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Assortative matching and search with labour supply and home production

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cemmap working paper CWP07/13

Assortative Matching and Search

with Labor Supply and Home Production*

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Abstract

We extend the search-matching model of the marriage market of Shimer and Smith (2000) to allow for labor supply and home production. We characterize the steady-state equilibrium when exogenous divorce is the only source of risk. We study nonparametric identification using cross-section data on wages and hours worked, and we develop a nonparametric estimator. The estimated matching probabilities that can be derived from the steady-state flow conditions are strongly increasing in male and female wages. We estimate the expected share of marriage surplus appropriated by each spouse as a function of wages. The model allows to infer the specialization of female spouses in home production from observations on wages and hours worked.

Keywords: Search-matching, sorting, assortative matching, collective labor supply, structural estimation.

JEL classification: C78, D83, J12, J22.

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1 Introduction

One of the key issues in understanding how tax policies affect labor supply is the intra-household allocation of time and consumption. This is in particular the case of welfare benefits, such as the Working Family Tax Credit program in the U.K. and the Earned Income Tax Credit in the U.S., aimed at providing work incentives and a safety net against poverty at the same time. The models used to address these issues typically take the household as a unit with unitary preferences (e.g. Eissa and Hoynes, 2004); and while the collective models of the family (Chiappori 1988, 1992¹) offer a solution for improvement by modeling intrahousehold resource allocation, the interest of this framework for policy evaluation is hampered by its inability to predict the impact of welfare policies on the sharing rule.² The evaluation of welfare policies for the family thus ultimately requires an equilibrium model of match formation and intra-household resource allocation. The present paper offers an attempt at constructing and estimating such a model.

A very popular approach uses Nash bargaining³ within a search-matching equilibrium framework in order to endogenize the threat points. They aim to analyze and explain a number of stylized socio-demographic trends such as declining marriage rates or increasing female college graduation rates, and various policies affecting the family.⁴ This literature focusses on long term trends and individual heterogeneity is usually minimal in these models. In parallel, a very important theoretical effort was made to understand the conditions for existence and unicity of the equilibrium, and the conditions for sorting

¹See Chiappori and Donni (2009) for a recent survey of non-unitary models of the household. Chiappori's seminal contributions generated a long list of papers building on the model of the family as a Pareto equilibrium. We can only cite a few of them: Browning, Bourguignon, Chiappori, and Lechene (1994), Fortin and Lacroix (1997), Browning and Chiappori (1998), Chiappori, Fortin, and Lacroix (2002), Mazzocco (2004, 2007), Blundell, Chiappori, Magnac, and Meghir (2007), etc. Note also that the assumption of efficient allocations within the family has been disputed, in particular, by Del Boca and Flinn (2005, 2006, 2009).

²Yet, the factors influencing the sharing rule, such as sex ratios or rules about divorce, are now well understood, and they can be influenced by policy. See Grossbard-Shechtman (1984), Brien (1997), Lundberg, Pollak, and Wales (1997), Chiappori, Fortin, and Lacroix (2002), Del Boca and Flinn (2005), Amuedo-Dorantes and Grossbard (2007), Seitz (2009).

³Nash bargaining was used to model intra-household resource allocation before Chiappori's collective framework outmoded this approach. See Manser and Brown, 1980, McElroy and Horney, 1981, Lundberg and Pollak, 1993, 1996).

⁴See e.g. Aiyagari, Greenwood, and Guner (2000), Greenwood, Guner, Knowles, Greenwood, Guner, and Knowles (2000), Caucutt, Guner, and Knowles (2002), Gould and Paserman (2003), Fernandez, Guner, and Knowles (2005), Chiappori and Weiss (2006, 2007), Chiappori and Oreffice (2008).

in a search-matching model with two-sided heterogeneity.⁵ Up to now, there has been very little empirical work using this approach. We are only aware of Wong (2003), who estimates the Shimer-Smith model assuming that individuals differ in a single index of observables.

Two-sided matching has initially been conceived in a perfect information framework (see Roth and Sotomayor, 1992, for a survey). This approach was very successfully applied to designing practical matching mechanisms such as the National Resident Matching Program. However, it is only recently that, long after Becker’s seminal work (Becker, 1973, 1974, 1981), the perfect-information assignment framework of Shapley and Shubik (1971) was used to address empirical matching issues.⁶ These papers feature essentially static matching models with discrete individual types.

This paper builds on Shimer and Smith (2000) and designs a search-matching model of the marriage market with labor supply. There are many reasons for engineering a search model of the marriage market. First, casual experience seems to suggest that it takes time and trial to find the right partner, and that mismatch is not a rare event. Second, forward looking behavior and risk are natural ingredients of search-matching models. In our model as in collective models, spouses’ labor supply are chosen efficiently along the Pareto frontier of the achievable set. The outside option is the value of remaining single, which is equal to the instantaneous utility of the wage plus the option value of an eventual future marriage. Couples are formed if an excess of public good is produced in the association. The resulting surplus is split between spouses by Nash bargaining. As a result, the model generalizes both the collective labor supply literature, to which we add an explicit mechanism behind spouses bargaining power, and marriage market models, to which we provide a structural foundation for the surplus function driving matching decisions.

Despite a rather complicated structure the model remains tractable thanks to the

⁵See e.g. Sattinger (1995), Lu and McAfee (1996), Burdett and Coles (1997), Shimer and Smith (2000), Sattinger (2003), Eeckhout (1999), Eeckhout and Kircher (2010b,a).

⁶See Choo and Siow (2006), Choo, Seitz, and Siow (2008b,a), Chiappori, Iyigun, and Weiss (2008), Siow (2009), Chiappori, Iyigun, and Weiss (2009), Chiappori, Salanie, and Weiss (2010), Galichon and Salanié (2010), Salanié and Galichon (2011), Chiappori and Oreffice (2008), Chiappori, Oreffice, and Quintana-Domeque (2010a,b).

steady state assumption. Although an important application of the matching framework is understanding long-term demographic changes such as increasing divorce and remarriage rates, we assume that these changes are slow enough for a steady state to hold at least approximately at all times. In other words, flows can vary over time in a trended or cyclical way, but net flows must remain small compared to gross flows. We show that steady-state flow conditions deliver important identifying restrictions on matching probabilities, and indirectly on the relationship between intrahousehold transfers and wages.

Under the steady-state restriction, a lot of information can be drawn from cross-section data. The matching probability is thus linked to cross-section wage distributions in a simple way. Despite a small correlation between spouses' wages (30% using SIPP data) we find that the matching probability increases in both wages at a very fast rate. Its shape is rather flat for most wages but very steep when both wages cross the median. We take this result as a strong indication of positive assortative matching, albeit unconventional. We also observe that married men work more than single men and single women, who work more than married women. The model is able to explain these facts without naively assuming that men have a preference for work and women a preference for leisure. We decompose the time that is not spent working for a wage into leisure and home-production time uses. Then, we show how a comparative advantage of women for home-production in marriage betrays itself in recognizable patterns of wage distributions, and explains married women's specialization in household production.

The labor supply literature has been faced with the problem of spurious correlations between hours worked and wages. The traditional approach is to look for instruments. We are not claiming here that we can solve the problem without instruments. In fact, we will see that even without any correlation between preference for leisure and ability the identification of income effects rests on one non-identified parameter, the variance of match-specific shocks. Yet we believe that this framework is useful in that it shows how one can relate empirical matching regularities to unobservable time uses that play an important role in the measure of individual welfare.

The layout of the paper is as follows. First we construct the model. Second we study identification. Third we estimate the model non parametrically. The last section concludes. An appendix details the numerical techniques used to perform the nonparametric analyses.

2 The model

The model builds on Shimer and Smith (2000), which we extend to allow for labor supply decisions and home production. So-doing, we considerably enrich the nature of the resource allocation process at work inside the household. Moreover, in Shimer-Smith, couples are formed because of complementarities in the match production function. In our model, agents egotistically derive utility from their own consumption and leisure. However, the consumption good can be either purchased or produced at home. Complementarities show up in household production, which is the externality that generates surplus and explains marriage.

2.1 Individual types

We consider a marriage market with L_m males and L_f females. The number of married couples is denoted by N and the respective numbers of single males and single females are $U_m = L_m - N$ and $U_f = L_f - N$.

Individuals differ in ability, $x \in [\underline{x}, \bar{x}]$ for males and $y \in [\underline{y}, \bar{y}]$ for females, later assumed to be equal to labor market wages. In this paper, individual ability is the only permanent source of individual heterogeneity, as in Becker's seminal work on marriage.⁷ Let $e_m(x)$ and $e_f(y)$ denote the density functions of the (continuous) measures of males of type x and females of type y , with $L_m = \int e_m(x) dx$ and $L_f = \int e_f(y) dy$. The corresponding densities for the sub-populations of singles are $u_m(x)$ and $u_f(y)$, with $U_m = \int u_m(x) dx$ and $U_f = \int u_f(y) dy$. The density of couples of type (x, y) is $n(x, y)$,

⁷This is clearly the first dimension of heterogeneity to consider as one is interested in modeling family labor supply. However, we shall soon introduce a match-specific heterogeneity component.

with $N = \iint n(x, y) \, dx \, dy$ and

$$e_m(x) = \int n(x, y) \, dy + u_m(x), \quad e_f(y) = \int n(x, y) \, dx + u_f(y). \quad (1)$$

2.2 Equilibrium flows

We assume that only singles search for a partner, ruling out meetings with alternative partners during marriage. The number of meetings per period is a function of the numbers of male and female singles, $M(U_m, U_f)$, and $\lambda_i = \frac{M(U_m, U_f)}{U_i}$ is the instantaneous probability that a searching individual of gender $i = m, f$ meets with a single of the other sex. We also denote $\lambda = \frac{M(U_m, U_f)}{U_m U_f}$.

All datings do not end up in wedlock. We assume that there exists a function $\alpha(x, y) \in [0, 1]$ equal to the probability that a match (x, y) be consummated. This matching probability is an equilibrium outcome that will later be derived from fundamentals. The matching set is the support of α . Matches are exogenously dissolved with instantaneous probability δ .

In steady state, flows in and out of the stocks of married couples of each type must exactly balance each other out. This means that, for all (x, y) ,

$$\delta n(x, y) = u_m(x) \lambda_m \frac{u_f(y)}{U_f} \alpha(x, y) = \lambda u_m(x) u_f(y) \alpha(x, y). \quad (2)$$

The left-hand side is the flow of divorces. The right-hand side is the flow of new (x, y) -marriages. It has three components: a single male of type x , out of the $u_m(x)$ ones, meets a single female with probability λ_m ; this woman is of type y with probability $u_f(y)/U_f$; the knot is tied with probability $\alpha(x, y)$.

Integrating equation (2) over y , we obtain the following steady-state flow condition for the stock of all married males of type x ,

$$\delta \int n(x, y) \, dy = \lambda u_m(x) \int u_f(y) \alpha(x, y) \, dy.$$

Using equation (1) to substitute $\int n(x, y) dy$ out of this equation yields

$$\delta [e_m(x) - u_m(x)] = \lambda u_m(x) \int u_f(y) \alpha(x, y) dy,$$

or, equivalently,

$$u_m(x) = \frac{\delta e_m(x)}{\delta + \lambda \int u_f(y) \alpha(x, y) dy}. \quad (3)$$

By symmetry, the equation defining the equilibrium distribution of wages in the population of single females is

$$u_f(y) = \frac{\delta e_f(y)}{\delta + \lambda \int u_m(x) \alpha(x, y) dx}. \quad (4)$$

2.3 Time use for singles

Individuals draw utility from consumption and leisure. Consider a woman (with similar definitions for males) with direct utility $V_f(c, \ell)$ for consumption of a unique good c and leisure ℓ . The consumption good can be either purchased or produced at home. Let k denote the time devoted to home production. Assuming that ability, labor productivity and wage are all equal, as in a competitive labor market, a female consumer seeks to solve the optimization problem: $\max_{c, \ell, h, k > 0} V_f(c, \ell)$, subject to the budget constraint $c = yh + H_f(k)$, for hours worked $h = T - k - \ell$, where T is total time endowment and $H_f(k)$ is the home production function for a single women. For simplicity, we only consider interior solutions, ruling out labor market non participation, and we assume that time is the only input for home production.

Let $k_f^0(y)$ denote the solution to the home production problem:

$$C_f(y) = \max_k H_f(k) - yk. \quad (5)$$

Then, optimal leisure is $\ell_f^0(y) \equiv \ell_f(y, yT + C_f(y))$ solving the problem

$$v_f(y, yT + C_f(y)) = \max_{\ell > 0} \{V_f(c, \ell) \mid c = y(T - \ell) + C_f(y)\}. \quad (6)$$

Linear utility. Specifically, we will assume the following functional form for the indirect utility function:

$$v_f(y, F) = \frac{R - A_f(y)}{B_f(y)}, \quad (7)$$

where R denotes total income, $B_f(y)$ is an aggregate price index, with $B_f(0) = 1$, and $A_f(y)$ is a minimum expenditure level necessary to attain positive utility, with $A_f(0) = 0$. Linearity with respect to total income will induce a simple rent sharing mechanism. But other specifications are possible.

The leisure demand function, $\ell_f(y, R)$, then follows from indirect utility by application of Roy's identity,

$$\ell_f(y, R) = -\frac{\partial_y}{\partial_R} v_f(y, R) = A'_f(y) + b'_f(y)[R - A_f(y)], \quad (8)$$

where ∂_y and ∂_R denote partial derivatives, a prime (such as in b'_f and A'_f) denotes a derivative, and $b_f(y) = \log B_f(y)$. A standard specification is $A_f(y) = ya_f$ and $B_f(y) = y^{b_f}$, yielding the linear expenditure system,

$$y\ell_f = ya_f + b_f(R - ya_f).$$

2.4 Time use for married individuals

Marriage allows individuals to benefit from economies of scale and task specialization. At the household level, let home production be $H(k_m, k_f) + z$, a function of the time spent in home production by both spouses, k_m, k_f . We also introduce a source of match heterogeneity, z , which is a match-specific shock that is drawn at the first meeting from a zero-mean distribution denoted G . It aims at capturing in a tractable way all other dimensions of mutual attractiveness but labor market productivity.⁸ Designing empirically tractable multidimensional matching models with random search is definitely a promising

⁸From a technical point of view, it also allows to smooth out the discontinuity at the boundary of the matching set.

area for further research.⁹

Let $k_m^1(x, y), k_f^1(x, y)$ denote optimal home production time uses for male and female partners, defined as the solution to the problem

$$C(x, y) = \max_{k_m, k_f} \{H(k_m, k_f) - xk_m - yk_f\}.$$

Determining leisure for each household member can be done if we know how home production output, $C(x, y) + z$, is shared between them. Let t_m, t_f denote a particular allocation, with

$$t_m + t_f = C(x, y) + z.$$

Then, leisure follows as $\ell_m^1(x, y, z) \equiv \ell_m(x, xT + t_m)$ and $\ell_f^1(x, y, z) \equiv \ell_f(y, yT + t_f)$, as in equation (8), assuming that individual preferences for consumption and leisure are independent of marriage status.

The next section shows how transfers t_m, t_f are agreed upon by both spouses.

2.5 Optimal rent sharing between spouses

Let $W_m^1(v, x)$ denote the present value of marriage for a married male of type x receiving a flow utility v and let $W_m^0(x)$ denote the value of singlehood (derived in the next subsection). The flow value of a marriage contract delivering v utils is

$$rW_m^1(v, x) = v + \delta [W_m^0(x) - W_m^1(v, x)],$$

where r is the discount rate and the second term of the right-hand side is the option value of divorce. We define the marriage surplus for males as

$$S_m(v, x) = W_m^1(v, x) - W_m^0(x) = \frac{v - rW_m^0(x)}{r + \delta}, \quad (9)$$

with a similar definition for females.

⁹Wong (2003) aggregates individual characteristics into one single index in a Shimer-Smith model with positive assortative mating. An obvious extension of our model would add to z a linear index of individual characteristics.

Spouses have to decide on a particular partition (t_m, t_f) of the home-produced good. Although transfers can be positive or negative, both should be positive in equilibrium, otherwise one individual is better off remaining single. We assume that spouses share resources cooperatively using Generalized Nash Bargaining, with bargaining coefficient β , whereby transfers t_m and t_f solve

$$\max_{t_m, t_f} S_m(v_m(x, xT + t_m), x)^\beta S_f(v_f(y, yT + t_f), y)^{1-\beta}$$

subject to the condition $t_m + t_f \leq C(x, y) + z$.

Linear utility. With linear indirect utility (equation (7)) the solution is trivially found to be

$$t_m(x, y, z) = s_m(x) + \beta[C(x, y) + z - s_m(x) - s_f(y)], \quad (10)$$

$$t_f(x, y, z) = s_f(y) + (1 - \beta)[C(x, y) + z - s_m(x) - s_f(y)], \quad (11)$$

where we denote

$$s_m(x) = B_m(x)rW_m^0(x) - xT + A_m(x), \quad s_f(y) = B_f(y)rW_f^0(y) - yT + A_f(y). \quad (12)$$

Functions s_m and s_f are the non labor income equivalents of annuities $rW_m^0(x), rW_f^0(y)$.¹⁰

Two dating bachelors decide to match if the total surplus is positive, i.e. $s(x, y) + z > 0$ with $s(x, y) = C(x, y) - s_m(x) - s_f(y)$. The matching probability then follows as

$$\alpha(x, y) \equiv \Pr\{s(x, y) + z > 0 | x, y\} = 1 - G[-s(x, y)]. \quad (13)$$

¹⁰Indeed, $rW_m^0(x) = \frac{xT + s_m(x) - A_m(x)}{B_m(x)} = v_m(x, xT + s_m(x))$.

2.6 The value of singlehood

The value of being single solves the option-value equation,

$$rW_m^0(x) = v_m[x, xT + C_m(x)] + \lambda_m \iint \max\{S_m[v_m[x, xT + t_m(x, y, z)], x], 0\} dG(z) \frac{u_f(y)}{U_f} dy,$$

where the second term of the right-hand side is the option value of meeting a female single.

Using the expression for marriage surplus (9), and substituting $s_m(x)$ for $rW_m^0(x)$ using (12), we obtain

$$s_m(x) = C_m(x) + \frac{\lambda\beta}{r+\delta} \iint \max\{C(x, y) + z - s_m(x) - s_f(y), 0\} dG(z) u_f(y) dy. \quad (14)$$

A similar expression can be derived for females,

$$s_f(y) = C_f(y) + \frac{\lambda(1-\beta)}{r+\delta} \iint \max\{C(x, y) + z - s_m(x) - s_f(y), 0\} dG(z) u_m(x) dx. \quad (15)$$

2.7 Equilibrium

An equilibrium is a fixed point (u_m, u_f, s_m, s_f) of the following system of equations, where the first two equations determine equilibrium wage distributions for singles (derived from (3) and (4)), and the last two equations determine equilibrium values of singlehood:

$$u_m(x) = \frac{e_m(x)}{1 + \frac{\lambda}{\delta} \int u_f(y) \alpha(x, y) dy}, \quad (16)$$

$$u_f(y) = \frac{e_f(y)}{1 + \frac{\lambda}{\delta} \int u_m(x) \alpha(x, y) dx}, \quad (17)$$

$$s_m(x) = \frac{C_m(x) + \frac{\lambda\beta}{r+\delta} \iint \max\{z + C(x, y) - s_f(y), s_m(x)\} dG(z) u_f(y) dy}{1 + \frac{\lambda\beta}{r+\delta} U_f}, \quad (18)$$

$$s_f(y) = \frac{C_f(y) + \frac{\lambda(1-\beta)}{r+\delta} \iint \max\{z + C(x, y) - s_m(x), s_f(y)\} dG(z) u_m(x) dx}{1 + \frac{\lambda(1-\beta)}{r+\delta} U_m}, \quad (19)$$

with $U_m = \int u_m(x) dx$, $U_f = \int u_f(y) dy$, $\lambda = \frac{M(U_m, U_f)}{U_m U_f}$, and

$$\alpha(x, y) = 1 - G[-C(x, y) + s_m(x) + s_f(y)].$$

We write equations (18), (19) in that form so that s_m and s_f become fixed points of contracting operators given u_m and u_f (see Shimer and Smith, 2000).

Shimer and Smith prove the existence of an equilibrium for a simpler version of the model. They consider a symmetric equilibrium with a quadratic matching function (i.e. λ constant). The common distribution of singles ($u = u_m = u_f$) is then the solution to an equation similar to equations (3) or (4),

$$u(x) = \frac{e(x)}{1 + \frac{\lambda}{\delta} \int u(y) \alpha(x, y) dy}, \quad (20)$$

that can be shown to be contracting once u is reparameterized as $v = \log(u)$. However, the general equilibrium fixed-point operator that involves α as well as u is not globally contracting. Shimer and Smith show that an equilibrium exists but it is not necessarily unique. Troeger and Noeldeke (2009) prove the existence of an equilibrium in u for all α (the first step of Shimer and Smith's proof) for the linear matching case ($\lambda = 1/\sqrt{U}$).

In the following sections, we use SIPP data to explore the empirical content of this equilibrium, i.e. we want to know how much on the exogenous parameters can be learnt from cross-section data on individual wages and labor supply by marriage status.

3 Data

In this section we present the data used in estimation, and we emphasize a few salient facts on wage and hour distributions that the model is challenged to replicate.

3.1 Demography of marriages and divorces

In the U.S. in 2001, 30.1% of men (24.6% of women), 15 years and plus, were not married and 21 % (23.1%) were divorcees (Kreider, 2005). The median age at first marriage was

Table 1: Percent ever married by age

Age	Men				Women			
	1945 to 1949	1950 to 1954	1955 to 1959	1960 to 1964	1945 to 1949	1950 to 1954	1955 to 1959	1960 to 1964
20	20.4	23.0	17.6	15.8	44.8	40.5	36.6	30.2
25	66.6	59.2	49.9	45.0	78.7	70.1	66.0	59.5
30	79.7	74.0	68.8	65.6	85.4	80.7	78.1	74.4
35	86.2	81.7	78.5	76.6	88.3	86.2	84.5	83.0
40	89.6	85.9	83.6		90.9	89.1	87.7	
45	91.5	88.2			92.1	90.6		
50	93.1				93.0			

Note: Percentage of men and women who had married at least once, by age (in row) and cohort (in column).

Source: Kreider (2005), U.S. Census Bureau, SIPP, 2001 Panel, Wave 2 Topical Module.

24 for men and 21.8 for women. Table 1 displays the percents of men and women of various cohorts who had married at least once at different ages. People are generally getting married later, but women persistently earlier than men.

In 2001, the median duration of first marriages was 8.2 and 7.9 years, respectively, for men and women. The median duration between first divorce and remarriage, for those married two times, was 3.3 years and 3.5, and second marriages lasted 9.2 and 8.1 years on average. About 75-80% of first marriages, depending on cohorts, reached 10 years, 60-65% 20 years, 50-60% 30 years. This indicates a separation rate of around 2.5% per year. For second marriages, 70-80% reached 10 years, 55% 15 years, and 50% 20 years, consistently with a slightly higher separation rate, around 3% annual.

According to survival data the median marriage duration should therefore be of 23-28 years instead of 8-9 years. The Poisson assumption is at odds with the data because a large proportion of marriages never end, and those who end in divorce do it relatively fast, in the first two years. One way of making divorce rates non stationary in the model is to permit z to change rapidly. Thus, marriages resulting from a very large z would end fast if new, likely lower values were soon drawn.

3.2 Wage and labor supply data

We use the U.S. Survey of Income and Program Participation (SIPP) from 1996-1999. For every quarter that an individual spends in the panel we collect information on the labor market state at the time of the survey, wages if employed, the number of hours worked, gender, and the corresponding information for the respondent's spouse if married. Our sample is restricted to individuals who are not self-employed or in the military, between the ages of 21 and 65.

In order to improve the reliability of the assumption that observed distributions are the stationary ones, we calculate individual mean wages over employment spells, and mean hours worked over all quarters including non-employment spells. By this way, we somewhat reduce the transitory noise in wages and hours, and we reduce the number of labor-market non-participation spells (with declared hours equal to zero). Then we drop all observations with zero hours worked (individuals and individuals' spouses never employed in the 4-year period). This is definitely not a satisfactory procedure but we thus avoid the huge additional complexity of corner solutions. Because the model is stationary we also stationarize wage and hour data by defining x and y as the residuals of the regression of male and female wages and hours on the age and age squared of the individuals. An interesting development of the current research will be to incorporate aging and differential mortality for men and women into the model. We finally trim the 1% top and bottom wages.

In the resulting sample, $2N/(2N + U_m + U_f) = 50.3\%$ of the population is married, out of $L_m = 13,223$ males and $L_f = 13,925$ females. There is a slight deficit of single males vis-a-vis single females: $U_m/U_f = 0.9$ ($N = 6,827, U_m = 6,386, U_f = 7,098$).

3.3 Wage distributions

Figure 1a shows the Gaussian kernel density estimates of wage distributions by gender and marriage status. Married males have higher and more dispersed wages than single males. Single and married females exhibit strikingly similar wage distributions. Panel (b) displays the corresponding CDFs. The wage scale is in logs so as to emphasize the

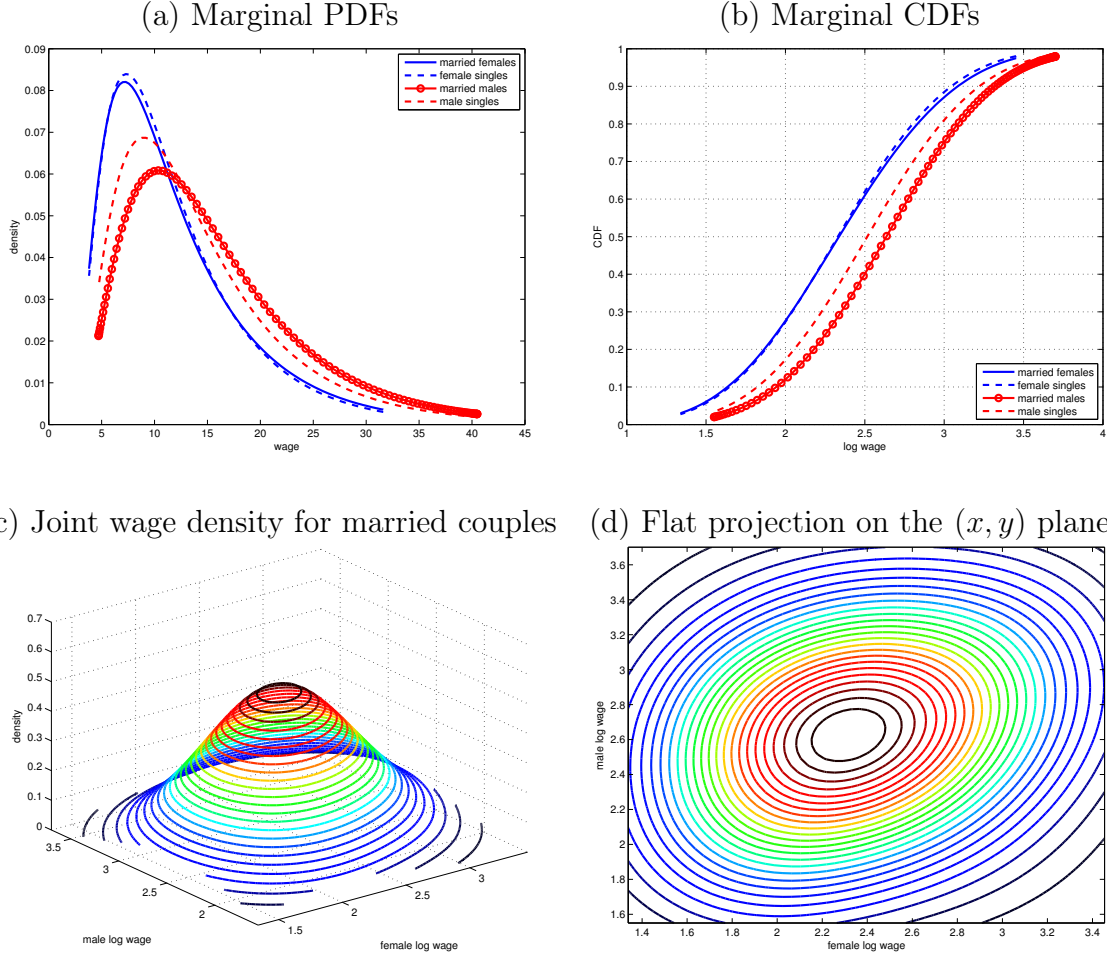


Figure 1: Wage distributions

non normality of the distributions: both tails are fatter than with a normal distribution.

Panel (c) plots the joint distribution of wages among married couples, also estimated using a Gaussian kernel density.¹¹ The most salient feature of this distribution is its very large support. Virtually all wage configurations, like a low male wage and a high female wage or vice versa, exist in the sample. Spouses' wages are only weakly correlated (about 30%), but the wage density is clearly oriented along the dominant diagonal (see the flat projection in Panel (d)). These patterns (wide support, low correlation) justify the introduction of the match-specific externality component z : it allows for imperfect sorting due to unobserved matching characteristics and explains both the low correlation and the large support.

¹¹We considerably oversmooth the nonparametric kernel estimates (by a factor 4) by comparison to the rule of thumb. Identification rests here on a sequence of non linear transformations, which tend to work nicely only when applied to smooth functions.

At this stage, the data seem to promise a low return for an economic theory of matching. Such a small correlation between x and y tends to indicate a very little amount of sorting based on wages. However, the estimation of the model has some interesting surprises in store.

3.4 Hours

Figure 2 displays nonparametric kernel estimates of mean hours (per month) given own wage for single and married individuals. A clear ordering appears: married males work more than single males and females, who work much more than married females. Marriage seems to allow men to specialize in wage-work and women in household-production. However, when males' wages increase the difference in hours worked between married males and singles gets smaller, and the reverse is true for women.

Figure 3 plots conditional mean hour estimates given both wages for married males and married females. There is some evidence of complementarity: individuals tend to work less when their spouse earns more. Now, this phenomenon is observed for male hours only at very high female wages. Male hours tend to be otherwise mostly independent of female wages.

Lastly, we compare total earnings for married couples to earnings for singles by calculating earnings differentials, i.e. the difference between couples' earnings and singles' earnings at same wages, i.e. $x(h_m^1 - h_m^0) + y(h_f^1 - h_f^0)$ (see Figure (4)). The earnings differential is non monotone in female wages. The highest differential is obtained for high-wage men and low-wage women. The lowest differential is for high-wage women and low-wage men. For most of the distribution of couples' wages, the differential is positive, but for high-wage females married with low-wage males it is negative.

4 Steady state and matching probabilities

In this section, we use the steady-state restriction (2) to identify and estimate the shape of the matching probability $\alpha(x, y)$ as a function of wages. Before that, we first argue

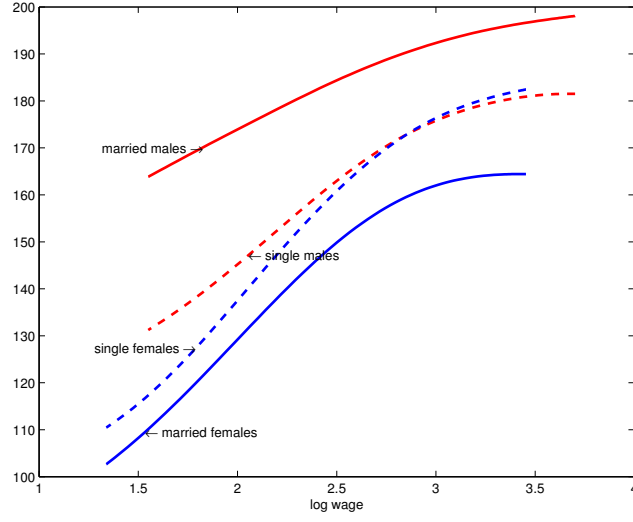


Figure 2: Mean hours given own wage

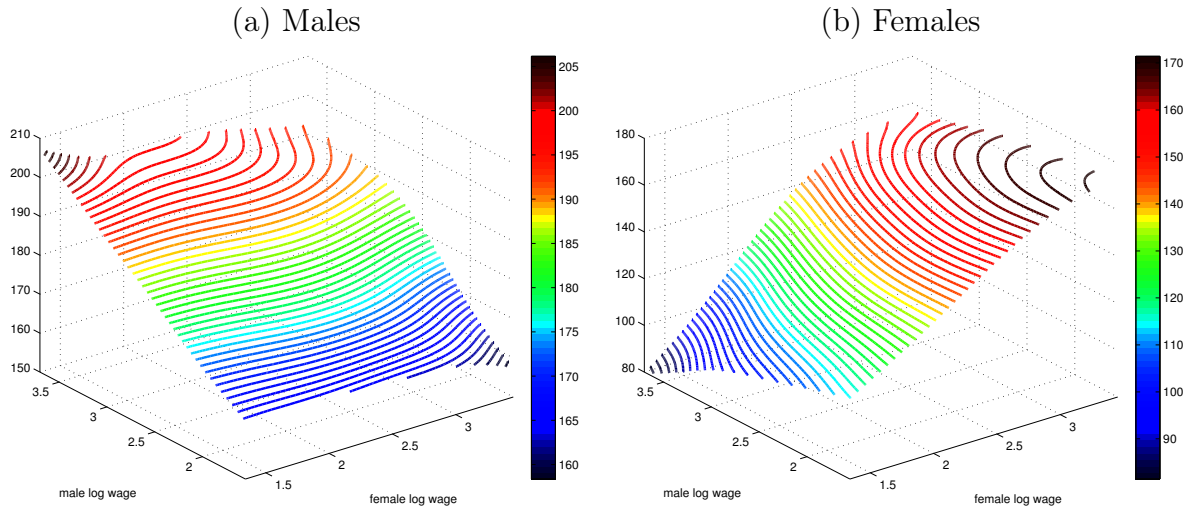


Figure 3: Mean hours for married individuals

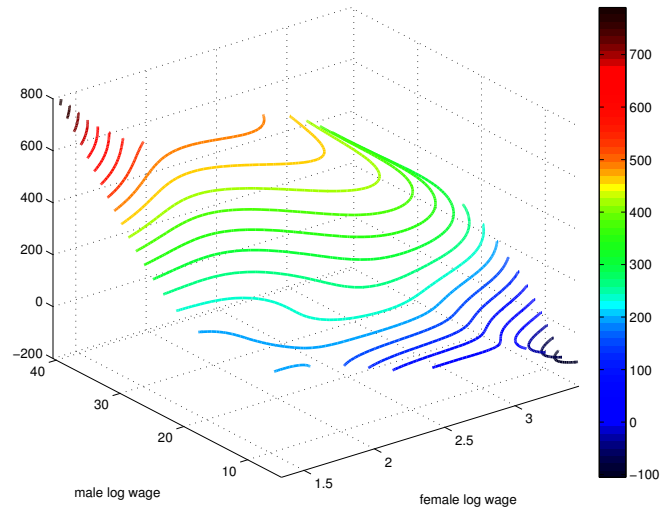


Figure 4: Household earnings differential between married and unmarried, $x(h_m^1 - h_m^0) + y(h_f^1 - h_f^0)$

that the steady-state assumption is an acceptable first-order approximation, even in a changing macro-environment, and we discuss a calibration for the divorce rate.

4.1 Divorce rates

The steady-state equation (2), by relating marriages to divorces, makes it possible to use data on first marriage ages to learn about marriage duration and divorce frequency. In addition it can help to tell us which estimate in the bracket $[2.5, 8]$ percent per year to choose for δ in the estimation of other structural parameters.

The average probability for a single man of type x to randomly meeting a single woman and marry her is defined by

$$\mu_m(x) \equiv \lambda_m \int \frac{u_f(y)}{U_f} \alpha(x, y) dy.$$

At the steady-state equilibrium described by equation (2), we have

$$\mu_m(x) = \delta \int \frac{n(x, y)}{u_m(x)} dy = \delta \frac{e_m(x) - u_m(x)}{u_m(x)},$$

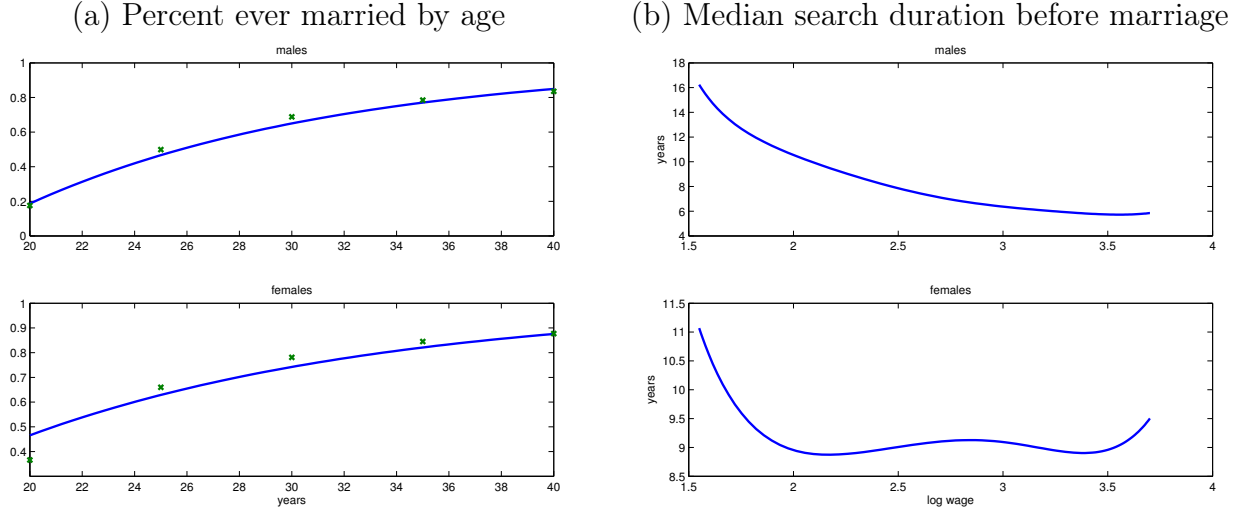
with a similar formula for single women. The average marriage rate among single men is the expectation of this quantity:

$$\mu_m \equiv \int \mu_m(x) \frac{u_m(x)}{U_m} dx = \delta \frac{L_m - U_m}{U_m}.$$

Using the data displayed in Table 1 on the distribution of first marriage age of the 1955-1959 cohort we estimate both the age at which individuals start searching for a partner (age_{0m} and age_{0f}) and δ by running joint regressions of log survival probabilities on search durations:

$$\log S_m(t) = -\delta(t - age_{0m}), \quad \log S_f(t) = -\delta(t - age_{0f}),$$

where $S_m(t), S_f(t)$ denotes the probability of not being married by age t respectively for



Note: x-marks show 1955-59 cohort data.

Figure 5: Duration of singlehood

men and women. We estimate starting ages $age_{0m} = 17.3$ years and $age_{0f} = 11.8$ years. The estimated divorce rate is $\delta = 8.0\%$ annual. Figure 5a shows the fit of this simple model, which is good. The implied median first marriage duration is 8.7 years (mean of 12.6 years) and is remarkably similar to the value that can be directly estimated from divorce data.

The median waiting time before marriage is estimated 8.1 years for men (mean of 11.7) and 9.0 years for women (mean of 13.0). Figure 5b plots the implied average search durations by gender and wage (*i.e.* $\log(2)/\mu_i(x)$, $i = m, f$). Low wage individuals have to wait for a very long time, and women more than men. The waiting time decreases with the wage. So, women get married before men but start searching earlier and take longer.

We obtain this result because there are more female singles (7,098) than male singles (6,386) in the sample. Given its simplicity, the model can only explain this difference from different wage distributions and different effects of wages on preferences and marriage externality. Of course, many other factors can explain the relative male scarcity in the marriage market, such as a higher mortality rate.

In the end, we conclude that the steady-state assumption is a reasonable approximation because estimates of marriage duration from data on wedlock age are consistent with direct observation. However, Table 1 seems to indicate that marriage and divorce habits do change over time, younger cohorts marrying both later and divorcing more often. This

also happens with unemployment rates, for example, which do fluctuate over time (between 4% and 10% for the U.S.). Still the steady-state approximation—job destruction rate divided by job destruction rate plus job finding rate—is a very good approximation. This is because inflows and outflows may fluctuate over time, yet they do so in such a way that net flows remain small compared to gross flows in all circumstances, which is what the steady-state restriction effectively means.

4.2 The matching probability

The equilibrium flow condition (2) implies that

$$\lambda\alpha(x, y) = \delta \frac{n(x, y)}{u_m(x)u_f(y)}. \quad (21)$$

So the matching probability is identified up to the multiplicative factor λ (or λ/δ) from the observed distributions of wages among singles and couples.

In absence of (good) data on datings, it is extremely difficult to separate the meeting probability from the probability of matching given meeting. In order to show the implied shape of $\alpha(x, y)$ we thus arbitrarily choose λ so that the meeting rate is twice a year for men ($\lambda_m = 1/6$). The shape of the implied matching probability, as a function of wages, is unaffected by this choice. Note that it is likely that no wage combination (x, y) can induce marriage for sure: $\alpha(x, y) \leq 1$ for all (x, y) . This condition imposes the lower bound $\max_{x,y} \frac{n(x,y)}{u_m(x)u_f(y)} = 1.37e - 03$ on λ/δ , or a minimum meeting rate for men of $\lambda_m = .065$, or a maximum of 15 datings per year.

Figure 6 displays the estimated matching probability function obtained by replacing wage densities in equation (21) by the estimates plotted in Figure 1. It is unambiguously increasing in both wages. More precisely, the matching probability increases at accelerated rate with both wages. It is rather flat for wages below the median with high-wage women having a very low probability of matching with anybody else but a high-wage man. This pattern indicates that positive assortative mating is definitely at work in the marriage market albeit mostly in the upper tail of the wage distribution (hence the low

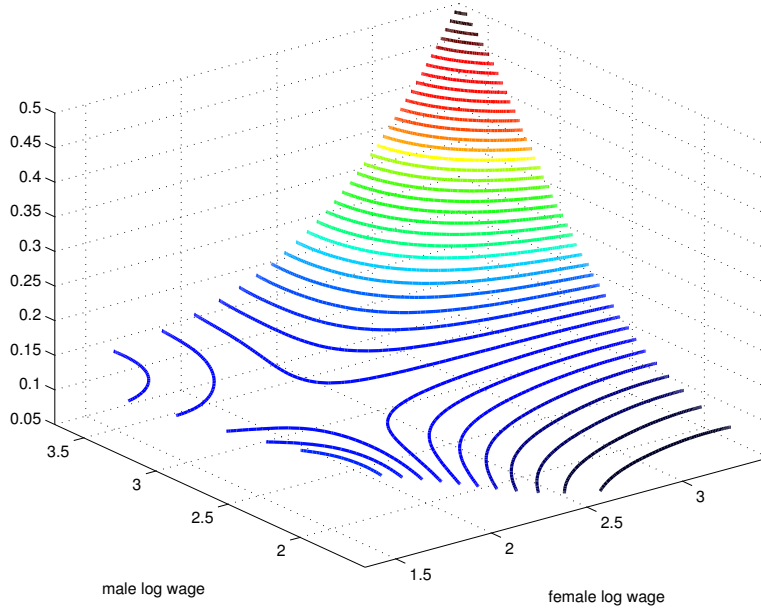


Figure 6: Matching probability, $\alpha(x, y)$ (with meeting frequency twice a year for men)

overall correlation between spouses' wages).

5 Transfers and sharing rule

We now turn to the identification of transfers t_m, t_f , and the sharing rule in particular, defined by the proportion $t_m/(t_m + t_f)$.

Let σ be the standard deviation of the distribution of the match-specific component z and define G_0 as the distribution of z/σ , that is $G(z) = G_0(z/\sigma)$.

5.1 Average transfers

First step. By inverting the equilibrium relationship between $\alpha(x, y)$ and $s(x, y)$ in equation (13) (i.e. $\alpha = 1 - G(-s)$), we obtain

$$s(x, y) = -\sigma G_0^{-1}(1 - \alpha(x, y)). \quad (22)$$

Then, noting that

$$\int \max\{z + s, 0\} dG(z) = s[1 - G(-s)] + \int_{-s}^{+\infty} z dG(z),$$

for all s , equation (14) relates $s_m(x)$ to total surplus $s(x, y)$ as

$$s_m(x) = C_m(x) + \beta\sigma \frac{\lambda}{r + \delta} \int \mu_{G_0}(\alpha(x, y')) u_f(y') dy', \quad (23)$$

with

$$\mu_{G_0}(\alpha) = -\alpha G_0^{-1}(1 - \alpha) + \int_{G_0^{-1}(1-\alpha)}^{+\infty} v dG_0(v).$$

And by symmetry,

$$s_f(y) = C_f(y) + (1 - \beta)\sigma \frac{\lambda}{r + \delta} \int \mu_{G_0}(\alpha(x', y)) u_m(x') dx'. \quad (24)$$

It follows that $\frac{s(x, y)}{\sigma}$, $\frac{s_m(x) - C_m(x)}{\beta\sigma}$ and $\frac{s_f(y) - C_f(y)}{(1-\beta)\sigma}$ are identified given G_0 and λ .

Second step. Actual transfers depend on the realized value of the match specific component of the public good z , which we never observe. Define expected transfers as

$$\begin{aligned} \bar{t}_m(x, y) &\equiv \mathbb{E}[t_m(x, y, z) | x, y, s(x, y) + z > 0], \\ \bar{t}_f(x, y) &\equiv \mathbb{E}[t_f(x, y, z) | x, y, s(x, y) + z > 0], \end{aligned}$$

where the expectation is of course conditional on marriage being consummated: $s(x, y) + z > 0$. Equations (10) and (11) for equilibrium transfers imply that

$$\begin{aligned} \bar{t}_m(x, y) &= s_m(x) + \beta \mathbb{E}[s(x, y) + z | x, y, s(x, y) + z > 0] \\ &= C_m(x) + \beta\sigma \left[\frac{\lambda}{r + \delta} \int \mu_{G_0}(\alpha(x, y')) u_f(y') dy' + \frac{\mu_{G_0}(\alpha(x, y))}{\alpha(x, y)} \right] \end{aligned} \quad (25)$$

and

$$\begin{aligned}\bar{t}_f(x, y) &= s_f(y) + (1 - \beta)\mathbb{E}[s(x, y) + z | x, y, s(x, y) + z > 0] \\ &= C_f(y) + (1 - \beta)\sigma \left[\frac{\lambda}{r + \delta} \int \mu_{G_0}(\alpha(x', y)) u_m(x') dx' + \frac{\mu_{G_0}(\alpha(x, y))}{\alpha(x, y)} \right].\end{aligned}\quad (26)$$

Hence, $\frac{\bar{t}_m(x, y) - C_m(x)}{\beta\sigma}$ and $\frac{\bar{t}_f(x, y) - C_f(y)}{(1 - \beta)\sigma}$ are in turn also identified given G_0 and λ on the support of $\alpha(x, y)$ (the matching set).

Discussion. Mean transfers (net of home production for singles) are proportional to the bargaining power coefficient (β for men, $1 - \beta$ for women). Clearly enough, the same transfer can be obtained with a better outside option and a lower β . Collective models do not separate these two sources of bargaining power within the family. Indeed, there is a one-to-one relationship between the minimal utility that the Pareto program assigns to household members and the equivalent utility weight (or the Kuhn-Tucker multiplier). In a bargaining model, however, the weight of each individual (log) surplus in the Nash program (β and $1 - \beta$) is structurally independent of the minimal utility levels (or outside options). This superior flexibility calls for more data, as identifying β effectively requires separate data on the size of the cake to be shared between parties and the shares themselves.¹²

With data on wages and matching, it is not possible to identify σ either. This is because the only information that is used to identify transfers is the frequency of marriage for any particular wage configuration. Marriage occurs in the model when

$$t_m - C_m + t_f - C_f = C - C_m - C_f + z > 0.$$

In absence of any additional information on the component $C(x, y)$ of household production, allowing to anchor it on some known level, we can divide all terms of this equation by any positive number and the inequality remains true for all x, y .

¹²For example, Cahuc, Postel-Vinay, and Robin (2006) use data on firm value-added and wages to identify the bargaining power of workers in an equilibrium search-matching model.

5.2 Average total net transfers

Total consumption is the sum of earnings and home production. The consumption differential between married couples and singles is the sum of the earnings differential (already displayed in Figure 4) and total net transfers, $C - C_m - C_f + z$. From the preceding subsection, mean total net transfers follow as

$$\bar{t}_m - C_m + \bar{t}_f - C_f = C - C_m - C_f + \mathbb{E}[z|s + z > 0],$$

and are thus identified up to the scaling factor σ given β, λ and G_0 . To use equations (25) and (26) for estimation, we set $\beta = 0.5$ and $\sigma = 1000$, the order of magnitude of monthly earnings. Moreover, we set G_0 equal to the CDF of a standard normal distribution ($G_0 = \Phi$).¹³ Computational details on the empirical implementation can be found in the appendix.

Figure 7 shows the decomposition of mean total net transfers ($\bar{t}_m - C_m + \bar{t}_f - C_f$) into the effect of home production ($C - C_m - C_f$) and the effect of the match-specific “love” effect ($\mathbb{E}[z|s + z > 0]$). The two surfaces move in opposite directions. This is because $s = C - s_m - s_f$ moves essentially like $C - C_m - C_f$. Yet the opposite match effect does not change the shape of the home production differential in relation to the total effect. There is strong assortative mating because $C - C_m - C_f$ is doubly increasing (increasing in both directions of x and y). We also find that in absence of the match-specific effect, most marriages would not occur, as $C - C_m - C_f$ is negative for most (x, y) combinations (all but the highest wages).¹⁴ Thus, the effect of wages on couple formation is limited, which is in a way reassuring.

5.3 Home production time

In our model, marriage is entirely determined by home production. The graph of $C - C_m - C_f$ only indirectly helps characterize home production functions. What does it tell us about $H - H_m - H_f$? By the Envelope theorem, home production times are related

¹³With $G_0 = \Phi$, $\mu_\Phi(\alpha) = -\alpha\Phi^{-1}(1 - \alpha) + \varphi \circ \Phi^{-1}(1 - \alpha)$.

¹⁴Note that this result follows from the sign of $C - C_m - C_f$; it is thus independent of the choice of σ .

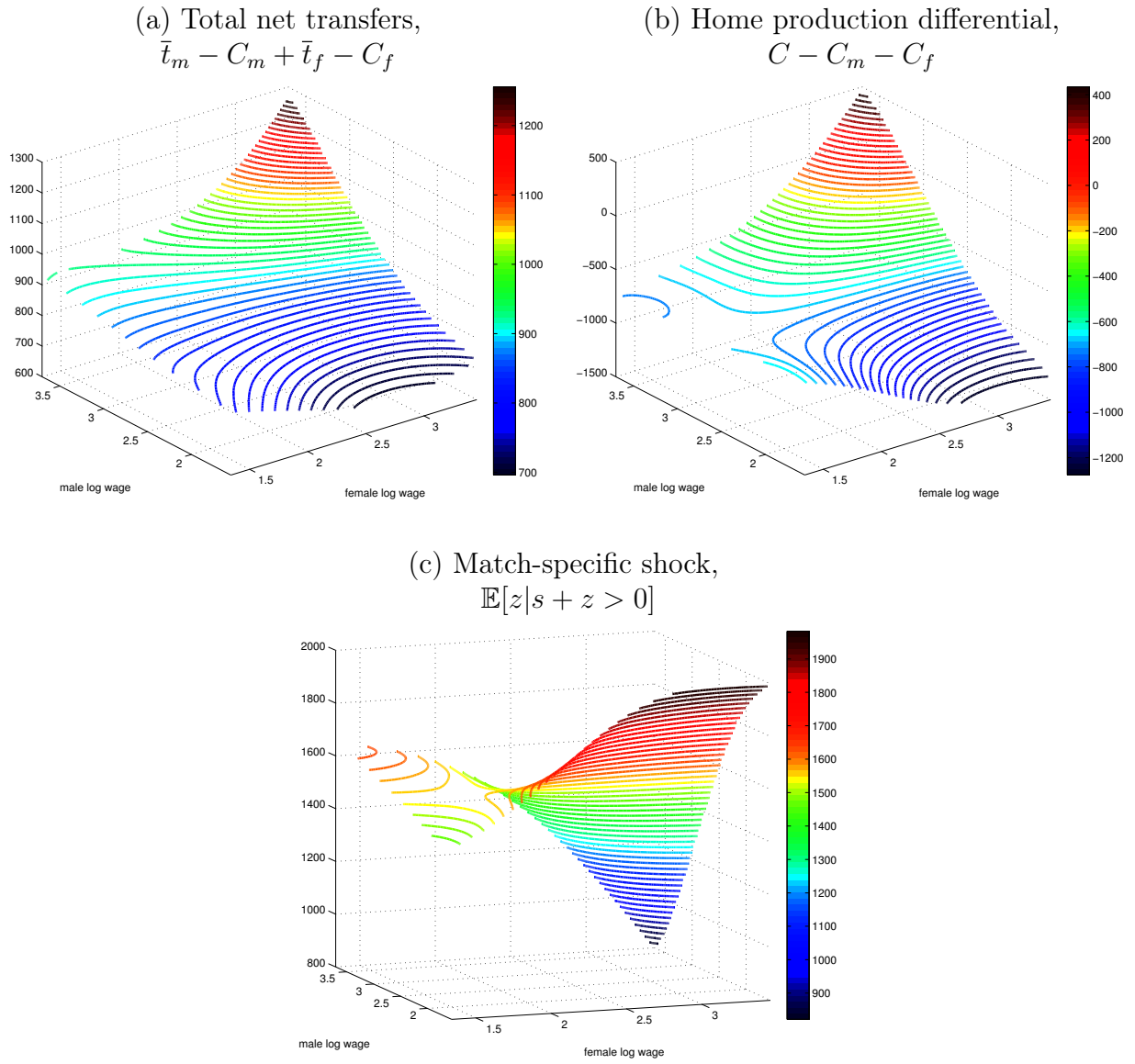


Figure 7: Total net transfers

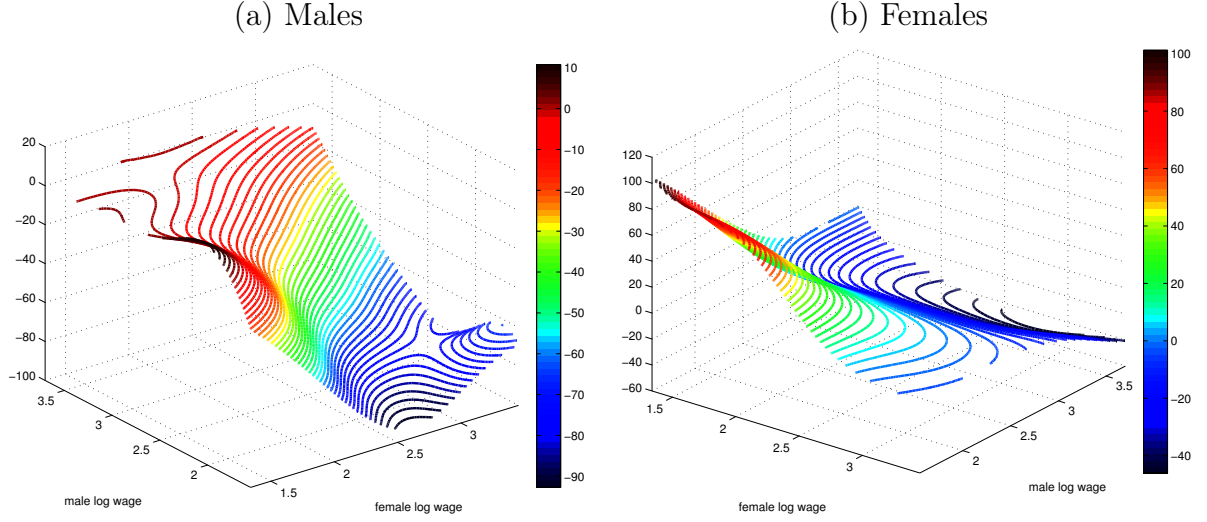


Figure 8: Home production time differentials

to C , C_m and C_f as

$$\begin{aligned} k_m^1(x, y) &= \partial_x C(x, y), & k_m^0(x) &= C'_m(x), \\ k_f^1(x, y) &= \partial_y C(x, y), & k_f^0(y) &= C'_f(y). \end{aligned}$$

Hence

$$\begin{aligned} k_m^1(x, y) - k_m^0(x) &= \partial_x [C - C_m - C_f](x, y), \\ k_f^1(x, y) - k_f^0(y) &= \partial_y [C - C_m - C_f](x, y). \end{aligned}$$

Figure 8 shows the maps of these differentials. They are not very far from planes with the following monotonicity properties:

- $k_m^1 - k_m^0$ increases in x and decreases in y ,
- $k_f^1 - k_f^0$ decreases in both x and y .

To get a sense of what this means, suppose that

$$H(k_m, k_f) = A k_m^{a_1} k_f^{b_1}, \quad H_m(k_m) = A_m k_m^{a_0}, \quad H_f(k_f) = A_f k_f^{b_0},$$

with positive elasticities and with $a_1 + b_1 \leq 1$ and $a_0, b_0 \leq 1$.

Then, simple algebra yields

$$\begin{aligned}\ln k_m^1(x, y) &= c_m^1 - \frac{1 - b_1}{1 - a_1 - b_1} \ln x - \frac{b_1}{1 - a_1 - b_1} \ln y, \\ \ln k_f^1(x, y) &= c_f^1 - \frac{a_1}{1 - a_1 - b_1} \ln x - \frac{1 - a_1}{1 - a_1 - b_1} \ln y, \\ \ln k_m^0(x) &= c_m^0 - \frac{1}{1 - a_0} \ln x, \\ \ln k_f^0(y) &= c_f^0 - \frac{1}{1 - b_0} \ln y,\end{aligned}$$

where $c_m^1, c_f^1, c_m^0, c_f^0$ are constants. It then follows that

- $\ln k_m^1 - \ln k_m^0$ decreases with y and increases with x if $\frac{a_1}{1-b_1} < a_0$,
- $\ln k_f^1 - \ln k_f^0$ decreases with x and decreases with y if $\frac{b_1}{1-a_1} > b_0$.

Hence, one can expect $k_m^1 - k_m^0$ to increase with x and $k_f^1 - k_f^0$ to decrease with y if $b_1 - b_0 > 0$ and if $a_0 - a_1 > a_0 b_1$. In other words, the observed pattern for $C - C_m - C_f$ is consistent with home production functions being such that married women are more productive at home than single women and married men are significantly less productive than single men. In addition, women have a strong comparative advantage in home production in marriage ($b_1 - a_1 > b_0 - a_0 + a_0 b_1$). It is quite remarkable that we can thus deduce the specialization of married women in household production just by looking at wage distributions.

Lastly, note that the direct observation of home production time would allow us to identify σ and β , as the range of values of $C - C_m - C_f$ obviously varies with σ (proportionality) and with β , to a lesser extent.

5.4 The sharing rule

We define the sharing rule as $\frac{\bar{t}_m - C_m}{\bar{t}_m - C_m + \bar{t}_f - C_f}$. It is identified given β, λ and G_0 , irrespective of σ . Figure 9 plots its estimate using the same calibrations of β, λ and G_0 as in the preceding sections. The median share of total transfers that goes to a married man with a median wage (2.6 in logs) is about one half, with some variance depending on his wife's wage. The same is true for a married woman with a median wage (around 2.4

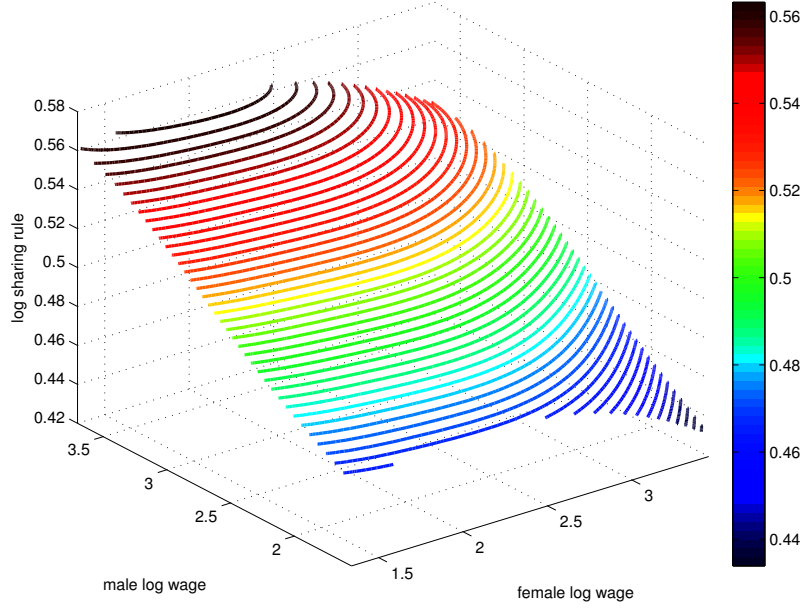


Figure 9: Sharing rule: share of total net transfers that goes to man, $\frac{\bar{t}_m - C_m}{\bar{t}_m - C_m + \bar{t}_f - C_f}$

in logs). This is expected given the arbitrary choice of $1/2$ for β . However, it is most remarkable that the share of total net transfers that is appropriated by the man is much more responsive to his own wage or ability than to his wife's wage. Looking at the figure more closely, we see that a linear function of x and y would not be a bad approximation of the sharing rule, with a positive and steep slope in the x -direction (male wages), and flat in the y -direction, with a slope becoming negative only for high female wages.

5.5 Leisure

More able married men thus tend to get a bigger share of the surplus. Yet they work more than women. Without home production, the model could only make sense of these observations by assuming that men have a preference for work (i.e. leisure is an inferior good for them) and women have a preference for leisure. By decomposing non wage-labor time separately into leisure and home production activities, we can understand this result by the specialization of married women in home production.

Unfortunately, in absence of data on home production time we can only identify $k_m^1 - k_m^0$ and $k_f^1 - k_f^0$ up to scale. Assuming a value for β ($1/2$) and σ (1000), we now

show one can proceed to identify and estimate income effects $b_m(x)$ and $b_f(y)$.

First, given $k_m^1 - k_m^0$ and $k_f^1 - k_f^0$, the difference in leisure time between married men and single men is

$$\ell_m^1(x, y, z) - \ell_m^0(x) = -[h_m^1(x, y, z) - h_m^0(x) + k_m^1(x, y) - k_m^0(x)].$$

Second, from equation (8), we deduce that

$$\ell_m^1(x, y, z) - \ell_m^0(x) = b'_m(x) [t_m(x, y, z) - C_m(x)].$$

Averaging the match component z out, we obtain that

$$\Delta \ell_m(x, y) \equiv \mathbb{E} [\ell_m^1|x, y] - \ell_m^0(x) = b'_m(x) [\bar{t}_m(x, y) - C_m(x)].$$

We will thus estimate income effects $b'_m(x)$ by regressing $\Delta \ell_m(x, y)$ on $\bar{t}_m(x, y) - C_m(x)$.

The corresponding population parameter is

$$b'_m(x) = \frac{\int \Delta \ell_m(x, y) [\bar{t}_m(x, y) - C_m(x)] n(x, y) dy}{\int [\bar{t}_m(x, y) - C_m(x)]^2 n(x, y) dy},$$

with a similar expression for women.

Figure 10 shows the estimated income effects, $b'_m(x)$ and $b'_f(y)$, obtained with $\beta = 0.5$ and $\sigma = 1000$, together with $B_m(x)$ and $B_f(y)$ (see appendix for estimation details). A low-order polynomial approximation is shown (dashed curves) for comparison. We thus find that leisure is a normal good for both men and women. However, for lower values of σ (say 500) we would obtain negative values for $b'_m(x)$. If σ is too small then $k_m^1 - k_m^0$ is too small to compensate for the negative wedge in hours worked between married men and single men.

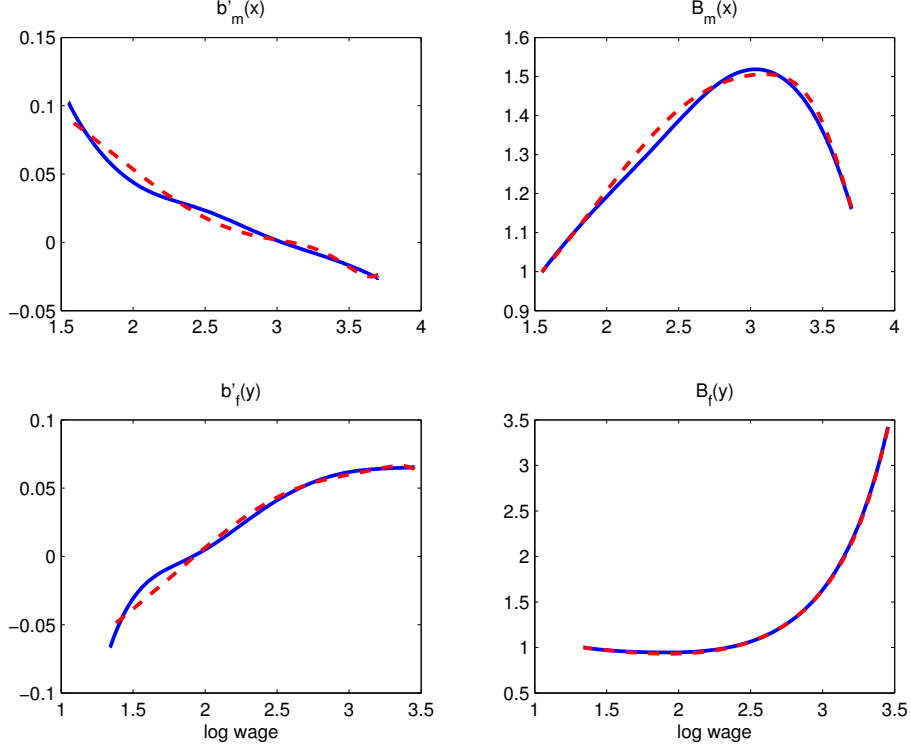


Figure 10: Preference parameters (the dotted lines correspond to 4th order approximations)

6 Conclusion

In this paper, we have developed a prototypical version of a search-matching model of the marriage market with labor supply and home production. Our model extends the existing theoretical and empirical literature in two ways. By incorporating labor supply decisions and household production in Shimer and Smith's (2000) search-matching model, we provide a structural foundation for the surplus function driving the marriage decision. Inside households, the surplus is shared according to the balance of powers that is induced by external opportunities on the marriage market. This is a way of endogenizing the sharing rule driving decisions on time uses in Chiappori's collective model of the household. We derive the steady-state equilibrium and study the nonparametric identification of exogenous parameters from a cross-section of data on wages and hours worked, and we show that most of the underlying unknown parameters of interest can be identified. The model is rich of interesting lessons. Despite a low correlation between spouses' wages, we estimate a matching probability function that is strongly increasing

in both wages, and we show how to learn from matching probabilities on the sharing rule and the response of home production to ability differences.

Many possible extensions of the model easily come to mind, like endogenizing divorce, either through shocks to z or via on-the-marriage search, or like allowing for other dimensions of heterogeneity but wages, or introducing public goods such and children (not substitutable to market goods). As far as heterogeneity is concerned, one issue is completely ignored in this paper, which for that reason is essentially a methodological contribution, namely unobserved heterogeneity correlating preferences and wages. This needs to be done if one credibly wants to evaluate such policies as WFTC or EITC.

Appendix - Computational Details

This appendix shortly describes the numerical tools used in estimation.

First, we discretize continuous functions on a compact domain using Chebyshev grids.¹⁵ For example, let $[\underline{x}, \bar{x}]$ denote the support of male wages, we construct a grid of $n + 1$ points as

$$x_j = \frac{\bar{x} + \underline{x}}{2} + \frac{\bar{x} - \underline{x}}{2} \cos \frac{j\pi}{n}, \quad j = 0, \dots, n.$$

Second, to estimate wage densities $n(x, y)/N$, $u_m(x)/U_m$ and $u_f(y)/U_f$ on those grids we use kernel density estimators with a lot of smoothing. This is important as, for instance, we divide n by $u_m u_f$ to calculate α according to (21).

Third, many equations involve integrals. Given Chebyshev grids, it is natural to use Clenshaw-Curtis quadrature to approximate these integrals:

$$\int_{\underline{x}}^{\bar{x}} f(x) dx \simeq \frac{\bar{x} - \underline{x}}{2} \sum_{j=0}^n w_j f(x_j),$$

where the weights w_j can be easily computed using Fast Fourier Transform (FFT). The following MATLAB code can be used to implement CC quadrature (Waldvogel, 2006):

```
function [nodes,wcc] = cc(n)
nodes = cos(pi*(0:n)/n);
N=[1:2:n-1]'; l=length(N); m=n-1;
v0=[2./N./(N-2); 1/N(end); zeros(m,1)];
v2=-v0(1:end-1)-v0(end:-1:2);
g0=-ones(n,1); g0(1+l)=g0(1+l)+n; g0(1+m)=g0(1+m)+n;
g=g0/(n^2-1+mod(n,2)); wcc=real(ifft(v2+g));
wcc=[wcc;wcc(1)];
```

Note that, although Gaussian quadrature provides exact evaluations of integrals for higher order polynomials than CC, in practice CC works as well as Gaussian. On the other hand, quadrature weights are much more difficult to calculate for Gaussian quadrature. See Trefethen (2008).

Fourth, we need to solve functional fixed point equations. The standard algorithm to calculate the fixed point $u(x) = T[u](x)$ is to iterate $u_{p+1}(x) = Tu_p(x)$ on a grid. If the fixed point operator T involves integrals, we simply iterate the finite dimensional operator \hat{T} obtained by replacing the integrals by their approximations at grid points. For example, an equation like

$$u(x) = T[u](x) = \frac{\ell(x)}{1 + \rho \int_{\underline{x}}^{\bar{x}} u(y) \alpha(x, y) dy}$$

becomes

$$\mathbf{u} = [u(x_j)]_{j=0, \dots, n} = \hat{T}(\mathbf{u}) = \left[\frac{\ell(x_j)}{1 + \rho \sum_{k=0}^n w_k u(x_k) \alpha(x_j, x_k)} \right]_{j=0, \dots, n}.$$

It was sometimes necessary to “shrink” steps by using iterations of the form $u_{p+1} = u_p + \theta(Tu_p - u_p)$ with $\theta \in (0, 1]$. A stepsize $\theta < 1$ may help if T is not everywhere strictly contracting.

Fifth, the fact that CC quadrature relies on Chebyshev polynomials of the first kind also allows to interpolate functions very easily between points $y_0 = f(x_0), \dots, y_n = f(x_n)$ using

¹⁵It can be shown that the error associated to a polynomial approximation (of any order) of an unknown function at any point x is proportional to $\prod_{j=0}^n (x - x_j)$. The Chebyshev points are the $\{x_j\}_{j=0, \dots, n}$ minimizing this quantity.

Discrete Cosine Transform (DCT):

$$f(x) = \sum_{k=0}^n Y_k \cdot T_k(x), \quad (\text{A.1})$$

where Y_k are the OLS estimates of the regression of $y = (y_0, \dots, y_n)$ on Chebishev polynomials

$$T_k(x) = \cos \left(k \arccos \left(\frac{x - \frac{\bar{x} + \underline{x}}{2}}{\frac{\bar{x} - \underline{x}}{2}} \right) \right),$$

but are more effectively calculated using FFT. A MATLAB code for DCT is, with $y = (y_0, \dots, y_n)$:

```
Y = y([1:n+1 n:-1:2],:);
Y = real(fft(Y/2/n));
Y = [Y(1,:); Y(2:n,:)+Y(2*n:-1:n+2,:); Y(n+1,:)];
f = @(x) cos(acos((2*x-(xmin+xmax))/(xmax-xmin))
*(0:n))*Y(1:n+1);
```

A bidimensional version is

```
Y = y([1:n+1 n:-1:2],:);
Y = real(fft(Y/2/n));
Y = [Y(1,:); Y(2:n,:)+Y(2*n:-1:n+2,:); Y(n+1,:)];
Y = Y(:, [1:n+1 n:-1:2]);
X = real(fft(X'/2/n));
Y = [Y(1,:); Y(2:n,:)+Y(2*n:-1:n+2,:); Y(n+1,:)]';
f=@(x,y) cos(acos((2*x-(xmin+xmax))/(xmax-xmin))*(0:n))...
*Y(1:n+1,1:n+1)...
*cos((0:n)'*acos((2*y'-(ymin+ymax))/(ymax-ymin))));
```

The fact that the grid (x_0, \dots, x_n) is not uniform and is denser towards the edges of the support interval allows to minimize the interpolation error and thus avoids the standard problem of strong oscillations at the edges of the interpolation interval (Runge's phenomenon).

Another advantage of DCT is that, having calculated Y_0, \dots, Y_n , then polynomial projections of $y = (y_0, \dots, y_n)$ of any order $p \leq n$ are obtained by stopping the summation in (A.1) at $k = p$. Finally, it is easy to approximate the derivative f' or the primitive $\int f$ simply by differentiating or integrating Chebyshev polynomials using

$$\cos(k \arccos x)' = \frac{k \sin(k \arccos x)}{\sin(\arccos x)},$$

and

$$\int \cos(k \arccos x) dx = \begin{cases} x & \text{if } k = 0, \\ \frac{x^2}{2} & \text{if } k = 1, \\ \frac{\cos(k+1)x}{2(k+1)} - \frac{\cos(k-1)x}{2(k-1)} & \text{if } k \geq 2. \end{cases}$$

In calculating an approximation of the derivative, it is useful to smoothen the function by summing over only a few polynomials. Derivatives are otherwise badly calculated near the boundary. Moreover, our experience is that the approximation:

$$\int_{\underline{x}}^x \mathbf{1}\{t \leq x\} f(t) dt \simeq \sum_{k=0}^n w_k \mathbf{1}\{t \leq x_k\} f(x_k)$$

gave similar results as integrating the interpolated function.

We implemented these procedures with numbers of grid points such as $n = 50, 100, 500$ on a laptop without running into any memory or computing time difficulty.

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