

Beissel-Durrant, Gabriele; Skinner, Chris

**Working Paper**

## Estimation of the distribution of hourly pay from household survey data: The use of missing data methods to handle measurement error

cemmap working paper, No. CWP12/03

**Provided in Cooperation with:**

The Institute for Fiscal Studies (IFS), London

*Suggested Citation:* Beissel-Durrant, Gabriele; Skinner, Chris (2003) : Estimation of the distribution of hourly pay from household survey data: The use of missing data methods to handle measurement error, cemmap working paper, No. CWP12/03, Centre for Microdata Methods and Practice (cemmap), London,  
<https://doi.org/10.1920/wp.cem.2003.1203>

This Version is available at:

<https://hdl.handle.net/10419/79268>

**Standard-Nutzungsbedingungen:**

Die Dokumente auf EconStor dürfen zu eigenen wissenschaftlichen Zwecken und zum Privatgebrauch gespeichert und kopiert werden.

Sie dürfen die Dokumente nicht für öffentliche oder kommerzielle Zwecke vervielfältigen, öffentlich ausstellen, öffentlich zugänglich machen, vertreiben oder anderweitig nutzen.

Sofern die Verfasser die Dokumente unter Open-Content-Lizenzen (insbesondere CC-Lizenzen) zur Verfügung gestellt haben sollten, gelten abweichend von diesen Nutzungsbedingungen die in der dort genannten Lizenz gewährten Nutzungsrechte.

**Terms of use:**

*Documents in EconStor may be saved and copied for your personal and scholarly purposes.*

*You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.*

*If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.*



# ESTIMATION OF THE DISTRIBUTION OF HOURLY PAY FROM HOUSEHOLD SURVEY DATA:

THE USE OF MISSING DATA METHODS TO HANDLE MEASUREMENT ERROR

---

*Gabriele Beissel-Durrant*  
*Chris Skinner*

THE INSTITUTE FOR FISCAL STUDIES  
DEPARTMENT OF ECONOMICS, UCL  
cemmap working paper CWP12/03

# **Estimation of the Distribution of Hourly Pay from Household Survey Data: The Use of Missing Data Methods to Handle Measurement Error**

**Gabriele Beissel-Durrant and Chris Skinner**

**University of Southampton**

## **Abstract**

Measurement errors in survey data on hourly pay may lead to serious upward bias in low pay estimates. We consider how to correct for this bias when auxiliary accurately measured data are available for a subsample. An application to the UK Labour Force Survey is described. The use of fractional imputation, nearest neighbour imputation, predictive mean matching and propensity score weighting are considered. Properties of point estimators are compared both theoretically and by simulation. A fractional predictive mean matching imputation approach is advocated. It performs similarly to propensity score weighting, but displays slight advantages of robustness and efficiency.

**Key Words** donor imputation; fractional imputation; hot deck imputation; nearest neighbour imputation; predictive mean matching; propensity score weighting.

## 1. INTRODUCTION

A national minimum wage was introduced in the United Kingdom (UK) in April 1999 and there is considerable interest in how the lower end of the distribution of hourly pay has changed since then, both overall and within subgroups, such as by gender. The UK Labour Force Survey (LFS) provides an important source of estimates of this distribution (Stuttard and Jenkins 2001). A major problem with the use of household surveys to produce such estimates is the difficulty in measuring hourly pay accurately (Rodgers, Brown and Duncan 1993; Moore, Stinson and Welniak 2000). Measurement error may lead to biased estimates of distribution functions, especially at the extremes (Fuller 1995). For example, the bold line in Figure 1 represents a standard estimate of the lower end of the distribution function of hourly pay using LFS data from the June-August 1999 ignoring measurement error. We suggest that this estimate is seriously biased upwards and that improved estimates, using methods to be described in this paper, are given by the three lower lines. These results suggest that the proportion of jobs paid at or below the national minimum wage rate may be overestimated by four or five times if measurement error is ignored.

[Figure 1 about here]

When a variable is measured with error, it is sometimes possible, as in our application, to measure the variable more accurately for a subsample. In these circumstances, if we assume that the variable measured accurately on the subsample is the true variable, inference about the distribution of this variable becomes a missing data problem. The variable measured erroneously on the whole sample is treated as an auxiliary variable. The case when the subsample is selected using a randomised scheme is well studied and referred to as double sampling or two phase sampling (e.g. Tenenbein 1970). In this case, unbiased estimates can be constructed from the subsample alone, but use of data on the correlated proxy variable for the whole sample may improve efficiency. See, for example, Luo, Stokes and Sager

(1998). In the application in this paper, the selection of the subsample is not randomised and we shall just assume that the accurate variable is missing at random (MAR) (Little and Rubin 2002) conditional on variables measured on the whole sample. Because the aim is to estimate a distribution function, which is unlikely to follow exactly a standard parametric form, we avoid approaches which make parametric assumptions about the true distribution, as for example in Buonaccorsi (1990). It is also desirable in our application to avoid strong assumptions about the measurement error model, for example that it is additive with zero mean and constant variance as in the SIMEX method of Luo et al. (1998). Instead, we consider the application of various imputation and weighting methods from the missing data literature to our measurement error problem. The main aim of this paper is to investigate how best to design these methods to improve the quality of point estimators of the distribution function of hourly pay, as measured by bias, efficiency and robustness to model assumptions.

The paper is structured as follows. The application and the estimation problem are introduced in section 2. Imputation and weighting approaches are set out in sections 3 and 4 respectively and their properties are studied and compared theoretically in section 5 and via a simulation study in section 7. The primary focus is on point estimation, but we briefly consider variance estimation in section 6. Some concluding remarks are given in section 8.

The basic measurement error problem considered in this paper was described by Skinner, Stuttard, Beissel-Durrant and Jenkins (2003), who also proposed the use of one imputation method. This paper extends that work by considering a wider class of approaches to missing data, by comparing their properties both theoretically and via simulation and by considering variance estimation. The imputation approach developed in this paper, which extends that considered by Skinner et al. (2003), has now been implemented by the Office for National Statistics in the United Kingdom as a new approach to producing low pay estimates.

## 2. THE ESTIMATION PROBLEM

Our aim is to estimate the distribution of hourly pay from LFS data. This inference problem requires consideration of both (i) sampling and unit nonresponse of employees and (ii) measurement error and item nonresponse for hourly pay. We outline the basic set-up for both (i) and (ii) in this section. The main focus of the paper will be the choice of methods to address (ii). Standard procedures will be used to handle (i).

The LFS is a quarterly survey of households selected from a national file of postal addresses with equal probabilities by stratified systematic sampling. All adults in selected households are included in the sample. The resulting sample is clustered by household but not otherwise by geography. Each selected household is retained in the sample for interview on five successive quarters and then rotated out and replaced. The questions underlying the hourly pay variables, described below, are asked in just the first and fifth interviews, giving information on hourly pay on about 17,000 employees per quarter. Survey weights are constructed by a raking procedure to compensate for differential unit nonresponse. Separate weights are constructed for earnings data (ONS, 1999).

The traditional method of measuring hourly pay in the LFS is (a) to ask employees questions about their main job to determine earnings over a reference period, (b) to ask questions to determine hours worked over the reference period and (c) to divide the result of (a) by the result of (b). We refer to the result of (c) as the *derived hourly pay* variable, since it is derived from answers to several questions. This is the variable used to produce the bold line in Figure 1. Skinner et al. (2003) describe and provide empirical evidence of many sources of measurement error in this variable. A more recent method of measuring hourly pay is first to ask whether the respondent is paid a fixed hourly rate and then, if the answer is positive, to ask respondents what this (basic) rate is. We refer to the resulting measure of hourly pay as the *direct variable*. Skinner et al. (2003) conclude from their study that the direct variable

measures hourly pay much more accurately than the derived variable and a key working assumption of this paper is that the direct variable measures hourly pay without error.

The problem with the direct variable is that it is missing for respondents who state that they are not paid at a fixed hourly rate (and for item nonrespondents) and this missingness may be expected to be positively associated with hourly pay. The proportion of LFS respondents with a (main) job who provide a response to the direct question is about 43%. This proportion tends to be higher for lower paid employees, for example the rate is 72% among those in the bottom decile of the derived variable. The direct variable is not collected for second (and further) jobs and we therefore restrict attention only to first jobs.

This paper addresses the following missing data problem. We wish to estimate the distribution of hourly pay defined as:

$$F(y) = N^{-1} \sum_{i \in U} I(y_i < y) \quad (1)$$

where  $U$  is the population of  $N$  (first) jobs,  $y_i$  is (basic) hourly pay for job  $i$  and  $y$  may take any specified value. Our data consist of values  $y_i^*$ ,  $x_i$  and  $r_i$  for  $i \in s$  and values  $y_i$  for  $i \in s$  when  $r_i = 1$ , where  $s$  is the set of (first) jobs for unit respondents in the sample drawn from  $U$ ,  $y_i^*$  is the value of the derived variable,  $y_i$  is the value of the direct variable assumed identical to the hourly pay variable of interest,  $r_i = 1$  if  $y_i$  is measured and  $r_i = 0$  if not and  $x_i$  is a vector of other variables measured in the survey.

We assume that inference from the sample  $s$  to the population  $U$  can be made using standard methods of survey sampling. Our primary concern is with the missingness of  $y_i$ . We consider two approaches to handle this missingness:

- (i) imputation of  $y_i$  for cases where  $r_i = 0$  ( $i \in s$ ), using the values  $y_i^*$  and  $x_i$  as auxiliary information;

- (ii) weighting of an estimator based upon the subsample  $s_1 = \{i \in s; r_i = 1\}$ , in particular, the use of propensity score weighting (Little 1986).

These approaches to estimating  $F(y)$  will be discussed in the following two sections.

### 3. IMPUTATION APPROACHES

We shall construct imputation methods based upon the assumption that the population values  $(y_i, y_i^*, x_i, r_i)$ ,  $i \in U$ , are independently and identically (IID) distributed. To allow for the LFS sampling design and unit nonresponse, we propose to incorporate the survey weights in the resulting point estimator of  $F(y)$ , in the same way that a pseudo-likelihood approach (Skinner, 1989) weights estimators based upon an IID assumption. We do not attempt to take account of the weights or the complex design directly in the imputation methods. It is, of course, desirable that allowance is made for the weights and complex design in variance estimation and this is referred to briefly in section 6.

Under the IID assumption and the assumption that sampling is ignorable (that is that the distribution of  $(y_i, y_i^*, x_i, r_i)$  is the same whether or not  $i \in s$ ), if it were possible to observe  $y_i$  for  $i \in s$ ,

$$\hat{F}(y) = n^{-1} \sum_{i=1}^n I(y_i < y) \quad (2)$$

would be an unbiased estimator of  $F(y)$  (in the sense that  $E[\hat{F}(y) - F(y)] = 0$  for all  $y$ ), where we write  $s = \{1, \dots, n\}$ . We assume that this estimator remains unbiased under the actual sampling design and unit nonresponse if the mean in (2) is weighted by the survey weights. The IID assumption used in the remainder of this section may be interpreted as holding condition on inclusion in  $s$ , with the implicit assumption that survey weighting will also be required to handle the selection of  $s$  from  $U$ .



To address the problem that  $y_i$  is missing when  $r_i = 0$ , we first consider a single imputation approach where  $y_i$  is replaced in (2) by a single imputed value  $y_i^f$  when  $r_i = 0$  (and  $i \in s$ ) and let  $\tilde{y}_i = y_i$  if  $r_i = 1$  and  $\tilde{y}_i = y_i^f$  otherwise. We assume that  $y_i^f$  is determined in a specified way using the data  $D = \{[y_i^*, x_i, r_i; i \in s], [y_i; r_i = 1, i \in s]\}$  and perhaps a stochastic mechanism. The resulting estimator of  $F(y)$  is

$$\tilde{F}(y) = n^{-1} \sum_{i=1}^n I(\tilde{y}_i < y). \quad (3)$$

A sufficient condition for  $\tilde{F}(y)$  to be an unbiased estimator of  $F(y)$  is that the conditional distribution of  $y_i^f$  given  $r_i = 0$ , denoted  $[y_i^f | r_i = 0]$ , is the same as the conditional distribution  $[y_i | r_i = 0]$ . However, since  $y_i$  is only observed when  $r_i = 1$ , the data provide no direct information about  $[y_i | r_i = 0]$  without further assumptions (Little and Rubin 2002). We consider two possible assumptions. The first assumption is common in the missing data literature (Little and Rubin 2002).

**Assumption (MAR):**  $r_i$  and  $y_i$  are conditionally independent given  $y_i^*$  and  $x_i$ .

The second assumption is that the measurement error model, defined as the conditional distribution of  $y_i^*$  given  $y_i$  and  $x_i$ , is the same for respondents ( $r_i = 1$ ) and nonrespondents ( $r_i = 0$ ), which may be expressed as follows.

**Assumption (Common Measurement Error Model):**  $r_i$  and  $y_i^*$  are conditionally independent given  $y_i$  and  $x_i$ .

The first assumption is the standard one made when using imputation or weighting and is the one which we shall make. We shall use the second assumption in the simulation study in section 7 to assess robustness of MAR-based procedures. Inference under the second assumption could be made under strong assumptions on the measurement error model, for example the additive error assumption in methods in Carroll, Ruppert and Stefanski (1995,

sect. 12.1.2.) and Luo et al. (1998). It does not appear straightforward to make inference under the second assumption for a measurement error model which is realistic for our application and we do not pursue this possibility further in this paper. The plausibility of these assumptions is discussed further in Skinner et al. (2003).

Under the MAR assumption we have  $[y_i | y_i^*, x_i, r_i = 0] = [y_i | y_i^*, x_i, r_i = 1]$  and a sufficient condition for  $\tilde{F}(Y)$  to estimate  $F(Y)$  unbiasedly is that

$$[y_i^I | y_i^*, x_i, r_i = 0] = [y_i | y_i^*, x_i, r_i = 1]. \quad (4)$$

We therefore consider an imputation approach where the conditional distribution of  $y$  given  $y^*$  and  $x$  is ‘fitted’ to the respondent ( $r_i = 1$ ) data and then the imputed values  $y_i^I$  are ‘drawn from’ this fitted distribution at the values  $y_i^*$  and  $x_i$  observed for the nonrespondents. We consider representing the conditional distribution  $[y_i | y_i^*, x_i, r_i = 1]$  by a parametric regression model:

$$g(y_i) = h(y_i^*, x_i; \beta) + e_i, \quad E(e_i | y_i^*, x_i) = 0 \quad (5)$$

where  $g(\cdot)$  and  $h(\cdot)$  are given functions and  $\beta$  is a vector of regression parameters. A simple point predictor of  $y_i$ , given an estimator  $\hat{\beta}$  of  $\beta$  based on respondent data, is

$$\hat{y}_i = g^{-1}[h(y_i^*, x_i; \hat{\beta})]. \quad (6)$$

Using  $\hat{y}_i$  for imputation may, however, lead to serious underestimation of  $F(Y)$  for low values of  $y$ , since such simple regression imputation may be expected to reduce the variation in  $F(Y)$  artificially (Little and Rubin 2002, p. 64). This effect might be avoided by taking  $y_i^I = g^{-1}[h(y_i^*, x_i; \hat{\beta}) + \hat{e}_i]$ , where  $\hat{e}_i$  is a randomly selected empirical residual (Little and Rubin 2002, p. 65). Our experience is, however, that this approach fails to generate imputed values which reproduce the ‘spiky’ behaviour of hourly pay distributions, for example around a minimum wage or rounded pay rates, and this may lead to bias around these spikes.

We prefer therefore to consider donor imputation methods, which set  $y_i^I = y_{d(i)}$  ( $r_i = 0$ )

for some donor respondent  $j = d(i)$  for which  $r_j = 1$ . The imputed value from a donor will always be a genuine value, as reported by the donor respondent, and will thus respect the spiky behaviour these values display. The basic donor imputation method we consider is predictive mean matching (Little 1988), that is nearest neighbour imputation with respect to  $\hat{y}_i$ , i.e.

$$|\hat{y}_i - \hat{y}_{d(i)}| = \min_{j:r_j=1} |\hat{y}_i - \hat{y}_j| \quad (7)$$

where  $r_i = 0$  and  $r_{d(i)} = 1$ .

Some conditions for the resulting estimator  $\tilde{F}(Y)$  to be approximately unbiased for  $F(Y)$  follow from Corollary 2 of Theorem 1 of Chen and Shao (2000). First, we require that  $y_i$  is missing at random (MAR) conditional on  $z_i = g^{-1}[h(y_i^*, x_i; \beta)]$ , where  $\beta = \text{plim}(\hat{\beta})$ . This condition seems reasonable if the MAR assumption above holds and if the distribution of  $y_i$  only depends on  $y_i^*$  and  $x_i$  via  $z_i$ . Second, we require that the conditional expectation of  $y_i$  given  $z_i$  is monotonic and continuous in  $z_i$ , which seems reasonable if  $y_i^*$  is a good proxy for  $y_i$ . Third, we require that  $z_i$  and  $E(y_i | z_i)$  have finite third moments which seems reasonable if we restrict attention to the lower part of the pay distribution. Fourth, we require the probability of response given  $z$  to be bounded above zero, which again seems reasonable if we restrict attention to the lower part of the pay distribution. Finally, Chen and Shao's (2000) result needs to be adapted for the fact that the nearest neighbour is defined with respect to  $\hat{\beta}$  whereas the above conditions are with respect to  $\beta$ . Again, it seems reasonable that this can be done if  $\hat{\beta}$  converges to a limit  $\beta = \text{plim}(\hat{\beta})$  and close neighbours with respect to  $\hat{y}_i = g^{-1}[h(y_i^*, x_i; \hat{\beta})]$  are also close neighbours with respect to  $z_i = g^{-1}[h(y_i^*, x_i; \beta)]$ .

There are thus theoretical grounds that nearest neighbour imputation with respect to  $\hat{y}_i$  will lead to an approximately unbiased estimator of  $F(Y)$ , subject to the MAR assumption and

certain additional plausible conditions. It is also of interest, however, to consider the efficiency of  $\tilde{F}(Y)$ . The variance of  $\tilde{F}(Y)$  for nearest neighbour imputation may be expected to be inflated, in particular because certain donors may be used much more frequently than others. We consider a number of approaches to reducing this variance inflation effect.

First, we may smooth the number of times that respondents are used as donors by defining imputation classes by disjoint intervals of values of  $\hat{y}_i$  and drawing donors for a recipient by simple random sampling from the class within which the recipient's value of  $\hat{y}_i$  falls. The smoothing will be greatest if we draw donors without replacement. We denote this hot deck method HDIWR or HDIWOR, depending on whether sampling is with or without replacement. A second approach is to undertake donor selection sequentially and to penalize the distance function employed for determining the nearest neighbour  $d(i)$  as follows

$$|\hat{y}_i - y_{d(i)}| = \min_{j:r_j=1} \{|\hat{y}_i - y_j| * (1 + \lambda t_j)\}, \quad (8)$$

where  $\lambda \in \mathbb{R}^+$  is a penalty factor,  $t_j$  is the number of times the respondent  $j$  has already been used as a donor,  $r_i = 0$  and  $r_{d(i)} = 1$  (Kalton 1983). A third approach is to employ repeated imputed values  $y_i^{(m)}$ ,  $m = 1, \dots, M$ , determined for each recipient  $i \in s$  such that  $r_i = 0$ . The resulting estimator of  $F(Y)$  is  $M^{-1} \sum_m \tilde{F}^{(m)}(y)$ , the mean of the resulting estimators  $\tilde{F}^{(m)}(y)$ , or equivalently is obtained by multiplying the weight for each imputed value by a factor  $1/M$ . We refer to the third approach as fractional imputation (Kalton and Kish 1984; Fay 1996) rather than multiple imputation (Rubin 1996), since we do not require the imputation method to be 'proper', that is to fulfil conditions which ensure that the multiple imputation variance estimator is consistent. We do not stipulate this requirement because our primary objective is point estimation and an alternative variance estimator is available (section 6). In our use of fractional imputation we aim to select donors  $d(i, m)$ ,

$m=1, \dots, M$ , each a close neighbour to  $i$ , so that  $\tilde{F}^{(m)}(y)$  remains approximately unbiased for  $F(Y)$ . We consider the following variations of this approach.

- (i) The  $M/2$  nearest neighbours above and below  $\hat{y}_i$  are taken, for  $M=2$  or  $10$ , denoted NN2 and NN10 respectively.
- (ii)  $M/2$  donors are selected by simple random sampling with replacement from the  $M$  respondents above and from the  $M$  respondents below  $\hat{y}_i$ , for  $M=2$  or  $10$ , denoted NN2(4) and NN10(20) respectively.
- (iii)  $M=10$  donors are selected by simple random sampling with or without replacement from the imputation classes referred to in the HDIWR and HDIWR methods described above. We refer to these as the HDIWR10 and HDIWR10 methods.

For comparison we also consider the Approximate Bayesian Bootstrap method of multiple imputation (Rubin and Schenker 1986), denoted ABB10, defined with respect to the imputation classes referred to in the HDIWR and HDIWR methods.

#### 4. WEIGHTED ESTIMATION

The estimator  $\tilde{F}(y)$  implied by the different imputation approaches considered in the previous section may be expressed in weighted form as:

$$\tilde{F}(y) = \sum_{i \in s_1} w_i I(y_i < y) / \sum_{i \in s_1} w_i \quad (9)$$

where  $s_1 = \{i \in s; r_i = 1\}$  is the set of respondents and  $w_i = 1 + d_i / M$ , where  $d_i$  is the total number of times that respondent  $i$  is used as a donor over the  $M$  repeated imputations. Note that  $\sum_{s_1} w_i = n$ . The weight  $w_i$  may be multiplied by the survey weight to allow for unit nonresponse. Other choices of weight  $w_i$  may also be considered. In particular, we may set  $w_i$  equal to the reciprocal of an estimated value of the propensity score,  $Pr(r_i = 1 | y_i^*, x_i)$  (Little 1986). This approach has been proposed for the hourly pay application in this paper

by Dickens and Manning (2002). This propensity score might be estimated, for example, under a logistic regression model relating  $r_i$  to  $y_i^*$  and  $x_i$ . Under the MAR assumption, the resulting estimator  $\tilde{F}(y)$  will be approximately unbiased assuming validity of the model for the conditional distribution  $[r_i | y_i^*, x_i]$  and some regularity conditions, such as those described in section 3 for the imputed estimator. Note that the need to model  $[r_i | y_i^*, x_i]$  replaces the need to model  $[y_i | y_i^*, x_i]$  in the imputation approach.

## 5. THEORETICAL PROPERTIES OF IMPUTATION AND WEIGHTING APPROACHES

In this section we investigate and compare the properties of the imputation and propensity score weighting approaches introduced in the previous two sections under various simplifying assumptions. We fix  $y$  and set  $u_i = I(y_i < y)$ . Letting  $N \rightarrow \infty$  we suppose that the parameter of interest is  $\theta = E(u_i)$ . We consider the imputation approach first and suppose that  $y_i$  depends upon  $y_i^*$  and  $x_i$  only via  $z_i = g^{-1}[h(y_i^*, x_i; \beta)]$  and that  $y_i$  is missing at random given  $z_i$ . Ignoring the difference between  $\beta$  and  $\hat{\beta}$  for large samples we consider nearest neighbour imputation with respect to  $z_i$ . As in (9) the imputed estimator of  $\theta$  may be expressed as

$$\hat{\theta}_{IMP} = \sum_{i \in s_1} w_i u_i / \sum_{i \in s_1} w_i \quad (10)$$

where  $w_i = 1 + d_i / M$  (and  $\sum_{s_1} w_i = n$ ). We write the corresponding expression for propensity score weighting as  $\hat{\theta}_{PS}$  with  $w_i$  replaced by  $w_{PSi}$ . Let  $z_{PSi}$  be the scalar function of  $y_i^*, x_i$  upon which  $r_i$  depends and write:

$$Pr(r_i = 1 | y_i^*, x_i) = \pi(z_{PSi}). \quad (11)$$

Just as we ignored the difference between  $\beta$  and  $\hat{\beta}$ , we ignore error in estimating  $\pi(z_{PSi})$  and write  $w_{PSi} = \pi(z_{PSi})^{-1}$ .

The imputation and propensity score weighting approaches may be expected to give similar estimation results if  $z_i$  and  $z_{PSi}$  are similar, that is they are close to deterministic functions of each other, and  $M$  is large. To see this, consider a simple example of the imputation approach, where the donor is drawn randomly from an imputation class  $c$  of close neighbours with respect to  $z_i$ , containing  $m_c$  respondents and  $n_c - m_c$  nonrespondents, as described in section 3, then  $w_i$  will approach  $1 + (n_c - m_c)/m_c = n_c/m_c$  as  $M \rightarrow \infty$  and this is the inverse of the response rate within the class (David, Little, Samuhel and Triest 1983). More generally, with the other nearest neighbour imputation approaches considered in section 3, the weight  $w_i = 1 + d_i/M$  may be interpreted as a local (with respect to  $z_i$ ) nonparametric estimate of  $\Pr(r_i = 1 | z_i)^{-1}$  and thus may be expected to lead to similar estimation results to propensity score weighting if  $z_i$  and  $z_{PSi}$  are deterministic functions of each other. In general, however, this will not be the case. Since  $\Pr(r_i = 1 | z_i)$  may be expressed as an average of  $\Pr(r_i = 1 | y^*, x)$  across values of  $y^*$  and  $x$  for which  $z = z_i$ , we may interpret  $w_i$  as a smoothed version of  $w_{PSi}$  and may expect it to show less dispersion. This suggests that it may be possible to use imputation to improve upon the efficiency of estimates based upon propensity score weighting, as also discussed by David et al. (1983) and Rubin (1996, sect. 4.6). To investigate this further, let us now make the MAR assumption and the other assumptions in sections 3 and 4 upon which the approaches are based. In this case both imputation and weighting approaches lead to approximately unbiased estimation of  $F(y)$  and we may focus our comparison on relative efficiency. It follows from equation (3.3) of Chen and Shao (2000), under their regularity conditions, that the variance of  $\hat{\theta}_{IMP}$  may be approximated for large  $n$  by

$$\text{var}(\hat{\theta}_{IMP}) \approx n^{-2} E[\sum_{s_1} w_i^2 V(u_i | z_i)] + n^{-1} V[\psi(z_i)] \quad (12)$$

where  $\psi(z_i) = E(u_i | z_i)$ . Note that Chen and Shao (2000) consider single imputation with  $M=1$  but their proof of this result carries through if  $M > 1$ . It is convenient to reexpress this result using

$$V[\psi(z_i)] = \sigma^2 - E[V(u_i | z_i)], \quad (13)$$

where  $\sigma^2 = V(u_i)$  and a corollary of Chen and Shao's (2000) Theorem 1 that

$$E[n^{-1} \sum_{s_1} w_i V(u_i | z_i)] = E[V(u_i | z_i)] + o_p(n^{-1/2}). \quad (14)$$

It follows that to the same order of approximation as in (12)

$$\text{var}(\hat{\theta}_{IMP}) \approx n^{-1} \sigma^2 + n^{-2} E[\sum_{s_1} (w_i^2 - w_i) V(u_i | z_i)]. \quad (15)$$

Note that  $w_i^2 - w_i = (d_i / M)(1 + d_i / M) \geq 0$ . This expression may be interpreted from both 'missing data' and 'measurement error' perspectives. From a missing data perspective, the first term in (15) is just the variance of  $\hat{\theta}$  in the absence of missing data and the second term represents the inflation of this variance due to imputation error. From a measurement error perspective, we may consider limiting properties under 'small measurement error asymptotics' (Chesher 1991), that is where  $y_i^*$  becomes a better measure of  $y_i$  and  $V(u_i | z_i)$  approaches zero. In this case, the second term also approaches zero and  $\hat{\theta}_{IMP}$  becomes 'fully efficient', i.e. its variance approaches  $\sigma^2 / n$ .

Let us now consider propensity score weighting. We make the corresponding assumption that  $y_i$  is missing at random given  $z_{PSi}$ . Linearising the ratio in (9) and using the fact that

$E(\sum_{s_1} w_{PSi}) = n$  we may write

$$\begin{aligned} \text{var}(\hat{\theta}_{PS}) &\approx n^{-2} \text{var}[\sum_{s_1} w_{PSi} (u_i - \theta)] \\ &= n^{-1} E[w_{PSi} (u_i - \theta)^2] \end{aligned} \quad (16)$$

which may be expressed alternatively as



$$\text{var}(\hat{\theta}_{PS}) \approx n^{-2} E[\sum_{s_1} w_{PSi}^2 V(u_i | z_{PSi})] + n^{-1} E\{w_{PSi} [\psi(z_{PSi}) - \theta]^2\}. \quad (17)$$

To compare the efficiency of weighting and imputation it is convenient to use (13) and (14) (which hold also with  $w_{PSi}$  in place of  $w_i$ ) to obtain

$$\begin{aligned} \text{var}(\hat{\theta}_{PS}) &\approx n^{-1} \sigma^2 + n^{-2} E[\sum_{s_1} (w_{PSi}^2 - w_{PSi}) V(u_i | z_{PSi})] \\ &\quad + n^{-1} E\{\sum_{s_1} [w_{PSi} - 1] [\psi(z_{PSi}) - \theta]^2\}. \end{aligned} \quad (18)$$

Note that, in comparison with (15), this involves a third term, which does not necessarily converge to zero as  $y_i^*$  approaches  $y_i$  and  $V(u_i | z_{PSi}) \rightarrow 0$ . Hence propensity score weighting does not become fully efficient as the measurement error disappears.

The second term of (18) may also be expected to dominate the second term of (15) when  $V(u_i | z_i)$  and  $V(u_i | z_{PSi})$  are constant and equal, since, recalling that  $\sum_{s_1} w_i = E(\sum_{s_1} w_{PSi}) = n$ , these second terms are primarily determined by the variances of the weights  $w_i$  and  $w_{PSi}$ , and, provided  $M$  is sufficiently large, we may expect  $w_i$  to display less variation than  $w_{PSi}$ , as argued above. In general, however, it does not appear that  $\hat{\theta}_{IMP}$  is necessarily more efficient than  $\hat{\theta}_{PS}$  and we look to the simulation study in section 7 for numerical evidence.

Let us finally consider the impact of departures from the MAR assumption. Under small measurement error asymptotics where  $V(u_i | z_i) \rightarrow 0$  and  $y_i^l \rightarrow y_i$ , the imputation approach will provide consistent inference about  $\theta$  even if the MAR assumption fails. This is not the case for the propensity score weighting approach. This suggests that the imputation approach may display more robustness to departures from the MAR assumption if the amount of measurement error is relatively small.

## 6. VARIANCE ESTIMATION

Point estimation is the priority in our application, but we do now consider variance estimation briefly. Ideally, variance estimation should take account of the survey weights and complex design. Some methods have been developed for nearest neighbour imputation, allowing for weighting and stratification (Chen and Shao, 2001). The treatment of household clustering seems less well understood. We are currently exploring the application of replication methods developed by Kim and Fuller (2002). In this section we simply describe how delta method estimators can be constructed under the assumption of IID observations and ignorable sampling (see section 3) ignoring finite population corrections, i.e. treating  $N$  as effectively infinite. See Rancourt (1999) and Fay (1999) for other variance estimation approaches for nearest neighbour imputation and Little and Rubin (2002) for multiple imputation approaches.

The delta method is applied most simply to the estimator  $\hat{\theta}_{PS}$  obtained from propensity score weighting. From equation (16), a delta-method estimator of  $\hat{\theta}_{PS}$  is given by

$$\hat{V}(\hat{\theta}_{PS}) = (\sum_{s_1} w_{PSi})^{-2} \sum_{s_1} w_{PSi}^2 (u_i - \hat{\theta}_{PS})^2. \quad (19)$$

We next consider the single and fractional imputation methods in section 3 based upon nearest neighbour imputation and use the expression for the variance of  $\hat{\theta}_{IMP}$  in (15).

The simple estimator of the first term  $\sigma^2 / n$  :

$$n^{-1} \hat{\sigma}^2 = n^{-2} \sum_{s_1} w_i (u_i - \hat{\theta}_{IMP})^2 \quad (20)$$

is approximately unbiased from Corollary 1 of Chen and Shao (2000). It follows that an approximately unbiased estimator of  $\text{var}(\hat{\theta}_{IMP})$  is

$$\hat{V}(\hat{\theta}_{IMP}) = n^{-1} \hat{\sigma}^2 + n^{-2} \sum_{s_1} (w_i^2 - w_i) \hat{V}(u_i | z_i) \quad (21)$$

if we can construct an approximately unbiased estimator  $\hat{V}(u_i | z_i)$  of  $V(u_i | z_i)$ . Various approaches to estimating  $V(u_i | z_i)$  seem possible. Following Fay (1999), we might consider

the sample variance of  $u_j$  values for responding neighbours near to  $i$  with respect to  $z$ . With  $u_i$  binary this may be a rather unstable estimator, however, if the number of neighbours is small and might be biased if the number of neighbours is large. We have therefore considered instead a model-based approach in which a model is fitted to  $\psi(z_i) = E(u_i | z_i)$  for  $i \in s$  giving  $\hat{\psi}(z_i)$  and we set  $\hat{V}(u_i | z_i) = \hat{\psi}(z_i)[1 - \hat{\psi}(z_i)]$ . We have considered nonparametric methods of fitting  $\psi(z_i)$ , but have found with the LFS data that these lead to very similar values of  $\hat{V}(\hat{\theta}_{IMP})$  as a logistic regression model for  $\psi(z_i)$ .

## 7. SIMULATION STUDY

The aim of the study is to generate independent repeated samples  $s^{(h)}$ ,  $h = 1, \dots, H$ , with realistic values  $y_i, y_i^*, x_i, r_i$ ,  $i \in s^{(h)}$ , to compute the corresponding estimates  $\tilde{F}^{(h)}(y)$  for alternative approaches to missing data and values of  $y$  and to assess the performance of the estimators  $\tilde{F}(y)$  empirically. In order to employ realistic values, the samples  $s^{(h)}$  of size  $n$  were drawn with replacement (i.e. using the bootstrap) from an actual sample of about 16,000 jobs for the March-May 2000 quarter of the LFS (only main jobs of employees aged 18+ were considered and the very small number of cases with missing values on  $y_i^*$  or  $x_i$  were omitted). The effective assumption that the population size is infinite seems reasonably given the small sampling fraction of the LFS. The assumption of (simple) random sampling neglects the clustering of the sample into households, although the impact of this simplification on the relative properties of estimators is expected to be slight. The values of  $x_i$  for each sample  $s^{(h)}$  were taken directly from the values in the LFS sample. Variables were chosen for inclusion in  $x_i$  if they were either related to hourly pay, measurement error in  $y_i^*$  or response  $r_i$  (see Skinner et al. 2003) and included age, gender, household position, qualifications, occupation, duration of employment, full-time/ part-time, industry and region

(several of these variables were represented by dummy variables). We set  $n=15,000$  and  $H=1000$  and generated the values of  $y_i$ ,  $y_i^*$  and  $r_i$  for each sample  $s^{(h)}$  from models, rather than directly from the LFS data, for the following reasons.

$y_i$ : these values were generated from a model because they were frequently missing in the LFS. A linear regression model was used, relating  $\ln(y_i)$  to  $\ln(y_i^*)$  and  $x_i$  with a normal error and with 20 covariates including squared terms in  $\ln(y_i^*)$  and age and interactions between  $\ln(y_i^*)$  and 5 components of  $x_i$ . The model was fitted to the roughly 7000 cases where  $y_i$  was observed.

$y_i^*$ : these values were generated from a model to avoid duplicate values of  $(y_i^*, x_i)$  within each  $s^{(h)}$ , which it was considered might lead to an unrealistic distribution of distances between units for the nearest neighbour method. The model was a linear regression model relating  $\ln(y_i^*)$  to  $x_i$  with a normal error and with 12 covariates, including a squared term in age and one interaction, fitted to the LFS data.

$r_i$ : these values were generated from a model to ensure that the missing data mechanism was known. Several models were fitted. The only one reported here is a logistic regression relating  $r_i$  to  $\ln(y_i^*)$  and  $x_i$  with 17 covariates including squared  $\ln(y_i^*)$  and interactions between  $\ln(y_i^*)$  and two covariates. The model was fitted to the LFS data. Note that this missing data mechanism is MAR given the  $y_i^*$  and  $x_i$ . An alternative non-MAR assumption was also used – see the next section.

Estimates  $\hat{\theta}_t^{(h)}$  of two parameters ( $t = 1, 2$ ) were obtained for each sample  $s^{(h)}$ ,

$\theta_1$  = proportion with pay below the national minimum wage (=£3.00 per hour age 18-21, £3.60 per hour aged 22+)

$\theta_2$  = proportion with pay between minimum wage and £5/ hour.

The true values are  $\theta_1=0.056$  and  $\theta_2= 0.185$ . The bias and standard error were estimated as

$$\hat{bias}(\hat{\theta}_t) = \bar{\theta}_t - \theta_t \text{ and } \hat{s.e.}(\hat{\theta}_t) = [H^{-1} \sum_{h=1}^H (\hat{\theta}_t^{(h)} - \bar{\theta}_t)^2]^{1/2}, \text{ where } \bar{\theta}_t = H^{-1} \sum_h \theta_t^{(h)}.$$

We first compare results for the alternative imputation approaches. Table 1 presents estimates of the biases of estimators of  $\theta_1$  and  $\theta_2$  for different imputation methods, for a MAR missing data mechanism. There is no evidence of significant biases for any of the nearest neighbour (NN) methods. The bias/ standard error ratios are small and may be expected to be even smaller for estimates within domains e.g. regions or age groups. We conclude that there is no evidence of important bias for these methods, provided the MAR mechanism holds and the model is correctly specified.

There is some evidence of statistically significant biases for each of the three methods based on imputation classes (HDIWR10, HDIWOR10, ABB10) perhaps because of the width of the classes, although the bias appears to be small relative to the standard error. Given the additional disadvantage of these methods, that the specification of the boundaries of the classes is arbitrary, these methods appear to be less attractive than the nearest neighbour methods. This finding contrasts with the preference sometimes expressed (e.g. Brick and Kalton, 1996, p. 227) for stochastic methods of imputation, such as the HDI methods, compared to deterministic methods, such as nearest neighbour imputation, when estimating distributional parameters.

[Table 1 about here]

Corresponding estimates of standard errors are given in Table 2. We find as expected that the greatest standard error occurs for the single NN1 imputation method. The variance is reduced by around 10% using the penalty function method (NN1P). About 10-20% reduction arises from using two imputations (NN2 or NN2(4)) and around 20% reduction from using ten imputations (NN10, NN10(20)), HDIWR10, HDIWOR10, ABB10). For a given number of imputations (2 or 10) there seem to be no obvious systematic effects of using a stochastic method (NN2(4) or NN10(20)) versus a deterministic method (NN2 or

NN10). We conclude that NN10 is the most promising approach, avoiding the bias of the imputation class methods and having appreciable efficiency gains over the methods generating one or two imputations.

[Table 2 about here]

We next compare the NN10 imputation approach with propensity score weighting. We consider not only the case when the specification of the model used for imputation or weighting corresponds to the model used in the simulation, as in Table 1, but also some cases of misspecification. To ensure a fair comparison of weighting and imputation we use the same covariates when fitting both the models generating  $y_i$  and  $r_i$ . We first consider the estimated biases in Table 3. When the model for imputation (NN10) or the propensity scores is correctly specified neither method demonstrates any significant bias in the estimation of  $\theta_1$  or  $\theta_2$ . Significant bias does arise, however, in both cases if the model is misspecified by failing to include covariates used in the simulation. The amount of bias is noticeably greater for the weighting approach. Corresponding estimated standard errors of  $\hat{\theta}_1$  and  $\hat{\theta}_2$  are given in Table 4. These also tend to be greater for the weighting approach with the increase of mean squared error ranging from 20% to 28% for the six values in Table 4. At least under the MAR assumption, the NN10 imputation approach appears to be preferable to propensity score weighting in terms of bias and variance.

[Table 3 and 4 about here]

Finally, we compare the properties of imputation (NN10) and propensity score weighting when the MAR assumption fails. We now simulate missingness according to the Common Measurement Error model assumption of section 3. The same logistic model with the same coefficients as in the previous simulation except that  $y_i^*$  is replaced as a covariate by  $y_i$ . Simulation estimates of biases and standard errors are presented in Table 5. We observe a non-negligible significant relative bias of around 5% for the imputation approach and a little higher for the propensity score weighting approach. The positive direction of the bias of  $\hat{\theta}_1$

is as expected from arguments in Dickens and Manning (2002) and Skinner et al. (2003). The relative bias of 5% of the NN10 approach does not, however, appear to make the resulting estimates unusable.

[Table 5 about here]

## 8. CONCLUSIONS

Measurement error may lead to serious upward bias in the estimation of proportions with low pay. Missing data methods have been used to correct for this bias. Figure 1 compares an estimated distribution which ignores measurement error (the bold line) with estimates based on three missing data methods (the three dotted lines). We suggest the latter estimates are more approximately unbiased than the former estimate. Corresponding estimates of two low pay proportions of interest are presented in Table 6. The estimates in both Figure 1 and Table 6 employ survey weights. Note that the estimates presented here might differ slightly from official UK estimates since, for example, the official estimates are based on different imputation models, treating outliers differently or imputing differently for certain professions.

[Table 6 about here]

The ‘missing data adjustments’ have a substantial impact. The differences between the missing data methods are much smaller. Among imputation methods, nearest neighbour methods have performed most promisingly in terms of bias. These deterministic methods display no evidence of greater bias than stochastic imputation methods. Fractional imputation has shown appreciable efficiency gains compared to single imputation and appears more effective than penalizing the distance function or sampling without replacement with single imputation. In comparison to a propensity score weighting approach, the fractional nearest neighbour imputation has performed similarly, but has demonstrated slight advantages of robustness and efficiency.

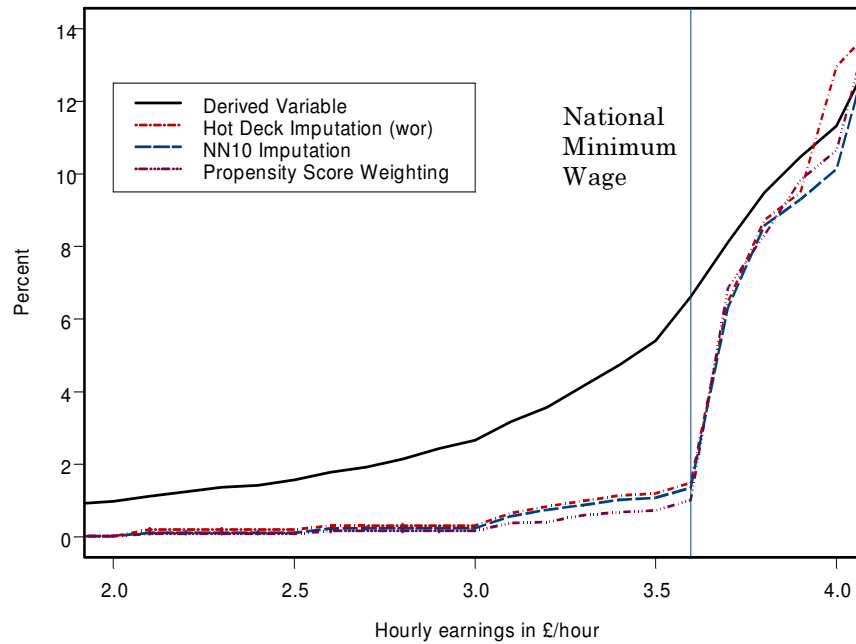
## REFERENCES

- Brick, J. M and Kalton, G. (1996), "Handling Missing Data in Survey Research", *Statistical Methods in Medical Research*, **5**, 215-238.
- Buonaccorsi, J. P. (1990), "Double Sampling for Exact Values in Some Multivariate Measurement Error Problems", *Journal of the American Statistical Association*, **85**, 1075-1082.
- Carroll, R. J., Ruppert, D. and Stefanski, L.A. (1995), *Measurement Error in Nonlinear Models*, London, Chapman and Hall.
- Chen, J. and Shao, J. (2000), "Nearest Neighbour Imputation for Survey Data", *Journal of Official Statistics*, **16**, 2, 113-131.
- Chen, J. and Shao, J. (2001), "Jackknife Variance Estimation for Nearest Neighbour Imputation", *Journal of the American Statistical Association*, **96**, 453, 260-269.
- Chesher, A. (1991), "The Effect of Measurement Error", *Biometrika*, **78**, 451-462.
- David, M. H., Little, R., Samuhel, M. and Triest, R. (1983), "Imputation Models Based on the Propensity to Respond", *Proceedings of the Business and Economic Statistics Section*, American Statistical Association, 168-173.
- Dickens, R. and Manning, A. (2002), "Has the National Minimum Wage Reduced UK Wage Inequality?", Discussion Paper 533, Centre for Economic Performance, London School of Economics, <http://cep.lse.ac.uk/pubs/download/dp0533.pdf>.
- Fay, R. E. (1996), "Alternative Paradigms for the Analysis of Imputed Survey Data", *Journal of the American Statistical Association*, **91**, 434, 490-498.
- Fay, R. E. (1999), "Theory and Application of Nearest Neighbour Imputation in Census 2000", *Proceedings of the Survey Research Methods Section*, *American Statistical Association*, 112-121.
- Fuller, W. A. (1995), "Estimation in the Presence of Measurement Error", *International Statistical Review*, **63**, 121-141.
- Kalton, G. (1983), *Compensating for Missing Survey Data*, Michigan.
- Kalton, G. and Kish, L. (1984), "Some Efficient Random Imputation Methods", *Communications in Statistics, Part A, Theory and Methods*, **13**, 1919-1939.
- Kim, J-K. and Fuller, W. A. (2002), "Variance Estimation for Nearest Neighbour Imputation", unpublished manuscript.
- Little, R. J. A. (1986), "Survey Nonresponse Adjustments for Estimates of Means", *International Statistical Review*, **54**, 2, 139-157.
- Little, R. J. A. (1988), "Missing-Data Adjustments in Large Surveys", *Journal of Business and Economic Statistics*, **6**, 3, 287-301.
- Little, R. J. A. and Rubin, D. B. (2002), *Statistical Analysis with Missing Data*, New York, Wiley.



- Luo, M., Stokes, L. and Sager, T. (1998), "Estimation of the CDF of a Finite Population in the Presence of a Calibration Sample", *Environmental and Ecological Statistics*, **5**, 277-289.
- Moore, J. C., Stinson, L. L. and Welniak, E. J. (2000), "Income Measurement Error in Surveys: A Review", *Journal of Official Statistics*, **16**, 331-361.
- ONS (1999), *Labour Force Survey*, User Guide, Volume 1, Background and Methodology, London.
- Rancourt, E. (1999), "Estimation with Nearest Neighbour Imputation at Statistics Canada", *Proceedings of the Survey Research Methods Section, American Statistical Association*, 131-138.
- Rodgers, W. L., Brown, C. and Duncan, G. J. (1993), "Errors in Survey Reports of Earnings, Hours Worked and Hourly Wages", *Journal of the American Statistical Association*, **88**, 1208-1218.
- Rubin, D. B. (1996), "Multiple Imputation After 18+ Years", *Journal of the American Statistical Association*, **91**, 434, 473-489.
- Rubin, D. B. and Schenker N. (1986), "Multiple Imputation for Interval Estimation from Simple Random Samples With Ignorable Nonresponse", *Journal of the American Statistical Association*, **81**, 394, 366-374.
- Skinner, C. J. (1989) Domain Means, Regression and Multivariate Analysis", in C. J. Skinner, D. Holt and T. M. F. Smith eds., *Analysis of Complex Surveys*, Chichester, Wiley.
- Skinner, C., Stuttard, N., Beissel-Durrant, G. and Jenkins, J. (2003), "The Measurement of Low Pay in the UK Labour Force Survey", *Oxford Bulletin of Economics and Statistics*. (to appear) [This paper may be downloaded as Working Paper M03/04 from <http://www.ssrc.soton.ac.uk/publications/methodology.html>]
- Stuttard, N. and Jenkins, J. (2001), "Measuring Low Pay Using the New Earnings Survey and the Labour Force Survey", *Labour Market Trends*, January 2001, 55-66.
- Tenenbein, A. (1970), "A Double Sampling Scheme for Estimating from Binary Data with Misclassifications", *Journal of the American Statistical Association*, **65**, 1350-1361.

**Figure 1. Alternative Estimates of the Distribution of Hourly Earnings From £ 2 to £ 4 for Age Group 22+, June-August 1999.**



**Table 1. Simulation Estimates of Biases of Estimators of  $\theta_1$  and  $\theta_2$  for Different Imputation Methods, Assuming MAR and Correct Covariates.**

| <b>Imputation Method</b> | <b>Bias of <math>\hat{\theta}_1</math></b> | <b>Rel. Bias of <math>\hat{\theta}_1</math></b> | <b>Bias of <math>\hat{\theta}_2</math></b> | <b>Rel. Bias of <math>\hat{\theta}_2</math></b> |
|--------------------------|--|---|--|---|
| NN1                      | $1.2*10^{-4}$<br>( $0.9*10^{-4}$ )         | 0.2 %   | $0.9*10^{-4}$<br>( $1.7*10^{-4}$ )         | 0.0 %   |
| NN1P <sup>1</sup>        | $4.4*10^{-4}$<br>( $2.6*10^{-4}$ )         | 0.8 %   | $0.3*10^{-4}$<br>( $5.1*10^{-4}$ )         | 0.0 %   |
| NN2                      | $0.6*10^{-4}$<br>( $8.5*10^{-4}$ )         | 0.1 %   | $1.6*10^{-4}$<br>( $1.5*10^{-4}$ )         | 0.0 %   |
| NN2(4)                   | $1.4*10^{-4}$<br>( $0.9*10^{-4}$ )         | 0.2 %   | $-2.5*10^{-4}$<br>( $1.5*10^{-4}$ )        | -0.1 %  |
| NN10                     | $0.2*10^{-4}$<br>( $6.5*10^{-4}$ )         | 0.0 %   | $-1.2*10^{-4}$<br>( $1.5*10^{-4}$ )        | -0.1 %  |
| NN10(20)                 | $0.2*10^{-4}$<br>( $0.8*10^{-4}$ )         | 0.0 %   | $0.7*10^{-4}$<br>( $1.5*10^{-4}$ )         | 0.0 %   |
| HDIWR10                  | $2.8*10^{-4}$<br>( $0.7*10^{-4}$ )         | 0.5 %   | $26.2*10^{-4}$<br>( $1.5*10^{-4}$ )        | 1.4 %   |
| HDIWOR10                 | $2.5*10^{-4}$<br>( $0.7*10^{-4}$ )         | 0.4 %   | $28.0*10^{-4}$<br>( $1.2*10^{-4}$ )        | 1.5 %   |
| ABB10                    | $4.6*10^{-4}$<br>( $0.8*10^{-4}$ )         | 0.8 %   | $29.8*10^{-4}$<br>( $1.5*10^{-4}$ )        | 1.6 %   |

Standard errors of bias estimates are below the estimates in parentheses.

<sup>1</sup> Note:  $H=100$  iterations were used due to computing time.

**Table 2. Simulation Estimates of Standard Errors of Estimators of  $\theta_1$  and  $\theta_2$  for Different Imputation Methods, Assuming MAR and Correct Covariates.**

| <b>Imputation Method</b> | $s.e.(\hat{\theta}_1)$ | $s.e.(\hat{\theta}_2)$ | $\frac{V(\hat{\theta}_1)}{V_{NN1}(\hat{\theta}_1)}$ | $\frac{V(\hat{\theta}_2)}{V_{NN1}(\hat{\theta}_2)}$ |
|--------------------------|------------------------|------------------------|---|---|
| NN1                      | $2.79*10^{-3}$         | $5.43*10^{-3}$         | 1   | 1   |
| NN1P <sup>2</sup>        | $2.60*10^{-3}$         | $5.15*10^{-3}$         | 0.87  | 0.91  |
| NN2                      | $2.68*10^{-3}$         | $5.05*10^{-3}$         | 0.91  | 0.86  |
| NN2(4)                   | $2.73*10^{-3}$         | $4.88*10^{-3}$         | 0.94  | 0.80  |
| NN10                     | $2.56*10^{-3}$         | $4.88*10^{-3}$         | 0.83  | 0.81  |
| NN10(20)                 | $2.57*10^{-3}$         | $4.79*10^{-3}$         | 0.84  | 0.77  |
| HDIWR10                  | $2.52*10^{-3}$         | $4.66*10^{-3}$         | 0.82  | 0.74  |
| HDIWOR10                 | $2.48*10^{-3}$         | $4.72*10^{-3}$         | 0.78  | 0.76  |
| ABB10                    | $2.63*10^{-3}$         | $4.87*10^{-3}$         | 0.88  | 0.80  |

<sup>2</sup> Note:  $H=100$  iterations were used due to computing time.

**Table 3. Simulation Estimates of Biases of Estimators of  $\theta_1$  and  $\theta_2$  for Nearest Neighbour Imputation (NN10) and Propensity Score Weighting, Assuming MAR and Correct and Misspecified Covariates.**

| Method                     | Assumed Covariates | Bias of $\hat{\theta}_1$              | Rel. Bias of $\hat{\theta}_1$ | Bias of $\hat{\theta}_2$              | Rel. Bias of $\hat{\theta}_2$ |
|----------------------------|--------------------|---------------------------------------|-------------------------------|---------------------------------------|-------------------------------|
| NN10                       | A1 (correct)       | $-0.18*10^{-4}$<br>( $0.64*10^{-4}$ ) | -0.03 %                       | $-5.8*10^{-4}$<br>( $1.20*10^{-4}$ )  | -0.31 %                       |
|                            | A2                 | $-1.31*10^{-4}$<br>( $0.65*10^{-4}$ ) | -0.24 %                       | $-4.74*10^{-4}$<br>( $1.23*10^{-4}$ ) | -0.25 %                       |
|                            | A3                 | $-1.66*10^{-4}$<br>( $0.63*10^{-4}$ ) | -0.30 %                       | $-10.6*10^{-4}$<br>( $1.23*10^{-4}$ ) | -0.57 %                       |
| Propensity Score Weighting | A1 (correct)       | $0.15*10^{-4}$<br>( $0.72*10^{-4}$ )  | 0.03 %                        | $-2.62*10^{-4}$<br>( $1.35*10^{-4}$ ) | -0.14 %                       |
|                            | A2                 | $-8.96*10^{-4}$<br>( $0.68*10^{-4}$ ) | -1.64 %                       | $70.2*10^{-4}$<br>( $1.40*10^{-4}$ )  | 3.80 %                        |
|                            | A3                 | $-5.02*10^{-4}$<br>( $0.68*10^{-4}$ ) | -0.92 %                       | $67.8*10^{-4}$<br>( $1.41*10^{-4}$ )  | 3.66 %                        |

Note: A1 is the correct model

A2 excludes the interactions and the square terms from the correct model

A3 drops further covariates from model A2.

**Table 4. Simulation Estimates of Standard Errors of Estimators of  $\theta_1$  and  $\theta_2$  for Nearest Neighbour Imputation (NN10) and Propensity Score Weighting, Assuming MAR and Correct and Misspecified Covariates.**

| Method                     | Assumed Covariates | $s.e.(\hat{\theta}_1)$ | $s.e.(\hat{\theta}_2)$ | $MSE(\hat{\theta}_1)$ | $MSE(\hat{\theta}_2)$ |
|----------------------------|--------------------|------------------------|------------------------|-----------------------|-----------------------|
| NN10                       | A1 (correct)       | $2.02*10^{-3}$         | $3.80*10^{-3}$         | $4.10*10^{-6}$        | $1.49*10^{-5}$        |
|                            | A2                 | $2.06*10^{-3}$         | $3.88*10^{-3}$         | $4.29*10^{-6}$        | $1.54*10^{-5}$        |
|                            | A3                 | $2.01*10^{-3}$         | $3.89*10^{-3}$         | $4.10*10^{-6}$        | $1.63*10^{-5}$        |
| Propensity Score Weighting | A1 (correct)       | $2.27*10^{-3}$         | $4.27*10^{-3}$         | $5.16*10^{-6}$        | $1.83*10^{-5}$        |
|                            | A2                 | $2.17*10^{-3}$         | $4.42*10^{-3}$         | $5.51*10^{-6}$        | $6.90*10^{-5}$        |
|                            | A3                 | $2.16*10^{-3}$         | $4.46*10^{-3}$         | $4.94*10^{-6}$        | $6.59*10^{-5}$        |

**Table 5. Simulation Estimates of Biases and Standard Errors of Estimators of  $\theta_1$  and  $\theta_2$  for Nearest Neighbour Imputation (NN10) and Propensity Score Weighting. Under the (non-MAR) Common Measurement Error Model.**

| Method                     | Bias of $\hat{\theta}_1$                           | Rel. Bias of $\hat{\theta}_1$ | Bias of $\hat{\theta}_2$                           | Rel. Bias of $\hat{\theta}_2$ | $s.e.(\hat{\theta}_1)$ | $s.e.(\hat{\theta}_2)$ |
|----------------------------|--|-------------------------------|--|-------------------------------|------------------------|------------------------|
| NN10                       | $29.0 \times 10^{-4}$<br>( $0.8 \times 10^{-4}$ )  | 5.1 %                         | $92.0 \times 10^{-4}$<br>( $1.48 \times 10^{-4}$ ) | 5.0 %                         | $2.53 \times 10^{-3}$  | $4.70 \times 10^{-3}$  |
| Propensity Score Weighting | $32.3 \times 10^{-4}$<br>( $0.73 \times 10^{-4}$ ) | 5.7 %                         | $100 \times 10^{-4}$<br>( $1.40 \times 10^{-4}$ )  | 5.7 %                         | $2.31 \times 10^{-3}$  | $4.42 \times 10^{-3}$  |

**Table 6. Estimates of  $\hat{\theta}_1$  and  $\hat{\theta}_2$  (Weighted) for 18+ Using Different Propensity Score Models and Imputation Models Applied to LFS, June-August 1999.**

| Method                     | Propensity Score Model or Imputation Model | (Weighted)<br>$\hat{\theta}_1$ in % | (Weighted)<br>$\hat{\theta}_2$ in % |
|----------------------------|--|-------------------------------------|-------------------------------------|
| Derived Variable           | -  | 7.13                                | 20.5                                |
| Propensity Score Weighting | A1   | 0.96                                | 34.5                                |
|                            | A2   | 1.08                                | 38.4                                |
|                            | A3   | 1.08                                | 38.4                                |
| HDIWOR10                   | A1   | 1.44                                | 32.1                                |
|                            | A2   | 1.41                                | 32.9                                |
|                            | A3   | 1.50                                | 33.2                                |
| NN10                       | A1   | 1.32                                | 32.6                                |
|                            | A2   | 1.44                                | 32.8                                |
|                            | A3   | 1.50                                | 33.0                                |