying out the conditional dominance test is available from the web site <https://sites.google.com/site/oliverlinton/oliver-linton>.

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1 Introduction

The so-called leverage hypothesis, Black (1976) and Christie (1982), is essentially that negative shocks to stock prices affect their volatility more than equal magnitude positive shocks. Whether this is attributable to changing financial leverage or is a result of the volatility feedback effect, French, Schwert, and Stambaugh (1987) and Campbell and Hentschel (1992), is still subject to dispute (Engle and Ng (1993), Figlewski and Wang (2000), Bekaert and Wu (2000), Bollerslev, Litvinova, and Tauchen (2006), and Dufour, Garcia, and Taamouti (2012)), but the terminology is in wide use. There are many statistical tests of the leverage hypothesis using discrete time data. These typically involve fitting of a general parametric or semiparametric model to conditional volatility and then testing the implied restrictions on parameters or curves, see for example Nelson (1991), Engle and Ng (1993), Linton and Mammen (2005), and Rodriguez and Ruiz (2012). Most authors have found that the parameters governing asymmetric volatility response in daily individual stock returns and in indexes to be statistically significant.

A theoretical justification of the leverage effect is given in Christie (1982) inside a continuous time model, and recently there has been an important literature on measuring leverage effects in high frequency data. Aït-Sahalia, Fan, and Li (2013) investigate the leverage effect “puzzle” within the continuous time framework. The puzzle is that natural estimators of the leverage effect based on high frequency data are usually very small and insignificant. They take apart the sources of this finding and interpret it as bias due to microstructure noise issues, and they propose a solution to this based on a bias correction. Empirically their method seems to uncover a stronger leverage effect. Wang and Mykland (2013) propose a nonparametric estimator of a class of leverage parameters inside a very general class of continuous time stochastic processes. They propose an estimator that is quite simple and easily studied and provide its limiting properties. They extend the theory to allow for measurement error and therefore more complicated estimators of volatility and leverage. Their modified procedure is consistent and asymptotically mixed normal in this case too, although the rate of convergence is slower. They provide the means to conduct inference about the leverage parameter, although their application is more towards prediction of volatility and they demonstrate the value added that their leverage effect has in this purpose.

Our focus is on the low frequency (daily) volatility and return relationship, although our methods could also be applied to higher frequency data. We propose a way of testing the leverage hypothesis nonparametrically without requiring a specific parametric or semiparametric model. In that way

our inference is robust to the model choices that many previous studies have adopted. In fact, we test a “strong leverage” hypothesis. Our null hypothesis is that the conditional distribution of volatility given negative returns and past volatility stochastically dominates in the first order sense the distribution of volatility given positive returns and past volatility. This hypothesis is stronger in some sense than those considered previously since we refer to the distribution rather than just the mean of the outcome.¹ If our null hypothesis is satisfied then any investor who values volatility negatively would prefer the distribution of volatility that arises after positive shocks to returns to the distribution that arises after negative shocks, Levy (2006). A further advantage of formulating our hypothesis in terms of distributions is that the tests are less sensitive to the existence of moments. A lot of informal evidence around the leverage effect is reported based on cross correlations between squared returns and lags and leads of returns, see for example Bouchaud, Matacz, and Potters (2001). As Mikosch and Starica (2000) have shown, the asymptotic behaviour of sample correlograms can be badly affected by heavy tails, which themselves have been widely documented in daily stock returns. Therefore, confidence intervals and hypothesis tests under these circumstances need to be evaluated with care. Our distribution theory builds on work of Linton, Maasoumi, and Whang (2005) who considered tests of unconditional stochastic dominance for time series data. Linton, Song, and Whang (2010) consider conditional dominance tests but inside specific semiparametric models. We allow for a general stationary and mixing process for both returns and volatility and impose some smoothness conditions needed for our asymptotic approximations, but otherwise our test is model-free. We obtain the limiting distribution of our test statistic: it is a functional of a Gaussian process. Since the limit distribution depends in a complicated way on nuisance parameters, we propose an inference method based on subsampling (Politis and Romano, 1994). Our test is consistent against a general class of alternatives.

We apply our testing methodology to stock returns. We focus on whether there is a leverage effect between daily volatility and daily lagged returns on the S&P500 (cash) index and on individual stocks. The stocks we consider are five constituents of the Dow Jones Industrial Average. The sample period covers 1993 to the end of 2009, which includes several very volatile episodes as well as some more tranquil ones. We measure daily volatility using realized volatility (computed from one minute and five minute intraday transactions data) and a transform of the realized range, which only requires

¹Although Wang and Mykland (2013) also allow for the leverage effect to be defined through any (given) function F of volatility.

daily high and low prices. These data are widely available both for indexes and individual stocks. Dufour, Garcia, and Taamouti (2012) in their study of S&P500 futures data used also the VIX index of implied volatility but this type of traded volatility instruments are not available for individual stocks for the long time span we consider. Our methodology sits between discrete time econometrics and continuous time econometrics, since we use concepts from both literatures. If the volatility measure we use can be interpreted as an unbiased estimator of ex ante volatility, then our hypothesis can be interpreted inside the typical discrete time framework. We find strong evidence in favour of the strong leverage effect in this data. We also carry out several robustness checks and compare our results with some newly developed methods, and find these further results lend support to our conclusions.

2 Hypotheses of interest

We suppose that we observe a stationary and weakly dependent process $\{y_t, x_t, r_t\}_{t=1}^T$, where $x_t \in \mathbb{R}^{d_x}$ for some d_x and $y_t \in \mathbb{R}$. Let

$$\begin{aligned} F^+(y|x) &= \Pr(y_t \leq y \mid r_{t-1} \geq 0, x_t = x) \\ F^-(y|x) &= \Pr(y_t \leq y \mid r_{t-1} < 0, x_{t-1} = x). \end{aligned}$$

We consider the hypothesis

$$\begin{aligned} H_0 &: F^+(y|x) \geq F^-(y|x) \text{ a.s. for all } (y, x) \in \mathcal{Y} \times \mathcal{X} \\ H_1 &: F^+(y|x) < F^-(y|x) \text{ for some } (y, x) \in \mathcal{Y} \times \mathcal{X}, \end{aligned}$$

where $\mathcal{Y} \subset \mathbb{R}$ denotes the support of y_t and $\mathcal{X} \subset \mathbb{R}^{d_x}$ denotes the support of x_t .

A leading example is where $y_t = \sigma_t^2$ and $x_t = \sigma_{t-1}^2$, in which case the hypothesis is that bad news on returns ($r_{t-1} < 0$) leads to a bigger effect on the conditional distribution of future volatility than good news ($r_{t-1} \geq 0$) whatever the current level of volatility. In this case, we can take $\mathcal{Y} = \mathcal{X} \subset \mathbb{R}_+$. Suppose that σ_t^2 was generated from a Glosten, Jagannathan and Runkle (1993) process, henceforth GJR, i.e.,

$$\sigma_t^2 = \omega + \beta\sigma_{t-1}^2 + \gamma_+ r_{t-1}^2 1(r_{t-1} > 0) + \gamma_- r_{t-1}^2 1(r_{t-1} \leq 0). \quad (1)$$

The case where $\gamma_- > \gamma_+$ corresponds to the presence of a leverage effect. In this case the distribution $F^-(y|x)$ first order dominates $F^+(y|x)$ for all x . The same dominance relation can be found inside the Nelson (1991) model taking $y_t = \ln \sigma_t^2$ and $x_t = \ln \sigma_{t-1}^2$.

We allow for a more general formulation than $y_t = \sigma_t^2$ and $x_t = \sigma_{t-1}^2$ for practical reasons. In view of the possible strong dependence in volatility we might consider conditioning on a vector of past volatilities $x_t = (\sigma_{t-1}^2, \dots, \sigma_{t-p}^2) \in \mathbb{R}^p$ instead of just on σ_{t-1}^2 . In practice, however, this is likely to work poorly for large p because of the curse of dimensionality. We consider a compromise approach in which we condition on a lower dimensional transform of a vector of lagged volatilities. Specifically, let $h : \mathbb{R}^p \rightarrow \mathbb{R}^{d_x}$ for $d_x < p \leq \infty$ be a measurable function and replace σ_{t-1}^2 by $x_t = h(\sigma_{t-1}^2, \dots, \sigma_{t-p}^2)$. For example, $h(x_1, \dots, x_p) = \sum_{j=1}^p c_j x_j$ for known c_1, \dots, c_p . In this case we consider the conditional distributions $F^+(y|x) = \Pr(\sigma_t^2 \leq y \mid r_{t-1} \geq 0, h(\sigma_{t-1}^2, \dots, \sigma_{t-p}^2) = x)$ and $F^-(y|x) = \Pr(\sigma_t^2 \leq y \mid r_{t-1} < 0, h(\sigma_{t-1}^2, \dots, \sigma_{t-p}^2) = x)$.

We next rewrite the null hypothesis in a way that we will use for testing. Letting

$$\begin{aligned}\pi_0^+(x) &= \Pr(r_{t-1} \geq 0 \mid x_t = x) \\ \pi_0^-(x) &= \Pr(r_{t-1} < 0 \mid x_t = x),\end{aligned}$$

we can write the above hypotheses by the conditional moment inequalities:

$$\begin{aligned}H_0 &: E \left[1(y_t \leq y) \left(\frac{1(r_{t-1} < 0)}{\pi_0^-(x_t)} - \frac{1(r_{t-1} \geq 0)}{\pi_0^+(x_t)} \right) \middle| x_t = x \right] \leq 0 \text{ for all } (y, x) \in \mathcal{Y} \times \mathcal{X}, \\ H_1 &: E \left[1(y_t \leq y) \left(\frac{1(r_{t-1} < 0)}{\pi_0^-(x_t)} - \frac{1(r_{t-1} \geq 0)}{\pi_0^+(x_t)} \right) \middle| x_t = x \right] > 0 \text{ for some } (y, x) \in \mathcal{Y} \times \mathcal{X},\end{aligned}$$

or equivalently,

$$\begin{aligned}H_0 &: E [1(y_t \leq y) \{ \pi_0^+(x_t) - 1(r_{t-1} \geq 0) \} \mid x_t = x] \leq 0 \text{ for all } (y, x) \in \mathcal{Y} \times \mathcal{X} \\ H_1 &: E [1(y_t \leq y) \{ \pi_0^+(x_t) - 1(r_{t-1} \geq 0) \} \mid x_t = x] > 0 \text{ for some } (y, x) \in \mathcal{Y} \times \mathcal{X},\end{aligned}$$

using the fact $\pi_0^+(x) = 1 - \pi_0^-(x) > 0$ for all x . It is well known that the hypotheses of H_0 and H_1 can be equivalently stated using the unconditional moment inequalities

$$H_0 : E [1(y_t \leq y) g(x_t) \{ \pi_0^+(x_t) - 1(r_{t-1} \geq 0) \}] \leq 0 \text{ for all } (y, g) \in \mathcal{Y} \times \mathcal{G}, \quad (2)$$

$$H_1 : E [1(y_t \leq y) g(x_t) \{ \pi_0^+(x_t) - 1(r_{t-1} \geq 0) \}] > 0 \text{ for some } (y, g) \in \mathcal{Y} \times \mathcal{G}, \quad (3)$$

where g is an instrument that depends on the conditioning variable x_t and \mathcal{G} is the collection of instruments, see, e.g., Andrews and Shi (2013) and the references therein. In this paper, we take

$$\mathcal{G} = \{ g_{a,b} : g_{a,b}(x) = \prod_{i=1}^{d_x} 1(a_i < x_i \leq b_i) \text{ for some } a, b \in \mathcal{X} \},$$

see Andrews and Shi (2013) for more examples of instruments. We will use the relation (2) to generate a test statistic.

We emphasize the null hypothesis of a leverage effect. Instead, one might take the hypothesis to be the absence of a leverage effect. Specifically, we might consider the conditional independence hypothesis, i.e.,

$$\sigma_t^2 \text{ is independent of } \text{sign}(r_{t-1}) \text{ given } \sigma_{t-1}^2. \quad (4)$$

This hypothesis would be consistent with a GARCH(1,1) process for σ_t^2 , namely, $\sigma_t^2 = \omega + \beta\sigma_{t-1}^2 + \gamma r_{t-1}^2$ for positive parameters ω, γ, β . The GJR process (1) is incompatible with this hypothesis. In fact, the GJR process is incompatible with (4) whenever $\gamma_- \neq \gamma_+$. In general, the alternative hypothesis to (4) contains many processes that do not represent what we think a leverage effect should be, and we should properly define the null hypothesis in a much more complicated way (in the parametric case this is straightforward, but not so in the nonparametric case), which is why we do not pursue this hypothesis here further.

3 Test Statistic

We next define empirical versions of the moment inequalities. Let $\hat{\pi}^+$ be nonparametric kernel estimators of π_0^+ , i.e.,

$$\hat{\pi}^+(x) = \frac{\sum_{t=2}^T 1(r_{t-1} \geq 0) K_h(x - x_t)}{\sum_{t=2}^T K_h(x - x_t)},$$

where $K : \mathbb{R}^{d_x} \rightarrow \mathbb{R}$ is a kernel function and $K_h(\cdot) = K(\cdot/h)/h$ and h is a bandwidth parameter satisfying the assumptions below. Now the hypothesis can be tested based on the following statistic

$$\bar{m}_T(y, g, \pi) = \frac{1}{T} \sum_{t=1}^T 1(y_t \leq y) g(x_t) \{ \pi(x_t) - 1(r_{t-1} \geq 0) \}.$$

We consider Kolmogorov Smirnov-type (KS) and Cramér-von Mises-type (CM) test statistics, defined by

$$\begin{aligned} S_T &= \sup_{(y,g) \in \mathcal{Y} \times \mathcal{G}} \sqrt{T} \bar{m}_T(y, g, \hat{\pi}^+) \text{ and} \\ S_T^* &= \int \max \left\{ (\sqrt{T} \bar{m}_T(y, g, \hat{\pi}^+), 0) \right\}^2 dQ(y, g), \end{aligned}$$

respectively, where Q is a weighting function (i.e., probability measure) on $\mathcal{Y} \times \mathcal{G}$ and the integral is over $\mathcal{Y} \times \mathcal{G}$. These statistics are relatively easy to compute, they require some choices regarding kernel, bandwidth and function class \mathcal{G} , but these will be discussed below. For brevity, we mainly discuss the asymptotic properties of S_T , but we expect that analogous results hold for S_T^* .

4 Asymptotic Theory

We suppose that we observe a process $\{y_t, x_t, r_t\}_{t=1}^T$. In practice, we have only an estimate $\widehat{\sigma}_t^2$ of σ_t^2 computed from high frequency data. Barndorff-Nielsen and Shephard (2002) have shown that the realized volatility consistently estimates integrated volatility σ_t^2 at rate $n_t^{-1/2}$, where n_t is the number of high frequency observations within day t . We assume that n_t is (effectively) very large relative to T such that we may safely ignore the fact that volatility is estimated.² In some contexts this may not be a good assumption. Ghosh and Linton (2008) worked with monthly volatility estimates computed from daily data. They developed a bias correction method suitable for a special class of moment condition asset pricing models that takes account of estimation error in volatility. This type of analysis is rather difficult to conduct in this context because of the lack of smoothness (indicator functions). We note that the volatility measures we consider are widely used in empirical studies, see for example French, Schwert, and Stambaugh (1987), and typically are used without any adjustment for estimation error.

4.1 The null distribution

Let \mathcal{X}_ε be an ε -neighborhood of \mathcal{X} for some $\varepsilon > 0$. For some constant $B < \infty$, let

$$\Pi = \left\{ \pi : \|\pi(\cdot)\|_{q, \mathcal{X}_\varepsilon} \leq B \right\}, \quad (5)$$

where q is an integer that satisfies $q > d_x/2$. For nonnegative integers k, λ and ω with $\omega \geq \lambda$, we define the following class of kernels:

²Corradi, Distaso and Fernandes (2012) established the “necessary” rate conditions relating n_t to T such that for a somewhat similar statistic for testing conditional independence one can do this.

$$\begin{aligned}
\mathcal{K}_{k,\lambda,\omega} = \{ & K(\cdot) : \mathbb{R}^k \rightarrow \mathbb{R} : \int K(x)dx = 1, \int x^\mu K(x)dx = 0 \\
& \forall 1 \leq |\mu| \leq \omega - \lambda - 1, \int |x^\mu K(x)|dx < \infty \forall |\mu| = \omega - \lambda, \\
& D^\mu K(x) \rightarrow 0 \text{ as } \|x\| \rightarrow \infty \forall \mu \text{ with } |\mu| < \lambda, \\
& \sup_{z \in \mathbb{R}^k} |D^{\mu+e_j} K(x)|(\|x\| \vee 1) < \infty \forall \mu \text{ with } |\mu| \leq \lambda \forall j = 1, \dots, k, \\
& \text{and } K(\cdot) \text{ is zero outside a bounded set in } \mathbb{R}^{d_x}, \\
& \text{where } e_j \text{ denotes the } j\text{-th elementary } d_x\text{-vector.} \} .
\end{aligned}$$

ASSUMPTION A

1. (i) $\{(y_t, x_t, r_t) : t \geq 1\}$ is a sequence of strictly stationary strong mixing random variables with mixing numbers of size $-2(4d_x + 5)(d_x + 2)$. (ii) \mathcal{X} is an open bounded subset of \mathbb{R}^{d_x} with minimally smooth boundary.
2. (i) The distribution of x_t is absolutely continuous with respect to Lebesgue measure with density $f(x)$. (ii) $\inf_{x \in \mathcal{X}_\varepsilon} f(x) > 0$, $D^\mu f(x)$ exists and is continuous on \mathbb{R}^{d_x} and $\sup_{x \in \mathcal{X}_\varepsilon} |D^\mu f(x)| < \infty \forall \mu$ with $|\mu| \leq \max\{\omega, q\}$, where ω is a positive integer that also appears in the other assumptions below. (iii) The conditional distribution $F(y|x)$ of y_t given $x_t = x$ has bounded density $f(y|x)$ for almost all $x \in \mathbb{R}^{d_x}$.
3. $D^\mu [\pi_0^+(x)f(x)]$ exists and are continuous on \mathbb{R}^{d_x} and $\sup_{x \in \mathcal{X}_\varepsilon} |D^\mu [\pi_0^+(x)f(x)]| < \infty \forall \mu$ with $|\mu| \leq \max\{\omega, q\}$.
4. $K(\cdot) \in \mathcal{K}_{d_x,0,\omega} \cap \mathcal{K}_{d_x,q,q}$.
5. The bandwidth parameter h satisfies $T^{\min\{\frac{1}{2(d_x+q)}, \frac{1}{4d_x}\}} h \rightarrow \infty$ and $T^{\frac{1}{2\omega}} h \rightarrow 0$.

Assumption A1 requires that x_t lies in an open bounded set with minimally smooth boundary. Examples of sets with minimally smooth boundaries include open bounded sets that are convex or whose boundaries are C^1 -embedded in \mathbb{R}^{d_x} . Finite unions of aforementioned type whose closures are disjoint also have minimally smooth boundaries. The boundedness assumption is not restrictive, because, if needed, we can transform the values of x_t into a compact interval, say $[0, 1]^{d_x}$, via strictly

increasing transformation. Assumptions A2 and A3 imposes smoothness on f and π_0^+ . They are needed to ensure that the realization of $\hat{\pi}^+$ are smooth with probability tending to one and therefore the stochastic equicontinuity condition of a stochastic process $\{\bar{\nu}_T(\cdot, \cdot, \cdot) : T \geq 1\}$ that appears in our proof can be verified. The use of higher-order kernel $K(\cdot)$ in Assumption A4 is due to the need to establish T^κ convergence of the kernel estimators \hat{f} , $\hat{\pi}^+(x)$ (see (11) and (12) in Appendix) for some sufficiently large $\kappa \geq 1/4$. Assumption A5 imposes some conditions on the rate of convergence of bandwidth to zero. The conditions are compatible if ω is sufficiently large. These conditions can be relaxed, if needed, to allow for data-dependent methods of choosing bandwidth parameters, e.g. cross-validation or plug-in procedures.

We now derive the asymptotic of the test statistic under the null hypothesis. Define the empirical processes in $(y, g) \in \mathbb{R} \times \mathcal{G}$

$$\nu_T(y, g) = \sqrt{T} \{ \xi_T(y, g) - E \xi_T(y, g) \}, \quad (6)$$

where

$$\xi_T(y, g) = \frac{1}{T} \sum_{t=1}^T \{ 1(y_t \leq y) - F(y|x_t) \} g(x_t) \{ \pi_0^+(x_t) - 1(r_{t-1} \geq 0) \}. \quad (7)$$

Let $\nu(y, g)$ be a mean zero Gaussian process with covariance function given by

$$C((y_1, g_1), (y_2, g_2)) = \lim_{T \rightarrow \infty} \text{cov}(\nu_T(y_1, g_1), \nu_T(y_2, g_2)).$$

The limiting null distribution of our test statistic is given in the following theorem.

Theorem 1. *Suppose that Assumption A holds. Then, under the null hypothesis H_0 ,*

$$S_T \Rightarrow \begin{cases} \sup_{(y,g) \in \mathcal{B}} [\nu(y, g)] & \text{if } \mathcal{B} \neq \emptyset \\ -\infty & \text{if } \mathcal{B} = \emptyset \end{cases},$$

where $\mathcal{B} = \{(y, g) \in \mathcal{Y} \times \mathcal{G} : E [1(y_t \leq y)g(x_t) \{ \pi_0^+(x_t) - 1(r_{t-1} \geq 0) \}] = 0\}$.

Theorem 1 shows that our test statistic has a non-degenerate limiting distribution on the boundary of the null hypothesis, i.e. the case where the ‘‘contact set’’ (i.e., the subset of $\mathcal{Y} \times \mathcal{G}$ where the null hypothesis (2) holds with equality) is non-empty. Since the distribution depends on the true data generating process, we cannot tabulate it once and for all. We suggest estimating the critical values by a subsampling procedure.

4.2 Critical Values and Consistency

We first define the subsampling procedure. With some abuse of notation, the test statistic S_T can be re-written as a function of the data $\{W_t : t = 1, \dots, T\}$:

$$S_T = \sqrt{T} \tau_T(W_1, \dots, W_T),$$

where $\tau_T(W_1, \dots, W_T)$ is given by $\sup_{(y,g) \in \mathcal{Y} \times \mathcal{G}} \bar{m}_T(y, g, \hat{\pi}^+)$. Let

$$G_T(\cdot) = \Pr \left(\sqrt{T} \tau_T(W_1, \dots, W_T) \leq \cdot \right) \quad (8)$$

denote the distribution function of S_T . Let $\tau_{T,b,t}$ be equal to the statistic τ_b evaluated at the subsample $\{W_t, \dots, W_{t+b-1}\}$ of size b , i.e.,

$$\tau_{T,b,t} = \tau(W_t, W_{t+1}, \dots, W_{t+b-1}) \text{ for } t = 1, \dots, T - b + 1.$$

We note that each subsample of size b (taken *without replacement* from the original data) is indeed a sample of size b from the true sampling distribution of the original data. Hence, it is clear that one can approximate the sampling distribution of S_T using the distribution of the values of $\tau_{T,b,t}$ computed over $T - b + 1$ different subsamples of size b . That is, we approximate the sampling distribution G_T of S_T by

$$\hat{G}_{T,b}(\cdot) = \frac{1}{T - b + 1} \sum_{t=1}^{T-b+1} 1 \left(\sqrt{b} \tau_{T,b,t} \leq \cdot \right).$$

Let $g_{T,b}(1 - \alpha)$ denote the $(1 - \alpha)$ -th sample quantile of $\hat{G}_{T,b}(\cdot)$, i.e.,

$$g_{T,b}(1 - \alpha) = \inf \{ w : \hat{G}_{T,b}(w) \geq 1 - \alpha \}.$$

We call it the *subsample critical value* of significance level α . Thus, we reject the null hypothesis at the significance level α if $S_T > g_{T,b}(1 - \alpha)$. The computation of this critical value is not particularly onerous, although it depends on how big b is. The subsampling method has been proposed in Politis and Romano (1994) and is thoroughly reviewed in Politis, Romano, and Wolf (1999). It works in many cases where the standard bootstrap fails: in heavy tailed distributions, in unit root cases, in cases where the parameter is on the boundary of its space, etc.

We now show that our subsampling procedure works under a very weak condition on b . In many practical situations, the choice of b will be data-dependent, see Linton, Maasoumi and Whang (2005,

Section 5.2) for some methodology for choosing b . To accommodate such possibilities, we assume that $b = \hat{b}_T$ is a data-dependent sequence satisfying

Assumption B: $\Pr[l_T \leq \hat{b}_T \leq u_T] \rightarrow 1$ where l_T and u_T are integers satisfying $1 \leq l_T \leq u_T \leq T$, $l_T \rightarrow \infty$ and $u_T/T \rightarrow 0$ as $T \rightarrow \infty$.

The following theorem shows that our test based on the subsample critical value has asymptotically correct size:

Theorem 2. *Suppose Assumptions A and B hold. Then, under the null hypothesis H_0 ,*

$$\lim_{T \rightarrow \infty} \Pr[S_T > g_{T, \hat{b}_T}(1 - \alpha)] \leq \alpha,$$

with equality holding if $\mathcal{B} \neq \emptyset$, where \mathcal{B} is defined in Theorem 1.

Theorem 2 shows that our test based on the subsampling critical values has asymptotically valid size under the null hypothesis and has asymptotically exact size on the boundary of the null hypothesis. Under additional regularity conditions, we can extend this pointwise result to establish that our test has asymptotically correct size uniformly over the distributions under the null hypothesis, using the arguments of Andrews and Shi (2013) and Linton, Song and Whang (2010). For brevity, we do not discuss the details of this issue in this paper.

We next establish that the test S_T based on the subsampling critical values is consistent against the fixed alternative H_1 .

Theorem 3. *Suppose that Assumptions A and B hold. Then, under the alternative hypothesis H_1 ,*

$$\lim_{T \rightarrow \infty} \Pr[S_T > g_{T, \hat{b}_T}(1 - \alpha)] = 1.$$

5 Empirical Results

In this section we test the leverage hypothesis with stock return data, the S&P500 (cash) index and five constituents of the Dow Jones Industrial Average (DJIA): Microsoft (MSFT), IBM (IBM), General Electronic (GE), Procter& Gamble (PG) and 3M (MMM). The samples used for the test span from Jan-04-1993 to Dec-31-2009 (4,283 trading days). We focus on whether there is a strong leverage effect between daily volatility and lagged return. We first introduce two estimators for estimating the daily volatility and detail how we construct the test statistic with the estimated daily volatility and present the empirical results.

5.1 Estimating The Daily Volatility

We consider two methods to estimate the daily volatility. The first one is the realized variance estimator RV_t

$$RV_t = \sum_{i=1}^{n_t} r_{i,t}^2,$$

where $r_{i,t} = \log P(t - 1 + \frac{i}{n_t}) - \log P(t - 1 + \frac{i-1}{n_t})$ is the i th intraday log return on day t , n_t is the total number of intraday log return observations on day t , and $P(t - 1 + i/n_t)$ is the intraday asset price at time stamp $t - 1 + i/n_t$. The second estimator we consider for estimating daily volatility is the squared intraday range RG_t^2 (Garman and Klass, 1980; Parkinson, 1980):

$$RG_t^2 = \frac{IG^2}{4 \log 2},$$

$$IG = \max_{t-1 \leq \tau < t} \log P(\tau) - \min_{t-1 \leq \tau < t} \log P(\tau),$$

where $P(\tau)$ is the intraday asset price at time stamp τ on day t , $t - 1 \leq \tau < t$. The constant $4 \log 2$ is an adjustment factor to scale IG^2 in order to obtain an unbiased estimate, and together with other mild regular conditions, RG_t^2 will be a conditionally unbiased estimator for the daily volatility.

To obtain the daily realized variance RV_t , we need intraday high frequency data of asset prices on day t . For the squared intraday range RG_t^2 , we need only the highest and lowest price data on day t . The two estimators thus contain different information of data from different sampling frequencies. One of the reasons we use the two estimators is that we would like to see whether the daily volatility estimated from using high frequency intraday or low frequency daily data can affect the test results. In addition, the two estimators are easy to use, and they also provide comparable performances on accurate estimation of the daily volatility as other volatility estimators. It is known that the squared intraday range estimator is more robust (to microstructure noise) than the squared daily return on capturing daily volatility dynamics when daily data are used. When high frequency intraday data are used, Liu, Patton and Sheppard (2013) show that over a wide range classes of assets and long sample period, there is strong evidence that more sophisticated daily volatility estimators do not outperform the realized variance estimator (with 5-min data) in term of accuracy of daily volatility estimations.

A detailed discussion of how we filter the high frequency data before they were used for the estimations can be found in Appendix B. Let RV_t^{1min} and RV_t^{5min} denote the realized variances

evaluated from using 1 and 5 minute intraday return data. Figure 1 shows time series plots of: $\sqrt{RV_t^{1min}}$, $\sqrt{RV_t^{5min}}$, RG_t and the daily return R_t for the S&P500 index and the five constituents of DJIA. Here the R_t of S&P500 is daily index return, and of the DJIA stocks are daily holding period returns excluding dividends. From the Figure, during the 2008 financial crisis period, all the five stocks and S&P500 show huge fluctuations in daily returns and volatilities, while in other periods, these fluctuations are relatively mild.

Table 1 shows some summary statistics of the daily returns and estimated daily volatilities. Since the initial estimations on the daily volatilities are very small, we scale them by 100 before we calculate the summary statistics. From the Table, for the individual stocks, it can be seen that their RG_t has a lower mean value but a higher standard deviation than their $\sqrt{RV_t}$, which may suggest that the intraday range estimator has a downward bias. The intraday range estimator also has a lower first order autocorrelation (denoted by $ACF(1)$) than the square root of the realized variance. Comparing means of square root of the realized variances of the five stocks, we can see rank of the average price fluctuations are consistent. On average, MSFT is the most volatile and MMM is least volatile stocks among the five stocks. The mean of the intraday range estimator, however, shows that PG is the least volatile stock, although it still indicates that MSFT is the most volatile stock. Comparing with the individual stocks, S&P500 on average has a less volatile return during the sample period. Its estimated daily volatilities all have smaller mean values than the five individual stocks have. The mean values of the three estimated daily volatilities suggest that on average, the daily return of S&P500 and the five stocks have different degrees of fluctuations. However, as can be seen in this Table, $\sqrt{RV_t}$ and RG_t of S&P500 overall have higher sample skewness and kurtosis than the individual stocks (except for GE), which indicates that shapes of unconditional distributions of $\sqrt{RV_t}$ and RG_t are more right-skewed and fat-tailed for S&P500.

5.2 Unconditional Cross Correlation

We next see how the daily volatility correlates with the daily return. Figure 2 shows the sample (unconditional) cross correlation ρ_j between the estimated daily volatility and daily return of the five stocks of the DJIA and S&P500 index. We set lag length $j = -10, \dots, 10$ (negative j means lead return and lagged volatility). The horizontal dash lines in these plots are the 95% confident bands for the sample cross correlation under the null of zero correlation.

From Figure 2 it can be seen that as $j < 0$, the sample cross correlation ρ_j gradually increases

as j approaches to zero, and then drops sharply from positive to negative as j becomes positive. As for $j > 0$, the negative ρ_j instead converges slowly to zero as j grows. It also can be seen that ρ_j 's for the cases of $j > 0$ are overall larger in magnitude and more statistically significant than those for the cases of $j < 0$, which suggests that the cross correlations are asymmetric and the unconditional leverage effect may exist for the five DJIA stocks and S&P500 index.

The leading effect of volatility on asset return is another volatility asymmetry, which is called the volatility feedback effect. The reason why the effect might exist is that if the volatility is priced, an expectation of volatility increase would raise the required return to compensate an investor who bears such volatility risk, which in turn leads to a decline in stock price (then a higher future return). Thus the causality induced from the volatility feedback effect runs from volatility to return, which is opposite to that induced from the leverage effect. Both the leverage and volatility feedback effects have been documented empirically, but the leverage effect seems to be more supported. Figure 2 shows that the cross correlations are overall higher for lagged return and lead volatility than those for lead return and lagged volatility, which suggests that if the two effects really exist, the leverage effect is stronger than the volatility feedback effect. More detailed discussions on the empirical evidence for the lead-lag effects of return and volatility with high frequency data estimations can be found in Bollerslev, Litvinova and Tauchen (2006) and Dufour, Garcia and Taamouti (2012).

5.3 Conducting the Leverage Hypothesis Test

In the previous section, we have shown that the sample unconditional cross correlations between the lead estimated daily volatilities and lagged returns of the five stocks of the DJIA and S&P500 index are negative and statistically significant, which suggests that the unconditional leverage effect may exist. Will such a leverage effect still exist when we explicitly take the lagged volatility into account and when we also require it to hold in distribution and not just in averages? With the estimated daily volatility, we can empirically construct the test statistic to test the conditional leverage hypothesis. In this section, we formally test the null hypothesis of (2) and use the test statistic S_T described in section 3 for the test. We plug the estimated daily volatility $\hat{\sigma}_t^2 = y_t$, lagged one period estimated daily volatility $\hat{\sigma}_{t-1}^2 = x_t$ and lagged j period daily return $R_{t-j} = r_t$, $j > 0$ into the function \bar{m}_T shown in section 3. We will use the three estimated daily volatilities RV_t^{1min} , RV_t^{5min} and RG_t^2 in the test.

Let

$$\begin{aligned}\tau_T^1(y, g, \hat{\pi}^+) &= \frac{1}{T} \sum_{t=1}^T 1\{y_t \leq y\} 1\{r_t < 0\} g(x_t) \hat{\pi}^+(x_t), \\ \tau_T^2(y, g, \hat{\pi}^-) &= \frac{1}{T} \sum_{t=1}^T 1\{y_t \leq y\} 1\{r_t \geq 0\} g(x_t) \hat{\pi}^-(x_t),\end{aligned}$$

where $\hat{\pi}^+$ and $\hat{\pi}^-$ are nonparametric kernel estimators of π_0^+ and π_0^- . It can be shown that

$$\bar{m}_T(y, g, \hat{\pi}^+) = \tau_T^1(y, g, \hat{\pi}^+) - \tau_T^2(y, g, \hat{\pi}^-).$$

Let \hat{m}_T , $\hat{\tau}_T^1$ and $\hat{\tau}_T^2$ denote empirical versions of \bar{m}_T , τ_T^1 and τ_T^2 when the daily return and estimated volatility are used for the evaluations. Note that $\hat{m}_T(y, g, \hat{\pi}^+) = \hat{\tau}_T^1(y, g, \hat{\pi}^+) - \hat{\tau}_T^2(y, g, \hat{\pi}^-)$ also holds. We use Gaussian kernel for $\hat{\pi}^+$ and $\hat{\pi}^-$. Let

$$\hat{S}_T = \sup_{(y, g) \in \mathcal{Y} \times \mathcal{G}} \sqrt{T} \hat{m}_T(y, g, \hat{\pi}^+).$$

The quantity \hat{S}_T is an empirical version of S_T obtained when $\sqrt{T} \hat{m}_T(\cdot)$ is used. To practically evaluate \hat{S}_T , we apply the following procedures. First, we set the instrument function $g(x_t) = 1\{\underline{\sigma} < x_t \leq \bar{\sigma}\}$, where $\underline{\sigma}$ and $\bar{\sigma}$ are lower and upper bounds for x_t . We fix the lower bound $\underline{\sigma} = 0$ for reducing computational burden. We then seek a combination of $(y, \bar{\sigma})$ to maximize the function $\hat{m}_T(\cdot)$, where $y \in [\min_{t=2, \dots, T} \hat{\sigma}_t^2, 1.1 \times \max_{t=2, \dots, T} \hat{\sigma}_t^2]$ and $\bar{\sigma} \in [\min_{t=2, \dots, T} \hat{\sigma}_{t-1}^2, 1.1 \times \max_{t=2, \dots, T} \hat{\sigma}_{t-1}^2]$, and \sqrt{T} times the maximum value of $\hat{m}_T(\cdot)$ is the feasible test statistic \hat{S}_T . The same procedures apply when we use the subsampling scheme to construct the approximated sample distribution for \hat{S}_T . Since we will plug different lagged returns R_{t-j} in the functions, for distinguishing the different evaluations and ease of notations, we will use \hat{m}_{T_j} , $\hat{\tau}_{T_j}^1$, $\hat{\tau}_{T_j}^2$ and \hat{S}_{T_j} to denote the corresponding quantities when the j period lagged return is used.

To illustrate how the test statistic behaves, we use MSFT as an example. Figure 3 shows plots of the surfaces $\hat{\tau}_{T_j}^1(y, g, \hat{\pi}^+)$, $\hat{\tau}_{T_j}^2(y, g, \hat{\pi}^-)$, and $\hat{m}_{T_j}(y, g, \hat{\pi}^+)$ against $(y, \bar{\sigma})$ when one and five period lagged returns are used ($j = 1$ and 5). Here RV_t^{5min} is used as the estimated daily volatility. From the Figure, the surfaces $\hat{\tau}_{T_j}^1(y, g, \hat{\pi}^+)$ and $\hat{\tau}_{T_j}^2(y, g, \hat{\pi}^-)$ are smooth and monotonically increasing with y and $\bar{\sigma}$, and visually they look almost the same. It also can be seen that the surfaces of $\hat{m}_{T_j}(y, g, \hat{\pi}^+)$ are not everywhere nonnegative, and searching for their maximum values, we find \hat{S}_{T_1} and \hat{S}_{T_5} are around 0.0916 and 0.1101 respectively.

With the same configurations as in Figure 3, we show the empirical critical values over different subsample sizes in Figure 4. The subsample $(1 - \alpha)$ -th quantile at different significant levels α often decrease as the subsample size b increases. They also become more concentrated as b becomes large. This is expected, since as the subsample size approaches the full sample size, the approximated sample distribution will converge to a point mass. Figure 5 shows the corresponding empirical p-values over different subsample sizes. The empirical p-value of $j = 5$ is larger and more stable than that of $j = 1$ as the subsample size varies. The empirical p-values for both j 's also show decreasing trends as b increases, which is consistent with the property that the approximated sample distribution becomes more concentrated as b approaches the full sample size. Over different subsample sizes, the empirical p-value ranges from 0.977 to 0.340 for $j = 1$ and from 0.986 to 0.851 for $j = 5$, which lends support to the claim that the leverage effect exists for MSFT.

As for more empirical results, Table 2 to 4 show the subsample critical values for the five stocks and S&P500 when different estimated daily volatilities are used. We focus on the cases when the subsample size $b = 500, 1000$ and 2000 and lag lengths of returns $j = 1$ and 5 . In each table we also report the test statistic \hat{S}_{T_j} . Table 5 shows their corresponding empirical p-values. From the Tables, it can be seen that only in a few cases the conditional leverage hypothesis can be rejected at $\alpha \leq 0.1$. For example, as RV_t^{1min} is used, GE has $\hat{S}_{T_1} = 0.1996$ and the null hypothesis of (2) can be rejected at $\alpha = 0.05$ when $b = 2000$. Another example is IBM. When RV_t^{1min} and RG^2 are used, its \hat{S}_{T_5} are around 0.248 and 0.186 and the corresponding empirical p-values are around 0.062 and 0.074 as $b = 2000$. For the remaining cases, the results shown are consistent: the null hypothesis of (2) cannot be rejected for each j at $\alpha = 0.1$, which suggests that controlling for lagged volatility, the strong leverage effect may still exist for the five DJIA stocks and the S&P500 index.

5.4 Robustness Checks

We conducted some robustness checks for our empirical results shown in the previous section. Due to similar results from using different daily volatility estimators and lag length of returns, in the following we only focus on results for the cases of RV_t^{1min} and $j = 1$.

In section 5.3 we show that the leverage effect may exist in our data over the full sample period (from Jan-04-1993 to Dec-31-2009). To see whether our results still hold within other time periods, we divide the sample into two subperiods: From Jan-04-1993 to Nov-30-2001 (2,248 trading days) and Dec-03-2001 to Dec-31-2009 (2,035 trading days), and conduct the conditional test for each of

them. The division of sample period is based on the U.S. recessions identified by the NBER: The first subsample period spans the 2001 recession ending in Nov-2001, while the second one spans the 2007-2009 recession ending in June-2009. Due to a smaller sample size in each subperiod, here we set $b = 500$ and 1000 for the subsampling scheme. We show the results in Table 6. It can be seen that except for MSFT with $b = 1000$ (with p-value equal to 0.0753), the leverage hypothesis cannot be rejected at the significant level 0.26. The results for the subperiods overall are consistent with those for the whole sample period, and for the five DJIA stocks and S&P500 index, we conclude that the leverage effects may still exist within the two subperiods.

In section 5.3 we simply set $x_t = h(\sigma_{t-1}^2, \dots, \sigma_{t-p}^2) = \sigma_{t-1}^2$ instead of considering a more general functional form of lagged daily volatility. To see whether a more complicated functional form of $h(\sigma_{t-1}^2, \dots, \sigma_{t-p}^2)$ affects the test results, here we specify it as

$$h_1(\sigma_{t-1}^2, \dots, \sigma_{t-p}^2) = \frac{2}{p+1} \sum_{i=0}^{p-1} \left(\frac{p-1}{p+1}\right)^i \sigma_{t-1-i}^2, \quad (9)$$

which is an approximation for the exponential weighted moving average of the daily volatility on day t up to day $t-1$, $EWMA(\sigma_{t-1}^2)$, i.e.,

$$EWMA(\sigma_{t-1}^2) = \frac{2}{p+1} \sigma_{t-1}^2 + \left(1 - \frac{2}{p+1}\right) EWMA(\sigma_{t-2}^2),$$

where the term $2/(p+1)$ is called the smoothing ratio. We set the parameter p equal to 5, 10 and 22, which correspond to using daily observations in previous one week, two weeks and one month respectively. Daily RV_{t-i}^{1min} , $i = 1, \dots, p$ are used to replace σ_{t-i}^2 , $i = 1, \dots, p$ in (9) in the test. Upper panel of Table 7 shows results of the conditional leverage hypothesis test conditioning on $h_1(RV_{t-1}^{1min}, \dots, RV_{t-p}^{1min})$ and one day lagged return R_{t-1} with the window length $b = 2000$ for the subsampling scheme. It can be seen that except for MSFT with $p = 10$ (which has empirical p-value equal to 0.0039), the results still hold as in the previous section: The null hypothesis cannot be rejected at a moderately significant level and there is evidence for the presence of the leverage effect.

We can also specify $h(\sigma_{t-1}^2, \dots, \sigma_{t-p}^2)$ as a predictive equation on σ_t^2 as $\sigma_{t-1}^2, \dots, \sigma_{t-p}^2$ are used, i.e., $h(\sigma_{t-1}^2, \dots, \sigma_{t-p}^2) = \mathbb{E}_{t-1}(\sigma_t^2 | \sigma_{t-1}^2, \dots, \sigma_{t-p}^2)$. With the realized variances at hand, a practical way to constructing the predictive equation is to estimate the HAR-RV (heterogeneous autoregressive realized variance) regression

$$RV_t = \alpha_D + \beta_{RD} RV_{t-1} + \beta_{RW} RV_{t-1, week} + \beta_{RM} RV_{t-1, month} + \varepsilon_t, \quad (10)$$

The variables in the above regression

$$\begin{aligned}
 RV_{t-1,week} &= \frac{1}{5} \sum_{i=0}^4 RV_{t-1-i}, \\
 RV_{t-1,month} &= \frac{1}{22} \sum_{i=0}^{21} RV_{t-1-i}
 \end{aligned}$$

are called normalized weekly and monthly realized variances. We run the HAR-RV regression (10) in the fashion of real time forecasting with RV_t^{1min} as the inputs. The predictive regression at each period t is estimated by using an expanding window scheme with initial window length equal to 22. At time $t - 1$, the real time projected realized variance for time t , denote by \widehat{RV}_t^{1min} , is a function of $RV_{t-1}^{1min}, \dots, RV_{t-22}^{1min}$. The bottom panel of Table 7 shows the leverage effect test conditioning on \widehat{RV}_t^{1min} and one day lagged return R_{t-1} with the subsample window length $b = 2000$. The values of the test statistic are all small (with range from 0.0087 to 0.1433) and the empirical p-values are all far larger than the frequently used rejection levels. The evidence shown here supports that the leverage effect may exist even when we include information for predictions of future volatility.

In the Appendix we further consider three alternative methods on detecting the leverage effect in our data: (1) Estimating HAR-RV type models with the leverage effect, and two methods for estimating the leverage parameters proposed by (2) Wang and Mykland (2013) and (3) Ait-Sahalia, Fan and Li (2013). We find results from these methods still support our analysis. A detailed discussion of these methods and their results can be found in Appendix C.

6 Conclusion and Extensions

We have found strong evidence in favour of a leverage effect in daily stock returns both at the individual stock level and the index level. The null hypothesis we consider is quite strong, namely first order distributional dominance. Therefore, it is quite powerful that the data do not reject this hypothesis. Investors care not just about the level of volatility but also about the volatility of volatility and indeed about its entire conditional distribution, which is why this result may be of value. Our empirical evidence is robust along a number of directions. For example, our results still hold for subperiods and for different specifications on the functional form of the aggregator of lagged volatilities. In addition, several recently developed alternative methods are used on our data and their results also support our findings.

On the theoretical side, we have considered stationary processes, but this can be relaxed along the lines of Dahlhaus (1997) and Dahlhaus and Subba Rao (2006). We may also weaken the restrictions on the amount of dependence to be consistent with some evidence on the time series properties of realized volatility, see for example Andersen, Bollerslev, Christoffersen, and Diebold (2011), although to allow long memory processes would be technically challenging.

Our testing methodology may be useful for other applications, i.e., for other choices of y_t, x_t and binary variable r_t . The case of first order dominance between the conditional distributions $F^-(y|x), F^+(y|x)$ may be quite strong in other datasets or applications, and one might consider the weaker concept of second order or third order dominance. The theory for these tests follows along the lines considered here and in Linton, Maasoumi, and Whang (2005).

Appendix A: Proofs

Consider the empirical process $\bar{\nu}_T(y, g, \pi)$ indexed by $(y, g, \pi) \in \mathbb{R} \times \mathcal{G} \times \Pi$

$$\bar{\nu}_T(y, g, \pi) = \sqrt{T} \{ \bar{m}_T(y, g, \pi) - E \bar{m}_T(y, g, \pi) \}.$$

The following Lemma establishes stochastic equicontinuity of $\{ \bar{\nu}_T(\cdot, \cdot, \cdot) : T \geq 1 \}$.

Lemma 1. Suppose that Assumption A holds. Then, for each $\varepsilon > 0$ and $\eta > 0$, there exists $\delta > 0$ such that

$$\overline{\lim}_{T \rightarrow \infty} P^* \left[\sup_{\rho((y_1, g_1, \pi_1), (y_2, g_2, \pi_2)) < \delta} | \bar{\nu}_T(y_1, g_1, \pi_1) - \bar{\nu}_T(y_2, g_2, \pi_2) | > \eta \right] < \varepsilon,$$

where Pr^* denotes P -outer measure and

$$\begin{aligned} \rho((y_1, g_1, \pi_1), (y_2, g_2, \pi_2)) &= \rho_a((y_1, g_1), (y_2, g_2)) \vee \rho_b(\pi_1, \pi_2) \\ \rho_a((y_1, g_1), (y_2, g_2)) &= \left(\int_{\mathcal{Y}} \int_{\mathcal{Y}} \{ [1(w_1 \leq y) - F(y|w_2)] g(w_2) \right. \\ &\quad \left. - [1(w_1 \leq y) - F(y|w_2)] g(w_2) \} dw_1 dw_2 \right)^{1/2} \\ \rho_b(\pi_1, \pi_2) &= (E(\pi_1(x_t) - \pi_2(x_t))^2)^{1/2} \end{aligned}$$

PROOF OF LEMMA 1. The result of Lemma 1 follows from the stochastic equicontinuity results of Andrews (1989, Theorem 7) that are applicable to classes of functions that are products of smooth functions from an infinite dimensional class and a Type IV class of uniformly bounded functions.

It suffices to verify Assumption E of the latter paper. (Assumption) E(i) holds by taking W_{aTt} , W_{bTt} , $\tau_a(\cdot)$, $\tau_b(\cdot)$, $m_a(W_{aTt}, \tau_a)$, and $m_b(W_{bTt}, \tau_b)$ to be x_t , (y_t, x_t) , $\pi(\cdot)$, $1(\cdot \leq y)g(\cdot)$, $\pi(x_t)$ and $1(y_t \leq y)g(x_t)$, respectively. E(ii) holds by Assumption A1(ii) with \mathcal{W}_a^* given by \mathcal{X} . E(iii) follows from Assumptions A2-A3 and the definition of Π in (5). E(iv) is irrelevant to our case. E(v) holds since $\{1(\cdot \leq y)g(\cdot) : y \in \mathcal{Y}, g \in \mathcal{G}\}$ is a type IV class of uniformly bounded functions with index $p = 2$, constant $\psi = 1/2$, and dimension $d = d_x + 1$. Finally, E(vi) holds by Assumption A1(i). \blacksquare

PROOF OF THEOREM 1. To prove Theorem 1, we first establish the following results:

$$T^{1/4} \sup_{x \in \mathcal{X}} |\hat{\pi}^+(x) - \pi_0^+(x)| \xrightarrow{P} 0, \quad (11)$$

$$T^{1/4} \sup_{x \in \mathbb{R}^{d_x}} |\hat{f}(x) - f(x)| \xrightarrow{P} 0, \quad (12)$$

$$\sup_{x \in \mathcal{X}_\varepsilon} |D^\mu \hat{\pi}^+(x) - D^\mu \pi_0^+(x)| \xrightarrow{P} 0 \quad \forall \mu \text{ with } 1 \leq \mu < q, \quad (13)$$

$$\sup_{(y,g) \in \mathcal{Y} \times \mathcal{G}} |\bar{\nu}_T(y, g, \hat{\pi}^+) - \bar{\nu}_T(y, g, \pi_0^+)| \xrightarrow{P} 0, \quad (14)$$

$$\sup_{(y,g) \in \mathcal{Y} \times \mathcal{G}} \left| \sqrt{T} E \bar{m}_T(y, g, \pi) |_{\pi = \hat{\pi}^+} - \sqrt{T} R_T(y, g) \right| \xrightarrow{P} 0, \quad (15)$$

where

$$\begin{aligned} R_T(y, g) &= \frac{1}{\sqrt{T}} \sum_t \{1(r_{t-1} \geq 0) - \pi_0^+(x_t)\} F(y|x_t)g(x_t) \\ &\quad + \sqrt{T} \int [F(y|x) - F^+(y|x)] g(x) \pi_0^+(x) f(x) dx. \end{aligned}$$

Equations (11)- (13) can be established using Theorem 1 of Andrews (1994) by verifying its Assumptions NP1-NP5. Notice that NP1-NP3 are implied by Assumptions A1-A3 with $\eta = \beta = \infty$ and $|\lambda| = \mu$ and Y_t , X_t , $f_t(x)$, and $g(x)$ given by $1(r_{t-1} \geq 0)$, x_t , $f(x)$, and $\pi_0^+(x)$, respectively. NP4 (a) and (c) holds by Assumption A4 with $\Omega = \hat{\Omega} = 1$ and NP4(b) is not relevant in our case, see Comment 5 to Theorem 1 of Andrews (1994). Finally NP5 is implied by Assumption A5. This establishes (11) - (13).

Equation (14) holds by a standard argument (see p.2257 of Andrews (1994)) because $\{\bar{\nu}_T(\cdot, \cdot, \cdot) : T \geq 1\}$ is stochastically equicontinuous by Lemma 1 and $\Pr(\hat{\pi}^+ \in \Pi) \rightarrow 1$ and $\rho_b(\hat{\pi}^+, \pi_0^+) \xrightarrow{P} 0$ using (13) and Assumption A3.

To establish (15), Write

$$\begin{aligned}
\sqrt{T} E\bar{m}_T(y, g, \pi)|_{\pi=\hat{\pi}^+} &= \sqrt{T} E1(y_t \leq y)g(x_t) \{\pi(x_t) - 1(r_{t-1} \geq 0)\}|_{\pi=\hat{\pi}^+} \\
&= \sqrt{T} \int F(y|x)g(x) [\hat{\pi}^+(x) - \pi_0^+(x)] f(x)dx \\
&\quad + \sqrt{T} \int [F(y|x) - F^+(y|x)] g(x) \pi_0^+(x) f(x)dx,
\end{aligned} \tag{16}$$

where the second equality holds by rearranging terms and applying law of iterated expectations. Consider the first term on the right hand side of (16). We have

$$\begin{aligned}
&\sqrt{T} \int F(y|x)g(x) [\hat{\pi}^+(x) - \pi_0^+(x)] f(x)dx \\
&= \sqrt{T} \int F(y|x)g(x) [\hat{\pi}^+(x) - \pi_0^+(x)] \hat{f}(x)dx \\
&\quad + \sqrt{T} \int F(y|x)g(x) [\hat{\pi}^+(x) - \pi_0^+(x)] [f(x) - \hat{f}(x)] dx \\
&= A_{1T} + A_{2T}, \text{ say.}
\end{aligned} \tag{17}$$

The term A_{2T} is asymptotically negligible because it is bounded uniformly over $(y, g) \in \mathcal{Y} \times \mathcal{G}$ by

$$T^{1/4} \sup_{x \in \mathcal{X}} |\hat{\pi}^+(x) - \pi_0^+(x)| \times T^{1/4} \sup_{x \in \mathbb{R}^{d_x}} |f(x) - \hat{f}(x)| \xrightarrow{p} 0 \tag{18}$$

using (11) and (12). Now, consider A_{1T} . Write

$$\begin{aligned}
[\hat{\pi}^+(x) - \pi_0^+(x)] \hat{f}(x) &= \frac{1}{T} \sum_t K_h(x - x_t) \{1(r_{t-1} \geq 0) - \pi_0^+(x_t)\} \\
&\quad + \frac{1}{T} \sum_t K_h(x - x_t) \{\pi_0^+(x_t) - \pi_0^+(x)\}.
\end{aligned}$$

Notice that

$$\begin{aligned}
&\sqrt{T} \int F(y|x)g(x) \left[\frac{1}{T} \sum_t K_h(x - x_t) \{1(r_{t-1} \geq 0) - \pi_0^+(x_t)\} \right] dx \\
&= \frac{1}{\sqrt{T}} \sum_t \{1(r_{t-1} \geq 0) - \pi_0^+(x_t)\} \int F(y|x)g(x) K_h(x - x_t) dx \\
&= \frac{1}{\sqrt{T}} \sum_t \{1(r_{t-1} \geq 0) - \pi_0^+(x_t)\} [F(y|x_t)g(x_t) \\
&\quad + \int \{F(y|x_t + uh)g(x_t + uh) - F(y|x_t)g(x_t)\} K(u) du] \\
&= \frac{1}{\sqrt{T}} \sum_t \{1(r_{t-1} \geq 0) - \pi_0^+(x_t)\} F(y|x_t)g(x_t) + o_p(1),
\end{aligned} \tag{19}$$

uniformly over $(y, g) \in \mathcal{Y} \times \mathcal{G}$, where the last equality holds by the following argument. For each $(y, g) \in \mathcal{Y} \times \mathcal{G}$ and some $\delta > 0$, we have

$$\begin{aligned}
& E \left(\frac{1}{\sqrt{T}} \sum_t \{1(r_{t-1} \geq 0) - \pi_0^+(x_t)\} \right. \\
& \quad \times \left. \int \{F(y|x_t + uh)g(x_t + uh) - F(y|x_t)g(x_t)\} K(u) du \right)^2 \\
& \leq C \left(E \left| \int \{F(y|x_t + uh)g(x_t + uh) - F(y|x_t)g(x_t)\} K(u) du \right|^{2+\delta} \right)^{2/(2+\delta)} \\
& \rightarrow 0,
\end{aligned} \tag{20}$$

where the inequality follows by the moment inequality for sums of strong mixing random variables (see Lemma 3.1 of Dehling and Philipp (2002)) and Assumption A1(i), and convergence to zero holds by the fact that $F(y|\cdot)g(\cdot)$ is a bounded function using a well known convergence result for convolutions of functions in an L_p -space (with $p = 2 + \delta$) (see Theorem 8.14 (a) of Folland (1984)). Using a stochastic equicontinuity argument as in Lemma 1, we can show that the convergence to zero holds uniformly over $(y, g) \in \mathcal{Y} \times \mathcal{G}$. This establishes (19). Furthermore, we have

$$\begin{aligned}
& \sup_{(y,g) \in \mathcal{Y} \times \mathcal{G}} \left| \sqrt{T} \int F(y|x)g(x) \left[\frac{1}{T} \sum_t K_h(x - x_t) \{ \pi_0^+(x_t) - \pi_0^+(x) \} \right] dx \right| \\
& = \sup_{(y,g) \in \mathcal{Y} \times \mathcal{G}} \left| \frac{1}{\sqrt{T}} \sum_t \int F(y|x_t + uh)g(x_t + uh) \{ \pi_0^+(x_t) - \pi_0^+(x_t + uh) \} K(u) du \right| \\
& \leq O_p(T^{1/2}h^\omega) \xrightarrow{p} 0,
\end{aligned} \tag{21}$$

where the equality holds by a change of variables, the inequality holds by an ω -term Taylor expansion using Assumptions A3 and A4, and the last convergence to zero holds by Assumption A5. Now, the result (15) is established by combining (16)-(21).

We now establish Theorem 1. We have

$$\begin{aligned}
\sqrt{T}\bar{m}_T(y, g, \hat{\pi}^+) &= \bar{\nu}_T(y, g, \hat{\pi}^+) + \sqrt{T} E\bar{m}_T(y, g, \pi)|_{\pi=\hat{\pi}^+} \\
&= \bar{\nu}_T(y, g, \pi_0^+) + \sqrt{T} E\bar{m}_T(y, g, \pi)|_{\pi=\hat{\pi}^+} + o_p(1)
\end{aligned} \tag{22}$$

$$= \nu_T(y, g) + \sqrt{T} \int [F(y|x) - F^+(y|x)] g(x) \pi_0^+(x) f(x) dx + o_p(1), \tag{23}$$

where the second equation hold by (14) and the last equality holds by (15). Notice that, under the null hypothesis, we have $\int [F(y|x) - F^+(y|x)] g(x) \pi_0^+(x) f(x) dx = 0$ for all $(y, g) \in \mathcal{B}$, while

$\int [F(y|x) - F^+(y|x)] g(x) \pi_0^+(x) f(x) dx < 0$ if $(y, g) \notin \mathcal{B}$. Furthermore, we can show that

$$\nu_T(\cdot, \cdot) \Rightarrow \nu(\cdot, \cdot) \quad (24)$$

with the sample paths of $\nu(\cdot, \cdot)$ uniformly continuous with respect to pseudometric ρ_a on $\mathcal{Y} \times \mathcal{G}$ with probability one. The latter holds by a standard argument because Lemma 1 implies that the pseudometric space $(\mathcal{Y} \times \mathcal{G}, \rho_a)$ is totally bounded, $\{\nu_T(\cdot, \cdot) : T \geq 1\}$ is stochastically equicontinuous, and finite dimensional convergence in distribution holds using a CLT for bounded strong mixing random variables (see Corollary 5.1 of Hall and Heyde (1980)). Therefore, using the same arguments as those in Linton, Maasoumi and Whang (2005, Proof of Theorem 1) and continuous mapping theorem, Theorem 1 is now established as desired. ■

PROOF OF THEOREM 2. The proof is similar to the proof of Theorem 2 of Linton, Maasoumi and Whang (2005). ■

PROOF OF THEOREM 3. The proof is similar to the proof of Theorem 3 of Linton, Maasoumi and Whang (2005). ■

Appendix B: Data Descriptions and Constructions

The data for estimating the daily volatility come from different sources. For the realized variances of S&P500, we use intraday high frequency data provided by tickdata.com, which consist of 1-minute and 5-minute index prices in regular trading time. The squared intraday range RG_t^2 of S&P500 is evaluated by using data of the highest and lowest trading prices during a day, which come from yahoo finance. For the five DJIA stocks, their RG_t^2 are evaluated with daily highest and lowest price data from the CRSP, and their RV_t are evaluated with the intraday trade prices from the TAQ database.

The raw data of the high frequency observations from the TAQ database contain noises. In order to obtain a more accurate realized variance estimation, we adopt the following procedures, which are suggested by Barndorff-Nielsen et al. (2009), to clean the high frequency data of the individual stocks:

- (1) We keep the data points between 09:30AM to 16:00PM (regular trading time), and delete data points with a time stamp outside this time interval.
- (2) We delete the data points which prices are zero.

- (3) We keep data points which the trade occurred on AMEX (A), NYSE (N), NASD (T/Q), and delete the rest data points.
- (4) Data points which are corrected traded are deleted (their Correction Indicator is not zero, CORR!= 0).
- (5) Data points which trades are not in abnormal sale condition are kept (The entries with the column COND which does not has a letter code,or has the letter "F" or "E").
- (6) If multiple trades have the same time stamp, we use their median price.
- (7) We delete the data point in which the absolute difference between its price and median of 50 neighborhood observations is larger than five times mean absolute deviation from the median.

Note that (7) is to replace rule T4 in Barndorff-Nielsen et al. (2009) for cleaning outliers in the high frequency trade data. The rule T4 uses the quote data to discipline the trade data: If the trade prices are above the ask plus bid-ask spread or below the bid minus the bid-ask spread, they will be deleted. However, it can be shown that such rule in practice is rarely activated. Since our raw data are the trade data, in order to more efficiently implementing the cleaning procedures without using the quote data, we use a more viable rule such as (7) for dealing with the outliers.

The time unit of the cleaned data is one second, but the data points are not equally-spaced. We then transform the cleaned data to equally-spaced data. We set the time intervals for the equally-spaced data equal to two frequently used choices: 1 minute and 5 minute. Then the last-tick method is used to construct the equally-spaced data. Finally the equal-spaced data are used to calculate the intraday log returns and realized variances.

Appendix C: Alternative Methods

In this appendix we consider three alternative methods on detecting the leverage effect to check whether our results in the main text still hold. The first method is based on estimating HAR-RV type models with the leverage effect. The other two methods are recently developed by Wang and Mykland (2013) and Ait-Sahalia, Fan and Li (2013), which are based on continuous time finance and high frequency data estimations. We find results from these methods lend further support to our previous analysis.

C1: HAR-RV Type Models with the Leverage Effect

We first use a simple and direct way to test whether the level effects exist. Following Corsi and Renò (2012), we incorporate the leverage effect into the HAR-RV model:

$$\begin{aligned} RV_t = & \alpha_D + \beta_{RD}RV_{t-1} + \beta_{RW}RV_{t-1,week} + \beta_{RM}RV_{t-1,month} + \\ & \gamma_+R_{t-1}^2 \times 1\{R_{t-1} \geq 0\} + \gamma_-R_{t-1}^2 \times 1\{R_{t-1} < 0\} + \varepsilon_t. \end{aligned} \quad (25)$$

To see whether the leverage effect exists, we can test whether $\gamma_- > \gamma_+$. We specify the null hypothesis as $H_0 : \gamma_- \leq \gamma_+$, and if it is rejected, we have evidence to say that there is the presence of leverage effects.

It is well known that a realized variance can be decomposed into continuous (non-jump) and jump components (Barndorff-Nielsen and Shephard, 2006). One of the most commonly used estimators for the continuous component is the realized bi-power variation BV_t :

$$BV_t = \frac{\pi}{2} \left(\frac{n_t}{n_t - 1} \right) \sum_{i=1}^{n_t-1} |r_{i,t}| |r_{i+1,t}|.$$

With the realized bi-power variation BV_t , the jump variation JV_t is given by

$$JV_t = RV_t - BV_t.$$

We thus can modify the right hand side of the above HAR-RV model as a linear combination of BV and JV , and such modification may improve performances of the HAR-RV type regression on predicting future realized variances (Andersen et al, 2007). The modified HAR-RV model is called HAR-RV-CJ (heterogeneous autoregressive realized variance model controlling for the continuous and jump components) model. With the leverage effect, the HAR-RV-CJ model has the following form:

$$\begin{aligned} RV_t = & \alpha_D + \beta_{BD}BV_{t-1} + \beta_{BW}BV_{t-1,week} + \beta_{BM}BV_{t-1,month} + \\ & \beta_{JD}JV_{t-1} + \beta_{JW}JV_{t-1,week} + \beta_{JM}JV_{t-1,month} + \\ & \gamma_+R_{t-1}^2 \times 1\{R_{t-1} \geq 0\} + \gamma_-R_{t-1}^2 \times 1\{R_{t-1} < 0\} + \varepsilon_t, \end{aligned} \quad (26)$$

where $BV_{t-1,week}$ ($JV_{t-1,week}$) and $BV_{t-1,month}$ ($JV_{t-1,month}$) are the normalized weekly and monthly realized bi-power (jump) variation.

For the terms of the leverage effects, besides R_{t-1}^2 , we also use $|R_{t-1}|$ in the regressions. The OLS estimation results for the HAR-RV and HAR-RV-CJ augmented with the leverage effects are shown in Table 8 and 9. In the parenthesis under the estimated coefficients are t-statistics obtained with Newey-West standard errors. The in-sample fittings have high adjusted R^2 (all above 0.41), which is one of the most documented features of the HAR-RV type models. In the Tables, it can be seen that most of the estimated γ_- show strong statistical significance. The estimated γ_+ , however, are not statistically significant in most of the cases, and some of them are even negative. The results of the statistical significance (or insignificance) of the estimated γ_- and γ_+ are consistent over different specifications for the regressions when either R_{t-1}^2 or $|R_{t-1}|$ is used. Comparing the estimated γ_- under the HAR-RV and HAR-RV-CJ models, their values are very similar within each asset. The same results also hold for the estimated γ_+ , but they have much smaller values than γ_- . When R_{t-1}^2 is used, the estimated γ_- ranges from 0.0176 (PG) to 0.2016 (GE) under the HAR-RV model and from 0.0197 (PG) to 0.2038 (GE) under the HAR-RV-CJ model. For the case of $|R_{t-1}|$, the same estimates range from 0.0029 (IBM) to 0.0115 (GE) under the HAR-RV model and from 0.0032 (IBM) to 0.0119 (GE) under the HAR-RV-CJ model. The results suggest that among the cases, realized variance of GE reacts most when its lagged return receives a negative impact.

We show results of testing $H_0 : \gamma_- \leq \gamma_+$ in the last two columns of the Table 8 and 9. Except for PG and MMM, the t test statistics are all above 2.37, which suggests the hypothesis can be rejected at the significant level 0.009. As for PG and MMM, the hypothesis can also be well rejected at the significant level 0.05. The evidence shown here indicates that the hypothesis at least can be rejected at a moderate significant level. There is evidence to say that negative shocks to asset returns have more impacts on the realized variances than do equal positive shocks, and the leverage effects may exist.

C2: Wang and Mkyland (2013)

We then use the quadratic co-variation approach proposed by Wang and Mykland (2013) to verify the existence of leverage effects in intraday log returns and volatilities. Considering the following data generating process for the log price $X_t := \log P_t$ and volatility σ_t :

$$\begin{aligned} dX_t &= \mu_t dt + \sigma_t dW_t, \\ d\sigma_t &= a_t dt + f_t dW_t + g_t dB_t, \end{aligned} \tag{27}$$

where W_t and B_t are two mutually independent Brownian motions. Wang and Mykland (2013) propose to use the quadratic co-variation between X_t and $F(\sigma_t^2)$ as a quantitative measure of the contemporaneous leverage effect:

$$\langle X, F(\sigma^2) \rangle_T = 2 \int_0^T F'(\sigma_t^2) \sigma_t^2 f_t dt. \quad (28)$$

The function $F(\cdot)$ is twice differentiable and monotonic on $(0, \infty)$, and in the following we assume either $F(x) = x$ or $F(x) = 1/2 \log(x)$.

Recall that in this paper we define the leverage effects as negative shocks to prices/returns affect volatility more than equal positive shocks, which is somehow different from the contemporaneous leverage effect defined in (28). While (28) only evaluates covariation between X_t and $F(\sigma_t^2)$, it does not tell whether the negative or positive shocks have more effects on the volatility. One way to link (28) and our definition of leverage effects is to require the parameter f_t to be negative: If $f_t < 0$, negative (positive) shocks to the log returns increase (decrease) volatility.

Suppose within the time interval $[0, T]$, the log price process X_t is observed at equally spaced time stamps; i.e., r_t is observed every $\Delta t_{n,i+1} = T/n$ units of time. To empirically estimate $\langle X, F(\sigma^2) \rangle_T$, we first divide the observed X_t 's into different blocks. Suppose the number of such blocks is K_n and each block contains $M_n = \lfloor c\sqrt{n} \rfloor$ observations, where c is some constant. Wang and Mykland (2013) propose to use

$$\langle X, \widehat{F(\sigma^2)} \rangle_T = 2 \sum_{i=0}^{K_n-2} (X_{\tau_{n,i+1}} - X_{\tau_{n,i}}) \left(F(\widehat{\sigma}_{\tau_{n,i+1}}^2) - F(\widehat{\sigma}_{\tau_{n,i}}^2) \right)$$

to estimate $\langle X, F(\sigma^2) \rangle_T$, where $\tau_{n,i}$, $i = 0, \dots, K_n - 1$ is the lower bound of the i th block, and

$$\widehat{\sigma}_{\tau_{n,i+1}}^2 = \frac{n}{M_n \times T} \sum_{t_{n,j} \in (\tau_{n,i}, \tau_{n,i+1}]} (X_{t_{n,j+1}} - X_{t_{n,j}})^2.$$

is an estimate for the integrate variance within the block $(\tau_{n,i}, \tau_{n,i+1}]$. They show that

$$n^{\frac{1}{4}} \left(\langle X, \widehat{F(\sigma^2)} \rangle_T - \langle X, F(\sigma^2) \rangle_T \right)$$

converges in law to $Z \times B(c, T)$, where Z is a standard normal random variable and independent of any information up to time T and

$$B(c, T) = \sqrt{\frac{16}{c} \int_0^T (F'(\sigma_t^2))^2 \sigma_t^6 dt + cT \int_0^T (F'(\sigma_t^2))^2 \sigma_t^4 \left(\frac{44}{3} f_t^2 + \frac{22}{3} g_t^2 \right) dt}.$$

For empirically estimating $B(c, T)$, one can use $\sqrt{G_n^1 + G_n^2}$, where

$$\begin{aligned} G_n^1 &= 2\sqrt{n} \sum_i (X_{\tau_{n,i+1}} - X_{\tau_{n,i}})^2 \left(F(\widehat{\sigma}_{\tau_{n,i+1}}^2) - F(\widehat{\sigma}_{\tau_{n,i}}^2) \right)^2, \\ G_n^2 &= 2\frac{M_n T}{\sqrt{n}} \sum_i \widehat{\sigma}_{\tau_{n,i}}^2 \left(F(\widehat{\sigma}_{\tau_{n,i+1}}^2) - F(\widehat{\sigma}_{\tau_{n,i}}^2) \right)^2. \end{aligned}$$

It can be shown that $G_n^1 + G_n^2$ converges in probability to $B(c, T)^2$. One also can use

$$\widetilde{\sigma}_{\tau_{n,i+1}}^2 = \frac{n}{M_n \times T} \sum_{t_{n,j} \in (\tau_{n,i}, \tau_{n,i+1}]} (X_{t_{n,j+1}} - \overline{\Delta X_{\tau_{n,i+1}}})^2.$$

to replace $\widehat{\sigma}_{\tau_{n,i+1}}^2$ in the estimation, where $\overline{\Delta X_{\tau_{n,i+1}}} = 1/M_n (X_{\tau_{n,i+1}} - X_{\tau_{n,i}})$ is an average of log return changes within the time interval $(\tau_{n,i}, \tau_{n,i+1}]$. Let $\langle X, \widehat{F}(\sigma^2) \rangle_T$ denote the leverage estimate when $\widetilde{\sigma}_{\tau_{n,i+1}}^2$ is used. As shown in Wang and Mykland (2013), $\widehat{\sigma}_{\tau_{n,i+1}}^2$ and $\widetilde{\sigma}_{\tau_{n,i+1}}^2$ are asymptotic equivalent, and therefore $\langle X, \widehat{F}(\sigma^2) \rangle_T$ and $\langle X, \widetilde{F}(\sigma^2) \rangle_T$ are also asymptotic equivalent. Furthermore, the following two test statistics can be used to detect local leverage effects,

$$L_1 := \frac{n^{\frac{1}{4}} \left(\langle X, \widehat{F}(\sigma^2) \rangle_T - \langle X, F(\sigma^2) \rangle_T \right)}{\sqrt{G_n^1 + G_n^2}}, \quad (29)$$

$$L_2 := \frac{n^{\frac{1}{4}} \left(\langle X, \widetilde{F}(\sigma^2) \rangle_T - \langle X, F(\sigma^2) \rangle_T \right)}{\sqrt{G_n^1 + G_n^2}}. \quad (30)$$

It can be shown that L_1 and L_2 both converge in law to standard normal.

We use 1-min equally spaced data, and set $T =$ one day and $M_n = 30$ for the estimations. Figure 6 shows time series plots of daily standardized quadratic co-variations, which are just daily L_1 and L_2 with $\langle X, F(\sigma^2) \rangle_T = 0$. Table 10 reports some statistics of these standardized quadratic co-variations. It can be seen that except MMM, all the rest four DJIA stocks and S&P500 index on average have negative L_1 and L_2 . Without considering signs of the estimates, we compare absolute values of the daily L_1 and L_2 with two critical values 1.96 and 2.58 (corresponding to significant levels $\alpha = 0.05$ and 0.01 of standard normal). We find very few days have significant L_1 and L_2 : Among the 4,283 days, the number of significant days ranges from 18 to 257 for $\alpha = 0.05$ and 0 to 13 for $\alpha = 0.01$. If considering significant negativity only, the numbers of significant days range from 60 to 392 for $\alpha = 0.05$ and 0 to 42 days $\alpha = 0.01$ (with critical values equal to -1.64 and -2.33 of standard normal). S&P500 index on average has lower standardized quadratic co-variations and more significant days

than the five DJIA stocks. Overall, the results suggest that the leverage effects may still exist in the five DJIA stocks and S&P500 index when log returns and volatilities are estimated at the intraday level, but only in certain periods, these leverage effects are strong enough to be detected by the quadratic co-variation approach.

C3: Aït-Sahalia, Fan and Li (2013)

We finally turn to the method in Aït-Sahalia, Fan and Li et al. (2013). In their paper, the following CIR process are considered for the squared volatility:

$$dv_t = \alpha_v (\theta - v_t) + \kappa \sqrt{v_t} dB_t,$$

where $v_t := \sigma_t^2$, and $2\alpha_v \theta > \kappa^2$. For the log price process, they assume it follows the same process as in (27) except now $\mathbb{E}(dB_t dW_t) = f dt$, where f is a constant. It can be shown that the parameter f is the limit of correlation between $v_{t+l} - v_t$ and $X_{t+l} - X_t$ when the time interval l approaches zero, i.e.,

$$f = \lim_{l \rightarrow 0} \text{Corr}(v_{t+l} - v_t, X_{t+l} - X_t). \quad (31)$$

Aït-Sahalia, Fan and Li (2013), use the limit correlation above as a measure for the leverage effect, which is different from Wang and Mykland (2013) and our definition. To link the limit correlation (31) to our definition of the leverage effect, again we may assume f to be negative.

Let \hat{f}_k and f_k be the sample and true correlations between the difference of the estimated integrated variances and difference of log prices at time $t + k\Delta$ and t . Here Δ is the time unit and we follow Aït-Sahalia, Fan and Li (2013), to assume it as one day for our cases³. Under some regular conditions, Aït-Sahalia, Fan and Li (2013) show that f_k and f satisfy the following linear relationship:

$$f_k = f + b \times k + o(k\Delta),$$

which provides an easily-implemented way to identify the limit correlation f : That is, running a linear regression of f_k (or \hat{f}_k if f_k is unknown) on the intercept term and k , and the estimated intercept term can be used as an estimate of f . Furthermore, Aït-Sahalia, Fan and Li (2013) propose the following data driven procedures for practically estimate the linear regression:

³Note that in Aït-Sahalia, Fan and Li (2013), the basic time unit for t is one year, so $\Delta = \text{one day} = 1/252$.

1. For each $k = 1, \dots, K$, we calculate \hat{f}_k , and rank the \hat{f}'_k s for $k = k_0, \dots, \lfloor K/2 \rfloor$. Then we take the three smallest values of these ranked \hat{f}'_k s. Let $k_{(1)}$, $k_{(2)}$, and $k_{(3)}$ be the corresponding indices k 's that the three smallest \hat{f}'_k s have. Let $\underline{k}^* = \max(k_{(1)}, k_{(2)}, k_{(3)})$.
2. Regressing \hat{f}_k on k with $k = \underline{k}^*, \dots, \underline{k}^* + m$, where $m = a_0, \dots, K - \underline{k}^*$. Let m^* denote the value of m that the regression yields the highest (unadjusted) R^2 . Then let $\bar{k}^* = \underline{k}^* + m^*$. The estimated intercept term of the regression with the data $\left\{k, \hat{f}_k\right\}_{k=\underline{k}^*}^{\bar{k}^*}$ is the final estimate of f .

We follow Ait-Sahalia, Fan and Li (2013) to set $K = 252$, $k_0 = 6$ and $a_0 = 11$. As shown in Ait-Sahalia, Fan and Li (2013), the sample correlation \hat{f}_k is a bias estimation for the true f_k . To improve performance of the data driven approach above, we can replace the sample correlation \hat{f}_k with the following bias corrected estimation in the above data driven procedures:

$$\hat{f}_k^{bc} = \gamma \frac{2\sqrt{k^2 - k/3}}{2k - 1} \hat{f}_k, \quad (32)$$

where

$$\gamma = \left(1 - \frac{4\Delta^2 \mathbb{E}(v_t^2)}{n \text{Var}(RV_{t+k\Delta} - RV_t)}\right)^{-\frac{1}{2}}.$$

Here the parameter n is the number of log return observations used to estimate the realized variance. For estimating $\Delta^2 \mathbb{E}(v_t^2)$, we can first estimate the realized quarticity:

$$QV_t = \frac{n_t}{3} \sum_{i=1}^{n_t} r_{i,t}^4,$$

and then calculate its sample mean. In the following we use

$$\hat{\gamma} = \left(1 - \frac{4 \times \text{sample mean of } QV_t}{\text{sample mean of } n_t \times \text{sample variance of } RV_t}\right)^{-\frac{1}{2}} \quad (33)$$

to replace γ for \hat{f}_k^{bc} .

All the following empirical analysis is in daily basis: We use daily RV_t^{1min} and R_t to calculate \hat{f}_k , and the 1-min log returns to estimate the daily realized quarticities in $\hat{\gamma}$. Figure 7 plots \hat{f}_k and \hat{f}_k^{bc} against k for the five DJIA stocks and S&P500 index. We use 1-min log returns to estimate the realized quarticities. For all the cases, the two estimated correlations are negative over k , and the bias corrected correlations are constantly lower than the uncorrected ones. Except for small k , the two correlation estimations follow extremely similar patterns within each case. Comparing

the results over different cases, however, the patterns of the correlation estimations are somehow different, but they often have a higher value as $k = 1$ and then suddenly drop to a lower value as k deviates from one. It also can be seen that as $k > 100$, the estimations gradually become stable and all of them steadily move either up or down as k becomes large. Overall, the estimated correlations vary substantially as the time interval k changes, no matter whether they are bias corrected or not; and the smaller the time interval k , the more possible that we will get a higher estimated correlation.

To estimate f , we may use \hat{f}_1 or \hat{f}_1^{bc} . As shown above, however, it is very likely that we get a higher estimated correlation with $k = 1$ than with $k > 1$, and the high \hat{f}_1 or \hat{f}_1^{bc} perhaps is an upward bias estimate for the limit correlation f . To obtain a more accurately estimated f , we use the data driven procedures introduced above. In Table 11 we report the final estimate of f based on the estimated intercept term from running linear regressions of \hat{f}_k (or \hat{f}_k^{bc}) on $k \in [\underline{k}^*, \bar{k}^*]$, and the unadjusted R^2 of the regression. The R^2 is very high for all of the six cases, and \underline{k}^* and \bar{k}^* in each case are similar when either \hat{f}_k or \hat{f}_k^{bc} is used. For all the six cases, the estimated f is moderately negative, with range from -0.24 (IBM) to -0.48 (GE)⁴. The negativity of the estimated f implies that on average a positive shock to the log price has a smaller impact on the volatility than does an equal negative shock, and the leverage effects may exist. It also can be seen that the estimated f from using \hat{f}_k^{bc} and \hat{f}_k are qualitatively similar, but the former is slightly lower than the later.

⁴One thing worth to note is that, the estimated f of MSFT and S&P500 shown here are different from those shown in Ait-Sahalia, Fan and Li (2013). It is mainly because we use different sample periods and different realized variance estimators. In Ait-Sahalia, Fan and Li (2013), they used the pre-averaging approach (Jacod et al., 2009) to estimate the integrated variances. With RV_t^{1min} , we re-estimate the f of MSFT with the data driven method over the same sample period as theirs (Jan-2005 to June-2007). The estimated f from using \hat{f}_k^{bc} (\hat{f}_k) is -0.90 (-0.87) and $[\underline{k}^*, \bar{k}^*] = [125, 165]$ ($[125, 165]$). As for S&P500 (sample period is from Jan-2004 to Dec-2007), the estimated f from using \hat{f}_k^{bc} (\hat{f}_k) is -0.62 (-0.60), and $[\underline{k}^*, \bar{k}^*] = [22, 101]$ ($[24, 101]$).

Table 1: The table shows summary statistics of daily 1-min realized volatility $\sqrt{RV_t^{1min}}$, 5-min realized volatility $\sqrt{RV_t^{5min}}$, intraday range estimator RG_t , and stock return R_t for S&P500 index and five stocks from Dow Jones Industrial Averages: Microsoft (MSFT), International Business Machines Corporation (IBM), General Electric (GE), Procter & Gamble (PG) and 3M (MMM). We scale $\sqrt{RV_t^{1min}}$, $\sqrt{RV_t^{5min}}$, and RG_t by 100 before we calculate the statistics. The sample period is from Jan-04-1993 to Dec-31-2009 (4,283 trading days).

MSFT							
	Min.	Mean	Max.	Std.	Skew.	Kurt.	ACF(1)
$\sqrt{RV_t^{1min}}$	0.3643	2.0820	9.2090	1.0199	1.1783	1.5501	0.9126
$\sqrt{RV_t^{5min}}$	0.3528	1.7520	7.7680	0.7936	1.3869	4.0803	0.8020
RG_t	0.2814	1.6050	9.9750	0.9070	1.9047	7.1083	0.5422
R_t	-0.1560	0.0008	0.1957	0.0221	0.2258	5.8996	-0.0333
IBM							
	Min.	Mean	Max.	Std.	Skew.	Kurt.	ACF(1)
$\sqrt{RV_t^{1min}}$	0.3694	1.6720	10.5900	0.7748	1.7240	7.8765	0.8480
$\sqrt{RV_t^{5min}}$	0.3282	1.5190	8.6820	0.7358	2.0632	8.7406	0.7626
RG_t	0.1758	1.4250	9.8130	0.8503	2.1492	8.4899	0.5345
R_t	-0.1554	0.0007	0.1316	0.0198	0.3747	6.3703	-0.0328
GE							
	Min.	Mean	Max.	Std.	Skew.	Kurt.	ACF(1)
$\sqrt{RV_t^{1min}}$	0.3737	1.7840	12.7800	0.9724	2.9976	17.9588	0.8709
$\sqrt{RV_t^{5min}}$	0.3319	1.5630	13.4200	0.9483	3.5067	21.6421	0.8158
RG_t	0.1952	1.4130	14.1500	1.0491	3.2342	17.1989	0.6692
R_t	-0.1279	0.0004	0.1970	0.0196	0.3141	8.7333	-0.0127
PG							
	Min.	Mean	Max.	Std.	Skew.	Kurt.	ACF(1)
$\sqrt{RV_t^{1min}}$	0.3809	1.5520	10.7800	0.6875	1.7891	13.0114	0.8341
$\sqrt{RV_t^{5min}}$	0.3284	1.3890	8.9300	0.6449	2.2713	12.7597	0.7711
RG_t	0.2144	1.1830	8.3060	0.7112	2.7397	14.1404	0.5479
R_t	-0.3138	0.0005	0.1021	0.0160	-1.8188	38.2772	-0.0546
MMM							
	Min.	Mean	Max.	Std.	Skew.	Kurt.	ACF(1)
$\sqrt{RV_t^{1min}}$	0.4162	1.4730	10.2100	0.6253	2.3454	15.7073	0.8057
$\sqrt{RV_t^{5min}}$	0.2834	1.3790	9.8280	0.6366	2.6075	15.3994	0.7498
RG_t	0.1392	1.2220	10.7400	0.7326	2.5815	13.8397	0.5137
R_t	-0.0959	0.0004	0.1107	0.0159	0.1628	4.3600	-0.0348
S&P500							
	Min.	Mean	Max.	Std.	Skew.	Kurt.	ACF(1)
$\sqrt{RV_t^{1min}}$	0.1276	0.6726	8.5770	0.4972	3.9649	30.9178	0.8797
$\sqrt{RV_t^{5min}}$	0.1649	0.7788	7.6050	0.5303	3.2613	19.9993	0.8385
RG_t	0.1066	0.8304	6.5490 ³²	0.6139	3.1255	16.7762	0.6363
R_t	-0.0904	0.0003	0.1158	0.0122	-0.0024	9.1608	-0.0668

Table 2: The table shows the subsample critical values for the conditional leverage hypothesis test at four different significant levels α for S&P500 index and five stocks from Dow Jones Industrial Averages: Microsoft (MSFT), International Business Machines Corporation (IBM), General Electric (GE), Procter & Gamble (PG) and 3M (MMM). We set lag lengths of daily returns $j = 1$ and 5 and subsample sizes $b = 500, 1000$ and 2000. Daily volatility is estimated by the daily realized variance with 1-min log returns. The sample period is from Jan-04-1993 to Dec-31-2009 (4,283 trading days).

		$j = 1$				$j = 5$					
		α				α					
	b	0.1	0.05	0.01	0.001		0.1	0.05	0.01	0.001	
$\hat{S}_{T_1} = 0.0825$	MSFT	500	0.2612	0.2810	0.3515	0.3735	$\hat{S}_{T_5} = 0.0823$	0.3550	0.3727	0.4182	0.4598
		1000	0.2085	0.2345	0.2553	0.2683		0.2776	0.2857	0.3017	0.3170
		2000	0.1557	0.1641	0.1758	0.1807		0.1890	0.1944	0.2055	0.2069
$\hat{S}_{T_1} = 0.0138$	IBM	500	0.2669	0.2966	0.3415	0.3585	$\hat{S}_{T_5} = 0.2479$	0.4320	0.4743	0.5836	0.6580
		1000	0.2406	0.2550	0.2741	0.2991		0.3759	0.4355	0.4874	0.5087
		2000	0.1452	0.1539	0.1627	0.1671		0.2306	0.2551	0.2661	0.2813
$\hat{S}_{T_1} = 0.1996$	GE	500	0.3275	0.3596	0.4135	0.4445	$\hat{S}_{T_5} = 0.1885$	0.3314	0.3830	0.4819	0.5294
		1000	0.2939	0.3365	0.4402	0.4707		0.2953	0.3223	0.3406	0.3556
		2000	0.1779	0.1853	0.2185	0.2471		0.3167	0.3248	0.3496	0.3989
$\hat{S}_{T_1} = 0.0622$	PG	500	0.2813	0.3021	0.3311	0.3506	$\hat{S}_{T_5} = 0.1073$	0.2896	0.3231	0.3917	0.4121
		1000	0.2318	0.2795	0.3575	0.3769		0.2453	0.2748	0.3274	0.3833
		2000	0.1554	0.1887	0.2155	0.2240		0.177	0.1897	0.2127	0.2157
$\hat{S}_{T_1} = 0.0595$	MMM	500	0.3360	0.3872	0.5014	0.5454	$\hat{S}_{T_5} = 0.1242$	0.3548	0.4010	0.4813	0.5120
		1000	0.3256	0.3529	0.3823	0.4055		0.2773	0.3074	0.3552	0.3660
		2000	0.1813	0.1927	0.2062	0.2245		0.1695	0.1802	0.1947	0.2038
$\hat{S}_{T_1} = 0.0096$	S&P500	500	0.2203	0.2615	0.3288	0.3547	$\hat{S}_{T_5} = 0.1480$	0.3675	0.4464	0.498	0.5252
		1000	0.1997	0.2152	0.2371	0.2508		0.3481	0.3777	0.4177	0.4689
		2000	0.2041	0.2140	0.2241	0.2292		0.2613	0.3255	0.3958	0.4243

Table 3: The table shows the subsample critical values for the conditional leverage hypothesis test at four different significant levels α for S&P500 index and five stocks from Dow Jones Industrial Averages: Microsoft (MSFT), International Business Machines Corporation (IBM), General Electric (GE), Procter & Gamble (PG) and 3M (MMM). We set lag lengths of daily returns $j = 1$ and 5 and subsample sizes $b = 500, 1000$ and 2000. Daily volatility is estimated by the daily realized variance with 5-min log returns. The sample period is from Jan-04-1993 to Dec-31-2009 (4,283 trading days).

		$j = 1$				$j = 5$					
		α				α					
	b	0.1	0.05	0.01	0.001		0.1	0.05	0.01	0.001	
$\hat{S}_{T_1} = 0.0916$	MSFT	500	0.3407	0.3635	0.3981	0.4522	$\hat{S}_{T_5} = 0.1101$	0.3825	0.4282	0.4821	0.4996
		1000	0.2622	0.2829	0.3106	0.3228		0.2911	0.3148	0.3548	0.3709
		2000	0.1454	0.1590	0.1630	0.1692		0.1845	0.1966	0.2085	0.2136
$\hat{S}_{T_1} = 0.0247$	IBM	500	0.2849	0.3361	0.3875	0.4905	$\hat{S}_{T_5} = 0.1970$	0.4443	0.4936	0.5745	0.6150
		1000	0.2510	0.3144	0.3687	0.3881		0.3549	0.4161	0.4809	0.5051
		2000	0.1096	0.1130	0.1214	0.1438		0.2238	0.2347	0.2572	0.2764
$\hat{S}_{T_1} = 0.0940$	GE	500	0.3433	0.3776	0.4344	0.4622	$\hat{S}_{T_5} = 0.1342$	0.3466	0.3785	0.4565	0.4803
		1000	0.2465	0.2699	0.3045	0.3220		0.2931	0.3240	0.3624	0.3769
		2000	0.1531	0.1823	0.2342	0.2489		0.1953	0.2104	0.2212	0.2279
$\hat{S}_{T_1} = 0.0544$	PG	500	0.2646	0.2963	0.3311	0.3535	$\hat{S}_{T_5} = 0.0776$	0.2862	0.3252	0.3718	0.3859
		1000	0.1785	0.2019	0.2547	0.2967		0.2079	0.2359	0.2843	0.291
		2000	0.1287	0.1402	0.1539	0.1601		0.1438	0.1669	0.2002	0.2185
$\hat{S}_{T_1} = 0.0637$	MMM	500	0.3679	0.5158	0.6259	0.7201	$\hat{S}_{T_5} = 0.1058$	0.3430	0.3817	0.4175	0.4508
		1000	0.4355	0.5008	0.5299	0.5677		0.2587	0.3023	0.3734	0.3938
		2000	0.2514	0.2727	0.3131	0.3401		0.1412	0.1509	0.1660	0.1741
$\hat{S}_{T_1} = 0.0000$	S&P500	500	0.1903	0.2110	0.2739	0.2947	$\hat{S}_{T_5} = 0.1160$	0.3524	0.4850	0.5389	0.5830
		1000	0.1158	0.1242	0.1556	0.1922		0.3561	0.3858	0.4409	0.4660
		2000	0.0897	0.0953	0.1146	0.1248		0.1886	0.2116	0.2571	0.2706

Table 4: The table shows the subsample critical values for the conditional leverage hypothesis test at four different significant levels α for S&P500 index and five stocks from Dow Jones Industrial Averages: Microsoft (MSFT), International Business Machines Corporation (IBM), General Electric (GE), Procter & Gamble (PG) and 3M (MMM). We set lag lengths of daily returns $j = 1$ and 5 and subsample sizes $b = 500, 1000$ and 2000. Daily volatility is estimated by the intraday range estimator. The sample period is from Jan-04-1993 to Dec-31-2009 (4,283 trading days).

		$j = 1$				$j = 5$				
		α				α				
	b	0.1	0.05	0.01	0.001		0.1	0.05	0.01	0.001
MSFT $\hat{S}_{T_1} = 0.0940$	500	0.4644	0.5648	0.7774	0.8167	$\hat{S}_{T_5} = 0.1534$	0.4478	0.5330	0.5944	0.6220
	1000	0.4412	0.4959	0.5720	0.5954		0.3699	0.4190	0.5083	0.5238
	2000	0.3001	0.3102	0.3308	0.3403		0.3140	0.3222	0.3328	0.3389
IBM $\hat{S}_{T_1} = 0.1354$	500	0.2890	0.3374	0.3864	0.4246	$\hat{S}_{T_5} = 0.1855$	0.3381	0.3781	0.4443	0.4841
	1000	0.2349	0.2517	0.2879	0.3031		0.2573	0.2841	0.3264	0.3433
	2000	0.1853	0.1916	0.2055	0.2092		0.1809	0.1923	0.2090	0.2301
GE $\hat{S}_{T_1} = 0.0575$	500	0.5426	0.6059	0.7410	0.8486	$\hat{S}_{T_5} = 0.0912$	0.3487	0.4119	0.5107	0.5821
	1000	0.4323	0.4668	0.5725	0.5999		0.2726	0.2891	0.3174	0.3351
	2000	0.2902	0.3095	0.3328	0.3493		0.2197	0.2472	0.3121	0.3254
PG $\hat{S}_{T_1} = 0.0837$	500	0.2644	0.3006	0.3464	0.3733	$\hat{S}_{T_5} = 0.1117$	0.3586	0.3913	0.4558	0.5071
	1000	0.1468	0.1595	0.1762	0.1865		0.3018	0.3405	0.3952	0.4364
	2000	0.1110	0.1175	0.1265	0.1281		0.1986	0.2075	0.2201	0.2233
MMM $\hat{S}_{T_1} = 0.0773$	500	0.3645	0.4466	0.6223	0.6821	$\hat{S}_{T_5} = 0.0609$	0.4895	0.536	0.5965	0.6332
	1000	0.2584	0.2792	0.3107	0.3545		0.3465	0.3777	0.4176	0.4383
	2000	0.2150	0.2213	0.2315	0.2383		0.1694	0.1856	0.2193	0.2291
S&P500 $\hat{S}_{T_1} = 0.0392$	500	0.3126	0.3525	0.4024	0.4240	$\hat{S}_{T_5} = 0.0751$	0.4060	0.4707	0.5518	0.5825
	1000	0.2606	0.2812	0.3033	0.3144		0.3203	0.4497	0.4980	0.5078
	2000	0.1843	0.1980	0.2301	0.2416		0.1216	0.1741	0.2233	0.2340

Table 5: The table shows the empirical p-values of the conditional test statistic \hat{S}_{T_j} for S&P500 index and five stocks from Dow Jones Industrial Averages: Microsoft (MSFT), International Business Machines Corporation (IBM), General Electric (GE), Procter & Gamble (PG) and 3M (MMM). Three different estimated daily volatilities are used. We set lag lengths of daily returns $j = 1$ and 5 and subsample sizes $b = 500, 1000$ and 2000. The sample period is from Jan-04-1993 to Dec-31-2009 (4,283 trading days).

	j	RV_t^{1min}			RV_t^{5min}			RG_t^2		
		b			b			b		
		500	1000	2000	500	1000	2000	500	1000	2000
MSFT	1	0.9101	0.9138	0.8608	0.8956	0.7914	0.3936	0.9160	0.8916	0.9698
	5	1.0000	0.9936	1.0000	0.9849	0.9662	0.8507	0.7389	0.6653	0.9343
IBM	1	0.9834	0.9769	0.8643	1.0000	0.9568	0.8958	0.6847	0.4674	0.4435
	5	0.2994	0.1687	0.0617	0.5938	0.4141	0.2421	0.6155	0.4562	0.0744
GE	1	0.5264	0.2500	0.0201	0.8356	0.6544	0.6975	0.9884	0.8952	0.8148
	5	0.6580	0.4568	0.3914	0.8597	0.9354	0.7329	0.9318	0.9814	0.9186
PG	1	0.9625	0.9656	0.8481	0.9765	0.7667	0.8117	0.7149	0.5356	0.4374
	5	0.8776	0.8441	0.6856	0.9284	0.7174	0.5416	0.8198	0.5627	0.4247
MMM	1	0.9730	0.9900	0.8643	0.9514	0.9650	0.8783	0.9236	0.9062	0.8927
	5	0.9086	0.7485	0.3827	0.8338	0.7211	0.4002	0.9374	0.7619	0.9387
S&P500	1	0.9860	0.9939	1.0000	0.9987	0.9915	1.0000	0.9323	0.9178	0.7877
	5	0.6786	0.5612	0.8494	0.8890	0.6559	0.4952	0.9294	0.7555	0.5066

Table 6: The table shows value of test statistic \hat{S}_{T_j} , empirical p-value and the subsample critical values for the conditional leverage hypothesis test at four different levels of α for SP500 index and five stocks from Dow Jones Industrial Averages: Microsoft (MSFT), International Business Machines Corporation (IBM), General Electric (GE), Procter & Gamble (PG) and 3M (MMM). Daily volatility is estimated by the daily realized variance with 1-min log returns. We set lag lengths of stock returns $j = 1$ and subsample sizes $b = 500$ and 1000 . The sample period is divided into two subperiods: 1) from Jan-04-1993 to Nov-30-2001 (2,248 trading days). 2) from Dec-03-2001 to Dec-31-2009 (2,035 trading days).

	Period	\hat{S}_{T_1}	b	p-value	α			
					0.1	0.05	0.01	0.0001
MSFT	Jan.93-Nov.01	0.1401	500	0.6049	0.2257	0.2662	0.3446	0.3610
			1000	0.0753	0.1370	0.1434	0.1647	0.1724
	Dec.01-Dec.09	0.0754	500	0.9150	0.2722	0.2839	0.3566	0.3780
			1000	0.9828	0.2284	0.2430	0.2584	0.2703
IBM	Jan.93-Nov.01	0.1458	500	0.8834	0.3014	0.3251	0.3461	0.3634
			1000	0.8343	0.2576	0.2652	0.2829	0.3008
	Dec.01-Dec.09	0.0214	500	0.9651	0.2140	0.2340	0.2541	0.2725
			1000	0.9445	0.1517	0.1662	0.1880	0.1997
GE	Jan.93-Nov.01	0.1622	500	0.7216	0.3519	0.3900	0.4354	0.4448
			1000	0.2562	0.3391	0.3980	0.4555	0.4738
	Dec.01-Dec.09	0.1472	500	0.6388	0.3157	0.3342	0.3652	0.3810
			1000	0.5155	0.2464	0.3204	0.3389	0.3448
PG	Jan.93-Nov.01	0.1075	500	0.8159	0.2597	0.2878	0.3207	0.3329
			1000	0.7150	0.1814	0.1881	0.2002	0.2047
	Dec.01-Dec.09	0.0470	500	0.9764	0.2942	0.3075	0.3339	0.3511
			1000	0.9695	0.2712	0.3285	0.3621	0.3803
MMM	Jan.93-Nov.01	0.1459	500	0.9989	0.3949	0.4506	0.5296	0.5523
			1000	1.0000	0.3584	0.3720	0.3949	0.4069
	Dec.01-Dec.09	0.0238	500	0.9951	0.2018	0.2294	0.2776	0.2971
			1000	1.0000	0.1995	0.2282	0.2501	0.2549
S&P500	Jan.93-Nov.01	0.0902	500	0.5638	0.2712	0.3069	0.3406	0.3603
			1000	0.4516	0.2111	0.2243	0.2386	0.2526
	Dec.01-Dec.09	0.0598	500	0.9391	0.2025	0.2200	0.2401	0.2574
			1000	0.8634	0.1839	0.2045	0.2333	0.2454

Table 7: The table shows the test statistic \hat{S}_{T_j} , the empirical p-values, and the subsample critical values for the conditional leverage hypothesis test at four different levels of α for S&P500 index and five stocks from Dow Jones Industrial Averages: Microsoft (MSFT), International Business Machines Corporation (IBM), General Electric (GE), Procter& Gamble (PG) and 3M (MMM). In addition to the lagged one day return R_{t-1} , the test is also conditioning on more general forms of $h(\sigma_{t-1}^2, \dots, \sigma_{t-p}^2)$. Upper panel shows results of the test conditioning on $h(\cdot)$ as a finite approximation for the exponential moving average in (9) and bottom panel shows results of the test conditioning on $h(\cdot)$ as a real time forecast for RV_t^{1min} from the HAR-RV model in (10). Daily volatility is estimated by daily realized variance with 1-min log returns. We set lag lengths of stock returns $j = 1$ and subsample size $b = 2000$. The sample period is from Jan-04-1993 to Dec-31-2009 (4,283 trading days).

Finite Exponential Moving Average							
				α			
	p	\hat{S}_{T_1}	p-value	0.1	0.05	0.01	0.001
MSFT	5	0.1078	0.5236	0.1804	0.1961	0.2059	0.2107
	10	0.2082	0.0039	0.1886	0.1928	0.1985	0.2162
	22	0.2027	0.2194	0.2129	0.2214	0.2284	0.2291
IBM	5	0.0817	0.9098	0.1784	0.2181	0.2381	0.2525
	10	0.0716	0.8717	0.1543	0.1736	0.1959	0.2043
	22	0.0636	0.8126	0.1913	0.1954	0.2186	0.2230
GE	5	0.1274	0.4882	0.2332	0.2416	0.2452	0.2455
	10	0.1194	0.4593	0.2271	0.2490	0.2714	0.2788
	22	0.1401	0.4286	0.2604	0.2918	0.3162	0.3254
PG	5	0.0281	0.9904	0.1099	0.1147	0.1409	0.1530
	10	0.0450	0.7412	0.1168	0.1416	0.1468	0.1504
	22	0.0537	0.6668	0.1247	0.1293	0.1396	0.1443
MMM	5	0.1329	0.5263	0.1731	0.1877	0.1981	0.2161
	10	0.1163	0.6734	0.2279	0.2340	0.2443	0.2650
	22	0.1220	0.9934	0.2009	0.2160	0.2237	0.2260
S&P500	5	0.0361	0.7601	0.0947	0.1022	0.1235	0.1336
	10	0.0086	1.0000	0.1001	0.1033	0.1178	0.1193
	22	0.0148	0.9834	0.1415	0.1449	0.1501	0.1812
Real Time Forecast from the HAR-RV model							
				α			
	p	\hat{S}_{T_1}	p-value	0.1	0.05	0.01	0.001
MSFT	22	0.1166	0.6607	0.1349	0.1658	0.1719	0.1912
IBM	22	0.0710	0.7469	0.1189	0.1405	0.1522	0.1672
GE	22	0.1433	0.3529	0.0928	0.2152	0.2269	0.2458
PG	22	0.0117	0.9400	0.0596	0.1057	0.1151	0.1236
MMM	22	0.0442	0.9308	0.0901	0.1795	0.1925	0.2111
S&P500	22	0.0087	1.0000	0.0502	0.1182	0.1258	0.1456

Table 8: The Table shows the OLS estimation results of the HAR-RV model augmented with the terms for the leverage effects (25) and results of testing the hypothesis $H_0 : \gamma_- \leq \gamma_+$. The 1-min realized variance RV_t^{1min} is used in the OLS fittings. In the parenthesis under the estimated coefficients are t-statistics obtained with Newey-West standard errors with 18 lag periods. The cases considered here are S&P500 index and five stocks from Dow Jones Industrial Averages: Microsoft (MSFT), International Business Machines Corporation (IBM), General Electric (GE), Procter & Gamble (PG) and 3M (MMM). The sample period is from Jan-04-1993 to Dec-31-2009 (4,283 trading days).

								$H_0 : \gamma_- \leq \gamma_+$		
		α_D	β_{RD}	β_{RW}	β_{RM}	γ_+	γ_-	Adj R^2	t-statistic	p-value
MSFT	R_{t-1}^2	0.0000 (1.5109)	0.3080 (3.4832)	0.4009 (3.4076)	0.2520 (4.8492)	0.0054 (1.0721)	0.0463 (3.1394)	0.7996	2.88	0.0020
	$ R_{t-1} $	-0.0000 (-2.1753)	0.2982 (3.3179)	0.4043 (3.4144)	0.2530 (4.8945)	0.0007 (1.9027)	0.0040 (5.5406)	0.8017	5.38	0.0000
IBM	R_{t-1}^2	0.0000 (3.1423)	0.1908 (1.2951)	0.5788 (2.7290)	0.1399 (2.1214)	-0.0131 (-1.4009)	0.0361 (2.9700)	0.5970	3.00	0.0014
	$ R_{t-1} $	0.0000 (1.8065)	0.1953 (1.4204)	0.5534 (2.9082)	0.1459 (2.4100)	0.0000 (-0.0355)	0.0029 (3.9841)	0.5974	3.67	0.0001
GE	R_{t-1}^2	0.0000 (2.7765)	0.2581 (3.7433)	0.4724 (6.0651)	0.1006 (1.5677)	-0.0140 (-0.7284)	0.2016 (3.4924)	0.6300	3.42	0.0003
	$ R_{t-1} $	-0.0000 (-0.7339)	0.2739 (3.6260)	0.4597 (5.8477)	0.0999 (1.6040)	0.0011 (1.0789)	0.0115 (3.8243)	0.6175	3.44	0.0003
PG	R_{t-1}^2	0.0000 (3.5922)	0.2021 (1.9651)	0.2902 (2.7418)	0.3967 (4.4417)	-0.0123 (-1.1327)	0.0176 (0.8961)	0.4187	1.91	0.0282
	$ R_{t-1} $	0.0000 (0.5923)	0.1732 (1.7321)	0.2616 (2.3486)	0.3915 (4.6627)	0.0014 (1.1906)	0.0065 (1.9860)	0.4433	2.33	0.0101
MMM	R_{t-1}^2	0.0000 (2.5989)	0.1103 (0.8507)	0.5569 (3.8863)	0.1881 (2.8876)	0.0074 (0.7038)	0.1306 (1.6649)	0.5429	1.66	0.0490
	$ R_{t-1} $	0.0000 (0.0432)	0.1142 (0.8813)	0.5667 (3.5723)	0.1707 (2.4630)	0.0012 (1.6301)	0.0056 (2.0489)	0.5360	2.02	0.0215
S&P500	R_{t-1}^2	0.0000 (1.1027)	0.1014 (0.8112)	0.5021 (3.3468)	0.1377 (2.6276)	0.0199 (1.2872)	0.1952 (2.8746)	0.6496	2.75	0.0030
	$ R_{t-1} $	-0.0000 (-1.6314)	0.0935 (0.6226)	0.5880 (3.1857)	0.1207 (1.7781)	0.0011 (1.6719)	0.0063 (2.3395)	0.5995	2.37	0.0089

Table 9: The Table shows the OLS estimation results of the HAR-RV-CJ model augmented with the terms for the leverage effects (26) and results of testing the hypothesis $H_0 : \gamma_- \leq \gamma_+$. The 1-min realized variance RV_t^{1min} is used in the OLS fittings. In the parenthesis under the estimated coefficients are t-statistics obtained with Newey-West standard errors with 18 lag periods. The cases considered here are S&P500 index and five stocks from Dow Jones Industrial Averages: Microsoft (MSFT), International Business Machines Corporation (IBM), General Electric (GE), Procter & Gamble (PG) and 3M (MMM). The sample period is from Jan-04-1993 to Dec-31-2009 (4,283 trading days).

		α_D	β_{BD}	β_{BW}	β_{BM}	β_{JD}	β_{JW}	β_{JM}	γ_+	γ_-	Adj R^2	t -statistic	p-value
		$H_0 : \gamma_- \leq \gamma_+$											
MSFT	R_{t-1}^2	0.0000 (2.1119)	0.2999 (2.8715)	0.4329 (3.5586)	0.2525 (4.2595)	-0.0313 (-0.3367)	0.3389 (2.0899)	0.9365 (4.7025)	0.0055 (1.0799)	0.0475 (3.0443)	0.7950	2.86	0.0021
	$ R_{t-1} $	0.0000 (0.0322)	0.2883 (2.7402)	0.4362 (3.5820)	0.2451 (4.1166)	-0.0287 (-0.3086)	0.3450 (2.1076)	0.9789 (4.8550)	0.0009 (1.8815)	0.0042 (5.1591)	0.7974	5.29	0.0000
IBM	R_{t-1}^2	0.0000 (3.9448)	0.1845 (1.1346)	0.5967 (2.6655)	0.1408 (2.1168)	0.0011 (0.0159)	0.2239 (0.5862)	0.7517 (2.0366)	-0.0106 (-1.1125)	0.0387 (3.0450)	0.5869	2.94	0.0016
	$ R_{t-1} $	0.0000 (2.5105)	0.1875 (1.2241)	0.5739 (2.8251)	0.1412 (2.2809)	0.0123 (0.1824)	0.1320 (0.2948)	0.8368 (2.0337)	0.0003 (0.5110)	0.0032 (4.1115)	0.5881	3.65	0.0001
GE	R_{t-1}^2	0.0000 (3.6073)	0.2816 (4.3197)	0.4995 (6.1839)	0.0842 (1.5749)	0.0133 (0.0515)	0.4664 (1.1606)	0.5403 (1.8748)	-0.0107 (-0.5542)	0.2038 (3.4998)	0.6373	3.37	0.0004
	$ R_{t-1} $	-0.0000 (-1.4288)	0.2981 (4.3399)	0.4864 (5.8685)	0.0805 (1.5416)	-0.0098 (-0.0373)	0.3522 (0.8862)	0.7051 (2.4516)	0.0016 (1.6488)	0.0119 (3.9520)	0.6256	3.43	0.0003
PG	R_{t-1}^2	0.0000 (3.8307)	0.1727 (1.2404)	0.3839 (6.7152)	0.3571 (2.6765)	0.1209 (3.6550)	-0.0834 (-0.3955)	0.8698 (4.1265)	-0.0090 (-0.7830)	0.0197 (0.9524)	0.4118	1.74	0.0412
	$ R_{t-1} $	0.0000 (0.6036)	0.1349 (1.0996)	0.3530 (5.8374)	0.3350 (2.9011)	0.1376 (4.2783)	-0.1197 (-0.5279)	0.8877 (4.1579)	0.0018 (1.4862)	0.0070 (2.0842)	0.4394	2.32	0.0101
MMM	R_{t-1}^2	0.0000 (3.9076)	0.1089 (0.7835)	0.6092 (4.0426)	0.1477 (2.2613)	0.0130 (0.0624)	-0.0012 (-0.0069)	0.8853 (3.0911)	0.0094 (0.9267)	0.1376 (1.7609)	0.5459	1.72	0.0430
	$ R_{t-1} $	0.0000 (0.0660)	0.1016 (0.7176)	0.6302 (3.7460)	0.1309 (1.8819)	0.1073 (0.7681)	-0.0501 (-0.2799)	0.8321 (3.5467)	0.0014 (1.9428)	0.0059 (2.1791)	0.5386	2.07	0.0191
S&P500	R_{t-1}^2	0.0000 (1.1751)	0.1096 (0.9227)	0.4774 (3.3274)	0.1542 (2.9598)	0.0212 (0.0974)	-0.4703 (-0.6704)	1.4289 (1.1810)	0.0220 (1.4051)	0.1972 (2.8945)	0.6479	2.78	0.0027
	$ R_{t-1} $	-0.0000 (-1.5487)	0.0978 (0.7030)	0.5675 (3.3248)	0.1374 (2.0635)	0.0702 (0.3511)	-0.5867 (-0.7494)	1.5699 (1.1615)	0.0012 (1.7194)	0.0064 (2.3504)	0.5971	2.39	0.0084

Table 10: The table shows minimum, mean and maximum values of daily standardized quadratic co-variations between intraday log price X_t and function of spot variance $F(\sigma_t^2)$ obtained by using the method in Wang and Mykland (2013), and number of significant days when comparing the daily standardized quadratic co-variations with different critical values. The standardized quadratic co-variations are defined as L_1 and L_2 in (29) and (30) with $\langle X, F(\sigma^2) \rangle_T = 0$. We use 1-min equally spaced data, and set $T =$ one day and $M_n = 30$ for the estimations. We assume $F(x) = x$ and $F(x) = 1/2 \log(x)$. The cases considered here are S&P500 index and five stocks from Dow Jones Industrial Averages: Microsoft (MSFT), International Business Machines Corporation (IBM), General Electric (GE), Procter & Gamble (PG) and 3M (MMM). The sample period is from Jan-04-1993 to Dec-31-2009 (4,283 trading days).

MSFT					IBM			
	$L_1 - x$	$L_1 - \frac{1}{2} \log(x)$	$L_2 - x$	$L_2 - \frac{1}{2} \log(x)$	$L_1 - x$	$L_1 - \frac{1}{2} \log(x)$	$L_2 - x$	$L_2 - \frac{1}{2} \log(x)$
Min.	-2.807	-2.561	-2.906	-2.520	-2.421	-2.691	-2.509	-2.709
Mean	-0.059	-0.065	-0.058	-0.065	-0.014	-0.017	-0.015	-0.019
Max.	2.224	2.498	2.429	2.716	2.584	2.728	2.763	2.728
$ \cdot \geq 1.96$	29	46	32	47	39	51	40	54
$\cdot \leq -1.64$	85	102	88	98	66	94	70	99
$ \cdot \geq 2.58$	2	0	1	1	1	2	1	3
$\cdot \leq -2.33$	2	3	3	5	1	3	1	3
GE					PG			
	$L_1 - x$	$L_1 - \frac{1}{2} \log(x)$	$L_2 - x$	$L_2 - \frac{1}{2} \log(x)$	$L_1 - x$	$L_1 - \frac{1}{2} \log(x)$	$L_2 - x$	$L_2 - \frac{1}{2} \log(x)$
Min.	-2.322	-2.613	-2.51	-2.601	-2.416	-2.354	-2.426	-2.403
Mean	-0.038	-0.039	-0.042	-0.042	-0.037	-0.038	-0.041	-0.043
Max.	2.815	2.765	2.790	2.851	2.263	2.475	2.260	2.501
$ \cdot \geq 1.96$	42	63	42	70	18	35	20	37
$\cdot \leq -1.64$	83	108	86	109	60	95	66	101
$ \cdot \geq 2.58$	1	2	1	2	0	0	0	0
$\cdot \leq -2.33$	0	1	1	2	1	2	1	2
MMM					S&P500			
	$L_1 - x$	$L_1 - \frac{1}{2} \log(x)$	$L_2 - x$	$L_2 - \frac{1}{2} \log(x)$	$L_1 - x$	$L_1 - \frac{1}{2} \log(x)$	$L_2 - x$	$L_2 - \frac{1}{2} \log(x)$
Min.	-2.362	-2.638	-2.383	-2.646	-2.605	-2.735	-2.620	-2.987
Mean	0.001	0.003	0.000	0.003	-0.191	-0.186	-0.186	-0.186
Max.	2.496	2.471	2.504	2.434	2.850	2.785	2.781	2.969
$ \cdot \geq 1.96$	43	64	50	71	148	244	157	257
$\cdot \leq -1.64$	88	99	87	101	308	379	307	392
$ \cdot \geq 2.58$	0	1	0	1	2	8	4	13
$\cdot \leq -2.33$	1	6	1	6	15	39	19	42

Table 11: The table shows estimated $f = \lim_{l \rightarrow 0} \text{Corr}(v_{t+l} - v_t, X_{t+l} - X_t)$ by using the data driven method in Aït-Sahalia, Fan and Li (2013). Here $X_t := \log P_t$ and $v_t := \sigma_t^2$. The estimates are based on sample correlation and bias corrected sample correlation (denoted by \hat{f}_k and \hat{f}_k^{bc}) between daily returns R_t and the difference of the realized variances RV_t^{1min} . We also report the upper and lower bounds \bar{k}^* and \underline{k}^* for $\left\{k, \hat{f}_k\right\}_{k=\underline{k}^*}^{\bar{k}^*}$ used for estimating the regression, and the (unadjusted) R^2 of the regression. The cases considered here are S&P500 index and five stocks from Dow Jones Industrial Averages: Microsoft (MSFT), International Business Machines Corporation (IBM), General Electric (GE), Procter & Gamble (PG) and 3M (MMM). The sample period is from Jan-04-1993 to Dec-31-2009 (4,283 trading days).

Uncorrected \hat{f}_k						
	IBM	MSFT	GE	PG	MMM	S&P500
f	-0.24	-0.31	-0.48	-0.32	-0.28	-0.36
\bar{k}^*	15	21	44	29	17	17
\underline{k}^*	55	53	87	91	54	94
R^2	0.97	0.92	0.96	0.98	0.94	0.89
Bias Corrected \hat{f}_k^{bc}						
	IBM	MSFT	GE	PG	MMM	S&P500
f	-0.25	-0.32	-0.48	-0.34	-0.30	-0.37
\bar{k}^*	15	20	31	29	17	15
\underline{k}^*	55	53	88	88	54	93
R^2	0.97	0.93	0.97	0.98	0.95	0.91

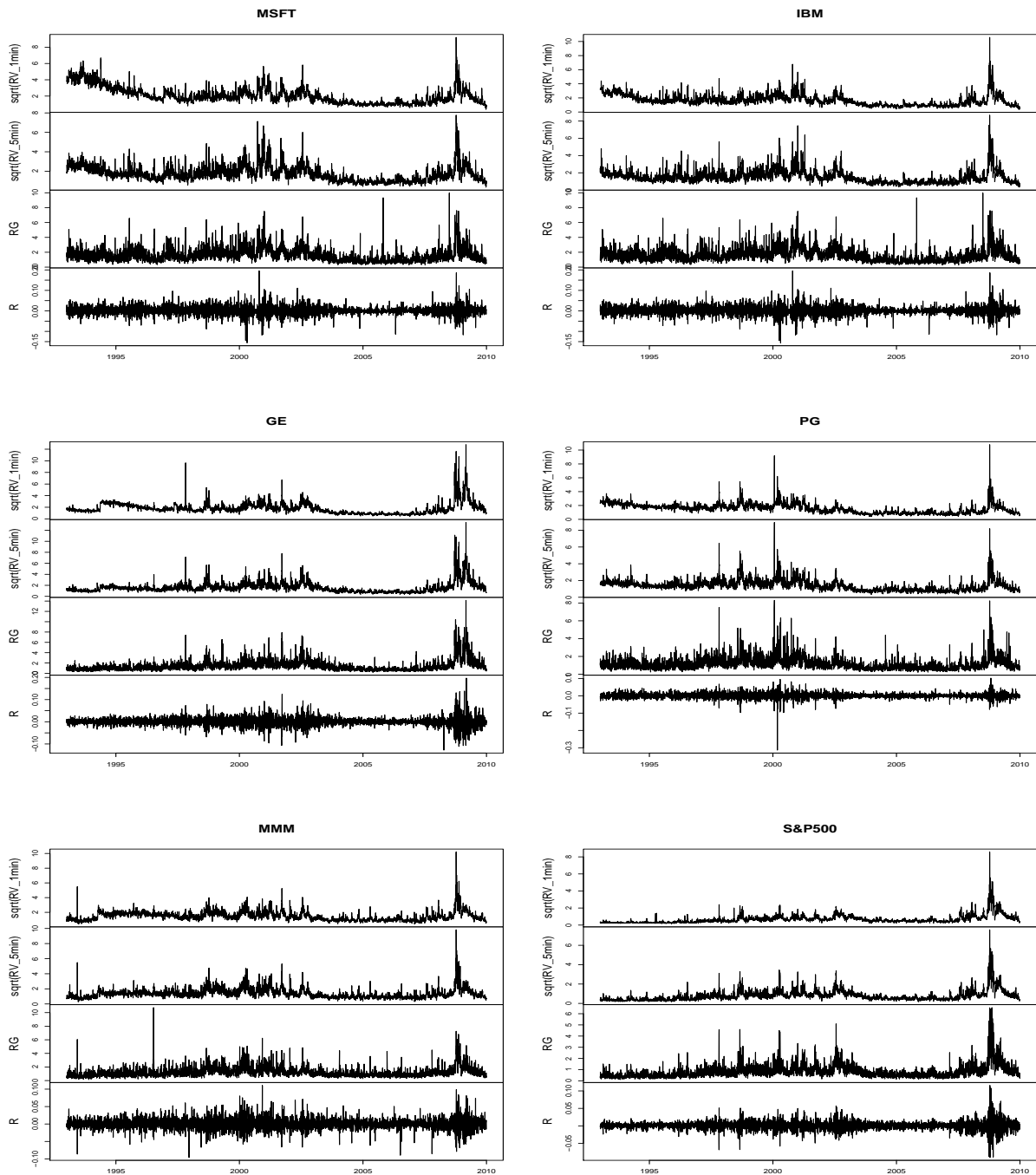


Figure 1: Time series plots of daily $\sqrt{RV_t^{1min}}$, $\sqrt{RV_t^{5min}}$, RG_t , and R_t for S&P500 index and five stocks from Dow Jones Industrial Averages: Microsoft (MSFT), International Business Machines Corporation (IBM), General Electric (GE), Procter & Gamble (PG) and 3M (MMM). The quantities of $\sqrt{RV_t^{1min}}$, $\sqrt{RV_t^{5min}}$, and RG_t shown here are scaled by 100. The sample period is from Jan-04-1993 to Dec-31-2009 (4,283 trading days).

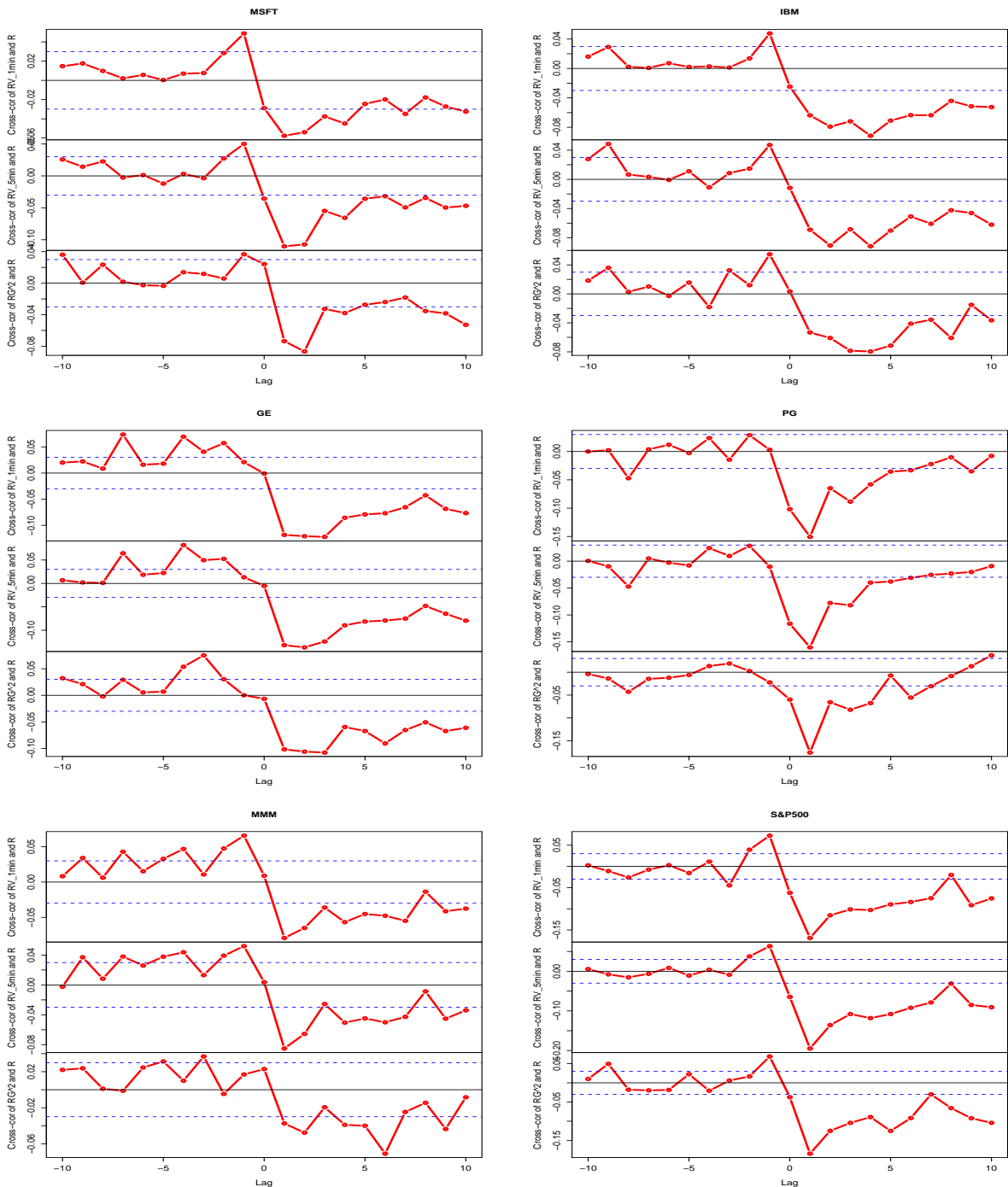


Figure 2: Cross correlations of daily RV_t^{1min} , RV_t^{5min} , RG_t^2 , and R_{t-j} for S&P500 index and five stocks from Dow Jones Industrial Averages: Microsoft (MSFT), International Business Machines Corporation (IBM), General Electric (GE), Procter & Gamble (PG) and 3M (MMM). We set lag length $j = -10, \dots, 10$, and the sample period for the calculations is from Jan-04-1993 to Dec-31-2009 (4,283 trading days).

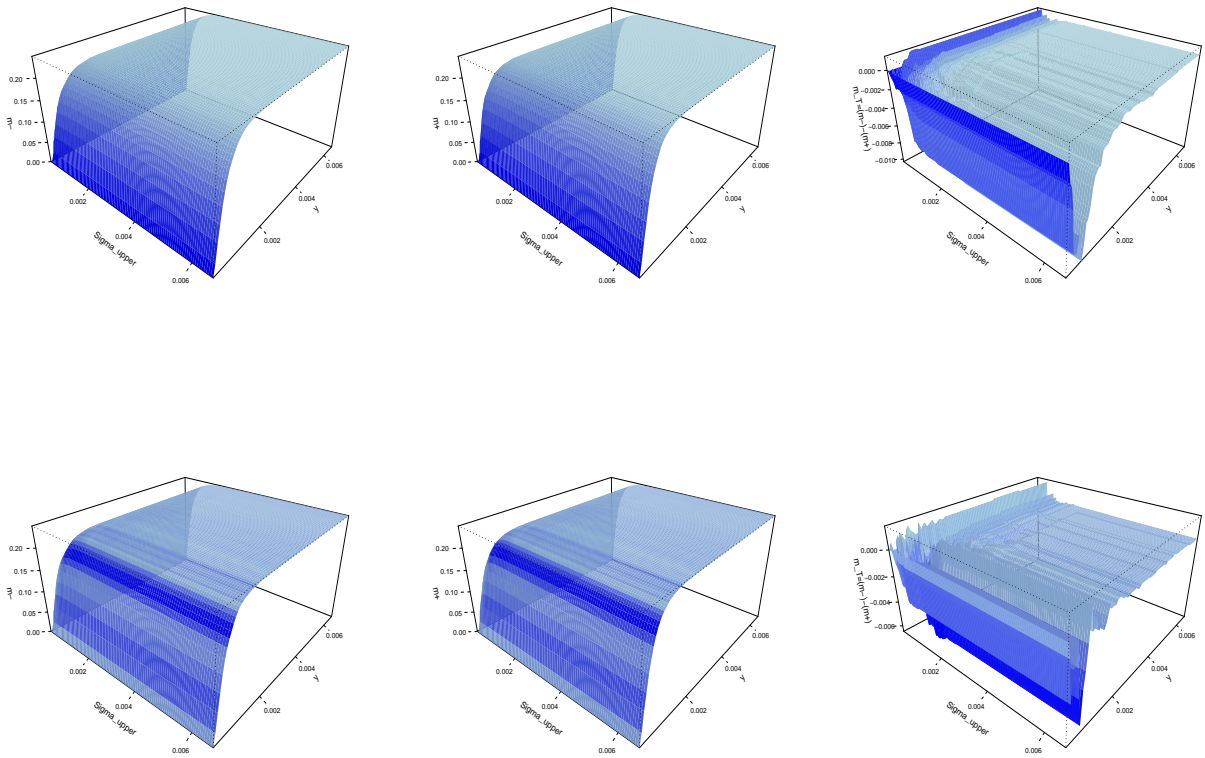


Figure 3: Plots of surfaces $\hat{m}_T(y, g, \hat{\pi}^+)$ (left), $\hat{\tau}_T^1(y, g, \hat{\pi}^+)$ (middle) and $\hat{\tau}_T^2(y, g, \hat{\pi}^-)$ (left) of MSFT. Note that $\hat{m}_T(y, g, \hat{\pi}^+) = \hat{\tau}_T^1(y, g, \hat{\pi}^+) - \hat{\tau}_T^2(y, g, \hat{\pi}^-)$. Upper: $j = 1$. Bottom: $j = 5$. Here RV_t^{5min} is the estimated daily volatility.

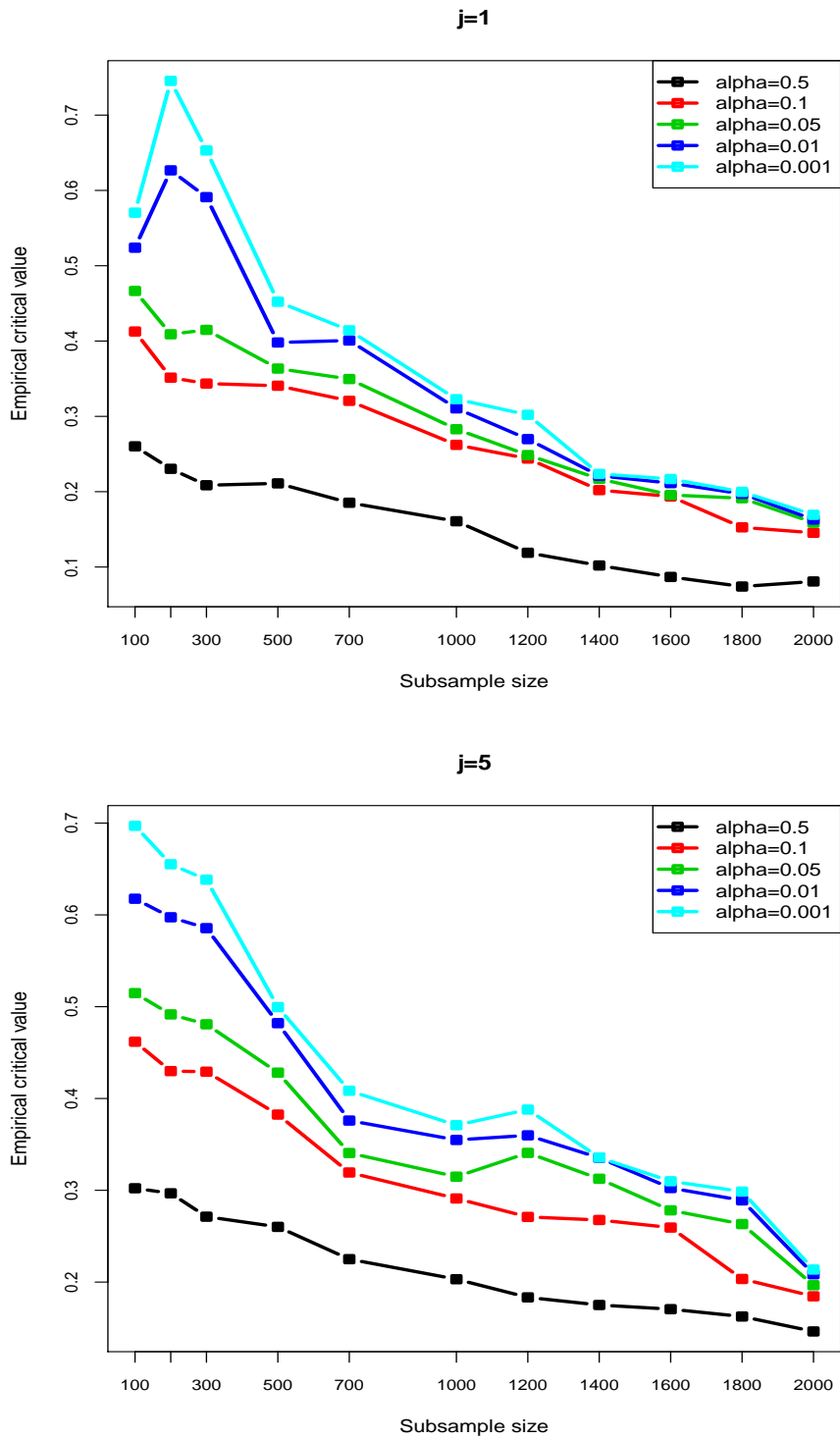


Figure 4: The subsample critical values of MSFT. Upper: $j = 1$. Bottom: $j = 5$. Here RV_t^{5min} is the estimated daily volatility.

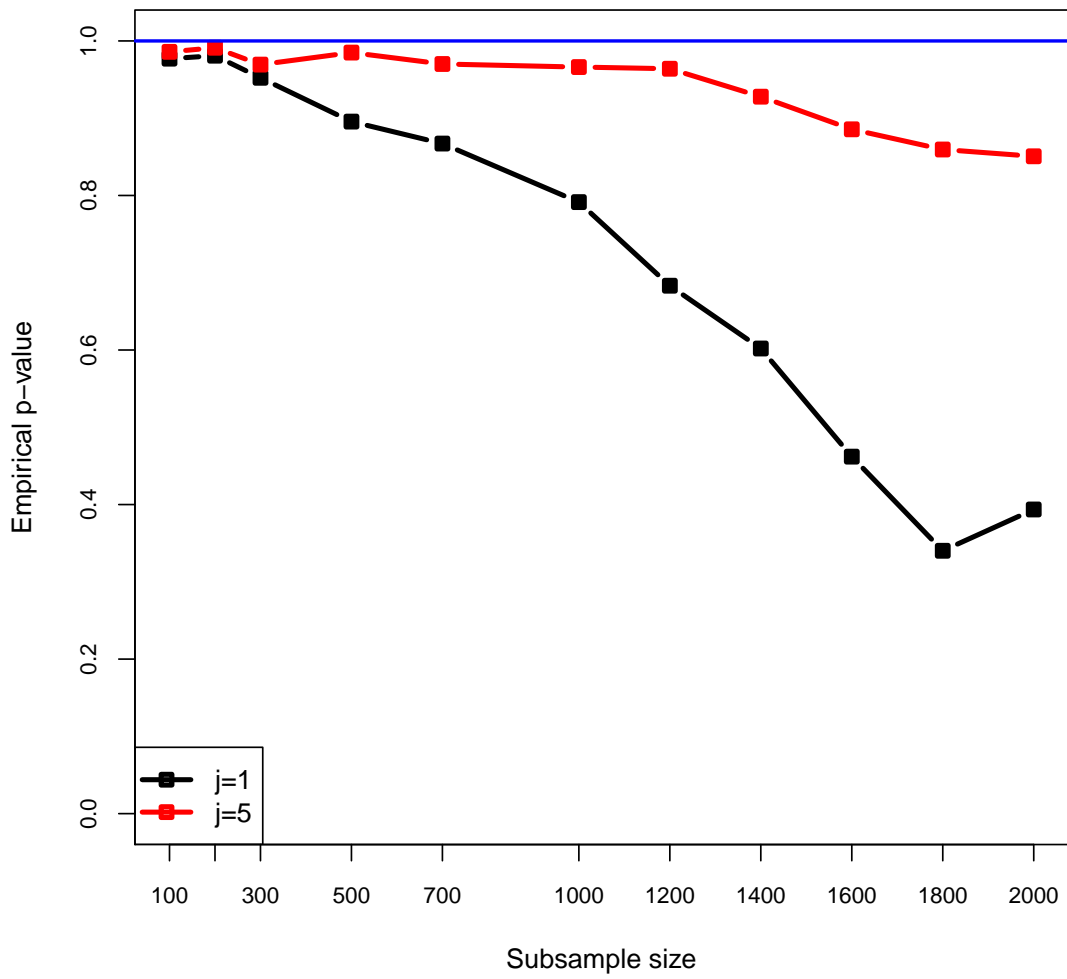


Figure 5: The subsample empirical p-values for \hat{S}_{T_j} of MSFT. Here RV_t^{5min} is the estimated daily volatility.

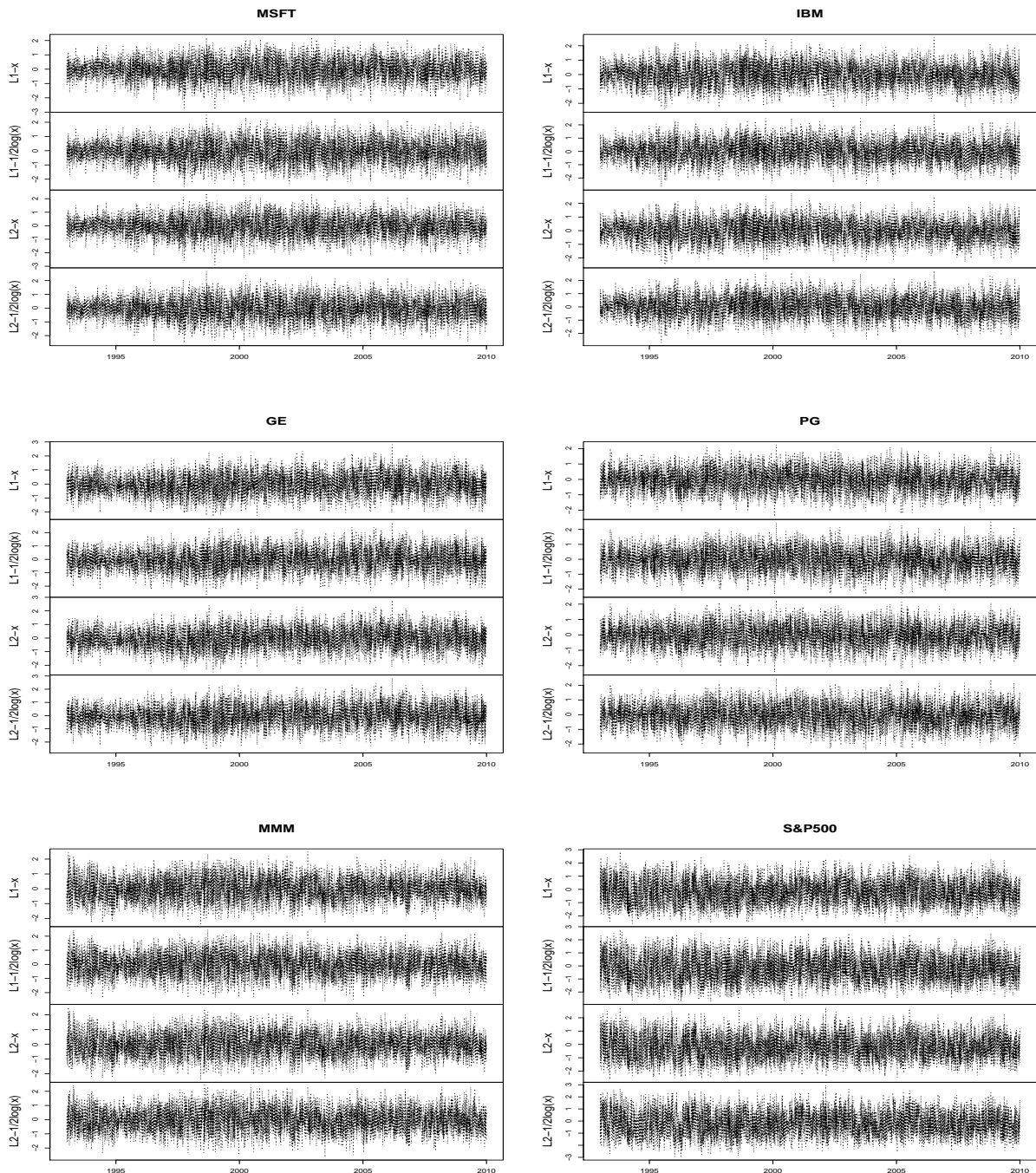


Figure 6: Time series plots of daily standardized quadratic co-variations between intraday log return X_t and function of spot variance $F(\sigma_t^2)$ obtained by using the method in Wang and Mykland (2013). The standardized quadratic co-variations are defined as L_1 and L_2 in (29) and (30) with $\langle X, F(\sigma^2) \rangle_T = 0$. We use 1-min equally spaced data, and set $T =$ one day and $M_n = 30$ for the estimations. We assume $F(x) = x$ and $F(x) = 1/2 \log(x)$. The cases considered here are S&P500 index and five stocks from Dow Jones Industrial Averages: Microsoft (MSFT), International Business Machines Corporation (IBM), General Electric (GE), Procter & Gamble (PG) and 3M (MMM). The sample period is from Jan-04-1993 to Dec-31-2009 (4,283 trading days).

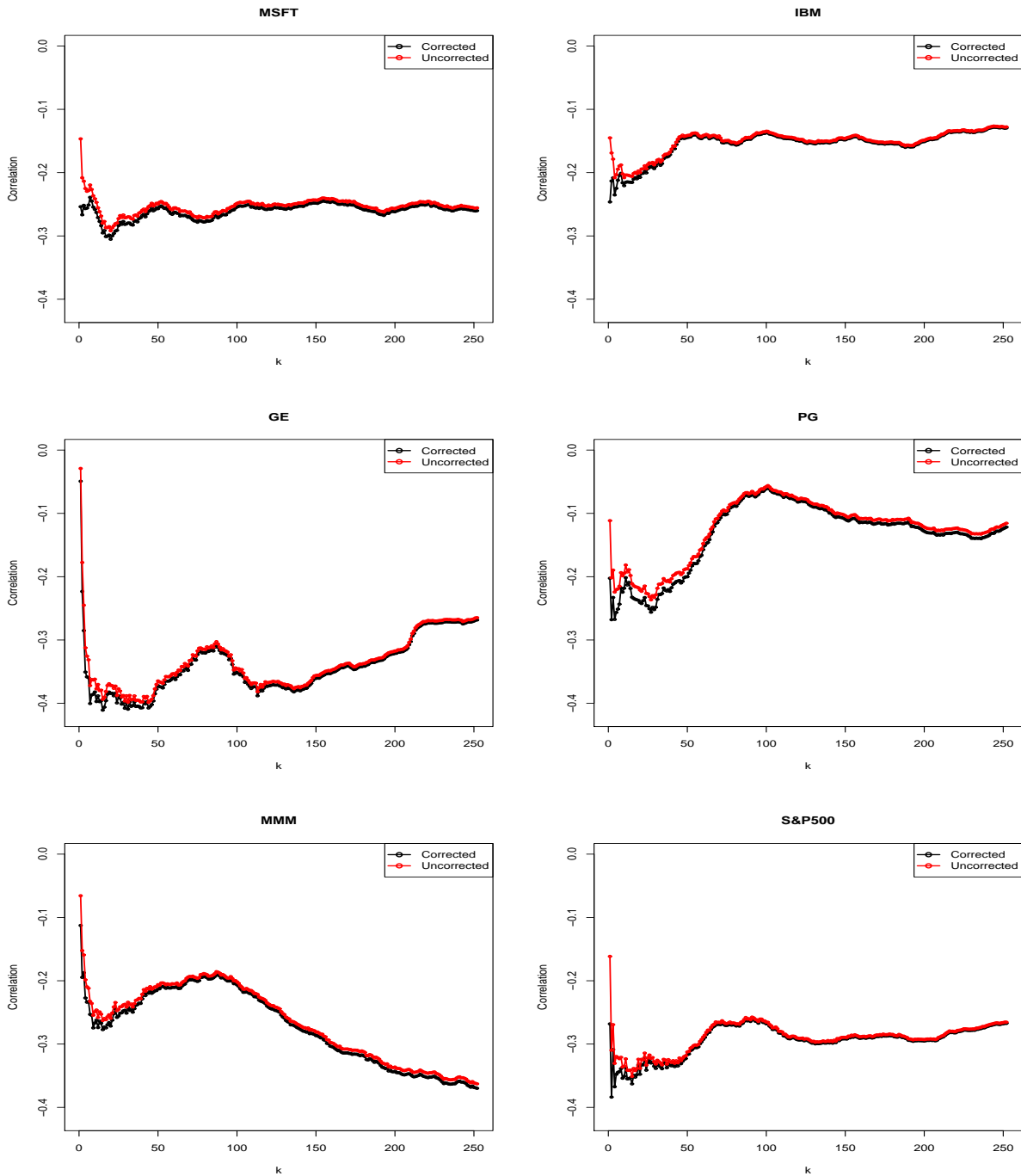


Figure 7: Plots of sample correlation between returns and difference of the daily realized variances, \hat{f}_k and bias corrected sample correlation \hat{f}_k^{bc} in (32) against k . The bias corrected sample correlation \hat{f}_k^{bc} is obtained by using method in Ait-Sahalia, Fan and Li (2013). We replace γ in (32) with $\hat{\gamma}$ in (33) which is estimated with mean of the realized quartilities and sample variance of the realized variances. We use 1-min equally spaced log returns to estimate the daily realized variances and quartilities. The cases considered here are S&P500 index and five stocks from Dow Jones Industrial Averages: Microsoft (MSFT), International Business Machines Corporation (IBM), General Electric (GE), Procter & Gamble (PG) and 3M (MMM). The sample period is from Jan-04-1993 to Dec-31-2009 (4,283 trading days).

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