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Uncertainty and Contradiction: An Essay on the Business Cycle*

by

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Abstract

The present paper presents a discussion of the forces at play behind the economic fluctuations in the medium run and their relation with the short-run macroeconomic equilibrium. The business cycle is the result of two separate phenomena. On the one hand there is the instability, which is caused by the discrepancy between the expected and the realized outcomes. On the other hand, this instability is contained by the inherent contradictions of capitalism; the upswing carries “within it the seeds of its own destruction”. The same happens with the downswing. A formal exposition of these insights is provided. It is discussed how the formulation of this mechanism resembles the simple harmonic motion of Classical mechanics. Finally, an empirical evaluation is provided.

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JEL Classification Codes: B22, E11, E12, E33
1 Prolegomena

In a recent paper co-authored with Duncan Foley (Nikiforos and Foley, 2012) we argue that the wage share presents a U-shaped behavior along the business cycle. For low levels of utilization the wage share decreases as utilization increases and for higher levels of utilization the wage share increases together with utilization.

Figure 1: Wage Share and Utilization in the United States, 1947-2010

Figure 1, which is taken from that paper, presents the actual data from the US economy for the period 1947-2010, which confirm—at least prima facie—this kind of behavior for distribution. However the evidence of a scatter plot, like the one of figure 1 cannot be conclusive because of the so-called endogeneity problem. Distribution and utilization are determined together through channels where the causality runs both ways. More precisely within the Structuralist (or Kaleckian, or Post-Keynesian) theoretical framework, they are determined through the interaction of the demand and distribution schedules. In the first one, the causality runs from distribution to utilization while in the second one it runs the other way around. In Nikiforos and Foley (2012), we go to great length to try to solve this problems. We use Instrumental Variables within a Two Stages Least Squares framework and we verify econometrically the U-shaped behavior of distribution.

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1For the source and the method of compilation of the data of the figure the reader can refer to the appendix of Nikiforos and Foley (2012).
This statistical conclusion taken together with the observations of figure 1 imply a mechanism for the business cycle. The cycle is driven by changes in demand, which interacts with a stable—at least in the medium run—U-shaped distributive schedule. This kind of mechanism is presented in figure 2. Equilibrium is the result of the interaction of distribution and demand; for instance, when demand is at the level $D_0$ equilibrium is at point $A$. The stable distributive schedule is the base of the cycle, which is driven by shifts of the demand schedule. In the upswing the demand schedule shifts to the right: in our graph shifts from $D_0$ to $D_1$, the new equilibrium in this case is $B$. In the downswing demand shifts to the left, to $D_2$, and equilibrium settles at low utilization levels at point $C$.

![Figure 2: A sketch of the business cycle](image)

The present paper is (yet) an(other) effort to answer these questions. Towards this direction, we combine the canonical structuralist analysis of growth and distribution, with the *instability principle*, set forth by Roy Harrod (1939) and the Marxian insights about the contradictory nature of the capitalist system. The business cycle is the result of two separate phenomena. On the one hand
there is the instability, which is caused by the discrepancy between the expected and the realized outcomes. On the other hand, this instability is contained by the inherent contradictions of capitalism; the upswing carries “within it the seeds of its own destruction”, and the same happens with the downswing. This paper focuses on one of these self-contradictory processes, the depletion of the reserve army of labor and the resulting profit squeeze. However, the analytical framework can be easily extended to incorporate other processes such as the financial instability hypothesis of Hyman Minsky (1975; 1986).

In the course of the discussion of our model it will also become obvious that at this level of abstraction, the mechanics of the business cycle resemble these of a simple harmonic oscillator. Section 6 below highlights the analogies between the two. Finally, in section 7 we discuss our model in relation to the actual observations on distribution and utilization from the US economy.

A warning is in order at this point. The economic phenomena are in reality much more complex than any economic model presents them to be, and this is especially true when it comes to business cycle and growth models. Thus the presentation here is necessarily—to use the words of the first sentence in Goodwin (1967)—“starkly schematized and hence quite unrealistic”[emphasis added]. Nevertheless, it can provide some intuition about the forces at play behind economic fluctuations.

2 Instability & Contradiction

We can start our discussion with Harrod and the contradictory nature of the capitalist economy. The setup and discussion of the canonical Structuralist/Kaleckian model of growth and distribution will be postponed until the next section.

The basic message of Harrod’s (1939) paper is that the capitalist economy is fundamentally unstable and will tend to over-expand or to plunge into recessions. The paper was published in March of 1939; the world had already been 10 years into the greatest economic recession of its history and was close to World War II. It is thus no wonder, that an economist of the time would have such a vision of capitalism. This comes in stark contrast with the vision that prevailed in mainstream post-WWII macroeconomics, where the system is perceived as stable and tending towards—full employment—equilibrium. In Harrod’s world minor perturbations can lead the macroeconomy off track. It is interesting that similar worries have started to prevail since 2008. For example, the fears that the default of a minor European peripheral country could derail the global economy into a double-dip recession is an indication of this state of mind.

Instability is not present only in the periods of big recessions; it is always there and is one of the major determinants of the economic cycle, no matter how shallow or deep this is. According

\footnote{Characteristic examples are the Solow (1956) model, or the assumed stability of the saddle path systems of more recent neoclassical economic models.}
to Harrod, this is due the discrepancy between the expected and the realized outcomes in the economy, the *ex ante* and the *ex post* magnitudes of the economic variables. *Ex ante* the entrepreneurs form certain expectations about the (growth rate of the) demand for their product and their profitability. However, the *ex post* actual realized (growth rate of the) demand and profitability might be different. In fact they will not be different only by fluke. If the realized demand is higher than the expected one, then there will be an “undue depletion of stock or shortage of equipment” (Harrod, 1939, p.22) and of course the realized profitability will be higher than the expected one. This will induce the entrepreneurs to invest more to cover this undesired depletion of stock and shortage of equipment. One could add that also their animal spirits would be stimulated. As a result the “system will be stimulated to further expansion”. If for the moment we assume unrealistically that despite all this euphoria the expectations of the entrepreneurs do not change, the system will explode; period after period the realized demand will be more and more distant from the expected one, causing an ever-growing expansion. The opposite happens in the case of a hysteresis of the realized demand and profitability from the expected ones: the system will plunge into a bigger and bigger recession.

Harrod was not so naive as to believe that expectations would not change. Expectations, he says, will *chase* the realized rates in either direction. “The maximum rates of advance or recession may be expected to occur at the moment when the chase is successful” (p.28). In other words, the business cycle is the result of this chase between expectations and reality. As long as reality exceeds expectations the economy will grow. If and when reality does not manage to meet the expectations the downswing begins (and so on and so forth).

The above discussion makes clear that Harrod is exploring the dynamic dimension of the world described by Keynes three years earlier with *The General Theory*. Output is determined from the demand side and fundamental uncertainty is prevalent. The uncertainty leads to the falsification of expectations, which (the falsification) then has an impact on demand and the output, as described in the previous paragraph. It is probably redundant to say how different is this world from a neoclassical supply-led rational expectations world. If expectations were never disappointed and output is determined on the supply side, the only way to have cycles is with random and inexplicable shocks to (total factor) productivity and thus supply.

The question that follows Harrod’s *instability principle*, is what stabilizes the system? Using the phraseology of the previous paragraphs, what makes expectations catch up with reality? The answer to that is given by Marx who throughout his work stresses the contradictory character of the capitalist system. Regarding the business cycle, because of these internal contradictions—to paraphrase the famous aphorism—each stage of the cycle contains the seeds to its own destruction.

It is beyond the scope of this paper to provide an extensive list of the inherent contradictions of the capitalist system. In this paper we will focus on the (so-called) profit-squeeze, the increase of the share of the wages for high levels of utilization, which exerts a negative influence on invest-
ment and accumulation. In the words of Richard Goodwin (1967, p.58) “the improved profitability carries the seed of its own destruction by engendering a too vigorous expansion of output and employment, thus destroying the reserve army of labour and strengthening labour’s bargaining power”. In other words, the increased profitability will stimulate growth. Higher growth means that employment grows, the reserve army of labor is destroyed and the labor market becomes tighter and that in turn leads to and increase in the wages and decrease in profitability. This decrease in profitability after a certain point will push the economy downwards, the growth will fall, the reserve army of labor will be (wo)manned again, the wages will thus fall and the profitability will increase again. Figure 1 confirms this squeeze on the profit share at high level of growth and utilization.  

Another “contradiction” which is particularly relevant for understanding the fluctuations of the last two decades and especially the recent economic recession is the financial instability hypothesis of Minsky (1975, 1986). “In a capitalist environment, stability is destabilizing” (Minsky, 1985, p.12). In this paper we will not formally pursue the Minskyan argument, although it would be straightforward to incorporate in our analysis.

Finally, it is worth mentioning that except for these inherent contradictions of capitalism, stabilization can come through policy. If, for example, the Central Banks increases its interest rate in the face of increasing utilization and the interest rate exerts a negative effect on investment, there would be a similar effect with the profit-squeeze.

To sum up, these two aspects of the business cycle can be treated as complementary to each other. On the one hand, Harrod offers the important insight of demand-led instability. On the other, because of the contradictory nature of capitalism, each stage of the cycle creates the seeds of its own destruction. These are the two forces that create the economic fluctuations.

3 The Short Run

We can now turn to the Structuralist-Kaleckian model of growth and distribution. We distinguish between the short and the medium run. The crucial difference between them is that in the short run expectations have been formed and are given. In the medium run the economic actors observe

3 Goodwin formalized an argument which was first formulated by Marx in Chapter 25 of The Capital (1976). A similar behavior of distribution has been proposed among others by Barbosa-Filho and Taylor (2006); Bowles and Boyer (1988); Davidson (1972); Foley (2003); Garegnani (1992); Kurz (1994); Gordon (1995); Taylor (2004); Shapiro and Stiglitz (1984), although there are stark differences in the rationale behind it.

4 This kind of policy-originated stabilization is assumed by Dumenil and Levy (1999).

5 The origins of the the structuralist analysis can be found in the classical political economists, John Maynard Keynes and Michal Kalecki (e.g.1971). In its modern form it has been developed by Steindl (1952); Rowthorn (1981); Taylor (1983, 1990, 2004); Dutt (1984, 1990); Amadeo (1986); Kurz (1990) and Marglin and Bhaduri (1990).
the short run outcomes and modify their expectations.\footnote{This distinction between different time horizons is common in the literature. See for example Dutt (1997), Lavoie (1995), Skott (2010).} Note that Harrod himself makes a similar distinction when he talks about dynamic and static equilibrium. He writes that “normally the latter is stable and the former unstable” (1939, p.21). In the context of our discussion static equilibrium resembles what we call short run, while the dynamic equilibrium is our medium run when expectations change and instability prevails. An advantage of this exposition is that it follows the dictum of Kalecki that the medium run is nothing more than a sequence of short-run equilibria. As we will show below expectations are the connecting link between the short run equilibria.

The setup of the model is standard. As was mentioned in the introduction the economy evolves around the demand and distributive schedules. In a closed economy without government sector, demand is determined by the saving behavior of workers and capitalists and the investment behavior of the firms. The income of the economy is distributed between wages and profits.

Investment (normalized for capital stock) can be defined as \( g^i = I(\psi, u) \), where \( \psi \) is the wage share, \( Y \) and \( \bar{Y} \) is output and potential output respectively and finally \( u = Y/\bar{Y} \) is capacity utilization with \( I_\pi > 0 \) and \( I_u > 0 \) (the subscript stands for the partial derivative for this variable). On the other hand, total saving (normalized for the capital stock) is \( g^s = S(\pi, u) \). \( S_u \) and \( S_\pi \) are positive.\footnote{It beyond the scope of this paper to go into the details of the Kaleckian model, which are widely known. A recent summary is provided among others by Nikiforos and Foley (2012).}

For the purposes of the present paper we will assume a simple linear functional form for the investment function

\[
g^i = \gamma + \alpha_1 u - \alpha_2 \psi \tag{1}
\]

where \( \alpha_1, \alpha_2 > 0 \) and \( \gamma \) symbolizes the expectations of the entrepreneurs about the growth rate.\footnote{From a formal point of view a correct specification for investment would \( g^i = \gamma' + \alpha_1 (u - \bar{u}) - \alpha_2 (\psi - \psi(\bar{u})) \), where \( \bar{u} \) is a the desired or normal utilization rate, which is constant in the short and medium run. In this case \( \gamma' \) can be thought of as the growth rate that would prevail when utilization would be equal to its desired level and \( \gamma \) in equation (1) is equal to \( \gamma' - \alpha_1 \bar{u} - \alpha_2 \psi \).} High expectation for the future growth rate boost investment and demand. The opposite happens when the future looks bleak. If someone is willing to put animal spirits in algebraic form, \( \gamma \) is a simple way to do it. A spontaneous urge of the entrepreneurs to action rather than inaction, would then be expressed with a high \( \gamma \), and would of course increase investment and growth. The opposite would happen in a crisis-period like ours when usually inaction is preferred over action. It is important to keep in mind that in the short run \( \gamma \) is exogenous.

Moreover, for reasons of convenience, we will assume a simple linear form for the saving function

\[
g^s = s_0 + su \tag{2}
\]
where $s$ is the marginal propensity to save and $s_0$ a constant.

Equations (1) and (2) define the demand side of the economy. At equilibrium investment is equal to saving, thus

$$g_i = g_s \Leftrightarrow \gamma + \alpha_1 u - \alpha_2 \psi = s_0 + su \Leftrightarrow$$

$$\gamma - \alpha_2 \psi - s_0 = (s - \alpha_1) u$$

(3)

Following Harrod it is also assumed that this equilibrium is stable, or that $s - \alpha_1 > 0$, what is usually called Keynesian stability condition\(^9\). Under this setup demand is profit-led, since we have assumed that distribution of income does not affect the saving rate. Graphically, equation (3) is the downward sloping demand curve in figure 3.

On the distribution side, the non-linearities will be temporarily ignored. The profit-squeeze can be expressed with a simple linear equation

$$\psi = \psi_0 + \beta u$$

(4)

\(^9\)As was mentioned at the beginning of this section (Harrod, 1939, p.21) writes that the short-run, static equilibrium is “usually stable”. He continues in the next paragraph: “Some recent writers have been disposed to urge that the static equilibrium is not so stable as is sometimes claimed....I have the impression that this type of criticism exaggerates the importance of this problem, and constitutes to some extent a failure to see the wood for the trees, and that on its own ground the theory of static equilibrium is well able to hold its own.”
where $\beta > 0$ and $\psi_0 \leq 0$. An increase of utilization will tighten the labor market and will increase the share of wages. Graphically, equation (4) is the upward sloping distribution curve in figure 3.

The short-run equilibrium $A$ will be at the intersection of the two curves. Algebraically, the equilibrium levels of utilization and growth rate can be derived using equations (3) and (4). The short run equilibrium value of utilization is:

$$u^* = \frac{1}{s - \alpha_1 + \alpha_2 \beta} \gamma - \frac{\alpha_2 \psi_0 + s_0}{s - \alpha_1 + \alpha_2 \beta}$$

and the equilibrium growth rate is:

$$g^* = s_0 + \frac{s}{s - \alpha_1 + \alpha_2 \beta} \gamma - \frac{s(\alpha_2 \psi_0 + s_0)}{s - \alpha_1 + \alpha_2 \beta}$$

It is also straightforward to find the equilibrium rate of the wage share

$$\psi^* = \psi_0 + \frac{\beta}{s - \alpha_1 + \alpha_2 \beta} \gamma - \frac{\beta(\alpha_2 \psi_0 + s_0)}{s - \alpha_1 + \alpha_2 \beta}$$

These results are standard and do not need further analysis.

4 The Medium Run

In the medium run economic actors observe the short run outcomes, as described in equations (5) to (7); they contrast them with their previous expectations and form their expectations about the future. In the model this is expressed with the determination of $\gamma$ of the investment function. A possible change of $\gamma$ would affect the short run equilibrium of the next period, which would then lead to a new formation of expectations, and so on and so forth.

This process is usually expressed with the following equation

$$\dot{\gamma} = \rho[(g^*(\gamma, t) - \gamma(t)]$$

where $\rho > 0$ and $t$ stands for time. The dot symbolizes the time derivative, thus $\dot{\gamma} = d\gamma/dt$. If the actual growth rate exceeds the expected growth rate, the expected growth rate increases. This is the chase game described by Harrod. Note that if $\partial g^*/\partial \gamma > 1$ the instability principle prevails. Expectations will never manage to catch up with realized outcomes and the system explodes.

However, economic agents form their expectations based not only on the realized outcome of the last period, but also previous realized outcomes. For example if there is “undue depletion of stock or shortage of equipment” for many consecutive periods before a certain period, say $t$, we would expect this to play a role in the formation of the expectations in period $t$. Harrod was well aware of
these lagged effects, although he chooses to ignore them for reasons of economy of exposition. He writes: “the study of these lags is of undoubted importance, but a division of labour in analysis is indispensable, and in this case the neglect is necessary in order to get the clearest possible view of the forces determining the trend and its influence as such” (Harrod, 1939, p.20).

Algebraically, these lagged effects can be expressed by substituting equation (8) with the following one

\[ \dot{\gamma} = \int_{0}^{t} \rho[(g^*(s) - \gamma(s)]ds \]  

(9)

By time differentiating both sides, equation (9) is transformed into:

\[ \ddot{\gamma} = \rho[g^*(t) - \gamma(t)] \]

(10)

where the double dot stands for the second time derivative. Then, by using the short run equilibrium growth rate from equation (6), equation (10) can be rewritten as

\[ \ddot{\gamma} + \frac{\alpha_2\beta - \alpha_1}{s - \alpha_1 + \alpha_2\beta}\gamma = \rho s_0 - \frac{s(\alpha_2\psi_0 + s_0)}{s - \alpha_1 + \alpha_2\beta} \]  

(11)

This equation describes the behavior of the economy in the medium run. With reference to it and equations (5) to (7), we can infer the behavior of the other variables of the system, utilization, growth rate and distribution.

The important term in equation (11) is the coefficient of \( \gamma \), that is \( \rho \frac{\alpha_2\beta - \alpha_1}{s - \alpha_1 + \alpha_2\beta} \). If this term is negative then \( \gamma \) will monotonically diverge from its steady state value. Together with \( \gamma \), utilization, the growth rate and wage share will also diverge. On the other hand if the term is positive \( \gamma \) will oscillate around its state state value with constant amplitude. We provide a brief discussion of the second order differential equations in the Appendix.

The denominator of this term is positive because of the Keynesian stability condition, and so is by definition \( \rho \). Thus the question about the sign of the term boils down to the sign of the numerator. It is not hard to see that the numerator is positive if and only if \( \alpha_2\beta > \alpha_1 \). In other words, if there is a relatively strong profit squeeze, if investment reacts strongly to changes in distribution or reacts weakly to changes in utilization (or some combination of the three), the term is positive and \( \gamma \) presents cyclical fluctuations. An increase in the growth rate will stimulate expectations and will further increase the growth rate. However, as growth and utilization increase profitability is squeezed and investment slows down. This will finally help the expectations to catch up with the realized growth rate. Then the opposite process begins. The growth rate decreases and expectations follow, the labor market loosens and the size of the reserve army of labor increases. This puts pressure on the wage share and slows down the decrease of the growth rate. Again this helps the ex-
pectations to catch up with the realized growth rate. This chase of expectations and realized growth continues \textit{ad infinitum}.

![Graph of Growth Rate and Wage Share](image)

Figure 4: Trajectories of Growth Rate and the Wage Share ($s = 0.1$, $\alpha_1 = 0.02$, $\alpha_2 = 0.6$, $\psi_0 = -8k = 1$, $\rho = 0.8$, $s_0 = -0.83$, $A_1 = 0.1$, $A_2 = 0.1$)

The second order differential equation (11) can be solved for $\gamma(t)$. The complete solution is

$$\gamma(t) = \gamma^{ss} + A_1 \cos(\theta t) + A_2 \sin(\theta t)$$

(12)

where $\theta = -\left[\rho \frac{\alpha_2 \beta - \alpha_1}{s - \alpha_1 + \alpha_2 \beta}\right]^2/2$ and $\gamma^{ss} = \frac{s - \alpha_1 + \alpha_2 \beta}{\alpha_2 \beta - \alpha_1} \rho s - \frac{s(\alpha_2 \psi_0 + s_0)}{\alpha_2 \beta - \alpha_1}$. Alternatively the solution can be written as
\( \gamma(t) = \gamma^s + A \cos(\theta t + \phi) \)  \hspace{1cm} (13)

The dynamic behavior of \( \gamma \) defines also the behavior of utilization, the growth and the wage share in the long run. Formally, this can be derived by substituting the solutions of equation (12) (or 13) into equations (5) to (7). Figure 4 presents the trajectories of the growth rate and the wage share for certain values of the parameters. The oscillation of \( \gamma \) drive the growth rate and the wage share, which oscillate with the same frequency.

5 Non-linear distribution

The discussion so far has not dealt with non-linearities in distribution. Our system is like in figure (3). The discussion of the previous section implies that in the medium run, the changes in \( \gamma \) as described by equation (12) (or 13) will shift the demand curve on the base of a linear distribution curve. How then can this system describe the non-linearities that were discussed in the introduction?

An answer to this can be given by overhead labor. A certain proportion of labor input is necessary for the production, irrespectively of the level of economic activity and it does not react to changes in the level of utilization\(^{10}\). The share of the wages of the overhead labor in total income \( (\psi_F) \), can be written as

\[
\psi_F = \bar{\psi}_F / u
\]

where \( \bar{\psi}_F \) is a constant. In this case \( \psi \) is the share of the wages of variable labor in income. Thus the total share of wages in income is

\[
\psi_T = \psi + \psi_F
\]

Profit squeeze continues to hold, thus the share of variable labor behaves like in equation (4). Equation (15) can be restated as

\[
\psi_T = \psi_0 + ku + \bar{\psi}_F / u
\]

The total wage share is U-shaped; it decreases as utilization increases for low levels of utilization and increases at higher levels. Under the same specification of the investment function \( (g^i) \) is a function of \( \psi \) and not \( \psi_T \), the dynamics of the system are similar with those of figure 2.

\(^{10}\)Kalecki (1971, ch.6) makes special mention of salaries, which (as opposed to wages) “because of their “overhead” character are likely to fall less during the depression and rise less during the boom”.

11
The formulation of the previous section reminds, not surprisingly, the formulation of a simple harmonic oscillator in Classical mechanics. Harmonic oscillators can be found in many different areas of mechanics and physics. For the purpose of the present paper we will focus on the simple “ideal” spring-mass system.

A simple mass spring system is presented in figure 5. A single mass \( m \) is attached to a spring and moves on one dimension on a horizontal surface without friction. The system has an equilibrium-rest position \( A \). If the mass is displaced from its equilibrium position, the spring exercises a restoring force on the mass. The force is proportional to the distance of the mass from the equilibrium position, \( x \), and can be described with the following equation

\[
F = -kx
\]  

(17)

where \( F \) is the force exercised by the spring on the mass, and \( k \) is the spring constant. Equation (17) expresses the so-called Hooke’s law.

From the second law of Newton, we know that

\[
F = m \frac{d^2x}{dt^2} = m \ddot{x}
\]  

(18)

From equations (17) and (18) it is straightforward that

\[
\ddot{x} = -\frac{k}{m}x
\]  

(19)

This equation is analogous to the equation (11) in section (4). It can be solved for \( x(t) \)
\[ x(t) = A_1 \cos(\omega t) + A_2 \sin(\omega t) \]  

where \( \omega = \sqrt{k/m} \) is the angular frequency.\(^{11}\)

A comparison of equation (11) with equation (19) shows that the term \( \rho(\alpha_2 \beta - \alpha_1) \) of the former is analogous to the Hooke’s law spring constant \( k \). This constant expresses the restoring force of the spring when the mass is displaced from its rest position.

In the case of our model, when utilization (and the growth rate) increases two forces are triggered. One is the accelerator force expressed with the variable \( \alpha_1 \); increasing utilization, increases the growth rate, which in turn with the mediation of the medium-run adjustment mechanism of equation (9) will increase \( \gamma \) which will increase again utilization. This is a force that draws the short run equilibrium away from its center of gravitation, its medium-run steady state.

The opposite happens in the case of the second force, the profit-squeeze, which is expressed with term \( \alpha_2 \beta \). An increase of utilization triggers an increase of the wage share which in turn tends to decrease the growth rate which in turn with the mediation of the medium-run adjustment mechanism of equation (9) will decrease \( \gamma \) and utilization. This is a restoring force. In both cases the mechanism passes through the medium-run adjustment mechanism of equation (9) and this is the reason for the presence of the term \( \rho \).

Thus, the net force that is exerted on the short-run equilibrium when utilization moves away from its steady state value is equal to \( -\rho(\alpha_2 \beta - \alpha_1) \) times the distance of the the short-run equilibrium from its steady state. The condition we stated above that cyclical fluctuations require \( \alpha_2 \beta > \alpha_1 \) is tantamount to saying that the net force exerted on the system when this drifts away from its medium-run equilibrium is restoring.

Another interesting point that comes from the comparison of equations (11) and (19) is related with their denominators. In the latter equation we see that the denominator is equal to the inertial mass. This inertial mass then is negatively correlated with the frequency of the oscillation. Inertia is a property of the matter to resist changes in its velocity. Similarly, \( s - \alpha_1 + \alpha_2 \beta \) expresses the “inertia” of the economy of the model, the resistance of the level of utilization, the growth rate and distribution to change. Since the economy is demand-driven, saving increases the resistance to change; the higher the saving rate is the smaller effect on the system of an increase in utilization. A similar role is played by the profit squeeze (this is expressed with the term \( \alpha_2 \beta \)), since demand is profit-led. On the other hand, the role of utilization as stimulant of investment (expressed with \( \alpha_1 \)) acts as a factor against inertia; an increase in utilization will have a higher effect on the system the higher \( \alpha_1 \) is. This is another way to think about what is usually called the Keynesian stability condition.

\(^{11}\)Alternatively, equation (20) can be written as \( x(t) = A \cos(\omega t + \phi) \)
7 Cycles and U-shapes; an empirical and theoretical evaluation

Several contributions to the theory and the empirics of the business cycle over the last decade have shown that capacity utilization and the wage share follow counter-clockwise paths in the \( < u, \psi > \) space in a manner similar to that outlined by Goodwin (1967).\(^\text{12}\) The “Goodwin-cycles” have come to be considered as a stylized fact for at least the post-war US economy. In this section we compare our exposition with this literature.

![Figure 6: A hypothetical example](image)

An obvious argument against the U-shaped distribution schedule is that it does not produce cycles. The “northern” part of the cycle is not present in a “U”. How then is compatible the framework of this paper with the stylized facts of the US economy? We can start our discussion with a fictional example, as presented in figure 6. Each period is denoted with the number on the right of the dot. The distribution is U-shaped and we can imagine the cycle being driven by changes in demand. As we can see the economy of this fictional example starts in period 1 from a low level of utilization and relatively high wage share; as the economy expands the wage share decreases until period 4 and

\(^{12}\)These contributions include among others Barbosa-Filho and Taylor (2006); Mohun and Veneziani (2008); Zipperer and Skott (2011) and several chapters in Flaschel and Landesmann (2008).
then it increases until period 8. Then a large decrease in demand leads to a low utilization equilibrium, on the downwards-sloping part of the distribution schedule. If we connect the dots, the path of this cycle appears as a counterclockwise cycle. In other words, certain paths of wage share and utilization that appear as Goodwin-type cycles, if seen from a different perspective are described better by the mechanism that was outlined in the previous sections of this paper.

In fact, a pattern similar to this fictional example emerges if we look at the actual US of the period 1973 to 1979.\textsuperscript{13} The 1970s were a period of profitability crisis. There had preceded two and a half decades of consistently high growth rates, very low levels of unemployment and increase of real wages in tandem with productivity. As we can see in figure 7 1973 and 1974 are marked by high wage shares, almost 10% higher than the wage share at the beginning of the crisis in 2008. The profit squeeze together with the oil-crisis that took place at the same period led to a decrease in utilization of around 10 percentage points in 1975. The economy recovered from this crisis. This recovery was accompanied by an initial decrease in the wage share in 1976 and then an increase until 1979. The steep increase of the wage share between 1978 and 1979 highlights the pressures on profitability at the time.

In sub-figure 7a we show that if we connect the dots of the actual data we end up with a counterclockwise cycle. However, the same data can be explained with a U-shaped distributive curve as in sub-figure 7b.

Moreover, the cyclical behavior of the wage share and utilization à la Goodwin can be explained by lagged effects of utilization on distribution. To a large extent the wage contracts for a certain period, say $t$, are agreed upon and signed in the previous period, $t-1$, based on the available information in $t-1$. The same applies to the hiring decisions of the firms. To a large extent the employment level in period $t$ is determined by the hiring decisions in period $t-1$. The product of nominal wage times employment is the wage bill, which is then divided by nominal product gives the wage share. Therefore, the wage share for each period $t$ depends on the conditions in the period $t-1$. If the model of section 3 was in discrete time we could capture this effect by substituting equation (4) with an equation with lags (in its simplest form it would be something like $\psi_t = \psi_0 + \beta u_{t-1}$).\textsuperscript{14} In such a case when the utilization would start decreasing after the peak of the cycle, the wage share would keep increasing for a period of time and that would create a visual image of a cycle, although the un-

\textsuperscript{13}One important issue for the examination of Goodwin cycles is the source of the data. A detailed discussion is provided in Mohun and Veneziani (2008) and Zipperer and Skott (2011). The series for utilization presented in this section is the total index capacity utilization rate from the Federal Reserve Board (series G17/CAPUTL/CAPUTL.B50001.A). For the wage share we use the nonfarm business labor share index as published from the Bureau of Labor Statistics (id: PRS85006173).

\textsuperscript{14}More precisely two separate possibilities exist. One is that at the peak of the cycle the wage bill continues to increase, so in the next period when the downswing has begun, the wage share is higher. However, the wage share in this first period of the downswing will be higher even if the wage bill is lower than at the peak of the cycle, as long as its decrease is relatively smaller than the fall in output.
Figure 7: Utilization and the wage share in the 1970s
Figure 8: Utilization and the wage share, 1968-1974

(a) A cycle

(b) Profit squeeze
derlying process would not be essentially different from an upward sloping or U-shaped distributive schedule as in our model.

To make this point more clear in figure 8 we present data for the period 1967 to 1974, the cycle before the one we already presented in figure 7 above. The conditions were similar, high utilization and growth rates, low unemployment and pressure on profits. The vertical increase in the wage share in the period 1967 until 1969 is telling. The profit-squeeze of 1969 reduces utilization in 1970 and then in 1971. However, in the former year the wage share keeps increasing and decreases only in the latter. In 1972 utilization recovers, but the wage share keeps falling and finally in 1973 the increasing utilization is accompanied by an increase in the wage share. If we connect the dots as in figure 8a we get a “perfect” counter-clockwise cycle.

If we take into account the lagged effects of utilization on the wage share, the picture changes. In figure 8b we present the data for the same period, but we match the wage share of each year with the utilization rate of the previous year. Instead of the “perfect” counter-clockwise cycle we now get a “perfect” linear upward sloping trajectory. Note that this cycle took place on high utilization level (the trough of the cycle is with utilization around 80%), so the downward-sloping segment of the distributive schedule was never reached. In conclusion, if there is a profit squeeze, if the distributive schedule is upward sloping for high levels of utilization, the lagged effects on distribution will naturally tend to create counterclockwise cycles. However, the basic mechanism of the cycle remains the same; fluctuations of demand on top of quasi-stable distribution.

This way of looking at the cycle remains valid if there is not a profit-squeeze. For example, it is well known that in the last thirty years there has been a gradual decrease of the wage share as a result of the weakening bargaining power of the workers vis-à-vis the capitalists. The weakening position of workers is not only captured through downward shifts of the distributive schedule (a lower $\psi_0$ in equation 4) but also with a weaker effect of utilization on the wage share, a weaker squeeze on the profits (in terms of equation 4 that means a lower $\beta$). As a result, the effect of overhead labor dominates the behavior of distribution and thus the downward sloping part of the distributive curve extends to higher levels of utilization and flattens out at high levels of utilization. Finally, the economy over the same period has been running on average at lower levels of utilization.

In figure 9 we present data for the period 2001 to 2012. It is clear that the recovery after the recession of 2001-2002 did not lead to a profit squeeze. Instead as demand increased and the economy recovered the wage share decreased and then stabilized in 2007. This is a starkly different picture than the previous figures with the steep upward-sloping distribution. The crisis of 2008 contributed to a further downward shift of the distributive schedule. The recovery of the last four years has been

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15This decrease becomes even starker if we account for the increase in the inequality within the distribution of wages.
taking place on top of the downward sloping part of this lower distributive schedule.¹⁶

![Figure 9: Utilization and the wage share after 2000](image)

In this period there is an absence of counterclockwise cycles since there is no profit squeeze. Unlike the 1970’s the explanation of the recent recession has to be sought in the destabilizing effects of finance. Nevertheless the theoretical framework we outlined above remains valid, although the profit-squeeze has to be supplemented or replaced with a Minskyan destabilizing mechanism.

8 Epilogue

The present paper is hardly the first one to combine the insights of (Keynes,) Kalecki and Harrod with the Marxian idea of a profit squeeze. The idea that an unstable economic process can be contained with a counteracting stabilizing force is behind most of the theories of economic fluctuations. For example, Skott (2010, section 4.2) derives the counter-clockwise cycles of Goodwin by combining the instability principle of Harrod with the depletion of the reserve army of labor; an argument

¹⁶It is worth noting that the from this point of view the lower-right sub-figure in figure 1 is mistaken. The observations for 2011 and 2012—which were not available when Nikiforos and Foley (2012) was written—make the current interpretation more plausible.
similar to the one of the previous sections. Counterclockwise cycles are also derived by Schoder (2012), who combines a short run Kaleckian specification with Harrodian dynamics and various stabilization mechanisms.

What is then the contribution of this paper to this large corpus of work on economic fluctuations? In my opinion it is four-fold. First, it combines a clear connection between the short-run and the medium-run, based on the famous aphorism of Kalecki (1968), that the “long-run trend [medium run in this paper] is but a slowly changing component of a chain of short-period situations”. It was explained in detail how the expectations for the future (as formally expressed with \( \gamma \)) are the link and the driving force between the successive short runs. It was also explained how the discrepancy of the expectations from the realized outcomes, taken together with the so-called profit squeeze, leads to endogenous fluctuations.

Moreover, the way that these fluctuations are formalized is flexible to accommodate other causes of economic fluctuations, e.g. a changing saving rate at different phases of the cycle, technological factors or the role of financial crises à la Minsky and Kindleberger. These different sources can be combined at different frequencies, as happens in reality. For example, the Minskyan transition from the hedge to speculative and Ponzi finance, takes a longer period of time to develop than the ordinary business cycle.\(^\text{17}\) The approach followed here can provide a formalization of these various sources, and their intertwined but distinctive frequencies. This formalization can gain intuition and tools from the analogy of the cycle with a classical harmonic oscillator.

Finally, the empirical discussion of the last section is also—to the best of my knowledge—novel, and shows how the counterclockwise cycles can be understood as simple fluctuations on top of a quasi-stable U-shaped or upward slopping distributive schedule. It also shows that the analytical framework of the paper is valid and can be easily extended to interpret cycles that do not involve profit-squeeze or counterclockwise cycles.

\(^\text{17}\)A formal exposition of long Minskyan waves combined with short-run Harrodian cycles is given by Ryoo (2010, 2013).
References


Appendix

A Second Order, Linear, Differential Equations

A second order, linear, autonomous equation can be written as

\[ \ddot{y} + a_1 \dot{y} + a_2 y = b \]  \hspace{1cm} (21)

The so-called characteristic equation of this second order differential equation can be written as

\[ r^2 + a_1 r + a_2 = 0 \]  \hspace{1cm} (22)

The eigenvalues of equation (22) are:

\[ r_1, r_2 = \frac{-a_1 \pm \sqrt{a_1^2 - 4a_2}}{2} \]  \hspace{1cm} (23)

The eigenvalues can be either real or complex numbers, depending on the discriminant of equation (23), \( \Delta = a_1^2 - 4a_2 \). If \( \Delta \geq 0 \) the eigenvalues are real, while if \( \Delta < 0 \) the eigenvalues are complex.

The complete solution for equation (21) is

\[ y(t) = \frac{b}{a_2} + A_1 e^{r_1 t} + A_2 e^{r_2 t} \]  \hspace{1cm} (24)

where \( A_1 \) and \( A_2 \) are arbitrary constants of integration. If the eigenvalues are complex equation (23) can be written as \( r_1, r_2 = h \pm \theta i \), where \( h = -a_1/2 \) and \( \theta = \sqrt{4a_2 - a_1^2} \). In this case, by using Euler’s formula\(^{18} \) and some manipulation, the complete solution of equation (24) can be restated as

\[ y(t) = \frac{b}{a_2} + e^{ht} (A_3 \cos \theta t + A_4 \sin \theta t) \]  \hspace{1cm} (25)

where again \( A_3 \) and \( A_4 \) are arbitrary constants of integration.

A.1 A special case: \( a_1 = 0 \)

In the special case that \( a_1 = 0 \) equation (21) can be rewritten as

\[ \ddot{y} + a_2 y = b \]  \hspace{1cm} (26)

and the solutions of the characteristic equation (22) are:

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\(^{18}\)Euler’s formula states that \( e^{ix} = \cos x + i \sin x \) for any real number \( x \).
If $a_2$ is negative then $r_1, r_2$ are real numbers and $y(t)$ is unstable. On the other hand if $a_2$ term is positive, the discriminant becomes $\Delta = -4a_2 < 0$. Thus the solutions are complex numbers without a real part (or with a real part equal to zero):

$$r_1, r_2 = \pm i\sqrt{a_2}$$  \hfill (28)

Equation (25) can be restated as

$$y(t) = \frac{b}{a_2} + (A_3 \cos \theta t + A_4 \sin \theta t)$$  \hfill (29)

In this case $y(t)$ will oscillate around its state state value with constant amplitude.

Using some basic trigonometric identities we can write the solution in yet another way

$$y(t) = \frac{b}{a_2} + A \cos(\theta t + \phi)$$  \hfill (30)

where $\theta$ is the initial phase angle, the angle for $t = 0$, and $A$ is the amplitude of the oscillation.

Finally, in both cases $\theta$ is the angular frequency and is equal to $\theta = 2\pi f$, where $f$ is the frequency of the oscillation, the number of cycles per unit of time. Using $f$ we can derive the period $T$, the time required for a complete cycle.