

Escudé, Guillermo J.

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A DSGE Model for a SOE with Systematic Interest and Foreign Exchange Policies in Which Policymakers Exploit the Risk Premium for Stabilization Purposes

Guillermo J. Escudé

Abstract

This paper builds a DSGE model for a small open economy (SOE) in which the central bank intervenes the domestic currency bond and FX markets using two policy rules: a Taylor-type rule and a rule that determines the rate of nominal depreciation. The 2 'corner' regimes, in which only one policy rule is used, are particular cases. The model is calibrated and implemented in Dynare for simple and optimal simple policy rules, and optimal policy under commitment. Numerical losses are obtained for ad-hoc loss functions for different sets of central bank preferences. The results show that the losses are lower when both policy rules are used. This is due to the central bank's enhanced ability, when it uses the two policy rules, to influence private capital flows through the effects of its actions on the endogenous risk premium in the risk-adjusted uncovered interest parity equation.

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Keywords DSGE models; small open economy; exchange rate policy; optimal policy

Authors

Guillermo J. Escudé, ✉ Central Bank of Argentina, Ciudad Autónoma de Buenos Aires, Argentina, gescude@bcra.gov.ar

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1 Introduction¹

According to John Williamson ‘the overwhelming conventional view in the profession is that it is a mistake to try to manage exchange rates’ (Williamson 2007), although he does not subscribe this view. He finds that ‘most of the time the only monetary policy objective that may merit consideration—other than inflation targeting—is the maintenance of a sufficiently competitive exchange rate to preserve the incentive to invest’. He also argues that ‘the government can expect to reduce misalignments by a policy of intervention. The question is how those interventions should be structured: whether they should be ad hoc or systematic and, if the latter, how the system should be designed.’ This paper attempts to deal with these issues in a novel way, integrating the usual Taylor rule approach with a policy of systematic intervention in the foreign exchange market.

Although there doesn’t seem to be any justification to having to choose between a policy that uses an operational target for the nominal interest rate (often with an inflation target in mind) and a policy that uses foreign exchange (FX) market intervention to target the exchange rate, it is not easy to escape this dichotomy in the absence of an adequate and accepted theoretical framework (see, however, Wollmerhäuser 2003, Bofinger and Wollmershäuser 2001, Kim 2003, Aguirre and Grosman 2010). This absence may be due to the pervasive preference of modelers (theoreticians) to ‘sweep under the rug’ some of the Central Bank (CB) ‘nuts and bolts’ that are necessary to achieve a more general theory. Such ‘nuts and bolts’ as the CB and other financial institutions’ balance sheets (and the financial assets and liabilities within them) are detailed and analyzed in any IMF Article IV mission report pertaining to developing countries. However, when it comes to modeling the macroeconomy such aspects are simply omitted in both academic, central bank, and IMF models. What makes such an omission possible, of course, is that if one accepts the dichotomy in question, an argument of system decomposability allows one to focus on the central block of equations. However, if one does not accept the dichotomy, the need to include such ‘nuts and bolts’ arises merely to ensure a consistent policy model.

¹A previous version of this paper was presented to the 7th Dynare Conference at the Federal Reserve Bank of Atlanta, September 9-10, 2011, under the title "Optimal (and simultaneous) Interest and Foreign Exchange feedback policies in a DSGE model for a small open economy".

This paper, and the model on which it is based, builds such a consistent policy model and uses it within various policy frameworks: simple policy rules, optimal simple policy rules, and optimal policy under commitment, implementing a first order approximation to the model through Dynare. It finds strong evidence that a proper systematic use by CBs of small open economies (SOEs) of two operational targets, one for the nominal interest rate and another for the rate of nominal depreciation, outperforms the ‘corner’ regimes which either control the nominal interest with a floating exchange rate or control the rate of nominal depreciation with a floating interest rate. The basic difference between the model used here and the workhorse DSGE model of the profession is the inclusion of more detail in the modeling of the institutional structure that comes closer to the way most CBs (at least those in developing economies) implement their interest and FX policies. However, as far as the author is aware no CB implements its FX policy using the type of model used in this paper. When FX policy is systematic, there tends to be an exchange rate level-related target that is discretionally moved around and this departs from the rule-based approach used here. And when there is an explicit framework of controlling the nominal interest rate (including ‘inflation targeting’), FX policy tends to be even more discretionary and opaque. One of the conclusions of this paper is that it is perfectly possible to articulate a consistent model which conserves the systematic interest rate policy rule that prevails in the literature (Taylor rule models) yet incorporates an additional policy rule to represent FX policy. Furthermore, the paper shows that when optimal simple rules or optimal policy under commitment are introduced through an ad hoc CB loss function, significant gains are obtained using two policy rules (or two control variables in the optimal control framework) for the usual CB preferences (i.e. combinations of weights for inflation and output, and possibly the real exchange rate (RER)).

The model used for this paper (ARGEMmin) is a smaller version of the models described in two previous models: ARGEM in Escudé (2007) and ARGEMmy in Escudé (2009). They can all represent the simultaneous (i.e. within the same quarterly period) intervention in the FX and the domestic currency bond markets. The simultaneous use of two policy rules is a generalization of standard models that are limited to having either a Taylor rule for the interest rate with a pure currency float or a pure pegged regime in which there is usually no feedback and the interest rate floats. The fact that most CBs of developing economies intervene

regularly in both markets should make this generalization of practical interest.² And a model that only adds the essential features that are needed to include FX policy without excluding interest rate policy should help in obtaining intuition as to why the CB can better achieve its objectives, whatever they may be, by the use of two policy rules instead of one. It is shown that the gains the CB obtains using the two instruments are basically due its increased ability to exploit the foreign investors' risk premium function that constrains the decision problems of some sector of the economy.

In this paper, it is the household decision problem that delivers the risk-adjusted uncovered interest parity (UIP) equation.³ The use of an endogenous risk premium function that Rest of the World (RW) agents use to determine the interest rate at which they are willing to purchase the economy's foreign currency bonds plays a fundamental role in the model's dynamics of capital flows. The use of a risk premium for foreign debt has a long history in open economy macroeconomics basically due to its realism (see Bhandari et al. 1990 and the papers there cited, and Agenor 1997) since it has long been accepted as an empirical fact and measured econometrically, although various candidate variables can be statistically significant in affecting the premium.⁴ In the DSGE strand, Schmitt-Grohe and Uribe (2003) noted that the simplest SOE models with incomplete asset markets used the assumption that the subjective discount rate equals the average real interest rate and, hence, presented equilibrium dynamics with a random walk component. They considered several alternative modifications that have been used

²IMF (2011), for example, notes that 'on average about one-third of the countries in the region (Latin America) intervened in any given day'. Indeed, their Table 3.1 (Stylized facts of FX Purchases, 2004–10) shows that Colombia and Peru intervened in 32% and 39% of working days, respectively. This table also contains interesting information on other regions: in the same period, Australia and Turkey intervened in 62% and 66% of working days, respectively, while Israel intervened 24% of working days but with a cumulative intervention that represented 22.3% of GDP.

³This differs from the author's two previous (and larger) models, where it was the decision of banks that delivered the model's UIP equation. The simplification in this paper seeks to obtain a model that is sufficiently close to the standard workhorse model so that the specific difference in modeling policy is highlighted.

⁴Outside of the open economy context, it goes back at least to Kalecki (1937), who says that 'the entrepreneur who has invested in equipment his reserves (cash, deposits, securities) and taken "too much credit" is obliged to borrow at a rate of interest which is higher than the market one' and attributes this to 'the danger of "illiquidity."'

to eliminate this random walk component and showed that they have quite similar dynamics. Among these modifications is the complete assets market model (i.e., doing away with the incomplete asset markets assumption altogether) and, more relevant for this paper, the use of a risk premium function by which the interest rate on foreign funds responds to the amount of foreign debt outstanding.

In the present paper's framework, combining the non-stochastic steady state (NSS) versions of the Euler and UIP equations gives $[(1 + i^*) / \pi^*] \varphi_D(\cdot) = 1/\beta$, where β is the intertemporal discount factor, $(1 + i^*) / \pi^*$ is the RW's real interest rate, and $\varphi_D(\cdot)$ is an endogenous premium function that combines the exogenous risk premium function $\tau_D(\cdot)$ that households face when getting funds abroad (d) and their first order condition pertaining to d . Hence, it is a function of d (and possibly other endogenous variables). This equation determines the long run foreign debt level d as a function of model parameters (including those that define the risk premium function and possibly other endogenous variables). Lubik (2007) adds that even if there is an exogenous risk premium function, to avoid the unit root problem it is necessary that it be fully internalized by the individual households, i.e., that each household take into account that other households' decisions are the same as its own and, hence, that the risk premium it faces is a function of the aggregate (and not its individual) foreign debt. In this paper the risk premium function $\tau_D(\cdot)$ is assumed to be a function of the foreign debt to GDP ratio: ed/Y (where e is the SOE's RER and Y is its GDP). Furthermore, there is an additional multiplicative shock ϕ^* that may represent either an exogenous component of the risk function or an international liquidity shock (or both).⁵ In an extension of the model, the risk premium function is made to also respond, but negatively, to the CB's international reserves to GDP ratio er/Y (see Fouejieu and Roger (2013) for econometric measurements).

For convenience, the policy framework where the CB uses two simultaneous policy rules is called a Managed Exchange Rate (MER) regime. The instruments that the CB uses for its intervention in the two markets are explicitly included as model variables, and the CB balance sheet that binds them is a model equation.

⁵In addition to ϕ^* , there are three more RW shocks that impinge on the SOE: the world nominal riskfree interest rate $1 + i^*$ and the rates of inflation of imported and exported goods. There are also two domestic shocks: a transitory productivity shock in the domestic output sector and a government expenditure ratio (to GDP) shock.

It has cash m_t and CB-issued domestic currency bonds b_t on the liabilities side, and foreign currency reserves r_t on the asset side. To make sure that there are no loose ends, the CB's flow budget constraint is explicitly considered and it is assumed that the institutional framework is such that any 'quasi-fiscal' surplus (or deficit) is handed over to (financed by) the Treasury, defining 'quasi-fiscal surplus' as financial flows (specifically, those related to interest earned and capital gains on international reserves, and the interest paid on CB bonds) that could make the CB net worth different from zero. Hence, while there is overall fiscal consistency (since the Treasury is assumed to be able to collect enough lump-sum taxes each period to finance its expenditures in excess of the quasi-fiscal surplus), the CB has a constraint each period on its two instruments (r_t and b_t) given by its balance sheet: $e_t r_t = m_t + b_t$. This equation implicitly defines how much the CB 'sterilizes' (through the issuance of domestic currency bonds) any unwanted monetary effect of its simultaneous and systematic monetary and exchange policy. The expression 'sterilized intervention' (in the FX market) is avoided because it implicitly gives the exchange rate policy a subordinate role (the undesired effects of which must be 'sterilized' to avoid disrupting the monetary equilibrium that is achieved through the use of conventional monetary policy). Generality is best preserved treating both interventions in a symmetrical way, neither of which 'sterilizes' the effects of the other. When the CB intervenes in both the 'money' and FX markets, it is subject to the set of constraints given by the equations of the model, among which is monetary equilibrium and the assumed institutional constraint that the CB's net worth is kept at zero. Clearly, other similar constraints could be used for the same purpose of endogenizing the CB's 'sterilization' policy. The one used here has the virtue of simplicity. The important point is that the overall means that the CB has available be made explicit. To further ensure consistency, the model includes the balance of payments (where both household foreign debt and CB reserves play relevant roles) and the fiscal equation.

Since the 2008 financial meltdown and the consequent introduction of 'unconventional' monetary policies, it has become customary to stress the importance of central bank balance sheets in the sense that huge purchases of financial assets by central banks get reflected in their assets as well as their liabilities. At the theoretical level, Curdia and Woodford (2010) study the CB balance sheet as an instrument of monetary policy in a very elaborate closed economy model in which

households are heterogenous in their spending opportunities. In a policy paper, Caruana (2011) stresses the need to start normalizing the situation before the risk of monetizing debts gets out of hand. In this paper the point is made that inclusion of the central bank balance sheet and its composition is important even in a more ‘normal’ world with short term interest rates that are well above zero and CB assets and liabilities that are closer to normal levels. Here, ‘normal’ levels are defined by the long run CB reserves/GDP ratio, and actual CB reserves fluctuate around the corresponding long run level. Hence, a return to normal levels is automatically guaranteed whenever the model is dynamically stable. But the explicit consideration of the CB’s balance sheet opens the door for modeling other policy combinations in open economies that may include, say, taxes on foreign debt to address a (possible) ‘trinity’ of interest rate, FX, and capital control policies. This, however, is for future research.

The rest of the paper has the following structure. In Section 2 the model is set up. Section 3 addresses the functioning of the model under simple policy rules, optimal simple policy rules, and optimal policy under commitment and full information and shows that there are indeed gains from using these two simultaneous policies instead of only one of the ‘corner’ regimes. In Section 4 it is shown that such gains are basically due to the central bank’s enhanced ability to influence the risk premium in the UIP equation when it uses the two policy rules. This section also includes intuition (based on impulse response functions) on the expanded range of policy possibilities when the second simple policy rule is included, and extends the model to include CB reserves in the endogenous risk premium. Section 5 makes some additional robustness checks based on the sensitivity of CB losses to various parameter calibrations, and Section 6 concludes. Appendix I shows how the model parameters and the NSS were jointly calibrated. Finally, Appendix 2 shows a selection of the impulse response functions in the context of optimal policy under commitment.

2 The model

2.1 Households

The household optimization problem

Infinitely lived identical households consume a CES bundle of domestic and imported goods and hold financial wealth in the form of domestic currency cash (M_t) and domestic currency denominated one period nominal bonds issued by the CB (B_t) that pay a nominal interest rate i_t . They also issue one period foreign currency bonds (D_t) in the international capital market that pay a nominal (foreign currency) interest rate i_t^D . It is assumed that the CB fully and credibly insures investors in CB bonds, so the domestic currency nominal rate is considered riskfree. However, foreign investors are only willing to hold the SOE's foreign currency bonds if they receive a risk premium over the international riskfree rate i_t^* . Since the RW is not modeled, the premium function is exogenously given. It has an exogenous stochastic and time-varying component ϕ_t^* (that can represent general liquidity conditions in the international market) as well as an endogenous (more country risk-related) component $\tau_D(\cdot)$ that is an increasing convex function of the aggregate foreign debt to GDP ratio. Individual households are assumed to fully internalize the dependence of the interest rate they face on the aggregate (instead of individual) foreign debt because they know that all households are (at least in this aspect) identical (Lubik 2007). The foreign currency gross interest rate households face is:

$$1 + i_t^D = (1 + i_t^*)\phi_t^* \tau_D(\gamma_t^D), \quad (1)$$

where γ_t^D , e_t , and d_t , are the foreign debt to GDP ratio, the RER, and real foreign debt (in terms of foreign prices), respectively:

$$\gamma_t^D = \frac{S_t D_t}{P_t Y_t} = \frac{e_t d_t}{Y_t}, \quad e_t \equiv \frac{S_t P_t^*}{P_t}, \quad d_t \equiv \frac{D_t}{P_t^*}, \quad (2)$$

S_t is the nominal exchange rate, P_t is the domestic goods price index, P_t^* is the price index of the goods the SOE imports, and Y_t is GDP. The gross risk premium function $\tau_D(\gamma_t^D)$ is assumed to be increasing and convex ($\tau_D \equiv 1 + \bar{\tau}_D > 1$, $\tau_D' > 0$ and $\tau_D'' > 0$).

The household holds cash M_t because doing so reduces its transaction costs. Transaction frictions are assumed to result in a loss of purchasing power (through the non-utility generating consumption of domestic goods) when households purchase consumption goods, and that this cost can be ameliorated using cash.⁶ To purchase quantity C_t of the consumption bundle, households must spend $\tau_M(\gamma_t^M) P_t^C C_t$, where P_t^C is the price index of the consumption bundle. All price indexes are in monetary units. The gross transactions cost function $\tau_M(\gamma_t^M)$ is assumed to be a decreasing and convex function ($\tau_M \equiv 1 + \bar{\tau}_M > 1$, $\tau'_M < 0$, $\tau''_M > 0$) of the cash/consumption ratio γ_t^M :

$$\gamma_t^M \equiv \frac{M_t}{P_t^C C_t} = \frac{m_t}{p_t^C C_t}, \quad (3)$$

where

$$p_t^C \equiv \frac{P_t^C}{P_t}, \quad m_t \equiv \frac{M_t}{P_t} \quad (4)$$

are the relative price of consumption goods and real cash.

The representative household maximizes an inter-temporal utility function which is additively separable in (constant relative risk aversion subutility functions of) goods C_t and labor N_t :

$$E_t \sum_{j=0}^{\infty} \beta^j \left\{ \frac{C_{t+j}^{1-\sigma^C}}{1-\sigma^C} - \xi^N \frac{N_{t+j}^{1+\sigma^N}}{1+\sigma^N} \right\}, \quad (5)$$

where β is the intertemporal discount factor, σ^C , and σ^N are the constant relative risk aversion coefficients for goods and labor, respectively, and ξ^N is a parameter.

The household receives income from profits, wages, and interests, and spends on consumption, interests, and taxes. Its nominal budget constraint in period t is:

$$\begin{aligned} \tau_M(\gamma_t^M) P_t^C C_t + M_t + B_t - S_t D_t &= W_t N_t + \Pi_t - Tax_t \\ + M_{t-1} + (1 + i_{t-1}) B_{t-1} - (1 + i_{t-1}^D) S_t D_{t-1} \end{aligned} \quad (6)$$

⁶The introduction of money is similar to the theoretical treatment in Montiel (1999), and also to the numerically implemented treatment in Schmitt-Grohe and Uribe (2004). It differs from the latter in that instead of velocity its inverse is used (the cash/consumption ratio), and there is a different specification of the transactions cost function.

where i_t is the interest rate that CB bonds pay each quarter, W_t is the nominal wage rate, Π_t is nominal profits, and Tax_t is lump sum taxes net of transfers. Introducing (1) in (6) and dividing by P_t , the real budget constraint is:

$$\begin{aligned} \tau_M (\gamma_t^M) p_t^C C_t + m_t + b_t - e_t d_t &= w_t N_t + \frac{\Pi_t}{P_t} - tax_t + \frac{m_{t-1}}{\pi_t} \\ + (1 + i_{t-1}) \frac{b_{t-1}}{\pi_t} - (1 + i_{t-1}^*) \phi_{t-1}^* \tau_D (\gamma_{t-1}^D) e_t \frac{d_{t-1}}{\pi_t^*}, \end{aligned} \quad (7)$$

where

$$b_t \equiv \frac{B_t}{P_t}, \quad w_t \equiv \frac{W_t}{P_t}, \quad tax_t \equiv \frac{Tax_t}{P_t}, \quad \pi_t \equiv \frac{P_t}{P_{t-1}}, \quad \pi_t^* \equiv \frac{P_t^*}{P_{t-1}^*}$$

are the real stock of domestic currency bonds, the real wage (in terms of domestic goods), real lump sum tax collection, and the gross rates of quarterly inflation for domestic goods and foreign goods, respectively.

The household chooses the sequence $\{C_{t+j}, m_{t+j}, b_{t+j}, d_{t+j}, N_{t+j}\}$ that maximizes (5) subject to its sequence of budget constraints (7) (and initial values for the predetermined variables). The Lagrangian is hence:

$$\begin{aligned} E_t \sum_{j=0}^{\infty} \beta^j \left\{ \frac{C_{t+j}^{1-\sigma^C}}{1-\sigma^C} - \xi^N \frac{N_{t+j}^{1+\sigma^N}}{1+\sigma^N} + \lambda_{t+j} \left\{ w_{t+j} N_{t+j} + \frac{\Pi_{t+j}}{P_{t+j}} + \frac{m_{t-1+j}}{\pi_{t+j}} \right. \right. \\ + (1 + i_{t-1+j}) \frac{b_{t-1+j}}{\pi_{t+j}} - (1 + i_{t-1+j}^*) \phi_{t-1+j}^* \tau_D \left(\frac{e_{t-1+j} d_{t-1+j}}{Y_{t-1+j}} \right) \times e_{t+j} \frac{d_{t-1+j}}{\pi_{t+j}^*} \\ \left. \left. - \tau_M \left(\frac{m_{t+j}}{P_{t+j}^C C_{t+j}} \right) p_{t+j}^C C_{t+j} - m_{t+j} - b_{t+j} + e_{t+j} d_{t+j} - tax_{t+j} \right\} \right\} \end{aligned} \quad (8)$$

where $\beta^j \lambda_{t+j}$ are the Lagrange multipliers, and can be interpreted as the marginal utility of real income.⁷

⁷There is also a no-Ponzi game condition that is omitted for simplicity and yields the transversality condition $\lim_{t \rightarrow \infty} \beta^t d_t = 0$ that prevents households from incurring in Ponzi games.

The first order conditions for an optimum are the following:

$$C_t : C_t^{-\sigma^C} = \lambda_t p_t^C \varphi_M(m_t/p_t^C C_t) \quad (9)$$

$$m_t : \lambda_t [1 + \tau'_M(m_t/p_t^C C_t)] = \beta E_t(\lambda_{t+1}/\pi_{t+1}) \quad (10)$$

$$b_t : \lambda_t = \beta(1 + i_t) E_t(\lambda_{t+1}/\pi_{t+1}) \quad (11)$$

$$d_t : \lambda_t e_t = \beta(1 + i_t^*) \phi_t^* \varphi_D(e_t d_t/Y_t) E_t(\lambda_{t+1} e_{t+1}/\pi_{t+1}^*) \quad (12)$$

$$N_t : \xi^N N_t^{\sigma^N} = \lambda_t w_t \quad (13)$$

Notice that in (9) and (12) the auxiliary functions φ_M and φ_D have been introduced merely to obtain a more compact notation:

$$\varphi_D(\gamma^D) \equiv \tau_D(\gamma^D) + \gamma^D \tau'_D(\gamma^D), \quad (14)$$

$$\varphi_M(\gamma^M) \equiv \tau_M(\gamma^M) - \gamma^M \tau'_M(\gamma^M).$$

Combining (10) and (11) implicitly gives the demand for cash as a function of the nominal interest rate and consumption expenditure:

$$-\tau'_M(m_t/p_t^C C_t) = 1 - \frac{1}{1 + i_t}. \quad (15)$$

Inverting $-\tau'_M$ gives the explicit demand function for cash as a vehicle for transactions (or 'liquidity preference' function):

$$m_t = \mathcal{L}(1 + i_t) p_t^C C_t, \quad (16)$$

where $\mathcal{L}(\cdot)$ is defined as:

$$\mathcal{L}(1 + i_t) \equiv (-\tau'_M)^{-1} \left(1 - \frac{1}{1 + i_t} \right), \quad (17)$$

and is strictly decreasing, since:

$$\mathcal{L}'(1 + i_t) = \left[-\tau''_M(\mathcal{L}(1 + i_t)) (1 + i_t)^2 \right]^{-1} < 0.$$

Under the assumption that the CB always satisfies cash demand, from now on (16) is called the money market clearing condition.

Using (9) to eliminate λ_t from (11) yields a version of the classical Euler equation that reflects the additional influence of the use of money on transactions costs:

$$\frac{C_t^{-\sigma^C}}{\varphi_M(m_t/p_t^C C_t)} = \beta (1 + i_t) E_t \left(\frac{C_{t+1}^{-\sigma^C}}{\varphi_M(m_{t+1}/p_{t+1}^C C_{t+1})} \frac{1}{\pi_{t+1}^C} \right), \quad (18)$$

where $\pi_t^C \equiv P_t^C/P_{t-1}^C$ is the gross rate of inflation of the basket of consumption goods and the identity:

$$\frac{p_t^C}{p_{t-1}^C} = \frac{\pi_t^C}{\pi_t} \quad (19)$$

has been used (based on the definition of p_t^C in (4)) to eliminate π_{t+1} .

The definition of the RER in (2) gives the following identity:

$$\frac{e_t}{e_{t-1}} = \frac{\delta_t \pi_t^*}{\pi_t}, \quad (20)$$

where $\delta_t \equiv S_t/S_{t-1}$ is the rate of nominal depreciation of the domestic currency. Hence, (12) can be written as:

$$1 = \beta (1 + i_t^*) \phi_t^* \varphi_D \left(\frac{e_t d_t}{Y_t} \right) E_t \left(\frac{\lambda_{t+1} \delta_{t+1}}{\lambda_t \pi_{t+1}} \right).$$

Also, multiplying both sides of (11) by δ_{t+1} and applying the expectations operator gives (up to a first order approximation):

$$E_t \delta_{t+1} = \beta (1 + i_t) E_t \left(\frac{\lambda_{t+1} \delta_{t+1}}{\lambda_t \pi_{t+1}} \right).$$

Combining the last two equations yields the risk-adjusted UIP equation:

$$1 + i_t = (1 + i_t^*) \phi_t^* \varphi_D \left(\frac{e_t d_t}{Y_t} \right) E_t \delta_{t+1}. \quad (21)$$

Finally, eliminating λ_t from (13) gives the household's labor supply:

$$N_t = \left(\frac{w_t}{\xi^N p_t^C C_t^{\sigma^C} \varphi_M(m_t/p_t^C C_t)} \right)^{\frac{1}{\sigma^N}}. \quad (22)$$

Domestic and imported consumption

The consumption index used in the household optimization problem is a constant elasticity of substitution (CES) aggregate consumption index of domestic (C_t^D) and imported (C_t^N) goods:

$$C_t = \left(a_D \frac{1}{\theta^C} (C_t^D)^{\frac{\theta^C-1}{\theta^C}} + a_N \frac{1}{\theta^C} (C_t^N)^{\frac{\theta^C-1}{\theta^C}} \right)^{\frac{\theta^C}{\theta^C-1}}, \quad a_D + a_N = 1. \quad (23)$$

$\theta^C (\geq 0)$ is the elasticity of substitution between domestic and imported goods. Total consumption expenditure is:

$$P_t^C C_t = P_t C_t^D + P_t^N C_t^N, \quad (24)$$

where P_t^N is the domestic currency price of imported goods. Then minimization of (24) subject to (23) for a given C_t , yields the following relations:

$$P_t = P_t^C \left(\frac{C_t^D}{a_D C_t} \right)^{-\frac{1}{\theta^C}} \quad (25)$$

$$P_t^N = P_t^C \left(\frac{C_t^N}{a_N C_t} \right)^{-\frac{1}{\theta^C}}. \quad (26)$$

Introducing these in (23) yields the consumption price index:

$$P_t^C = \left(a_D (P_t)^{1-\theta^C} + a_N (P_t^N)^{1-\theta^C} \right)^{\frac{1}{1-\theta^C}}. \quad (27)$$

Dividing (27) through by P_t yields a relation between the relative prices of consumption and imported goods (both in terms of domestic goods):

$$p_t^C = \left(a_D + (1 - a_D) (p_t^N)^{1-\theta^C} \right)^{\frac{1}{1-\theta^C}}, \quad (28)$$

where

$$p_t^N \equiv \frac{P_t^N}{P_t}.$$

For simplicity, the Law of One Price is assumed to hold. Hence, the domestic price of (the aggregate of) imported goods is simply $P_t^N = S_t P_t^*$. This implies that the domestic relative price of imports is simply the RER:

$$p_t^N = \frac{P_t^N}{P_t} = \frac{S_t P_t^*}{P_t} = e_t \quad (29)$$

and hence the relative price of the consumption bundle (28) is:

$$p_t^C = \left(a_D + (1 - a_D) e_t^{1-\theta^C} \right)^{\frac{1}{1-\theta^C}}. \quad (30)$$

(25) and (26) show that a_D and $a_N = 1 - a_D$ in (23) are directly related to the shares of domestic and imported consumption in total consumption expenditures. In fact, the shares are:

$$\frac{C_t^D}{p_t^C C_t} = a_D \frac{1}{(p_t^C)^{1-\theta^C}} \quad (31)$$

$$\frac{e_t C_t^N}{p_t^C C_t} = (1 - a_D) \left(\frac{e_t}{p_t^C} \right)^{1-\theta^C} \quad (32)$$

It is assumed that there is a bias for domestic goods, i.e., $a_D > 1/2 > a_N$, and that $\theta^C > 1$.⁸

C_t^D is a CES aggregate of an infinite number of domestic varieties of goods, each produced by a monopolist under monopolistic competition:

$$C_t^D = \left(\int_0^1 C_t^D(i)^{\frac{\theta-1}{\theta}} di \right)^{\frac{\theta}{\theta-1}}, \quad \theta > 1 \quad (33)$$

where θ is the elasticity of substitution between varieties of domestic goods in household expenditure.

⁸In the Cobb-Douglas case ($\theta^C = 1$) the shares are a_D and $a_N = 1 - a_D$ (and hence are time invariant). But in this case the relative demand of domestic to imported goods is independent of p_t^N (and hence, the RER), which is something not too desirable. With $\theta^C > 1$ an increase in the relative price of imported goods increases the relative demand for domestic goods.

Conditions (25), and (26) are necessary for the optimal allocation of household expenditures across domestic and imported bundles of goods. Similarly, for the optimal allocation across varieties of domestic goods within the first of these classes, use of (33) yields the following necessary conditions:

$$P_t(i) = P_t \left(\frac{C_t^D(i)}{C_t^D} \right)^{-\frac{1}{\theta}}.$$

2.2 Domestic goods firms

The representative final goods firm

There is perfect competition in the production (or bundling) of final domestic output Q_t , with the output of intermediate firms as inputs. A representative final domestic output firm uses the following CES technology:

$$Q_t = \left(\int_0^1 Q_t(i)^{\frac{\theta-1}{\theta}} di \right)^{\frac{\theta}{\theta-1}}, \quad \theta > 1 \quad (34)$$

where $Q_t(i)$ is the output of the intermediate domestic good i . The final domestic output representative firm solves the following problem each period:

$$\max_{Q_t(i)} P_t \left(\int_0^1 Q_t(i)^{\frac{\theta-1}{\theta}} di \right)^{\frac{\theta}{\theta-1}} - \int_0^1 P_t(i) Q_t(i) di, \quad (35)$$

the solution of which is the demand for each type of domestic good (as an input):

$$Q_t(i) = Q_t \left(\frac{P_t(i)}{P_t} \right)^{-\theta}. \quad (36)$$

Introducing (36) in (34) and simplifying, it is readily seen that the domestic goods price index is:

$$P_t = \left(\int_0^1 P_t(i)^{1-\theta} di \right)^{\frac{1}{1-\theta}}. \quad (37)$$

Also, introducing (36) into the cost part of (35) yields:

$$\int_0^1 P_t(i) Q_t(i) di = P_t Q_t.$$

The monopolistically competitive firms

A continuum of monopolistically competitive firms produce the intermediate domestic goods (that the final goods producer bundles) using homogenous labor, with no entry or exit. The production function of each firm is:

$$Q_t(i) = \varepsilon_t N_t(i) \quad (38)$$

where ε_t is an industry-wide transitory productivity shock.

Since $N_t(i)$ is firm i 's labor demand, using (38) and (36) and integrating yields aggregate labor demand:

$$N_t^D = \int_0^1 N_t(i) di = \int_0^1 \frac{Q_t(i)}{\varepsilon_t} di = \frac{1}{\varepsilon_t} \int_0^1 Q_t \left(\frac{P_t(i)}{P_t} \right)^{-\theta} di = \frac{Q_t}{\varepsilon_t} \Delta_t \quad (39)$$

where (as in Schmitt-Grohe and Uribe 2004 and 2005) a measure of price dispersion at period t has been defined :

$$\Delta_t \equiv \int_0^1 \left(\frac{P_t(i)}{P_t} \right)^{-\theta} di \geq 1. \quad (40)$$

Notice that $\Delta_t = 1$ when all prices are the same and $\Delta_t > 1$ otherwise (Schmitt-Grohe and Uribe 2005).

Equating labor supply (22) and demand (39) gives the labor market equilibrium real wage (in terms of domestic goods):

$$w_t = \xi^N \left(\frac{Q_t}{\varepsilon_t} \Delta_t \right)^{\sigma^N} p_t^C C_t^{\sigma^C} \varphi_M (m_t / p_t^C C_t) \quad (41)$$

Each firm's cost is $W_t N_t(i) = (W_t / \varepsilon_t) Q_t(i)$. Hence, its marginal cost is W_t / ε_t and its real marginal cost (in terms of domestic goods) is:

$$mc_t = \frac{w_t}{\varepsilon_t}. \quad (42)$$

Notice that all firms face the same marginal cost. Also, (41) shows that increases in price dispersion raise the equilibrium real wage and hence the real marginal cost of firms. This is due to the positive effect of increased price dispersion on aggregate labor demand (see (39)) and, given the level of supply, on the equilibrium real wage. Furthermore, tighter monetary conditions increase marginal cost because an increase in i_t makes households economize on cash (see (16)), lowering $m_t/p_t^C C_t$. Because $\phi'_M = -\gamma^M \tau''_M < 0$, this has a positive effect on ϕ_M , lowering labor supply (see (22)) and hence increasing the equilibrium real wage.

The dynamics of inflation and price dispersion

Firms make pricing decisions taking the aggregate price and quantity indexes as parametric. Every period, each firm has a probability $1 - \alpha$ of being able to set the optimum price for its specific type of good (Calvo 1983). The firms that can't optimize must leave the same price they had last period. The pricing problem of firms that get to optimize is:

$$\max_{P_t(i)} E_t \sum_{j=0}^{\infty} \alpha^j \Lambda_{t,t+j} Q_{t+j}(i) \left\{ \frac{P_t(i)}{P_{t+j}} - mc_{t+j} \right\} \quad (43)$$

subject to the demand they will face until they can again optimize:

$$Q_{t+j}(i) = Q_{t+j} \left(\frac{P_t(i)}{P_{t+j}} \right)^{-\theta}. \quad (44)$$

$\Lambda_{t,t+j}$ is the pricing kernel used by domestic firms for discounting, which, since firms are owned by households and respond to their preferences, is equal to households' intertemporal marginal rate of substitution in the consumption of domestic goods between periods $t + j$ and t :

$$\Lambda_{t,t+j} \equiv \beta^j \frac{U_{C^D,t+j}}{U_{C^D,t}},$$

where $U(C_{t+j}, N_{t+j})$ is the function within brackets in (5). Notice that the marginal utility of consuming domestic goods can be obtained from the marginal util-

ity of consuming the aggregate bundle of (domestic and imported) goods. Specifically:

$$U_{CD,t} = U_{C,t} \frac{dC_t}{dC_t^D} = U_{C,t} a_D^{\frac{1}{\theta^C}} \left(\frac{C_t^D}{C_t} \right)^{-\frac{1}{\theta^C}} = C_t^{-\sigma^C} \frac{P_t}{P_t^C} = \frac{1}{p_t^C C_t^{\sigma^C}},$$

where the second equality is obtained by differentiating (23) with respect to C_t^D , and the third comes from using (25). Hence, the pricing kernel of domestic firms is:

$$\Lambda_{t,t+j} \equiv \beta^j \frac{p_t^C C_t^{\sigma^C}}{p_{t+j}^C C_{t+j}^{\sigma^C}}. \quad (45)$$

Introducing (44) and (45) in (43) (and eliminating irrelevant multiplying terms that refer to time t) gives

$$\max_{P_t(i)} E_t \sum_{j=0}^{\infty} (\beta \alpha)^j \frac{Q_{t+j}}{p_{t+j}^C C_{t+j}^{\sigma^C}} \left\{ \left(\frac{P_t(i)}{P_{t+j}} \right)^{1-\theta} - mc_{t+j} \left(\frac{P_t(i)}{P_{t+j}} \right)^{-\theta} \right\}.$$

Since by symmetry all optimizing firms make the same decision the optimum price can be denominated \tilde{P}_t (dropping the firm index). Hence, the firm's first order condition is the following:

$$0 = E_t \sum_{j=0}^{\infty} (\beta \alpha)^j \frac{Q_{t+j}}{p_{t+j}^C C_{t+j}^{\sigma^C}} \left(\frac{P_{t+j}}{P_t} \right)^{\theta} \left\{ \tilde{p}_t \frac{P_t}{P_{t+j}} - \frac{\theta}{\theta-1} mc_{t+j} \right\} \quad (46)$$

where $\tilde{p}_t \equiv \tilde{P}_t/P_t$ is the relative price of firms that optimize and the general price level (which includes the prices of both optimizers and non-optimizers). In the Calvo setup, because optimizers (and hence non-optimizers) are randomly chosen from the population, the average price in $t-1$ of non-optimizers (which must keep their price constant) is equal to the overall price index in $t-1$ no matter when they optimized for the last time. Hence, (37) implies the following law of motion for the aggregate domestic goods price index:

$$P_t^{1-\theta} = \alpha (P_{t-1})^{1-\theta} + (1-\alpha) \tilde{P}_t^{1-\theta}. \quad (47)$$

Dividing through by $P_t^{1-\theta}$ and rearranging yields the relative price of optimizers as an increasing function of the inflation rate:

$$\tilde{p}_t = \left(\frac{1 - \alpha \pi_t^{\theta-1}}{1 - \alpha} \right)^{\frac{-1}{\theta-1}} \equiv \tilde{p}(\pi_t). \quad (48)$$

Hence, using this in (46) gives the (non-linear) Phillips equation that determines the dynamics of domestic inflation:

$$0 = E_t \sum_{j=0}^{\infty} (\beta \alpha)^j \frac{Q_{t+j}}{p_{t+j}^C C_{t+j}^{\sigma^C}} \left(\frac{P_{t+j}}{P_t} \right)^{\theta} \left\{ \tilde{p}(\pi_t) \frac{P_t}{P_{t+j}} - \frac{\theta}{\theta-1} m c_{t+j} \right\}. \quad (49)$$

In order to implement the Phillips equation in Dynare this equation is now expressed in a recursive (nonlinear) form. Define:

$$\begin{aligned} \Gamma_t &= E_t \sum_{j=0}^{\infty} (\beta \alpha)^j \frac{Q_{t+j}}{p_{t+j}^C C_{t+j}^{\sigma^C}} \left(\frac{P_{t+j}}{P_t} \right)^{\theta-1} \\ \Psi_t &= \frac{\theta}{\theta-1} E_t \sum_{j=0}^{\infty} (\beta \alpha)^j \frac{Q_{t+j}}{p_{t+j}^C C_{t+j}^{\sigma^C}} \left(\frac{P_{t+j}}{P_t} \right)^{\theta} m c_{t+j} \end{aligned} \quad (50)$$

and express (49) as:

$$\tilde{p}(\pi_t) \Gamma_t = \Psi_t.$$

Now write Γ_t and Ψ_t recursively as follows:

$$\begin{aligned} \Gamma_t &= \left(Q_t / p_t^C C_t^{\sigma^C} \right) + \beta \alpha E_t \pi_{t+1}^{\theta-1} \Gamma_{t+1} \\ \Psi_t &= \frac{\theta}{\theta-1} \left(Q_t / p_t^C C_t^{\sigma^C} \right) m c_t + \beta \alpha E_t \pi_{t+1}^{\theta} \Psi_{t+1}. \end{aligned}$$

Hence, the complicated Phillips equation (with infinite summations) is transformed into these three simple nonlinear equations. Notice that collapsing the log-linear approximations of these equations yields the usual log-linearized Phillips equation:

$$\hat{\pi}_t = \frac{(1 - \beta \alpha)(1 - \alpha)}{\alpha} \hat{m} c_t + \beta E_t \hat{\pi}_{t+1}.$$

Δ_t is an additional variable in the model, which hence needs an additional equation. A recursive equation for the dynamics of this variable is now derived in three steps. First, separate the set of non-optimizing firms N from the set of optimizing firms O and notice that in a given period the latter all set the same price \tilde{P}_t and have mass $1 - \alpha$:

$$\Delta_t \equiv \int_{i \in N} \left(\frac{P_t(i)}{P_t} \right)^{-\theta} di + \int_{i \in O} \left(\frac{P_t(i)}{P_t} \right)^{-\theta} di = \alpha \Delta_t^N + (1 - \alpha) \tilde{p}_t^{-\theta} \quad (51)$$

where an analogous measure of price dispersion for non-optimizers is used (see (40)):

$$\Delta_t^N \equiv \int_{i \in N} \frac{1}{\alpha} \left(\frac{P_t(i)}{P_t} \right)^{-\theta} di.$$

Second, write Δ_t^N recursively using the fact that non-optimizers maintain in t the same price as in $t - 1$:

$$\Delta_t^N \equiv \int_{i \in N} \frac{1}{\alpha} \left(\frac{P_{t-1}(i)}{P_{t-1}} \frac{1}{\pi_t} \right)^{-\theta} di = \pi_t^\theta \int_{i \in N} \frac{1}{\alpha} \left(\frac{P_{t-1}(i)}{P_{t-1}} \right)^{-\theta} di = \pi_t^\theta \Delta_{t-1}^N$$

and use this and (48) in (51) to get:

$$\Delta_t = \alpha \pi_t^\theta \Delta_{t-1}^N + (1 - \alpha) \tilde{p}(\pi_t)^{-\theta}.$$

Third, since non-optimizers (as well as optimizers) are selected randomly from the set of all firms, the dispersion of non-optimizers in $t - 1$ is equal to the dispersion of the population: $\Delta_{t-1}^N = \Delta_{t-1}$. The new model equation is therefore:

$$\Delta_t = \alpha \pi_t^\theta \Delta_{t-1} + (1 - \alpha) \tilde{p}(\pi_t)^{-\theta}. \quad (52)$$

The log-linear approximation of this equation is:

$$\begin{aligned} \widehat{\Delta}_t &= \alpha \pi^\theta \widehat{\Delta}_{t-1} + \theta \varepsilon^{\tilde{p}} (\pi - 1) \widehat{\pi}_t \\ \varepsilon^{\tilde{p}} &\equiv \frac{\alpha \pi^{\theta-1}}{1 - \alpha \pi^{\theta-1}}. \end{aligned}$$

Hence, if in the NSS there is domestic price stability ($\pi = 1$) and therefore no domestic price dispersion ($\Delta = 1$), a linear approximation of the model will not give any dynamics for $\hat{\Delta}_t$ if initially there is no price dispersion (Schmitt-Grohe and Uribe 2005). Since this paper does not go beyond a first order approximation of the model, to see the dynamics of price dispersion in IRFs (that show the responses of the log-linear deviations of the variables from the NSS values to shocks when they are initially at the NSS) it is necessary to assume that the target gross inflation is different from one. And since it is also realistic to assume a positive target rate of inflation, in the calibrations of Appendix I a target rate of 1.015 (1.5% quarterly inflation, i.e., 6.1% annual inflation), is assumed.

2.3 Foreign trade, the public sector, and the balance of payments

Firms in the export sector use domestic goods and the composite of goods that defines GDP. It is assumed that the export good is a single homogenous primary good (a commodity). Firms in this sector sell their output in the international market at the foreign currency price P_t^{*X} . They are price takers in factor and product markets. The price of primary goods in terms of the domestic currency is merely the exogenous international price multiplied by the nominal exchange rate: $S_t P_t^{*X}$. Let the production function employed by firms in the export sector be the following:

$$X_t^* = (Q_t^X)^{b^A} Y_t^{1-b^A}, \quad 0 < b^A < 1, \quad (53)$$

where Q_t^X is the amount of domestic goods used as input in the export sector. These firms maximize profit $S_t P_t^{*X} X_t^* - P_t Q_t^X$ subject to (53). In terms of domestic goods, they maximize:

$$\frac{\Pi_t^X}{P_t} = e_t p_t^* (Q_t^X)^{b^A} Y_t^{1-b^A} - Q_t^X$$

where the SOE's external terms of trade (XTT) is defined as:

$$p_t^* \equiv \frac{P_t^{*X}}{P_t^*},$$

where P_t^* is the price index of the foreign currency price of the SOE's imports. Notice that the XTT is a ratio of two price indexes determined in the RW. Hence, the follow identity relates the rates of foreign inflation of exported and imported goods to the XTT (giving the dynamics of the XTT):

$$\frac{p_t^*}{P_{t-1}^*} = \frac{\pi_t^{*X}}{\pi_t^*}, \quad \text{where } \pi_t^{*X} \equiv \frac{P_t^{*X}}{P_{t-1}^{*X}}.$$

The first order condition for profit maximization yields the export sector's (factor) demand for domestic goods:

$$Q_t^X = (b^A e_t p_t^*)^{\frac{1}{1-b^A}} Y_t. \quad (54)$$

Also, inserting this in (53) shows that optimal exports vary directly with the product of the RER and the XTT and with GDP:

$$X_t^* = (b^A e_t p_t^*)^{\frac{b^A}{1-b^A}} Y_t. \quad (55)$$

The real value of exports in terms of domestic goods is:

$$X_t = \frac{S_t P_t^{*X} X_t^*}{P_t} = e_t p_t^* X_t^* = e_t p_t^* (b^A e_t p_t^*)^{\frac{b^A}{1-b^A}} Y_t = \kappa_X (e_t p_t^*)^{b_X} Y_t \quad (56)$$

where for simplicity of notation the following parameters are defined:

$$b_X \equiv \frac{1}{1-b^A}, \quad \kappa_X \equiv (b^A)^{\frac{b^A}{1-b^A}}.$$

Government expenditure is assumed to be a time-varying and stochastic fraction \bar{G}_t of private consumption expenditure. Define the gross government expenditure fraction as: $G_t \equiv 1 + \bar{G}_t$. Hence, using (31) and (32), GDP in terms of domestic goods is:

$$\begin{aligned} Y_t &= G_t \tau_M (\gamma_t^M) p_t^C C_t + X_t - (1 - a_D) e_t^{1-\theta^C} G_t \tau_M (\gamma_t^M) (p_t^C)^{\theta^C} C_t \\ &= a_D G_t \tau_M (\gamma_t^M) (p_t^C)^{\theta^C} C_t + X_t. \end{aligned} \quad (57)$$

In the domestic goods market, the output of domestic firms Q_t must satisfy final demand from households (including the resources for transactions), the government, and the export sector:⁹

$$Q_t = a_D G_t \tau_M (\gamma_t^M) (p_t^C)^{\theta^C} C_t + Q_t^X = Y_t - (1 - b^A) X_t. \quad (58)$$

The public sector includes the Government and the CB. The latter issues currency (M_t) and domestic currency bonds (B_t), and holds international reserves (R_t) in the form of foreign currency denominated riskfree bonds issued by the RW. It is assumed that the CB has no operational costs and that CB bonds are only held by domestic residents. The (flow) budget constraint of the CB is:

$$\begin{aligned} M_t + B_t - S_t R_t &= M_{t-1} + (1 + i_{t-1}) B_{t-1} - (1 + i_{t-1}^*) S_t R_{t-1} \\ &= [M_{t-1} + B_{t-1} - S_{t-1} R_{t-1}] - QF_t. \end{aligned} \quad (59)$$

where

$$\begin{aligned} QF_t &= i_{t-1}^* S_t R_{t-1} + (S_t - S_{t-1}) R_{t-1} - i_{t-1} B_{t-1} \\ &= [i_{t-1}^* + (1 - 1/\delta_t)] S_t R_{t-1} - i_{t-1} B_{t-1} \end{aligned}$$

is the CB's quasi-fiscal surplus, which includes interest earned and capital gains on international reserves minus the interest paid on its bonds. It is assumed that the CB transfers its quasi-fiscal surplus (or deficit) to the Government every period. Hence, its net wealth is constant. Furthermore, assuming for convenience that the CB's net worth is zero, the following holds for all t :

$$M_t + B_t - S_t R_t = M_{t-1} + B_{t-1} - S_{t-1} R_{t-1} = 0. \quad (60)$$

⁹Notice that intermediate output in the export sector (54) can be written as:

$$Q_t^X = (b^A)^{\frac{1}{1-b^A}} (e_t p_t^*)^{b_X} Y_t = b^A (b^A)^{\frac{b^A}{1-b^A}} (e_t p_t^*)^{b_X} Y_t = b^A X_t$$

Hence, rearranging the second equality in (58) shows that GDP is the sum of the outputs of the domestic and export sectors, minus the intermediate use of domestic goods in the export sector $Y_t = Q_t + X_t - b^A X_t$.

The CB supplies whatever amount of cash is demanded by households, and can influence these supplies by changing R_t or B_t , i.e. intervening in the FX market or in the domestic currency bond market. In terms of domestic goods, the CB balance, for all t , is:

$$m_t + b_t = e_t r_t. \quad (61)$$

This equation provides a constraint on the CB's ability to simultaneously intervene in the FX market (through sales and purchases of foreign reserves r_t) and in the domestic bonds market (through sales and purchases of domestic currency CB bonds b_t).¹⁰

The Government spends on goods, receives the quasi-fiscal surplus (or finances the deficit) of the CB, and collects taxes. It is assumed that fiscal policy consists of an exogenous autoregressive path for real government expenditures as a (gross) fraction of private consumption (G_t) and collecting whatever lump-sum taxes are needed to balance the budget each period. The Public Sector flow budget constraint is hence:

$$Tax_t = \bar{G}_t \tau_M (\gamma_t^M) P_t^C C_t - QF_t. \quad (62)$$

So in real terms:

$$\begin{aligned} tax_t &= \bar{G}_t \tau_M (\gamma_t^M) p_t^C C_t - qf_t, \\ qf_t &= (1 + i_{t-1}^* - 1/\delta_t) \frac{e_t r_{t-1}}{\pi_t^*} - ((1 + i_{t-1}) - 1) \frac{b_{t-1}}{\pi_t}. \end{aligned} \quad (63)$$

Inserting

$$Y_t = w_t N_t + \frac{\Pi_t}{P_t},$$

¹⁰In the present setup this equation can be interpreted as an institutional constraint that the CB must preserve a 'full backing' of its domestic currency liabilities with (the domestic currency value of) its foreign reserves. But it is obviously unnecessary to restrict the CB net wealth to zero (or to full backing). Any constant amount would do. Moreover, there is clearly the possibility of adding a degree of freedom for a more general model in which the CB net wealth can vary (perhaps stochastically) or even be used as an additional control variable. The latter would require additional modeling, such as market perceptions of CB risk. For the purpose of modeling the simultaneous use of the interest rate and the rate of nominal depreciation as control variables, the simplest assumption of zero CB net wealth is sufficient.

in the household budget constraint (7) and consolidating the household, CB and government budget constraints yields the balance of payments equation:

$$r_t - d_t = CA_t + r_{t-1} - d_{t-1},$$

where the current account (in foreign currency) is:

$$CA_t = \left(\frac{1 + i_{t-1}^*}{\pi_t^*} - 1 \right) r_{t-1} - \left[\frac{1 + i_{t-1}^*}{\pi_t^*} \phi_{t-1}^* \tau_D \left(\frac{e_{t-1} d_{t-1}}{Y_{t-1}} \right) - 1 \right] d_{t-1} + TB_t$$

and, using (32) and (57), the trade balance (in foreign currency) is:

$$\begin{aligned} TB_t &= \frac{1}{e_t} [X_t - e_t G_t \tau_M (\gamma_t^M) C_t^N] \\ &= \frac{1}{e_t} \left[X_t - (1 - a_D) e_t^{1-\theta^C} (p_t^C)^{\theta^C} G_t \tau_M (\gamma_t^M) C_t \right] \\ &= \frac{1}{e_t} \left[X_t - \frac{1 - a_D}{a_D} e_t^{1-\theta^C} (Y_t - X_t) \right] \\ &= \frac{1}{a_D e_t} \left[(p_t^C)^{1-\theta^C} X_t - (1 - a_D) e_t^{1-\theta^C} Y_t \right]. \end{aligned}$$

2.4 Monetary and exchange rate policy

In this paper the CB uses either simple policy rules or optimal policy under commitment (OPC) and full information (Svensson and Woodford 2003). The policy rules are simple, i.e., they respond to a limited number of endogenous variables through constant coefficients, and these coefficients may be calibrated directly or obtained by minimizing a loss function. The rule for the nominal interest rate usually involves feedback (as in the typical Taylor-like rule) and the rule for nominal depreciation may or may not involve feedback. In the case of optimal simple rules, the CB is assumed to minimize a weighted average of the variances of some of the endogenous variables ('target' variables). In the case of OPC, the CB is assumed to minimize the expected discounted value of future losses for a suitably defined quadratic loss function of some of the endogenous variables.

In any of these three cases, the CB can operate under one of three alternative monetary regimes. The expression 'monetary regime' is here used broadly. It

expresses the combination of the CB's operating procedures concerning the issuance of base money and the intervention it may have in the bond and FX markets to influence the nominal interest rate and the rate of nominal currency depreciation. Money issuance is whatever is needed to satisfy money market once the other two policies are defined. For convenience, the three monetary regimes are denominated: I) a Managed Exchange Rate (MER) regime, in which the CB uses both rules (or both instruments in the case of OPC), II) a Floating Exchange Rate (FER) regime, in which the CB only uses the Taylor-like rule (or only uses the interest rate as an instrument—in the case of OPC), and III) a Pegged Exchange Rate (PER) regime, in which the CB only uses the rule for the rate of nominal depreciation (or only uses the rate of nominal depreciation as an instrument, in the case of OPC).

In the MER regime, through its regular and systematic interventions in the domestic currency bond (or 'money') market and in the FX market, the CB aims for the achievement of two operational targets: one for the interest rate i_t , and another for the rate of nominal depreciation δ_t . When there are simple policy rules (whether they are optimal or not), the CB can respond to deviations of the consumption inflation rate (π_t^C) from a target (π^T) which is the NSS value of this variable, to deviations of GDP from its NSS value, and to deviations of the RER from its NSS value. The rate of nominal depreciation can respond to the same variables and additionally to the deviations of the CB's international reserves (IRs) ratio (to GDP) from a long run target (γ^R). There may be history dependence (or inertia) in one or both of the two simple rules through the presence of the lagged operational target variable. The simple rules are the following:

$$\frac{1+i_t}{1+i} = \left(\frac{1+i_{t-1}}{1+i} \right)^{h_0} \left(\frac{\pi_t^C}{\pi_t^T} \right)^{h_1} \left(\frac{Y_t}{Y} \right)^{h_2} \left(\frac{e_t}{e} \right)^{h_3} \quad (64)$$

$$\frac{\delta_t}{\delta} = \left(\frac{\delta_{t-1}}{\delta} \right)^{k_0} \left(\frac{\pi_t^C}{\pi_t^T} \right)^{k_1} \left(\frac{Y_t}{Y} \right)^{k_2} \left(\frac{e_t}{e} \right)^{k_3} \left(\frac{e_t r_t / Y_t}{\gamma^R} \right)^{k_4}, \quad (65)$$

where $h_1 \neq 0$ and $k_4 \neq 0$ and variables without time subscripts denote NSS values. The first of these is used in the MER and FER regimes, and the second is used in

the MER and PER regimes. In a floating exchange rate regime (FER), the CB abstains from intervening in the FX market. Hence, the international reserves that appear in its balance sheet remain constant. For simplicity, it is assumed that they remain constant at the NSS value r of the general model (with MER regime). In a pegged exchange rate regime (PER), the CB abstains from intervening in the domestic currency bond market. Hence, its stock of bonds remains constant, and it is assumed that they remain at the NSS value b of the MER regime. In both of the corner cases, one of the policy rules is dropped and one of the endogenous variables is turned into an exogenous parameter.¹¹

The FER and PER regimes are extreme cases ('corner regimes') in which the CB chooses not to use one of its potential instruments. In the case of OPC this means that the optimal policy under any one of the 'corner' regimes cannot dominate the optimal policy under the MER regime. One can define these regimes as cases in which the CB imposes an additional restriction on itself ('ties its hands') and relinquishes its use of one of its 'control' variables. Hence that variable turns into a 'non-control' variable.¹² Notice that the PER regime could alternatively, and perhaps more appropriately, be denominated a Floating Interest Rate (FIR) regime. Here it has been preferred to emphasize the exchange rate policy aspect, but the FER vs. FIR denominations directly point to the two extreme non-interventionist (or passive) policies that are possible in this theory.¹³

To obtain a generalization of the standard DSGE monetary policy model, the instruments that the CB uses when it intervenes in each of the two markets are specified and included in the model. The CB purchases or sells domestic currency bonds, and thus changes its stock of bonds b_t , to intervene with high frequency in

¹¹One must bear in mind that here the nominal and real exchange rates are (in spirit) multilateral. If we modeled a multicountry RW, the nominal exchange rate would be the domestic currency price of a basket of the nominal exchange rates of the SOE's trade partners, with weights equal to the shares in trade. Hence, our peg is completely different from pegging against the currency of a country with which only a small part of the SOE's trade is done (as was the case of Argentina's ill fated 'Convertibility').

¹²The term 'state variable' is avoided here because in this model δ is a non-predetermined (or jump) variable and it is usual to call predetermined variables 'state variables'.

¹³If one uses the FIR (instead of PER) denomination, the MER regime could then be called an Active Interest and Exchange Rates (AIXR) regime, pointing to the non-passivity in the two markets.

this market in order to attain its operational target for the interest rate as determined by (64).¹⁴ And it purchases or sells reserves to intervene in the FX market, thereby changing its stock of international reserves r_t , in order to attain its operational target for the rate of nominal depreciation as determined by (65). While at high frequency (hours, days, weeks) the CB is active changing b_t and/or r_t , at low frequency (quarters in this paper) these variables passively adapt to accommodate i_t and δ_t as given by the feedback policy rules and the rest of the model equations.

To represent the constraints that the CB faces it is necessary to broaden the usual policy model to include the CB balance sheet (61) and its arrangement with the rest of the government (Treasury) as to the use of the fiscal dimension of the CB's flow budget constraint (called CB quasi-fiscal surplus qf_t above). By assuming that the CB's arrangement with the Treasury is that it hands over its quasi-fiscal surplus (or receives automatic finance for its quasi-fiscal deficit) period by period, the CB balance sheet equation is maintained period by period in the sense that the CB's net worth is constant. This can be seen as a simple device for defining the CB's 'sterilization' policy, i.e. the value of b_t , given the values of m_t ('determined' by money market balance), and the values of e_t and r_t . But it is probably more adequate to think more symmetrically that (61) imposes a constraint on the simultaneous use of b_t and r_t . From this vantage point, one should think of the 'corner' regimes as the imposition of an additional constraint (instead of the dropping of an endogenous variable). In the case of the FER regime, the additional constraint is $r_t = r$ (an equation that replaces (65)). And in the case of the PER regime, the additional constraint is $b_t = b$ (an equation that replaces (64)). In terms of an optimal control framework (as is OPC), any one of the 'corner' regimes imposes an additional constraint on the policymaker and, simultaneously, converts one of the 'controls' (δ_t in the case of the FER regime and i_t in the case of the PER regime) into a non-control variable. Hence, it is quite evident that the MER regime cannot be inferior to any of the two 'corner' regimes (in the sense of generating a larger loss). With the same loss function and the same (basic) model equations and endogenous variables, but with one additional constraint (equation)

¹⁴Notice that this high-frequency action may be modeled in different ways. But in the quarterly frequency of the model the instruments, operational target variables, and the rest of the model variables are related through the model equations that any higher frequency model must respect if it is designed to be consistent with the quarterly model.

and one less ‘control’, the expected discounted loss cannot be lower. Indeed, it is shown below that it is quite higher for all of the usual CB preferences (represented through weights for inflation and output deviations).

2.5 Functional forms for auxiliary functions

For calibrations it is convenient to define the **net** functions:

$$\begin{aligned}\bar{\tau}_D(\gamma_t^D) &= \tau_D(\gamma_t^D) - 1, & \bar{\varphi}_D(\gamma_t^D) &= \varphi_D(\gamma_t^D) - 1 \\ \bar{\tau}_M(\gamma_t^M) &= \tau_M(\gamma_t^M) - 1, & \bar{\varphi}_M(\gamma_t^M) &= \varphi_M(\gamma_t^M) - 1.\end{aligned}\quad (66)$$

The following functional forms are used:¹⁵

$$\bar{\tau}_D(\gamma_t^D) \equiv \frac{\alpha_1}{1 - \alpha_2 \gamma_t^D}, \quad \alpha_1, \alpha_2 > 0, \quad (67)$$

$$\bar{\tau}_M(\gamma_t^M) \equiv \frac{\beta_1}{(1 + \beta_2 \gamma_t^M)^{\beta_3}}, \quad \beta_1, \beta_2, \beta_3 > 0, \quad (68)$$

which, according to definitions (14), give:

$$\begin{aligned}\bar{\varphi}_D(\gamma_t^D) &= \frac{\alpha_1}{(1 - \alpha_2 \gamma_t^D)^2} = \bar{\tau}_D(\gamma_t^D) \frac{1}{1 - \alpha_2 \gamma_t^D}, \\ \bar{\varphi}_M(\gamma_t^M) &= \frac{\beta_1}{(1 + \beta_2 \gamma_t^M)^{\beta_3}} \left(1 + \beta_3 \frac{\beta_2 \gamma_t^M}{1 + \beta_2 \gamma_t^M} \right) = \bar{\tau}_M(\gamma_t^M) \left(1 + \beta_3 \frac{\beta_2 \gamma_t^M}{1 + \beta_2 \gamma_t^M} \right).\end{aligned}\quad (69)$$

The liquidity preference function (17) that results from (68) is:

$$\frac{m_t}{p_t^C C_t} \equiv \gamma_t^M = \mathcal{L}(1 + i_t) = \frac{1}{\beta_2} \left[\left(\frac{\beta_1 \beta_2 \beta_3}{1 - \frac{1}{1+i_t}} \right)^{\frac{1}{\beta_3+1}} - 1 \right]. \quad (70)$$

¹⁵In calibrating the model parameters it was found important to include a third parameter in the transactions cost function. Otherwise realistic money demand interest elasticities could not be calibrated and the variability of the instruments was excessive. Notice that adding the analogous parameter α_3 in (67) can be of use in any refining of the calibrations or econometric fitting of the model to data.

And to get a more compact notation in some of the equations the following auxiliary variables and equations are introduced:

$$\begin{aligned}\tau_{D,t} &= 1 + \frac{\alpha_1}{1 - \alpha_2 \gamma_t^D}, & \varphi_{D,t} &= 1 + (\tau_{D,t} - 1) \left(1 + \frac{\alpha_2 \gamma_t^D}{1 - \alpha_2 \gamma_t^D} \right) \\ \tau_{M,t} &= 1 + \frac{\beta_1}{(1 + \beta_2 \gamma_t^M)^{\beta_3}}, & \varphi_{M,t} &= 1 + (\tau_{M,t} - 1) \left(1 + \beta_3 \frac{\beta_2 \gamma_t^M}{1 + \beta_2 \gamma_t^M} \right).\end{aligned}$$

2.6 The nonlinear system of equations

This section lists the model equations for simple feedback rules in a MER regime.

Interest rate feedback rule:

$$\frac{1 + i_t}{1 + i} = \left(\frac{1 + i_{t-1}}{1 + i} \right)^{h_0} \left(\frac{\pi_t^C}{\pi_t^T} \right)^{h_1} \left(\frac{Y_t}{Y} \right)^{h_2} \left(\frac{e_t}{e} \right)^{h_3} \quad (71)$$

Nominal depreciation feedback rule:

$$\frac{\delta_t}{\delta} = \left(\frac{\delta_{t-1}}{\delta} \right)^{k_0} \left(\frac{\pi_t^C}{\pi_t^T} \right)^{k_1} \left(\frac{Y_t}{Y} \right)^{k_2} \left(\frac{e_t}{e} \right)^{k_3} \left(\frac{\gamma_t^R}{\gamma^R} \right)^{k_4} \quad (72)$$

Consumption:

$$\frac{C_t^{-\sigma^C}}{\varphi_{M,t}} = \beta (1 + i_t) E_t \left(\frac{C_{t+1}^{-\sigma^C}}{\varphi_{M,t+1}} \frac{1}{\pi_{t+1}^C} \right)$$

Risk-adjusted UIP:

$$1 + i_t = (1 + i_t^*) \phi_t^* \varphi_{D,t} E_t \delta_{t+1} \quad (73)$$

Phillips equations:

$$\begin{aligned}\Gamma_t &= \frac{Q_t}{p_t^C C_t^{\sigma^C}} + \beta \alpha E_t \pi_{t+1}^{\theta-1} \Gamma_{t+1} \\ \Psi_t &= \frac{\theta}{\theta - 1} \frac{Q_t}{p_t^C C_t^{\sigma^C}} m c_t + \beta \alpha E_t \pi_{t+1}^\theta \Psi_{t+1} \\ \Psi_t &= \left(\frac{1 - \alpha \pi_t^{\theta-1}}{1 - \alpha} \right)^{\frac{-1}{\theta-1}} \Gamma_t\end{aligned}$$

Price dispersion:

$$\Delta_t = \alpha \pi_t^\theta \Delta_{t-1} + (1 - \alpha) \left(\frac{1 - \alpha \pi_t^{\theta-1}}{1 - \alpha} \right)^{\frac{\theta}{\theta-1}}$$

Exports:

$$X_t = \kappa_X (e_t p_t^*)^{b_X} Y_t$$

Trade Balance:

$$TB_t = \frac{1}{a_D e_t} \left[(p_t^C)^{1-\theta^C} X_t - (1 - a_D) e_t^{1-\theta^C} Y_t \right]$$

Current Account:

$$CA_t = \left(\frac{1 + i_{t-1}^*}{\pi_t^*} - 1 \right) r_{t-1} - \left(\frac{1 + i_{t-1}^*}{\pi_t^*} \phi_{t-1}^* \tau_{D,t-1} - 1 \right) d_{t-1} + TB_t.$$

Balance of Payments:

$$r_t - d_t = CA_t + r_{t-1} - d_{t-1}$$

Real marginal cost:

$$mc_t = \frac{w_t}{\varepsilon_t}$$

Labor market clearing:

$$w_t = \xi^N p_t^C C_t^{\sigma^C} \varphi_{M,t} N_t^{\sigma^N}$$

Hours worked:

$$N_t = \frac{Q_t}{\varepsilon_t} \Delta_t$$

Domestic goods market clearing:

$$Q_t = Y_t - (1 - b^A) X_t$$

GDP:

$$Y_t = a_D \tau_{M,t} G_t (p_t^C)^{\theta^C} C_t + X_t$$

Consumption relative price:

$$p_t^C = \left(a_D + (1 - a_D) e_t^{1-\theta^C} \right)^{\frac{1}{1-\theta^C}}$$

Money market clearing:

$$m_t = \frac{1}{\beta_2} \left[\left(\frac{\beta_1 \beta_2 \beta_3}{1 - \frac{1}{1+i_t}} \right)^{\frac{1}{\beta_3+1}} - 1 \right] p_t^C C_t,$$

CB balance sheet:

$$b_t = e_t r_t - m_t$$

Tax collection:

$$tax_t = (G_t - 1) \tau_{M,t} p_t^C C_t - qf_t$$

Quasi-fiscal surplus:

$$qf_t = (1 + i_{t-1}^* - 1/\delta_t) \frac{e_t r_{t-1}}{\pi_t^*} - ((1 + i_{t-1}) - 1) \frac{b_{t-1}}{\pi_t}$$

Identities:

$$\frac{e_t}{e_{t-1}} = \frac{\delta_t \pi_t^*}{\pi_t}, \quad \frac{p_t^C}{p_{t-1}^C} = \frac{\pi_t^C}{\pi_t}, \quad \frac{p_t^*}{p_{t-1}^*} = \frac{\pi_t^{*X}}{\pi_t^*} \quad (76)$$

Great ratios:

$$\gamma_t^D = \frac{e_t d_t}{Y_t}, \quad \gamma_t^M = \frac{m_t}{p_t^C C_t}, \quad \gamma_t^R = \frac{e_t r_t}{Y_t},$$

Auxiliary functions:

$$\begin{aligned}\tau_{D,t} &= 1 + \frac{\alpha_1}{1 - \alpha_2 \gamma_t^D}, & \varphi_{D,t} &= 1 + (\tau_{D,t} - 1) \left(1 + \frac{\alpha_2 \gamma_t^D}{1 - \alpha_2 \gamma_t^D} \right) \\ \tau_{M,t} &= 1 + \frac{\beta_1}{(1 + \beta_2 \gamma_t^M)^{\beta_3}}, & \varphi_{M,t} &= 1 + (\tau_{M,t} - 1) \left(1 + \beta_3 \frac{\beta_2 \gamma_t^M}{1 + \beta_2 \gamma_t^M} \right).\end{aligned}$$

Notice that b_t nor r_t are not constrained to be non-negative, which may be quite unrealistic. Negative international reserves would mean borrowing from abroad and, in the context of this model, would require a risk premium as in the case of households. And many CBs are institutionally constrained in lending to the non-financial private sector, making b_t non-negative. Here, it is assumed that the CB's target for reserves γ^R is sufficiently high and the household's steady state demand for cash is sufficiently low to ensure that these non-negativity constraints hold for all t and all relevant stochastic shocks.¹⁶

In addition to these equations there are those that are subject to stochastic shocks, most of which are simple AR(1) processes. The external terms of trade (XTT) is a particularly important external effect for most SOE's. This justified giving the calibration of its components a careful treatment. As a working hypothesis, it was assumed that the inflation rates for imported and exported goods are interrelated in such a way that a shock to one of them affects the other through the dynamics of the XTT (which is the ratio of the two corresponding foreign price levels). Hence, the following equations are assumed:

$$\begin{aligned}\pi_t^{*X} &= (\pi_{t-1}^{*X})^{\rho^{\pi^{*X}}} (\pi^*)^{1-\rho^{\pi^{*X}}} (p_{t-1}^*)^{\alpha_{\pi^{*X}}} \exp(\sigma^{\pi^{*X}} \varepsilon_t^{\pi^{*X}}), & (77) \\ \pi_t^* &= (\pi_{t-1}^*)^{\rho^{\pi^*}} (\pi^*)^{1-\rho^{\pi^*}} (p_{t-1}^*)^{\alpha_{\pi^*}} \exp(\sigma^{\pi^*} \varepsilon_t^{\pi^*}), \\ p_t^* &= p_{t-1}^* \frac{\pi_t^{*X}}{(\pi_t^*)^{\beta_{\pi^*}}}.\end{aligned}$$

Notice that if the two price indexes are non-stationary, this implies that they are cointegrated. The XTT variable p_t^* plays the role of a cointegration error term,

¹⁶In the parent model ARGEM (Escudé 2007), it is banks that invest in domestic currency bonds and usually Central Banks do have the institutional ability to assist banks, though usually with limitations.

$\alpha_{\pi^{*X}} \leq 0$, $\alpha_{\pi^*} > 0$ are the speeds of adjustment and $(1, -\beta_{\pi^*})$ plays the role of a cointegrating vector, with $\beta_{\pi^*} = 1$ as in the last identity in (76). In Appendix I, these equations are estimated using data for Argentina and evidence is found for the cointegration hypothesis with an additional influence of π_{t-1}^{*X} on π_t^* , as in the equation below. The equations subject to stochastic shocks are hence the following (where the NSS values ε , π^* , π^{*X} are assumed equal to one):

Productivity shock:

$$\varepsilon_t = (\varepsilon_{t-1})^{\rho^\varepsilon} \exp(\sigma^\varepsilon \varepsilon_t^\varepsilon)$$

Government expenditure shock:

$$G_t = (G_{t-1})^{\rho^G} G^{1-\rho^G} \exp(\sigma^G \varepsilon_t^G)$$

Riskfree interest rate shock:

$$1 + i_t^* = (1 + i_{t-1}^*)^{\rho^{i^*}} (1 + i^*)^{1-\rho^{i^*}} \exp(\sigma^{i^*} \varepsilon_t^{i^*})$$

Financing risk/liquidity shock:

$$\phi_t^* = (\phi_{t-1}^*)^{\rho^{\phi^*}} (\phi^*)^{1-\rho^{\phi^*}} \exp(\sigma^{\phi^*} \varepsilon_t^{\phi^*})$$

Exports inflation shock:

$$\pi_t^{*X} = (\pi_{t-1}^{*X})^{\rho^{\pi^{*X}}} (p_{t-1}^*)^{\alpha_{\pi^{*X}}} \exp(\sigma^{\pi^{*X}} \varepsilon_t^{\pi^{*X}})$$

Imported inflation shock:

$$\pi_t^* = (\pi_{t-1}^*)^{\rho^{\pi^*}} (p_{t-1}^*)^{\alpha_{\pi^*}} (\pi_{t-1}^{*X})^{\rho^{\pi^{*XN}}} \exp(\sigma^{\pi^*} \varepsilon_t^{\pi^*}).$$

3 Numerical solution in Dynare

A detailed calibration of the parameters and derivation of the NSS values of the endogenous variables can be found in Appendix 1. This section studies the stabilizing role of the two policy rules under the different monetary and exchange

rate regimes, mainly by studying the volatilities (standard deviations) of the main endogenous variables in the model. The policy parameter ranges that guarantee the Blanchard-Kahn (BK) stability conditions are also explored. Table 1 summarizes the calibrated values of the main model parameters, and compares them with parameter values used in two other relevant SOE models.

Table 1: Calibrated values of main model parameters

Parameters		Values	G-M	De P
β	Intertemporal discount factor	0.99	0.99	0.99
σ^C	Relative risk aversion for goods	1.5	1	1
σ^N	Relative risk aversion for labor	0.5	3	0.47
α	Probability of not adjusting price	0.66	0.75	0.66
θ	E. of substitution between domestic goods	6	6	10
θ^C	E. of substitution, domestic vs. imported goods	1.5	1	3
a_D	Coef. for share of domestic goods	0.86	0.6	0.6
b^A	Coef. in production function for commodities	0.5	1	
$\varepsilon_{\bar{\tau}_D}$	E. of net risk function $\bar{\tau}_D(ed/Y)$	10		
$\varepsilon_{\mathcal{L}}$	E. of inverse cash velocity $\mathcal{L}(1+i)$	1.02		
E.: Elasticity; G_M='Gali and Monacelli (2002)'; De P='De Paoli (2006)'.				

The standard errors and persistence parameters used for the six shock variables are given in Table 2. They were calibrated taking into account the available time series for Argentina and the RW during the period 1994.1–2009.2: public consumption to GDP (σ^G, ρ^G), imported and exported goods inflation as they conform Argentina's XTT ($\sigma^{\pi^*}, \sigma^{\pi^{*X}}, \rho^{\pi^*}, \rho^{\pi^{*X}}, \rho^{\pi^{*XN}}$), Libor 3 months (σ^{i^*}, ρ^{i^*}), and balance of payments information on private sector foreign debts and interest payments for the calculation of the spread over Libor 3 months ($\sigma^{\phi^*}, \rho^{\phi^*}$). The standard deviations were not always taken exactly according to the data. Some were calibrated using both the data and the theoretical standard deviation and variance decomposition for GDP resulting from a baseline calibration of the two policy rules ($h_1 = 0.8, h_2 = 0.8, k_4 = -0.8$, and the rest of the coefficients zero). This implied diminishing the observed standard deviation of G (from 0.054 in a simple AR(1) estimation, from which the persistence parameter ρ^G was used),

which seemed to weigh too heavily in the volatility of Y , and increasing the standard deviation of ϕ^* (from 0.0034), which seemed not to weigh enough. The value of σ^ε was chosen so that the resulting theoretical standard deviation of Y was similar to the data for detrended and s.a. GDP for Argentina leaving out the crisis years 2001/2002.

Table 2: Calibration of shock variables

Standard deviations						
σ^ε	σ^G	σ^{i^*}	σ^{ϕ^*}	σ^{π^*}	$\sigma^{\pi^{*X}}$	
0.01	0.03	0.0046	0.05	0.0295	0.0424	
Persistence						
ρ^ε	ρ^G	ρ^{i^*}	ρ^{ϕ^*}	ρ^{π^*}	$\rho^{\pi^{*X}}$	$\rho^{\pi^{*XN}}$
0.8	0.85	0.7	0.3	0.2	0.41	0.18
Speeds of adjustment						
				α_{π^*}	$\alpha_{\pi^{*X}}$	
				0.181	-0.255	

3.1 Basic Blanchard-Kahn stability analysis

First some of the general stability properties of the model are studied in relation to the parameters of the two simple policy rules in the MER regime. The coefficients on the policy rules not explicitly mentioned below are made equal to zero. When a particular configuration of parameters are said to give stability it means that all the requirements for determinacy and non-explosiveness are met, including the rank condition and absence of unit roots.

Because the present model with 2 simple policy rules (or MER regime) is a generalization of the standard DSGE New Keynesian model, it is of some interest to explore how it stands in relation to the most characteristic stability requirement for the standard model: the Taylor Principle (in the generalization of Woodford 2003, Proposition 4.4), which states that Blanchard-Kahn (BK) stability requires that the sum of the inertial and inflation coefficients in the interest rate feedback rule be greater than one ($h_0 + h_1 > 1$). Obtaining necessary and sufficient conditions for BK stability is too complex, given the number of parameters and generalized

eigenvalues in the model. But the following observations concerning BK stability under the MER regime gives some idea of the richness of the model.

1) It is not necessary that any of the 4 k_i be different from zero to have BK stability. Notice that when all 4 of the k_i are zero, the second policy rule in log deviation form is $\widehat{\delta}_t = 0$, which means that the operating target for the rate of nominal depreciation is its NSS value. In this case, two alternative sufficient (additional) conditions for BK stability are A) all h_i except h_3 are zero and h_3 is positive (at any level), and B) all h_i except h_2 are zero and h_2 is positive (at any level). Hence, two viable policy regimes are $\widehat{\delta}_t = 0$ and either $\widehat{i}_t = h_3 \widehat{e}_t$ (with $h_3 > 0$), or $\widehat{i}_t = h_3 \widehat{Y}_t$ (with $h_2 > 0$). In particular, this shows that the Taylor Principle is not necessary for BK stability.

In the PER case (where the Taylor rule is substituted by $b_t = b$) there is also BK stability when all the coefficients are zero ($k_j = 0, j = 0, 1, 2, 3, 4$). In this case the policy rule implies intervening in the FX market sufficiently to keep the nominal exchange crawling at the NSS rate δ^T , but otherwise letting the economy run its course, and not responding to international reserves (since they nevertheless return to their NSS ratio to GDP). More generally, in the PER regime BK stability is obtained if $k_j = 0, j = 0, 1, 2, 3$, and $k_4 \in [-1.6, 1.6]$, which includes the previous case but also allows for explicitly responding to gaps in the CB reserves ratio.

2) Going back to the MER regime, it is not necessary that any of the 3 h_i be different from zero to have BK stability. For sufficiently small positive values of k_4 , a policy regime in which all the rest of the coefficients are zero is feasible. Hence, a policy regime where $\widehat{i}_t = 0$ and $\widehat{\delta}_t = k_4 (\widehat{e}_t + \widehat{r}_t - \widehat{Y}_t)$ with $k_4 \in [0.00001, 0.0073]$ is viable. In such a regime, the operating target for the interest rate is its NSS value and the operational target for the rate of currency depreciation is a small fraction of the deviation of CB reserves ratio (relative to GDP) from the long run target. This again shows that the Taylor Principle is not necessary for BK stability.

Perhaps even more surprising is that the Taylor Principle does not even hold in the FER regime. For example, (with $r_t = r$) the two alternative policy rules defined by $h_3 = h_4 = 0$, and either $h_0 = -2, h_1 = 1$ or $h_0 = 0.8, h_1 = -10$ are both feasible. If neither of h_0 and h_1 is negative, however, then their sum must be greater than one (Taylor Principle). But if it is not true that $h_3 = h_4 = 0$, then even a rule where all the coefficients are negative may be feasible. For example,

the following policy rule satisfies the BK conditions in the FER regime:

$$\widehat{i}_t = -0.5\widehat{i}_{t-1} - 0.5\widehat{\pi}_t^C - 0.5\widehat{Y}_t - 0.5\widehat{e}_t.$$

Before advancing any further, a few observations related to the previous points are worthwhile. First, if in the MER regime all 9 policy rule coefficients are zero the model generates a unit root and hence is not BK stable. Second, if (as in 1) above) all 4 of the k_i are zero, there is a very active exchange rate policy: the CB is permanently intervening in the FX market to make the exchange rate crawl at the long run rate. In contrast, under the FER regime the CB lets the exchange rate float, not intervening in the exchange market at all and hence keeping its international reserves constant. Third, if (as in 2) above) all 3 of the h_i are zero, there is a very active interest rate policy: the CB is permanently intervening in the domestic currency bond market to keep the interest rate at the long run level. In contrast, under the PER (or FIR) regime the CB lets the interest rate float, not intervening in the bond market at all and hence maintaining its stock of domestic currency bonds constant. Notice that in the standard New Keynesian model a FIR regime would never be feasible due to the validity of the Taylor Principle.

3) If in the MER regime all the k_i except k_4 are zero and all the h_i except h_1 and h_2 are zero, then sufficient (additional) conditions for BK stability are that either a) $k_4 < 0$ and $h_0 + h_1 > 1$ or b) $k_4 > 0$ and $h_0 + h_1 < 1$. Notice that in the second case the Taylor Principle is turned on its head. For example, the following sets of policy rules are BK stable:

$$\widehat{i}_t = 0.5\widehat{i}_{t-1} + 0.51\widehat{\pi}_t^C \quad \text{and} \quad \widehat{\delta}_t = -0.001 \left(\widehat{e}_t + \widehat{r}_t - \widehat{Y}_t \right)$$

$$\widehat{i}_t = 0.5\widehat{i}_{t-1} + 0.49\widehat{\pi}_t^C \quad \text{and} \quad \widehat{\delta}_t = 0.001 \left(\widehat{e}_t + \widehat{r}_t - \widehat{Y}_t \right)$$

but neither of the following are:

$$\widehat{i}_t = 0.5\widehat{i}_{t-1} + 0.51\widehat{\pi}_t^C \quad \text{and} \quad \widehat{\delta}_t = 0.001 \left(\widehat{e}_t + \widehat{r}_t - \widehat{Y}_t \right)$$

$$\widehat{i}_t = 0.5\widehat{i}_{t-1} + 0.49\widehat{\pi}_t^C \quad \text{and} \quad \widehat{\delta}_t = -0.001 \left(\widehat{e}_t + \widehat{r}_t - \widehat{Y}_t \right).$$

4) The sign of k_4 plays a complex role in BK stability which is not always intuitive. If $k_0 = k_1 = k_2 = k_3 = 0$ and $k_4 < 0$, whenever there are insufficient

reserves ($e_t r_t / Y_t < \gamma^R$ and hence) $\widehat{e}_t + \widehat{r}_t - \widehat{Y}_t < 0$, the CB depreciates the currency at a rate greater than in the NSS:

$$\widehat{\delta}_t = \log \left(\frac{\delta_t}{\delta} \right) = k_4 \left(\widehat{e}_t + \widehat{r}_t - \widehat{Y}_t \right) > 0.$$

Since a purchase of reserves (increase in r_t) expands the money supply (*ceteris paribus*) one tends to associate it with a currency depreciation. But things are more complex here. First, it is the ratio between the real domestic value of reserves ($e_t r_t$) and GDP that must increase if initially $e_t r_t / Y_t < \gamma^R$. Second, that increase must take place in the long run, so the direction of movement may be the opposite during a transition period. In fact, in Section 3.3 below (in the context of optimal simple rules) a positive k_4 is at times optimal.

5) To get a feeling for the range within each coefficient can vary without impairing BK stability, a baseline calibration for the coefficients in the two policy feedback rules is defined and the intervals within which each of the coefficients can be moved individually (leaving the rest at the baseline value) without impairing stability are found. The search is restricted to two decimal points accuracy and only checked for parameter values at most 10 in absolute value. The following is the baseline calibration for this exercise:

Baseline calibration								
h_0	h_1	h_2	h_3	k_0	k_1	k_2	k_3	k_4
0.8	0.8	0.0	0.0	0.0	0.0	0.0	0.0	-0.8

The results for the three policy regimes are shown in the Table 3. Starting with the MER regime, both of the inertial coefficient intervals of stability are quite wide, both going into high superinertial levels (of 10). Because unity is included in the feasible intervals for h_0 and k_0 , one or both of the simple policy rules can be implemented as the feedback response of the first difference (in the interest rate or the depreciation rate, respectively) to the various arguments on the r.h.s. In the case of the interest rate rule, there are no upper bounds for the reactions to inflation or the RER, but, perhaps surprisingly, there is an upper bound of only 1.04 for the response to GDP. There is much more room for diminishing the interest rate (up to -3.03) when GDP is high. In the case of the nominal depreciation rule, there

are no upper or lower bounds for the reactions to inflation, GDP, or the RER. In the case of k_4 , the only restriction is that it must be outside of a small interval around zero. The fact that there is a relatively low upper bound for the interest rate response to GDP while there is no bound for the nominal depreciation response to the same variable is interesting, since the stabilization of GDP is, of course, of primary interest in most CBs (along with the stabilization of inflation). The wide negative intervals for h_0 and h_1 are also very interesting, since they invalidate the Taylor principle. For example a regime that combines either $\hat{i}_t = -5\hat{i}_{t-1} + 0.8\hat{\pi}_t^C$ or $\hat{i}_t = 0.8\hat{i}_{t-1} - 9\hat{\pi}_t^C$ with $\hat{\delta}_t = -0.8(\hat{e}_t + \hat{r}_t - \hat{Y}_t)$ is BK stable.

The FER regime shows stability ranges that are very similar to those of the first policy rule of the MER regime. There is a narrowing of the negative range in the case of h_3 . But again those wide negative intervals for h_0 and h_1 that invalidate the Taylor Principle show up. The narrowing of the range of stability is more significant in the case of the PER regime, especially in the cases of the positive and negative ranges for k_0 , k_2 and k_4 and the positive range for k_3 . On the other hand, in contrast to the MER regime, in the PER regime the stability range for k_4 includes 0.

Table 3: Stability ranges for individual coefficients of policy rules

	MER	FER	PER
Interest rate rule			
$h_0 \in$	$[-10, -1.17] \cup [0.21, 10]$	$[-10, -1.16] \cup [0.21, 10]$	
$h_1 \in$	$[-10, -8.71] \cup [0.21, 10]$	$[-10, -8.65] \cup [0.21, 10]$	
$h_2 \in$	$[-3.03, 1.04]$	$[-3.02, 1.03]$	
$h_3 \in$	$[-4.16, 10]$	$[-1.97, 10]$	
Nominal depreciation rule			
$k_0 \in$	$[-10, 10]$		$[-1.18, 0.67]$
$k_1 \in$	$[-10, 10]$		$[-10, 10]$
$k_2 \in$	$[-10, 10]$		$[-1.16, 1.80]$
$k_3 \in$	$[-10, 10]$		$[-10, 2.47]$
$k_4 \in$	$[-10, -0.01] \cup [0.01, 10]$		$[-1.61, 4.34]$

3.2 Effects of policy coefficients on the volatility of selected variables

This subsection focuses on the MER regime and looks into the effects of changes in some of the simple policy rule coefficients on the volatilities (standard deviations) of the main endogenous variables, leaving the rest of the policy rule coefficients at the baseline values used in the previous subsection.¹⁷ The results are in Table 4, where the minimum value in each row is highlighted in bold and the maximum is in italics. The ratio between the maximum and minimum volatility is also shown in the last column. Below the horizontal line in each panel are the volatilities of the operational targets and their corresponding instruments. In 3 of the panels, the maximum volatilities tend to be in the extremes while the minimum volatilities are more scattered.

In the first panel are shown the volatilities for increasing values of h_1 . It is clear that increases in the interest rate response to higher than desired inflation (h_1) monotonically reduce the volatility of inflation (piC in the Dynare file) and utility. But while the effect on the volatility of inflation is strong (the max/min ratio is 2.31), the effect on utility is weak (the max/min ratio is merely 1.06). The effect on the RER and GDP, however, are not monotonous. While the volatility of GDP first falls and then (after $h_1 = 2$) rises, the volatility of the RER first falls, jumps upward to its maximum level at $h_1 = 2$, and then starts to fall again. Clearly, the effects of the individual coefficients on the volatilities of the usual target variables is quite complex. Although attention is usually focused on the volatility of Y , it is C and N that enter the aggregate utility of households. The table shows that the volatilities of these two variables respond quite differently to increases in h_1 . While both C and N have their minimum in the interior of the interval ($h_1 = 2$ in the case of C and $h_1 = 3$ in the case of N), the volatility of period utility (Utility) steadily diminishes as h_1 increases. Looking at the intermediate targets and instruments, while the volatility of i (ii in the Dynare file) is monotonically increasing, the volatility of the second operational target δ ($delta$ in the Dynare file) is steadily decreasing. Furthermore, the volatility of the variables that the CB actually uses as instruments on a day by day basis, b and r , both diminish monotonically. The volatility of b varies in the opposite direction to its corresponding target variable

¹⁷Amato and Laubach (2003) do a similar analysis for the case of sticky prices and wages when only an interest rate rule is used.

i as h_1 increases. To achieve a substantial reduction in the volatility of inflation, h_1 increases from 0.6 to 5, which also increases the volatility of the operational target i (from 0.009 to 0.03). In this process, the volatility of the instrument used (b) actually diminishes somewhat (from 0.0196 to 0.0156), while the volatilities of δ and r both decrease.

The second block of Table 4 shows a similar exercise except that it is h_0 that increases. The volatilities of π^C , Y , e , N , and *Utility* are highest for the lowest value of h_0 , but while the volatilities of π^C and N fall to a minimum and then start increasing, those of Y , e , and *Utility* fall monotonically. On the other hand, the volatility of C increases steadily with h_0 . As to the intermediate targets and the instruments, increases in the ‘inertial’ coefficient h_0 have the effect of strongly reducing the volatility of i (the max/min ratio is 4.45), while the volatility of the corresponding instrument b hardly changes. Increases in h_0 also have the effect of diminishing the volatilities of the remaining operational target variable δ and its corresponding instrument r .

The third block of Table 4 shows the effects of gradual increases in the speed k_4 with which the CB seeks to attain its long run target for international reserves through its nominal depreciation response, starting from a negative level (-0.1) and going up to positive levels (0.1). This case has the peculiarity that all the maximum volatilities are achieved in the middle ground (either $k_4 = -0.01$ or $k_4 = 0.01$). The minimum volatilities, however, are reached at different levels, both negative (π^C , Y , N) and positive (e , C , and *Utility*). Both operational targets reach minimum volatility for $k_4 = -0.01$ and maximum volatility for $k_4 = 0.01$. As k_4 increases from -0.1 to -0.01 , a small reduction in the volatility of δ is achieved with an extremely large increase in the volatility of the corresponding instrument r (from 0.247 to 2.080). And while the volatility of i practically stays the same, that of b increases very substantially (from 0.140 to 1.234).

Table 4: Standard deviations of main variables for different values of h_1, h_0, k_4, k_2 .

Note: unless otherwise specified, $h_0=0.8, h_1=0.8, h_2=h_3=k_0=k_1=k_2=k_3=0$ and $k_4=-0.8$.										
VARIABLE	MEAN	STANDARD DEVIATIONS								
		VALUES OF h_1								
		$h_1=0.6$	$h_1=0.8$	$h_1=1.0$	$h_1=1.5$	$h_1=2$	$h_1=3$	$h_1=4$	$h_1=5$	max/min
piC	1.0150	0.0150	0.0141	0.0133	0.0117	0.0114	0.0087	0.0074	0.0065	2.31
Y	1.4430	0.0819	0.0803	0.0790	0.0769	0.0690	0.0749	0.0747	0.0748	1.19
e	0.5951	0.0480	0.0470	0.0461	0.0443	0.0488	0.0410	0.0398	0.0389	1.25
C	1.3108	0.0276	0.0295	0.0310	0.0342	0.0273	0.0402	0.0427	0.0446	1.63
N	1.3220	0.0660	0.0645	0.0635	0.0622	0.0664	0.0617	0.0621	0.0627	1.08
Utility	-2.2744	0.0543	0.0538	0.0535	0.0528	0.0533	0.0518	0.0514	0.0511	1.06
ii	1.0253	0.0091	0.0111	0.0130	0.0168	0.0256	0.0242	0.0274	0.0298	3.27
b	0.0722	0.0196	0.0192	0.0188	0.0180	0.0157	0.0165	0.0160	0.0156	1.26
delta	1.0150	0.0874	0.0846	0.0822	0.0772	0.0653	0.0677	0.0639	0.0610	1.43
r	0.3152	0.0489	0.0482	0.0477	0.0465	0.0460	0.0444	0.0435	0.0429	1.14
VALUES OF h_0										
		$h_0=0.4$	$h_0=0.6$	$h_0=0.8$	$h_0=1.0$	$h_0=1.5$	$h_0=2$	$h_0=3$	$h_0=4$	max/min
piC	1.0150	0.0172	0.0153	0.0141	0.0134	0.0130	0.0133	0.0142	0.0149	1.32
Y	1.4430	0.0839	0.0819	0.0803	0.0791	0.0775	0.0767	0.0761	0.0758	1.11
e	0.5951	0.0492	0.0480	0.0470	0.0462	0.0451	0.0446	0.0440	0.0437	1.13
C	1.3108	0.0265	0.0280	0.0295	0.0307	0.0331	0.0345	0.0362	0.0371	1.40
N	1.3220	0.0699	0.0665	0.0645	0.0633	0.0622	0.0620	0.0623	0.0626	1.13
Utility	-2.2744	0.0562	0.0547	0.0538	0.0532	0.0524	0.0520	0.0516	0.0514	1.09
ii	1.0253	0.0147	0.0126	0.0111	0.0099	0.0076	0.0061	0.0043	0.0033	4.45
b	0.0722	0.0192	0.0192	0.0192	0.0191	0.0190	0.0190	0.0189	0.0189	1.02
delta	1.0150	0.0883	0.0862	0.0846	0.0834	0.0816	0.0806	0.0797	0.0792	1.11
r	0.3152	0.0493	0.0488	0.0482	0.0478	0.0469	0.0465	0.0459	0.0456	1.08
VALUES OF k_4										
		$k_4=-0.1$	$k_4=-0.05$	$k_4=-0.02$	$k_4=-0.01$	$k_4=0.01$	$k_4=0.02$	$k_4=0.05$	$k_4=0.1$	max/min
piC	1.0150	0.0161	0.0158	0.0153	0.0148	0.0257	0.0187	0.0170	0.0167	1.74
Y	1.4430	0.0755	0.0755	0.0766	0.0833	0.0871	0.0777	0.0760	0.0757	1.15
e	0.5951	0.0437	0.0444	0.0474	0.0571	0.0505	0.0430	0.0427	0.0429	1.34
C	1.3108	0.0390	0.0394	0.0411	0.0455	0.0370	0.0371	0.0379	0.0382	1.23
N	1.3220	0.0632	0.0631	0.0630	0.0643	0.0695	0.0649	0.0639	0.0636	1.10
Utility	-2.2744	0.0513	0.0516	0.0528	0.0572	0.0543	0.0511	0.0509	0.0510	1.12
ii	1.0253	0.0017	0.0017	0.0017	0.0017	0.0025	0.0019	0.0018	0.0018	1.47
b	0.0722	0.1402	0.2762	0.6567	1.2339	2.5338	0.8741	0.3125	0.1532	18.07
delta	1.0150	0.0770	0.0753	0.0712	0.0668	0.1367	0.0942	0.0837	0.0811	2.05
r	0.3152	0.2465	0.4741	1.1119	2.0795	4.2369	1.4552	0.5138	0.2470	17.19
VALUES OF k_2										
		$k_2=-10.0$	$k_2=-8.0$	$k_2=-4.0$	$k_2=-2.0$	$k_2=0.0$	$k_2=2.0$	$k_2=4.0$	$k_2=6.0$	max/min
piC	1.0150	0.0161	0.0162	0.0162	0.0163	0.0163	0.0163	0.0164	0.0164	1.02
Y	1.4430	0.0730	0.0735	0.0745	0.0750	0.0755	0.0760	0.0765	0.0770	1.05
e	0.5951	0.0437	0.0436	0.0434	0.0433	0.0433	0.0433	0.0433	0.0433	1.01
C	1.3108	0.0381	0.0382	0.0384	0.0385	0.0386	0.0387	0.0389	0.0390	1.02
N	1.3220	0.0632	0.0632	0.0633	0.0634	0.0634	0.0634	0.0635	0.0635	1.005
Utility	-2.2744	0.0496	0.0499	0.0505	0.0508	0.0511	0.0515	0.0518	0.0521	1.05
ii	1.0253	0.0017	0.0017	0.0017	0.0017	0.0017	0.0018	0.0018	0.0018	1.06
b	0.0722	0.1141	0.0913	0.0458	0.0254	0.0189	0.0359	0.0590	0.0836	6.04
delta	1.0150	0.0775	0.0777	0.0782	0.0784	0.0787	0.0789	0.0792	0.0795	1.03
r	0.3152	0.1963	0.1590	0.0863	0.0559	0.0452	0.0658	0.1006	0.1398	4.34

The relatively narrow range of stability for the coefficient on the interest rate response to GDP deviations (h_2) in the MER case, along with the boundless range of stability for the corresponding coefficient in the second policy rule (k_2) naturally raises the question of the effects of the latter coefficient on the volatilities. The fourth block of Table 4 shows these effects. All of the variables shown reach minimum volatilities for non-positive values of k_2 . And for a number of very significant variables such as π^C , Y , C , N , and $Utility$, the minimum is reached for the most negative value shown ($k_2 = -10$). Although at this value the volatility of e is maximum, there is in fact very little difference between the minimum and maximum volatilities of this variable (the max/min ratio is 1.01). Hence, reducing k_2 from zero to -10 (which implies aggressively reducing the rate of nominal depreciation—or perhaps even appreciating the currency—when GDP is above its NSS level) has an effective but small (the max/min ratios are at most 1.05) stabilizing role for most of the variables of interest. Notice that this implies substantially increasing the volatility of both instruments and slightly increasing the volatility of δ .

This same exercise could be repeated for the remaining policy rule coefficients. However, this would be quite tedious and furthermore it would be extremely difficult to obtain clear cut conclusions. Hence, the following subsection follows a more systematic approach by assuming the CB has definite weights (preferences) for the volatilities of certain target variables.

3.3 Optimal simple rules

This subsection enquires what the optimal simple policy rules coefficients are when using an objective function that represents the CB's priorities with respect to the volatilities it wants to minimize. For this a quadratic (loss) function is defined. Dynare's 'osr' command invokes a search engine that must be initialized by giving initial numerical values for these coefficients. However, there is no guarantee that the coefficients found give a global optimum. Indeed, with different initial values the command often finds a different set of optimal coefficients and a different loss. The tables below were constructed using various sets of initial values for the

policy rule coefficients and always choosing the coefficients obtained that gave the lowest loss.¹⁸

Assuming that the CB minimizes a linear combination of the variances of target variables that policymakers are typically concerned about:

$$\operatorname{argmin}_{h_i, k_i} \{ \omega_\pi \operatorname{Var}(\pi_t^C) + \omega_Y \operatorname{Var}(Y_t) + \omega_e \operatorname{Var}(e_t) + \omega_{\Delta i} \operatorname{Var}(\Delta i_t) + \omega_{\Delta \delta} \operatorname{Var}(\Delta \delta_t) \}$$

Aside from the usual terms (with weights ω_π , ω_Y , $\omega_{\Delta i}$), this loss function also allows for CB preferences with respect to the variances of the RER and the changes in the rate of nominal depreciation (with weights ω_e , $\omega_{\Delta \delta}$). Table 5 defines four different CB styles (or preferences), A-D, according to the combinations of weights in each. All of them have the same weight (50) for the changes in each of the operational targets because the role of inertia in the operational targets is not of central concern here. Also, zeros have been avoided by giving a (relatively very small) weight of 1 to target variables of little importance for the style defined. Hence, it can be said loosely that in style A only inflation matters and in style B only GDP matters, whereas both matter equally in style C and in style D the RER matters as much as inflation and GDP.

Table 5: Definition of CB styles

Weights	Styles			
	A	B	C	D
ω_π	100	1	100	100
ω_Y	1	100	100	100
ω_e	1	1	1	100
$\omega_{\Delta i}$	50	50	50	50
$\omega_{\Delta \delta}$	50	50	50	50

Using Dynare’s ‘osr’ command, the optimal coefficients for the simple policy rules and corresponding losses for each of the CB styles and each policy regime are shown in Table 6.

¹⁸The possibility that a local but not global minimum has been found is one of the reasons for additionally using the optimal policy under commitment framework (in Section 3.4 below), where the optimum found is necessarily global and unique. More thorough searches for the optimal simple rules can be performed. The references given by one of the anonymous referees on looping over the parameter space are appreciated and may be used in a future paper.

Table 6: Optimal simple policy rules and losses for the 3 regimes and 4 CB styles

	OPTIMAL SIMPLE POLICY RULES											
	A			B			C			D		
	MER	FER	PER	MER	FER	PER	MER	FER	PER	MER	FER	PER
h_0	1.34	-0.34		1.22	0.87		6.74	-0.38		3.10	-0.37	
h_1	0.66	-2.66		-0.35	0.20		-1.31	-1.49		-0.76	-1.58	
h_2	-0.01	0.02		3.64	-1.39		-4.31	-0.99		-4.08	-0.93	
h_3	-0.01	-0.26		0.10	-0.29		0.21	0.11		-0.02	0.04	
k_0	-0.15		-0.61	1.91		-0.61	-0.02		-0.77	-0.23		-0.79
k_1	-0.02		-1.19	-0.74		-2.25	1.28		-5.02	1.31		-5.14
k_2	-0.13		-0.24	-2.61		-3.00	-0.80		-2.48	-1.43		-2.30
k_3	-0.15		-0.35	0.44		0.05	-0.24		0.06	-0.85		-0.24
k_4	-0.001		-0.02	0.001		1.30	-0.001		0.70	-0.003		0.75
	LOSS											
	0.013	0.224	0.233	0.027	0.552	0.710	0.149	0.771	0.788	0.203	0.899	0.915
	RELATIVE LOSS											
	1	17.12	17.80	1	20.79	26.72	1	5.16	5.27	1	4.44	4.51

Looking at the optimal coefficients, in the case of the MER regime the inertial coefficient for the interest rate (h_0) is greater than 1 (‘superinertial’) for all styles and the interest rate response to inflation deviations (h_1) is positive (and less than 1) for style A and negative for styles B, C, and D. In styles A, C, and D, $h_0 + h_1 > 1$ and $k_4 < 0$. However, in style B (where only GDP matters) the opposite signs hold (the Taylor Rule is ‘turned on its head’) and the interest rate response to GDP (h_2) is highly positive. In styles C and D, however, the optimal interest rate response to GDP is very negative: it is optimal to reduce the interest rate when there is a positive GDP gap. In both of these cases there is a very high superinertial coefficient (h_0) in the first policy rule. The interest rate response to the RER (h_3) is quite moderate in all 4 styles. In the case of the second policy rule of the MER regime, k_0 is small and negative except for style B, in which it is highly superinertial. The depreciation rate response to inflation is negative for styles A and B and positive and greater than one for styles C and D. And the depreciation rate response to GDP is negative in all 4 cases: it is optimal to respond to a positive GDP gap with a reduction in the rate of depreciation.

In the FER regime, the inequality $h_0 + h_1 > 1$ only holds in the case of style B, and both coefficients are negative in styles A, C, and D. The inertial coefficient is always below one in absolute value, in contrast with its equivalent in the MER regime. The response to GDP is markedly negative in 3 of the styles and is only

(slightly) positive for style A. The interest rate response to the RER is negative for styles A and B, and positive for styles C and D.

Finally, in the PER regime, k_0 , k_1 , and k_2 , are negative for all 4 styles, with the inertial coefficient k_0 between -0.6 and -0.8 and k_1 always greater than one in absolute value (and in the case of styles C and D, above 5). k_3 is negative for styles A and D. Finally, k_4 is positive for styles B, C, and D. Hence, under the PER regime, both high inflation and high GDP demand lowering the rate of nominal depreciation, and the previous' period rate of nominal depreciation affects the present rate negatively.

A caveat is that many of these observations on the sign and magnitude of the optimal simple rule coefficients are highly sensitive to parameter calibrations.¹⁹ Hence, there is no claim here of generality for the results obtained. The important point is that simple characteristics of the standard New Keynesian model to which we have been accustomed (such as the Taylor Rule) do not survive the type of model generalization introduced here, even in the case of the FER regime.

3.4 Optimal policy under commitment

In this section Dynare's 'ramsey' command is used to obtain the optimal policy under commitment, i.e., the policy functions that yield the minimum expected value (conditional on the information at $t = t_0$, including given initial conditions for the predetermined variables) of a discounted ad hoc loss function:

$$\mathcal{L}_{t_0} = E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \frac{1}{2} L_t, \quad (78)$$

subject to all the non-policy model equations, where the period loss function L_t is given by²⁰:

$$L_t = \omega_{\pi} (\pi_t^C - \pi^T)^2 + \omega_Y (Y_t - Y)^2 + \omega_e (e_t - e)^2 + \omega_r (r_t - r)^2 + \omega_{\Delta i} (\Delta i_t)^2 + \omega_{\Delta \delta} (\Delta \delta_t)^2, \quad (79)$$

¹⁹This is quite evident if one compares with the results of the 'osr' exercise in the Discussion Paper, where the calibrations differ only in the NSS value of the elasticity of the foreign debt premium (which is here much lower).

²⁰See Section 3.4 below for a discussion on the desideratum between using ad hoc versus utility-based loss functions.

It is assumed that policymakers have the same intertemporal discount factor as households ($\beta = 0.99$). The same definition of CB styles as in the previous section are maintained except that a small preference for r has been introduced (with $\omega_r = 1$) in all the CB styles. Otherwise, to obtain BK stability it would be necessary to increase the policymaker discount factor (say to 0.999). Table 7 shows the losses and relative losses for the alternative CB styles (A-D) and policy regimes (MER, FER, PER).

As expected, the MER regime always dominates the two ‘corner’ regimes. The PER regime ranks above the FER regime when only inflation matters (style A) but in the other 3 CB styles the FER regime has a lower relative loss than the PER regime. The ‘corner’ regimes have losses between 0.8% and 4.7% higher than in the MER regime. For the baseline $\varepsilon_{\tau_D} = 10$ used, the increase in loss for forfeiting one of the policy rules does not appear to be very high. However, in Section 4 it is shown that for higher elasticities and for alternative calibrations of other parameters this increase in cost may be very substantial.

Table 7: Losses under optimal policy under commitment

STYLE	LOSS			RELATIVE LOSS		
	MER	FER	PER	MER	FER	PER
A	119.9	121.0	120.9	1	1.009	1.008
B	112.0	114.5	117.3	1	1.023	1.047
C	378.1	388.1	388.8	1	1.027	1.028
D	394.5	405.2	405.7	1	1.027	1.028

Table 8: Reduced form policy functions with optimal policy under commitment

REGIMES: STYLES:	MER								FER				PER			
	A		B		C		D		A	B	C	D	A	B	C	D
	<i>ii</i>	<i>delta</i>	<i>ii</i>	<i>delta</i>	<i>ii</i>	<i>delta</i>	<i>ii</i>	<i>delta</i>	<i>ii</i>	<i>ii</i>	<i>ii</i>	<i>ii</i>	<i>delta</i>	<i>delta</i>	<i>delta</i>	<i>delta</i>
<i>ii</i> (-1)	0.275	0.022	0.495	0.373	0.184	0.031	0.181	0.033	0.265	0.493	0.176	0.173	0.021	0.375	0.027	0.029
<i>delta</i> (-1)	0.022	0.377	0.373	0.562	0.031	0.179	0.033	0.174	0.022	0.375	0.027	0.029	0.375	0.559	0.172	0.168
<i>r</i> (-1)	-0.031	-0.020	0.016	-0.017	-0.008	-0.042	-0.007	-0.038	-0.032	0.016	-0.009	-0.008	-0.020	-0.016	-0.042	-0.040
<i>e</i> (-1)	-0.355	-0.484	0.452	-0.432	-0.175	-1.143	-0.164	-1.163	-0.366	0.454	-0.199	-0.187	-0.485	-0.442	-1.173	-1.188
<i>d</i> (-1)	0.031	0.020	-0.016	0.017	0.008	0.042	0.007	0.038	0.032	-0.016	0.009	0.008	0.020	0.016	0.042	0.040
Δ (-1)	-0.001	-0.008	-0.017	0.014	0.024	0.058	0.024	0.058	-0.001	-0.017	0.025	0.025	-0.009	0.014	0.059	0.059
τ_D (-1)	0.038	0.024	-0.019	0.020	0.010	0.051	0.008	0.046	0.039	-0.019	0.011	0.009	0.024	0.020	0.051	0.048
p^C (-1)	0.189	0.432	0.009	0.011	0.154	0.249	0.155	0.247	0.198	0.009	0.156	0.157	0.435	0.011	0.254	0.252
p^* (-1)	-0.111	-0.121	0.161	-0.152	-0.022	-0.350	-0.027	-0.337	-0.114	0.162	-0.030	-0.035	-0.121	-0.155	-0.360	-0.348
z_{π} (-1)	-0.068	-0.095	0.042	-0.043	-0.051	-0.151	-0.051	-0.164	-0.070	0.042	-0.054	-0.054	-0.095	-0.044	-0.155	-0.165
z_{π} X(-1)	-0.075	-0.077	0.086	-0.083	-0.045	-0.225	-0.044	-0.206	-0.076	0.086	-0.050	-0.049	-0.076	-0.084	-0.229	-0.215
z_G (-1)	-0.030	-0.039	0.182	-0.152	-0.093	-0.463	-0.091	-0.442	-0.031	0.182	-0.105	-0.103	-0.039	-0.158	-0.476	-0.460
z_{ϵ} (-1)	0.003	0.026	0.059	-0.047	-0.107	-0.233	-0.108	-0.234	0.004	0.059	-0.113	-0.114	0.026	-0.049	-0.236	-0.236
z_i (-1)	0.573	0.144	-0.042	0.051	0.398	0.534	0.396	0.475	0.587	-0.041	0.419	0.419	0.144	0.054	0.550	0.505
z_{ϕ} (-1)	0.231	0.028	0.013	-0.007	0.169	0.164	0.170	0.148	0.237	0.014	0.177	0.179	0.028	-0.006	0.169	0.157
mult ₁₀ (-1)	0.000	0.004	0.004	0.006	0.000	0.002	0.000	0.002	0.000	0.004	0.000	0.000	0.004	0.006	0.002	0.002
mult ₁₇ (-1)	0.026	0.027	0.137	0.152	0.024	0.027	0.024	0.027	0.027	0.137	0.024	0.024	0.028	0.152	0.027	0.027
mult ₁₈ (-1)	0.036	0.042	0.179	0.200	0.032	0.036	0.032	0.036	0.037	0.179	0.032	0.032	0.042	0.200	0.037	0.037
mult ₂₄ (-1)	0.000	-0.003	-0.003	-0.004	0.000	-0.001	0.000	-0.001	-0.001	-0.003	0.000	0.000	-0.003	-0.004	-0.001	-0.001
eps _{epsilon}	0.004	0.032	0.074	-0.059	-0.134	-0.291	-0.135	-0.292	0.005	0.074	-0.141	-0.142	0.033	-0.061	-0.295	-0.295
eps _G	-0.035	-0.046	0.214	-0.179	-0.110	-0.544	-0.106	-0.520	-0.037	0.214	-0.123	-0.121	-0.046	-0.186	-0.560	-0.541
eps _i (-1)	0.779	0.180	-0.039	0.051	0.558	0.709	0.557	0.631	0.798	-0.038	0.586	0.588	0.180	0.057	0.732	0.671
eps _{phi} (-1)	-0.643	-0.013	-0.107	0.089	-0.530	-0.376	-0.540	-0.341	-0.661	-0.110	-0.554	-0.566	-0.012	0.086	-0.393	-0.363
eps _{pi} (-1)	-0.179	-0.249	0.111	-0.114	-0.133	-0.397	-0.135	-0.432	-0.185	0.112	-0.141	-0.141	-0.250	-0.117	-0.408	-0.434
eps _{pi} X(-1)	-0.182	-0.187	0.209	-0.202	-0.111	-0.549	-0.108	-0.504	-0.187	0.210	-0.122	-0.120	-0.187	-0.206	-0.560	-0.524

Table 8 shows the coefficients of the policy functions in the reduced form (or ‘solution’) that correspond to the instrument variables (in the sense of optimal control theory), i.e., the operational targets (in the economic sense) of the three alternative regimes. These variables²¹ are linear functions of the 9 non-shock predetermined variables (i , δ , r , e , d , Δ , τ_D , p^C , p^*), the 6 shock variables, and the Lagrange multipliers corresponding to the 4 equations with forward-looking terms (the UIP equation, the two dynamic Phillips equations, and the consumption Euler equation).²² In all of the CB styles there is substantial inertia in the interest rate policy function and in the nominal depreciation policy function. This is hardly

²¹Notice that the variables are shown as they appear in the Dynare output. However, it is necessary to ‘read’ the variables (contemporaneous or lagged) as their log-linear deviations with respect to their NSS values.

²²The real interest rate has been eliminated from the model for the construction of this table to avoid having an additional and unnecessary Lagrange multiplier variable.

surprising since all these CB styles have been defined to show a significant preference for policy inertia. What is perhaps more surprising is the dispersion in the inertial coefficients, given that they all have the same weight for preference for inertia ($\omega_{\Delta i} = \omega_{\Delta \delta} = 50$). The coefficients on the Lagrange multipliers are relatively small, implying that the policy function coefficients (for the rest of the variables) do not vary much from quarter to quarter when these effects are cumulated (attributable to the commitment to never again re-optimize).

Table 9 shows the variance decomposition for the 4 CB styles in the case of the MER regime. The corresponding variance decompositions for the FER and PER regimes are very similar and hence not shown. The table shows that the substantial shocks in explaining the variances of the target and operational target variables are G , ϕ^* , π^* , and π^{*X} . This is not surprising considering the assumed standard errors for the shocks (which are lowest for ε and i^*). The shock to export price inflation explains around 50% of the variance of the RER for all CB styles. While 65% of the variance of inflation is explained by the exogenous risk/liquidity shock ϕ^* when inflation is the CB priority (style A), this drops to around 17% in styles C and D, where inflation is equally important as GDP or both GDP and the RER. This shock also explains as much as 83% of the variance of the nominal interest rate under style A but only 29% under style B. Almost 60% of the variance of GDP is explained by this shock when the CB gives priority to stabilizing GDP, but only around 35% for the other CB styles. The shock to G has its highest relative effect on inflation under styles C and D (around 65%) and also has high effect on GDP for styles A, C and D (around 40%). The shock to G hardly explains any of the variance of inflation under style A. Between 19% and 26% of the variance of GDP is explained by the shock to export price inflation π^{*X} in all the CB styles.

Table 9: Variance decomposition (MER; ‘ramsey’)

	eps_epsilon	eps_G	eps_iStar	eps_phiStar	eps_piStar	eps_piStarX
CB style A						
piC	0.26	0.27	1.36	64.87	19.42	13.82
Y	2.36	35.29	0.63	33.49	1.85	26.38
e	0.62	2.01	1.65	21.23	28.5	45.99
ii	0.09	0.38	1.66	82.77	3.86	11.25
delta	0.04	0.73	0.37	56.00	18.34	24.53
CB style B						
piC	1.57	29.51	0.75	44.04	5.53	18.61
Y	0.07	9.9	1.33	57.97	4.4	26.33
e	0.00	7.39	1.45	29.01	11.31	50.84
ii	2.07	48.16	0.32	29.14	3.21	17.11
delta	1.52	31.02	0.86	49.53	2.30	14.78
CB style C						
piC	4.76	63.67	0.39	17.13	5.06	8.99
Y	2.01	42.3	0.75	34.17	2.15	18.62
e	0.00	7.57	1.46	28.29	12.28	50.39
ii	1.84	44.39	0.62	47.63	1.66	3.85
delta	1.48	29.78	0.78	38.19	6.35	23.41
CB style D						
piC	4.88	65.43	0.40	16.86	4.91	7.54
Y	1.66	36.44	0.81	36.20	1.41	23.47
e	0.00	7.20	1.42	28.90	12.09	50.39
ii	1.82	43.95	0.67	49.43	1.37	2.77
delta	1.64	31.57	0.71	37.17	7.78	21.13

Table 10 explores to what extent the ranking of policy regimes depends on the shocks considered by eliminating 5 of the shocks and maintaining the same value for the standard error of the remaining shock. The table shows that the superiority of the MER regime is robust to any of the shocks taken separately. It also shows that with only one exception, the (ramsey-optimal) pure exchange rate float is superior to the (ramsey-optimal) pure peg (or interest rate pure float) for any of the individual shocks and CB preferences. The one exception is the case of the shock to the exogenous risk/liquidity premium ϕ^* under CB style A (in which only inflation matters). It is to be noted that when all the 6 shocks are used (as in Table 7) the pattern of this one exception is repeated and the relative losses are

very similar to those of the central set of columns of Table 10. This points to the importance of the exogenous risk/liquidity shock in the overall model.

Table 10: Losses for each individual shock

STYLE	LOSS														
	epsilon			G			phiStar			piStar			piStarX		
	MER	FER	PER	MER	FER	PER	MER	FER	PER	MER	FER	PER	MER	FER	PER
A	103.4	104.0	104.1	103.7	104.3	104.5	116.5	117.5	117.5	104.6	105.2	105.3	104.9	105.5	105.6
B	88.5	90.4	93.2	90.7	92.5	95.5	104.1	106.4	109.1	89.1	91.0	93.8	93.3	95.2	98.0
C	333.7	342.0	343.2	344.5	353.0	354.3	355.8	365.3	366.1	334.8	343.2	344.4	341.0	349.6	350.7
D	337.5	345.2	346.3	349.3	357.3	358.4	363.3	372.6	373.2	340.2	347.9	348.9	351.2	359.8	360.6
	RELATIVE LOSS														
A		1.006	1.007		1.006	1.007		1.009	1.008		1.006	1.007		1.006	1.007
B		1.021	1.053		1.021	1.053		1.023	1.049		1.021	1.053		1.021	1.051
C		1.025	1.029		1.025	1.028		1.027	1.029		1.025	1.029		1.025	1.028
D		1.023	1.026		1.023	1.026		1.026	1.027		1.022	1.026		1.024	1.027

3.5 Ad hoc vs. utility based loss functions

In this paper policymakers are assumed to want to reduce the volatility of (a weighted average of) certain target variables that are deemed to be important for the success of their stabilizing efforts. These variables, inflation, GDP, or the RER, are periodically measured by statistical agencies in most countries and their evolution is well publicized. Furthermore, most policymakers have an understanding of basic (more or less sophisticated) macroeconomic theory that links these variables in a unified framework. Any macroeconomic policy model will typically include the household decision problem in terms of a utility function that simply expresses in a mathematical way that people like to consume more and work less. Most macroeconomic policy models have tended to simplify reality in the dimension of household heterogeneity going to the extreme of postulating a ‘representative household’ and thus completely overlooking aspects of the policy process, such as the consequences of policy decisions on sectorial income and risk distribution, that are in fact considered important by households and firms.

An important strand of macro policy DSGE models, without going beyond the household homogeneity assumption, looks for the optimal policy that would deliver the highest expected household intertemporal utility. Without actually modeling policymakers as agents that have preferences and constraints, this research looks for the policy that would be followed if these policymakers were ‘benevolent’ or ‘altruistic’ in the sense that they unanimously decide to maximize the

utility of the representative household. A typical way of doing this is to obtain a second order approximation of the discounted intertemporal household utility, find a way of doing away with the first order terms to ensure that the resultant approximation adequately orders the losses obtained with different policy regimes, and use this, along with first order approximations of the non-policy model equations to implement optimal policy under commitment in a linear-quadratic optimal control framework. Doing this is a very tedious process when one has a moderately complex model, but it does enrich the analysis in a sense. Not only does the optimum policy reflect household utility maximization but also the loss function can be expressed in terms of welfare relevant gaps which depart from the simple gaps with respect to the NSS that are used in this paper. However, this enrichment is based on very strong assumptions that need not be universally accepted as true. First, the fact that the model used overlooks household heterogeneity is an important limitation. If instead of one class of households, the model had two classes that have sufficiently different sources of income or risk, then the ‘benevolence’ assumption loses meaning and some assumption has to be made as to what policymaker preferences are concerning the distributional consequences of their policy actions. Second, even if the household homogeneity assumption is maintained, it is at least controversial to assume that actual policymakers are not only homogeneous but also ‘benevolent’. Indeed, it is quite paradoxical that the combined assumptions that a) policytaking households care only for themselves and b) (non-modeled) policymakers care only for others, should exert such fascination.

The ad hoc loss function approach can be considered more general since, if one has adequate target values in the loss function, the loss function that would be obtained from, say, a second order approximation to household utility is a particular case, i.e., gives specific (utility-related) values to the exogenous weights used in the ad hoc procedure. One can argue that if a sufficiently varied set of exogenous weights were used, one of them would be close to the one that could be obtained through a second order approximation of utility. The ad hoc procedure is also more general in that it does not need to assume that policymakers are ‘benevolent’. Making monetary and exchange rate decisions is a complex process where many people intervene, with differing views with respect to the ‘correct’ model and the ‘preferred’ outcome of the policymaking. Usually, more than one

model is used in the process. One of the less defensible aspect of typical DSGE models is that they usually ignore distributive aspects by making households homogeneous or only heterogeneous in certain technical details. Distributive aspects are usually very important in policymaking, both in developed and less developed economies. The world economy has been recently hit by a crisis that many attribute to the lack of regulatory actions that could possibly have prevented the building up of bubbles in real estate sectors and financial system vulnerability to the risks posed by insufficiently understood derivatives. Monetary and exchange rate policy is an integral part of the political process in both developed and less developed economies. Unless a large amount of research effort is invested in trying to reflect household heterogeneity, not much is actually gained by obtaining the policies that maximize the welfare of a fictitious ‘representative household’ that is used in the model, presumably to avoid complexity. And if realistic household heterogeneity is actually reflected in the model there remains the fact that there is also policymaker heterogeneity (say, between different members of the central bank board or between the central bank and the treasury). Different participants in the decision process may have different preferences with respect to the outcomes of the policy decisions.

The principal objective of this paper is to show that using a relatively standard and simple SOE model in which there is an endogenous risk premium that affects the interest rate at which the private sector can borrow funds abroad there are significant gains from simultaneously using interest and exchange rate policies (instead of only one of them) no matter what the specific policymaker preferences are. Although various caveats have been stated above with respect to using a utility based loss function, it is nevertheless a useful complement of the approach followed here since it can be used to make conditional statements such as, "if policymakers were homogeneous, had a high degree of confidence in the appropriateness of the model used, and were only concerned with maximizing the welfare of the model’s representative household, they would ..." However, such research clearly goes beyond the scope of the present paper.

4 Monetary and exchange rate policy and capital flows in the SOE

4.1 The effectiveness of two simple policy rules in managing private capital flows for stabilization

It has been shown that for a broad set of policy preferences the CB can better achieve its goals by simultaneously using interest and exchange rate policies. It remains to be seen what aspects of the model account for this. This subsection assumes the CB uses simple policy rules and starts by conjecturing that the gain in using two policy rules is related to the CB's ability to influence, to a certain extent, households' foreign debt ratio. The latter determines the endogenous risk premium that foreign agents charge over the international interest rate and which, through the UIP equation, is a primary ingredient in determining the relation between the interest rate differential and the expected rate of nominal depreciation. The basic idea is that the corner regimes amount to forfeiting a part of the CB ability to affect this crucial relation. To substantiate this idea, consider the log-linear approximations of the UIP equation and the two simple policy rules equations under the MER regime:

$$\widehat{i}_t = E_t \widehat{\delta}_{t+1} + \widehat{i}_t^* + \widehat{\phi}_t^* + \varepsilon_{\varphi_D} \left(\widehat{d}_t + \widehat{e}_t - \widehat{Y}_t \right) \quad (80)$$

$$\widehat{i}_t = h_0 \widehat{i}_{t-1} + h_1 \widehat{\pi}_t^C + h_2 \widehat{Y}_t + h_3 \widehat{e}_t \quad (81)$$

$$\widehat{\delta}_t = k_0 \widehat{\delta}_{t-1} + k_1 \widehat{\pi}_t^C + k_2 \widehat{Y}_t + k_3 \widehat{e}_t + k_4 \left(\widehat{r}_t + \widehat{e}_t - \widehat{Y}_t \right). \quad (82)$$

Leading the third equation, subtracting the resulting equation from the second, and using the first, gives the following equation:

$$\begin{aligned} \widehat{i}_t^* + \widehat{\phi}_t^* + \varepsilon_{\varphi_D} \left(\widehat{d}_t + \widehat{e}_t - \widehat{Y}_t \right) &= \left(h_0 \widehat{i}_{t-1} - k_0 \widehat{\delta}_t \right) + \left(h_1 \widehat{\pi}_t^C - k_1 E_t \widehat{\pi}_{t+1}^C \right) \\ &+ \left(h_2 \widehat{Y}_t - k_2 E_t \widehat{Y}_{t+1} \right) + \left(h_3 \widehat{e}_t - k_3 E_t \widehat{e}_{t+1} \right) - k_4 \left(E_t \widehat{r}_{t+1} + E_t \widehat{e}_{t+1} - E_t \widehat{Y}_{t+1} \right). \end{aligned} \quad (83)$$

On the l.h.s. is the (log-linear deviation from the NSS of the) of the foreign currency riskless interest rate plus the risk premium in the UIP (with exogenous and endogenous components). On the r.h.s. is a complex term that exclusively depends on the log-linear deviations of the variables the CB uses for its simple policy rules

and the exogenous coefficients in the simple policy rules. Changes in the coefficients on the CB policy rules can thus modify a crucial relation between the households' foreign debt ratio and a linear combination of lagged, current and expected endogenous variables to which the CB responds. The policy coefficients thus have an important role in determining what households' foreign debt is in each period, given the values of the international interest rate and risk/liquidity premium ($\widehat{i}_t^* + \widehat{\phi}_t^*$), both exogenous. For example, when one of the latter is shocked, the policy coefficients help in determining the effects on the households' foreign debt and, hence, international capital flows. The constraints that the respective 'corner' regimes impose (i.e., the constancy of one of the potential CB instruments: either $b_t = b, \forall t$, or $r_t = r, \forall t$, each replacing one of the simple policy rules), imply that the CB has less leeway in affecting international capital flows in the direction that helps it stabilize the economy according to its preferences (or style).

Under the FER regime, in which (82) is replaced by $\widehat{r}_t = 0$, (83) is reduced to:

$$\widehat{i}_t^* + \widehat{\phi}_t^* + \varepsilon_{\varphi_D} \left(\widehat{d}_t + \widehat{e}_t - \widehat{Y}_t \right) = h_0 \widehat{i}_{t-1} + h_1 \widehat{\pi}_t^C + h_2 \widehat{Y}_t + h_3 \widehat{e}_t - E_t \widehat{\delta}_{t+1}$$

and under the PER regime, in which (81) is replaced by $\widehat{b}_t = 0$, it is reduced to:²³

$$\begin{aligned} \widehat{i}_t^* + \widehat{\phi}_t^* + \varepsilon_{\varphi_D} \left(\widehat{d}_t + \widehat{e}_t - \widehat{Y}_t \right) &= \widehat{i}_t - k_0 \widehat{\delta}_t - E_t \left[k_1 \widehat{\pi}_{t+1}^C + k_2 \widehat{Y}_{t+1} + k_3 \widehat{e}_{t+1} \right. \\ &\quad \left. + k_4 \left(\widehat{r}_{t+1} + \widehat{e}_{t+1} - \widehat{Y}_{t+1} \right) \right]. \end{aligned}$$

In both of the corner cases, the CB affects the foreign debt ratio through its interest rate or exchange rate policy, respectively. It therefore also affects the endogenous part of the risk/liquidity premium, and hence the (domestic) foreign currency interest rate that impinges on the economy. The flexibility that the CB achieves by using two simultaneous policy rules generates gains that, at least for the most usual CB styles, can be significant. Such gains have been measured above, in the context of this particular model and optimal simple rules, as the reductions in expected loss obtained from using the MER regime instead of any of the corner regimes. Although this argument is more clearly valid in the case of optimal simple

²³Notice that in the particular PER regime in which there is no feedback, the r.h.s. of this equation is simply $\widehat{i}_t - k_0 \widehat{\delta}_t$, and in the fixed exchange rate policy it reduces to \widehat{i}_t .

rules, in which the optimal coefficients in the simple policy rules are obtained, a similar argument is also valid for the Ramsey case, where the additional constraints (either $b_t = b, \forall t$, or $r_t = r, \forall t$) which the corner regimes impose also imply less leeway for CB optimal action.

To see if this conjecture can be validated (or refuted) the optimal policy under commitment framework is now used to study the sensitivity of the expected intertemporal loss to the elasticity of $\bar{\tau}_D$ ($\varepsilon_{\bar{\tau}_D}$).

Table 11 shows that, as conjectured, the magnitude of $\varepsilon_{\bar{\tau}_D}$ is very relevant in the determination of the excess loss which the two corner regimes generate. For each CB style, 1) the corner regimes imply higher losses than the MER regime; 2) the lower is $\varepsilon_{\bar{\tau}_D}$, the lower is the excess loss which the corner regimes imply and these are extremely low when $\varepsilon_{\bar{\tau}_D} = 0.1$, especially for the FER regime; 3) for $\varepsilon_{\bar{\tau}_D} = 100$ the corner regimes have losses 40-69% higher than the MER regime; 4) for all the values of $\varepsilon_{\bar{\tau}_D}$ shown, the FER regime achieves a lower loss than the PER regime for CB styles B, C, and D, but in the case of style A (in which only inflation matters), the PER regime is second best when $\varepsilon_{\bar{\tau}_D}$ is 10 or higher.

This exercise confirms the conjecture that the ability of the CB to better affect household indebtedness behavior for the purpose of getting nearer to its objectives is based on its capacity to influence household foreign debt. It is important to stress that ultimately it is the elasticity ε_{φ_D} (not $\varepsilon_{\bar{\tau}_D}$) that determines the effect of household debt through the UIP equation. The Appendix shows that the relation between the two elasticities is:

$$\varepsilon_{\varphi_D} = 2 [1 - \beta \phi^* (1 + i^*) / \pi^*] \varepsilon_{\bar{\tau}_D} = 0.0028255 \varepsilon_{\bar{\tau}_D},$$

where the second equality is given by the calibrated values of the parameters involved. Changing $\varepsilon_{\bar{\tau}_D}$ individually (without changing other calibrated parameters such as γ^D) does not change the NSS value of φ_D but does change the value of its elasticity ε_{φ_D} . Notice that a relatively high elasticity of the net foreign investors' risk premium such as $\varepsilon_{\bar{\tau}_D} = 100$ only generates an elasticity of the UIP risk premium $\varepsilon_{\varphi_D} = 0.28255$. Hence, high elasticities $\varepsilon_{\bar{\tau}_D}$ may be empirically important, and it is these that generate the most substantial losses from restricting CB action to the corner regimes, as shown in Table 11. Nevertheless, a moderate elasticity $\varepsilon_{\varphi_D} = 0.028255$ has been used in this paper as a baseline.

Table 11: Sensitivity of losses to alternative elasticities $\varepsilon_{\bar{\tau}_D}$ under Ramsey

STYLE	LOSS														
	ELAST $\tau_{BarD}=0.1$			ELAST $\tau_{BarD}=1$			ELAST $\tau_{BarD}=10$			ELAST $\tau_{BarD}=50$			ELAST $\tau_{BarD}=100$		
	MER	FER	PER	MER	FER	PER	MER	FER	PER	MER	FER	PER	MER	FER	PER
A	113.1	113.1	113.2	114.7	114.7	114.8	119.9	121.0	120.9	118.5	138.4	136.4	109.0	159.8	152.8
B	127.5	127.5	128.1	124.0	124.1	125.1	112.0	114.5	117.3	78.1	100.0	106.1	57.9	91.2	98.8
C	396.3	396.3	396.4	395.8	395.9	396.1	378.1	388.1	388.8	268.0	363.8	366.6	205.0	341.3	346.4
D	410.2	410.3	410.3	410.0	410.2	410.3	394.5	405.2	405.7	294.2	387.0	388.6	242.5	368.8	371.7
	RELATIVE LOSS														
A	1.0000	1.0010		1.0001	1.0010		1.009	1.008		1.169	1.151		1.466	1.401	
B	1.00003	1.0052		1.0006	1.0088		1.023	1.047		1.280	1.358		1.576	1.707	
C	1.00001	1.0003		1.0004	1.0008		1.027	1.028		1.357	1.368		1.665	1.689	
D	1.00001	1.0002		1.0005	1.0008		1.027	1.028		1.315	1.321		1.521	1.533	

4.2 Intuition on the superiority of using two policy rules through IRFs

The interest rate rule can be seen as basically addressing nominal rigidity and the exchange rate depreciation rule as basically addressing the external sector. Each of them has a more direct impact on one of the two most important relative prices in any open economy model: the real interest rate and the RER, respectively. But the fact that the model is quite rich and there are so many possible cases makes it difficult to present clear cut intuitions on why the use of two simultaneous rules is normally to be preferred in terms of diminishing an ad hoc policymaker loss function. Table 11 shows that whenever the endogenous risk premium is sufficiently elastic policymakers have leverage on capital flows, and can hence influence them to suit their purposes. Such changes in household foreign debt are a particular case of more general portfolio shifts. This paper shows that even in a linearized DSGE model such policy-induced portfolio shifts can be easily represented in the case of a SOE. A more general portfolio model would need higher than first order approximations for obtaining solutions. In Escudé (2007 and 2009) it is banks that change the level and composition of their liabilities, through the deposit vs. foreign debt funding of their domestic loans. But the basic mechanisms are the same as here.

To gain some intuition on what the second policy rule may add to the traditional interest rate rule the graphs below show the IRFs to the 4 most significant shocks in the model, for a FER regime where the interest rate rule responds only to the lagged interest rate and inflation (with nonzero coefficients $h_0 = 0.2$, $h_1 = 1.2$) and for a MER regime that adds to this rule a nominal rate of depreciation rule

that responds only to its lag, GDP, and the CB's reserves ratio (γ^R) (with nonzero coefficients $k_0 = 0.2$, $k_2 = -0.5$, $k_4 = -0.02$). In log deviations from the NSS the two policy rules are:

$$\begin{aligned}\hat{i}_t &= 0.2\hat{i}_{t-1} + 1.2\hat{\pi}_t^C \\ \hat{\delta}_t &= 0.2\hat{\delta}_{t-1} - 0.5\hat{Y}_t - 0.02(\hat{r}_t + \hat{e}_t - \hat{Y}_t).\end{aligned}$$

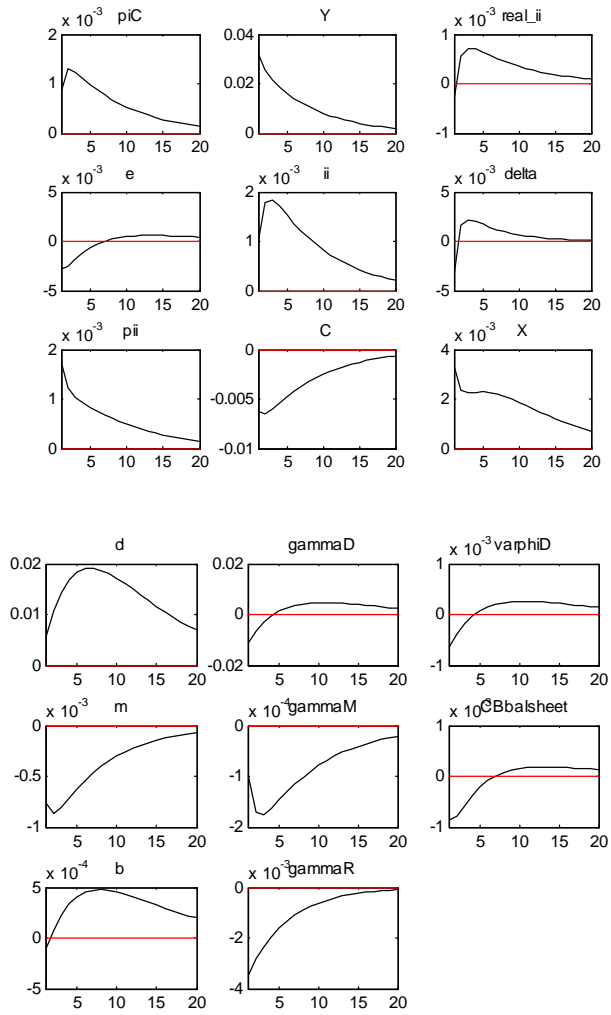
A new variable has been defined to capture the response of the size of the CB's balance sheet: $CBbalsheet = e * r (= m + b)$. Consider first the case of a positive shock to G (Figure 1). Under the FER regime, the increase in government expenditure is expansionary and inflationary and generates real currency appreciation. Consumption is crowded out and falls,²⁴ but the increase in government expenditures and in exports more than compensates and GDP increases initially. Households obtain funds abroad to avoid a further fall in consumption, also responding to the fall in the UIP risk premium (due to the reduction in their debt ratio γ^D i.e. γ^D), since the fall in e and the increase in Y more than compensate for the increase in d). When the second policy rule is introduced for the MER regime, the shock is still expansionary and inflationary for domestic goods (pii). However, it is deflationary overall (piC) and slightly less expansionary. The negative coefficient on GDP in the second rule makes the rate of nominal depreciation fall substantially (from 1.015 to around 1.005). This, along with CB sales of reserves, helps to generate a stronger real appreciation and makes the consumption inflation (π^C) fall on impact (and later increase less). This effect on π^C makes the CB target a lower nominal interest rate, and hence the latter falls initially, making the expected real interest rate also fall initially and consumption fall less initially. The stronger initial real appreciation also makes exports fall initially. Notice the marked change in the dynamics of d , which on impact increases more then with

²⁴Notice that this is consumption of private goods. To abstain from departing more significantly from the standard New Keynesian model, this paper repeats the usual absurd representation of government expenditures as pure waste instead of financing (more or less efficiently) the production of public goods. This could easily be remedied by introducing the production of public goods, the consumption of which affect household utility in an additively separable way (hence not needing to change the first order conditions of the household decision problem). However, having public roads probably increases the utility derived from one's car, so this would only scratch the surface of a very important and relevant topic.

the FER regime, allowing households to ameliorate their reduction in consumption taking advantage of the large reduction in the real interest rate. But already in the second quarter the household rapidly reduces its foreign debt, as the CB is by then selling reserves in order to induce the greater real currency appreciation. The role of the second policy rule is clearly stabilizing, at least for the most usual CB preferences (that target inflation or GDP).

Under the FER regime, the negative shock to ϕ^* (Figure 2) generates an exogenous availability of foreign funds that households take advantage of by increasing d . The capital inflow is deflationary and hence the action of the interest rate rule facilitates the reduction in the nominal interest rate on impact. The real interest rate falls even more because after the initial reduction in π^C it is expected to increase. The inflow of funds also generates a real currency appreciation which reduces exports and GDP even though consumption initially increases. In the MER regime the action of the nominal depreciation rule makes the CB initially purchase foreign reserves, which ameliorates the real appreciation, as well as the fall in exports and GDP. Households obtain a much greater quantity of funds abroad initially, when the CB is purchasing reserves. However, they quickly start to reduce their debt when the CB starts to sell reserves, overshooting the original (NSS) level. The action of the second rule hence reduces the impact and volatility of inflation, GDP and the RER.

Figure 1: Positive shock to G
FER



MER

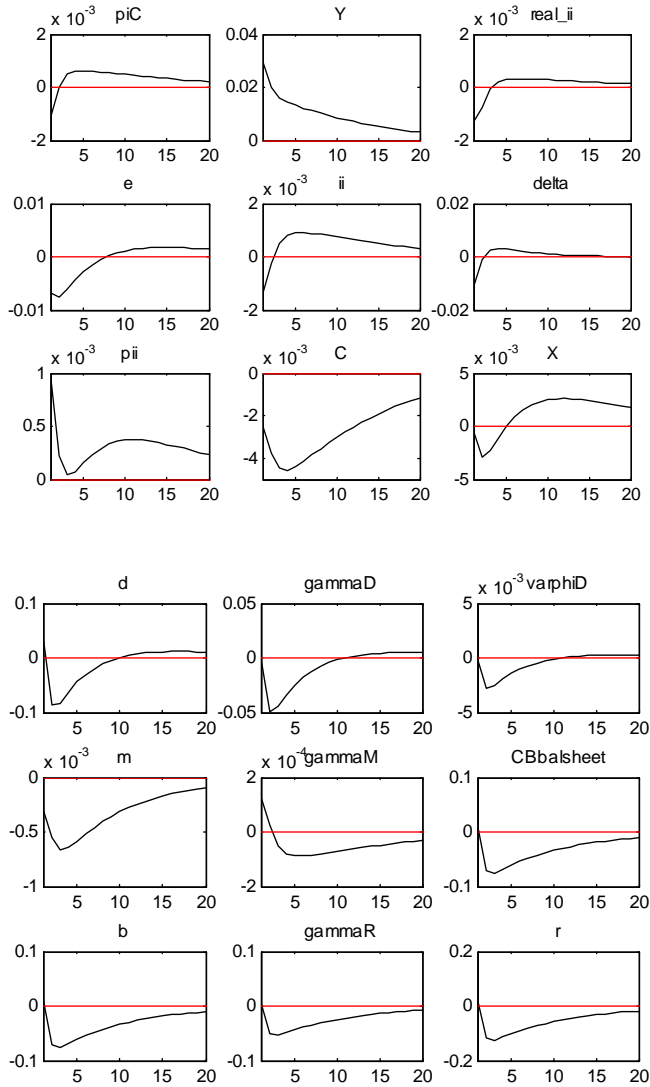
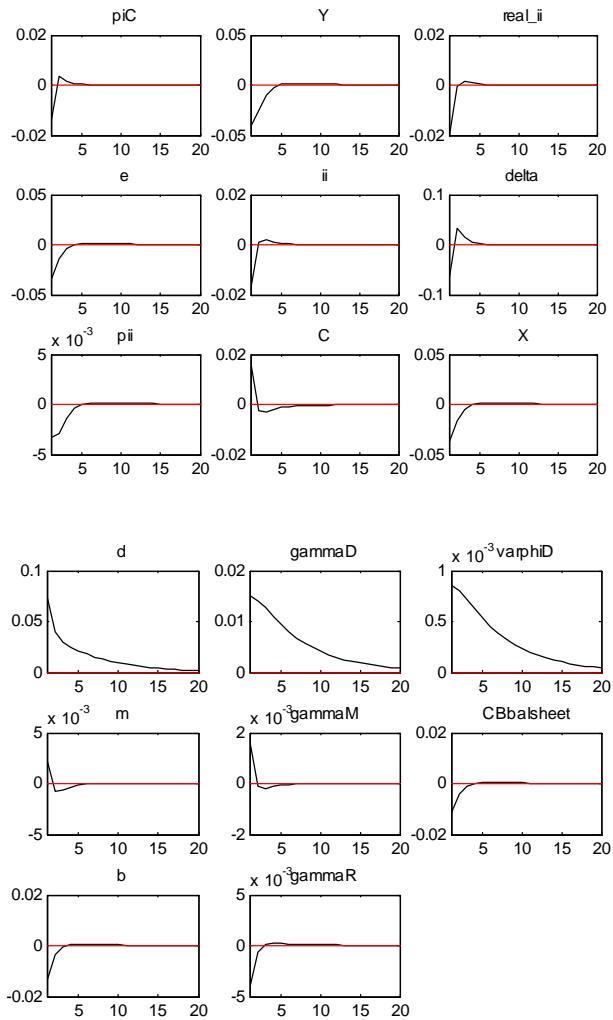


Figure 2: Negative shock to ϕ^*
FER



MER

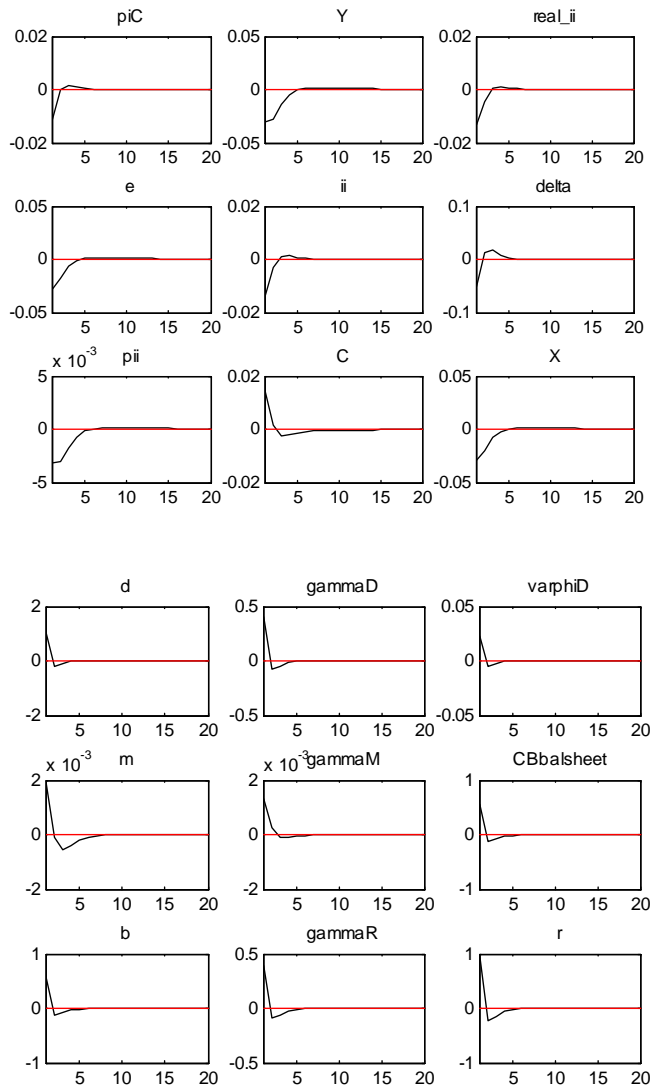
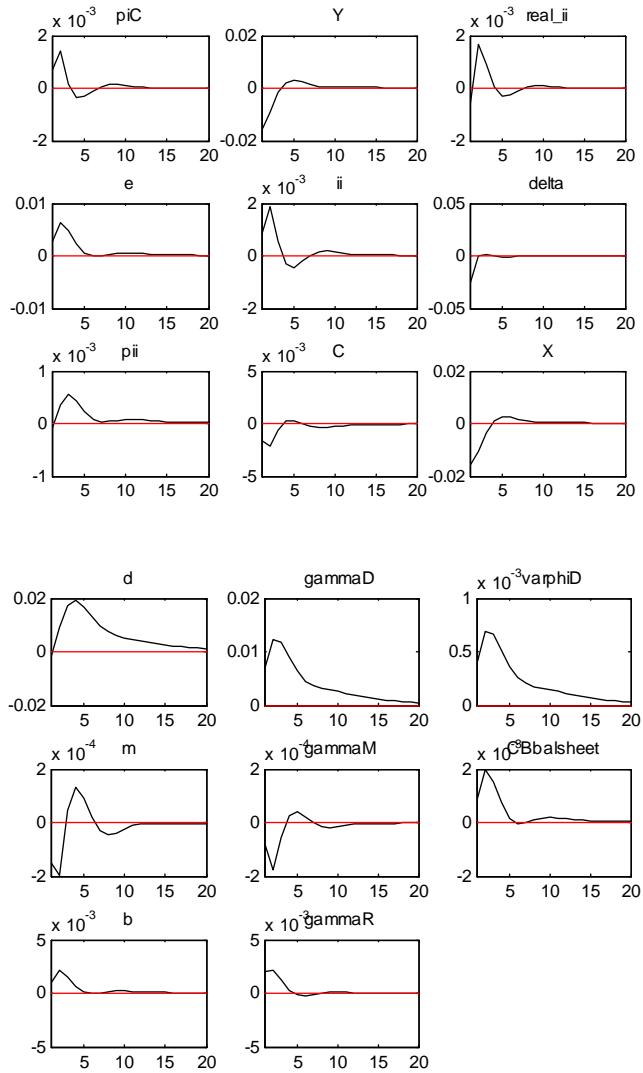


Figure 3: Positive shock to π^*
FER



MER

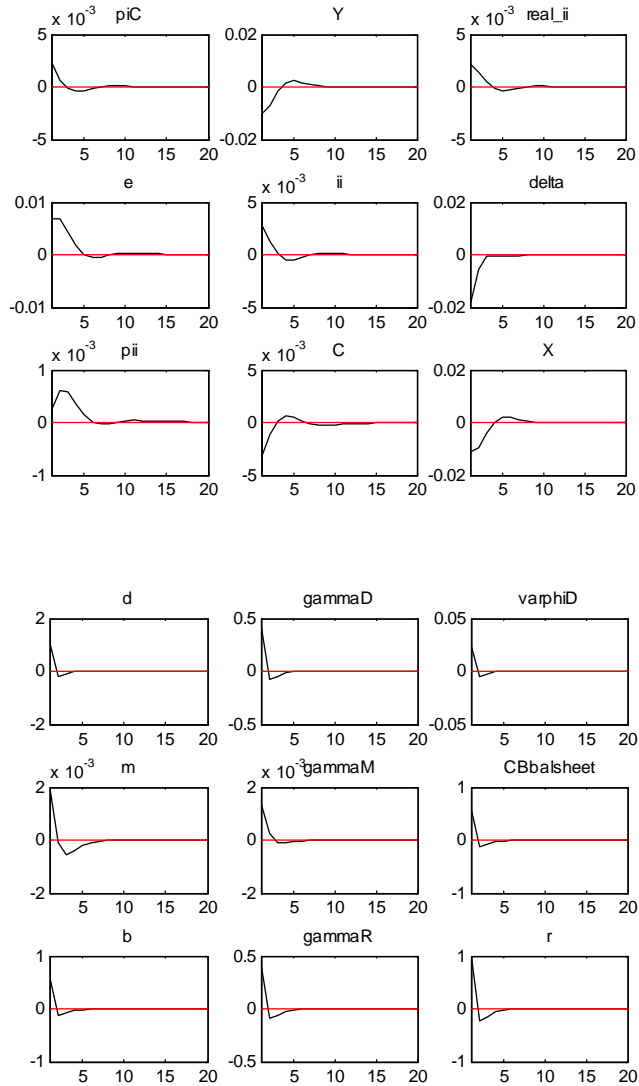
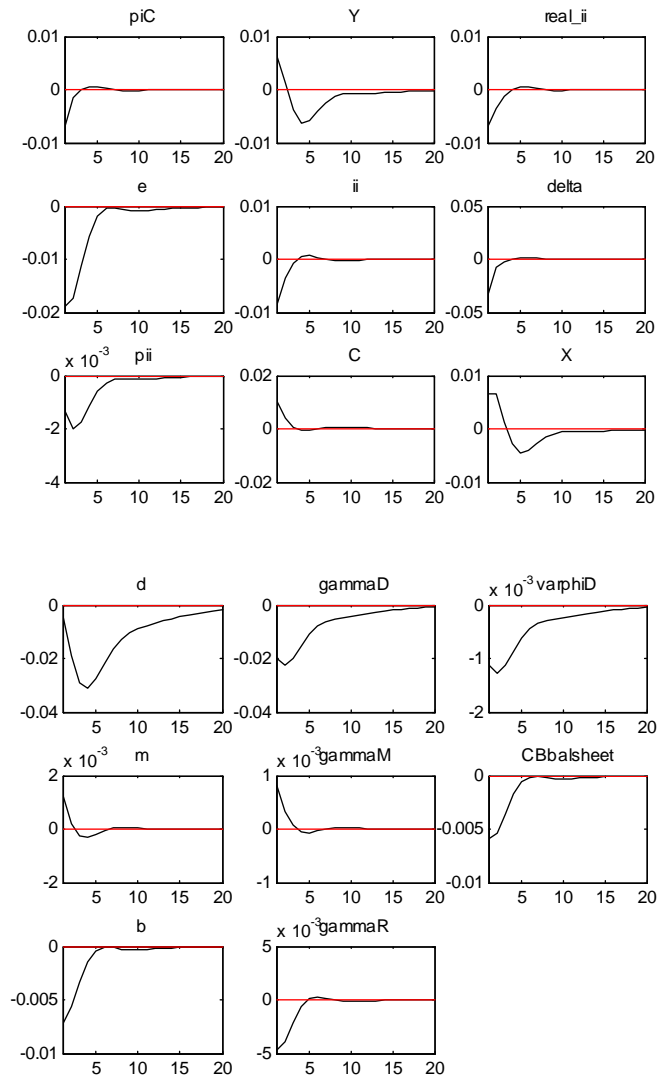
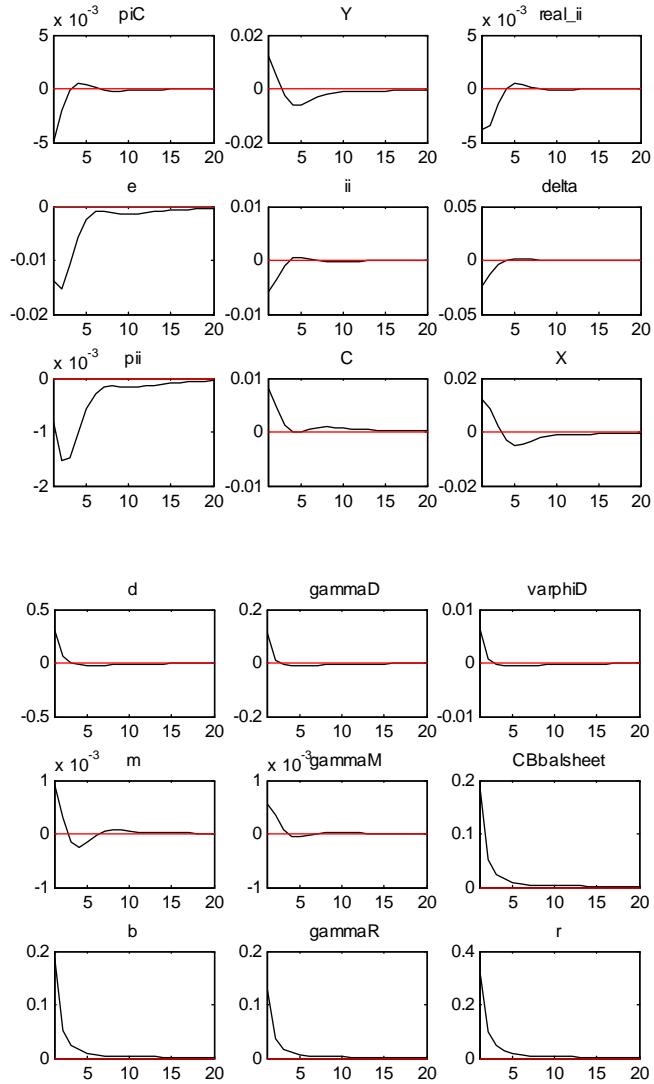


Figure 4: Positive shock to π^{*X}
FER



MER



In the case of shocks to G or ϕ^* , the MER regime has been seen to be superior to the FER regime for any of the usual CB preferences. For the following two shocks, the effect of introducing the second policy rule will only be positive for some CB preferences but not all. The shock to imported goods inflation π^* (Figure 3) under a FER regime generates nominal currency depreciation, increasing consumption inflation on impact through its imported component. Hence, consumption falls, dragging GDP with it. This makes exports fall even though there is real depreciation. The CB, following its interest rate rule, increases the nominal interest rate. The fall in GDP makes the foreign debt ratio increase on impact (even though d does not change and the RER has increased) and hence the foreign currency interest rate households face when obtaining funds abroad also increases, which is consonant with the increased operational target for the domestic currency nominal interest rate. Households subsequently ameliorate their reduction in consumption by obtaining funds abroad. Under the MER regime, the second policy rule makes the CB purchase reserves on impact, generating a larger initial real depreciation. This makes exports and GDP fall less than in the FER regime and consumption fall more since there is greater inflation (for both domestic and imported goods) and the expected real interest rate rises on impact. Households now increase their foreign debt on impact (as the CB is purchasing reserves) but thereafter reduce it along with the CB's rapid reversal of its purchases. Hence, the use of the second rule makes the shock less recessionary, but it also makes it more inflationary and generate more real depreciation. Hence, in this case the MER regime should be favored over the FER regime whenever the CB cares more about stabilizing GDP than inflation or the RER.

Finally, a shock to exported goods inflation π^{*X} (Figure 4) boosts exports and generates nominal and real appreciation. Inflation falls on impact, boosting consumption and GDP, the increase in consumption being facilitated by the reduction in the target nominal interest rate (which makes the real interest rate fall). Under the MER regime, the action of the second policy rule makes the CB purchase reserves to obtain a lower reduction in the rate of nominal depreciation, which yields a lower real appreciation. Consequently, the shock is less deflationary, more expansionary and generates less real appreciation. Hence the MER regimes should be favored over the FER regime for CB preferences that care more for stabilizing inflation and the RER than stabilizing GDP.

4.3 An extension: CB international reserves as an additional influence on the risk premium

The debt premium has so far been modeled as depending only on household foreign debt (as an endogenous variable). However, although a positive correlation between foreign debt and the risk premium is typically found in empirical research (Bellás et al. 2010, Di Cesare et al. 2012), a negative correlation between CB reserves and the risk premium is also measured.²⁵ Fouejieu and Roger (2013), for example, place Gross external debt and Foreign exchange reserves (both as a ratio to GDP) at the top of their potential determinants of country risk and use system GMM estimation with annual data from 40 emerging and high income countries in the 1989 to 2010 period. In their Table 2, they report statistically significant (at 1%) effects of both variables, and the positive influence of foreign debt on country risk is around three times (the absolute value of) the negative influence of CB international reserves.

To address this additional influence, the functional form of the endogenous risk premium $\bar{\tau}_D$ is here extended to additionally include the negative influence of CB international reserves (as a ratio of GDP). The new definition is hence:

$$\bar{\tau}_D(\gamma_t^D, \gamma_t^R) \equiv \frac{\alpha_1}{1 - \alpha_2 \gamma_t^D + \alpha_3 \gamma_t^R}, \quad \alpha_1, \alpha_2, > 0, \quad \alpha_3 \geq 0. \quad (84)$$

The sections above have centered in the special case $\alpha_3 = 0$. More generally, there are now two partial elasticities:²⁶

$$\begin{aligned} \varepsilon_{\bar{\tau}_D,1} &= \frac{\alpha_2 \gamma^D}{1 - \alpha_2 \gamma^D + \alpha_3 \gamma^R} \\ \varepsilon_{\bar{\tau}_D,2} &= \frac{-\alpha_3 \gamma^R}{1 - \alpha_2 \gamma_t^D + \alpha_3 \gamma_t^R}. \end{aligned} \quad (85)$$

The conditions under which α_2 and α_3 are equal may be deemed of special interest because in that case $\bar{\tau}_D(\cdot)$ is a function of net foreign liabilities (as a ratio

²⁵I thank one of my anonymous referees for suggesting the expansion of the analysis in this direction.

²⁶Notice that the Marshallian convention that makes the elasticities always positive is not used.

to GDP) $\gamma_t^D - \gamma_t^R$. They are easily obtained:

$$\alpha_2 = \alpha_3 \Leftrightarrow \frac{\varepsilon_{\bar{\tau}_{D,1}}}{\gamma^D} = \frac{-\varepsilon_{\bar{\tau}_{D,2}}}{\gamma^R} \Leftrightarrow -\varepsilon_{\bar{\tau}_{D,2}} = \frac{\gamma^R}{\gamma^D} \varepsilon_{\bar{\tau}_{D,1}} = \frac{0.13}{0.5} \varepsilon_{\bar{\tau}_{D,1}} = 0.26 \varepsilon_{\bar{\tau}_{D,1}}.$$

Hence, if $\varepsilon_{\bar{\tau}_{D,1}} = 10$ then $-\varepsilon_{\bar{\tau}_{D,2}} = 2.6$. However, notice that the risk function in the UIP equation continues to be a function of the two ratios individually, since the CB international reserves r_t is not a household decision variable (whereas d_t is). The following table assumes $\varepsilon_{\bar{\tau}_{D,1}} = 10$ (as in the central set of columns of Table 11) and makes different assumptions with respect to $\varepsilon_{\bar{\tau}_{D,2}}$ (including $\varepsilon_{\bar{\tau}_{D,2}} = -2.6$).²⁷

The second set of columns shows that $\varepsilon_{\bar{\tau}_{D,2}} = -2.6$ gives the lowest relative advantage of the MER regime (at least among those shown), but it is still positive. Both lower and higher values of $-\varepsilon_{\bar{\tau}_{D,2}}$ give higher advantages for using the two policy rules. Also, according to the value of $\varepsilon_{\bar{\tau}_{D,2}}$ either the FER or PER regime is second best. The important point is that the general argument is robust to the presence of the CB's international reserves in the risk premium.

Table 12: Sensitivity of losses to alternative elasticities $\varepsilon_{\bar{\tau}_{D,2}}$ under Ramsey ($\varepsilon_{\bar{\tau}_{D,1}} = 10$)

STYLE	LOSS														
	ELASTtauBarD 2=0:			ELASTtauBarD 2=-2.6:			ELASTtauBarD 2=-3.33:			LASTtauBarD 2=-6.6:			ELASTtauBarD 2=-10:		
	MER	FER	PER	MER	FER	PER	MER	FER	PER	MER	FER	PER	MER	FER	PER
A	119.9	121.0	120.9	120.207	120.210	120.31	119.95	120.00	120.15	117.2	119.1	119.4	112.6	118.2	118.7
B	112.0	114.5	117.3	114.3	114.3	115.2	114.1	114.3	114.6	109.2	114.0	112.0	99.6	113.7	109.4
C	378.1	388.1	388.8	387.1	387.1	387.3	386.3	386.8	386.9	366.7	385.6	384.9	331.8	384.3	382.9
D	394.5	405.2	405.7	404.0	404.1	404.2	403.2	403.7	403.8	382.4	402.3	402.0	348.5	400.8	400.1
	RELATIVE LOSS														
A		1.009	1.008		1.00002	1.0009		1.0004	1.0017		1.016	1.019		1.049	1.054
B		1.023	1.047		1.0001	1.0079		1.00101	1.00437		1.044	1.026		1.141	1.099
C		1.027	1.028		1.0001	1.0005		1.00128	1.00134		1.052	1.050		1.158	1.154
D		1.027	1.028		1.0001	1.0005		1.00133	1.00153		1.052	1.051		1.150	1.148

5 Some additional robustness checks

In this section the optimal policy under commitment framework ('ramsey') is used to obtain the sensitivity of the losses and relative losses to several additional parameters. In all the tables below the elasticities used are $\varepsilon_{\bar{\tau}_{D,1}} = 10$, $\varepsilon_{\bar{\tau}_{D,2}} = 0$.

²⁷In the Dynare code, $\varepsilon_{\bar{\tau}_{D,i}}$ is ELASTtauBarD_i ($i = 1, 2$).

Taking into account the importance in the model of the shock to the exogenous risk/liquidity premium ϕ^* , the next exercise gauges how the losses and relative losses are affected by different levels of the standard deviation (or standard error) of the risk/liquidity shock (σ^{ϕ^*}). The results are shown in Table 13 for a range of values of σ^{ϕ^*} that go from 0.01 to 0.15. As expected, the losses are monotonically increasing in σ^{ϕ^*} . The relative losses of the FER regime for all CB styles and of the PER regime for styles A, C, and D are also increasing in σ^{ϕ^*} . The relative loss in the PER regime for style B (where only GDP matters), however, is decreasing with σ^{ϕ^*} . In this case, the FER regime is always second best. For the rest of the CB styles, low values of σ^{ϕ^*} make the FER regime second best and high values make the PER regime second best.

Table 13: Losses for different values of σ^{ϕ^*}

STYLE	LOSS														
	stderr(eps_phiStar)=0.001			stderr(eps_phiStar)=0.01			stderr(eps_phiStar)=0.05			stderr(eps_phiStar)=0.1			stderr(eps_phiStar)=0.15		
	MER	FER	PER	MER	FER	PER	MER	FER	PER	MER	FER	PER	MER	FER	PER
A	106.7	107.4	107.5	107.2	107.9	108.0	119.9	121.0	120.9	159.6	161.8	161.1	225.6	229.9	228.1
B	96.4	98.4	101.3	97.1	99.1	102.0	112.0	114.5	117.3	158.7	162.9	165.2	236.6	243.5	245.0
C	355.3	364.2	365.3	356.2	365.1	366.2	378.1	388.1	388.8	446.4	459.9	459.4	560.4	579.5	577.1
D	368.1	377.1	378.0	369.2	378.2	379.1	394.5	405.2	405.7	473.8	489.6	488.6	605.9	630.2	626.9
	RELATIVE LOSS														
A		1.0059	1.0071		1.0060	1.0071		1.009	1.008		1.014	1.010		1.019	1.011
B		1.0206	1.0509		1.0207	1.0507		1.023	1.047		1.026	1.040		1.029	1.035
C		1.0251	1.0282		1.0251	1.0282		1.027	1.028		1.030	1.029		1.034	1.030
D		1.0245	1.0269		1.0246	1.0270		1.027	1.028		1.033	1.031		1.040	1.035

The degree of price stickiness (α) in the New Keynesian Phillips equation is often considered an important factor in determining the desirability of alternative exchange regimes. Table 14 shows the losses under each CB style and exchange rate regime for six alternative degrees of price stickiness, which go from practically no price stickiness ($\alpha=0.01$) to very high price stickiness ($\alpha=0.90$). Starting with $\alpha=0.3$, higher price stickiness generates higher losses for all regimes and styles. This is also true for lower values of α in the case of styles C and D. However, for (the extreme) styles A and B and very low values of price stickiness an increase in α generates a reduction in losses for the 3 regimes. As expected, for each CB style and value of α , the MER regime does significantly better than the corner regimes. The smallest advantage for the MER regime appears in the intermediate range of price stickiness (including the baseline value). For all CB styles, the PER regime is second best for low degrees of price stickiness and the FER

regime is second best for high degrees of price stickiness. Summing up, with or without price stickiness there is a gain from intervening in the FX market in the sense that the CB can better stabilize its target variables.

Table 14: Losses for different values of α

STYLE	LOSS														
	alfa=0.01			alfa=0.30			alfa=0.66			alfa=0.80			alfa=0.90		
	MER	FER	PER	MER	FER	PER	MER	FER	PER	MER	FER	PER	MER	FER	PER
A	125.6	129.5	129.3	115.7	117.9	117.7	119.9	121.0	120.9	153.7	155.0	154.8	210.2	212.3	212.6
B	89.8	96.6	94.1	90.9	92.5	91.5	112.0	114.5	117.3	124.0	127.8	131.5	142.9	147.8	153.0
C	211.1	220.9	219.9	239.9	245.4	244.8	378.1	388.1	388.8	400.6	417.8	419.4	443.3	465.1	466.7
D	233.8	248.4	247.0	257.9	266.0	265.0	394.5	405.2	405.7	417.2	434.5	435.8	460.1	481.7	483.0
	RELATIVE LOSS														
A		1.031	1.029		1.019	1.018		1.009	1.008		1.008	1.007		1.010	1.012
B		1.076	1.048		1.017	1.006		1.023	1.047		1.031	1.060		1.035	1.071
C		1.046	1.042		1.023	1.021		1.027	1.028		1.043	1.047		1.049	1.053
D		1.063	1.056		1.032	1.028		1.027	1.028		1.041	1.045		1.047	1.050

Table 15: Losses for different values of γ^D

STYLE	LOSS														
	gammaD=0.1			gammaD=0.25			gammaD=0.5			gammaD=1			gammaD=1.5		
	MER	FER	PER	MER	FER	PER	MER	FER	PER	MER	FER	PER	MER	FER	PER
A	127.6	146.0	143.7	125.2	129.0	128.5	119.9	121.0	120.9	115.0	115.3	115.4	112.9	113.0	113.2
B	104.0	131.0	136.5	113.7	121.3	125.0	112.0	114.5	117.3	106.4	107.2	109.5	102.2	102.6	104.9
C	306.3	416.4	415.5	364.3	396.7	397.2	378.1	388.1	388.8	384.8	387.7	388.4	394.8	396.2	396.9
D	319.5	429.0	427.7	376.1	409.6	409.9	394.5	405.2	405.7	415.9	419.7	419.9	446.7	449.4	449.4
	RELATIVE LOSS														
A		1.145	1.126		1.031	1.027		1.009	1.008		1.002	1.003		1.001	1.003
B		1.259	1.313		1.067	1.099		1.023	1.047		1.008	1.029		1.004	1.026
C		1.360	1.357		1.089	1.090		1.027	1.028		1.008	1.009		1.004	1.005
D		1.343	1.339		1.089	1.090		1.027	1.028		1.009	1.010		1.0061	1.0062

Another interesting parameter to move is the NSS value of the household foreign debt ratio γ^D . In the calibrations concerning the risk premium this parameter is always multiplied by α_2 and the calibration is such that any exogenous change in γ^D produces a compensating change in α_2 so that the product remains the same. However, γ^D enters the trade balance ratio (to GDP) by itself so the changes in losses below are solely due to the effects related to its impact in the (NSS equilibrium) RER and other real variables related to the external sector. Table 15 shows that, maintaining the rest of the independent parameters constant, lower values of γ^D imply higher losses for styles A (for all 3 regimes) and B (for all 3 regimes except for the very last reduction shown in the case of the MER regime). For styles C and D, however, lower values of γ^D imply lower losses for the MER regime, but there is no monotonicity for the corner regimes. More importantly, lower γ^D

generate higher relative costs from using the corner regimes, which become very high for very low values of γ^D . As to the ranking of the corner regimes, except for style B in which the FER regime is always second best, low values of γ^D make the PER regime second best while high values of this parameter make the FER regime second best.

6 Conclusion

This paper tries to bridge the gap between the fact that many central banks systematically intervene both in the domestic bond market (trying to impose a nominal interest rate) and in the FX market (trying to influence the path of the exchange rate), and the absence of any generally accepted model for the representation and analysis of this practice. This paper takes a core structure from previous papers of the author to build a simple New Keynesian model in which the CB can simultaneously intervene in the FX and bond markets, varying its outstanding bond liabilities and reserve assets in order to achieve two operational targets: one for the interest rate and another for the rate of nominal depreciation. For this, the DSGE model includes financial variables and institutional practices ('nuts and bolts' of central banking) that are left out of conventional modeling in which only the extreme policy regimes of a pure float or a pure peg are considered, but cannot be left out when trying to build a more general model. The resulting model has a core that is little more than the typical DSGE workhorse of the profession, but extends it in directions which allow for a richer policy framework. The model parameters and steady state values of endogenous variables are calibrated in detail in an Appendix, and the model is implemented in Dynare.

Three alternative policy regimes are considered: the general, two rules regime (denominated Managed Exchange Rate -MER- regime), and the two 'corner' regimes of Floating Exchange Rate -FER- and Pegged Exchange Rate -PER- (both of which use a single simple policy rule or a single control variable). The alternative policy regimes are studied under simple policy rules, optimal simple policy rules (where the coefficients are obtained by minimizing a linear combination of the variances of certain target variables), and optimal policy in a linear-quadratic optimal control framework under commitment and full information. First there is

a study of the effects of moving individual coefficients of the simple policy rules on the standard deviations of the typical target variables under the MER regime. Then the minimum losses are obtained in the two optimal policy frameworks for a range of alternative ad hoc CB preferences (or styles) and the three alternative policy regimes. Because the optimal simple rules framework in Dynare does not necessarily yield a global optimum, the bulk of the analysis is done within the linear-quadratic optimal control framework. It is shown that the use of two policy rules (or control variables) systematically outperforms any of the corner regimes. For the central bank preferences usually considered (that seek low variability of inflation and/or output) substantially better results are achieved when two control variables are used. The reason for this outperformance is shown to derive from the added leverage the CB obtains in exploiting its ability to influence the foreign debt of households, which is an important determinant of the risk premium function in the risk-adjusted uncovered interest parity equation. By using its interventions to obtain operational targets for both the domestic interest rate and the (actual and expected) rate of nominal depreciation, the CB has greater influence on the foreign debt ratio that determines the (endogenous part of the) risk premium in the UIP equation. The CB can thus achieve a lower loss when it intervenes in both markets. The analysis also gives the second best regime in each of the many cases considered. The sensitivity of the CB losses in each regime and style to different crucial parameters is gauged, including the assumed elasticity of the endogenous risk premium function, the assumed standard deviation of the exogenous risk/liquidity shock, the Calvo price stickiness parameter, and the steady state foreign debt to GDP ratio.

Concluding, a policy of systematically intervening in the foreign exchange market through a feedback rule for the rate of nominal depreciation is a valuable complement to any interest rate policy rule framework, and there are good reasons for defending a managed exchange rate regime as the baseline in any SOE modeling framework.

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Appendix

In this Appendix the calibrated values for the model's parameters are obtained as well as the corresponding NSS values of the model variables. There are always many ways of doing this. Some parameters, some ratios and some NSS values of endogenous variables are first calibrated and the rest are obtained sequentially using the static nonlinear equations so that a computer code can follow the same steps if one changes some of the calibrated values.

A.1 Calibration of parameters and derivation of the corresponding non-stochastic steady state

Calibration of the components of the external terms of trade

The terms of trade is a particularly important variable for any SOE. Hence, a preliminary investigation of the data pertaining to Argentina was made. To confront (77) with the data, notice that the first two of these equations can be written in terms of the (logs of) price indexes:

$$\begin{aligned}\Delta \log P_t^{*X} &= \rho^{\pi^{*X}} \Delta \log P_{t-1}^{*X} + (1 - \rho^{\pi^{*X}}) \log \pi^{*X} + \alpha_{\pi^{*X}} (\log P_{t-1}^{*X} - \log P_{t-1}^{*N}) \\ &\quad + \sigma^{\pi^{*X}} \varepsilon_t^{\pi^{*X}}, \\ \Delta \log P_t^{*N} &= \rho^{\pi^*} \Delta \log P_{t-1}^{*N} + (1 - \rho^{\pi^*}) \log \pi^{*N} + \alpha_{\pi^*} (\log P_{t-1}^{*X} - \log P_{t-1}^{*N}) \\ &\quad + \sigma^{\pi^*} \varepsilon_t^{\pi^*}.\end{aligned}$$

A quick estimation for cointegration of Argentina's trade price indexes during 1993Q3–2009Q2 gave the results in the table below (the notation should be obvious). Although empirically it was not possible to impose a coefficient of negative one for the second coefficient in the cointegrating relation, it was imposed in the calibration to be consistent with the definition of the terms of trade. The small deterministic trend in the cointegrating relation was also ignored, as well as the two time dummies (first and fourth quarters of 2008) that made the residuals normal, homoscedastic and devoid of serial correlation and the non-significant coefficients. Hence, the following specification was used in the model:

Vector Error Correction Estimates		
Sample (adjusted): 1993Q3 2009Q2		
Included observations: 64 after adjustments		
Standard errors in () & t-statistics in []		
Cointegrating Eq: CointEq1		
LPSTARXLEVEL(-1)	1.0000	
LPSTARNLEVEL(-1)	-1.4924 0.1263 [-11.8125]	
@TREND(93Q1)	-0.0044	
C	2.3074	
Error Correction: D(LPSTARXLEVEL) D(LPSTARNLEVEL)		
CointEq1	-0.25543 0.09767 [-2.61520]	0.18115 0.06597 [2.74597]
D(LPSTARXLEVEL(-1))	0.40776 0.13273 [3.07203]	0.17699 0.08965 [1.97414]
D(LPSTARNLEVEL(-1))	0.15719 0.17834 [0.88142]	0.20080 0.12046 [1.66697]
C	-0.00273 0.00838 [-0.32536]	-0.00498 0.00566 [-0.87938]
@TREND(93Q1)	0.00021 0.00023 [0.95374]	0.00018 0.00015 [1.17769]
D081	0.08543 0.03638 [2.34827]	0.00287 0.02457 [0.11686]
D084	-0.15245 0.03296 [-4.62518]	-0.12326 0.02226 [-5.53617]
R-squared	0.48888	0.52026
Adj. R-squared	0.43508	0.46976
Sum sq. resids	0.05778	0.02636
S.E. equation	0.03184	0.02151
F-statistic	9.08656	10.30235
Log likelihood	133.50707	158.62000
Akaike AIC	-3.95335	-4.73813
Schwarz SC	-3.71722	-4.50200
Mean dependent	0.00581	0.00029
S.D. dependent	0.04236	0.02953
Determinant resid covariance (dof adj.)		0.00000045
Determinant resid covariance		0.00000035
Log likelihood		293.76131
Akaike information criterion		-8.68004
Schwarz criterion		-8.14032

$$\begin{aligned}\Delta \log P_t^{*X} &= 0.41 \Delta \log P_{t-1}^{*X} + (1 - 0.41) \log \pi^{*X} - 0.25 (\log P_{t-1}^{*X} - \log P_{t-1}^{*N}) \\ &\quad + 0.0424 \varepsilon_t^{\pi^{**}}, \\ \Delta \log P_t^{*N} &= 0.20 \Delta \log P_{t-1}^{*N} + (1 - 0.20) \log \pi^{*N} + 0.18 (\log P_{t-1}^{*X} - \log P_{t-1}^{*N}) \\ &\quad + 0.18 \Delta \log P_{t-1}^{*X} + 0.0295 \varepsilon_t^{\pi^*},\end{aligned}$$

where, using the notation in (77), $\beta_{\pi^*} = 1$, and $\rho^{\pi^{*XN}} = 0.18$ is added for the effect of $\Delta \log P_{t-1}^{*X}$ on $\Delta \log P_t^{*N}$ (which did not appear in the original specification). Hence, the final specification of the XTT block (77) is:

$$\begin{aligned}\pi_t^{*X} &= (\pi_{t-1}^{*X})^{0.41} (\pi^{*X})^{1-0.41} (p_{t-1}^*)^{-0.25} \exp(0.0424 \varepsilon_t^{\pi^{**}}), \\ \pi_t^* &= (\pi_{t-1}^*)^{0.20} (\pi^*)^{1-0.20} (p_{t-1}^*)^{0.18} (\pi_t^{*X})^{0.18} \exp(0.0295 \varepsilon_t^{\pi^*}), \\ p_t^* &= p_{t-1}^* \frac{\pi_t^{*X}}{\pi_t^*}.\end{aligned}$$

The NSS relations between parameters and endogenous variables

Eliminating time indexes from the model equations and simplifying gives a set of nonlinear equations that involve both the parameters and NSS values of the endogenous variables. It is assumed that $\varepsilon = 1$ and $\pi^* = 1$. Several key ratios are used such as the target value for the CB reserves ratio $\gamma^R = er/Y$, the NSS household foreign debt ratio $\gamma^D = ed/Y$ and money/consumption ratio $\gamma^M = m/(p^C C)$. In some cases the equation is divided through by GDP.

Interest rate feedback rule:

$$1 = \left(\frac{\pi^C}{\pi^T} \right)^{h_1} \tag{86}$$

Nominal depreciation feedback rule:

$$1 = \left(\frac{\pi^C}{\pi^T} \right)^{k_1} \left(\frac{er/Y}{\gamma^R} \right)^{k_4} \tag{87}$$

Consumption:

$$\frac{1+i}{\pi^C} = \frac{1}{\beta} \quad (88)$$

Risk-adjusted UIP:

$$1+i = (1+i^*)\phi^*\varphi_D\delta \quad (89)$$

Phillips equations:

$$\Gamma = \frac{Q/(p^C C^{\sigma^C})}{1-\beta\alpha\pi^{\theta-1}} \quad (90)$$

$$\Psi = \frac{\theta}{\theta-1} mc \frac{Q/(p^C C^{\sigma^C})}{1-\beta\alpha\pi^{\theta}} \quad (91)$$

$$\frac{\Gamma}{\Psi} = \left(\frac{1-\alpha\pi^{\theta-1}}{1-\alpha} \right)^{\frac{1}{\theta-1}} \quad (92)$$

Price dispersion:

$$\Delta = \left(\frac{1-\alpha}{1-\alpha\pi^{\theta}} \right) \left(\frac{1-\alpha\pi^{\theta-1}}{1-\alpha} \right)^{\frac{\theta}{\theta-1}} \quad (93)$$

Exports:

$$\frac{X}{Y} = \kappa_X (ep^*)^{b_X} \quad (94)$$

Trade Balance:

$$TB \frac{e}{Y} = \frac{1}{a_D} \left[(p^C)^{1-\theta^C} \frac{X}{Y} - (1-a_D) e^{1-\theta^C} \right] \quad (95)$$

Current Account:

$$CA \frac{e}{Y} = \left(\frac{1+i^*}{\pi^*} - 1 \right) \gamma^R - \left(\frac{1+i^*}{\pi^*} \phi^* \tau_{D-1} - 1 \right) \gamma^D + TB \frac{e}{Y} \quad (96)$$

Balance of Payments:

$$CA = 0 \quad (97)$$

Real marginal cost:

$$mc = w \quad (98)$$

Labor market clearing:

$$w = \xi^N p^C C^{\sigma^C} \varphi_M N^{\sigma^N} \quad (99)$$

Hours worked:

$$N = Q\Delta \quad (100)$$

Domestic goods market clearing:

$$\frac{Q}{Y} = 1 - (1 - b^A) \frac{X}{Y} \quad (101)$$

GDP:

$$1 = a_D \frac{\tau_{MG} p^C C}{(p^C)^{1-\theta^C} Y} + \frac{X}{Y} \quad (102)$$

Consumption relative price:

$$p^C = \left(a_D + (1 - a_D) e^{1-\theta^C} \right)^{\frac{1}{1-\theta^C}} \quad (103)$$

Money market clearing:

$$\frac{m}{Y} = \frac{1}{\beta_2} \left[\left(\frac{\beta_1 \beta_2 \beta_3}{1 - \frac{1}{1+i}} \right)^{\frac{1}{\beta_3+1}} - 1 \right] \frac{p^C C}{Y}, \quad (104)$$

CB balance sheet:

$$\frac{b}{Y} = \gamma^R - \gamma^M \frac{p^C C}{Y} \quad (105)$$

Tax collection:

$$tax = (G - 1)p^C C - qf$$

Quasi-fiscal surplus:

$$qf = (1 + i^* - 1/\delta) \frac{er}{\pi^*} - ((1 + i) - 1) \frac{b}{\pi}$$

Identities:

$$\pi = \delta, \quad \pi = \pi^C, \quad 1 = \pi^{*X} \quad (106)$$

Great ratios:

$$\gamma^D = \frac{ed}{Y}, \quad \gamma^M = \frac{m}{p^C C}, \quad \gamma^R = \frac{er}{Y},$$

Auxiliary functions:

$$\begin{aligned} \tau_D &= 1 + \frac{\alpha_1}{1 - \alpha_2 \gamma^D + \alpha_3 \gamma^R}, & \varphi_D &= 1 + (\tau_D - 1) \left(1 + \frac{\alpha_2 \gamma^D}{1 - \alpha_2 \gamma^D + \alpha_3 \gamma^R} \right) \\ \tau_M &= 1 + \frac{\beta_1}{(1 + \beta_2 \gamma^M) \beta_3}, & \varphi_M &= 1 + (\tau_M - 1) \left(1 + \beta_3 \frac{\beta_2 \gamma^M}{1 + \beta_2 \gamma^M} \right). \end{aligned}$$

Exports inflation shock

$$1 = (p^*)^{\alpha_{\pi^{*X}}} \quad (107)$$

Imported inflation shock

$$1 = (p^*)^{\alpha_{\pi^{*X}}} (\pi^{*X})^{\rho \pi^{*XN}}. \quad (108)$$

The NSS values of the model's variables and the calibrated values of parameters are obtained sequentially as follows. (86) implies $\pi^C = \pi^T$, since $h_1 \neq 0$ is assumed. Inserting this in (106) yields $\pi = \delta = \pi^T$. Also, (107) implies that the XTT is $p^* = 1$, and hence (108) implies that $\pi^{*X} = 1$. Summing up,

$$\pi = \delta = \pi^C = \pi^T, \quad \text{and} \quad \pi^* = \pi^{*X} = p^* = 1.$$

Hence, (88) gives the nominal interest rate: $1 + i = \pi^T / \beta$.

It is assumed that $\beta = 0.99$. For the NSS GDP, Argentina's 2010 level (at 2010 prices and in trillions of pesos) is used: $Y = 1.443$. The gross exogenous risk/liquidity premium for households and the RW gross interest rate are assumed to be $\phi^* = 1.005^{0.25}$ and $1 + i^* = 1.03^{0.25}$, respectively. Also, the household ratios are calibrated to $\gamma^D \equiv ed/Y = 0.5$, $\gamma^M \equiv m/p^C C = 0.095522$, the CB international reserves/GDP ratio to $\gamma^R = 0.13$, and the Government to household consumption ratio to $G = 1.19$.

The home bias parameter (or share of domestic goods) in household consumption is calibrated to $a_D = 0.86$. The constant relative risk aversion for labor and consumption are: $\sigma^N = 0.5$ and $\sigma^C = 1.5$, respectively. Finally, it is assumed that the elasticity of substitution between varieties of domestic goods is $\theta = 6$ and the elasticity of substitution between the bundles of domestic and imported goods is $\theta^C = 1.5$. Assuming that the exogenous parameter in the export goods production function is $b_A = 0.5$, yields $b_X \equiv (1 - b^A)^{-1} = 2$ and $\kappa_X \equiv (b^A)^{b^A b^X} = 0.5$.

The endogenous risk premium

Using (88), (106) and (69) in the UIP equation (89) gives:²⁸

$$\frac{\alpha_1 (1 + \alpha_3 \gamma^R)}{(1 - \alpha_2 \gamma^D + \alpha_3 \gamma^R)^2} = \bar{\varphi}_D (\gamma^D, \gamma^R) = \frac{1}{\beta \phi^* (1 + i^*) / \pi^*} - 1. \quad (109)$$

The parameters on the r.h.s. have already been calibrated, as well as γ^D and γ^R . It is now necessary to calibrate the values of the exogenous parameters α_1 , α_2 and α_3 . Assuming first that $\alpha_3 = 0$, as in most of the paper, α_1 and α_2 can be expressed in terms of γ^D and $\varepsilon_{\bar{\tau}_D}$. First, notice that, according to (67), the elasticity of $\bar{\tau}_D$ is

$$\varepsilon_{\bar{\tau}_D} (\gamma_t^D) \equiv \frac{\alpha_2 \gamma_t^D}{1 - \alpha_2 \gamma_t^D}. \quad (110)$$

²⁸Notice that the extension of the endogenous risk premium of Section 5 is used here.

Hence, if the NSS values of $\varepsilon_{\bar{\tau}_D}$ and γ^D are calibrated (110) gives the value of α_2 :

$$\alpha_2 = \frac{1}{\gamma^D} \frac{\varepsilon_{\bar{\tau}_D}}{1 + \varepsilon_{\bar{\tau}_D}}. \quad (111)$$

And (109) gives:

$$\alpha_1 = (1 - \alpha_2 \gamma^D)^2 \left(\frac{1}{\beta \phi^* (1 + i^*) / \pi^*} - 1 \right) = \frac{\left(\frac{1}{\beta \phi^* (1 + i^*) / \pi^*} - 1 \right)}{(1 + \varepsilon_{\bar{\tau}_D})^2}. \quad (112)$$

More generally, when the endogenous risk premium also depends on the CB's international reserves, as in (84), there are two partial elasticities given by (85). In this case α_1 , α_2 and α_3 can be obtained by expressing them in terms of the 2 elasticities $\varepsilon_{\bar{\tau}_{D,1}}$, $\varepsilon_{\bar{\tau}_{D,2}}$, and the 2 great ratios γ^D , γ^R . First, notice that (85) implies:

$$1 + \varepsilon_{\bar{\tau}_{D,1}} + \varepsilon_{\bar{\tau}_{D,2}} = \frac{1}{1 - \alpha_2 \gamma^D + \alpha_3 \gamma^R}. \quad (113)$$

Hence, (85) implies:

$$1 + \varepsilon_{\bar{\tau}_{D,1}} = (1 + \alpha_3 \gamma^R) (1 + \varepsilon_{\bar{\tau}_{D,1}} + \varepsilon_{\bar{\tau}_{D,2}}) \quad (114)$$

$$1 + \varepsilon_{\bar{\tau}_{D,2}} = (1 - \alpha_2 \gamma^D) (1 + \varepsilon_{\bar{\tau}_{D,1}} + \varepsilon_{\bar{\tau}_{D,2}}). \quad (115)$$

Also, using (113) and (114) yields:

$$\bar{\varphi}_D(\gamma^D, \gamma^R) = \frac{\alpha_1 (1 + \alpha_3 \gamma^R)}{(1 - \alpha_2 \gamma^D + \alpha_3 \gamma^R)^2} = \alpha_1 (1 + \varepsilon_{\bar{\tau}_{D,1}}) (1 + \varepsilon_{\bar{\tau}_{D,1}} + \varepsilon_{\bar{\tau}_{D,2}}).$$

Hence, using (109) gives:

$$\alpha_1 (1 + \varepsilon_{\bar{\tau}_{D,1}}) (1 + \varepsilon_{\bar{\tau}_{D,1}} + \varepsilon_{\bar{\tau}_{D,2}}) = \frac{1}{\beta \phi^* (1 + i^*) / \pi^*} - 1. \quad (116)$$

Therefore, given the calibrated values of γ^D , γ^R , $\varepsilon_{\bar{\tau}_{D,1}}$ and $\varepsilon_{\bar{\tau}_{D,2}}$ (114), (115) and (116) yield the values of the three alphas:

$$\begin{aligned}\alpha_1 &= \frac{\left(\frac{1}{\beta\phi^*(1+i^*)/\pi^*} - 1\right)}{(1 + \varepsilon_{\bar{\tau}_{D,1}} + \varepsilon_{\bar{\tau}_{D,2}})(1 + \varepsilon_{\bar{\tau}_{D,1}})} \\ \alpha_2 &= \frac{1}{\gamma^D} \frac{\varepsilon_{\bar{\tau}_{D,1}}}{1 + \varepsilon_{\bar{\tau}_{D,1}} + \varepsilon_{\bar{\tau}_{D,2}}} \\ \alpha_3 &= \frac{1}{\gamma^R} \frac{-\varepsilon_{\bar{\tau}_{D,2}}}{1 + \varepsilon_{\bar{\tau}_{D,1}} + \varepsilon_{\bar{\tau}_{D,2}}}.\end{aligned}$$

With $\varepsilon_{\bar{\tau}_{D,2}} = 0$ (as in most of the paper) and assuming $\varepsilon_{\bar{\tau}_{D,1}} = 10$:

$$\begin{aligned}\alpha_1 &= \frac{\frac{1}{\beta\phi^*(1+i^*)/\pi^*} - 1}{(1+)^2} = \frac{\frac{1}{0.99(1.03^{0.25})1.005^{0.25}} - 1}{(1+10)^2} = 1.1692 \times 10^{-5} \\ \alpha_2 &= \frac{1}{\gamma^D} \frac{\varepsilon_{\bar{\tau}_{D,1}}}{1 + \varepsilon_{\bar{\tau}_{D,1}}} = \frac{1}{0.5} \frac{10}{1+10} = 1.8182 \\ \tau_D &= 1 + \frac{\alpha_1}{1 - \alpha_2\gamma^D + \alpha_3\gamma^R} = 1 + \alpha_1 (1 + \varepsilon_{\bar{\tau}_{D,1}} + \varepsilon_{\bar{\tau}_{D,2}}) \\ &= 1 + 1.1692 \times 10^{-5} (1 + 10) = 1.0001286\end{aligned}$$

The elasticity that appears in the log-linearized UIP equation varies linearly with $\varepsilon_{\bar{\tau}_D}$ as:

$$\begin{aligned}\varepsilon_{\varphi_D} &= \frac{\gamma^D}{\varphi_D} \frac{d\varphi_D}{d\gamma^D} = \frac{\bar{\varphi}_D}{\varphi_D} \left(\frac{\gamma^D}{\bar{\varphi}_D} \frac{d\bar{\varphi}_D}{d\gamma^D} \right) = [1 - \beta\phi^*(1+i^*)/\pi^*] \varepsilon_{\bar{\varphi}_D} \\ &= [1 - \beta\phi^*(1+i^*)/\pi^*] 2\varepsilon_{\bar{\tau}_D} = [1 - 0.99(1.03^{0.25})1.005^{0.25}] 2\varepsilon_{\bar{\tau}_D} \\ &= 0.0028255\varepsilon_{\bar{\tau}_D}.\end{aligned}$$

The balance of payments

Using the previous calibrations, (97) and (96) give the trade balance to GDP ratio necessary to sustain net interest payments abroad:

$$\begin{aligned} \frac{TB}{Y} \frac{e}{Y} &= \left(\frac{1+i^*}{\pi^*} \phi^* \tau_D - 1 \right) \gamma^D - \left(\frac{1+i^*}{\pi^*} - 1 \right) \gamma^R \\ &= \left(\frac{1.03^{0.25}}{1} 1.005^{0.25} (1.0001286) - 1 \right) 0.5 - \left(\frac{1.03^{0.25}}{1} - 1 \right) 0.13 \\ &= 0.0034376 \end{aligned}$$

Then, using (95), (94), and (103), one can obtain the RER necessary to generate this trade surplus:

$$\begin{aligned} \kappa_X (ep^*)^{b_X} \left[a_D + (1 - a_D) e^{1-\theta^C} \right] - (1 - a_D) e^{1-\theta^C} &= a_D \frac{TB}{Y} \frac{e}{Y} \\ 0.5e^2 \left[0.86 + (1 - 0.86) e^{1-1.5} \right] - (1 - 0.86) e^{1-1.5} &= 0.86 (0.003437639) \\ e &= 0.595128 \end{aligned}$$

and hence the exports to GDP ratio and p^C :

$$\begin{aligned} \frac{X}{Y} &= \kappa_X (ep^*)^{b_X} = 0.5 (0.595128)^2 = 0.177089, \\ p^C &= \left(0.86 + (1 - 0.86) (0.595128)^{1-1.5} \right)^{\frac{1}{1-1.5}} = 0.921935 \end{aligned}$$

The transactions cost function and money demand

The elasticity of $\mathcal{L}(1+i)$ (see (70)) can be shown to satisfy the following relation:

$$\varepsilon_{\mathcal{L}}(\gamma^M) = \frac{1}{(\beta_3 + 1)i} \left(1 + \frac{1}{\beta_2 \gamma^M} \right), \quad (117)$$

which gives:

$$\beta_2 = \frac{1}{\gamma^M} \frac{1}{(\beta_3 + 1) \varepsilon_{\mathcal{L}} i - 1}.$$

Also, reshuffling (70) yields:

$$\beta_1 = \frac{(1 + \beta_2 \gamma^M)^{\beta_3 + 1}}{\beta_2 \beta_3} \left(1 - \frac{1}{1+i}\right).$$

So using the last two expressions in (68) to eliminate β_1 and β_2 gives:

$$\bar{\tau}_M (\gamma^M) = \left(1 + \frac{1}{\beta_3}\right) \left(1 - \frac{1}{1+i}\right) i \varepsilon_{\mathcal{L}} (\gamma^M) \gamma^M. \quad (118)$$

Since transaction costs are dependent on the inflation rate (through the nominal interest rate) the three parameters β_1 , β_2 , and β_3 cannot be calibrated without first calibrating the inflation rate. Assume that the target inflation rate is $\pi^T = 1.015$. Hence, the nominal interest rate is given by (88): $1 + i = 1.015/0.99 = 1.0253$. Next, calibrate the value of the interest elasticity of money demand to, say, $\varepsilon_{\mathcal{L}} = 1.02$. Notice that to have β_2 positive, β_3 must be sufficiently high (and hence $\bar{\tau}_M$ sufficiently low):²⁹

$$\begin{aligned} \beta_3 &> \frac{1}{\varepsilon_{\mathcal{L}} i} - 1 = \frac{1}{1.02(1.015/0.99 - 1)} - 1 = 37.82352941 \\ \bar{\tau}_M &< \left(1 + \frac{1}{37.82352941}\right) \left(1 - \frac{1}{1.015/0.99}\right) \left(\frac{1.015}{0.99} - 1\right) 0.095522 * 1.02 \\ &= 0.00006220357. \end{aligned}$$

If, say, $\beta_3 = 160$, then:

$$\begin{aligned} \beta_2 &= \frac{1}{0.095522 (160 + 1) 1.02 \left(\frac{1.015}{0.99} - 1\right) - 1} = 3.3266 \\ \beta_1 &= \frac{(1 + 3.3266 * 0.095522)^{160+1}}{3.3266 * 160} \left(1 - \frac{0.99}{1.015}\right) = 9.1058 \times 10^{14}. \end{aligned}$$

²⁹Although this level of transaction costs may seem unrealistically low, in this paper transaction costs per se do not matter but only their effect on money demand. To have more realistic levels of transaction costs a different transaction costs function would be required.

Hence:

$$\begin{aligned}\bar{\tau}_M &= \frac{\beta_1}{(1 + \beta_2 \gamma^M)^{\beta_3}} = \frac{9.1058 \times 10^{14}}{(1 + 3.3266 * 0.095522)^{160}} = 6.098 \times 10^{-5} \\ \bar{\varphi}_M &= \bar{\tau}_M \left(1 + \beta_3 \frac{\beta_2 \gamma^M}{1 + \beta_2 \gamma^M} \right) = 6.098 \times 10^{-5} \left(1 + 160 \frac{3.3266 * 0.095522}{1 + 3.3266 * 0.095522} \right) \\ &= 2.4137 \times 10^{-3}.\end{aligned}$$

Finally, using (102), the consumption to GDP ratio is:

$$\begin{aligned}\frac{p^C C}{Y} &= \frac{(p^C)^{1-\theta^C}}{a_D \tau_M G} \left[1 - \kappa_X (ep^*)^{b^X} \right] \\ &= \frac{(0.921935)^{1-1.5}}{0.86 * 1.00006098 * 1.19} (1 - 0.177089) = 0.8374.\end{aligned}$$

Hence, C and Q can be obtained:

$$\begin{aligned}C &= \frac{p^C C}{Y} \frac{Y}{p^C} = 0.8374 * \frac{1.443}{0.921935} = 1.3107 \\ Q &= \left[1 - (1 - b^A) \frac{X}{Y} \right] Y = [1 - (1 - 0.5) 0.177089] 1.443 = 1.3152.\end{aligned}$$

Inflation, price dispersion and marginal cost

(93) shows NSS price dispersion as a function of the NSS inflation rate. It is easy to check that this function has a local minimum at $\pi = 1$, where there is price stability and no price dispersion ($\Delta = 1$). Given the above calibrations, the NSS value of price dispersion is:

$$\Delta = \frac{1 - 0.66}{1 - 0.66 (1.015)^6} \left(\frac{1 - 0.66 (1.015)^{6-1}}{1 - 0.66} \right)^{\frac{6}{6-1}} = 1.0051.$$

Hence, (101) gives the value of hours worked:

$$N = Q\Delta = 1.31526 * 1.0051 = 1.32197,$$

(90) gives the value of Γ :

$$\Gamma = \frac{Q / (p^C C^{\sigma^C})}{1 - \beta \alpha \pi^{\theta-1}} = \frac{1.3152 / (0.921935 * 1.3107^{1.5})}{1 - 0.99 * 0.66 * 1.015^{6-1}} = 3.210803,$$

(92) gives the value of Ψ :

$$\Psi = \Gamma \left(\frac{1 - \alpha}{1 - \alpha \pi^{\theta-1}} \right)^{\frac{1}{\theta-1}} = 3.21066 \left(\frac{1 - 0.66}{1 - 0.66 * 1.015^{6-1}} \right)^{\frac{1}{6-1}} = 3.316898,$$

and (91) gives the value of mc :

$$\begin{aligned} mc &= \Psi / \left(\frac{\theta}{\theta-1} \frac{Q / (p^C C^{\sigma^C})}{1 - \beta \alpha \pi^{\theta}} \right) \\ &= 3.210803 / \left(\frac{6}{6-1} \frac{1.3152 / (0.921935 * 1.3107^{1.5})}{1 - 0.99 * 0.66 * 1.015^6} \right) = 0.830172. \end{aligned}$$

Finally, (98) and (99) give the value of ξ^N :

$$\begin{aligned} \xi^N &= mc / (p^C C^{\sigma^C} \varphi_M N^{\sigma^N}) \\ &= 0.830172 / (0.921935 * 1.3107^{1.5} * 1.0024137 * 1.32197^{0.5}) = 0.52067, \end{aligned}$$

and the NSS value of period aggregate utility is:

$$\text{Utility} = \frac{1.3107^{1-1.5}}{1-1.5} - 0.52067 \frac{1.32197^{1+0.5}}{1+0.5} = -2.2745.$$

The fact that it is negative is irrelevant, since utility has only ordinal, not cardinal, significance.

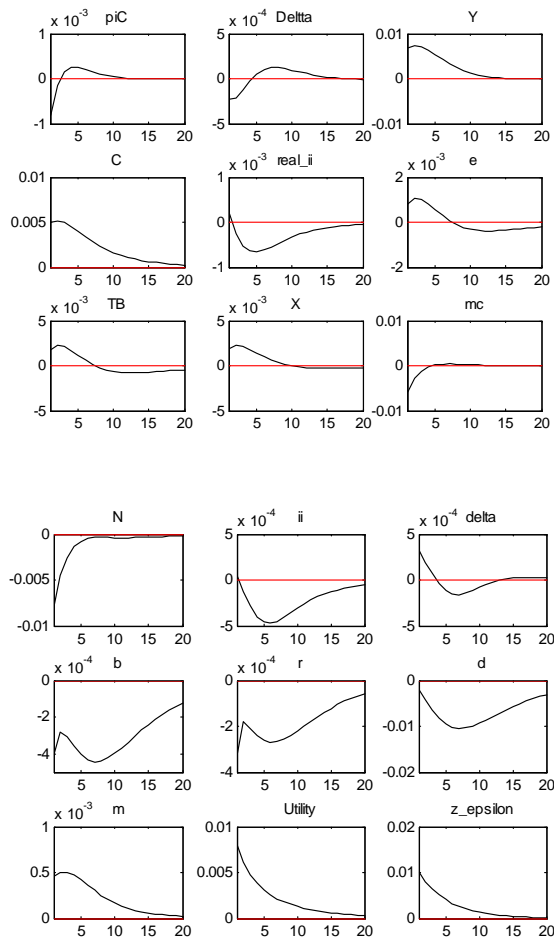
A.2 Impulse response functions for optimal policies under commitment (MER regime, CB styles A, B, and C)

All shocks in the IRFs below are positive and of 1 standard deviation. Shock variables are in logs in the nonlinear model. The IRFs correspond to the specified weights on the loss function:

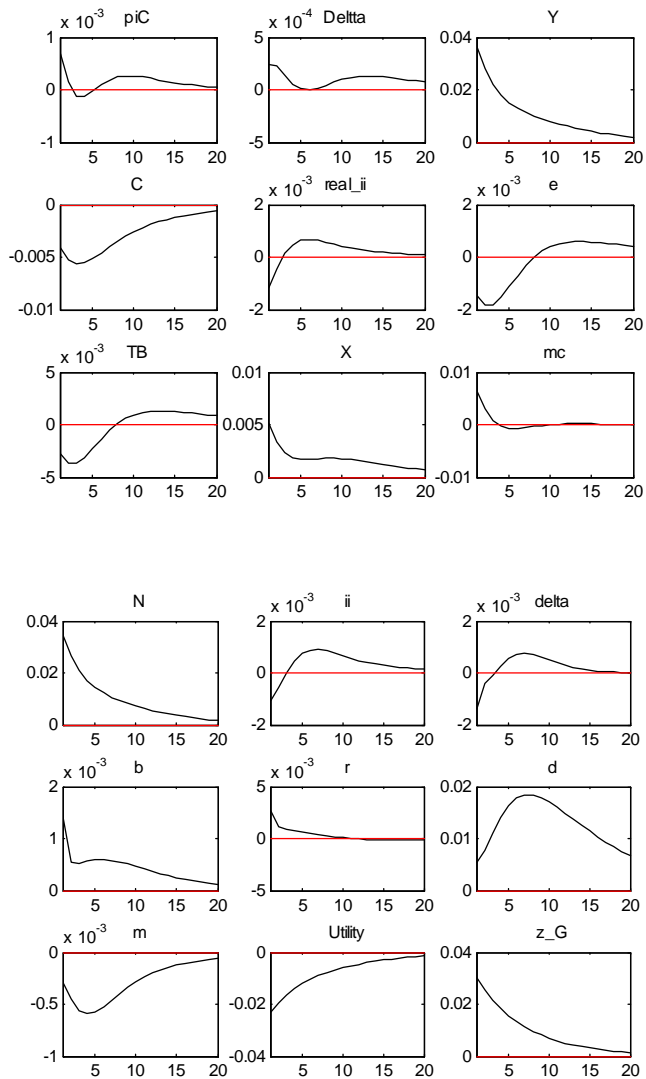
Central Bank style A

$$\omega_\pi = 100, \quad \omega_Y = 1, \quad \omega_e = 1, \quad \omega_r = 1, \quad \omega_{\Delta i} = 50, \quad \omega_{\Delta\delta} = 50.$$

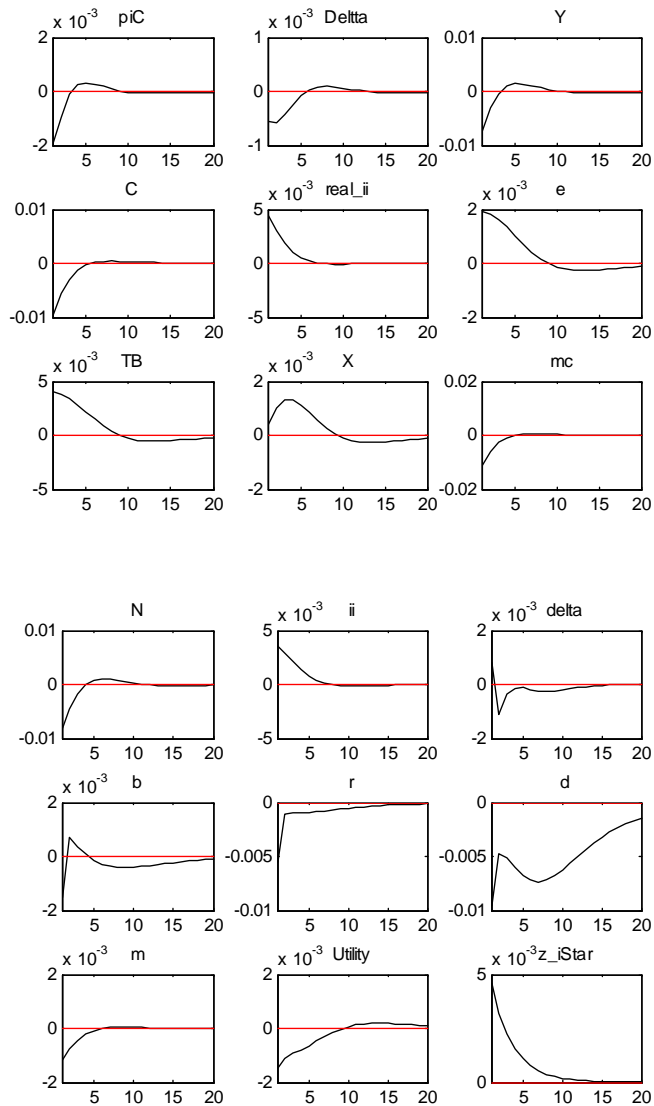
Response to a shock to domestic sector productivity: ε



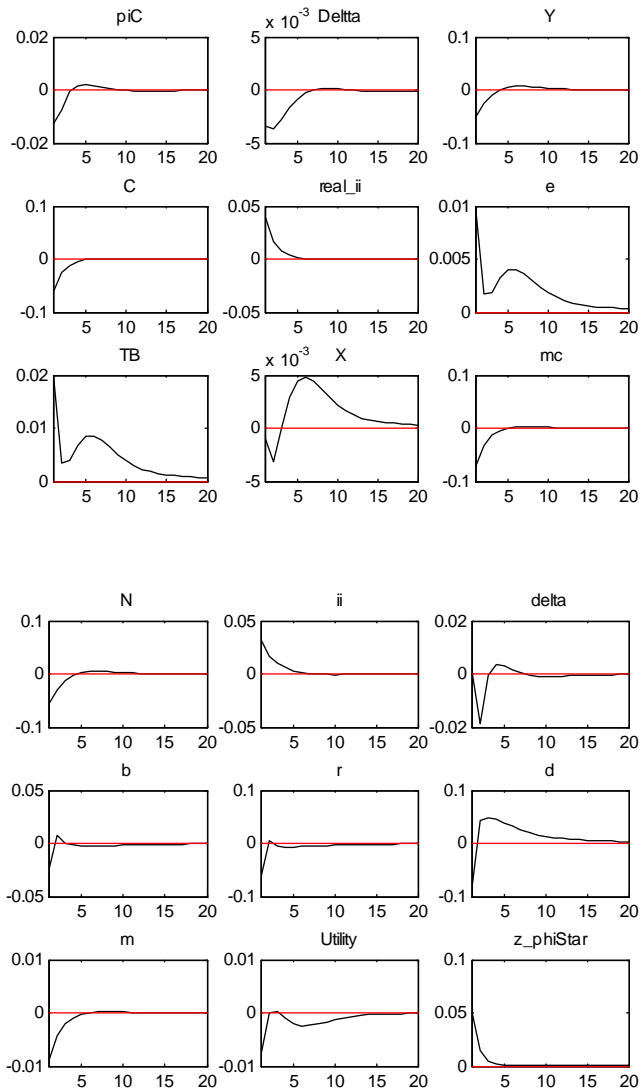
Response to a shock to government expenditures: G



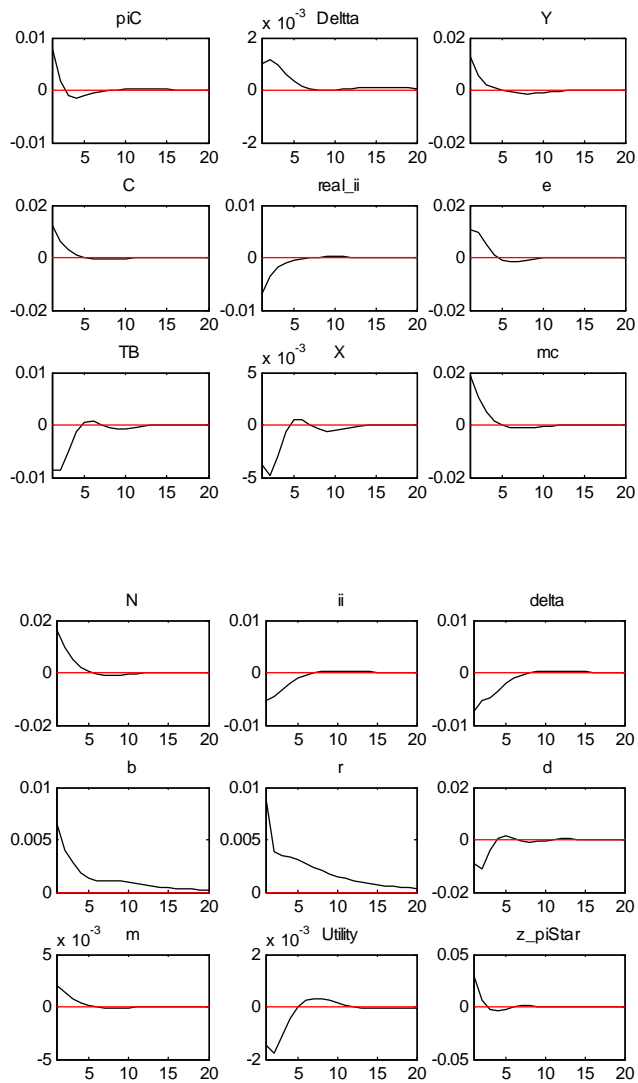
Response to a shock to the RW interest rate: i^*



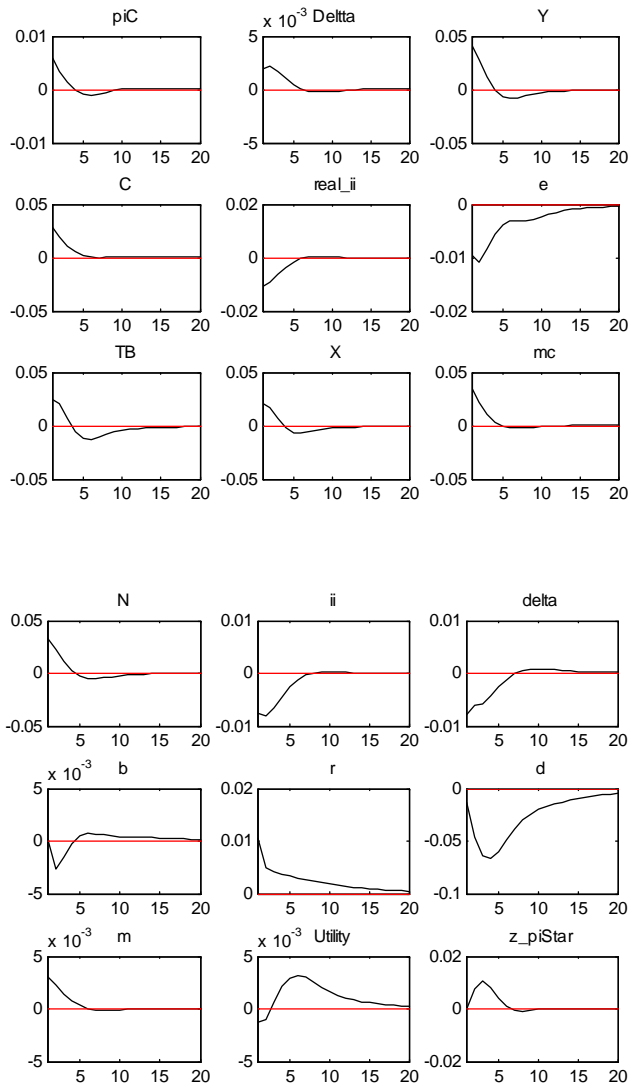
Response to a shock to the SOE's exogenous risk/liquidity premium: ϕ^*



Response to a shock to imports inflation: π^*



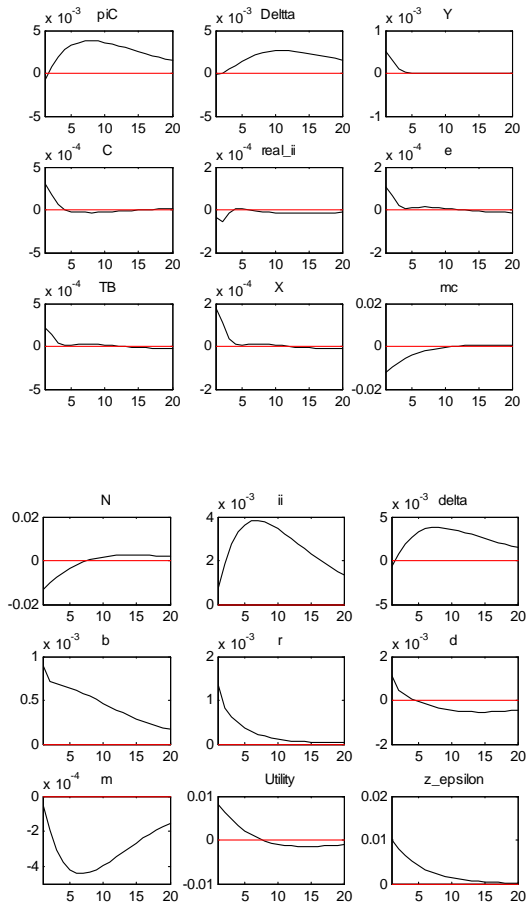
Response to a shock to exports inflation: π^*X



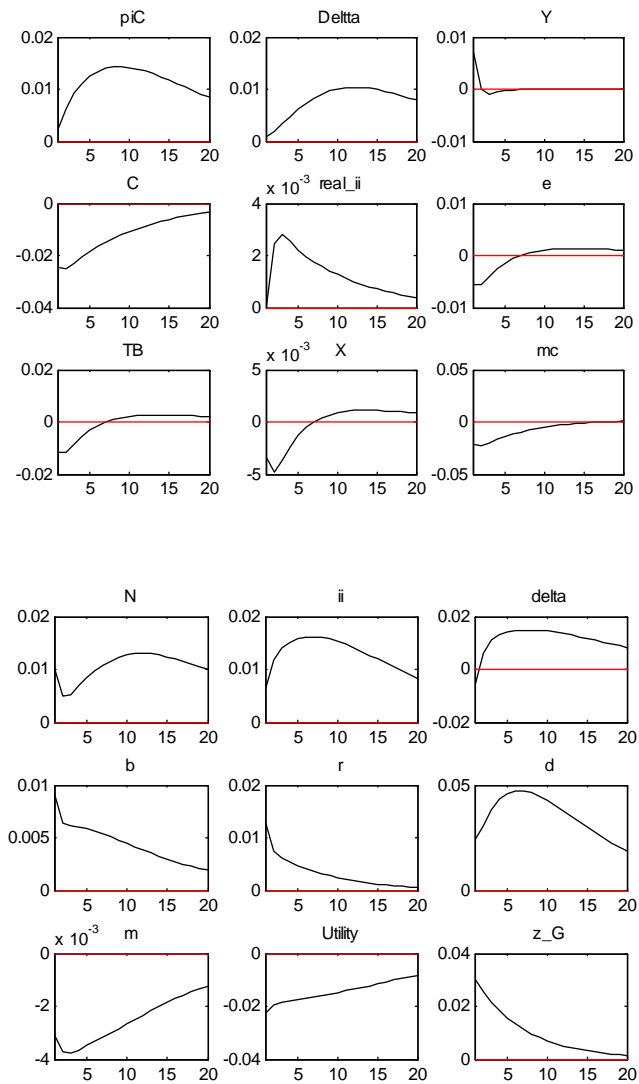
Central Bank style B

$$\omega_\pi = 1, \quad \omega_Y = 100, \quad \omega_e = 1, \quad \omega_r = 1, \quad \omega_{\Delta i} = 50, \quad \omega_{\Delta\delta} = 50.$$

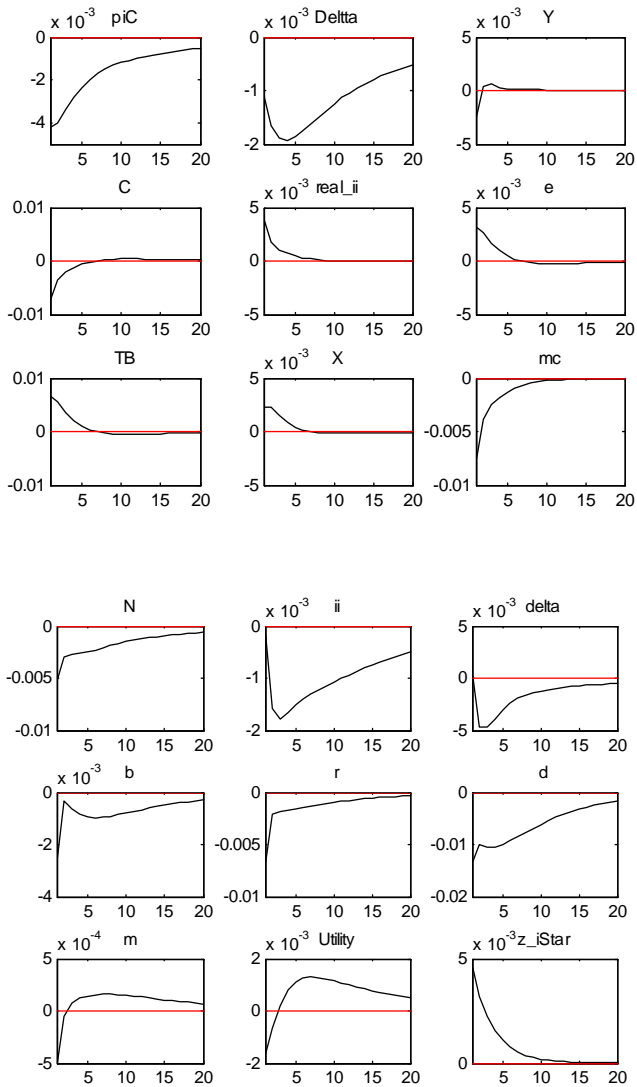
Response to a shock to domestic sector productivity: ε



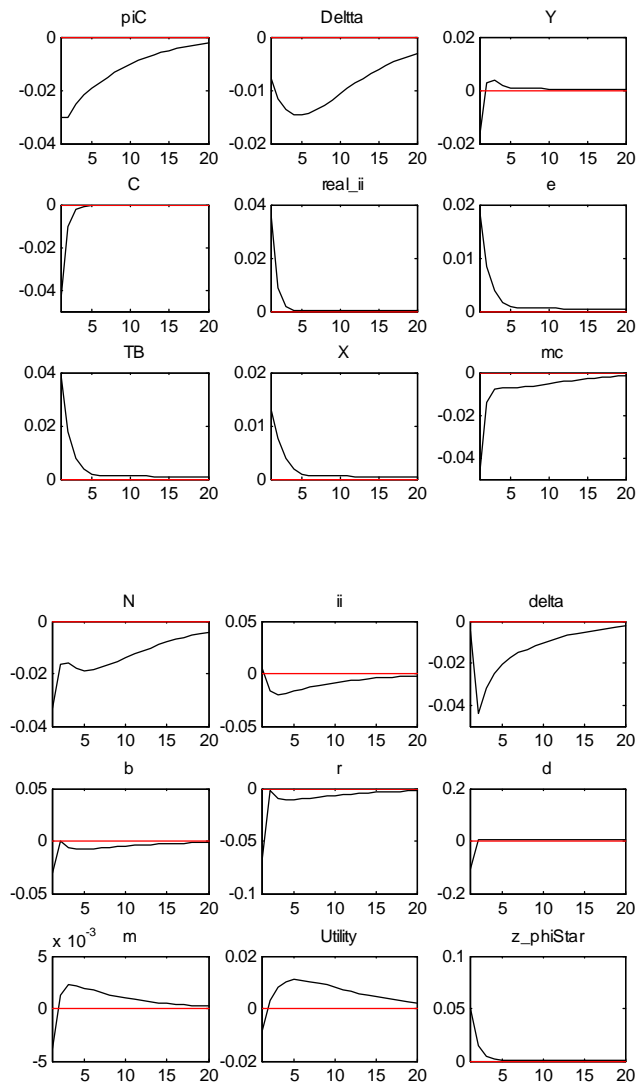
Response to a shock to government expenditures: G



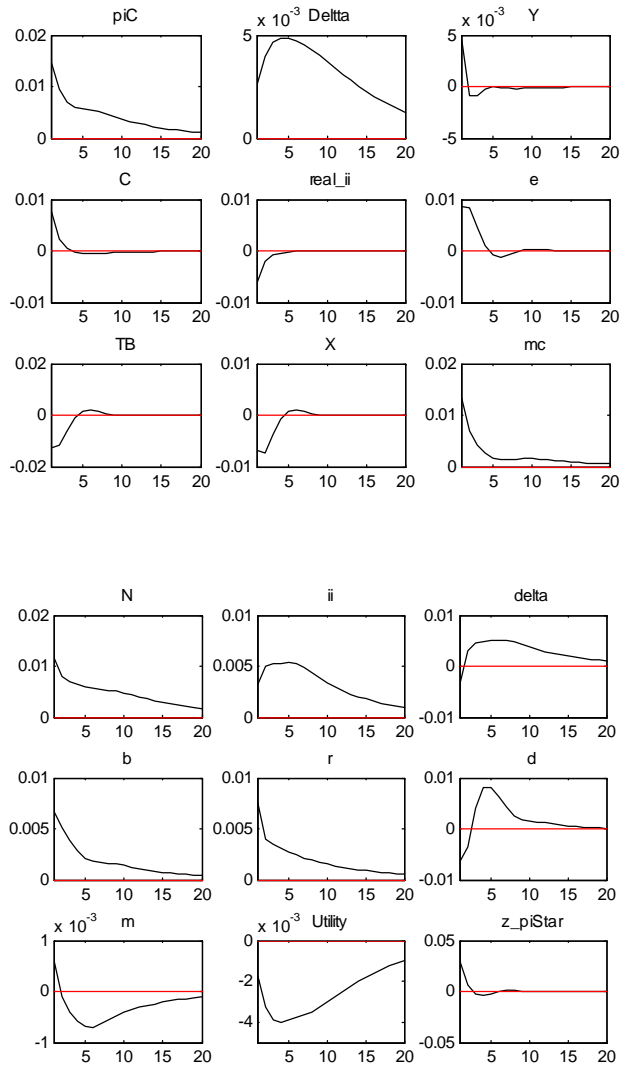
Response to a shock to the RW interest rate: i^*



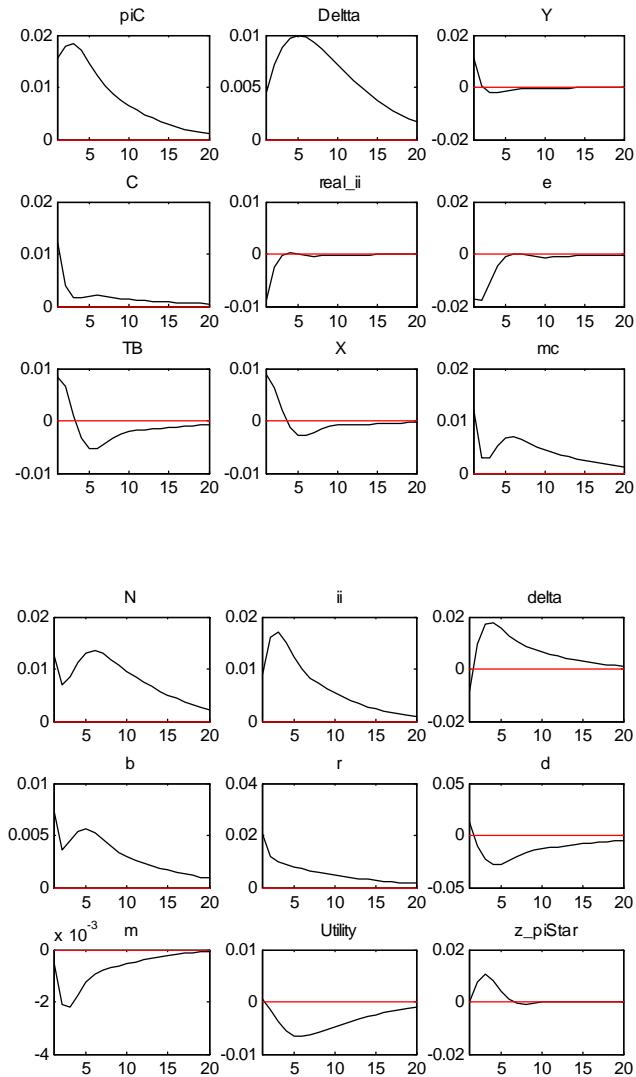
Response to a shock to the SOE's exogenous risk/liquidity premium: ϕ^*



Response to a shock to imports inflation: π^*



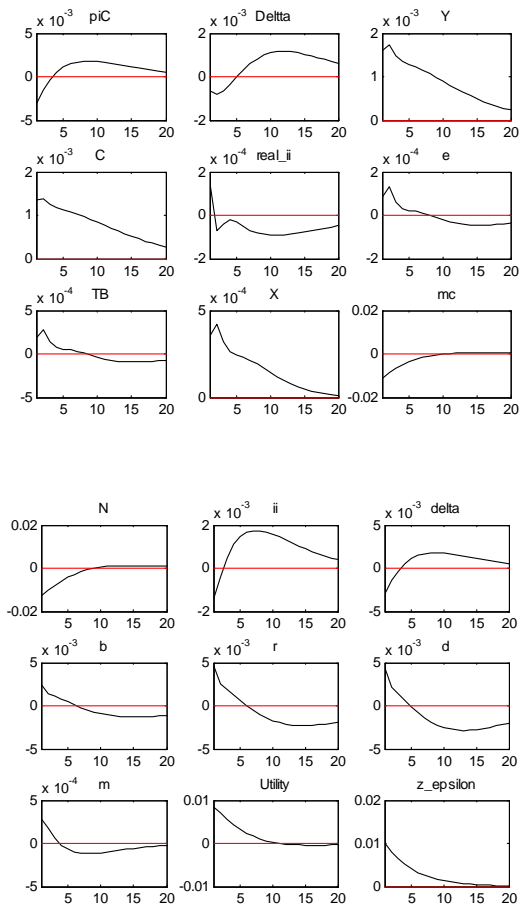
Response to a shock to exports inflation: π^*X



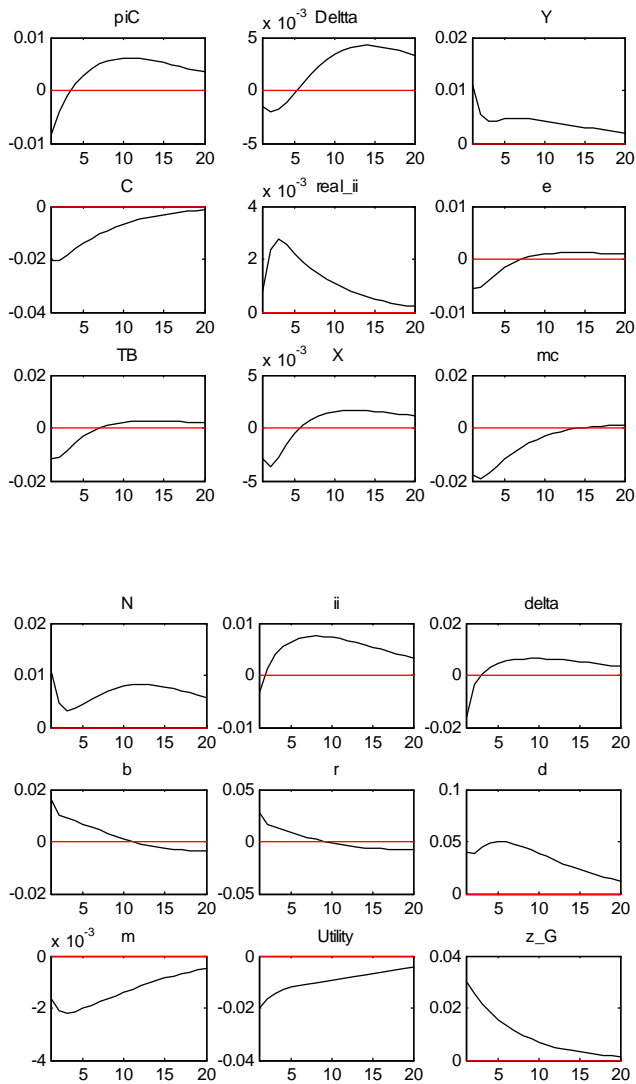
Central Bank style C

$$\omega_{\pi} = 100, \quad \omega_Y = 100, \quad \omega_e = 1, \quad \omega_r = 1, \quad \omega_{\Delta i} = 50, \quad \omega_{\Delta\delta} = 50.$$

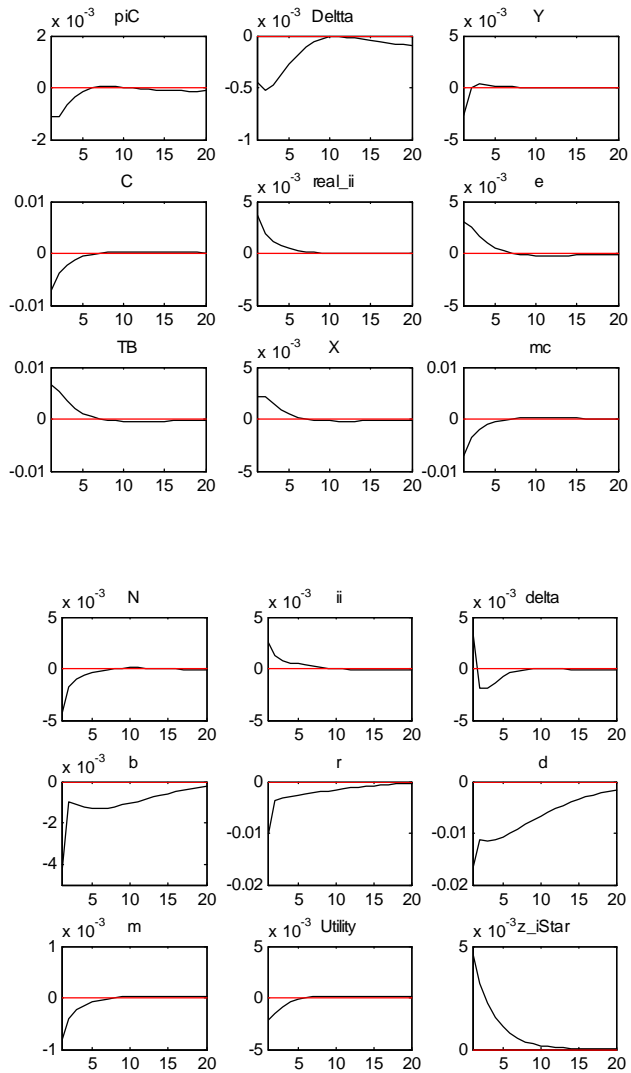
Response to a shock to domestic sector productivity: ε



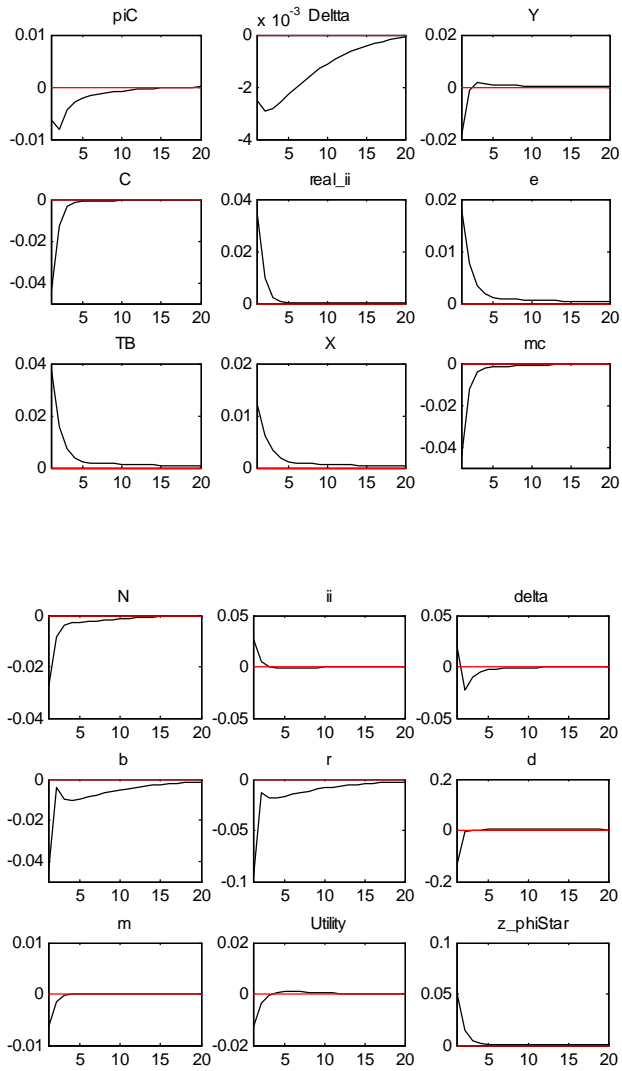
Response to a shock to government expenditures: G



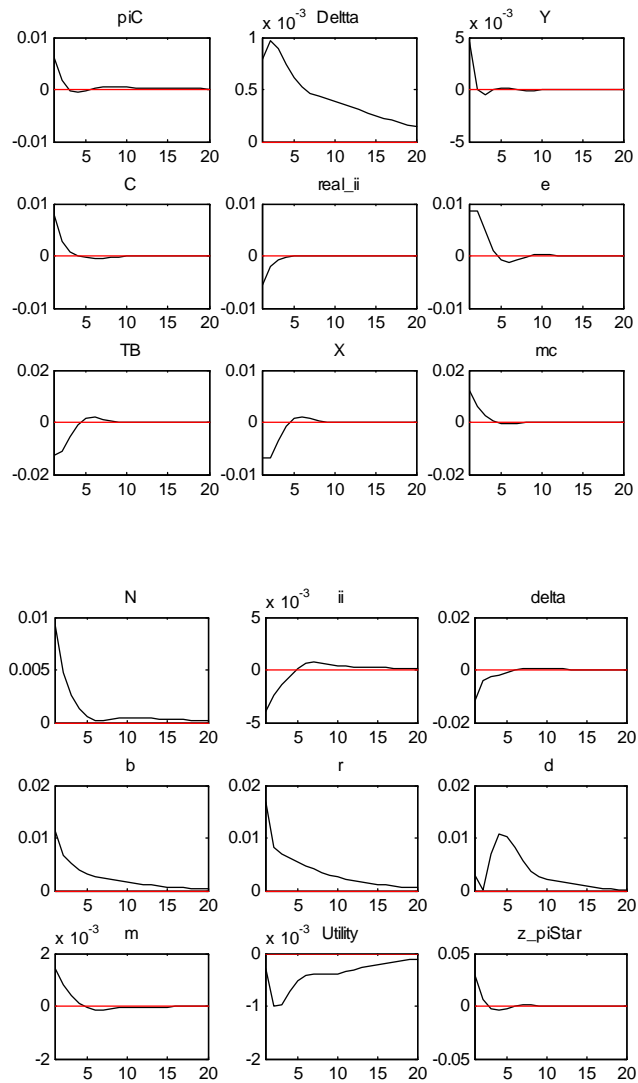
Response to a shock to the RW interest rate: i^*



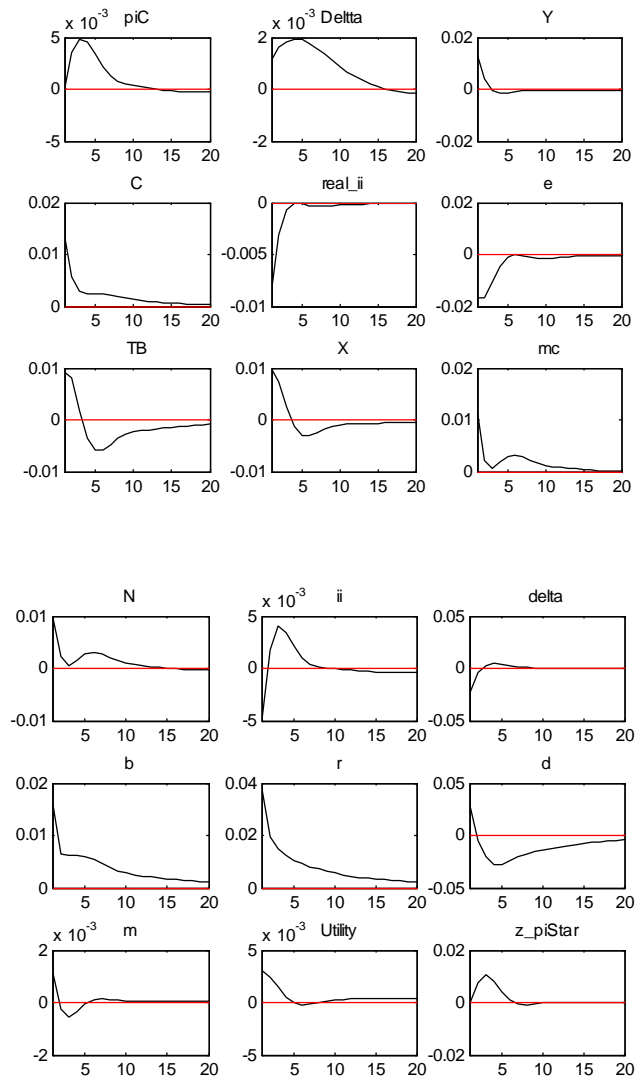
Response to a shock to the SOE's exogenous risk/liquidity premium: ϕ^*



Response to a shock to imports inflation: π^*



Response to a shock to exports inflation: π^*X



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