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Risk Taking in Asymmetric Tournaments*

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Abstract

In an asymmetric tournament model with endogenous risk choice by the agents it is shown that equilibrium efforts decrease (increase) with risk if abilities are sufficiently similar (different). Risk also affects winning probabilities. The interaction of both effects is analyzed.

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1 Introduction

Rank-order tournaments have been extensively discussed in labor economics, sports and other fields (see, e.g., Lazear and Rosen 1981; Nalebuff and Stiglitz 1983). In a tournament, agents compete against each other for given prizes that have been fixed in advance. When choosing their optimal efforts agents have to trade off an increased winning probability against a higher disutility of work. In practice, agents do not only choose efforts but have also to decide between more or less risky actions.

Previous literature has pointed at two effects when an endogenous choice of risk is considered: First, it has been claimed that in asymmetric tournaments agents with high abilities will prefer less risky actions to preserve their favorable positions, whereas agents with low abilities, who have nothing to lose, will prefer more risky actions to increase the likelihood of winning (e.g., Rosen 1988, p. 84, referring to Bronars 1986; Knoeber and Thurman 1994, p. 158). Second, in symmetric tournaments agents choose riskier strategies as a higher total variance of the outcome leads to lower equilibrium efforts and, therefore, lower costs (compare for instance Hvide 2000).

We analyze a simple asymmetric tournament where the agents can determine the risk of their strategy on the first stage and their effort level on the second stage. As a first contribution, we show that it is no longer true in general that equilibrium efforts decrease in total variance. In contrast, if the ability difference is large enough, high risk will lead to low efforts: For high ability differences a higher risk tends to bring back the less able agent into the race and therefore raises overall incentives to exert effort which is of course bad from the agents' point of view. On the other hand, the above mentioned "likelihood" effect continues to be of importance. Whereas the agents' interests are aligned with respect to reducing equilibrium effort they

are strongly opposed when the likelihood effect is considered. We study the interaction of both effects and characterized subgame perfect equilibria of the game.

2 Model and Results

We consider a two-stage tournament between the two risk neutral agents A and B . Agent i 's ($i = A, B$) production function can be described by $y_i = a_i + e_i + \varepsilon_i$ where a_i denotes ability, e_i effort and ε_i an individual noise term.¹ ε_A and ε_B are assumed to be stochastically independent with $\varepsilon_i \sim N(0, \sigma_{r_i}^2)$. Let $\Delta a_i = a_i - a_j$ and $\Delta a = |\Delta a_i| = |\Delta a_j|$. The agents' cost functions are $c(e_i) = \frac{c}{2}e_i^2$. On the first stage, both agents observe the given abilities and simultaneously choose the risk of their respective production technologies, r_i , with $r_i \in \{L, H\}$ and $\sigma_H^2 > \sigma_L^2$. On the second stage, each agent observes the chosen risks and decides about his effort e_i . The two agents compete for tournament prizes w_1 and w_2 with $w_1 > w_2 \geq 0$. The prize spread $w_1 - w_2$ is denoted by Δw . If $y_i > y_j$, agent i will receive the winner prize w_1 whereas agent j gets w_2 ($i, j = A, B; i \neq j$).

First we examine the tournament competition on the second stage.²

Proposition 1 *A pure strategy equilibrium on the second stage is symmetric. For given risk choices r_i both agents exert the following effort level:³*

$$e^*(r_i, r_j, \Delta a) = \frac{\Delta w}{c} \frac{1}{\sqrt{(\sigma_{r_i}^2 + \sigma_{r_j}^2)}} \phi \left(\frac{\Delta a}{\sqrt{\sigma_{r_i}^2 + \sigma_{r_j}^2}} \right).$$

¹Alternatively, we could model heterogeneous agents by using different cost functions. But the additive model has the advantage that we can interpret the ability difference also as an agent's lead in a homogeneous tournament.

²For the existence of pure-strategy equilibria see the discussion in Lazear and Rosen (1981), p. 845, fn. 2; Nalebuff and Stiglitz (1983).

³Here $\phi(\cdot)$ denotes the density of a standard normal distribution.

The effort is strictly increasing in Δw and strictly decreasing in Δa and c .

The effort is single peaked in $\sigma_{r_i}^2 + \sigma_{r_j}^2$ and highest at $\sigma_{r_i}^2 + \sigma_{r_j}^2 = \Delta a^2$.

Proof. On stage 2, agent i 's objective function is given by $u_i(e_i) = w_2 + \Delta w \cdot \text{pr}\{y_i > y_j\} - \frac{c}{2}e_i^2$ with $\text{pr}\{y_i > y_j\} = G(e_i - e_j + \Delta a_i; r_i, r_j)$ as i 's probability of winning where $G(\cdot; r_i, r_j)$ denotes the cdf of the composed random variable $\varepsilon_j - \varepsilon_i$ which is again normally distributed with $\varepsilon_j - \varepsilon_i \sim N(0, \sigma_{r_1}^2 + \sigma_{r_2}^2)$. Since $G(\cdot; r_i, r_j)$ is symmetric, we have $1 - G(e_i - e_j + \Delta a_i; r_i, r_j) = G(-e_i + e_j - \Delta a_i; r_i, r_j)$ for agent j 's winning probability. The first-order condition for agent i yields $g(e_i - e_j + \Delta a_i; r_i, r_j) \Delta w = ce_i$ with $g(\cdot; r_i, r_j) = G'(\cdot; r_i, r_j)$. Since $g(e_i - e_j + \Delta a_i; r_i, r_j) = g(-e_i + e_j - \Delta a_i; r_i, r_j)$ the left-hand sides of both agents' first-order conditions are identical which implies a symmetric equilibrium with $e^*(r_i, r_j; \Delta a) = g(\Delta a; r_i, r_j) \Delta w / c$. The last part is proved by substituting for the normal density and checking that $\partial e^* / \partial (\sigma_{r_i}^2 + \sigma_{r_j}^2) \gtrless 0$ iff $\sigma_{r_i}^2 + \sigma_{r_j}^2 \lesseqgtr \Delta a^2$. ■

Now we can go back to stage 1 and analyze the optimal risk choice by each agent. The variance affects the agents' efforts as well as their winning probabilities. We start by taking a closer look at the first aspect.

Lemma 1 For a given technology choice r_j of the other player there exists a threshold level for the ability difference, $\Delta \hat{a}$, so that $e_i^*(H, r_j; \Delta a) \gtrless e_i^*(L, r_j; \Delta a)$ for $\Delta a \gtrless \Delta \hat{a}(r_j)$. Furthermore $\Delta \hat{a}(H) > \Delta \hat{a}(L)$.

Proof. Compare,

$$e_i^*(H, r_j; \Delta a) \gtrless e_i^*(L, r_j; \Delta a) \Leftrightarrow \frac{\Delta w}{c} \frac{\exp\left(-\frac{\Delta a^2}{2(\sigma_H^2 + \sigma_{r_j}^2)}\right)}{\sqrt{2\pi(\sigma_H^2 + \sigma_{r_j}^2)}} \gtrless \frac{\Delta w}{c} \frac{\exp\left(-\frac{\Delta a^2}{2(\sigma_L^2 + \sigma_{r_j}^2)}\right)}{\sqrt{2\pi(\sigma_L^2 + \sigma_{r_j}^2)}} \Leftrightarrow$$

$$\Delta a^2 \geq \frac{2(\sigma_L^2 + \sigma_{r_j}^2)(\sigma_H^2 + \sigma_{r_j}^2)}{\sigma_H^2 - \sigma_L^2} \ln \sqrt{\frac{\sigma_H^2 + \sigma_{r_j}^2}{\sigma_L^2 + \sigma_{r_j}^2}}.$$

which proves the first part of the lemma. To see that $\Delta \hat{a}(H) > \Delta \hat{a}(L)$ compare

$$\frac{2(\sigma_L^2 + \sigma_H^2)(\sigma_H^2 + \sigma_H^2)}{\sigma_H^2 - \sigma_L^2} \ln \sqrt{\frac{\sigma_H^2 + \sigma_H^2}{\sigma_L^2 + \sigma_H^2}} > \frac{2(\sigma_L^2 + \sigma_L^2)(\sigma_H^2 + \sigma_L^2)}{\sigma_H^2 - \sigma_L^2} \ln \sqrt{\frac{\sigma_H^2 + \sigma_L^2}{\sigma_L^2 + \sigma_L^2}}.$$

Now let $\sigma_H^2 = k\sigma_L^2$ with $k > 1$ so that the inequality can be rewritten as

$$k \ln \sqrt{\frac{2k}{1+k}} > \ln \sqrt{\frac{k+1}{2}} \Leftrightarrow \Psi(k) := 2^{k+1}k^k - (1+k)^{(1+k)} > 0$$

which is true as $\Psi(1) = 0$ and $\Psi(k)$ monotonically increases for $k > 1$. ■

Lemma 1 describes the effort effect of risk taking. A riskier technology will reduce the equilibrium effort of both agents if and only if the ability difference Δa is sufficiently small. For large values of Δa a riskier technology increases the equilibrium effort of both agents. To understand this result on an intuitive level, suppose that the ability difference is very large and the risk of the technologies is low. In that case, the outcome of the tournament is largely determined by the abilities of both agents. On the one hand, the more able player knows that he will win the tournament with a high probability even with low effort. On the other hand, the less able player can affect his probability of winning only to a small extent. If the variance of the technologies increases, however, luck may compensate the ability difference which then will increase the impact of effort on the outcome of the tournament. This makes exerting effort more attractive for the low ability agent. That in turn forces the high ability agent to exert higher effort levels as well. For small values of Δa the outcome of the tournament is less dependent on the abilities of the players but on effort. In such a situation, choosing a risky strategy decreases the influence of effort on the

outcome. As in that way the marginal return of effort for both players is lowered, they both can commit to exert low efforts by selecting a high risk strategy.

On a more technical level, as we have seen in Proposition 1 there is a unique positive value for the total variance (namely Δa^2) at which the equilibrium effort is maximized. Hence, depending on the values of σ_H^2 and σ_L^2 relative to Δa either high risk choices or low risk choices by both agents minimize effort. If $\Delta a > \Delta \hat{a}(H)$ then Δa will be sufficiently large relative to σ_H^2 and σ_L^2 . Therefore, effort will be minimized with a low risk choice by both. On the other hand, if $\Delta a < \Delta \hat{a}(L)$ then Δa will be sufficiently small relative to σ_H^2 and σ_L^2 and effort will be minimized with a high risk choice by both. If $\Delta \hat{a}(L) < \Delta a < \Delta \hat{a}(H)$, however, the effort is highest if one player chooses a high risk, the other one a low risk strategy. Total effort is minimized when either both choose a high risk or a low risk strategy. If only considering the effort effect, the risk choices by both players are strategic complements. A coordination problem exists although both agents' interests are perfectly aligned.

As we have pointed out above risk taking influences as well each agent's probability of winning. This likelihood effect is characterized by Lemma 2:

Lemma 2 *Player i 's probability of winning will decrease (increase) in $\sigma_{r_i}^2 + \sigma_{r_j}^2$ if he is the agent with the higher (lower) ability. If both agents have the same ability, the winning probability will not be affected by risk.*

Proof. We have $G(\Delta a_i; r_i, r_j) = \Phi\left(\Delta a_i / \sqrt{\sigma_{r_i}^2 + \sigma_{r_j}^2}\right)$ with $\Phi(\cdot)$ as the cdf of the standardized normal distribution. Let $\hat{\sigma}^2 = \sigma_{r_i}^2 + \sigma_{r_j}^2$ and let $\hat{\sigma}_H^2 > \hat{\sigma}_L^2$ be two possible values of $\hat{\sigma}^2$. As $\Phi(\cdot)$ is monotonically increasing, we obtain $\Phi\left(\Delta a_i / \sqrt{\hat{\sigma}_H^2}\right) \begin{matrix} \leq \\ \geq \end{matrix} \Phi\left(\Delta a_i / \sqrt{\hat{\sigma}_L^2}\right)$ if $\Delta a_i \begin{matrix} \geq \\ \leq \end{matrix} 0$. ■

The winning probability of the agent with the higher ability decreases

and that of the agent with the lower ability increases with total variance. Hence, more able agents will prefer the less risky and less able agents the more risky technology if they only consider the probability of winning the tournament. Recall that when only considering the impact of the technology choice on the effort effect there is no conflict of interests between the two agents. If the ability difference Δa is high they both will prefer a low risk, and when Δa is low, they both want a high risk. However, when only the likelihood effect is considered there is a strong conflict of interests. The equilibrium outcome on stage 1 clearly depends on the relative importance of both effects.

For a given strategy r_j player i will prefer a high risk to a low risk if

$$\Delta w G(\Delta a_i; H, r_j) - c(e^*(H, r_j; \Delta a)) > \Delta w G(\Delta a_i; L, r_j) - c(e^*(L, r_j; \Delta a)) \Leftrightarrow$$

$$\begin{aligned} & \frac{\Delta w}{2c} \left[\left(\frac{\phi\left(\frac{\Delta a}{\sqrt{\sigma_L^2 + \sigma_{r_j}^2}}\right)}{\sqrt{\sigma_L^2 + \sigma_{r_j}^2}} \right)^2 - \left(\frac{\phi\left(\frac{\Delta a}{\sqrt{\sigma_H^2 + \sigma_{r_j}^2}}\right)}{\sqrt{\sigma_H^2 + \sigma_{r_j}^2}} \right)^2 \right] \\ & > \Phi\left(\frac{\Delta a_i}{\sqrt{\sigma_L^2 + \sigma_{r_j}^2}}\right) - \Phi\left(\frac{\Delta a_i}{\sqrt{\sigma_H^2 + \sigma_{r_j}^2}}\right). \end{aligned} \quad (1)$$

Hence, if there is no difference in abilities the equilibrium is straightforward:⁴

Proposition 2 *If $\Delta a = 0$ both players always choose the risky strategy, i.e. $(r_i, r_j) = (H, H)$.*

If both players have different abilities, effort and likelihood effect may work into opposite directions and the total effect clearly depends on the parameter values. As the left-hand side of inequality (1) is linearly increasing in $\Delta w/c$, all decisions are dominated by the left-hand side of the inequality for sufficiently large values of $\Delta w/c$. The expression in square brackets is

⁴This result has also been shown in Hvide (2000).

positive if and only if $e^*(H, r_j; \Delta a) < e^*(L, r_j; \Delta a)$. From Lemma 1 we know that this will be the case if and only if $\Delta a < \Delta \hat{a}(r_j)$. We can use these considerations to find all subgame perfect equilibria of the game:

Proposition 3 *(i) If $\Delta a > \Delta \hat{a}(H)$ the more able agent will always choose L , the less able agent will choose L if and only if $\Delta w/c$ is larger than a certain cutoff value. (ii) If $0 < \Delta a < \Delta \hat{a}(L)$ the less able agent will always choose H , the more able agent will choose H if and only if $\Delta w/c$ is larger than a certain cutoff value. (iii) If $\Delta a(L) < \Delta a < \Delta a(H)$ there will be an asymmetric equilibrium in which the more able agent chooses L and the less able agent chooses H if $\Delta w/c$ is sufficiently small. If $\Delta w/c$ is sufficiently large there will be two symmetric equilibria in which either both agents choose H or both L .*

Proof. (i) From Lemma 1 we know that L will reduce the effort and therefore the costs for both agents if $\Delta a > \Delta \hat{a}(H)$ no matter what the other agent does. From Lemma 2 we know that the winning probability of the high ability agent is also highest with L . Hence, L is the dominant strategy of the high ability agent. To see the best reply of the low ability agent inspect inequality (1). He will choose H if and only if this inequality holds for $r_j = L$. The right-hand side is negative for the low ability agent. The left-hand side is negative as well because $\Delta a > \Delta \hat{a}(H) > \Delta \hat{a}(L)$. Hence, the inequality will not hold if and only if $\Delta w/c$ is larger than a certain value. (ii) The proof proceeds analogously to (i).

(iii) If $\Delta w/c$ is sufficiently small, the likelihood effect will dominate the effort effect. The equilibrium then follows from Lemma 2. If $\Delta w/c$ is sufficiently large the effort effect will dominate the likelihood effect. In that case, we know from the discussion subsequent to Lemma 1 that there are

two symmetric equilibria.⁵ ■

If $\Delta w/c$ is large, the costs of effort will be relatively small in comparison with the winner prize. Hence, both agents will exert a high effort to win the prize. In that case, the choice of technology is dominated by the concern to keep the equilibrium effort level as low as possible at the second stage and only the effort effect matters for the agents' decisions. As we have seen in Lemma 1, for small values of Δa it will be beneficial for both agents to select high variances to limit the effort exerted, but for high values of Δa the contrary is true. For intermediate values of Δa , however, a coordination problem exists. If one agent chooses a more (less) risky strategy, the other one will prefer the same strategy since r_i and r_j are strategic complements in that respect.

If $\Delta w/c$ is small, the costs of effort will be high relative to the winner prize. Hence, both will not exert too much effort to win the tournament in any case. Therefore, risk choice is dominated by the likelihood effect. But Lemma 2 gives us a clear cut result in that case. The high ability agent will prefer a low risk to safeguard his position. The low ability agent chooses a high risk as this helps him to challenge the high ability agent.

3 Conclusion

Our analysis has pointed at two important effects of risk taking in tournaments: On the one hand, it affects the equilibrium effort levels, on the other hand, the winning probabilities. As we have shown, the impact of risk on

⁵Explicit cut-off values for $\Delta w/c$ for the three equilibrium types can be computed by rearranging inequality (1) for the different cases. Note that the cut-off values for the two symmetric equilibria will differ. If $\Delta w/c$ is larger than the highest of both cut-offs, the two symmetric equilibria will coexist.

effort levels crucially depends on the difference in abilities of both participants. Similar agents' efforts decrease in risk, but if talents are sufficiently different the opposite will hold.

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