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# Optimal Fiscal and Monetary Policy Under Imperfect Competition

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## Abstract

This paper studies optimal fiscal and monetary policy under imperfect competition in a stochastic, flexible-price, production economy without capital. It shows analytically that in this economy the nominal interest rate acts as an indirect tax on monopoly profits. Unless the social planner has access to a direct 100 percent tax on profits, he will always find it optimal to deviate from the Friedman rule by setting a positive and time-varying nominal interest rate. The dynamic properties of the Ramsey allocation are characterized numerically. As in the perfectly competitive case, the labor income tax is remarkably smooth, whereas inflation is highly volatile and serially uncorrelated. An exact numerical solution method to the Ramsey conditions is proposed. *JEL Classification:* E52, E61, E63.

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# 1 Introduction

In the existing literature on optimal monetary policy two distinct branches have developed that deliver diametrically opposed policy recommendations concerning the long-run and cyclical behavior of prices and interest rates. One branch follows the theoretical framework laid out in Lucas and Stokey (1983). It studies the joint determination of optimal fiscal and monetary policy in flexible-price environments with perfect competition in product and factor markets. In this group of papers, the government's problem consists in financing an exogenous stream of public spending by choosing the least disruptive combination of inflation and distortionary income taxes. The criterion under which policies are evaluated is the welfare of the representative private agent. A basic result of this literature is the optimality of the Friedman rule. A zero opportunity cost of money has been shown to be optimal under perfect-foresight in a variety of monetary models, including cash-in-advance, money-in-the-utility function, and shopping-time models.<sup>1</sup>

In a significant contribution to the literature, Chari et al. (1991) characterize optimal monetary and fiscal policy in stochastic environments. They prove that the Friedman rule is also optimal under uncertainty: the government finds it optimal to set the nominal interest rate at zero at all dates and all states of the world. In addition, Chari et al. show that income tax rates are remarkably stable over the business cycle, and that the inflation rate is highly volatile and serially uncorrelated. Under the Ramsey policy, the government uses unanticipated inflation as a lump-sum tax on financial wealth. The government is able to do this because public debt is assumed to be nominal and non-state-contingent. Thus, inflation plays the role of a shock absorber of unexpected adverse fiscal shocks.

On the other hand, a more recent literature focuses on characterizing optimal monetary policy in environments with nominal rigidities and imperfect competition.<sup>2</sup> Besides its emphasis on the role of price rigidities and market power, this literature differs from the earlier one described above in two important ways. First, it assumes, either explicitly or implicitly, that the government has access to (endogenous) lump-sum taxes to finance its budget. An important implication of this assumption is that there is no need to use unanticipated inflation as a lump-sum tax; regular lump-sum taxes take up this role. Second, the government is assumed to be able to implement a production (or employment) subsidy so as to eliminate the distortion introduced by the presence of monopoly power in product and factor markets.

A key result of this literature is that the optimal monetary policy features an inflation rate that is zero or close to zero at all dates and all states.<sup>3</sup> In addition, the nominal interest rate is not only different from zero, but also varies significantly over the business cycle. The reason why price stability turns out to be optimal in environments of the type described here is straightforward: the government keeps the price level constant in order to minimize

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<sup>1</sup>See, for example, Chari et al. (1991), Correia and Teles (1996), Guidotti and Végh (1993), and Kimbrough (1986).

<sup>2</sup>See, for example, Erceg, Henderson, and Levin (1999), Galí and Monacelli (2000), Khan, King, and Wolman (2000), and Rotemberg and Woodford (1999).

<sup>3</sup>In models where money is used exclusively as a medium of account or when money enters in an additively separable way in the utility function, the optimal inflation rate is typically strictly zero. Khan, King, and Wolman (2000) show that when a nontrivial transaction role for money is introduced, the optimal inflation rate lies between zero and the one called for by the Friedman rule. However, in calibrated model economies they find that the optimal rate of inflation is in fact very close to zero and smooth.

(or completely eliminate) the costs introduced by inflation under nominal rigidities.

This paper is the first step of a larger research project that aims to incorporate in a unified framework the essential elements of the two approaches to optimal policy described above. Specifically, in this paper we build a model that shares three elements with the earlier literature: (a) The only source of regular taxation available to the government is distortionary income taxes. In particular, the fiscal authority cannot adjust lump-sum taxes endogenously in financing its outlays. (b) Prices are fully flexible. (c) The government cannot implement production subsidies to undo distortions created by the presence of imperfect competition. At the same time, our model shares with the more recent body of work on optimal monetary policy the assumption that product markets are not perfectly competitive. In particular, we assume that each firm in the economy is the monopolistic producer of a differentiated good. An assumption maintained throughout this paper that is common to all of the papers cited above (except for Lucas and Stokey, 1983) is that the government has the ability to fully commit to the implementation of announced fiscal and monetary policies.

In our imperfectly competitive economy, profits represent the income to a fixed “factor,” namely, monopoly rights. It is therefore optimal for the Ramsey planner to tax profits at a 100 percent rate. Realistically, however, governments cannot implement a complete confiscation of this type of income. The main finding of our paper is that under this restriction the Friedman rule ceases to be optimal. The Ramsey planner resorts to a positive nominal interest rate as an indirect way to tax profits. The nominal interest rate represents an indirect tax on profits because households must hold (non-interest-bearing) fiat money in order to convert income into consumption. Indeed, we find that the Ramsey allocation features an increasing relationship between the nominal interest rate and monopoly profits. Under the assumption of no profit taxation and for plausible calibrations of other structural parameters of the model economy, we find that as the profit share increases from zero to 25 percent, the optimal average nominal interest rate increases continuously from zero to 8 percent per year. In addition, the interest rate is time varying and its volatility is increasing in the degree of monopoly power.

The second central result of our study is that while the first moments of inflation, the nominal interest rate, and tax rates are sensitive to the degree of market power in the Ramsey allocation, the cyclical properties of these variables under imperfect competition are similar to those arising in perfectly competitive environments. In particular, it is optimal for the government to smooth tax rates and to make the inflation rate highly volatile. Thus, as in the case of perfect competition, the government uses variations in the price level as a state-contingent tax on financial wealth.

The remainder of the paper is organized in 5 sections. Section 2 describes the economic environment and defines a competitive equilibrium. Section 3 presents the primal form of the equilibrium and shows that it is equivalent to the definition of competitive equilibrium given in section 2. Section 4 establishes the first central result of this paper. Namely, the fact that the Friedman rule is not optimal when monopoly profits cannot be fully confiscated. Section 5 analyzes the business-cycle properties of Ramsey allocations. It first presents a novel numerical method for solving the exact conditions of the Ramsey problem. It then describes the calibration of the model and presents the quantitative results. Section 6 presents some concluding remarks.

## 2 The model

In this section we develop a simple infinite-horizon production economy with imperfectly competitive product markets. Prices are assumed to be fully flexible and asset markets complete. The government finances an exogenous stream of purchases by levying distortionary income taxes and printing money.

### 2.1 The private sector

Consider an economy populated by a large number of identical households. Each household has preferences defined over processes of consumption and leisure and described by the utility function

$$E_0 \sum_{t=0}^{\infty} \beta^t U(c_t, h_t) \quad (1)$$

where  $c_t$  denotes consumption,  $h_t$  denotes labor effort,  $\beta \in (0, 1)$  denotes the subjective discount factor, and  $E_0$  denotes the mathematical expectation operator conditional on information available in period 0. The single period utility function  $U$  is assumed to be increasing in consumption, decreasing in effort, strictly concave, and twice continuously differentiable.

In each period  $t \geq 0$ , households can purchase two types of financial assets: fiat money,  $M_t$ , and one-period, state-contingent, nominal assets,  $D_{t+1}$ , that pay one unit of currency in a particular state of period  $t + 1$ . Money holdings are motivated by assuming that they facilitate consumption purchases. Specifically, consumption purchases are subject to a proportional transaction cost  $s(v_t)$  that depends on the household's money-to-consumption ratio, or consumption-based money velocity,

$$v_t = \frac{P_t c_t}{M_t}, \quad (2)$$

where  $P_t$  denotes the price of the consumption good in period  $t$ . The transaction cost function satisfies the following assumption:

**Assumption 1** *The function  $s(v)$  satisfies: (a)  $s(v)$  is nonnegative and twice continuously differentiable; (b) There exists a level of velocity  $\underline{v} > 0$ , to which we refer as the satiation level of money, such that  $s(\underline{v}) = s'(\underline{v}) = 0$ ; (c)  $(v - \underline{v})s'(v) > 0$  for  $v \neq \underline{v}$ ; and (d)  $2s'(v) + vs''(v) > 0$  for all  $v \geq \underline{v}$ .*

Assumption 1(a) states that the transaction cost is smooth. Assumption 1(b) ensures that the Friedman rule, i.e., a zero nominal interest rate, need not be associated with an infinite demand for money. It also implies that both the transaction cost and the distortion it introduces vanish when the nominal interest rate is zero. Assumption 1(c) guarantees that money velocity is always greater than or equal to the satiation level. As will become clear shortly, assumption 1(d) ensures that the demand for money is decreasing in the nominal interest rate. (Note that assumption 1(d) is weaker than the more common assumption of strict convexity of the transaction cost function.)

The consumption good  $c_t$  is assumed to be a composite good made of a continuum of intermediate differentiated goods. The aggregator function is of the Dixit-Stiglitz type. Each household is the monopolistic producer of one variety of intermediate goods. The intermediate goods are produced using a linear technology,  $z_t \tilde{h}_t$ , that takes labor,  $\tilde{h}_t$ , as the sole input and is subject to an exogenous productivity shock,  $z_t$ . The household hires labor from a perfectly competitive market. The demand for the intermediate input is of the form  $Y_t d(p_t)$ , where  $Y_t$  denotes the level of aggregate demand and  $p_t$  denotes the relative price of the intermediate good in terms of the composite consumption good. The demand function  $d(\cdot)$  is assumed to be decreasing and to satisfy  $d(1) = 1$  and  $d'(1) < -1$ .<sup>4</sup> The monopolist sets the price of the good it supplies taking the level of aggregate demand as given, and is constrained to satisfy demand at that price, that is,

$$z_t \tilde{h}_t \geq Y_t d(p_t). \quad (3)$$

Finally, we assume that each period the household receives profits from financial institutions in the amount  $\Pi_t$ . The household takes  $\Pi_t$  as exogenous. We introduce the variable  $\Pi_t$  because we want to allow for the possibility that in equilibrium only a fraction of the transactions costs be true resource (or shoe-leather) costs. We do so by assuming that part of the transaction cost is rebated to the public in a lump-sum fashion. We motivate this rebate by assuming that part of the transaction cost represent pure profits of financial institutions owned (in equal shares) by the public.

The flow budget constraint of the household in period  $t$  is then given by:

$$P_t c_t [1 + s(v_t)] + M_t + E_t r_{t+1} D_{t+1} \leq M_{t-1} + D_t + P_t [p_t Y_t d(p_t) - w_t \tilde{h}_t] + (1 - \tau_t) P_t w_t h_t + \Pi_t, \quad (4)$$

where  $w_t$  is the real wage rate and  $\tau_t$  is the labor income tax rate. The variable  $r_{t+1}$  denotes the period- $t$  price of a claim to one unit of currency in a particular state of period  $t+1$  divided by the probability of occurrence of that state conditional on information available in period  $t$ . The left-hand side of the budget constraint represents the uses of wealth: consumption spending, including transactions costs, money holdings, and purchases of interest bearing assets. The right-hand side shows the sources of wealth: money and contingent claims acquired in the previous period, profits from the sale of the differentiated good, after-tax labor income, and profits received from financial institutions.

In addition, the household is subject to the following borrowing constraint that prevents it from engaging in Ponzi schemes:

$$\lim_{j \rightarrow \infty} E_t q_{t+j+1} (D_{t+j+1} + M_{t+j}) \geq 0, \quad (5)$$

at all dates and under all contingencies. The variable  $q_t$  represents the period-zero price of one unit of currency to be delivered in a particular state of period  $t$  divided by the probability of occurrence of that state given information available at time 0 and is given by

$$q_t = r_1 r_2 \dots r_t,$$

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<sup>4</sup>The restrictions on  $d(1)$  and  $d'(1)$  are necessary for the existence of a symmetric equilibrium.

with  $q_0 \equiv 1$ .

The household chooses the set of processes  $\{c_t, h_t, \tilde{h}_t, p_t, v_t, M_t, D_{t+1}\}_{t=0}^{\infty}$ , so as to maximize (1) subject to (2)-(5), taking as given the set of processes  $\{Y_t, P_t, w_t, r_{t+1}, \tau_t, z_t\}_{t=0}^{\infty}$  and the initial condition  $M_{-1} + D_0$ .

Let the multiplier on the flow budget constraint be  $\lambda_t/P_t$ . Then the first-order conditions of the household's maximization problem are (2)-(5) holding with equality and

$$U_c(c_t, h_t) = \lambda_t[1 + s(v_t) + v_t s'(v_t)] \quad (6)$$

$$-\frac{U_h(c_t, h_t)}{U_c(c_t, h_t)} = \frac{(1 - \tau_t)w_t}{1 + s(v_t) + v_t s'(v_t)} \quad (7)$$

$$v_t^2 s'(v_t) = 1 - E_t r_{t+1} \quad (8)$$

$$\frac{\lambda_t}{P_t} r_{t+1} = \beta \frac{\lambda_{t+1}}{P_{t+1}} \quad (9)$$

$$p_t + \frac{d(p_t)}{d'(p_t)} = \frac{w_t}{z_t} \quad (10)$$

The interpretation of these optimality conditions is straightforward. The first-order condition (6) states that the transaction cost introduces a wedge between the marginal utility of consumption and the marginal utility of wealth. The assumed form of the transaction cost function ensures that this wedge is zero at the satiation point  $\underline{v}$  and increasing in money velocity for  $v > \underline{v}$ . Equation (7) shows that both the labor income tax rate and the transaction cost distort the consumption/leisure margin. Given the wage rate, households will tend to work less and consume less the higher is  $\tau$  or the smaller is  $v_t$ . Equation (8) implicitly defines the household's money demand function. Note that  $E_t r_{t+1}$  is the period- $t$  price of an asset that pays one unit of currency in every state in period  $t + 1$ . Thus  $E_t r_{t+1}$  represents the inverse of the risk-free gross nominal interest rate. Formally, letting  $R_t$  denote the gross risk-free nominal interest rate, we have

$$R_t = \frac{1}{E_t r_{t+1}}$$

Our assumptions about the form of the transactions cost function imply that the demand for money is strictly decreasing in the nominal interest rate and unit elastic in consumption. Equation (9) represents a standard pricing equation for one-step-ahead nominal contingent claims. Finally, equation (10) states that firms set prices so as to equate marginal revenue,  $p_t + d(p_t)/d'(p_t)$ , to marginal cost,  $w_t/z_t$ .

## 2.2 The government

The government faces a stream of public consumption, denoted by  $g_t$ , that is exogenous, stochastic, and unproductive. These expenditures are financed by levying labor income taxes at the rate  $\tau_t$ , by printing money, and by issuing one-period, risk-free, nominal obligations, which we denote by  $B_t$ . The government's sequential budget constraint is then given by

$$M_t + B_t = M_{t-1} + R_{t-1}B_{t-1} + P_t g_t - \tau_t P_t w_t h_t$$

for  $t \geq 0$ . The monetary/fiscal regime consists in the announcement of state-contingent plans for the nominal interest rate and the tax rate,  $\{R_t, \tau_t\}$ .

## 2.3 Equilibrium

We restrict attention to symmetric equilibria where all households charge the same price for the good they produce. As a result, we have that  $p_t = 1$  for all  $t$ . It then follows from the fact that all firms face the same wage rate, the same technology shock, and the same production technology, that they all hire the same amount of labor. That is,  $\tilde{h}_t = h_t$ . Also, because all firms charge the same price, we have that the marginal revenue of the individual monopolist, given by the left-hand side of (10), is constant and equal to  $1 + 1/d'(1)$ . Letting

$$\eta = d'(1)$$

denote the equilibrium value of the elasticity of demand faced by the monopolist and  $\mu$  the equilibrium gross markup of prices to marginal cost, we have that

$$\mu = \frac{\eta}{1 + \eta} \tag{11}$$

The equilibrium gross markup approaches unity as the demand elasticity becomes perfectly elastic ( $\eta \rightarrow -\infty$ ) and approaches infinity as the demand elasticity becomes unit elastic ( $\eta \rightarrow -1$ ).

Because all households are identical, in equilibrium there is no borrowing or lending among them. Thus, all interest-bearing asset holdings by private agents are in the form of government securities. That is,

$$D_t = R_{t-1}B_{t-1}$$

at all dates and all contingencies. In equilibrium, it must be the case that the nominal interest rate is non-negative,

$$R_t \geq 1.$$

Otherwise pure arbitrage opportunities would exist and households' demand for consumption would not be well defined.

Finally, as explained earlier, we assume that only a fraction  $\alpha \in [0, 1]$  of the transaction cost represents a true resource cost. The rest of the costs are assumed to be pure profits of financial institutions owned by the public. Thus, in equilibrium

$$\Pi_t = (1 - \alpha)c_t s(v_t)$$

We are now ready to define an equilibrium. A competitive equilibrium as a set of plans  $\{c_t, h_t, M_t, B_t, v_t, w_t, \lambda_t, P_t, q_t, r_{t+1}\}$  satisfying the following conditions:

$$U_c(c_t, h_t) = \lambda_t[1 + s(v_t) + v_t s'(v_t)] \quad (12)$$

$$-\frac{U_h(c_t, h_t)}{U_c(c_t, h_t)} = \frac{(1 - \tau_t)w_t}{1 + s(v_t) + v_t s'(v_t)} \quad (13)$$

$$v_t^2 s'(v_t) = \frac{R_t - 1}{R_t} \quad (14)$$

$$\lambda_t r_{t+1} = \beta \lambda_{t+1} \frac{P_t}{P_{t+1}} \quad (15)$$

$$R_t = \frac{1}{E_t r_{t+1}} \geq 1 \quad (16)$$

$$\frac{1 + \eta}{\eta} = \frac{w_t}{z_t}, \quad (17)$$

$$M_t + B_t + \tau_t P_t w_t h_t = R_{t-1} B_{t-1} + M_{t-1} + P_t g_t \quad (18)$$

$$\lim_{j \rightarrow \infty} E_t q_{t+j+1} (R_{t+j} B_{t+j} + M_{t+j}) = 0 \quad (19)$$

$$q_t = r_1 r_2 \dots r_t \quad \text{with } q_0 = 1 \quad (20)$$

$$[1 + \alpha s(v_t)]c_t + g_t = z_t h_t \quad (21)$$

$$v_t = P_t c_t / M_t, \quad (22)$$

given policies  $\{R_t, \tau_t\}$ , exogenous processes  $\{z_t, g_t\}$ , and the initial condition  $R_{-1} B_{-1} + M_{-1}$ . The optimal fiscal and monetary policy is the process  $\{R_t, \tau_t\}$  associated with the competitive equilibrium that yields the highest level of utility to the representative household, that is, that maximizes (1). As is well known, the planner will always find it optimal to confiscate the entire initial financial wealth of the household by choosing a policy consistent with an infinite initial price level,  $P_0 = \infty$ . This is because such a confiscation amounts to a nondistortionary lump-sum tax. To avoid this unrealistic feature of optimal policy, we restrict the initial price level to be arbitrarily given.

To find the optimal policy, it is convenient to use a simpler representation of the competitive equilibrium known as the primal form. We turn to this task next.

### 3 Ramsey allocations

In this section we analytically derive the first central result of this paper. Namely, that the Friedman rule ceases to be optimal when product markets are imperfectly competitive. We begin by deriving the primal form of the competitive equilibrium. Then we state the Ramsey problem. And finally we characterize optimal fiscal and monetary policy.

#### 3.1 The primal form

Following a long-standing tradition in Public Finance, we study optimal policy using the primal-form representation of the competitive equilibrium. Finding the primal form involves the elimination of all prices and tax rates from the equilibrium conditions, so that the resulting reduced form involves only real variables. In our economy, the real variables that appear in the primal form are consumption, hours, and money velocity. The primal form of the equilibrium conditions consists of two equations. One equation is a feasibility constraint, given by the resource constraint (21), which must hold at every date and under all contingencies. The other equation is a single, present-value constraint known as the implementability constraint. The implementability constraint guarantees that at the prices and quantities associated with every possible competitive equilibrium, the present discounted value of consolidated government surpluses equals the government's total initial liabilities. The following proposition presents the primal form of the competitive equilibrium and establishes that it is equivalent to the definition of competitive equilibrium given in section 2.3.

**Proposition 1** *Plans  $\{c_t, h_t, v_t\}_{t=0}^{\infty}$  satisfying*

$$z_t h_t = [1 + \alpha s(v_t)] c_t + g_t, \quad (23)$$

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ U_c(c_t, h_t) c_t \phi(v_t) + U_h(c_t, h_t) h_t + \frac{U_c(c_t, h_t) z_t h_t}{\gamma(v_t) \eta} \right\} = \frac{U_c(c_0, h_0)}{\gamma(v_0)} \frac{R_{-1} B_{-1} + M_{-1}}{P_0}, \quad (24)$$

$$v_t \geq \underline{v} \quad \text{and} \quad v_t^2 s'(v_t) < 1,$$

given  $(R_{-1} B_{-1} + M_{-1})$  and  $P_0$ , are the same as those satisfying (12)-(22), where

$$\gamma(v_t) \equiv 1 + s(v_t) + v_t s'(v_t)$$

and

$$\phi(v_t) \equiv [1 + \alpha s(v_t) + v_t s'(v_t)] / \gamma(v_t)$$

**Proof:** We first show that plans  $\{c_t, h_t, v_t\}$  satisfying (12)-(22) also satisfy (23), (24),  $v_t \geq \underline{v}$ , and  $v_t^2 s'(v_t) < 1$ .

Obviously, (21) implies (23). Furthermore, (14), (16), and assumption 1 together imply that  $v_t \geq \underline{v}$  and  $v_t^2 s'(v_t) < 1$ . Let  $W_{t+1} = R_t B_t + M_t$  and note that  $W_{t+1}$  is in the information set of time  $t$ . Use this expression to eliminate  $B_t$  from (18) and multiply by  $q_t$  to obtain

$$q_t M_t (1 - R_t^{-1}) + q_t E_t r_{t+1} W_{t+1} - q_t W_t = q_t [P_t g_t - \tau_t P_t w_t h_t],$$

where we use (16) to write  $R_t$  in terms of  $r_{t+1}$ . Take expectations conditional on information available at time zero and sum for  $t = 0$  to  $t = T$  to obtain

$$E_0 \sum_{t=0}^T [q_t M_t (1 - R_t^{-1}) - q_t (P_t g_t - \tau_t P_t w_t h_t)] = -E_0 q_{T+1} W_{T+1} + W_0.$$

In writing this expression, we use the fact that  $q_0 = 1$ . Take limits for  $T \rightarrow \infty$ . By (19) the limit of the right hand side is well defined and equal to  $W_0$ . Thus, the limit of the left-hand side exists. This yields:

$$E_0 \sum_{t=0}^{\infty} [q_t M_t (1 - R_t^{-1}) - q_t (P_t g_t - \tau_t P_t w_t h_t)] = W_0$$

By (15) we have that  $P_t q_t = \beta^t \lambda_t P_0 / \lambda_0$ . Use (12) to eliminate  $\lambda_t$  (22) to eliminate  $M_t / P_t$  to obtain

$$E_0 \sum_{t=0}^{\infty} \beta^t \frac{U_c(c_t, h_t)}{\gamma(v_t)} \left[ \frac{c_t}{v_t} (1 - R_t^{-1}) - (g_t - \tau_t w_t h_t) \right] = \frac{W_0}{P_0} \frac{U_c(c_0, h_0)}{\gamma(v_0)}$$

Solve (13) for  $\tau_t$  and (17) for  $w_t$ . Then  $\tau_t w_t h_t = (1 + \eta) / \eta z_t h_t + \gamma(v_t) / U_c(c_t, h_t) U_h(c_t, h_t) h_t$ . Use this in the above expression and replace  $g_t$  with (21). This yields

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[ U_c(c_t, h_t) c_t \frac{1 + \alpha s(v_t) + \frac{1 - R_t^{-1}}{v_t}}{\gamma(v_t)} + U_h(c_t, h_t) h_t + \frac{z_t h_t U_c(c_t, h_t)}{\eta \gamma(v_t)} \right] = \frac{W_0}{P_0} \frac{U_c(c_0, h_0)}{\gamma(v_0)}$$

Finally, use (14) to replace  $(1 - R_t^{-1}) / v_t$  with  $v_t s'(v_t)$  and use the definitions  $\phi(v_t) = \frac{1 + \alpha s(v_t) + v_t s'(v_t)}{\gamma(v_t)}$  and  $W_0 = R_{-1} B_{-1} + M_{-1}$  to get

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[ U_c(c_t, h_t) c_t \phi(v_t) + U_h(c_t, h_t) h_t + \frac{z_t h_t U_c(c_t, h_t)}{\eta \gamma(v_t)} \right] = \frac{R_{-1} B_{-1} + M_{-1}}{P_0} \frac{U_c(c_0, h_0)}{\gamma(v_0)}$$

which is (24).

Now we show that plans  $\{c_t, h_t, v_t\}$  that satisfy  $v_t \geq \underline{v}$ ,  $v_t^2 s'(v_t) < 1$ , (23), and (24) also satisfy (12)-(22) at all dates all contingencies.

Clearly, (23) implies (21). Given a plan  $\{c_t, h_t, v_t\}$  proceed as follows. Use (14) to construct  $R_t$  as  $1/[1 - v_t^2 s'(v_t)]$ . Note that our assumption that given assumption 1, the constraints  $v_t \geq \underline{v}$  and  $v_t^2 s'(v_t) < 1$  ensure that  $R_t \geq 1$ . Let  $\lambda_t$  be given by (12),  $w_t$  by (17) and  $\tau_t$  by (13). To construct plans for  $M_t$ ,  $P_{t+1}$ , and  $B_t$ , for  $t \geq 0$ , use the following iterative procedure: (a) Set  $t = 0$ ; (b) Use equation (22) to construct  $M_t$  (one can do this for  $t = 0$

because  $P_0$  is given); (c) Set  $B_t$  so as to satisfy equation (18); (d) Construct  $P_{t+1}$  (one price level for each state of the world in  $t + 1$ ) as the solution to:

$$E_{t+1} \sum_{j=0}^{\infty} \beta^j \left[ U_c(c_{t+j+1}, h_{t+j+1}) c_{t+j+1} \phi(v_{t+j+1}) + U_h(c_{t+j+1}, h_{t+j+1}) h_{t+j+1} + \frac{z_{t+j+1} h_{t+j+1}}{\eta} \frac{U_c(c_{t+j+1}, h_{t+j+1})}{\gamma(v_{t+j+1})} \right] = \frac{R_t B_t + M_t U_c(c_{t+1}, h_{t+1})}{P_{t+1} \gamma(v_{t+1})}. \quad (25)$$

(e) Increase  $t$  by 1 and repeat steps (b) to (e). This procedure yields plans for  $P_t$  and thus for the gross inflation rate  $\pi_t \equiv P_t/P_{t-1}$ . Of course, only if the resulting price level process is positive can the plan  $\{c_t, h_t, v_t\}$  be supported as a competitive equilibrium. Once the plan for  $\pi_t$  is known, construct a plan for  $r_{t+1}$  from (15) and let  $q_t$  be given by (20). It remains to be shown that (16) and (19) also hold. Use the definition of  $P_t$  to get for any  $t \geq 0$ :

$$\begin{aligned} & U_c(c_t, h_t) c_t \phi(v_t) + U_h(c_t, h_t) h_t + \frac{z_t h_t}{\eta} \frac{U_c(c_t, h_t)}{\gamma(v_t)} + \\ & E_t \sum_{j=1}^{\infty} \beta^j \left[ U_c(c_{t+j}, h_{t+j}) c_{t+j} \phi(v_{t+j}) + U_h(c_{t+j}, h_{t+j}) h_{t+j} + \frac{z_{t+j} h_{t+j}}{\eta} \frac{U_c(c_{t+j}, h_{t+j})}{\gamma(v_{t+j})} \right] \\ = & \frac{R_{t-1} B_{t-1} + M_{t-1} U_c(c_t, h_t)}{P_t \gamma(v_t)} \end{aligned}$$

Make a change of index. Let  $k = j - 1$  and use the definition of  $P_{t+1}$ . Then the above expression can be written as

$$\begin{aligned} & U_c(c_t, h_t) c_t \phi(v_t) + U_h(c_t, h_t) h_t + \frac{z_t h_t}{\eta} \frac{U_c(c_t, h_t)}{\gamma(v_t)} + \\ & \beta E_t \left[ \frac{U_c(c_{t+1}, h_{t+1})}{\gamma(v_{t+1})} \frac{R_t B_t + M_t}{P_{t+1}} \right] = \frac{R_{t-1} B_{t-1} + M_{t-1} U_c(c_t, h_t)}{P_t \gamma(v_t)} \end{aligned}$$

Multiplying by  $\gamma(v_t) P_t / U_c(c_t, h_t)$  yields

$$\begin{aligned} & P_t c_t (1 + (1 - R_t^{-1})/v_t) + U_h(c_t, h_t) / U_c(c_t, h_t) \gamma(v_t) P_t h_t + \frac{P_t z_t h_t}{\eta} \\ & + (R_t B_t + M_t) \beta E_t \left[ \frac{\gamma(v_t) P_t}{U_c(c_t, h_t)} \frac{U_c(c_{t+1}, h_{t+1})}{\gamma(v_{t+1}) P_{t+1}} \right] = R_{t-1} B_{t-1} + M_{t-1} \end{aligned}$$

Using (13), (15), and (21) this expression can be written as:

$$P_t z_t h_t - P_t g_t + M_t (1 - R_t^{-1}) - (1 - \tau_t) w_t P_t h_t + \frac{P_t z_t h_t}{\eta} + (R_t B_t + M_t) E_t r_{t+1} = R_{t-1} B_{t-1} + M_{t-1}$$

Then use (17) and (18) to simplify this expression to

$$(R_t B_t + M_t) E_t r_{t+1} = B_t + R_t^{-1} M_t,$$

which implies (16).

Finally, we must show that (19) holds. We have already established that (18) holds at every date and under every contingency. Multiply (18) in period  $t + j$  by  $q_{t+j}$  and take expectations conditional on information available at time  $t$  to get

$$E_t[q_{t+j}M_{t+j}(1 - r_{t+j+1}) + q_{t+j+1}W_{t+j+1}] = E_t[q_{t+j}W_{t+j} + q_{t+j}(P_{t+j}g_{t+j} - \tau_{t+j}P_{t+j}w_{t+j}h_{t+j})]$$

Now sum for  $j = 0$  to  $j = J$ .

$$E_t \sum_{j=0}^J [q_{t+j}M_{t+j}(1 - r_{t+j+1}) - q_{t+j}(P_{t+j}g_{t+j} - \tau_{t+j}P_{t+j}w_{t+j}h_{t+j})] = -E_t q_{t+J+1}W_{t+J+1} + q_t W_t$$

Divide by  $q_t P_t$

$$E_t \sum_{j=0}^J \frac{q_{t+j}P_{t+j}}{q_t P_t} [(c_{t+j}/v_{t+j})(1 - r_{t+j+1}) - (g_{t+j} - \tau_{t+j}w_{t+j}h_{t+j})] = -E_t q_{t+J+1}W_{t+J+1}/(q_t P_t) + \frac{W_t}{P_t}$$

Using the definition of  $P_t$  given by (25), we have already shown that the limit of the left-hand side of the above expression as  $J \rightarrow \infty$  is  $W_t/P_t$ . Hence the limit of the right-hand side is well defined. It then follows that

$$\lim_{J \rightarrow \infty} E_t q_{t+J+1}W_{t+J+1} = 0$$

for every date  $t$ . Using the definition of  $W_t$ , one obtains immediately (19). ■

### 3.2 The Ramsey Problem

It follows from proposition 1 that the Ramsey problem can be stated as choosing contingent plans  $c_t, h_t, v_t > 0$  so as to maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t U(c_t, h_t)$$

subject to

$$z_t h_t = [1 + \alpha s(v_t)]c_t + g_t, \tag{23}$$

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ U_c(c_t, h_t)c_t \phi(v_t) + U_h(c_t, h_t)h_t + \frac{U_c(c_t, h_t)}{\gamma(v_t)} \frac{z_t h_t}{\eta} \right\} = \frac{U_c(c_0, h_0)}{\gamma(v_0)} \frac{R_{-1}B_{-1} + M_{-1}}{P_0}, \tag{24}$$

and  $v_t^2 s'(v_t) < 1$ , taking as given  $(M_{-1} + R_{-1}B_{-1})/P_0$  and the exogenous stochastic processes  $g_t$  and  $z_t$ .

The Lagrangian of the Ramsey planner's problem is

$$\begin{aligned}\mathcal{L} = & E_0 \sum_{t=0}^{\infty} \beta^t [U(c_t, h_t) + \\ & \xi \left( U_c(c_t, h_t)c_t\phi(v_t) + U_h(c_t, h_t)h_t + \frac{U_c(c_t, h_t)}{\gamma(v_t)} \frac{z_t h_t}{\eta} \right) \\ & + \psi_t(z_t h_t - [1 + \alpha s(v_t)]c_t - g_t)] \\ & - \xi \frac{U_c(c_0, h_0)}{\gamma(v_0)} \frac{R_{-1}B_{-1} + M_{-1}}{P_0},\end{aligned}$$

where  $\psi_t$  and  $\xi$  are the Lagrange multipliers on the feasibility and implementability constraints, respectively. The partial derivatives of the Lagrangian with respect to  $v_t$ ,  $c_t$ ,  $h_t$ , for  $t > 0$  are:

$$\frac{\partial \mathcal{L}}{\partial v_t} = \xi U_c(c_t, h_t)c_t\phi'(v_t) - \xi \frac{z_t h_t}{\eta} U_c(c_t, h_t) \frac{\gamma'(v_t)}{\gamma(v_t)^2} - \psi_t \alpha s'(v_t)c_t$$

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial c_t} = & U_c(c_t, h_t)[1 + \xi\phi(v_t)] + \xi \left[ U_{cc}(c_t, h_t)c_t\phi(v_t) + U_{hc}(c_t, h_t)h_t + U_{cc}(c_t, h_t) \frac{z_t h_t}{\gamma(v_t)\eta} \right] \\ & - \psi_t[1 + \alpha s(v_t)]\end{aligned}$$

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial h_t} = & U_h(c_t, h_t) + \xi \left[ U_{ch}(c_t, h_t)c_t\phi(v_t) + U_h(c_t, h_t) + h_t U_{hh}(c_t, h_t) + U_{ch}(c_t, h_t) \frac{z_t h_t}{\gamma(v_t)\eta} \right. \\ & \left. + U_c(c_t, h_t) \frac{z_t}{\gamma(v_t)\eta} \right] + z_t \psi_t\end{aligned}$$

The partial derivatives of the Lagrangian with respect to  $c_0$ ,  $h_0$ , and  $v_0$  are:

$$\frac{\partial \mathcal{L}}{\partial v_0} = \xi U_c(c_0, h_0)c_0\phi'(v_0) - \xi U_c(c_0, h_0) \frac{\gamma'(v_0)}{\gamma(v_0)^2} \left[ \frac{z_0 h_0}{\eta} - \frac{R_{-1}B_{-1} + M_{-1}}{P_0} \right] - \psi_0 \alpha s'(v_0)c_0$$

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial c_0} = & U_c(c_0, h_0)[1 + \xi\phi(v_0)] + \xi \left[ U_{cc}(c_0, h_0)c_0\phi(v_0) + U_{hc}(c_0, h_0)h_0 + U_{cc}(c_0, h_0) \frac{z_0 h_0}{\gamma(v_0)\eta} \right. \\ & \left. - \frac{U_{cc}(c_0, h_0)}{\gamma(v_0)} \frac{R_{-1}B_{-1} + M_{-1}}{P_0} \right] - \psi_0[1 + \alpha s(v_0)]\end{aligned}$$

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial h_0} = & U_h(c_0, h_0) + \xi \left[ U_{ch}(c_0, h_0)c_0\phi(v_0) + U_h(c_0, h_0) + h_0 U_{hh}(c_0, h_0) + U_{ch}(c_0, h_0) \frac{z_0 h_0}{\gamma(v_0)\eta} \right. \\ & \left. + U_c(c_0, h_0) \frac{z_0}{\gamma(v_0)\eta} - \frac{U_{ch}(c_0, h_0)}{\gamma(v_0)} \frac{R_{-1}B_{-1} + M_{-1}}{P_0} \right] + z_0 \psi_0\end{aligned}$$

The first-order conditions associated with the Ramsey problem are: (23), (24),  $v_t \geq \underline{v}$ ,  $v_t^2 s'(v_t) < 1$ , and

$$\frac{\partial \mathcal{L}}{\partial v_t} \leq 0 \quad (= 0 \text{ if } v_t > \underline{v}) \quad \forall t \geq 0 \quad (26)$$

$$\frac{\partial \mathcal{L}}{\partial c_t} = 0 \quad \forall t \geq 0 \quad (27)$$

and

$$\frac{\partial \mathcal{L}}{\partial h_t} = 0 \quad \forall t \geq 0. \quad (28)$$

One must check that the solution also satisfies  $v_t^2 s'(v_t) < 1$ .

## 4 Non-optimality of the Friedman rule

In this section, we analyze conditions under which the Friedman rule, that is,  $R_t = 1$  for all  $t > 0$ , is optimal as well as conditions under which it fails to be optimal. To keep the presentation simple, we restrict attention to the case that transaction costs represent true resource costs to the economy ( $\alpha = 1$ ) and reserve the analysis of the case that only some part of the transactions cost are resource costs ( $0 \leq \alpha < 1$ ) for the appendix. Note that when  $\alpha = 1$ ,  $\phi(v_t) = 1$  at all times so that  $\phi'(v_t) = 0$ .

We begin by establishing that the Friedman rule is optimal in our economy in the absence of market power. While the optimality of the Friedman rule has been shown in a number of different flexible price monetary economies (see, for example, Chari et al., 1996; Correia and Teles, 1996), to our knowledge, it has not previously been established for a transactions cost based monetary economy like the one we consider; thus this result is of interest in its own.<sup>5</sup>

In our framework the case of perfect competition obtains when the demand function  $Y_t d(p_t)$  becomes perfectly elastic, that is, when  $\eta = -\infty$ . In this case the partial derivative of the Lagrangian with respect to  $v_t$  for  $t > 0$  becomes  $\partial \mathcal{L} / \partial v_t = -\psi_t s'(v_t) c_t$  so that the first-order condition of the Ramsey problem with respect to velocity is

$$-\psi_t s'(v_t) c_t \leq 0 \quad (= 0 \text{ if } v_t > \underline{v})$$

The only solution to this first-order condition is  $v_t = \underline{v}$ . To see this, note first that, given our maintained assumption of no satiation in consumption and leisure, the Lagrange multiplier on the feasibility constraint,  $\psi_t$ , is strictly positive. Also, by assumption 1,  $s'(v_t)$  is strictly positive for  $v_t > \underline{v}$ . It follows that the above first-order condition is violated when  $v_t > \underline{v}$ . Finally, note that by assumption 1,  $s'(\underline{v}) = 0$ . From the liquidity preference function (14) it then follows immediately that  $R_t = 1$  for all states and dates  $t > 0$ . We summarize this result in the following proposition.

**Proposition 2 (Optimality of the Friedman rule under perfect competition)** *Suppose product markets are perfectly competitive ( $\eta = -\infty$ ), transaction costs are not rebated ( $\alpha = 1$ ), and assumption 1 holds. Then, under the Ramsey allocation*

$$R_t = 1$$

for all  $t > 0$ .

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<sup>5</sup>Chari and Kehoe (1999) analyze three models of money in which the Friedman rule is optimal if the consumption elasticity of money demand is one. Specifically, they consider money-in-the-utility function, shopping-time, and cash-in-advance models. Thus, our result on the optimality of the Friedman rule under perfect competition adds one more model to the list of economies for which in the presence of a unit consumption elasticity of money demand a zero nominal interest rate is optimal.

We now turn to one of the central results of this paper, namely, that the Friedman rule fails to be optimal under imperfect competition. In this case the first-order condition of the Ramsey problem with respect to velocity for  $t > 0$  becomes

$$-\xi \frac{z_t h_t}{\eta} U_c(c_t, h_t) \frac{\gamma'(v_t)}{\gamma(v_t)^2} - \psi_t s'(v_t) c_t \leq 0 \quad (= 0 \text{ if } v_t > \underline{v}) \quad (29)$$

To see that the Friedman rule cannot be optimal, evaluate the above expression at  $v_t = \underline{v}$ . At  $v_t = \underline{v}$ ,  $s'(\underline{v}) = 0$ , thus the second term on the left-hand side of the above first-order condition vanishes. The Lagrange multiplier on the implementability constraint,  $\xi$ , is positive, otherwise an increase in initial government debt would be welfare improving. Since  $\eta$  is negative, it follows that the first-order condition can be satisfied only if  $\gamma'(\underline{v}) = \underline{v} s''(\underline{v}) \leq 0$ . However, given assumption 1, this can never be the case.<sup>6</sup> Thus, we have the following proposition

**Proposition 3 (Non-optimality of the Friedman rule under imperfect competition)** *Suppose that product markets are imperfectly competitive ( $-\infty < \eta < -1$ ), transaction costs are not rebated ( $\alpha = 1$ ), and assumption 1 holds. Then, if a Ramsey allocation exists, it must be the case that*

$$R_t > 1$$

for all  $t > 0$ .

## 4.1 Intuition

The intuition behind the breakdown of the Friedman rule under imperfect competition is the following: In the imperfectly competitive economy, part of income takes the form of pure monopoly rents. By definition, the labor income tax rate cannot tax profits. As a result, the social planner resorts to the inflation tax as an indirect way to tax profit income. Specifically, when the household transforms profit income into consumption, it must use fiat money, which is subject to the inflation tax.

Because profits represent payments to a fixed factor, namely monopoly rights, a tax on profits would be nondistortionary. Thus, when a benevolent government has the ability to tax profits, it would like to tax them at a hundred percent rate. Formally, this result can be shown as follows. In equilibrium, profits are given by  $z_t h_t - w_t h_t$ . Suppose the government

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<sup>6</sup>One may argue that assumption 1(d), which implies that the nominal interest rate is a strictly increasing function of  $v$  for all  $v \geq \underline{v}$ , or alternatively, that the elasticity of the liquidity preference function at a zero nominal interest rate is finite, is too restrictive. Suppose instead that assumption 1(d) is relaxed by assuming that it must hold only for  $v > \underline{v}$  but not at  $v = \underline{v}$ . In this case, a potential solution to (29) is  $v = \underline{v}$  provided  $s''(\underline{v}) = 0$ . However,  $v = \underline{v}$  may not be the only solution to this first-order necessary condition. It could very well be the case that there exists another  $v > \underline{v}$  such that  $s''(v) > 0$  and that (29) is satisfied. Then it has to be determined whether both solutions represent Ramsey allocations. If this were indeed found to be the case, then the Friedman rule would not be a necessary feature of the Ramsey allocation. To establish that the Friedman rule is a necessary characteristic of the Ramsey allocation, one would have to determine that the solution involving a strictly positive nominal interest rate does not represent a maximum and that the one involving the Friedman rule does. To establish such result it would be necessary to show that the Ramsey problem is concave or alternatively to consider second-order conditions.

has access to a proportional tax on profits,  $\tau_t^\pi \in [0, 1]$ . Then the Ramsey problem can be shown to be the same as above but for the implementability constraint (24), which now takes the form

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ U_c(c_t, h_t) c_t \phi(v_t) + U_h(c_t, h_t) h_t + (1 - \tau_t^\pi) \frac{U_c(c_t, h_t) z_t h_t}{\gamma(v_t) \eta} \right\} = \frac{U_c(c_0, h_0) R_{-1} B_{-1} + M_{-1}}{\gamma(v_0) P_0}. \quad (30)$$

Because the derivative of the Lagrangian associated with the Ramsey problem with respect to  $\tau_t^\pi$  is strictly positive, the optimal profit tax rate is one at all dates and under all contingencies. Note that when  $\tau_t^\pi = 1$ , the implementability constraint is the same as in the perfectly competitive economy. It then follows immediately that the Friedman rule reemerges as the optimal monetary policy. On the other hand, if the government is constrained to tax profits at a rate strictly less than unity ( $\tau_t^\pi < \bar{\tau}^\pi < 1$ ), which is clearly the case of greatest empirical interest, then the Ramsey planner chooses to set the profit tax rate at that upper bound ( $\tau_t^\pi = \bar{\tau}^\pi$  for all  $t$ ). In this case the implementability constraint is the same as that of an imperfectly competitive economy without profit taxes (equation (24)) but with an elasticity of demand of  $\eta/(1 - \bar{\tau}^\pi)$ . That is, the presence of profit taxes is equivalent, from a Ramsey point of view, to an economy without profit taxes but with less market power. It therefore follows directly from proposition 3 that as long as  $\bar{\tau}^\pi < 1$ , the Friedman rule is suboptimal.<sup>7</sup>

## 5 Dynamic properties of Ramsey allocations

In this section we characterize numerically the dynamic properties of Ramsey allocations. We begin by proposing a solution method that does not rely on any type of approximation of the nonlinear Ramsey conditions. Then we describe the calibration of the model. Finally, we present the quantitative results.

### 5.1 Solution method

We propose a method to find a numerical solution to the exact nonlinear equations describing the Ramsey planner's first-order conditions. We look for stochastic processes  $\{c_t, h_t, v_t, \psi_t\}$  and a positive scalar  $\xi$  satisfying

$$z_t h_t = [1 + \alpha s(v_t)] c_t + g_t \quad (31)$$

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ U_c(c_t, h_t) c_t \phi(v_t) + U_h(c_t, h_t) h_t + \frac{U_c(c_t, h_t) z_t h_t}{\gamma(v_t) \eta} \right\} = \frac{U_c(c_0, h_0)}{\gamma(v_0)} d_0 \quad (32)$$

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<sup>7</sup>One can show that if the Ramsey planner is constrained to apply a uniform tax rate to all forms of income, that is,  $\tau_t = \tau_t^\pi$  for all  $t$ , the Friedman rule is optimal. (The derivation of this result is available from the authors upon request.). However, this case is of marginal interest, for if the Ramsey planner had the ability to set both tax rates independently, he would never find it optimal to set the same tax rate for labor and profit income, thus neither to follow the Friedman rule.

$$\xi U_c(c_t, h_t) c_t \phi'(v_t) - \xi U_c(c_t, h_t) \frac{\gamma'(v_t)}{\gamma^2(v_t)} \left[ \frac{z_t h_t}{\eta} - I(t=0) d_0 \right] = \psi_t \alpha s'(v_t) c_t \quad (33)$$

$$U_c(c_t, h_t) [1 + \xi \phi(v_t)] + \xi \left[ U_{cc}(c_t, h_t) \left( c_t \phi(v_t) - I(t=0) \frac{d_0}{\gamma(v_t)} \right) + U_{hc}(c_t, h_t) h_t + U_{cc}(c_t, h_t) \frac{z_t h_t}{\gamma(v_t) \eta} \right] = \psi_t [1 + \alpha s(v_t)] \quad (34)$$

$$-z_t \psi_t = U_h(c_t, h_t) + \xi \left[ U_{ch}(c_t, h_t) \left( c_t \phi(v_t) - I(t=0) \frac{d_0}{\gamma(v_t)} \right) + U_h(c_t, h_t) + h_t U_{hh}(c_t, h_t) + U_{ch}(c_t, h_t) \frac{z_t h_t}{\gamma(v_t) \eta} + U_c(c_t, h_t) \frac{z_t}{\gamma(v_t) \eta} \right] \quad (35)$$

given exogenous processes  $\{g_t, z_t\}$  and the initial condition  $d_0 \equiv (R_{-1}B_{-1} + M_{-1})/P_0$ . In these expressions,  $I(t=0)$  is an indicator function that takes the value of one if  $t=0$  and zero otherwise. The following procedure describes a method to compute exact numerical solutions to the above system.

1. We assume that  $g_t$  and  $z_t$  follow independent 2-state symmetric Markov processes. Let  $g_t$  take on the values  $g^h$  and  $g^l$  and  $z_t$  the values  $z^h$  and  $z^l$ .<sup>8</sup> Let  $\phi^g = \text{Prob}(g_{t+1} = g^i | g_t = g^i)$  and  $\phi^z = \text{Prob}(z_{t+1} = z^i | z_t = z^i)$  for  $i = h, l$ . Then the possible states of the economy are described by the  $4 \times 1$  state vector  $S$ , where

$$S = \begin{bmatrix} s_1 \\ s_2 \\ s_3 \\ s_4 \end{bmatrix} = \begin{bmatrix} (g^h, z^h) \\ (g^h, z^l) \\ (g^l, z^h) \\ (g^l, z^l) \end{bmatrix}$$

Let  $\Phi$  denote the transition matrix of the state vector  $S$ . Then,

$$\Phi = \begin{bmatrix} \phi^g \phi^z & \phi^g (1 - \phi^z) & (1 - \phi^g) \phi^z & (1 - \phi^g)(1 - \phi^z) \\ \phi^g (1 - \phi^z) & \phi^g \phi^z & (1 - \phi^g)(1 - \phi^z) & (1 - \phi^g) \phi^z \\ (1 - \phi^g) \phi^z & (1 - \phi^g)(1 - \phi^z) & \phi^g \phi^z & \phi^g (1 - \phi^z) \\ (1 - \phi^g)(1 - \phi^z) & (1 - \phi^g) \phi^z & \phi^g (1 - \phi^z) & \phi^g \phi^z \end{bmatrix}$$

2. Choose an initial state  $s_0 \equiv (g_0, z_0)$ .
3. Choose a positive value for  $\xi$ . Note that there is one equilibrium value of  $\xi$  for each possible initial state.
4. For  $t > 0$ , given a value for  $\xi$  and a realization of the state of the economy  $s_i$ , equations (31) and (33)-(35) form a static system that can be solved for  $c$ ,  $v$ ,  $h$ , and  $\psi$  as functions of  $\xi$  and  $s_i$ . Since there are only 4 possible values for  $s_i$ , given  $\xi$ , the variables  $c$ ,  $v$ ,  $h$ , and  $\psi$  take only 4 different values, one for each possible state. Thus, for  $t > 0$  and

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<sup>8</sup>The generalization of the method to  $N$ -state Markov processes is trivial.

a given value of  $\xi$ , the solution to the Ramsey conditions can be written as  $c(\xi, s_i)$ ,  $h(\xi, s_i)$ ,  $v(\xi, s_i)$ , and  $\psi(\xi, s_i)$ .

Similarly, for  $t = 0$ , given an initial state,  $s_0 = (g_0, z_0)$  and a value for  $\xi$ , the variables  $c_0, h_0, \psi_0, v_0$  are the solution to (31) and (33)-(35) with  $I(t = 0) = 1$ . Thus, the solutions will be different from those obtained for  $t > 0$ . We denote the time 0 solution by  $c_0(\xi, s_0)$ ,  $h_0(\xi, s_0)$ ,  $v_0(\xi, s_0)$ , and  $\psi_0(\xi, s_0)$ .

We use a numerical nonlinear equation solver, such as the MATLAB routine `fsolve.m`, to find given  $\xi$  and  $s_0$ , the values of  $c(\xi, s_i)$ ,  $h(\xi, s_i)$ ,  $v(\xi, s_i)$ ,  $\psi(\xi, s_i)$  for  $i = 1, \dots, 4$  and of  $c_0(\xi, s_0)$ ,  $h_0(\xi, s_0)$ ,  $v_0(\xi, s_0)$ , and  $\psi_0(\xi, s_0)$ .

5. Having computed the values taken by  $c$ ,  $h$ ,  $\psi$ , and  $v$  at every state and date for a given guess of  $\xi$ , we now check whether this guess of  $\xi$  is the correct one by evaluating the implementability constraint, equation (32).

To do this, start by evaluating the left-hand side of the implementability constraint. For  $t > 0$ , let  $lhs(\xi, s_i)$  be defined as

$$\begin{aligned} lhs(\xi, s_i) = & U_c(c(\xi, s_i), h(\xi, s_i))c(\xi, s_i)\phi(v(\xi, s_i)) + \\ & U_h(c(\xi, s_i), h(\xi, s_i))h(\xi, s_i) + \\ & \frac{U_c(c(\xi, s_i), h(\xi, s_i))}{\gamma(v(\xi, s_i))} \frac{z(s_i)h(\xi, s_i)}{\eta} \end{aligned}$$

where  $z(s_i)$  is the value of  $z$  in state  $i$  for  $i = 1, \dots, 4$ . Let

$$lhs(\xi) = \begin{bmatrix} lhs(\xi, s_1) \\ lhs(\xi, s_2) \\ lhs(\xi, s_3) \\ lhs(\xi, s_4) \end{bmatrix}$$

Similarly, let  $lhs_0(\xi, s_0)$  be defined as

$$\begin{aligned} lhs_0(\xi, s_0) = & U_c(c_0(\xi, s_0), h_0(\xi, s_0))c_0(\xi, s_0)\phi(v_0(\xi, s_0)) + \\ & U_h(c_0(\xi, s_0), h_0(\xi, s_0))h_0(\xi, s_0) + \\ & \frac{U_c(c_0(\xi, s_0), h_0(\xi, s_0))}{\gamma(v_0(\xi, s_0))} \frac{z(s_0)h_0(\xi, s_0)}{\eta}, \end{aligned}$$

where  $z(s_0)$  is the value taken by the technology shock in the initial state  $s_0$ .

Using this notation the left-hand side of (32), which we denote by  $LHS(\xi, s_0)$ , can be written as

$$LHS(\xi, s_0) = lhs_0(\xi, s_0) + \beta\Phi(s_0)(I - \beta\Phi)^{-1}lhs(\xi)$$

where  $\Phi(s_0)$  is the row of the transition matrix  $\Phi$  corresponding to the state  $s_0$ . Now compute the right-hand side of (32)

$$RHS(\xi, s_0) = \frac{U_c(c_0(\xi, s_0), h_0(\xi, s_0))}{\gamma(v_0(\xi, s_0))} d_0$$

Then compute the difference

$$y(\xi, s_0) = LHS(\xi, s_0) - RHS(\xi, s_0)$$

6. Use a numerical solver, such as the MATLAB routine `fsolve.m`, to find  $\xi$  such that  $y(\xi, s_0) = 0$ . This yields the equilibrium value of  $\xi$  (and with it the equilibrium processes for  $c$ ,  $h$ ,  $v$ , and  $\psi$ .) for a given initial state  $s_0$ .
7. The computation of the inflation rate process  $\pi_t$  deserves special discussion. This process can be computed using the following steps.

- (a) Note that because  $P_0$  is assumed to be given, so is  $\pi_0 = P_0/P_{-1}$ .
- (b) Define the real value of outstanding liabilities at the beginning of period  $t$  as  $d_t \equiv (R_{t-1}B_{t-1} + M_{t-1})/P_t$ . Note that  $d_0$  is given.
- (c) Use the sequential budget constraint of the government to write

$$\pi_{t+1} = R_t/d_{t+1} [d_t + g_t - \tau_t w_t h_t - c_t/v_t(1 - R_t^{-1})]; \quad t \geq 0$$

- (d) From the proof of proposition 1 it follows that

$$d_{t+1} = \frac{\gamma(v_{t+1})}{U_c(t+1)} LHS(t+1)$$

where

$$LHS(t) = E_t \sum_{j=0}^{\infty} \beta^j \left[ U_c(t+j)c_{t+j}\phi(t+j) + U_h(t+j)h_{t+j} + \frac{z_{t+j}h_{t+j}U_c(t+j)}{\eta\gamma(t+j)} \right]$$

- (e) The construction of  $LHS(t)$  is similar to that of  $LHS(\xi, s_0)$  described above. Note that given  $\xi$ , the expression within square brackets in the above equation for LHS takes on only 4 distinct values, one for each of the 4 possible states of  $(g_t, z_t)$ . This means that, given  $\xi$ ,  $LHS(t)$  (for  $t > 0$ ) depends only on  $(g_t, z_t)$ .

We now put this method to work by computing the dynamic properties of the Ramsey allocation in a simple calibrated economy.

## 5.2 Calibration

We calibrate our model to the U.S. economy. The time unit is meant to be a year. We assume that up to period 0, the economy is in a steady state with an inflation rate of 4.2 percent per year, which is consistent with the average growth rate of the U.S. GDP deflator over the period 1960:Q1 to 1998:Q3 and a debt-to-GDP ratio of 0.44 percent, which corresponds to the figure observed in the United States in 1995 (see the 1997 Economic Report of the President, table B-79), and government expenditures equal to 20 percent of GDP, a figure that is in line postwar U.S. data. We follow Prescott (1986) and set the subjective discount rate  $\beta$  to 0.96 to be consistent with a steady-state real rate of return of 4 percent per year.

Drawing from the empirical study of Basu and Fernald (1997), we assign a value of 1.2 to the gross value added markup,  $\mu$ , defined by equation (11). These authors estimate gross output production functions and obtain estimates for the gross output markup of about 1.1. They show that their estimates correspond to values for the value added markup of 25 percent. We assume that the single-period utility index is of the form

$$U(c, h) = \ln(c) + \theta \ln(1 - h)$$

We set  $\theta$  so that in the steady state households allocate 20 percent of their time to work.<sup>9</sup>

We use the following specification for the transactions cost technology:

$$s(v) = Av + B/v - 2\sqrt{AB} \quad (36)$$

This functional form implies a satiation point for consumption-based money velocity,  $\underline{v}$ , equal to  $\sqrt{B/A}$ . The money demand function implied by the above transaction technology is of the form

$$v_t^2 = \frac{B}{A} + \frac{1}{A} \frac{R_t - 1}{R_t},$$

To identify the parameters  $A$  and  $B$ , we estimate this equation using quarterly U.S. data from 1960:1 to 1999:3. We measure  $v$  as the ratio of non-durable consumption and services expenditures to M1. The nominal interest rate is taken to be the three-month Treasury Bill rate. The OLS estimate implies that  $A = 0.0111$  and  $B = 0.07524$ .<sup>10</sup> In our baseline calibration we assume that transaction costs are true shoe leather costs, that is, we set  $\alpha = 1$ .

As in Chari et al. (1995), the processes for government spending and the technology shock are assumed to follow independent, symmetric, two-state Markov chains. Government consumption,  $g_t$ , has a mean value of 0.04, a standard deviation of 0.0028 (or 7 percent of mean  $g$ ), and a first-order serial correlation of 0.9. The technology shock,  $z_t$ , has a mean value of unity, a standard deviation of 0.04, and a serial correlation of 0.82. Table 1 summarizes the calibration of the economy.

### 5.3 Numerical results

Table 2 displays a number of unconditional moments of key macroeconomic variables under the Ramsey policy. Each panel corresponds to a different degree of market power, measured by  $\mu$ , the gross markup of prices over marginal cost. All economies are assumed to start out in period 0 with the same level of real total government liabilities  $(R_{-1}B_{-1} + M_{-1})/P_0$ . Thus, the moments shown on table 2 are unconditional with respect to the exogenous shocks  $g_t$  and  $z_t$ , but not with respect to the initial level of government liabilities.

The top panel shows the case of perfect competition ( $\mu = 1$ ). In this case, the nominal interest rate is constant and equal to zero, in line with our analytical results. Because under perfect competition the nominal interest rate is zero at all times, the distortion introduced

<sup>9</sup>See the appendix for a derivation of the exact relations used to identify  $\theta$ .

<sup>10</sup>The estimated equation is  $v_t^2 = 6.77 + 90.03(R_t - 1)/R_t$ . The  $t$ -statistics for the constant and slope of the regression are, respectively, 6.81 and 5.64; The  $\bar{R}^2$  of the regression is 0.16. Instrumental variable estimates using three lagged values of the dependent and independent variables yield similar estimates for  $A$  and  $B$ .

Table 1: Calibration

Symbol	Definition	Value	Description
<u>Calibrated Parameters:</u>			
$\beta$		0.96	Subjective discount factor
$\pi$		1.04	Gross inflation rate
$h$		0.2	Fraction of time allocated to work
$s_g$	$g/y$	0.2	Government consumption to GDP ratio
$s_b$	$B/(Py)$	0.44	Public debt to GDP ratio
$\mu$	$\eta/(1 + \eta)$	1.2	Gross value-added markup
$A$	$s(v) = Av + B/v - 2\sqrt{AB}$	0.01	Parameter of transaction cost function
$B$		0.08	Parameter of transaction cost function
$\alpha$		1	Fraction of transaction not rebated
$z^h$		1.04	High value of technology shock
$z^l$		0.96	Low value of technology shock
$g^h$		0.043	High value of gov't consumption shock
$g^l$		0.037	Low value of gov't consumption shock
$\phi^z$		0.91	$\text{Prob}(z_t = z^i   z_{t-1} = z^i) \ i = h, l$
$\phi^g$		0.95	$\text{Prob}(g_t = g^i   g_{t-1} = g^i) \ i = h, l$
<u>Implied Parameters:</u>			
$\theta$	$U(c, h) = \ln(c) + \theta \ln(1 - h)$	2.90	Preference parameter

Note. The time unit is a year. The variable  $y \equiv zh$  denotes steady-state output.

by the transaction cost is driven to zero in the Ramsey allocation ( $s(v) = s'(v) = 0$ ). On the other hand, distortionary income taxes are far from zero. The average value of the labor income tax rate is 18.8 percent. The Ramsey planner keeps this distortion smooth over the business cycle; the standard deviation of  $\tau$  is 0.05 percentage points. This means that a two-standard deviation interval around the mean tax rate is 18.7-18.9 percent.

In the Ramsey allocation with perfect competition, inflation is on average negative and equal to -3.4 percent per year. This value is close to the negative of the subjective discount rate of 4 percent. Average inflation does not exactly equal the subjective discount rate because the real interest rate (given by the intertemporal marginal rate of substitution in consumption) is correlated with the inflation rate. The inflation rate is highly volatile. A two-standard deviation band around the mean features a deflation rate of 10.9 percent at the lower end and inflation of 4.1 percent at the upper end. The Ramsey planner uses the inflation rate as a state-contingent lump-sum tax on households' financial wealth. This lump-sum tax appears to be used mainly in response to unanticipated changes in the state of the

economy. This is reflected in the fact that inflation displays a near zero serial correlation.<sup>11</sup> The high volatility and low persistence of the inflation rate stands in sharp contrast to the smooth and highly persistent behavior of the labor income tax rate. Our results on the dynamic properties of the Ramsey economy under perfect competition are consistent with those obtained by Chari et al. (1991).<sup>12</sup>

As soon as one moves away from the assumption of perfect competition, the Friedman rule ceases to be optimal. A central result of table 2 is that the average optimal nominal interest rate is an increasing function of the profit share, which in our economy is related to the markup by the function  $(\mu - 1)/\mu$ .<sup>13</sup> As the markup increases from 1 to 1.35, or the profit share increases from 0 to 25 percent, the average nominal interest rate increases from 0 to 7.8 percent per year.

Because when the interest rate increases so do interest savings from the issuance of money, one might be led to believe that as the markup increases the government might lower income tax rates to compensate for the increase in seignorage revenue. However, as table 2 shows, this is not the case here. The average labor income tax rate increases sharply from 18.8 to 33.3 percent as the markup rises from zero to 35 percent. The reason for the Ramsey planner's need to increase tax rates when the markup goes up is that the labor income tax base falls as both employment and wages fall as the economy becomes less competitive. Indeed employment falls from 0.24 to 0.17 and wages fall from 1 to 0.74 as  $\mu$  increases from 1 to 1.35.

Another result that emerges from inspection of table 2 is that, unlike under perfect competition, in the presence of market power the Ramsey planner chooses not to keep the nominal interest rate constant along the business cycle. Although small, the volatility of the nominal interest rate increases with the markup. The standard deviation of the nominal interest rate increases to 22 basis points as the degree of market power increases from zero to 35 percent. In addition, the nominal interest rate is highly persistent, with a serial correlation of 0.88, highly correlated with government purchases, with a correlation coefficient of 0.86, and negatively correlated with the technology shock, with a correlation of -0.5.

Finally as in the case of perfect competition, tax rates are little volatile and persistent and inflation is highly volatile and almost serially uncorrelated.

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<sup>11</sup>The observation that in the Ramsey equilibrium inflation acts as a lump-sum tax on wealth was, to our knowledge, first made by Chari et al. (1991) and has recently been stressed by Sims (2000).

<sup>12</sup>The main quantitative difference in results is that Chari et al. find a standard deviation of inflation of 19.93 percentage points which is more than twice the value we report in table 2. The source of this discrepancy may lie in the fact that Chari et al. use a different solution method. One might think that the disparity could be due to the fact that the models incorporate different motivations for the demand for money. We assume that money reduces transactions costs, whereas Chari et al. use a cash-credit goods model. However, since in both frameworks the Friedman rule is optimal, the monetary distortion vanishes in both setups and thus should not play any role for the properties of the Ramsey allocation. Of course, the two models might imply different values for the average ratio of money to GDP. Since money is one component of households' financial wealth, which in turn is the tax base of the state contingent lump-sum tax embodied in inflation, the different money demand specifications may in principle explain the difference in the volatility of inflation. However, we experimented using the steady-state money-to-GDP and the debt-to-GDP ratios used by Chari et al. and found that the volatility of inflation is not significantly different from the value we report in table 2.

<sup>13</sup>Profits and monopoly power need not be related in this way. For instance, in the presence of fixed costs the average profit share can be significantly smaller than  $(\mu - 1)/\mu$ .

## 6 Summary and conclusion

In this paper we have characterized optimal fiscal and monetary policy in an economy with market power in product markets. The study was conducted within a standard stochastic, dynamic, monetary economy with production but no capital. The production technology is assumed to be subject to exogenous stochastic productivity shocks. The government finances an exogenous and stochastic stream of government purchases by issuing money, levying distortionary income taxes, and issuing bonds. Public debt takes the form of nominal, non-state-contingent government obligations.

In this economy, under perfect competition the Friedman rule is optimal. The central result of this paper is that once pure profits are introduced through imperfect competition, the Friedman rule ceases to be optimal. Indeed, the nominal interest rate increases with the profit share. In addition, in the presence of pure monopoly rents, the optimal nominal interest rate is time varying and its unconditional volatility increases with the magnitude of such rents.

A number of important properties of the Ramsey allocation under perfect competition are, however, robust to the introduction of market power. In particular, regardless of the degree of monopoly power the income tax rate displays very little volatility and is highly persistent. By contrast, the inflation rate is highly volatile and nearly serially uncorrelated. This shows that as in the case of perfect competition, under monopoly power the government uses the inflation rate as a state-contingent, lump-sum tax on total financial wealth. This lump-sum tax allows the government to refrain from changing distortionary taxes in response to adverse government purchases or productivity shocks.

In conducting the analysis of optimal fiscal and monetary policy we restricted attention to a specific motivation for the demand for money. Namely, one in which money reduces transaction costs associated with purchases of final goods. We conjecture, however, that our central result regarding the breakdown of the Friedman rule in the presence of imperfect competition holds in any monetary model where inflation acts as a tax on income or consumption. This is the case because as long as profit tax rates are bounded away from one hundred percent, which is arguably the most realistic case, a benevolent government will have an incentive to use inflation as an indirect way to tax profits.

This paper can be extended in several directions. A natural one, which we pursue in Schmitt-Grohé and Uribe (2000), is to introduce nominal rigidities in the form of sticky prices. One motivation for considering such an extension is that under price flexibility the Ramsey allocation calls for highly volatile inflation rates. This aspect of the optimal policy regime is at odds with conventional wisdom about the desirability of price stability. Sticky prices may contribute to bringing down the optimal degree of inflation volatility. In Schmitt-Grohé and Uribe (2000), we show that the incorporation of sticky prices into a model like the one analyzed in this paper introduces a significant modification in the primal form of the Ramsey problem. Specifically, under sticky prices and non-state-contingent nominal government debt, it is no longer the case that the implementability constraint takes the form of a single intertemporal restriction. Instead, it is replaced by a sequence of constraints like (24), one for each date and state of the world. This modification of the Ramsey problem is similar to the one that takes place in real models in which real public debt is restricted to be non-state-contingent like the one studied by Marcet, Sargent, and Seppala (2000).

More broadly, because imperfect competition is an essential element of modern general equilibrium formulations of sticky-price models, the present study can be viewed as an intermediate step in the quest for understanding the properties of optimal fiscal and monetary policy in models with sluggish nominal price adjustment.

## 7 Appendix

### 7.1 Optimality of the Friedman Rule when transactions costs are not fully rebated ( $0 \leq \alpha < 1$ )

The first-order necessary condition of the Ramsey problem for the choice of  $v_t$  for  $t > 0$  in this case takes the form

$$\xi U_c(c_t, h_t) c_t \phi'(v_t) - \xi \frac{z_t h_t}{\eta} U_c(c_t, h_t) \frac{\gamma'(v_t)}{\gamma(v_t)^2} - \psi_t \alpha s'(v_t) c_t \leq 0 \quad (= 0 \text{ if } v_t > \underline{v}), \quad (37)$$

where

$$\phi'(v) = (1 - \alpha) \left( \frac{s(v)[s'(v) + v s''(v)] - s'(v)[1 + v s'(v)]}{\gamma^2} \right)$$

#### Perfect Competition ( $\eta = -\infty$ )

Suppose first that product markets are perfectly competitive ( $\eta = -\infty$ ). In this case, equation (37) reduces to

$$\xi U_c(c_t, h_t) c_t \phi'(v_t) - \psi_t \alpha s'(v_t) c_t \leq 0 \quad (= 0 \text{ if } v_t > \underline{v})$$

At  $v = \underline{v}$ ,  $\phi'(v) = 0$  as long as assumption 1 holds and  $s''(\underline{v})$  is finite. Therefore, the above first-order condition is satisfied. There may exist additional solutions  $v > \underline{v}$  to this first-order condition if  $\phi'(v) > 0$ . On the other hand, if  $\phi'(v) < 0$  for all  $v > \underline{v}$ , then  $v = \underline{v}$  is the only solution to the first-order condition, and thus, if a Ramsey equilibrium exists it will feature a zero nominal interest rate. An example of a transaction cost function that satisfies assumption 1 and for which  $\phi'(v) < 0$  for all  $v > \underline{v}$  is the given in equation (36), which we reproduce here for convenience:

$$s(v) = Av + B/v - 2\sqrt{AB} \quad A, B > 0$$

To see why, note that in this case the marginal transaction cost is given by

$$s'(v) = A - B/v^2$$

At the satiation point  $\underline{v} = \sqrt{B/A}$ , we have  $s(\underline{v}) = s'(\underline{v}) = 0$ , so that assumption 1(b) holds. Furthermore,

$$s''(v) = 2B/v^3 > 0$$

which implies that assumption 1(d) is satisfied. Finally, we wish to show that  $\phi'(\underline{v}) = 0$  and  $\phi'(v) < 0$  for all  $v > \underline{v}$ . Taking derivative of  $\phi(v)$  with respect to  $v$  yields:

$$\phi'(v) = (1 - \alpha) \frac{(A + B/v^2)(1 + 2Av - 2\sqrt{AB}) - 2A(1 + Av - B/v)}{(1 + 2Av - 2\sqrt{AB})^2}$$

Let  $N(v)$  and  $D(v)$  denote, respectively, the numerator and denominator of the fraction on the right-hand side of this expression. Note that  $N(\underline{v}) = 0$  and that  $D(\underline{v}) = 1$ . Thus,

$\phi'(\underline{v}) = 0$ . Also,  $D(v) > 0$  for  $v > \underline{v}$ . Finally,  $N'(v) = -B/v^2[1 + 2A(v - v^s)] < 0$  for  $v > \underline{v}$ . Thus,  $\phi'(v) < 0$  for all  $v > \underline{v}$ . This means that if a Ramsey equilibrium exists, then the associated nominal interest rate is zero for all  $t > 0$ . We formalize this result in the following proposition:

**Proposition 4 (Optimality of the Friedman rule under perfect competition without full rebate)** *Suppose product markets are perfectly competitive ( $\eta = -\infty$ ), transaction costs are partially rebated ( $0 \leq \alpha < 1$ ), and the transaction cost function is given by (36). Then*

$$R_t = 1$$

for all  $t > 0$ .

**Imperfect competition ( $-\infty < \eta < -1$ )**

Consider next the case of imperfectly competitive product markets,  $-\infty < \eta < -1$ . The analysis of this case is essentially the same as for the case that  $\alpha = 1$ . For in this case,  $\phi'(\underline{v}) = 0$ , so that the first-order condition of the Ramsey problem with respect to velocity, equation (37), can never be satisfied at  $v_t = \underline{v}$ . We summarize this result in the following proposition:

**Proposition 5 (Non-optimality of the Friedman rule under imperfect competition without full rebate)** *Suppose that product markets are imperfectly competitive ( $-\infty < \eta < -1$ ), transaction costs are partially rebated ( $0 \leq \alpha < 1$ ), and assumption 1 holds. Then, if a Ramsey allocation exists, it must be the case that*

$$R_t > 1$$

for all  $t > 0$ .

## 7.2 Calibration of $\theta$

To find the value of  $\theta$  we first need to compute the steady state value of  $\tau$ . To do this, write the government budget constraint (18) in real terms as:

$$m_t + b_t + \tau_t w_t h_t = \frac{R_{t-1}}{\pi_t} b_{t-1} + \frac{m_{t-1}}{\pi_t} + g_t$$

Divide through by  $h$  to express variables in terms of GDP (note that  $m_t = c_t/v_t$  and that  $w = (1 + \eta)/\eta = 1/\mu$ )

$$\frac{s_c}{(1 + s(v))v} + s_b + \tau_t/\mu = \frac{R}{\pi} s_b + \frac{s_c}{v(1 + s(v))\pi} + s_g$$

Where  $s_b = b/h$ ,  $s_c = (1 + s(v))c/h$  and  $s_g = g/h$ . Solving for  $\tau$  yields

$$\tau = \mu \left[ (R/\pi - 1)s_b + \frac{s_c}{v(1 + s(v))} \left( \frac{1}{\pi} - 1 \right) + s_g \right]$$

To obtain  $\theta$ , use equation (13) to get

$$\theta = \frac{1 - \tau}{\gamma(v)} \frac{1 + s(v)}{s_c} \frac{1 + \eta}{\eta} \frac{1 - h}{h}$$

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Table 2: Dynamic properties of the Ramsey allocation at different degrees of market power

Variable	Mean	Std. Dev.	Auto. corr.	Corr( $x, y$ )	Corr( $x, g$ )	Corr( $x, z$ )
$\mu = 1$						
$\tau$	18.8	0.0491	0.88	-0.276	0.865	-0.5
$\pi$	-3.39	7.47	-0.0279	-0.214	0.315	-0.299
$R$	0	0	0	0	0	0
$y$	0.241	0.0087	0.825	1	0.242	0.97
$h$	0.241	0.00243	0.88	-0.276	0.865	-0.5
$c$	0.201	0.00846	0.82	0.948	-0.0784	0.997
$\mu = 1.1$						
$\tau$	22.6	0.0296	0.88	-0.237	0.865	-0.5
$\pi$	-2.81	7.52	-0.0273	-0.199	0.317	-0.299
$R$	0.59	0.00938	0.88	-0.237	0.865	-0.5
$y$	0.219	0.00781	0.826	1	0.281	0.96
$h$	0.219	0.00254	0.88	-0.237	0.865	-0.5
$c$	0.179	0.00752	0.82	0.936	-0.0757	0.997
$\mu = 1.2$						
$\tau$	26.6	0.042	0.88	0.179	-0.864	0.5
$\pi$	-1.46	7.92	-0.0239	-0.174	0.323	-0.3
$R$	1.95	0.0369	0.88	-0.18	0.864	-0.5
$y$	0.199	0.00701	0.829	1	0.337	0.942
$h$	0.199	0.00273	0.88	-0.18	0.865	-0.5
$c$	0.159	0.00661	0.82	0.919	-0.0616	0.998
$\mu = 1.35$						
$\tau$	33	0.27	0.88	0.0447	-0.865	0.5
$\pi$	4.4	9.48	-0.00953	-0.11	0.349	-0.305
$R$	7.83	0.222	0.88	-0.0447	0.862	-0.499
$y$	0.172	0.00595	0.837	1	0.461	0.887
$h$	0.172	0.00318	0.88	-0.045	0.865	-0.5
$c$	0.131	0.00527	0.82	0.884	-0.00668	1

Note.  $\tau$ ,  $\pi$ , and  $R$  are expressed in percentage points and  $y$ ,  $h$ , and  $c$  in levels. The parameter  $\mu$  denotes the markup of prices to marginal cost.