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# The 'flypaper effect' is not an anomaly

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ABSTRACT

"The 'flypaper effect' is not an anomaly"

by

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An in-kind subsidy is equivalent, both theoretically and empirically, to an increase of income for an individual consumer. But the equivalence does not empirically carry over to in-kind grants by a central government to a local one: this has been seen as an anomaly and dubbed the "flypaper effect."

We argue that the "anomaly" label is incorrect: the nonequivalence of increases in grants and community income is predicted, almost everywhere, by models that understand collective decision as the outcome of electoral competition among political parties. In addition, we compute politico-economic equilibria for a model with two independent tax parameters and obtain numerical values that agree with the existing empirical literature.

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## 1. Introduction

We learn in introductory economics that an in-kind subsidy to a consumer is theoretically equivalent to an increase in the consumer's income (as long as the constraint of consuming at least the amount of the in-kind subsidy does not bind). Moreover, there is some empirical evidence of the equivalence.

Does the equivalence carry over to in-kind subsidies by a central government to a local one? More precisely, does an increase in the amount of local public goods financed by the federal government have the same effect as an increase in the income of the community on the total amount of the local public good eventually supplied? The empirical answer is a resounding "no": Hines and Thaler (1995, Table 1) report on ten studies where an additional dollar of a federal grant earmarked for the consumption of the public good increases the supply of local public goods, on the average, by 63.7 cents, an amount substantially larger than the increase in the supply of local public goods caused by an additional dollar in the community's income.

The gap has been dubbed the "flypaper effect," and, because it conflicts with the predictions of the individual decision model or simple extensions thereof, it has been labeled an anomaly: see Hines and Thaler (1995) and the references therein. It is the aim of the present paper to argue that this label is naïve, because the gap agrees with the predictions of sensible models of collective decision making.

More specifically, we discuss the predictions of politico-economic equilibrium models, where two parties compete. First, in the uni-dimensional policy context, we use the well-known median-voter model, which there applies, and discuss special cases where the equivalence obtains, such as the one presented by David Bradford and Wallace Oates (1971, Section III), as well as other cases where it does not. In a nutshell, the equivalence requires that the income formation

pattern and the tax rules precisely match, an unrealistic feature because they are determined by independent processes. Moreover, realistic tax policies are two-dimensional. We therefore proceed, in Section 3, to apply a concept, recently proposed by one of us, of politico-economic equilibrium in the case where policy spaces are multidimensional. We offer a simulation of a model with a two-dimensional space of income-tax regimes where an additional thousand dollars of a federal grant earmarked for the consumption of the public good increases the supply of local public goods, on average, by precisely $635 -- virtually identical the marginal propensity to consume out of public funds that Hines and Thaler report.

**2. Equilibrium with one-dimensional policies**

Example

There are $n$ consumer-citizens. The amount of the public good is denoted $G$, and the amount of the private good consumed by i is denoted $m^i$. Assume fixed income shares, i.e, $i$'s income is $?^i w$, where $w$ is aggregate income. The federal government's grant, earmarked for the provision of the public good, is denoted $s$. Let the price of both the public good and the private good be equal to one (i.e., the community can transform one unit of the private good into one unit of the public good). Assume that the public good must financed by a lumpsum ("head") tax denoted $h$ . Thus, the community's decision is unidimensional: the community has to decide on the level of the single variable $h$ with the understanding that the final amount of public good will be $s + nh$.

Let the utility function of consumer-citizen $i$ be $u^i(G, m^i) = ?^i \ln G + (1 - ?^i) \ln m^i$. Given $w$ and $s$, consumer-citizen $i$'s indirect utility depends only on level of the policy variable $h$, according to the function:

$$v^i(h; w, s) = ?^i \ln (s + nh) + (1 - ?^i) \ln (?^i w - h),$$

defined on the domain where $s + nh > 0$ and $?^i w > h$. We compute:

$$\frac{?v^i}{?h} ? ?^i \frac{n}{s ? nh} ? (1 ? ?^i)\frac{1}{?^i w ? h} ? \frac{?^i n?^i w ? ?^i nh ? s ? nh ? ?^i s ? ?^i nh}{(s ? nh)(?^i w ? h)} ? \frac{?^i n?^i w ? s ? nh ? ?^i s}{(s ? nh)(?^i w ? h)}.$$

The denominator is positive on the domain of $v^i$. The numerator is

positive for $h$? $[0, ?^i ?^i w - (1/n)(1- ?^i)s)$,

zero for $h = ?^i ?^i w - (1/n)(1- ?^i)s$, and

negative for $h > ?^i ?^i w - (1/n)(1- ?^i)s$.

Thus, for given $(w, s)$, $v^i(h; w, s)$ is a single peaked function of the policy variable $h$, with peak at $max\ \{0, ?^i ?^i w - (1/n)(1- ?^i)s\}$. Under several specifications of the nature of political competition between two parties, political equilibrium leads to the level of $h$ preferred by the median voter (i.e., the one with the median peak)[1]. Assume that consumer-citizen $j$ is the median voter both at $(w, s)$, and in a neighborhood of $(w, s)$, and that her preferred point is interior. Then the equilibrium head tax in that neighborhood is $\tilde{h}(w,s) \; ? \; ?^j ?^j w - (1/n)(1 - ?^j)s$, with partial derivatives $\tilde{h}_w(w,s) = ?^j ?^j$ and $\tilde{h}_s(w,s) = - (1-?^j)\ (1/n)$.

Denote by $\tilde{G}(w,s) = s + n\ \tilde{h}(w,s)$ the community's equilibrium level of the public good, as a function of $w$ and $s$, and by $\tilde{G}_w$ and $\tilde{G}_w$ its partial derivatives. Then

$$\frac{\tilde{G}_w}{\tilde{G}_s} ? \frac{n? ^j ?^j}{1 ? (1 ? ? ^j)} ? \frac{?^j}{\frac{1}{n}},$$

which is the ratio of the median voter's income to mean income. If income is always equally distributed, then $?^j ? \frac{1}{n}$, and the equivalence between income and federal grants holds. But this requires that the pattern of income formation, parametrized by $?^j$, match the contribution rule for the financing of the public good. If there is no match, as when the income of the median voter is less than average income, then the ratio of the two propensities to consume will not be one. If the parameters $?^j$ are equal, then the median voter is the one with the median income. A social marginal propensity to consume the public-good out of income lower than that out of federal grant

[1] It is well –known that this 'median voter theorem' holds when (1) the two 'parties' are merely two opportunistic candidates for office, each of whom desires solely to win the election, and (2) both candidates know with certainty the distribution of voter preferences (types). If we retain assumption (2), then a median-voter theorem continues to hold should the candidates have policy preferences, and each desires to maximize his expected utility, where no particular utility is attached to office-holding other than its provision of the opportunity to implement policy. For elaboration, see Roemer (In press, Chapter 1).

would then not "anomalous" in our example, but just the consequence of the common observation than median income is less than mean income.

The example has shown that the community's marginal propensity to spend in the public-good out of income differs from that out of federal grant if the income pattern fails to match the relevant features of the contribution scheme. This turns out to be true in general, as we show now by considering arbitrary utility functions, income patterns and one-dimensional contribution rules.

Denote by $u^i(G, m^i)$ $i$ 's utility function , $i = 1,\ldots, n$, assumed to be twice continuously differentiable.

An <u>income pattern</u> specifies the relation between individual income and aggregate income. It formally is a n-tuple of differentiable functions $\{\tilde{w}^i : \mathbf{R}_+ \to \mathbf{R} : i = 1,\ldots, n\}$ satisfying $\sum_i \tilde{w}^i(w) = w, \forall w$, which in turn implies

$$\sum_i \tilde{w}^{i\prime}(w) = 1, \quad \forall w, \qquad (2.1)$$

where $\tilde{w}^{i\prime}(w)$ denotes the derivative of $\tilde{w}^i(w)$. In the example, $\tilde{w}^i(w) = \theta^i w$.

The community has to choose the value of a single parameter, to be abstractly denoted $\theta$, in an otherwise exogenous contribution (or tax) rule that specifies $i$'s contribution towards the public good given $i$'s income and the value of $\theta$ ( $i = 1,\ldots, n$). A <u>one-dimensional contribution rule</u> is formally an $n$-tuple of functions $\{c^i: \mathbf{R}_+ \times \mathbf{R} \to \mathbf{R} : i = 1,\ldots, n\}$, which determines the total supply of the public good, given $s$, as $s + \sum_i c^i(w^i, \theta)$. In the example, $\theta$ was written $h$, $c^i(w^i, h) = h$, and the total supply of the public good was $s + \sum_i c^i(w^i, h) = s + nh$.

Consumer-citizen $i$'s indirect utility function is now

$v^i(\theta; w, s) = u^i(s + \sum_k c^k(\tilde{w}^k(w), \theta), \tilde{w}^i(w) - c^i(\tilde{w}^i(w), \theta))$.

Assume that, for $i = 1,\ldots, n$, and given $(w, s)$, $v^i$ is single peaked (in $\theta$). Let $j$ be the median voter in a neighborhood of $(w, s)$, and assume that $j$'s preferred point is interior. For $i = 1,\ldots, n$, let

$u^i_G$ denote the partial derivative of $u^i$ with respect to $G$

$u^i_m$ denote the partial derivative of $u^i$ with respect to $m^i$,

$c^i_w$ denote the partial derivative of $c^i$ with respect to $w^i$,

$c^i_\theta$ denote the partial derivative of $c^i$ with respect to $\theta$,

and write the partial derivative of $v^j$ with respect to $\theta$ as

$$L(\theta, w, s) = u^j_G(s + \theta_k c^k(\tilde{w}^k(w), \theta),\ \tilde{w}^j(w) - c^j(\tilde{w}^j(w), \theta))\ \theta_k c^k_\theta(\tilde{w}^k(w), \theta)$$
$$- \quad u^j_m(s + \theta_k c^k(\tilde{w}^k(w), \theta),\ \tilde{w}^j(w) - c^j(\tilde{w}^j(w), \theta))\ c^j_\theta(\tilde{w}^j(w), \theta). \qquad (2.2)$$

Let $L_\theta$, $L_w$ and $L_s$ denote the partial derivatives of $L$. If $L_\theta(\theta, w, s) < 0$ at $j$'s preferred point, then the first-order condition "$L(\theta, w, s) = 0$" implicitly defines the community's equilibrium level of the parameter $\theta$ locally as a function $\tilde{\theta}(w, s)$, with partial derivatives

$$\tilde{\theta}_w = -\frac{L_w}{L_\theta} \text{ and } \tilde{\theta}_s = -\frac{L_s}{L_\theta}.$$

Define $\tilde{G}(w, s) \equiv s + \theta_k c^k(\tilde{w}^k(w), \tilde{\theta}(w, s))$. The community's marginal propensity to consume the public good out of income is the partial derivative

$$\tilde{G}_w = \theta_k c^k_w \cdot \tilde{w}^k{}' - c^k_\theta \cdot \frac{L_w}{L_\theta},$$

and the marginal propensity to consume out of a federal grant is

$$\tilde{G}_s = 1 - \theta_k c^k_\theta \cdot \frac{L_s}{L_\theta}.$$

The difference between the two is, therefore

$$\tilde{G}_w - \tilde{G}_s = \frac{1}{L_\theta}\left[L_\theta \cdot \theta_k c^k_w \cdot \tilde{w}^k{}' - L_w \cdot \theta_k c^k_\theta\right] - \left[L_\theta - L_s \cdot \theta_k c^k_\theta\right] \qquad (2.3)$$

Denote by $\begin{bmatrix} u^j_{GG} & u^j_{Gm} \\ u^j_{mG} & u^j_{mm} \end{bmatrix}$ the Hessian matrix of the median voter's utility function $u^j$, and, for $i = 1, \ldots, n$, denote by $\begin{bmatrix} c^i_{ww} & c^i_{w\theta} \\ c^i_{w\theta} & c^i_{\theta\theta} \end{bmatrix}$ the Hessian matrix of $c^i$. By differentiating (2.2), we obtain the following expressions for $L_\theta$, $L_w$ and $L_s$, which can then be plugged into (2.3).

$$L_\theta = \left[u^j_{GG} \cdot \theta_k c^k_\theta + u^j_{Gm} \cdot c^j_\theta\right]\left[\theta_k c^k_\theta + \theta_k c^k_{\theta\theta} \cdot u^j_G + \left[u^j_{mG} \cdot \theta_k c^k_\theta + u^j_{mm} \cdot c^j_\theta\right]\left[c^j_\theta + c^j_{\theta\theta} u^j_m\right]; \qquad (2.4)$$

$$L_w = \left(u^j_{GG} + \sum_k c^k_w \tilde{w}^k\right)\left(u^j_{Gm} + \left(1 + c^j_w\right)\tilde{w}^j\right) + \sum_k c^k_\gamma \left(u^j_{Gm} + c^j_{\gamma w}\tilde{w}^j\right)u^j_G ; \quad (2.5)$$
$$+ \left(u^j_{mG} + \sum_k c^k_w \tilde{w}^k\right)\left(u^j_{mm} + \left(1 + c^j_w\right)\tilde{w}^j\right) + c^j_\gamma + c^j_{\gamma w}\tilde{w}^j u^j_m$$

$$L_s = u^j_{GG} + \sum_k c^k_\gamma + u^j_{mG} c^j_\gamma . \quad (2.6)$$

If we do plug (2.4-6) into (2.3), then we obtain a rather complex expression for $\tilde{G}_w - \tilde{G}_s$, which will typically differ from zero for arbitrary income patterns and one-dimensional contribution rules. We now show that, given a one-dimensional contribution rule, (2.3) will equal zero only if the income pattern exactly "matches" the contribution rule, as illustrated in the example. The match should be seen as an unusual occurrence, because the income formation process and the determination of contribution rules are independent phenomena, and only exceptionally will they match. We consider two forms of one-dimensional contribution rules.

Contribution form 1. An individual contributes an exogenously given fraction of expenditure

Assume (as in Bradford and Oates, 1971) that person *i* contributes a fixed fraction $\alpha^i$ of the public-good expenditure, i.e., the single parameter $\gamma$ is $G - s$, and *i's* contribution is:

$$c^i(w^i, \gamma) = \alpha^i \gamma, \; i = 1,\ldots,n,$$

where $(\alpha^1, \ldots, \alpha^n)$ are given, and satisfy $\sum_i \alpha^i = 1$. Note that the head tax of the example above is a special case, for $\alpha^i = 1/n$, $\forall$ i (and $nh = \gamma$). Now $c^i{}_w = 0$, $c^i{}_\gamma = \alpha^i$, and the second order partial derivatives of $c^i$ are zero. Expressions (2.3-6) become, respectively

$$\tilde{G}_w - \tilde{G}_s = \frac{1}{L_\gamma}\left(L_w - \left(L_\gamma - L_s\right)\tilde{w}^j\right), \quad (2.7)$$

$$L_\gamma = \left(u^j_{GG} + u^j_{Gm}\alpha^j\right) + \left(u^j_{mG} + u_{mm}\alpha^j\right)\tilde{h}^j \quad , \quad (2.8)$$

$$L_w = u^j_{Gm}\tilde{w}^j + u^j_{mm}\tilde{w}^j\tilde{h}^j , \quad (2.9)$$

$$L_s = u^j_{GG} + u^j_{mG}\alpha^j . \quad (2.10)$$

Then

$\tilde{G}_w - \tilde{G}_s = 0 \Rightarrow -L_w = L_\gamma - L_s \Rightarrow -\left(u^j_{Gm} + u^j_{mm}\frac{d\tilde{w}^j}{dw}\right) = -\left(u^j_{mG} + u^j_{mm}\frac{d\tilde{w}^j}{dw}\right)$

$\Rightarrow \tilde{w}^{j\prime} = \gamma^j$ (as long as $\tilde{G}_s > 0$, and hence, $-\left(u^j_{mG} + u^j_{mm}\frac{d\tilde{w}^j}{dw}\right) > 0$),

In words, the tax rule and the income pattern must match in the sense that, as total income increases, the income of the median voter must increase at the rate of her fixed contribution to the public good, i.e., $\tilde{w}^j(w) = k^j + \gamma^j w$, for some parameter $k^j$. This equality is (implicitly) postulated by Bradford and Oates (1971), and they accordingly obtain the equivalence result.[2]

The tax rule and the income pattern did not match in the example when, for the median voter, $\tilde{w}^j(w) = \gamma^j < 1/n = \gamma^j$. More generally, if $\tilde{w}^{j\prime}(w) = \gamma^j$ and $\gamma^j = 1/n$, then from (2.7-10) we obtain:

$$\frac{\tilde{G}_w}{\tilde{G}_s} = \frac{\left(u^j_{Gm} + u^j_{mm}\left(\frac{1}{n}\right)\gamma^j\right)}{\left(u^j_{mG} + u^j_{mm}\left(\frac{1}{n}\right)\frac{1}{n}\right)} = n\gamma^j,$$

as in the example. Therefore, independently of the utility function, $\frac{\tilde{G}_w}{\tilde{G}_s}$ equals the ratio of the median income to the mean income as long as the contribution is a head tax, and relative incomes remain constant.

Contribution form 2. A linear income tax

Suppose now that contributions are proportional to income, i.e., the parameter $\gamma$ is now the constant average (and marginal) rate of a linear income tax. Formally, $c^i(w^i, \gamma) = \gamma w^i$, $i = 1,\ldots,n$. Now $c^i_w = \gamma$, $c^i_\gamma = w^i$, and the Hessian $\begin{pmatrix} c^i_{ww} & c^i_{w\gamma} \\ c^i_{\gamma w} & c^i_{\gamma\gamma} \end{pmatrix}$ of $c^i$ is $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$.

[2] Bradford and Oates (1971, p. 419) acknowledge the lack of realism of such a rule. Their (implicit) assumption is $\tilde{w}^i(w) \equiv \bar{w}^i + \gamma^i\left(w - \sum_k \bar{w}^k\right)$, $i = 1,\ldots, n$, which guarantees a stronger form of equivalence, where not only the amount of the public good, but also the final allocation of the private good is identical under either an increase in income or a federal grant.

Using (2.1), expression (2.3) now becomes

$$\tilde{G}_w - \tilde{G}_s - \frac{1}{L_\rho}\left(L_\rho - wL_w\right)\left(L_\rho - wL_s\right). \qquad (2.11)$$

The first order condition "$L(\rho, w, s) = 0$" now reads

$$u_G^j(w)\,w - u_m^j(w)\,w^j = 0. \qquad (2.12)$$

Expressions (2.4-6) become, respectively

$$L_\rho = \left(u_{GG}^j(w) + u_{Gm}^j(1-\rho)\tilde{w}^{j-\rho}\right)w + \left(u_{mG}^j(w) + u_{mm}^j(1-\rho)\tilde{w}^{j-\rho}\right)w^j,$$

$$L_w = \left(u_{GG}^j\rho + u_{Gm}^j(1-\rho)\tilde{w}^{j-\rho}\right)w + u_G^j + \left(u_{mG}^j\rho + u_{mm}^j(1-\rho)\tilde{w}^{j-\rho}\right)w^j + \tilde{w}^{j-\rho}u_m^j$$
$$= \left(u_{GG}^j\rho + u_{Gm}^j(1-\rho)\tilde{w}^{j-\rho}\right)w + \left(u_{mG}^j\rho + u_{mm}^j(1-\rho)\tilde{w}^{j-\rho}\right)w^j + u_m^j\left(\frac{w^j}{w} + \tilde{w}^{j-\rho}\right),$$

where (2.12) has been used, and

$$L_s = u_{GG}^j(w) + u_{mG}^j(w)\,w^j.$$

Substituting the last three expressions into (2.11) we obtain:

$$\tilde{G}_w - \tilde{G}_s = 0 \quad \left(L_\rho - wL_w\right) - \left(L_\rho - wL_s\right) = 0 \quad (1-\rho)L_\rho - w(L_w - L_s) = 0$$

$$(1-\rho)\left(u_{GG}^j w w + u_{Gm}^j w^j w + u_{mG}^j w w^j + u_{mm}^j w^j w^j\right)$$
$$- w\left(u_{GG}^j\rho + u_{Gm}^j(1-\rho)\tilde{w}^{j-\rho}\right)w - w\left(u_{mG}^j\rho + u_{mm}^j(1-\rho)\tilde{w}^{j-\rho}\right)w^j - u_m^j\left(\frac{w^j}{w} + \tilde{w}^{j-\rho}\right)$$
$$- w\left(u_{GG}^j w + u_{mG}^j w^j\right) = 0$$

$$(1-\rho)\left(u_{GG}^j w w + u_{Gm}^j w^j w + u_{mm}^j w^j w^j\right) + u_{mG}^j w w^j - \rho\left(u_{GG}^j w w + u_{mG}^j w w^j\right) - (1-\rho)\left(u_{Gm}^j w w^j + u_{mm}^j w^j w^j\right)$$
$$- u_m^j\left(\frac{w^j}{w} + \tilde{w}^{j-\rho}\right) - u_{GG}^j w w - u_{mG}^j w w^j = 0$$

$$\rho\left(u_{GG}^j w w + u_{mG}^j w w^j\right) - (1-\rho)\left(u_{Gm}^j w w^j + u_{mm}^j w^j w^j\right) - u_m^j\left(\frac{w^j}{w} + \tilde{w}^{j-\rho}\right) - u_{GG}^j w w - u_{mG}^j w w^j = 0$$

$$? \quad (1 ? ?)\left[u^j_{GG} ?w ?w ? u^j_{Gm} ?w^j ?w ? u^j_{mm} ?w^j ?w^j\right]$$

$$? (1 ? ?)\left[? w ?u^j_{GG} ?w ? w ?u^j_{Gm} ?\tilde{w}^{j\prime} ?w ? w ?u^j_{mm} ?\tilde{w}^{j\prime} ?w^j\right] ? w ?u^j_m ?\left[\frac{w^j}{w} ? \tilde{w}^{j\prime}\right] ? 0$$

$$? \quad (1 ? ?)\left[? w ?u^j_{Gm} ?\left[\frac{w^j}{w} ? \tilde{w}^{j\prime}\right] ? w^j ?u^j_{mm} ?\left[\frac{w^j}{w} ? \tilde{w}^{j\prime}\right] ? u^j_m ?\left[\frac{w^j}{w} ? \tilde{w}^{j\prime}\right] ? 0\right]$$

$$? \quad \left[\frac{w^j}{w} ? \tilde{w}^{j\prime}\right]\left[(1 ? ?) ? w ?u^j_{Gm} ? w^j ?u^j_{mm} ? u^j_m\right] ? 0,$$

which, as long as $(1 ? ?)\left[? w ?u^j_{Gm} ? w^j ?u^j_{mm}\right] ? u^j_m ? 0$, is equivalent to $\tilde{w}^{j\prime}(w) ? \dfrac{\tilde{w}^j(w)}{w}$, i.e., to the income of the median voter (locally) being a linear function $\tilde{w}^j(w) ? ?^j w$ of aggregate income, as in the income pattern of the example. Again, the conventional wisdom of the equivalence between federal grants and income is valid here if the rules of income formation and taxation match.

The two types of tax rules just discussed are oversimplifications: existing effective income taxes are realistically approximated by a tax function with a constant marginal tax rate and a decreasing average tax rate. If we rule out negative taxes (in order to separate the financing of the local public good from redistributive taxation), then the amount paid by a consumer-citizen with income $w^i$ is

$$\max \{0,\ t\,(w^i - b)\}, \qquad (2.13)$$

where $t$ is the marginal tax rate, and $b$ is the income exempt from taxation. The tax rule is then described by two parameters, whereas the discussion of the previous section was restricted to one-dimensional policies (with only one parameter in the tax function). We consider two-dimensional policies next.

## 3. Equilibrium with multi-dimensional policies

### A. Theory

We now adopt the two-dimensional tax rule (2.13). Formally, we view a *policy* as a triple $? = (b, t, G)$: there is a set of feasible policies in $\mathbf{R}^3$, but a balanced budget constraint implies that the

policies are restricted to a two-dimensional manifold of $\mathbf{R}^3$. With a polity consisting of voters with heterogeneous incomes, there will in general be no Condorcet winner among these policies. Hence, there is no Nash equilibrium in the game between two candidates who each seek only to win the election – that is, no 'median voter theorem.' We require another theory of political competition to construct a coherent concept of political equilibrium. We shall here use the theory recently introduced in Roemer (1998, 1999), and further elaborated in Roemer (In press).

That theory, which we first review briefly and informally, conceives of political competition as taking place between two parties, in which:

1. each party 'represents' a coalition of citizens,
2. parties consist of factions with different interests, and
3. parties are uncertain about the exact distribution of voter types.

Here we assume that all voters have the same utility function, and differ only in their income capacity.[3] Imagine, for the moment, that all citizens whose income capacity is less than some number $w^*$ belong to the Left party, and all others belong to the Right party. Let T denote the set of feasible policies, i.e., such that $b \geq 0$, $0 \leq t \leq 1$, and the government balances its budget. As there is uncertainty, we denote by $?(?^L, ?^R)$ the probability that policy $?^L$ defeats another policy $?^R$. We denote the indirect utility of the type with income capacity $w$ over policies by $v(?, w)$. Consider, now, type $w_L$, defined as the average income capacity of those in party L. We define three factions within each party, which are called *opportunists, reformists,* and *militants.* The opportunists are concerned to maximize the probability of victory – they are the *dramatis personae* of the Downs (1957) model. The reformists wish to maximize the expected utility of the party's members: we take this to mean they aim to maximize the expected utility of the average member. The militants are not concerned with victory, at least this time around: they want the party to propose a policy as close as possible to the party's ideal policy, which we take to be the ideal of the average member.

To be precise, facing a policy $?^R$ proposed by the opposition (Right), the three factions in Left behave as follows:

[3] We introduce, in this section, a distinction between a worker's income capacity – his income if he works full time – and his income.

* the opportunists would like to respond with a policy ? that maximizes $?(?,?^R)$;

* the reformists would like to respond with a policy ? that maximizes

$$?(?,?^R)v(?,w_L) ? (1 ? ?(?,?^R))v(?^R,w_L);$$

* the militants would like to respond with a ?? that maximizes $v(?,w_L)$.

Similarly, there are three factions in Right, who have analogous interests.

We now define a *party-unanimity Nash equilibrium* (PUNE) as a pair of policies $(?^L,?^R)$ such that

* given that Right is playing $?^R$, there is no policy that *all three factions* in Left would weakly prefer to play, and that one would (strictly) prefer to play instead of $?^L$, and ,

* given that Left is playing $?^L$, there is no policy that all three of Right's factions would weakly prefer to play, and one would (strictly) prefer to play instead of $?^R$.

Thus, the policy pair is Nash, where any deviation by a party must be unanimously agreed upon by its three factions.

It is generally the case that PUNEs exist with multi-dimensional policy spaces. Indeed, there is generally a two-dimensional manifold of PUNEs in the cross-product of policy spaces T x T, -- regardless of the dimension of T!

Our equilibrium concept is not yet complete, as we have yet to determine the 'pivotal' type $w^*$ that determines party membership. To do this, we invoke a notion of stationarity in party membership: namely, in equilibrium, all members of each party should prefer the policy proposed by their own party to the policy of the other party. Were this not the case, then we would expect that dissidents would 'vote with their feet,' and move to the other party.

In sum, we define a *party-unanimity Nash equilibrium with endogenous parties*[4], which henceforth we call an *equilibrium*, as a triple of incomes $\{w^*, w_L, w_R\}$ and a pair of policies $\{?^L, ?^R\}$ such that:

[4] The original definition of this equilibrium concept is in Roemer (In press, Chapter 13).

1. $w_L$ is the average income among those incomes less than $w^*$, and $w_R$ is the average wage income among those greater than or equal to $w^*$;
2. $\{?^L, ?^R\}$ is a PUNE with respect to the parties defined by $\{w^*, w_L, w_R\}$;
3. For all $w < w^*$, $v(?^L, w) ? v(?^R, w)$, and for all $w > w^*$, $v(?^L, w) ? v(?^R, w)$.

It is likewise true, in general, that there is a two-dimensional manifold of such equilibria in the space T x T.

We next define the function ?. Our primitive is the distribution of income, which is given by a probability measure **F** on the non-negative real numbers, with associated distribution function $F$. For any pair of policies, denote by $? (?^L, ?^R)$ the set of types (incomes) who prefer $?^L$ to $?^R$. Then, were certainty to hold, fraction $\mathbf{F}(? (?^L, ?^R))$ of the polity would vote Left. We now posit that this fraction is subject to a uniformly distributed error: thus the parties both believe that the actual fraction of the polity that will vote Left is uniformly distributed on the interval $? ? = [\mathbf{F}(? (?^L, ?^R)) ? ?, \mathbf{F}(? (?^L, ?^R)) ? ?]$, where $? ?$ is some positive number less than one (assuming that $? ? [0, 1]$). Thus, the probability that Left wins at this policy pair is the probability that a random variable, uniformly distributed on the above interval, is greater than one-half. It is easily computed that this probability is

$$? (?^L, ?^R) ? \frac{\mathbf{F}(? (?^L, ?^R)) ? ? ? .5}{2?}. \qquad (3.1)$$

**B. Application**

We are now ready to apply the multi-dimensional concept of political equilibrium to study the flypaper effect. We take as the direct utility function of all citizens, over consumption (*m)* and the public good (*G*) [5] :

$$u(G, m) ? ? \ln(G ? ?) ? (1 ? ?) \ln m. \qquad (3.2)$$

[5] Note that here we postulate that all citizens have the same utility function, but, for the sake of realism, we depart from the Cobb-Douglas formulation of the example of Section 2 by introducing the term ?. As is well known, this realistically yields affine Engel curves.

To recapitulate, a policy ? = (*b, t, G*) provides the public good in value *G*, taxes all income of a citizen above an exemption of *b*, at constant marginal rate *t*. As proposed by Jonathan Hamilton (1986), we introduce a cost of taxation. We assume, in particular, that citizens do not supply labor inelastically, although their preferences for leisure are not formally modeled. Without microfoundations, we posit that if the tax rate is *t* , then an individual with an income capacity of *w* works long enough to produce an actual income of

?? *w*, if $w < b$

?? $b + (1 - ? t)(w - b)$, if $w \geq b$,

where ? ? [0, 1] is a constant independent of *w*, so that her tax bill is $max\ \{0, t(1 - ? t)(w - b)\}$, and her after tax income is $min\ \{w, b + (1 – t)(1 - ? t)(w - b)\}$. We therefore can write the balanced-budget constraint as $g(b,t,G;s) ? 0$, where

$$g(b,t,G;s) ? ? \int_{b}^{?} t(1 ? ?t)(w ? b)d\mathbf{F}(w) ? s ? G, \qquad (3.3)$$

and *s* is the public-good grant provided by the federal government. To be explicit, define the set T of feasible policies to be all policies such that:

$$g ? 0, b ? 0, 0 ? t ? 1. \qquad (3.4)$$

We define the indirect utility function:

$$v(b,t,G;w) ? ? \ln(G ? ?) ? (1 ? ?)\ln(\min\{w, b ? (1 ? t)(1 ? ?t)(w ? b)\}).$$

We next require a characterization of the set $? (?^L, ?^R)$. Indeed, this set, for an arbitrary pair of policies ($?^L$, $?^R$) is rather complicated. We shall compute it only for policies that we expect to be characteristic of equilibrium. We have:

Proposition *Let* $?^L$ *and* $?^R$ *be two policies such that*

*(a)* $b^R < b^L$,

*(b)* $G^L > G^R$,

*(c)* $K(1 ? t^L)(1 ? ?t^L) ? (1 ? t^R)(1 ? ?t^R) ? 0$,

*where* $K ? (\frac{G^L ? ?}{G^R ? ?})^{\frac{?}{1?}}$. *Then* $? (?^L, ?^R) = \{w ? ? (?^L, ?^R)\}$, *where*

$$? (?^L, ?^R) ? \frac{b^R(1 ? (1 ? t^R)(1 ? ?t^R)) ? Kb^L(1 ? (1 ? t^L)(1 ? ?t^L))}{K(1 ? t^L)(1 ? ?t^L) ? (1 ? t^R)(1 ? ?t^R)}.$$

Proof: See appendix.

Conditions (a) and (b) of the Proposition are easy to interpret: the Left wants to tax only the fairly rich and the Right wants to tax a larger segment of the citizenry ((a)), and the Left wants to spend more on public goods than the Right ((b)). The reader need not try to interpret condition (c); we shall show that at equilibrium conditions (a)-(c) hold, and hence, the Left consists of all types whose income capacities are smaller than the number $? (?^L, ?^R)$, while the Right consists of all those with larger income capacities.

It follows from the Proposition and (3.1) that if, $?^L$ and $?^R$ satisfy premises (a)-(c) of the Proposition, then we may write the function $?$ as:

$$?(?^L, ?^R) ? \frac{F(? (?^L, ?^R)) ? ? ? .5}{2?}.$$

A *political economy* is thus specified by the data vector $(?, ?, ?, ?, \mathbf{F}, s)$. Our study shall consist in calculating the equilibria for three political economies:

E1. $(?, ?, ?, ?, \mathbf{F}, s)$,

E2. $(?, ?, ?, ?, \mathbf{F}, s ? 1)$,

E3. $(?,?,?,?,\mathbf{F}^*,s)$,

where the mean of **F*** is one greater than the mean of **F.** Thus, the move from E1 to E2 is one where the federal subsidy to the state increases by one unit of income, and the move from E1 to E3 is one where that subsidy remains unchanged, but mean income capacity increases by one unit. The flypaper conjecture is the assertion that the increase in the equilibrium value of *G* in moving from E1 to E3 is substantially less than the increase in *G* in moving from E1 to economy E2.

Two modifications must be made to the above statement. First, an equilibrium in this section consists of a *pair* of policies, one for each party, and a *probability* that each party wins. We shall identify the predicted equilibrium value of *G* as the *expected value of G* at the equilibrium, that is, $G^{ave} = ?(?^L, ?^R)G^L \; ? \; (1 ? ?(?^L, ?^R))G^R$. There is a fairly natural interpretation of this move. Suppose that there are elections in many states, each of which is described by this model, and suppose that the draw on the random variable that determines which party wins is independent across states. Then, with a large number of states, the average value of implemented *G* should be close to $G^{ave}$. But other interpretations are possible. The second modification is that, as we shall see, there is a continuum of equilibria in our model. These equilibria are quite concentrated in the policy space, and so their average value gives a good approximation of any one of them. We shall compute this continuum, and identify the 'predicted' value of the public good as the average of $G^{\text{ave}}$, over this continuum.

We observe, finally, that for a pair of policies $(?^L, ?^R)$ to constitute a PUNE, it is necessary and sufficient that there be no agreeable deviation to both the militants and the opportunists in either party. For it is easy to see that if those two factions agree to

deviate, the reformists will agree to, as well. Thus, although reformists indeed exist in reality, their presence, at least in our model, does not influence the nature of political equilibrium.

We now proceed to give a constructive characterization of our political equilibrium. Let $(w^*, w_L, w_R, ?^L, ?^R)$ be an equilibrium for the political economy $(?, ?, ?, ?, \mathbf{F}, s)$. We suppose, for the purposes of illustration, that the equilibrium has the properties that the only constraints that bind (see (3.4)) are the budget constraints, that is:

$$g(?^L; s) ? 0, g(?^R; s) ? 0,$$

and that $b^J ? w^J$ for $J = L, R$. Denote by $? v(?, w)$ the gradient vector of the function $v$ with respect to $b$, $t$, and $G$, evaluated at $(?, w)$: thus, a 3-vector.[6] Likewise, denote by $? g$ the gradient of the function $g$ with respect to the same three arguments. The function ? has six arguments – the components of $(?^L, ?^R)$. Denote by $? _L ?$ the 3-gradient of ? with respect to $b^L$, $t^L$, and $G^L$, and by $? _R ?$ the 3-gradient of ? with respect to the three components of the R policy. Then it must be the case that:

$$? d ? \mathbf{R}^3 \quad ? v(?^L, w_L) ?d ? 0 \ \& \ ? g(?^L) ?d ? 0 ? \quad ? _L ?(?^L, ?^R) ?d ? 0. \qquad (3.5)$$

This condition says that if there is any direction $d$ in which Left can deviate at $?^L$ which increases the utility of its militants, and moves it into the feasible policy region, then that deviation must reduce the utility of Left's opportunists. Were this not the case, then there would be a feasible deviation agreeable to both Left's militant and opportunist factions, contradicting the requirement of PUNE. In like manner, by considering the requirement for deviation-proofness in Right, we have:

[6] Because we stay away from the case $b = w$, where $v$ has a kink, the gradients in the following expressions are well-defined.

$$? d ? \mathbf{R}^3 \quad ? v(?^R, w_R) ?d ? 0 \;\&\; ? g(?^R) ?d ? 0 ? \quad ? \;_R ?(?^L, ?^R) ?d ? 0. \qquad (3.6)$$

Now we can rewrite (3.5) as:

$$? d ? \mathbf{R}^3 \quad ? v(?^L, w_L) ?d ? 0 \;\&\; ? g(?^L) ?d ? 0 ? \quad ?? \;_L ?(?^L, ?^R) ?d ? 0. \qquad (3.5')$$

We now apply the separating hyperplane theorem (more precisely, its linear version: Farkas' Lemma) which tells us, from (3.5'), that the vector $?? \;_L ?(?^L, ?^R)$ must lie in the cone spanned by the vectors $? v(?^L, w_L)$ and $? g(?^L)$, which is to say:[7]

$$? x_1 \geq 0 \text{ and } x_2 \geq 0 \text{ such that } ?? \;_L ?(?^L, ?^R) = x_1 ? v(?^L, w_L) + x_2 ? g(?^L); \qquad (3.7)$$

and, similarly from (3.6):

$$? y_1 \geq 0 \text{ and } y_2 \geq 0 \text{ such that } ? \;_R ?(?^L, ?^R) ? \; y_1 ? v(?^R, w_R) + y_2 ? g(?^R). \qquad (3.8)$$

In addition we have the equations:

$$g(?^L) ? 0, \qquad (3.9)$$

$$g(?^R) ? 0, \qquad (3.10)$$

$$w^* ? ? (?^L, ?^R), \qquad (3.11)$$

$$w_L ? \frac{\int_0^{w^*} ? w \, d\mathbf{F}(w)}{F(w^*)}, \qquad (3.12)$$

and

$$w_R ? \frac{\int_{w^*}^{?} ? w \, d\mathbf{F}(w)}{1 ? F(w^*)}, \qquad (3.13)$$

and the inequalities:

$$b^L > 0, \; b^R > 0, \; 0 < t^L < 1, \; 0 < t^R < 1,$$

and inequalities (a), (b), (c) of the Proposition. Only equation (3.11) may require explanation: it, in conjunction with (3.12) and (3.13), guarantees that precisely those in

[7] To be precise, equation (3.7) follows from condition (3.6) as long as neither $? v$ nor $? g$ is the zero vector.

the Left party prefer $?^L$ to $?^R$, and precisely those in the Right party prefer $?^R$ to $?^L$. Thus, the characterization of our concept of political equilibrium is complete.

Now count equations: we have unknowns $b^L, t^L, G^L, b^R, t^R, G^R, w^*, w_L, w_R, x_1, x_2, y_1, y_2$ (13 of them) and equations (3.7abc), (3.8abc), (3.9), (3.10), (3.11), (3.12), and (3.13) (11 of them). Thus, we can expect that there are either no such equilibria, or a 2-manifold of them.

We are unable to construct these equilibria analytically; we must compute. Consequently, we specify an actual politico-economic environment as follows:

$$? \; ? \; 0.25, ? \; ? \; 5, ? \; ? \; 0.4, ? \; ? \; 0.75, s \; ? \; 3, \text{ and}$$

**F** the lognormal distribution with mean 40 and median 30. Thinking of the unit of income as one thousand dollars, the distribution **F** captures approximately the US income distribution in the early 1990's. The other parameters are self-explanatory. The value of ? may seem high: we are asserting that, at the time parties announce their policies, the uncertainty surrounding the vote fraction is ?40%.

Equations (3.7)-(3.13) now become completely specified. Using methods which need not detain us here, we find no solutions of this type.

We do however find equilibria where $b_L = w_L$, and the Left plays the *ideal policy* for its representative, $w_L$. Since $? v(?^L, w_L)$ is not defined, we substitute for equations (3.7abc) the two equations:

$$b^L = w_L, \qquad (3.14a)$$

and

$$t^L = min \{1, \frac{1}{2?}\}. \qquad (3.14b)$$

For $w < 35$, (3.l4ab) characterize $w$ 's ideal policy[8]. Equations (3.8) – (3.14) now comprise 10 equations in the 11 unknowns $b^L, t^L, G^L, b^R, t^R, G^R, w^*, w_L, w_R, y_1, y_2$, and so there are either no solutions satisfying the inequality constraints, or a unidimensional manifold of such solutions. We indeed find the latter.

Our method of finding equilibria is to computationally pave out the equilibrium policy manifold.[9] Figure 1 presents a graph of this manifold, projected onto the (*t, G*) plane. The vertical line on the right in the figure constitute Left (*t, G)* components at equilibrium policies (note that $t^L$ is constant at $\frac{1}{2?}$ ? .667); public expenditures vary between approximately 10.5 and 12 (thousand dollars) per capita. The curve in the left side of the figure represents the (*t, G*) components of Right policies in equilibrium. We see that the tax rate varies between .2 and .3, and public goods are provided at the rates of between 9 and 9.5 thousand dollars per capita. It is worth noting how concentrated the equilibrium policies are in the policy space – so we do not lose much predictability because of the non-uniqueness of equilibrium.

Indeed, when we simulate the economies described by E2 and E3 above, we also find equilibria precisely of this type. We defined the distribution **F*** of political economy E3 as the lognormal distribution in which the mean income capacity is 41, and median income capacity is $\frac{41}{40}30$; thus, the distribution in which mean income capacity has increased by one unit, and all income capacities have increased, from the distribution in political economy E1, in the same proportion.

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[8] This can be deduced analytically, or, more easily, seen by graphing contours of *v* in the (*t, b*) plane, and substituting out for *G*.

[9] The method is explained in detail in Roemer (In press).

We summarize the characteristics of equilibrium for these three politico-economies in Table 1. Each column in the table presents the average of values over the continuum of equilibria that exist for that economy. By examining the column $G^{ave}$, we see that, when mean income capacity of the economy increases by $1,000, the predicted increase in expenditure on public goods is $157, but when the federal subsidy increases by $1000, the predicted expenditure on public goods increases by $635. Thus, the increase in public expenditures induced by the federal subsidy is four times the increase engendered by a similar increase in mean income capacity: the flypaper conjecture holds.

**4. Conclusion**

The flypaper effect, generically understood as the nonequivalence of the effect on public spending of a federal grant and an increase in income of the same amount, is the rule in models of politico-economic equilibrium, and only for special and, we may add, unrealistic cases does equivalence obtain in such models.

This illustrates the fundamental difference between individual and collective decision making. An individual "flypaper effect" would indeed be an anomaly, a violation of individual rationality (to the extent that the relevant part of the budget constraint is invariant). But the nonequivalence of increases in grants and community income is predicted, almost everywhere, by models that understand collective decision as the outcome of electoral competition among political parties. In political-equilibrium models, it is the equivalence, rather than the "flypaper effect" that is, if not anomalous, at least exceptional.

The realistic specification and calibration of a model yielding the large "marginal propensity to spend in public goods out of a federal grant" magnitudes observed in empirical work (see Table 1 in Hines and Thaler, 1995) is a different issue. We understand realism to demand (i) empirically sensible assumptions on the distribution of income; (ii) progressive taxation, which in turn demands more than one parameter in the tax rule (in fact, it has repeatedly been observed that two parameters suffice). To that end, we have applied to this problem Roemer's (1998, 1999) model of the strategic interaction between two parties in multi-dimensional policy spaces, with the added feature of a cost of taxation (as proposed by Hamilton, 1986). No doubt due to luck, our first simulation, reported in section 4.B above, yielded a marginal propensity identical, up to two decimal places, to the marginal propensity averaged over the ten studies reported in Table 1 of Hines and Thaler.

| Econ'y | $G^L$ | $G^R$ | ?? | $G^{ave}$ | $t^R$ | $w^*$ | $w^L$ | $w^R$ |
|---|---|---|---|---|---|---|---|---|
| E1 | 11.14 | 9.18 | .44 | 9.971 | .28 | 26.76 | 16.42 | 58.78 |
| E2 | 12.31 | 9.59 | .40 | 10.606 | .24 | 25.31 | 15.97 | 57.65 |
| E3 | 11.38 | 9.30 | .42 | 10.128 | .28 | 27.20 | 16.84 | 60.25 |

Table: Summary of political equilibria for three politico-economic environments

Appendix: Proof of the Proposition:

Partition the set of types into the three subsets

$W_1 ? \{w? b^R\}, W_2 ? \{b^R ? w ? b^L\}, W_3 ? \{w ? b^L\}$.

1. It is obvious that $W_1 ? ? (?^L, ?^R)$. For any $w ? W_1$ is not taxed under either policy, and so such a type prefers the policy with greater public expenditure, which is $?^L$ .

2. We next show $W_2 ? ? (?^L, ?^R)$. This is true if and only if:

$? \ln(G^L ? ?) ? (1 ? ?)\ln w ? ? \ln(G^R ? ?) ? (1 ? ?)\ln(b^R ? ? (t^R)(w ? b^R))$ (A1)

where $? (t) ? (1 ? t)(1? ?t)$. (A1) reduces to:

$$w ? \frac{b^R(1 ? ? (t^R))}{K ? ? (t^R)}, \qquad \text{(A1')}$$

*if* $K ? ? (t^R) ? 0$. But this holds, because $K > 1$, and $? (t^R) ? 1$. Hence (A1') is true, since all $w$ in $W_2$ are greater than $b^R$. The claim of this paragraph follows.

3. $w ? W_3$ prefers $?^L$ to $?^R$ if and only if:

$$K ? \frac{b^R ? ? (t^R)(w ? b^R)}{b^L ? ? (t^L)(w ? b^L)}. \qquad \text{(A2)}$$

This reduces to:

$$w ? \frac{b^R(1 ? ? (t^R)) ? Kb^L(1? ? (t^L))}{K? (t^L) ? ? (t^R)}, \qquad \text{(A2')}$$

*if* $K? (t^L) ? ? (t^R) < 0$, which is premise (c) of the proposition. The expression on the r.h.s. of (A2') is just $? (?^L, ?^R)$. To conclude the proof we need only check that $? (?^L, ?^R) ? b^L$. Manipulation shows this inequality is equivalent to the statement:

$$\frac{b^R}{b^L} ? \frac{K ? ? (t^R)}{1 ? ? (t^R)}. \qquad \text{(A3)}$$

But (A3) holds, because its l.h.s. is less than one, while its r.h.s. is greater than one. ??

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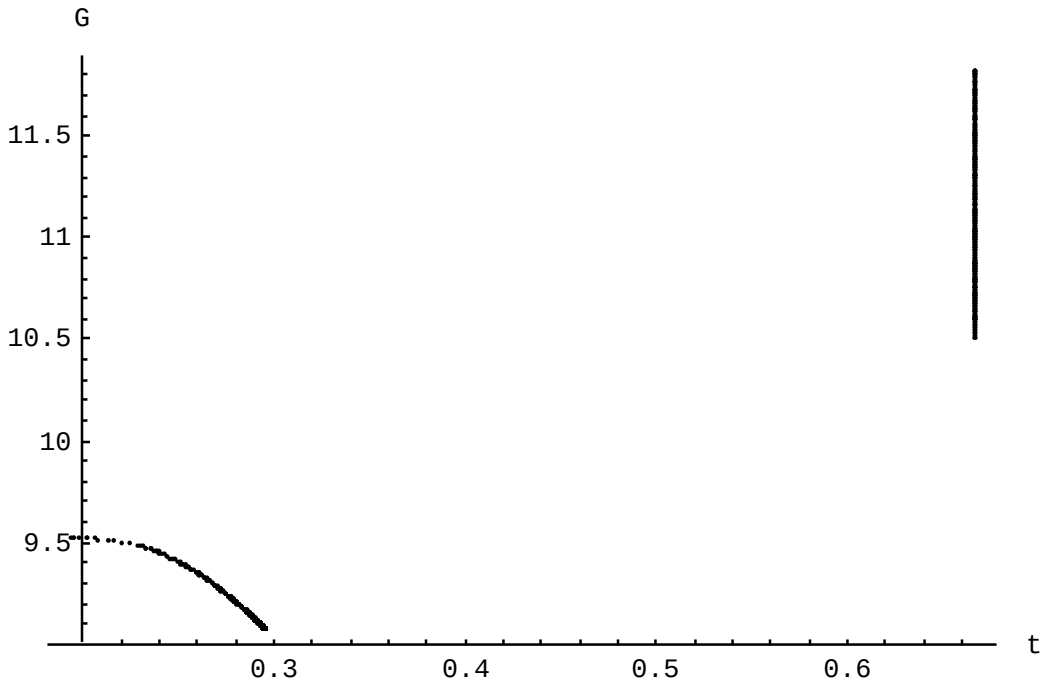
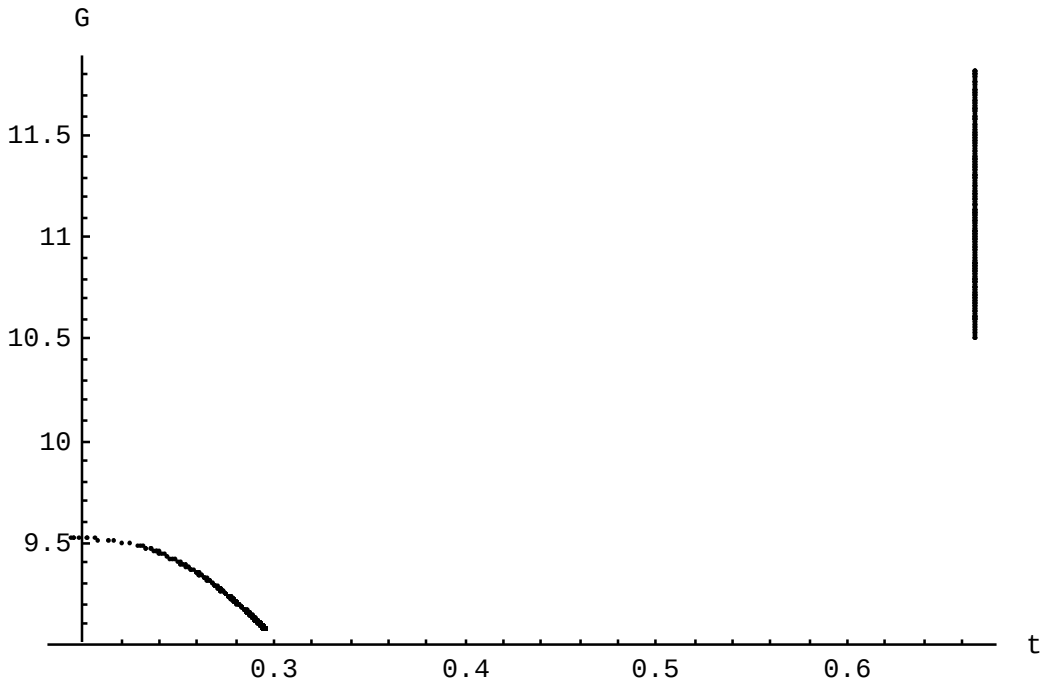



Figure 1. The equilibrium manifold projected onto (*t,G)* space

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