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by

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Economic Growth and Social Poverty: The Evolution of Social Participation

Angelo Antoci†  Pier Luigi Sacco‡  Paolo Vanin§

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Abstract

We develop an evolutionary model of growth in which agents choose how to allocate their time between private and social activities. We argue that a shift from social to private activities may foster market-based growth, but also generate social poverty. Within a formal framework that merges a game theoretic analysis of the evolution of social participation with a model of dynamic accumulation of its effects on social environment (i.e., of social capital accumulation), we show that growth and well-being may evolve in opposite directions (a plausible outcome for advanced and affluent societies).

JEL-Classification: C73, D62, I31, J22, O41, Z13

Key-words: Time Allocation, Social Capital, Relational Goods

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1 Introduction

Many social scientists would agree that economic growth and welfare are highly influenced by cultural and social factors. Such intuition remains though in most cases rather vague, due to the fact that cultures and social systems are polyhedral entities, difficult to capture with formal models. We provide a formal representation of the dynamics of a particular aspect of such processes: how growth and well-being may be influenced by a shift in the allocation of time between social and private activities.

It is intuitively clear, and well acknowledged by social psychologists [see e.g. Moscovici (1993)], that individual well-being depends crucially on social relations. In contrast, most economic models let it depend just on private consumption\(^1\) [see Cole, Mailath, Postlewaite (1992) for a discussion of why this is the case and Gui (2000) for an investigation of the interpersonal dimension of economic interaction]. Cultures and societies differ in the relative weight they attribute to these two sources of well-being. Single individuals face a choice between them as well, in that they may choose either to invest more time in social activities or to substitute them with private ones, which are less exposed to the influence of others’ behavior. This choice is highly conditioned, among other factors, by the quality of the social environment surrounding an individual: a poor social environment lowers the opportunities available to the time spent in social activities, and may render it more a source of frustration than of satisfaction. As a consequence, individuals may be led to defend themselves from social poverty by concentrating more on private activities, but if such behavior spreads over, it will further worsen the quality of social environment. The final effect may be an individualization of the basic sources of well-being, which fosters private activities (e.g. consumption and production of private goods) and hence stimulates economic growth, but at the expense of negative social externalities which accumulate their effects over time.

This extreme polarization needs not necessarily be the case, and one can imagine more balanced development paths, but the mechanism outlined seems indeed relevant and worth of investigation. In particular, a trend towards individualization is probably observable in many different countries (the extent to which this is true is an empirical question). Moreover, the possibility for economic growth to be accompanied by social poverty, and

\(^{1}\)For this reason we prefer to use the term well-being in spite of welfare.
the fact that it does not automatically bring about social development, is an old but still actual idea in many social sciences. We investigate it here with the tools of modern economics. Since ours is a theoretical enquiry, we do not want to insist on the empirical aspects of such phenomena, which are indeed more complicated than we are sketching them here, but we would rather like to make two general points: first, the substitution of social with private activities may constitute an engine of market-based growth and the lack of this substitution may represent an obstacle to growth; second, if we consider social relations relevant for individual well-being, there is the possibility for private growth not to be well-being improving. To capture these effects, we propose an evolutionary model of growth, which focuses on defensive choices and merges a game theoretic analysis [see Weibull (1995)] of the evolution of social participation with a model of dynamic accumulation of its effects on social environment.

In Section 2 we discuss how these cumulative effects on social environment may be thought of in terms of social capital and how they relate to social participation dynamics on one side and to the theory of endogenous growth on the other. In Section 3 we further investigate the process of substitution of social activities with private activities. Sections 4, 5 and 6 display the model. Section 7 discusses its outcomes in terms of well-being and growth. Section 8 concludes.

2 Social participation and social capital accumulation

Let us start to define what we mean by private and social activities. We call private those activities that either can be carried on by an individual alone or that are instrumental to them. In particular, we focus on consumption and production of private goods\(^2\). Unlike private activities, social activities cannot be carried on by an individual in isolation: if they are not shared,\(^2\)Consumption of public goods is a private activity as well, but, since the distinction between private and public goods is not relevant in the present context, we shall ignore it and speak henceforth just of private goods. Moreover, it is admittedly a simplification to consider the time spent in production just as a private activity, but it captures indeed a relevant aspect of labor, namely the fact that, in order to consume, one has to work. In the concluding section we discuss the consequences of considering the social dimension of production activities.
they simply do not exist. For this reason, their success or failure, and hence
the utility they provide to the individuals involved, depend at least on four
distinctive factors: first, the level of participation of other individuals (in
general, the more the participants, and the more the time they spend on
a social activity, the higher the utility of each one of them); second, an
individual’s own participation (most social activities, even in the case in
which they have positive external effects, provide though higher benefits to
the participants than to the non-participants; moreover, they are mostly
time intensive); third, the specific identity of the participants; finally, the
opportunities available in a certain social environment.

No widespread agreement has yet been reached about how to model these
effects. A relevant contribution, quite close in the spirit to our stress on so-
cial activities, is Uhlaner (1989), who introduces in economic literature the
notion of ‘relational goods’. She argues that traditional rational choice mod-
els cannot explain why people are willing to undertake costly actions such
as political participation and voting, despite their awareness that the actual
influence of their participation or of their vote is indeed negligible in terms
of the final outcome; on the contrary, such behaviors can be understood as
rational once we consider that people are enjoying a ‘relational good’. Rela-
tional goods are defined as a particular type of local public goods, that can
only be produced and consumed through the joint action of several individu-
als, whose identities become relevant. Corneo, Jeanne (1999a) refer to them
as ‘socially provided private goods’, but the essence does not change. Ex-
amples of relational goods are social approval, solidarity, friendship, sharing
life with another person, creating or reinforcing group identity or the sense
of affiliation to a group (possibly cultivated through the identification with
shared social or ethical norms). It is evident that such factors play a ma-
jor role in shaping individual identity. Akerlof, Kranton (2000) argue very
convincingly that identity explains a great variety of economic choices, but
they provide no explicit analysis of the social process through which identity
is shaped. Even if our focus is not directly on identity, we investigate some
channels of social interaction that contribute to determine individual identity
indirectly through the shaping of social environment.

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3 In some cases, even congestion increases utility. A convincing example is provided by Uhlaner’s (1989) observation that “a baseball game is more exciting when the park is full”.

4 One economically relevant dimension of a person’s social identity is, for instance, social status, on whose relevance we return in Section 3.
Since social interaction provides direct utility to the participants, following Uhlaner (1989) we call ‘relational goods’ ($B_i$) what an individual ($i$) enjoys when participating to some social activity. Relational goods depend on the following factors: $B_i = B(r_i, r_{-i}, I, K_s)$, where $r_i$ is individual $i$'s own participation, $r_{-i}$ is the others’ participation, $I$ is a vector of social identities of the participants, and $K_s$ measures the ‘quality’ of the social environment, which is inherited from past history. In Section 4, when presenting the formal model, we choose a particular specification of this function, which reduces to $B_i = s_i R(K_s, x)$, where $s_i$ is individual $i$’s investment of time in social activities and $R(K_s, x)$ represents the amount of social opportunities available ($x$ captures the overall participation, $K_s$ the accumulated effects of past interaction and, for simplicity, in the present study we ignore any identity-related issue$^5$).

Coming to the factor $K_s$, we interpret it as social capital. There is by now a huge literature on social capital, following in particular Coleman (1988, 1990) and Putnam (1993, 1995) [see World Bank’s (2001) electronic library on social capital]. The term social capital has been used by different authors with slightly different meanings [see in particular Grootaert (1998) and for a reconstruction and Bowles, Gintis (2000) for a criticism]. Coleman (1988, 1990) defines it as a set of long-lasting patterns of social relationships, which constitute a resource that actors can use to realize their interests [see also Granovetter (1973)]. These attributes of the social structure are referred to as a form of capital since they have the nature of a stock that is fed or depleted through time as relations evolve, and this stock may be seen as an input in an individual production function. Coleman also provides an articulated taxonomy of the different forms of social capital. Collier (1998) summarizes the concept of social capital as those ‘externality generating social interactions’ which either are themselves durable or whose effects are durable, and classifies the different possible external effects. Narayan (1999) acknowledges that, at the highest level of generality, the term social capital refers to ‘the glue that holds groups and societies together - bonds of shared

$^5$As it will be clear in Section 4, this expression for $B_i$ is derived as follows: with a continuous population of measure 1, we allow individuals to choose between spending more or less time in social activities (respectively, $s_H$ and $s_L$); hence, we identify $r_i = s_i$, $s_i \in \{s_L, s_H\}$, and, called $x$ the fraction of the population that chooses $s_H$, we let $r_{-i} = [s_H x + s_L (1-x)]$; further, we assume that social opportunities available take the form $R(K_s, x) = [s_H x + s_L (1-x)]^\gamma K_s^\mu$ and, after dropping $I$ since we presently ignore the effects of identity, we obtain $B_i = s_i R(K_s, x)$. 
values, norms and institutions’. More specifically, he provides the following
definition: “Social capital is defined as the norms and social relations em-
bedded in the social structures of societies that enable people to coordinate
action to achieve desired goals”. After discussing some related concepts, he
focuses attention on the fact that the same ties that link some individual
may result in social exclusion of those outside a specific group (or relational
network), and discusses how social ’cross-cutting ties’ may be complement or
substitute for the well functioning of the state. The empirical relevance of so-
cial capital for growth is pointed out, among the others, by Temple, Johnson
(1998), who show that a measure of ’social capability’ is a good predictor for
long run growth, and by Knack, Keefer (1997), who find that one standard
deviation change in a measure of trust is associated with a change in growth
of more than half a standard deviation.

From this literature it emerges that there are at least two useful ways
of thinking of social capital: one is in terms of evolution of customs and
social norms, the other is in terms of the formation of associations and other
social organizations. Sometimes the former is used as a definition of social
capital and the latter as a proxy for it [see Putnam (1993), who views it as
constituted by ’networks of civic engagement’ and associated norms which
facilitate coordination and cooperation and proposes the density of volun-
tary organizations as a proxy; in a similar vein see also Paldam, Svendsen
(2000), whose definition focuses though on trust, as Fukuyama’s (1995) one].
In any case, these are just two aspects of the same dynamic process of social
interaction. Moreover, accumulation of any form of social capital requires
investment in relations [see the experimental and survey study of Glaeser
et al. (1999)]. Hence, we do not distinguish any further among the pos-
sible forms of social capital, but rather try to capture the dynamics of its
accumulation at an aggregate level as a result of the evolution of social par-
ticipation. For simplicity, we focus on a homogeneous population, so that we
ignore here the problems of social stratification and social exclusion inves-
tigated by Narayan. We assume that social capital is accumulated when a
higher aggregate social participation renders more social opportunities avail-
able and strengthens social relations, shared norms and the level of trust.
Investment in social capital takes here the primary form of time investment
in social activities6 and its costs can be measured in terms of forgone private

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6Notice that the same activity, i.e. social participation, gives rise both to consumption of relational goods and to accumulation of social capital. The fact that consumption and
Since social capital has mainly the nature of a public good, a general problem of under-investment could be relevant. This point was already made by Coleman, but his emphasis was on the forgone positive externalities of social capital for private growth (in particular, through its contribution to the creation of human capital). In contrast, we look at under-investment in social capital from a different point of view: social poverty could generate higher instead of lower private growth, because agents are led to defend themselves from it through the substitution of social activities with private ones [see Antoci and Bartolini (1999) for an analysis of negative externalities and growth]. Nevertheless, we show formally that, under certain plausible conditions, growth and well-being may evolve in opposite directions. To our knowledge, these aspects have not yet been considered in the literature on social capital, at least not within the framework of a formal model. Moreover, while Coleman and others interpret social capital as an input in an individual production function, we rather focus on its relevance from an aggregate and dynamic point of view, that is to say, we are more interested in its importance in the process through which an economy as a whole (or a part of it) is able to reproduce its relational patterns, to modify its aggregate performance, and to alter the roots of people’s well-being.

A similar observation applies to the relationship between our model and those of endogenous growth [see Barro and Sala-i-Martin (1995) and Aghion and Howitt (1998)]. The theory of endogenous growth has deeply investigated the role of the accumulation of factors different from physical capital for investment are not opposed here should not surprise, since it is a common feature of the forms of non material capital (e.g., the use of knowledge increases its stock, rather than diminishing it). Such intuition goes back to the Aristotelian analysis of ethical virtues, whose seeds are still to be found in Nussbaum’s (1986) investigation of relational goods.

It will be interesting, in a future study, to consider the fact that social organizations require some form of physical capital as well, and hence individuals have to provide a material contribution to be able to enjoy the benefits from them.

Notice that this assumption strengthens our results. In the concluding section we discuss the consequences of allowing an impoverishment in social capital to be harmful for growth.

The idea that capital is not just a productive input, but rather the process of reproduction of certain economic relations, has been most famously stated by Marx, but it is not strictly dependent upon his entire theoretical system, and can be applied in evolutionary contexts in general [see also Paldam, Svendsen (2000) for a distinction of approaches based on production, transaction costs and monitoring costs].
an economy’s growth, but its main focus has been on the accumulation of
human capital. Though, it is interesting to notice that Lucas (1988) already
distinguished between internal and external effects of human capital, the
former basically constituted by the increased productivity due to privately
acquired knowledge, while the latter taking place to a relevant degree through
the relations among economic actors\textsuperscript{10}. While we extend the idea of such
external effects by modelling their dynamics, the main difference remains
that we are not adding a new input in a production function, but rather
focusing on how social dynamics may affect growth and well-being in the
medium-long run, possibly, as anticipated, in opposite directions\textsuperscript{11}.

Two remarks are in order. First, we admit that theoretically social capital
accumulation, and enjoyment of relational goods as well, might also take
place through market interactions; but the overall impact on them of an
expansion of the market may still be negative, in that the latter is private-
oriented and does not foster a collective orientation [see Polanyi (1977) for
an analysis of the market as a form of integration of economic activities;
more recently Anderson (1990) for a discussion of its ethical limitations;
Sacco, Zamagni (1997) for an inquiry into civil economy and the role of
reciprocity; Elster (1989) for an investigation of the problems generated by

\textsuperscript{10}In his words: "We know from ordinary experience that there are group interactions
that are central to individual productivity [. . . ] Certainly in our own profession, the benefits
of colleagues from whom we hope to learn are tangible enough to lead us to spend a
considerable fraction of our time fighting over who they shall be, and another fraction
travelling to talk with those we wish we could have as colleagues but cannot".

\textsuperscript{11}It may be still important to stress the difference between social capital and the concepts
of human capital and of technology, which constitute the main sources of growth in the
endogenous growth models. Romer (1989) presents such models as assuming "the existence
of an aggregate production function \( F(\cdot) \) that depends on a subset of the following list of
inputs: services from physical capital \( K \), labor services \( L \) from a person with a minimal
level of schooling and training, services from additional human capital \( H \), and measure of
the technology or state of the art \( A \)." He adds that "\( A \) is like a public good", in that
it represents public knowledge available at zero marginal cost, which is accumulated as
a side effect of the investments in physical capital. Social capital is similar to the latter
variable, in that it shares with it the nature of public good (at least locally, i.e. for those
who belong to a certain relational network), but it differs from it because it is the result of
a specific investment of time (and possibly of other resources) in building and maintaining
relations, while the variable \( A \) is generally seen as a side effect of the production activity.
Moreover, social capital differs essentially from \( H \) not only in that its services are largely
not privately appropriable, but also because it cannot even be built and accumulated by
a single individual, since it is rather the result of some form of reciprocal or collective
action.
collective action]. Second, we neglect here the role of investments in physical and human capital just for the sake of simplicity. Nevertheless, we believe that our results may be generalized to an economy with physical and human capital accumulation and this task is in our future agenda.

3 Substitution of social activities with private activities

We have already pointed out that the process of substitution of social activities with private ones, which constitutes the key mechanism of our model, may emerge as agents try to defend themselves from a poor social environment. Hirsch (1976) argues that this substitution may be due to another reason as well. In affluent societies, as the quantity of consumption goods increases, the time needed to consume them becomes both more scarce and more expensive. This effect is strengthened by positional competition, that forces each individuals to earn an increasing income if they want to keep their relative social position\[12\]. These two processes generate an increasing pressure on time, which leads to a substitution of time-saving (e.g. fast-foods) for time-intensive consumption (e.g. relational goods emerging from participation to social activities) [see the pioneering contribution of Becker (1965)]. In Hirsch’s own words:

As the subjective cost of time rises, pressure for specific balancing of personal advantage in social relationships will increase. [...] Perception of the time spent in social relationships as a cost is itself a product of privatized affluence. The effect is to whittle down the amount of friendship and social contact [...]. The huge increase in personal mobility in modern economies adds to the problem by making sociability more of a public and less of a private good. The more people move, the lower are the chances of social contacts being reciprocated directly on a bilateral basis. (p.80)

The relevance of the last point, i.e. of personal mobility, for the substitution of social by private activities, is also investigated by Schiff (1992) [see also

\[12\] There is by now a large literature on the economics of status seeking and on concerns for relative position in income or in consumption. Reviewing it is though beyond the scope of this paper.
Schiff (1999) for a general equilibrium model of labor mobility in the presence of social capital. He starts from the consideration of the following two striking aspects of U.S. society: "the higher level of wealth for a huge number of people, on one hand, and a weaker social support structure, on the other (including a higher crime rate, weaker interpersonal relations, and more isolation)". He next sheds light on the substitution process we are interested in by observing that

The need to cope with the high degree of isolation caused by the higher degree of geographic labor mobility may lead to the creation of alternative institutions where people who are not as close can interact (e.g., singles’ bars, dating services, nursing homes, insurance, and so on). These market activities enter into the gross national product (GNP) but do not necessarily imply higher welfare than in societies where some of these functions are carried out outside the market. (p.167-168)

Taken together, these observations show that, out of many different possible causes, the substitution process on which we are focusing is a relevant one. Its effects on growth and poverty are worth of formal analysis: in contrast with much of the literature on social capital, but together with Schiff, we stress that the decline in social capital brought about by this substitution could be accompanied by (or could be itself an engine of) private growth, registered in national accounting\textsuperscript{13}; on the other side, the overall effect in terms of well-being may be negative (so that we can speak of social poverty traps). This is due to different causes. First of all, an individual’s shift of time from social to private activities imposes a negative externality on the others, in terms of decreased opportunities of social interaction [see Collier (1998)]. Secondly, as we argued above, these externalities accumulate over time. Third, since we do not take as granted that agents are perfect optimizers, but we rather represent them as boundedly rational and their behaviors as involved in an evolutionary social selection dynamics, this social selection dynamics may act as a form of external pressure (possibly the same pointed out by Hirsch\textsuperscript{14}), which leads

\textsuperscript{13}Notice however that in developing countries, where welfare still depends to a substantial degree upon the satisfaction of primary needs, the converse problem may arise, since a highly relational-oriented culture might become an objective obstacle for a strong market-based growth, as people are relatively unwilling to give up part of the time spent in social activities to embrace income-generating ones.

\textsuperscript{14}Hirsch has stressed in particular the unavoidable aggregate dissatisfaction generated by positional competition, which is indeed a rat race (even if it may have indirect positive
individuals to adopt strategies that are presently or dynamically sub-optimal. All these effects are explicitly formalized in the next sections. In particular, the latter one opens the way to an analysis of different possible causes of social poverty traps in connection with the question of the optimality of individual strategies, an analysis that will be carried on in Section 7.

4 The model

We model an economy in which agents choose how to allocate their time between private and social activities. Participating to the latter ones requires time and forgoing some private consumption, but they provide an individual utility which depends both on her own and on aggregate participation, as well as on the opportunities available in the social environment. Agents may defend themselves from a poor social environment by shifting to private activities, less exposed to external effects. If this strategy spreads over, private activities will be fostered, but at the expense of social activities.

In particular, we assume a (homogeneous) continuous population of measure 1, whose well-being depends on three kinds of goods: a private subsistence good ($Y$), a relational good ($B$) and a private good which is perfect substitute of the relational good ($Y_s$). In every instant (say, every day, but we adopt a continuous specification), each individual has to choose how to allocate her time endowment (normalized to 1) between social and private activities (respectively, $s$ and $l = 1 - s$). For simplicity, we assume that agents have to choose between the two following pure strategies:

(R) a relational-oriented strategy, corresponding to the couple \( \{ s_H, l_L \} \), where \( l_L = 1 - s_H \);

(P) a private-oriented strategy, corresponding to the couple \( \{ s_L, l_H \} \), where \( l_H = 1 - s_L \) and \( 0 < s_L < s_H < 1 \) (so that \( 0 < l_L < l_H < 1 \)).

Notice that, for shortness, we can summarize strategies R and P as the choice of respectively \( s_H \) and \( s_L \).

welfare effects through the fostering of private growth [see Corneo, Jeanne(1999a, 1999b)]).

\(^{15}\)In what follows we shall assume for simplicity that there are just three single goods, but it would be easy to generalize to the case of three bundles of goods.

\(^{16}\)The assumption that \( Y_s \) is a perfect substitute for the relational good is obviously rather optimistic [see Anderson (1990)]. By such assumption, the results about well-being in the next section gain more relevance.
Private activities amount to the production and consumption of private goods, according to the following technology: under the relational strategy, the amount of time \( l_L \) serves to produce and consume a fixed quantity \( \bar{Y} \) of private subsistence goods; under the private strategy, agents still use \( l_L \) to produce and consume \( \bar{Y} \), but with the additional time \( (l_H - l_L) \) they produce and consume also an amount \( \bar{Y}_s \) of the private goods which are substitutes of relational goods.

Social activities provide utility through the enjoyment of relational goods. As pointed out in Section 2, relational goods enjoyed by individual \( i \) are specified as follows (notice that we can identify an individual with her chosen strategy, so that \( i \in \{R, P\} \)):

\[
B_i(K_s, x) = s_i R(K_s, x),
\]

where \( s_i \) is individual \( i \)'s investment of time in social activities (i.e. either \( s_L \) or \( s_H \)) and \( R(K_s, x) \) represents the amount of social opportunities available. The latter ones depend in turn on the level \( K_s \) of social capital and on the total amount of time devoted to social activities by the population \( [s_H x + s_L (1 - x)] \), where \( x \in [0, 1] \) is the fraction of individuals choosing the relational strategy. Notice that \( K_s \) captures the accumulated effect of past social participation, whereas \( x \) measures its present level\(^{17} \). \( R(K_s, x) \) is specified as follows:

\[
R(K_s, x) \equiv [s_H x + s_L (1 - x)]^{\beta K_s^\gamma},
\]

where \( \beta \) and \( \gamma \) are strictly positive parameters\(^{18} \). Notice that \( R(K_s, x) \) is an increasing function of \( x \). Notice as well that in our model the time not spent in production and consumption of private goods is not itself a final good, but rather an intermediate good, whose value in terms of enjoyed relational goods depends on the social participation of the rest of the population and on the amount of social opportunities available.

We can now write down the full expression of the payoffs of the two strategies. Following the relational strategy yields

\(^{17}\)Let us emphasize that \( x = 1 \) does not mean that individuals spend all of their time in social activities, but rather that all of them spend relatively more time in these activities and relatively less in private ones.

\(^{18}\)Since our results do not depend on \( \beta \), we could, without loss of generality, set it equal to 1.
\[ U_R(K_s, x) \equiv \ln \bar{Y} + \ln[B_R(K_s, x)] = \]
\[ = \ln \bar{Y} + \ln[s_H R(K_s, x)] = \]
\[ = \ln \bar{Y} + \ln\{s_H x + s_L(1 - x)^{\beta K_s^\gamma}\}. \tag{3} \]

The private strategy yields
\[ U_P(K_s, x) \equiv \ln \bar{Y} + \ln[B_P(K_s, x) + a \bar{Y}_s] = \]
\[ = \ln \bar{Y} + \ln[s_L R(K_s, x) + a \bar{Y}_s] = \]
\[ = \ln \bar{Y} + \ln\{s_L[s_H x + s_L(1 - x)]^{\beta K_s^\gamma} + a \bar{Y}_s\}, \tag{4} \]

where \( a \) is a strictly positive parameter which represents the marginal rate of substitution between \( B_P(K_s, x) \) and \( \bar{Y}_s \).

Notice that an increase of the proportion of individuals choosing strategy \( P \) (i.e. an increase of \( 1 - x \)) generates a negative externality which affects especially individuals that do not consume \( \bar{Y}_s \). Hence, by following strategy \( P \), individuals have the opportunity to defend themselves from this negative externality.

5 Evolution of social participation

We follow an evolutionary game approach and assume that the time derivative of \( x \), \( \dot{x} \equiv \frac{dx}{dt} \), is given by the so called ‘replicator equation’ [see Weibull (1995)]:
\[ \dot{x} = x[U_R(K_s, x) - \bar{U}(K_s, x)], \tag{5} \]

where \( \bar{U}(K_s, x) \) is the average payoff
\[ \bar{U}(K_s, x) \equiv U_R(K_s, x)x + U_P(K_s, x)(1 - x). \tag{6} \]

The choice of the replicator dynamics as social selection mechanism does not imply a real loss of generality in a two-strategy setting like the one of the present paper (although the dynamic interaction with the accumulation of social capital, discussed in the Section 6 below could, at least in principle). It must be pointed out, however, that the choice of the replicator equation as
a 'representative' form of selection dynamics is not arbitrary. As pointed out
\textit{e.g.} by Björnerstedt and Weibull (1996), every payoff-monotonic selection
dynamics can be represented in terms of the replicator dynamics (by means
of a suitable time and/or strategy dependent factor) and, moreover, such
dynamics is consistent with several realistic individual and social learning
mechanisms, such as simple forms of reinforcement of successful own behav-
iors or imitation of observed successful behaviors of others [see also Börgers,
Sarin (1997) and Schlag (1998) for deeper insights into the behavioral micro-
foundations of the replicator dynamics].

Equation (7) may be rewritten as follows:

\[
\dot{x} = x(1 - x)\Delta U(K_s, x), \tag{7}
\]

where \(\Delta U(K_s, x)\) is the payoff differential

\[
\Delta U(K_s, x) \equiv U_R(K_s, x) - U_P(K_s, x) = \ln \frac{s_H[s_L + (s_H - s_L)x]^\beta K_s^\gamma}{s_L[s_L + (s_H - s_L)x]^\beta K_s^\gamma + aY_s}. \tag{8}
\]

In general the evolution of social participation will depend on the dy-
namics of \(K_s\), but for expositional purposes it is worthwhile to start with
a separate analysis of \(\dot{x}\) when the stock of social capital is fixed. Through-
out the rest of this section we treat consequently \(K_s\) as a strictly positive
parameter.

The following proposition gives a classification of dynamics (7) when \(K_s\)
is constant.

\textbf{Proposition 1} Dynamics (7) can be classified as follows:

(i) If

\[
K_s \leq K_s^1 \equiv \left[ \frac{aY_s}{(s_H - s_L)s_H^\beta} \right]^{\frac{1}{\gamma}}, \tag{9}
\]

then, for every initial value \(x(0) \neq 1\), the adoption process converges
to the fixed point \(x = 0\), in which all individuals follow strategy \(P\) (see
figure 1.a).
(ii) If

\[ K_s \geq K_s^2 \equiv \left[ \frac{a\bar{Y}_s}{(s_H - s_L)s_L^{\beta}} \right]^{\frac{1}{\gamma}}, \]

(10)

where \( K_s^1 < K_s^2 \), then the opposite of case (i) holds (see figure 1.b).

(iii) If

\[ K_s^1 < K_s < K_s^2, \]

(11)

then both the fixed points \( x = 0 \) and \( x = 1 \) are locally attracting and their attraction basins are separated by the repulsive fixed point \( \bar{x} \in (0, 1) \), given by (see figure 1.c)

\[ \bar{x} \equiv \left[ \frac{a\bar{Y}_s}{(s_H - s_L)^{1+\beta}} \right]^{\frac{1}{\gamma}} \frac{1}{K_s^3} \left( \frac{s_H}{s_H - s_L} \right). \]

(12)

**Proof** Notice first that \( \dot{x} = 0 \iff \{ x = 0 \lor x = 1 \lor \Delta U(x) = 0 \} \).

Since \( \frac{d\Delta U(x)}{dx} > 0 \quad \forall x \in (0, 1) \), this implies that \( x = 0 \) and \( x = 1 \) are the only possible attracting fixed points.

Finally, from the observation that \( \Delta U(x) = 0 \iff x = \bar{x} \) [see equation (??)] and that \( \text{sgn}(\dot{x}) = \text{sgn}[\Delta U(x)] \) it immediately follows that

- \( \bar{x} \geq 1 \) [i.e. \( \Delta U(x) < 0 \quad \forall x \in (0, 1) \)] \iff (??);
- \( \bar{x} \leq 0 \) [i.e. \( \Delta U(x) > 0 \quad \forall x \in (0, 1) \)] \iff (??);
- \( \bar{x} \in (0, 1) \) (i.e. \( \exists \) interior repulsive fixed point) \iff (??). Q.E.D.

Notice that, by (??), the attraction basin of \( x = 1 \) expands if the value of \( K_s \) increases.

The next proposition concerns the value of \( \bar{U}(x) \) in the fixed points.

**Proposition 2** The fixed point \( x = 1 \) Pareto-dominates the fixed point \( x = 0 \) (i.e. \( \bar{U}(1) > \bar{U}(0) \)) if and only if

\[ K_s > K_s^3 \equiv \left[ \frac{a\bar{Y}_s}{s_H^{1+\beta} - s_L^{1+\beta}} \right]^{\frac{1}{\gamma}}, \]

(13)

where \( K_s^3 < K_s^1 \).
When the interior fixed point \( \bar{x} \) exists [see case (iii) of Proposition 1], then it holds

\[
\bar{U}(1) > \bar{U}(\bar{x}) > \bar{U}(0).
\]

Proof Notice that in \( x = 0 \), in \( x = 1 \) and in \( x = \bar{x} \) each individual has the same utility level \( \bar{U}(x) \), [defined in (??)], respectively equal to \( U_P(0) \), \( U_R(1) \), and both \( U_R(\bar{x}) \) and \( U_P(\bar{x}) \). Now, (??) just amounts to a re-writing of \( U_R(1) > U_P(0) \). Recalling that \( U_R(x) \), \( U_P(x) \) and \( \bar{U}(x) \) are all strictly increasing in \( x \), the last result follows immediately. Q.E.D.

Proposition 2 implies that if the fixed point \( x = 1 \) is locally attractive [cases (ii)-(iii)] it always Pareto-dominates the fixed point \( x = 0 \), even when the latter is locally attractive. Furthermore, even if \( x = 0 \) is the unique attracting fixed point [case (i)], it may be Pareto-dominated by \( x = 1 \); in particular, this is the case when \( K_s^3 < K_s \leq K_s^1 \).

If case (i) holds or if the initial distribution \( x \) of strategies belongs to the interval \([0, \bar{x})\) in case (iii), then \( x \) converges to 0. Along the trajectory that drives the economy to \( x = 0 \), an increasing proportion of individuals build their well-being on private rather than on social sources. Consequently, the aggregate production and consumption of private goods increases and the population experiences economic growth. Proposition 2 says that economic growth may be well-being worsening. In such case, economic growth is the undesirable effect of a coordination failure.

6 Evolution of social capital

In the above section, social capital \( K_s \) has been taken as a parameter; we have seen that such parameter plays a key role in determining the relative performance of pure strategies \( R \) and \( P \) and the well-being properties of attracting fixed points under dynamics (??). However, the assumption of stationarity of \( K_s \) is restrictive; therefore, in this section we augment dynamics (??) by an equation describing the evolution of social capital. More specifically, we assume that the accumulation of \( K_s \) builds on a learning-by-doing mechanism as follows:
\[
\dot{K}_s = \dot{B}_R(K_s, x)x + \dot{B}_P(K_s, x)(1 - x) - \delta K_s = \]
\[
= [s_H x + s_L(1 - x)]R(K_s, x) - \delta K_s, \tag{14}
\]

where \( \delta > 0 \) is the depreciation rate of \( K_s \).

Equation (14) assumes that social capital increases when available social opportunities are effectively exploited, i.e. individuals devote time to social activities and enjoy relational goods. We are closer here to an interpretation of social capital in terms of evolution of customs and of social norms rather then in terms of construction of associations and other social organization; however, as discussed in Section 2, these are just two aspects of a same process, so that our assumption does not appear to be very restrictive.

By plugging in (14) the expression given in (12) for \( R(K_s, x) \), we can write (14) as follows:

\[
\dot{K}_s = K_s\{[s_L + (s_H - s_L)x]^{1+\beta}K_s^{\gamma - 1} - \delta\}. \tag{15}
\]

We shall analyze dynamics (14), (15) in the region of the plane \( (K_s, x) \) in which \( K_s \geq 0 \) and \( 0 \leq x \leq 1 \). We work out the complete classification of the dynamics in the appendix. Figures 2, 3 and 4 (corresponding respectively to \( \gamma > 1 \), \( \gamma = 1 \) and \( \gamma < 1 \)) illustrate the different possible cases. The next proposition summarizes the main results.

**Proposition 3** There are two possible asymptotic attractors, \( (K^L_s, 0) \) and \( (K^H_s, 1) \), where \( K^L_s \) and \( K^H_s \) are defined as follows:

\[
K^L_s \equiv \begin{cases} 
0, & \text{if } \gamma \geq 1 \\
\dot{K}_s(0), & \text{if } \gamma < 1 
\end{cases} \quad K^H_s \equiv \begin{cases} 
\infty, & \text{if } \gamma \geq 1 \\
\dot{K}_s(1), & \text{if } \gamma < 1 
\end{cases} \tag{16}
\]

Both attractors are present in figures 2, 3.b and 4.c.

Only \( (K^H_s, 1) \) is present in figures 3.a and 4.a.

Only \( (K^L_s, 0) \) is present in figures 3.c and 4.c.

Along the trajectory leading to \( (K^L_s, 0) \), the economy experiences private growth at the expenses of social participation and ends up in a state of social

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\(^{19}\)In the figures, sinks are represented by full dots \( \bullet \), sources by open dots \( \circ \) and saddle points by drawing their stable and unstable manifolds.
poverty; along the path towards \((K^H_s, 1)\), expansion of social participation leads to social prosperity, but at the expenses of private growth.

**Proof** See appendix.

7 Well-being and social poverty traps

We next consider the well-being properties of the two asymptotic attractors \((K^L_s, 0)\) and \((K^H_s, 1)\) and of the other fixed points of dynamics \((??), (??)\).

**Proposition 4** When both attractors are present, \((K^H_s, 1)\) always Pareto-dominates \((K^L_s, 0)\).

Whenever present, \((K^H_s, 1)\) Pareto-dominates every other fixed point of the dynamics.

When only \((K^L_s, 0)\) is present, for \(\gamma = 1\) it Pareto-dominates the only other fixed \((0, 1)\); for \(\gamma < 1\) \((K^L_s, 0)\) may be Pareto-dominated by some other fixed point: in particular, \(\hat{K}_s(1) > K^3_s\) is a sufficient condition for \((K^L_s, 0)\) to be Pareto-dominated by \((K^L_s, 0)\) and \((\hat{K}_s(1), 1)\).

**Proof** We work out the proof only for the first sentence of Proposition 4. Since the proof of the other results works exactly the same way, we omit it. Recall from Proposition 2 that, given \(K_s\), \(\bar{U}(K_s, 1) > \bar{U}(K_s, 0) \iff K_s > K^3_s\). Notice then that, given \(x\), \(\bar{U}(K_s, x)\) is a strictly increasing function of \(K_s\). When both attractors are present, for \(\gamma \geq 1\) (case (a) and subcase (b.2)), \((K_s, x) \to (\infty, 1)\) for \(t \to \infty\) (see figures 2.a-2.c and 3.b) and along this trajectory the value of \(\bar{U}(K_s, x)\) becomes definitely higher than in any fixed point of the dynamics; for \(\gamma < 1\) (subcase (c.2), see figure 4.b), it is enough to notice that \(K^3_s < K^1_s = \hat{K}_s(1) < \hat{K}_s(1)\), so that \(\bar{U}(K_s(1), 1) > \bar{U}(K_s(1), 0) > \bar{U}(K_s(0), 0)\). Q.E.D.

As already mentioned, in \((K^L_s, 0)\) the economy reaches the highest level of expansion of private wealth at the expenses of social participation. Along the trajectories leading the economy to such traps we observe an economic growth process driven by the destruction of social opportunities and by their substitution by private goods.

The above considerations do not however imply that it is always 'optimal' for individuals to manage to coordinate their choices on the strategy \(R\). The problem is analogous to that of the interpretation of the well known Golden
Rule in Solow’s model as a normative device. To consider it, we have to study how individual behavior could rationally deviate from the dynamics assumed in the replicator equation (??). To this purpose, we first give the following definition.

**Definition 1** We shall say that the state \((K_s^H, 1)\) is achievable from a given initial value \(K_s^0\) of \(K_s\) if the trajectory passing through the point \((K_s^0, 1)\) converges to \((K_s^H, 1)\).

Definition 1 says that a state is achievable when the dynamics of \(K_s\) brings to it in presence of the highest possible social participation \((x = 1)\), i.e., when individuals, by coordinating on strategy \(R\), would be able to reach it. The next proposition sheds light on the ‘optimality’ of coordinating on strategy \(R\) and helps us to distinguish among different causes of social poverty traps.

**Proposition 5**

(i) If \(K_s^0 > K_s^3\) and \((K_s^H, 1)\) is achievable from \(K_s^0\), coordinating on strategy \(R\) is individually ‘optimal’ both in transition and in \((K_s^H, 1)\). Therefore, social poverty traps may be interpreted as the result of a coordination failure.

(ii) If \(K_s^0 < K_s^3\) and \((K_s^H, 1)\) is achievable from \(K_s^0\), coordinating on strategy \(R\) is individually ‘optimal’ only if agents are patient enough (or altruist toward future generations), and social poverty traps may be interpreted as the combined result of a coordination failure and of impatience (or of lack of altruism toward future generations).

(iii) If \((K_s^H, 1)\) is not achievable from \(K_s^0\), social poverty traps are the result of the technology of social capital accumulation.

**Proof** Notice first that, if \(\gamma < 1\), \((K_s^H, 1)\) is achievable whatever is \(K_s^0 > 0\) (see figures 4.a-4.c); if \(\gamma > 1\), \((K_s^H, 1)\) is achievable only if \(K_s^0 > \hat{K}_s(1)\) (see figures 2.a-2.c) and, if \(\gamma = 1\), \((K_s^H, 1)\) is achievable only if \(x^* > 1\) [see (??)] and \(K_s^0 > 0\) (see figures 3.a-3.c). If \((K_s^H, 1)\) is not achievable from a given initial value \(K_s^0\), then any trajectory passing through a point with \(K_s = K_s^0\) does not converge to \((K_s^H, 1)\) whatever individuals’ choices of strategy may be.

The results in cases (i) and (ii) of Proposition 5 follow from Proposition 2. Specifically, by simultaneously choosing strategy \(R\), in case (i) agents could
obtain in every instant of time a payoff higher than they get if they behave according to the replicator dynamics; in case (ii), on the contrary, they would have to face an initial reduction of their payoff. Therefore, in case (ii) the convenience to coordinate on strategy $R$ is associated to their discount factor as in Solow’s model. Finally, case (iii) is self-evident. Q.E.D.

Proposition 5 opens the way to some policy considerations. The first point to be made is that social arrangements matter for growth and well-being in a non-obvious way. The mentioned literature on social capital has already pointed out, theoretically and empirically, that an impoverishment in social capital is detrimental to growth [see in particular Knack, Keefer (1997), who show empirically that trust and norms of civic cooperation are more relevant for growth than associational activities]. Here we add that, even if this negative effect were compensated (to some extent) through defensive strategies and substitution channels which foster private growth, the overall effect in terms of well-being might still be negative\textsuperscript{20}. Hence, the first (easy) conclusion is that private growth is not a sufficient target for policy, but its social consequences should be taken into account as well. To go beyond this vague statement, it is opportune to distinguish [see Collier (1998)] between social capital provided by government and social capital provided by civil society. Indeed, policy conclusions are easier to understand for the former than for the latter, whereas the present paper is more concerned with the latter, i.e. with social capital arising from spontaneous social participation. Nevertheless, there may be the scope for some intervention in this field as well, as the government could act, through an adequate system of rights, laws, incentives and services, as the engine that allows individuals to overcome the coordination failure problem pointed out above. This means that government and civil society may be seen as complementary in generating the conditions for social capital accumulation. On the other hand, if either of them does not work properly, the other one may play, to a certain extent, a substitutive role [see Narayan (1999) for a deeper analysis of virtuous and vicious paths in the interaction between state and civil society], but this kind of substitution is unlikely to drive to social and economic prosperity. Examples of possible interventions, frequent in the literature, are promotion of association rights, improvement of communication systems and infrastructures and facilitating

\textsuperscript{20}This is especially relevant if measures of growth just take into consideration production and consumption of private goods.
'cross-cutting ties' among different social groups. What seems not possible for the state to do, according to Paldam, Svendsen (2000), is to enforce social capital top-down, since the latter emerges essentially out of a self-enforcing process, that can be incentivized, but not substituted for [see also Rose (1998) for insights in the case of Russia]. A deeper analysis of these policy implications is beyond the scope of this paper.

8 Concluding remarks

We developed an evolutionary model of growth in which agents choose how to allocate their time between private and social activities. Participating to the latter ones requires time and forgoing some private consumption, but they provide an individual utility which depends both on her own and on aggregate participation, as well as on the opportunities available in the social environment. Agents may defend themselves from a poor social environment by shifting to private activities, less exposed to external effects. If this strategy spreads over, private activities will be fostered, but at the expense of social activities. Since both effects accumulate over time, the outcome may be a joint occurrence of economic growth and social poverty. On one side, this is likely to increase social costs (from crime prevention to children and elder keeping [see Coleman (1990)], from schooling in most diseased areas [see Benabou (1993)] to monitoring and transaction costs for firms, to the lost of real opportunities provided by social links [see Granovetter (1973)]); on the other side (and most importantly, since higher private growth could in principle allow an economy to face higher social costs), economic growth needs not be optimal in terms of well-being. A possible alternative outcome is that of a large amount of time spent in social activities, which brings about a rich social environment (i.e. growth in the social opportunities available to the individuals), but may act as an obstacle to private growth. When both these outcomes are possible, the present framework shows that the latter is Pareto-superior to the former one (a plausible result in advanced societies).

Our analysis has introduced a certain number of innovative features, which call for a deeper investigation in future. The present model may be extended first of all by allowing for the possibility of a balanced growth between private and social activities. Two effects might be relevant from this point of view. On one side, social capital increases productivity in the private sector, as well established in the quoted literature, and this renders even more
serious the problems created by under-investment in social activities (social poverty traps), while, at the same time, it allows a balanced growth when social participation is high enough. On the other side, market activities might themselves contribute to create new relations, thus rendering less serious the problem of social poverty and, in turn, allowing a balanced growth even with a low social participation. Since the two effects are counterbalancing, our basic results should still hold under these extensions.

A second extension of the model would be to consider contexts in which, on one side, physical and human capital accumulation may be taken into account, and, on the other side, social capital accumulation may be more deeply investigated by regarding it as a socially differentiated process. It will also be interesting to compare the present results with the ones that can be obtained outside an evolutionary context, e.g. in an infinite-life agent model or in an OLG context. The analysis in terms of well-being may be extended as well, since social evolution not only determines how actual needs are satisfied, but it directly influences the formation of such needs too (an aspect that is particularly difficult to capture, and that can be legitimately ignored in a short run or even in a medium run horizon, but that appears crucially relevant for any long term analysis of well-being).

Finally, as the concepts of relational goods and of social capital are still unusual in the most known economic literature, it may be worth to conclude with a brief consideration about their methodological status. In particular, they fit well Granovetter’s (1985) program of considering together individual actions and social structures, and are well compatible with an extended form of methodological individualism, in which individuals are still the starting point, but they are no more seen as atoms isolated from one another [see Donzelli (1986) and Boland (1982)]. Moreover, the investigation of the relational dimension of individual choices is likely to lead us to extend the scope of economic analysis to consider how cultural and social factors influence needs and purposes of people. Of course, any strict disciple of Robbins’s (1935) epistemology would reply that economics deals with the allocation of scarce means to given alternative purposes and does not discuss the sense of these goals. In contrast, our opinion is that, as well-being and growth may depend to a relevant degree upon the specific motivational structure belonging to agents within a certain culture, it might be worth for economists to directly tackle their determination and evolution [for examples of analyses of this kind see Joireman et al. (1996), Menicucci and Sacco (1996), Sacco (1997), Sacco and Zamagni (1997)]. Indeed, this question develops somehow
the same core as methodological individualism (since its starting point is made of intentional choices), but it brings beyond it, because it recognizes that purposes themselves are not a primum of the analysis, as it is well acknowledged in other social sciences [see e.g. the category of sense, which is always presupposed by that of purpose, in the analyses of Heidegger (1927) for philosophy, of Habermas (1981) for sociology and of Greimas (1983) for semiotics].

Appendix: classification of dynamics (??), (??)

For simplicity, in the following classification we shall consider 'robust' cases only, i.e. those which do not correspond to equality conditions on parameters’ values (except for \( \gamma = 1 \)).

Let us first consider the locus \( \dot{K}_s = 0 \). To this end, notice that \( \dot{K}_s = 0 \) holds if \( K_s = 0 \) or if:

\[
[s_L + (s_H - s_L)x]^{1+\beta} K_s^{\gamma-1} = \delta. \tag{17}
\]

For \( \gamma = 1 \), equation (??) is satisfied if and only if

\[
x = x^* \equiv \frac{\delta 1+\beta}{\beta (s_H - s_L)} \tag{18}
\]

holds; in this context, \( \dot{K}_s < 0 \) if and only if \( x < x^* \) and \( \dot{K}_s > 0 \) if and only if \( x > x^* \).

For \( \gamma \neq 1 \), equation (??) defines a function

\[
K_s = \hat{K}_s(x) \equiv \left( \frac{\delta}{[s_L + (s_H - s_L)x]^{1+\beta}} \right)^{\gamma-1}, \tag{19}
\]

which is strictly increasing in \( x \) if \( \gamma < 1 \) and strictly decreasing if \( \gamma > 1 \). In both cases, \( \dot{K}_s(x) \) is such that\(^{21}\)

\[
\dot{K}_s(0) = \left[ \frac{\delta}{s_L^{1+\beta}} \right]^{\gamma-1}, \tag{20}
\]

\[
\dot{K}_s(1) = \left[ \frac{\delta}{s_H^{1+\beta}} \right]^{\gamma-1}. \tag{21}
\]

\(^{21}\)Notice that \( \lim_{s_L \to 0} \dot{K}_s(0) = +\infty \) if \( \gamma > 1 \) and \( \lim_{s_L \to 0} \dot{K}_s(0) = 0 \) if \( \gamma < 1 \).
If $\gamma < 1$, below (above) the graph of $\hat{K}_s(x)$ it holds $\dot{K}_s > 0$ (respectively, $\dot{K}_s < 0$), while the opposite holds if $\gamma > 1$.

Let us now consider the locus $\dot{x} = 0$. As discussed in Section 5, $\dot{x} = 0 \iff \{x = 0 \lor x = 1 \lor \Delta U(K_s, x) = 0\}$ and $\Delta U(K_s, x) = 0 \iff x = \bar{x}$ [see equation (??)], but now, being $K_s$ no more a fixed parameter, (??) defines a function

$$K_s = \tilde{K}_s(x) \equiv \left\{ \frac{a Y_s}{(s_H - s_L)[s_L + (s_H - s_L)x]^\beta} \right\}^{\frac{1}{\gamma}}$$

(22)

strictly decreasing in $x$ with $\tilde{K}_s(1) = K_s^1$ (see Proposition 1) and $\tilde{K}_s(0) = K_s^2$. Below (above) the graph of $\tilde{K}_s(x)$ it holds $\dot{x} < 0$ (respectively, $\dot{x} > 0$).

Notice that the points $(K_s, x) = (0, 0)$ and $(K_s, x) = (0, 1)$ are always fixed points under dynamics (??), (??). The points $(K_s, x) = (\hat{K}_s(0), 0)$ and $(K_s, x) = (\hat{K}_s(1), 1)$ are fixed points for $\gamma \neq 1$. In each of these fixed points, there is no coexistence of the two strategies. The existence of an 'interior' fixed point (i.e. a fixed point where $K_s > 0$ and $0 < x < 1$) depends on the shape of the graphs of $\tilde{K}_s(x)$ and $\hat{K}_s(x)$. It is easy to check that, if the graphs of $\tilde{K}_s(x)$ and $\hat{K}_s(x)$ cross each other, then at the intersection point it holds $\frac{d \tilde{K}_s(x)}{dx} > \frac{d \hat{K}_s(x)}{dx}$. Therefore, there exists at most an interior fixed point and, when existing, it is always an hyperbolic saddle

22The condition $\frac{d \tilde{K}_s(x)}{dx} > \frac{d \hat{K}_s(x)}{dx}$ implies that the determinant of the jacobian matrix evaluated at the interior fixed point is strictly negative; consequently, the associated eigenvalues are both different from zero (i.e. the fixed point is hyperbolic) and have opposite sign (i.e. the fixed point is a saddle point).

Let us classify dynamics (??), (??).

Case (a): $\gamma > 1$

There are three subcases:

(a.1) For $\hat{K}_s(1) > \tilde{K}_s(1)$, the interior saddle does not exist, $(0, 1)$ and $(\hat{K}_s(1), 1)$ are saddles
23, $(0, 0)$ is a sink (i.e. it is locally attractive) and $(\hat{K}_s(0), 0)$ is a source (i.e. it is repulsive). The stable manifold of $(\hat{K}_s(1), 1)$ separates the trajectories that approach $(0, 0)$ from those where $K_s$ goes to infinity and $x$ approaches 1 (see figure 2.a).

(a.2) For $\hat{K}_s(1) < \tilde{K}_s(1)$ and $\tilde{K}_s(1) > \hat{K}_s(0)$, there exists an interior saddle, $(0, 0)$ is a sink, $(0, 1)$ is a saddle, and both $(\hat{K}_s(0), 0)$ and $(\hat{K}_s(1), 1)$ are sources.

23The stability properties of the fixed points on the edges of the state space follow from a straightforward application of linearization techniques.
The stable manifold of the interior saddle separates the two dynamic regimes described in case (a.1) (see figure 2.b).

(a.3) For $\hat{K}_s(0) > \tilde{K}_s(0)$, the interior saddle does not exist, $(0,0)$ is a sink, both $(0,1)$ and $(\hat{K}_s(0),0)$ are saddles, and $(\hat{K}_s(1),1)$ is a source. The stable manifold of $(\hat{K}_s(0),0)$ separates the two dynamic regimes described in case (a.1). (see figure 2.c).

Case (b): $\gamma = 1$

There are three subcases:

(b.1) For $x^* > 1$ [see (??)], the interior saddle does not exist, $(0,0)$ is a saddle and $(0,1)$ is a source. Along almost every trajectory $x$ approaches the value 1 and $K_s$ goes to infinity (see figure 3a).

(b.2) For $0 < x^* < 1$, there exists the interior saddle, $(0,0)$ is a sink and $(0,1)$ is a source. The stable manifold of the interior saddle separates the two dynamic regimes described in (a.1) (see figure 3b).

(b.3) For $x^* < 0$, the interior saddle does not exist, $(0,0)$ is a sink and $(0,1)$ is a saddle. Almost all trajectories $x$ approach $(0,0)$ (see figure 3c).

Case (c): $\gamma < 1$

There are three subcases:

(c.1) For $\hat{K}_s(0) > \tilde{K}_s(0)$, the interior saddle does not exist, both $(0,0)$ and $(\hat{K}_s(0),0)$ are saddles, $(0,1)$ is a source and $(\hat{K}_s(1),1)$ is a sink. Almost all the trajectories approach the sink (see figure 4.a).

(c.2) For $\hat{K}_s(0) < \tilde{K}_s(0)$ and $\hat{K}_s(1) > \tilde{K}_s(0)$, there exists the interior saddle, $(0,0)$ is a saddle, $(0,1)$ is a source, both $(\hat{K}_s(0),0)$ and $(\hat{K}_s(1),1)$ are sinks. The stable manifold of the interior saddle separates the attraction basins of the two sinks (see figure 4.b).

(c.3) For $\hat{K}_s(1) < \tilde{K}_s(1)$, the interior saddle does not exist, both $(0,0)$ and $(\hat{K}_s(1),1)$ are saddles, $(0,1)$ is a source and $(\hat{K}_s(0),0)$ is a sink. Almost all the trajectories approach the sink (see figure 4.c).
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