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**Limited Liability and Option Contracts in Models  
with Sequential Investments**

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# Limited Liability and Option Contracts in Models with Sequential Investments

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## Summary

The paper investigates a model where two parties  $A$  and  $B$  invest sequentially in a joint project (an asset). Investments and the asset value are nonverifiable, and  $A$  is wealth constrained so that an initial outlay must be financed by either agent  $B$  or an external investor  $C$  (a bank). We show that an option contract in combination with a loan arrangement facilitates first best investments and any arbitrary distribution of surplus if renegotiation is infeasible. Moreover, the optimal strike price of the option is shown to differ across financing modes. If renegotiation is admitted, the first best can still be attained if  $A$ 's bargaining position is not too strong. In addition, either  $B$ -financing or  $C$ -financing may be strictly preferable, and a combination of multiple lenders may be optimal.

*Keywords:* Option Contracts, Sequential Investments, Double Moral Hazard, Corporate Finance.

*JEL-Classification:* D23, H57, L51.

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# 1 Introduction

Parties in an economic relationship often invest sequentially to enhance the value of a joint project. In a research joint venture between a large pharmaceutical company and a specialized biotechnology firm, the research unit is responsible for the early-stage development of a new product, while the subsequent manufacturing and sales tasks are assigned to the downstream company. Similarly, in high tech start-ups, the creativity and devotion of a founder is decisive for the firm's success in its early stages, while the skills of an experienced management team ( often brought in by a venture capitalist) becomes crucial in later periods.<sup>1</sup>

Typically, the investments of both partners are complex and hard to describe or contract upon. Therefore, the question arises of how to design a governance structure which assigns optimal investments incentives in an indirect manner. The literature has shown that - in contrast to straight or non-conditional ownership structures - option contracts may be best suited to attain this goal. Demski and Sappington (1991) were among the first to consider an agency model with sequential investments.<sup>2</sup> They show that if agent  $B$  (who invests subsequently) holds an option to sell the asset to agent  $A$  (who invests first) after both parties invested, both parties can be led to exert efficient effort and the first best is attained. Nöldeke and Schmidt (1998) extend this result to a setting where the agents can renegotiate their initial arrangement. They find that an option-to-own contract which grants  $B$  the right to buy the asset after both parties invested implements the first best. Moreover, renegotiation does not arise on the equilibrium path, and the first best often remains feasible in a stochastic environment where the asset value is subject to uncertainty.<sup>3</sup>

The present paper reconsiders this double moral hazard framework with sequential investments for a situation where agent  $A$  is wealth constrained. Then, two intertwined issues emerge that are well known from the corporate finance literature: First, who should finance an initial outlay that may be necessary to launch the relationship?

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<sup>1</sup>For a recent survey on research alliances, see Hagedoorn et al. (2000). Sahlman (1990) provides a thorough assessment of venture capital financing.

<sup>2</sup>See also Banerjee and Beggs (1989) who, however, focus on a specific production technology.

<sup>3</sup>Edlin and Hermalin (2000) consider a framework where party  $A$  is risk-averse, and focus on option-to-buy contracts with an exercise date after the first agent - but before the second agent - invested. For the special case of risk-neutral parties, their results imply that efficiency can then be attained if and only if the parties' investments are substitutive on the margin.

Second, how can the surplus from the relationship be distributed among the parties involved? Both questions seem relevant in a variety of economic situations where sequential investments play role. In biotechnology joint ventures, the small research firm often does not have the monetary endowment to finance the high-tech equipment necessary to conduct its research. Likewise, start-up firms often enter a relationship with a venture capital fund in order to obtain a seed financing. In these and other situations, it is interesting to ask whether wealth constraints interfere with incentive considerations, and which mode of financing generates optimal incentives. We investigate two basic possibilities. First, partner  $B$  may provide the necessary monetary resources (which we label 'internal financing'). Second,  $A$  and  $B$  can bring in an third party (a bank, say) to raise the initial outlay ('external financing').

A broad theme of the corporate finance literature is that limited liability may have an adverse impact on the outcome of economic relationships. In the present setting, however, we find that financing constraints are in fact irrelevant if renegotiation can be prevented. Internal as well as external financing implements the first best, and the optimal initial contract is a two-part arrangement: it consists of (a) a loan given by either agent  $B$  or an outside party  $C$ , and (b) an option-to-buy contract between  $A$  and  $B$ . Specifically, the optimal arrangement under internal financing resembles a convertible debt contract under which  $B$  can choose whether to insist on a repayment, or to exercise her option instead.<sup>4</sup> We also show that, under internal as well as external financing, any arbitrary distribution of surplus is compatible with the efficiency goal. Hence, each point along the Pareto frontier can be reached and distributional issues impose no constraint on the optimal solution.

Despite this congruence in results, we also find that that the optimal strike price of  $B$ 's option crucially depends on the mode of financing. In case of an external investor, the efficient option price makes agent  $B$  in equilibrium just indifferent between investing and exercising her option or not, and is shown to be the same as in a model where wealth constraints are absent. Conversely, under internal financing, the optimal option price is *ceteris paribus*, i.e., for a given distribution of joint surplus, *larger* than under

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<sup>4</sup>In our model, party  $B$  acquires the *entire* equity as is frequently observed in research joint ventures. According to Arora and Gambardella (1990), for example, many indicators suggest that biotechnology firms are often founded with the intent of later on being sold to a large corporation. In an empirical study on joint ventures, Bleeke and Ernst (1995) find that one partner buys out the other in almost 80% of their sample.

external financing. At first sight, this result is counterintuitive because  $B$  then strictly prefers a debt repayment over her conversion option when  $A$  invests efficiently. This puzzle is resolved by observing that, when agent  $A$  is wealth constrained,  $B$  can in many situations not credibly insist on a repayment when she forgoes own effort because the asset value is then smaller than her repayment claim. To enforce a debt repayment,  $B$  must then undertake some positive threshold effort which increases the asset value but reduces her payoff relative to a situation where  $A$  has a cash endowment. At a strike price which is optimal under external financing,  $B$  thus has a strict preference to exercise even if  $A$  underinvests, which leads  $A$  to defect. As a remedy, the strike price must be raised to a level where  $B$  is in equilibrium just indifferent between a repayment minus the accompanied default effort on the one hand, and efficient investments and debt conversion on the other.

We then consider a scenario where renegotiation is admitted. Now, the financing constraint may have allocative consequences and preclude an efficient outcome of the relationship. In particular,  $A$  may have an incentive to underinvest for strategic reasons in order to extract a larger portion of the surplus. Notably, this problem also appears when  $A$  has all the bargaining power *ex ante* so that the contractual arrangement allows him to appropriate the entire surplus from the relationship. Intuitively, if  $A$  underinvests, renegotiation becomes necessary in order to induce  $B$  to expend the conditionally efficient effort level. If the initial outlay is positive and/or if the initial contract promises  $B$  or  $C$  a large fraction of social surplus, a defection allows  $A$  to default on these claims. Then,  $A$  finds a defection indeed optimal if he is in a sufficiently strong bargaining position, and appropriates a large share of the bargaining surplus.

The efficiency properties of internal and external financing differ in a setting with renegotiation. In general, internal financing dominates when  $A$ 's bargaining power in renegotiations with  $B$  and  $C$  (which arises after a deviation under external financing) is not significantly smaller than in bilateral bargaining with only agent  $B$  (which arises under internal financing). Otherwise, however, bank financing renders it easier to mitigate the underinvestment problem so that there may be an efficiency-improving role for third parties. Finally, we argue that a combination of multiple lenders ( $B$  and  $C$ ) may be optimal if the first best cannot be attained in a arrangement with a single lender. In particular, multiple lenders combine the advantages of internal and external financing: while the lender's default payoff under internal financing is larger (because

$B$  will invest to increase this default payoff) which reduces the renegotiation surplus for party  $A$ , external financing may reduce  $A$ 's strength in renegotiations because he now faces two opponents.

Our results contribute to earlier findings on the optimal governance and financial structure of a wealth-constrained firm. Among others, Bolton and Scharfstein (1990) and Hart and Moore (1995) consider models where the asset (the firm) exists for two periods, in which the firm owner can expend noncontractible productive investments. At the end of each period, a non-verifiable cash flow is realized. If renegotiation is infeasible, the optimal debt contract gives the investor the right to liquidate (part of) the asset after the firm's default on repayments after the first period. While this liquidation is inefficient, it reduces the firm's incentives to default strategically. Berglöf and von Thadden (1994), Dewatripont and Tirole (1994) and Bolton and Scharfstein (1996) show that, if renegotiation is feasible, the optimal capital structure calls for a combination of long-term and short term investors with claims of different priority. In line with our results, the presence of multiple investors may reduce the firm's anticipated renegotiation gain after a default, which renders it less attractive to defect on a repayment obligation on short-term debt. The present paper thus draws on these previous contributions, but is also differs from them in several respects. Most importantly, we assume that two parties rather than one have to expend non-contractible investments which makes it efficient to change ownership titles during the course of the relationship.<sup>5</sup> Also, we consider a situation where the asset value materializes only at the end of the game, implying that repayment decisions cannot impose a shutdown threat on the firm owner. Despite these differences, we also find that the optimal contractual arrangement may resemble financing schemes that are observed in reality.

The remainder of the paper is organized as follows. Section 2 introduces the framework. In Section 3, we analyze the model for the case where renegotiation is infeasible, while Section 4 considers renegotiation. Section 5 contains some brief concluding remarks.

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<sup>5</sup>In independent work, Schmidt (2000) analyzes a similar model where a wealth constrained start-up entrepreneur  $E$  and a venture capitalist  $VC$  expend sequential investments. His model confines attention to the case of internal financing and (in contrast to the other papers in the literature) assumes the asset value to be verifiable which renders shared-equity arrangements feasible. Finally,  $VC$  can only undertake a binary rather than a continuous effort after  $E$  invested. Disregarding the possibility of strategic default which is the focus of the present paper, it is shown that convertible debt facilitates the first best in a variety of situations.

## 2 The Model

We consider a model with two risk-neutral agents  $A$  and  $B$  who start a relationship and sequentially invest into an asset. At *date 0*, the partners sign an initial contract, and a monetary seed investment  $K \geq 0$  has to be incurred. At *date 1*, party  $A$  can expend an idiosyncratic investment (which will be referred to as effort)  $a \in R_0^+$ . At *date 2*, the initial contract may be renegotiated, before agent  $B$  undertakes her own effort  $b \in R_0^+$  at *date 3*. Both investments are in physical capital so that the asset value neither depends on its final owner nor on the further engagement of either party. The asset value  $v(a, b)$  materializes at *date 4*, and the game ends at *date 5* where repayments are made, options may be exercised, and final payoffs are realized.

Figure 1 below illustrates the sequence of events.

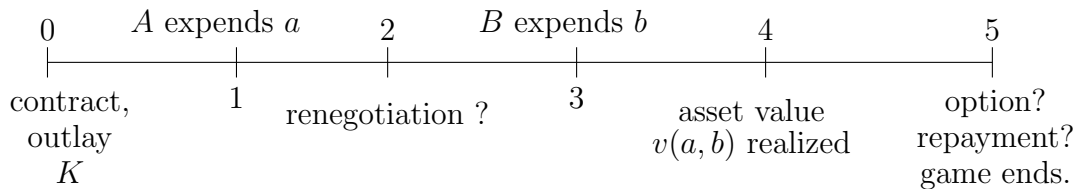


FIGURE 1

We make the following informational and contracting assumptions. Both parties have complete information throughout the game. Moreover, the monetary seed investment  $K$  is contractible, while the idiosyncratic effort levels  $a$  and  $b$  are not. Likewise, the asset value  $v(a, b)$  is non-verifiable to outsiders and thus cannot be contracted upon. In what follows, we also impose

**Assumption 1.** *The function  $v(\cdot)$  is twice continuously differentiable, strictly increasing in both arguments, strictly quasiconcave, and satisfies (subscripts denote derivatives)*

- a)  $v(a, b) > 0$  for all  $(a, b) > 0$  and  $v(0, 0) = 0$ .
- b)  $v_{ii}(\cdot) < 0$ ,  $\lim_{i \rightarrow 0} v_i(\cdot) = \infty$  and  $\lim_{i \rightarrow \infty} v_i(\cdot) = 0$  for  $i \in \{a, b\}$ .
- c)  $v_{ab}(\cdot) \geq 0$  for all  $(a, b)$ .



According to part a), the joint project has a non-negative gross value for any feasible combination  $(a, b)$ . If neither party expends effort, this value is normalized to zero. Part b) ensures that some positive but finite investment levels are optimal provided the project should be started. Finally, part c) states that investments are (weak) complements at the margin. We thus focus on the natural case where the return on, e.g., basic research is small if not combined with complementary skills such as production experience and marketing know-how, and vice versa.<sup>6</sup>

For subsequent reference, we compute the first-best investments  $(a^{FB}, b^{FB})$  which maximize the ex-ante surplus,

$$S(a, b) = v(a, b) - a - b - K. \quad (1)$$

Throughout, we suppose that the relationship should be started, i.e.,  $v(a^{FB}, b^{FB})$  is sufficiently larger than the monetary seed investment  $K$ . Then,  $(a^{FB}, b^{FB})$  are strictly positive and uniquely defined by the first-order conditions

$$v_a(a^{FB}, b^{FB}) = 1 \quad \text{and} \quad v_b(a^{FB}, b^{FB}) = 1. \quad (2)$$

Let  $\hat{S}(a, b) = v(a, b) - b$  be the joint continuation surplus after  $A$  invested. Also, define  $b^*(a) = \arg \max_b \hat{S}(a, b)$  as the ‘conditionally’ efficient investment level of party  $B$  for given  $a$ , so that  $b^*(a^{FB}) = b^{FB}$ . Notice that  $B$  will expend  $b^*(a)$  when she anticipates to be asset owner and therefore residual claimant for the return from her own effort at date 5.

After  $A$  invested, the parties may find it useful to rescind their initial contractual arrangement and write a new one. For convenience and in line with the literature, we suppose that the outcome of these renegotiations is described by the generalized Nash-bargaining solution. When renegotiations occur at date 2, the agents therefore share the efficiency gain above their respective default payoffs according to a linear sharing rule.<sup>7</sup> We parameterize  $A$ ’s bargaining strength in bilateral renegotiations with  $B$  by  $\gamma \in [0, 1]$ , while  $B$  has a relative bargaining power  $(1 - \gamma)$ .

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<sup>6</sup>As mentioned in the Introduction, this assumption implies that option contracts with an exercise date before  $B$  invested (i.e., at date 2) do not implement the first best even if limited liability is disregarded [see Edlin and Hermalin (2000)]. Most of our subsequent results extend to the case of substitutive investments as well. See the discussion in footnote 14 below.

<sup>7</sup>This efficiency gain is the difference between the maximal joint continuation surplus  $\hat{S}(a, b^*(a))$ , and the sum of the default payoffs.

As a useful starting point of analysis, let us first consider a situation where  $A$  is not wealth-constrained and finances the initial outlay  $K$  out of own funds. Suppose that  $A$  is asset owner at date 0 and  $A$  and  $B$  do not sign an initial contract.<sup>8</sup> After  $A$  invested  $a$  at date 1,  $B$  will at date 3 undertake the conditionally efficient investment  $b^*(a)$  only if she anticipates to become residual claimant. Hence, the parties will (if feasible) renegotiate the initial governance structure at date 2 where  $B$  buys the asset. Since  $B$  will clearly not invest when  $A$  retains ownership,  $A$ 's default payoff at the renegotiation date 2 is given as  $v(a, 0)$ . Accordingly, and presuming it is efficient to start the project, his maximization program at date 1 reads

$$\max U^A(a) = v(a, 0 + \gamma[v(a, b^*(a)) - b^*(a) - v(a, 0)] - K - a, \quad (3)$$

and the unique equilibrium effort  $\hat{a}$  is determined by the first-order condition

$$(1 - \gamma)v_a(\hat{a}, 0) + \gamma v_a(\hat{a}, b^*(\hat{a})) = 1. \quad (4)$$

This condition immediately reveals that  $\hat{a} < a^{FB}$  when investments are complementary, because  $v_a(a, 0) < v_a(a, b^*(a))$  in this case.<sup>9</sup> Only if investments are marginally independent,  $A$  invests efficiently and non-conditional ownership attains the first best provided renegotiation is feasible. In the next sections, we analyze option contracts and ask whether this contingent governance structure can overcome the inefficient outcome under non-contingent ownership. Thereby, we disregard the possibility of renegotiation in Section 3, while renegotiation is taken into account in the subsequent Section 4.

### 3 Equilibrium Analysis

In related papers, Demski and Sappington (1991) and Nöldeke and Schmidt (1998) have shown that option contracts generically facilitate an efficient outcome. These models suppose that both parties possess a sufficiently large monetary endowment and, consequently, limited liability is no matter of concern. Also, no monetary outlay is required at the start of the relationship. As we will see in subsection 3.1 below, an efficient outcome remains feasible if a seed investment is needed but  $A$  is not wealth constrained. After analyzing this benchmark, we incorporate a wealth constraint on

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<sup>8</sup>Notice that  $A$  will not expend any effort when  $B$  initially owns the asset.

<sup>9</sup>Similarly, if investments are substitutive, we have  $\hat{a} > a^{FB}$ .

$A$ 's side who then cannot finance  $K$ , and also cannot assure  $B$ 's participation in the venture by providing an upfront payment. Subsection 3.2 considers a situation where  $B$  finances the asset start up. Subsequently, in subsection 3.3, we examine external financing by a bank  $C$  which has no further productive role.

### 3.1 Wealthy Agents

Suppose  $A$  and  $B$  sign the following arrangement  $(L, R, p)$  at date 0. Under the terms of contract,  $A$  initially owns the asset and  $B$  provides some (possibly negative) monetary loan  $L$ . At date 5,  $B$  can then either insist on (or provide) a repayment  $R$ , or she can alternatively exercise an option-to-buy. If  $B$  exercises, she disburses a strike price  $p$  and acquires asset ownership. Notice that  $B$  will find it profitable to invest and to exercise her option if and only if  $v(a, b^*(a)) - b^*(a) - p \geq R$ . We now demonstrate that this conditional ownership arrangement in combination with a debt-like financial structure induces  $A$  to invest efficiently if  $A$  is not wealth constrained. Consider a strike price

$$p^* = v(a^{FB}, b^{FB}) - b^{FB} - R$$

for some arbitrary combination  $(L, R)$  that is compatible with each agent's participation constraint. Imagine  $A$  undertakes an effort  $a \geq a^{FB}$  at date 1. Then,  $B$  invests  $b^*(a)$  and exercises her option-to-buy at date 5 because her associated continuation payoff  $v(a, b^*(a)) - b^*(a) - p^*$  at least weakly exceeds  $R$ . Therefore,  $A$  will never invest more than  $a^{FB}$  because  $B$  reaps the return on any excess effort. We must also show that  $A$  will not invest less than  $a^{FB}$ . Suppose  $a < a^{FB}$ . Then,  $B$  refrains from any investment and insists on the repayment  $R$  at date 5 because  $R > v(a, b^*(a)) - b^*(a) - p^*$ . Hence,  $A$ 's payoff is  $U^A(a < a^{FB}) = S(a, 0) + L - R$  which is strictly smaller than  $U^A(a = a^{FB}) = S(a^{FB}, b^{FB}) + L - R$ . As a consequence,  $a < a^{FB}$  cannot be  $A$ 's preferred choice for strike price  $p^*$ . Notice that this result applies for any feasible loan and repayment levels. Thus, if  $A$  is not wealth-constrained,  $L$  as well as  $R$  can be chosen as desired to divide the ex-ante surplus  $S(a^{FB}, b^{FB})$  between both parties.

We can state the following proposition which is an extension of results found in Demski and Sappington (1991) and Nöldeke and Schmidt (1998).<sup>10</sup>

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<sup>10</sup>As mentioned in the Introduction, Demski and Sappington analyze option-to-sell rather than option-to-buy contracts. One can show that both contracting types have identical economic conse-

**Proposition 1.** *Suppose that  $A$  is not wealth-constrained. For any  $K \geq 0$ , an option-to-buy contract  $(L, R, p)$  with strike price  $p^* = v(a^{FB}, b^{FB}) - b^{FB} - R$  implements efficient investments and distributes the joint surplus in any desired way by appropriate choice of  $(R, L)$ .*

Proposition 1 shows that the start-up cost  $K$  *per se* does not hamper an efficient outcome of the relationship. If  $A$  is not wealth-constrained, it is in fact irrelevant which party bears the monetary seed investment. For example, if one switches from  $L = 0$  to  $L = K$  so that  $B$  finances the asset start up, a reduction of the option price by an amount  $R = K$  leaves the equilibrium payoffs of both parties unaffected, and does not distort their incentives to expend value-enhancing effort.

### 3.2 Financing Constraints

We now explore whether the implementation result of Proposition 1 carries over to a setting where  $A$  is wealth constrained, which seems relevant in many real-life situations (including venture capital financing and research joint ventures). Observe that Proposition 1 applies for any conceivable level of the initial monetary payment  $L$  that flows from  $B$  to  $A$ . Therefore, even if  $A$  is wealth-constrained,  $B$ 's participation and the asset start up can be ensured under an option contract with strike price  $p^*$  if both parties continue to invest efficiently. This reasoning seems to suggest that  $A$ 's lack of monetary resources does neither affect the optimal contract nor the implementable outcome. Perhaps counterintuitively, though, we show that an option contract with price  $p^*$  may lead party  $A$  to underinvest. Fortunately, a modified strike price is found to restore a first-best outcome under internal financing.

While a detailed formal analysis is relegated to the Appendix, it is useful to give an intuitive explanation for these results. Recall that if  $A$  is not wealth constrained, he will not underinvest under an option-to-buy contract with strike price  $p^*$  because  $B$  then refuses to invest  $b > 0$  and insists on a repayment  $R$ . Since  $A$  appropriates the maximum overall surplus  $S(a^{FB}, b^{FB})$  minus a constant  $(R - L)$  when he invests efficiently and  $B$ 's continuation payoff cannot fall below  $R$ , any deviation from  $a^{FB}$  can only hurt  $A$  and is thus self defeating.

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quences whether or not wealth constraints are admitted. Therefore, we can without loss of generality focus on option-to-buy contracts to shorten the exposition.

If  $A$  is wealth-constrained, however,  $B$ 's default strategy  $b = 0$  after observing  $a < a^{FB}$  may no longer be optimal. To see this, consider  $R \geq L = K$  and  $A$  expends an effort so small that  $v(a, 0) < R$ . Notice that effort levels with this property exist for any  $K > 0$  by Assumption 1.<sup>11</sup> Then,  $A$  has to default on repayment  $R$  when  $B$  exerts no effort, and  $B$  has a legal claim on the asset and becomes owner at date 5. However, if  $A$  goes indeed bankrupt and  $B$  seizes the collateral,  $b = 0$  cannot be her best response on  $A$ 's defective action. It is now useful to state the following definition.

**Definition 1.**

Let

$$\tilde{b}(a, R) = \begin{cases} \max\{b \mid v(a, b) \leq R\} & \text{if } v(a, 0) \leq R \\ 0 & \text{otherwise.} \end{cases}$$

The threshold investment  $\tilde{b}(a, R)$  represents the minimal effort which  $B$  must expend for given  $a$  to enforce the repayment  $R$ .<sup>12</sup> If  $\tilde{b}(a, R)$  is positive,  $B$ 's best reply is one of two actions: either, she undertakes the threshold investment  $\tilde{b}(\cdot)$  where  $A$  is just able to repay. If this effort level is the optimal response,  $B$  insists on a repayment and her continuation payoff becomes  $R - \tilde{b}(a, R)$ . The corresponding payoff of  $A$  is then strictly smaller than  $S(a^{FB}, b^{FB}) - (R - L)$  so that a deviation from  $a^{FB}$  cannot be profitable. Alternatively, the best continuation action for  $B$  may be to expend  $b^*(a)$  and to appropriate the asset. If  $b^*(a) < \tilde{b}(a, R)$ , this strategy is always dominant because  $b^*(a)$  by definition maximizes the continuation value of the asset, and  $B$  acquires it costlessly at date 5 where  $A$  is still unable to repay. Again, a deviation cannot be worthwhile for  $A$  who is left with a non-positive payoff. Finally, though, consider a situation where

$$\tilde{b}(a, R) < b^*(a) \tag{C1}$$

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<sup>11</sup>They also exist if  $B$  has to be promised a positive share of total surplus, i.e., if  $R > 0$  even if  $K = L = 0$ .

<sup>12</sup>Throughout the main text, we will without loss of generality concentrate on loan levels  $L = K$ . For  $L > K$  where the initial loan exceeds the seed investment,  $A$  retains a monetary endowment  $L - K$ . Hence, he can repay his debt for given  $(a, b)$  whenever  $(L - K) + v(a, b) \geq R$ . In order to leave  $B$ 's equilibrium surplus unaffected, an increase in  $L$  must be accompanied by an identical increase in  $R$ . Accordingly, the threshold investment  $\tilde{b}(a, L, R(L))$  does not vary in  $L$  for any given distribution of total surplus. All proofs in the Appendix allow for loan levels  $L > K$ , so that  $\tilde{b}(\cdot)$  is there defined as  $\max\{b \mid v(a, b) \leq R - (L - K)\}$  if  $v(a, 0) \leq R - (L - K)$ , and  $\tilde{b}(\cdot) = 0$  otherwise.

and notice that this condition holds (at least) for deviations  $a$  close to  $a^{FB}$ .<sup>13</sup> Clearly,  $B$  can then again exert  $\tilde{b}(a, R)$  and claim  $R$  (which, of course, remains optimal if  $\tilde{b}(\cdot) = 0$ ). Alternatively, however, she may undertake an investment  $b^*(a)$ , subsequently exercise her option-to-buy at strike price  $p^*$ , and obtain a continuation payoff  $v(a, b^*(a)) - b^*(a) - p^*$ . In this latter case,  $A$  appropriates a total payoff  $U^A(a) = p^* - a$  which is clearly larger than his utility when investing efficiently. One can easily check that, if (C1) is satisfied,  $b^*(a)$  is agent  $B$ 's optimal response on  $A$ 's defection if  $v(a, b^*(a)) - b^*(a) - p^* > R - \tilde{b}(a, R)$ . Inserting  $p^*$  and  $R$ , this latter condition translates into

$$[v(a^{FB}, b^*(a^{FB})) - b^{FB}] - [v(a, b^*(a)) - b^*(a)] < \tilde{b}(a, R). \quad (C2)$$

We can now state the following preliminary result.

**Lemma 1.** *Suppose  $A$  is wealth-constrained and the parties sign an option contract with strike price  $p^*$ . Then, this contract fails to implement the first best whenever there exists some investment level  $a$  which satisfies conditions (C1) and (C2). In particular, efficient investments cannot be implemented if  $v(a^{FB}, 0) < R$ .*

Importantly, the lemma states that agent  $A$  may deviate from efficient investments under strike price  $p^*$  even if the effort  $a^{FB}$  allows him to capture the entire social surplus, e.g., if  $R = K$ . If he is wealth constrained, he can 'force'  $B$  to expend a positive investment even after a defection.  $B$  may then even find it beneficial to expend  $b^*(a)$  and to buy the asset, although this reduces her overall surplus below  $(R - L)$ . In a word,  $A$  may sacrifice efficiency and exploit his limited liability strategically in order to extract a larger share of the social surplus.

It is easy to see that defection always arises if  $v(a^{FB}, 0) < R$ , i.e., if  $K > 0$  and/or  $B$  has some bargaining power ex ante, and if  $B$ 's investment is sufficiently important. In these cases,  $\tilde{b}(a, R)$  is strictly positive for any deviation  $a < a^{FB}$ . At least for a slight underinvestment of party  $A$ , agent  $B$  then strictly prefers to invest  $b^*(a) > \tilde{b}(a, R)$ , and buys the asset at price  $p^*$  because condition (C2) is satisfied. To illustrate this, consider the quasiconcave Cobb-Douglas production function  $v(a, b) = a^y b^y$ ,  $y < 1/2$ . For this functional form, we have  $v(a, 0) < K$  and  $\tilde{b}(a, R) = R^{1/y}/a$  for any positive  $a, K$ . Suppose  $A$  invests slightly less than  $a^{FB}$  and observe that  $\tilde{b}(a, R) < b^*(a)$  for

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<sup>13</sup>Since  $R < v(a^{FB}, b^{FB})$  is necessary to satisfy  $A$ 's participation constraint, we must have  $\tilde{b}(a \rightarrow a^{FB}, R) < b^*(a \rightarrow a^{FB}) = b^{FB}$ .

this deviation. For  $a \rightarrow a^{FB}$ , the left-hand side of (C2) converges to zero while  $\tilde{b}(a, R)$  converges to a strictly positive value for any  $R \geq K$ . Accordingly, an option contract with strike price  $p^*$  does not implement efficient investments.

We now show that a modified option price may nevertheless facilitate an efficient outcome of the relationship. Specifically, consider the strike price

$$p^{**} = v(a^{FB}, b^B) - b^{FB} - R + \tilde{b}(a^{FB}, R).$$

Then, we have

**Proposition 2.** *Consider internal financing. Then, an option-to-buy contract with strike price  $p^{**}$  generically implements efficient investments, and distributes (by proper choice of  $R \geq K$ ) the joint surplus  $S(a^{FB}, b^{FB})$  in any desired way. In particular, the optimal strike price is strictly larger than  $p^*$  unless  $\tilde{b}(a^{FB}, R) = 0$ .*

**Proof:** see the Appendix.

At first glance, one may wonder how  $B$  can be induced to invest and to exercise her option at a strike price  $p^{**} > p^*$ . Even if  $A$  invests efficiently,  $B$  at this larger strike price strictly prefers a repayment  $R$  over her option-to-buy and the accompanied investments. Using our previous arguments, though, this puzzle can easily be resolved. If  $A$  is wealth-constrained and  $v(a^{FB}, 0) < R$ ,  $B$  cannot recover  $R$  without expending own investments  $\tilde{b}(a, R)$ , which reduces her continuation payoff below  $R$ . At an option price  $p^*$  and for  $a = a^{FB}$ , agent  $B$  therefore has a *strict* rather than a weak preference to choose  $b^{FB}$  and to acquire the asset. Moreover,  $A$  will exploit this preference because he can underinvest (at least in some range) without running the risk that  $B$  does not exercise subsequently.

The steeper option price  $p^{**} = p^* + \tilde{b}(a^{FB}, R)$  resolves this problem. The argument proceeds in two steps. First, provided  $A$  invests efficiently,  $B$  is at price  $p^{**}$  just indifferent between investing efficiently and exercising her option, or to undertake the threshold investment  $\tilde{b}(a^{FB}, R)$  and to claim  $R$ . To see this, recall that  $B$  prefers the former strategy at strike price  $p$  if and only if

$$v(a, b^*(a)) - b^*(a) - p \geq R - \tilde{b}(a, R). \quad (5)$$

By construction of  $p^{**}$ , this condition is satisfied with equality for  $a = a^{FB}$ . Second, we must show that  $B$  will not exercise her option for any  $a < a^{FB}$ , but instead choose

the threshold investment  $\tilde{b}(a, R)$  and insist on a debt repayment. From (??), this behavior is indeed optimal if  $v(a, b^*(a)) - b^*(a) + \tilde{b}(a, R)$  is strictly increasing in  $a$ . In the Appendix, we demonstrate that this monotonicity condition is always satisfied if investments are independent or complementary at the margin, i.e., under Assumption 1c). Accordingly, since  $B$  will invest efficiently and disburse  $p^{**}$  only if  $a \geq a^{FB}$ , agent  $A$  will not deviate from  $a^{FB}$  and an efficient outcome is attained. We should emphasize that the above reasoning applies for any feasible repayment level  $R$ . Hence, one can choose  $R$  arbitrarily in order to distribute the social surplus  $S(a^{FB}, b^{FB})$  between both parties.

Under internal financing, limited liability thus does not preclude efficient sequential investments if renegotiation is infeasible.<sup>14</sup> In contrast to the framework where  $A$  has a monetary endowment, though, the optimal option price may be one which does not make party  $B$  indifferent between exercising her option on the one hand, and to claim a repayment on the other. Rather, the option price may be so large that she has strict preferences for a repayment but anticipates that an enforcement of this claim will require costly effort.

### 3.3 External Financing

We now suppose that  $A$  does not rely on  $B$  to finance the initial outlay, i.e.,  $L = 0$ . Instead, he signs a debt contract with an external investor  $C$  (e.g., a bank) which at date 0 provides a loan  $L^C \geq K$  and holds a repayment claim  $R^C$  to be exercised at date 5.

Specifically, consider the following arrangement:  $A$  signs an option-to-buy contract  $(p^*, L = R = 0)$  with  $B$  and a debt contract  $(L^C, R^C)$  with the external investor  $C$ . Suppose  $A$  invests efficiently. Then,  $B$  will expend  $b^{FB}$  and exercise her option at date

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<sup>14</sup>To show this, we relied on the assumption that investments are weakly complementary,  $v_{ab}(\cdot) \geq 0$ . While an inefficient outcome may possibly arise for substitutive investments, I was unable to construct an example where this is actually the case. Edlin and Hermalin (2000) have shown that, if wealth constraints are disregarded, an option contract with exercise date 2 facilitates the first best if (and only if) investments are substitutive on the margin. Hence, in order to prove that an option contract can trigger an inefficient outcome under internal financing with substitutive investments, one would also have to check this alternative type of contractual arrangement which is outside the scope of the present work. We should note that - with the exceptions of Proposition 2 and Proposition 4(b) below - all results in the present paper also hold in situations where investments are strict substitutes.



5, and  $A$  repays her debt  $R^C$  (which is smaller than  $p^*$ ) to agent  $C$ . It remains to show that  $A$  cannot gain by investing less than  $a^{FB}$ . If he does,  $B$  will expend no effort because this would yield a negative continuation payoff while she can guarantee herself a reservation payoff of zero by not investing. Accordingly,  $A$ 's payoff in case of a deviation is either zero [if  $v(a, 0) < R^C$  in which case  $C$  seizes the asset], or it is  $S(a, 0) - (R^C - L^C)$  [if  $v(a, 0) \geq R^C$  so that  $A$  can repay]. Either payoff is smaller than  $S(a^{FB}, b^{FB}) - (R^C - L^C)$  so that a defection cannot be profitable. Thus,

**Proposition 3.** *Suppose  $A$  is wealth-constrained and  $K$  is raised from an external investor  $C$ . Then, an option-to-buy contract  $p^*(R = 0)$  implements efficient investments and a first-best outcome prevails. Also, any distribution of surplus among all three parties can be ensured via a fixed payment from  $C$  to  $B$ .*

The logic behind this proposition is simple. If  $B$  does not finance the initial outlay, she is not locked into the relationship before expending own investments. Therefore,  $A$  cannot exploit his limited liability to hold up  $B$  and to force her to invest in order to protect her repayment claims after  $A$  defected. Relying on an external investor prevents hold up exactly because this investor has no productive role. In addition, the initial arrangement can assign parties  $B$  and  $C$  any arbitrary share of the joint surplus. To see this, consider a contract extension under which  $B$  receives from  $C$  an unconditional fixed payment, say  $T$ . Clearly, the size of this lump sum transfer has no effect on efficiency and the parties' equilibrium rents are  $U^B = T$ ,  $U^C = R^C - L^C - T$ , and  $U^A = S(a^{FB}, b^{FB}) - (R^C - L^C)$ .

A simple contractual arrangement among  $A$ ,  $B$  and an external investor  $C$  thus implements the first best if renegotiation is infeasible.<sup>15</sup> While this outcome replicates the implementation result under internal financing, the construction of the optimal strike price of  $B$ 's option differs across regimes. In particular, and in contrast to internal financing, the optimal strike price now exactly coincides with the one in a model where limited liability and financing issues do not arise. Consequently, when external investors are admitted and renegotiation can be prevented, there is no loss of generality in confining attention to an option-to-buy contract with strike price  $p^*(R = 0) = v(a^{FB}, b^{FB}) - b^{FB}$ .

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<sup>15</sup>Notice that external financing generically implements the first best even if investments are strict substitutes. Hence, this financing mode may strictly dominate internal financing for  $v_{ab}(\cdot) < 0$ . See also footnote 14.

## 4 Renegotiation

In our discussion of section 3, we found that the relationship between  $A$  and  $B$  yields an efficient outcome if renegotiation is infeasible after  $A$  expended his effort. It is now interesting to ask whether under which conditions this positive outcome extends to a scenario where renegotiation is allowed for. We obtain the following result.

**Proposition 4.** *Suppose the parties can renegotiate their initial arrangement after  $A$  invested. Then, we have:*

- (a) *If  $A$  is not wealth constrained, an option-to-buy contract with strike price  $p^*$  attains efficient investments and the initial contract is not renegotiated. Hence, Proposition 1 fully applies.*
- (b) *If  $A$  is wealth constrained and  $B$  provides the start-up financing, an option-to-buy contract  $p^{**}$  implements a first-best outcome unless there exists some effort level  $a$  with  $\tilde{b}(a, R) < b^*(a)$  such that*

$$p^{**} - a^{FB} < \gamma[v(a, b^*(a)) - b^*(a) - (R - \tilde{b}(a, R))] - a \quad (C3).$$

*Otherwise,  $A$  underinvests and a first-best outcome cannot be implemented.*

- (c) *If  $A$  is wealth constrained and an external investor  $C$  contributes the start-up financing as a loan with repayment claim  $R^C$ , an option-to-buy contract  $p^*(R = 0)$  between  $A$  and  $B$  facilitates a first-best outcome unless there exists some effort level  $a$  with  $\tilde{b}(a, R^C) > 0$  such that*

$$p^*(R = 0) - R^C - a^{FB} < \gamma_A[v(a, b^*(a)) - b^*(a) - v(a, 0)] - a \quad (C4),$$

*where  $\gamma_A$  is  $A$ 's Nash bargaining parameter in trilateral renegotiations with  $A$  and  $B$ . Otherwise,  $A$  underinvests under external financing.*

**Proof:** see the Appendix.

In a setting where  $A$  is not wealth constrained, our previous findings continue to apply if renegotiation is admitted. In fact, Proposition 4(a) is a straightforward extension of results in Nöldeke and Schmidt (1998). Even if renegotiation is feasible,  $B$  can always insist on a repayment so that her overall equilibrium utility cannot fall below

$(R - L)$ . Likewise, she cannot insist on renegotiation if  $A$  invests efficiently (see the proof of Lemma 1). Since  $A$  reaps the entire social surplus from the relationship minus the constant  $(R - L)$  when he expends efficient effort, any deviation cannot raise his surplus and efficiency prevails.

The other parts of the proposition, however, establish that the opportunity to renegotiate may hamper an efficient outcome of the relationship. Proposition 4(b) shows that in case of internal financing  $A$  may defect and choose an investment level smaller than efficient. In view of our previous arguments, this result is easily explained. Under an option contract with strike price  $p^{**}$  (which remains optimal),  $B$  will on a deviation never respond with a default investment  $b > \tilde{b}(a, R)$ . If  $b^*(a) \leq \tilde{b}(\cdot)$ , her preferred default effort is  $b^*(a)$  in which case she seizes the asset and  $A$  will not deviate because no renegotiation arises. Conversely, if  $b^*(a) > \tilde{b}(\cdot)$ ,  $B$  expends a default effort  $\tilde{b}(\cdot)$  which is smaller than the conditionally efficient level. Therefore, the parties renegotiate at date 2 and  $A$  reaps a fraction  $\gamma$  of the bargaining surplus  $[v(a, b^*(a)) - b^*(a) - (R - \tilde{b}(a, R))]$ . If this payoff net of the corresponding investment  $a$  exceeds  $p^{**} - a^{FB}$ ,  $A$  finds a defection profitable and an efficient outcome becomes infeasible. Intuitively,  $A$  may underinvest under internal financing because this forces  $B$  into renegotiation, and diminishes her overall utility significantly below  $(R - \tilde{b}(a^{FB}, R) - L)$  if  $A$ 's own bargaining position is sufficiently strong.<sup>16</sup>

Proposition 4(c) asserts that underinvestment may also arise under external financing when renegotiation is taken into account. The reason for a possible defection under  $C$ -financing is however quite different. If  $A$  underinvests, he cannot hurt  $B$  who will simply not invest and obtain her reservation utility. For any investment such that  $\tilde{b}(a, R^C) > 0$  which implies  $v(a, 0) < R^C$ , though,  $A$  causes an externality on  $C$  who then cannot recover his loan and seizes the collateral unless renegotiations are successful.<sup>17</sup> If these renegotiations proceed as Nash bargaining among all three involved parties,<sup>18</sup>  $A$  again captures a part of the negotiation gain  $[v(a, b^*(a)) - b^*(a) - v(a, 0)]$ . Provided (C4) does

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<sup>16</sup>To see that defection can indeed arise under internal financing, consider a situation where  $\tilde{b}(a^{FB}, R) > 0$  (e.g., the Cobb-Douglas value function). Since  $\tilde{b}(\cdot)$  decreases in  $a$ , the maximizer of the right-hand side of (C3) must be strictly smaller than  $a^{FB}$  for any  $\gamma \leq 1$ . Since (C3) is satisfied with equality for  $a = a^{FB}$  and  $\gamma = 1$ ,  $A$  will thus defect for  $\gamma$  sufficiently close to unity.

<sup>17</sup>Recall that  $B$  will not invest after observing  $a < a^{FB}$ , so that  $C$  seizes the asset and obtains a pre-renegotiation payoff  $v(a, 0) < R^C$ .

<sup>18</sup>While we consider Nash bargaining for simplicity, a qualitatively identical result is obtained for more sophisticated bargaining rules as, for instance, the Shapley value.

not apply,  $A$  does not deviate and a first-best outcome remains feasible. Conversely, if (C4) applies for some  $a$  where  $A$  is unable to repay his initial loan, a deviation cannot be avoided and the possibility of renegotiation imposes a binding constraint on the feasible outcome.

To further assess these findings, it is interesting to note that (C4) is never satisfied if  $A$ 's bargaining parameter in trilateral renegotiations with  $B$  and  $C$  is small. Hence, if  $\gamma_A$  is sufficiently smaller than  $\gamma$  because, for instance, the external investor has a strong bargaining position, external financing may facilitate the first best while internal financing does not.<sup>19</sup> On the other hand, suppose  $\gamma_A = \gamma$  so that  $A$ 's bargaining strength in negotiations with  $B$  and  $C$  is no smaller than under internal financing. Then, internal financing dominates external financing when renegotiation is admitted: since  $B$  expends no default investment after a deviation under external financing, but exerts a positive default effort under internal financing, the bargaining surplus is strictly smaller in the latter case.<sup>20</sup> In addition, the non-deviation payoff in (C4) is strictly larger than in (C3). Taken both effects together, internal financing renders it easier to implement efficient investments if  $\gamma_A = \gamma$  and thus becomes unambiguously preferable in this case.

Finally, we want to argue that a combination of internal and external financing may facilitate the first best if renegotiation is feasible and a single lender triggers a sub-optimal outcome.<sup>21</sup> Suppose  $B$  provides a loan of size  $\beta K$ ,  $\beta \in [0, 1]$ , and  $C$  one of size  $(1 - \beta)K$  to cover the start-up cost. For simplicity, suppose these loan levels coincide with the lenders' repayment claims  $R^B$  and  $R^C$  with  $R^B + R^C = R$ , respectively. Consider a situation where  $A$  defects under pure external ( $\beta = 0$ ) and pure internal

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<sup>19</sup>Similar observations on the virtues of external financing have been made in the literature. Aghion and Tirole (1994) show that external financing can improve the bargaining position of a client vis-a-vis the researcher (the agent) in a research project, and facilitate an efficient allocation of property rights even if the agent is wealth constrained.

<sup>20</sup>In technical terms, we have  $v(a, 0) < R - \tilde{b}(a, R) = v(a, \tilde{b}(\cdot)) - \tilde{b}(\cdot)$  because  $v(a, b) - b$  is increasing in  $b$  for any  $b < b^*(a)$ , while  $\tilde{b}(\cdot) < b^*(a)$  is a necessary condition for a deviation.

<sup>21</sup>Joint financing is empirically relevant. For example, the internet broker priceline.com recently announced its plans to expand into Europe [see Priceline Press release, June 28, 2000]. Priceline.com Europe is a new company in which priceline.com and the venture capital fund General Atlantic are investors and jointly fund the company. Under the terms of contract, priceline.com purchases a convertible note allowing the company to take up to a 50% equity stake in priceline.com Europe under certain conditions. Until that note is converted, priceline.com will not hold an equity stake in the new venture.

financing ( $\beta = 1$ ). Now, consider multiple lenders and suppose that  $A$  and  $B$  hold claims of the same priority. In addition, suppose  $A$  and  $B$  sign an option contract with strike price  $\tilde{p}^{**} = v(a^{FB}, b^{FB}) - b^{FB} - R^B + \hat{b}(a^{FB}, R^B)$  where  $\hat{b}(a, R^B)$  is implicitly defined by  $\beta v(a, \hat{b}) = \beta K = R^B$ . Notice that  $\hat{b}(a, R^B) = \tilde{b}(a, R)$  because agent  $B$ 's default effort is insensitive with respect to  $\beta$  (for  $\beta > 0$ ) if both claims have the same priority. If  $A$  deviates in a way that  $\tilde{b}(a, R^B) < b^*(a)$ , the default investment of agent  $B$  is therefore  $\tilde{b}(a, R)$ , and all three parties renegotiate in order to realize the bargaining surplus  $[v(a, b^*(a)) - b^*(a) - (R^B - \tilde{b}(a, R))]$ . Since  $A$  appropriates a fraction  $\gamma_A$  of this bargaining gain, underinvestment can be profitable only if

$$\tilde{p}^{**} - R^C - a^{FB} < \gamma_A[v(a, b^*(a)) - b^*(a) - (R^B - \tilde{b}(a, R))] - a \quad (C5).$$

Consider some  $a$  with  $\tilde{b}(a, R) < b^*(a)$  and  $\beta$  close to unity. For any such  $a$ , (C5) is harder to satisfy than (C4) because  $\tilde{p}^{**} - R^C > p^*(R=0) - R$  and  $R^B - \tilde{b}(a, R) > v(a, 0)$ . Moreover, for  $\gamma_A < \gamma$ , (C5) is also more demanding than condition (C3). Accordingly,  $A$  will find it less attractive to deviate if  $B$  and  $C$  jointly finance the initial outlay.<sup>22</sup>

## 5 Conclusion

This paper has analyzed a scenario where two parties  $A$  and  $B$  sequentially invest in an asset whose setup requires an initial outlay. The model is suited to represent, e.g., the relationship between a start-up firm and a venture capitalist, or between a biotech firm and a pharmaceutical company in a research joint venture. We assume that the idiosyncratic investments of both parties and the final asset value are noncontractible. In line with existing work, it was shown that an option-to-buy contract generically implements the first best if  $A$  is not wealth constrained. This outcome extends to a setting with wealth constrained agent if renegotiation can be prohibited after  $A$  expended his effort. In this case, a debt contract between  $A$  and either  $B$  or an external investor in combination with an option-to-buy contract facilitates efficient investments.

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<sup>22</sup>The literature has identified a variety of circumstances where multiple lenders can be beneficial. Dewatripont and Tirole (1994) show that multiple outside investors can alleviate the problem that a long-term project is stopped prematurely after first-period profits turned out low. Berglöf and von Thadden (1994) and Bolton and Scharfstein (1996) show that multiple (long-term and short-term) creditors reduce the firm's payoff in renegotiations and accordingly reduce its incentives to default strategically.

The optimal strike price of  $B$ 's option depends on which of those financing modes is chosen. Under certain conditions, a first best can also be attained if renegotiation is feasible. In general, however, the performance of internal and external financing now differs and we characterize conditions under which one financing form dominates the other. Finally, a combination of internal and external investors may restore an efficient outcome if renegotiation is feasible. Overall, our findings shed some light on the relative performance of internal and external financing in relationships with sequential investments. They also reinforce the by now well received wisdom that option-to-buy arrangements are often a proper tool to govern the incomplete-contracting relationship between parties who invest sequentially, and show how these arrangements should be augmented to account for financing constraints.

# Appendix

## Appendix A: Proof of Lemma 1

Consider the following arrangement  $(L, R, p^*)$ . At date 0,  $B$  contributes a payment  $L \geq K$ . At date 5, she can then insist on a repayment  $R \geq K$ , or can at that date exercise her option and acquire the asset at a price  $p^* = v(a^{FB}, b^{FB}) - b^{FB} - R$ .

Suppose  $A$  invested  $a^{FB}$  at date 1. We must distinguish between two subcases. If  $B$  expends  $b = b^{FB}$  at date 3 (note that a larger investment is never profitable), she will subsequently exercise her option-to-buy at date 5 because  $v(a^{FB}, b^{FB}) - p^*$  strictly exceeds  $R$ . Accordingly,  $B$ 's continuation payoff after date 0 is  $v(a^{FB}, b^{FB}) - b^{FB} - p^* = R$  and she attains an overall payoff  $U^B = R - L$ . Second, she may undertake an investment  $b < b^{FB}$  and insist on a repayment at date 5. If  $v(a^{FB}, b) + [R - L] \geq R$ ,  $A$  can meet his repayment obligation and  $B$ 's overall payoff is weakly smaller than  $R$  which cannot be optimal. Conversely, if  $v(a^{FB}, b) + [R - L] < R$ ,  $A$  cannot meet his repayment obligation. In this case,  $B$  seizes the asset as well as  $A$ 's remaining cash endowment  $[R - L]$  as a collateral. Again, her continuation payoff is then weakly smaller than  $R$  so that  $B$  will undertake  $b^{FB}$  at date 3 provided that  $A$  invests efficiently at date 1.

We now examine whether  $A$  gains by deviating from  $a^{FB}$ . Observe that his overall payoff for efficient investments is  $U^A = S(a^{FB}, b^{FB}) - [R - L]$ , and his participation constraint requires  $R < S(a^{FB}, b^{FB}) + L$ . To start with, notice that a deviation  $a > a^{FB}$  cannot be optimal because  $B$  still exercises her option-to-buy so that  $A$  recovers no return on any excess investment. Consider now an arbitrary deviation  $a < a^{FB}$ . Let  $\tilde{b}(a, R)$  be the threshold investment level as defined in Definition 1 for  $L = R$  and as defined in footnote 13 for  $L > K$ .  $A$  is able to repay his debt if  $B$  chooses an effort level  $b \geq \tilde{b}(\cdot)$ . Notice that  $A$  will never choose an investment  $a$  for which  $\tilde{b}(a, R) = 0$  because  $B$  can then insist on a repayment  $R$  without expending own effort. Hence, we can concentrate on deviations  $a$  where  $\tilde{b}(a, R)$  is positive.

Consider first deviations from  $a^{FB}$  where  $\tilde{b}(a, R) > b^*(a)$ . If  $B$  responds by some  $b < \tilde{b}(a, R)$ ,  $A$  cannot repay and  $B$  seizes the asset as well as  $A$ 's monetary endowment  $(L - K)$  at date 5. Moreover,  $b = b^*(a)$  is  $B$ 's optimal response because she appropriates the entire continuation surplus  $v(a, b^*(a)) - b^*(a)$  as well as  $A$ 's remaining cash endowment

when exerting  $b^*(a)$ . Since  $A$  is left with a non-positive surplus, she will never deviate from  $a^{FB}$  in a way that  $\tilde{b}(a, R) \geq b^*(a)$ .

Consider now deviations such that  $\tilde{b}(a, R) < b^*(a)$ . If  $B$  responds by some  $b < \tilde{b}(a, R)$ , she again seizes the asset at date 5. Her locally best reply is an effort level close to  $\tilde{b}(a, R)$ , which gives  $B$  a continuation payoff slightly smaller than  $v(a, \tilde{b}(\cdot)) - \tilde{b}(\cdot) + (L - K)$ . Alternatively,  $B$  can undertake an investment from the complementary interval  $b \geq \tilde{b}(a, R)$ . If she does, she can insist on a repayment in which case  $b = \tilde{b}(a, R)$  is her best reply, and her continuation payoff becomes  $R - \tilde{b}(a, R) = v(a, \tilde{b}(\cdot)) - \tilde{b}(\cdot) + (L - K)$ . Accordingly, a response  $b < \tilde{b}(a, R)$  is dominated, and  $A$  will not deviate whenever  $B$  chooses  $\tilde{b}(a, R)$  because his overall payoff is then non-positive. Finally, though,  $B$  may anticipate to exercise her option-to-buy at date 5 in which case she expends  $b^*(a) [ > \tilde{b}(a, R) ]$  at date 3. In this latter case, her continuation payoff becomes  $v(a, b^*(a)) - b^*(a) - p^*$ . For  $\tilde{b}(a, R) < b^*(a)$ , this last strategy is  $B$ 's best response if  $v(a, b^*(a)) - b^*(a) - p^* > R - \tilde{b}(a, R)$ , which translates into

$$[v(a^{FB}, b^{FB}) - b^{FB}] - [v(a, b^*(a)) - b^*(a)] < \tilde{b}(a, R). \quad (C2)$$

If  $\tilde{b}(a, R) < b^*(a)$  and (C2) is satisfied (which implies  $\tilde{b}(a, R) > 0$ ),  $b^*(a)$  is  $B$ 's optimal reaction on a deviation  $a < a^{FB}$  and she exercises her option at price  $p^*$ . Then,  $A$  accrues an overall payoff  $p^* + (L - K) - a$  which strictly exceeds  $S(a^{FB}, b^{FB}) - (R - L)$ , so that  $A$  deviates from  $a^{FB}$  if and only if conditions (C1) and (C2) are violated for some  $a < a^{FB}$ . Finally, we show that defection generically occurs if  $v(a^{FB}, 0) < R$  so that  $\tilde{b}(a, R) > 0$  for any  $a \leq a^{FB}$ . Consider  $a \rightarrow a^{FB}$ . Then,  $\tilde{b}(a, R) < b^*(a)$  because  $A$ 's participation constraint implies  $R < v(a^{FB}, b^{FB})$ . In addition, the left-hand side of (C2) converges to zero while the right-hand side is strictly positive. Accordingly,  $A$  deviates and underinvestment generically occurs at strike price  $p^*$  if  $v(a^{FB}, 0) < R$ .  $\square$

## Appendix B: Proof of Proposition 2

Consider an option price

$$p^{**} = v(a^{FB}, b^{FB}) - b^{FB} - R + \tilde{b}(a^{FB}, R) \quad (6)$$

and notice that  $\tilde{b}(a^{FB}, R) > 0$  iff  $v(a^{FB}, 0) < R + (L - K)$ . Suppose first that  $A$  expends an effort that satisfies  $\tilde{b}(a, R) \geq b^*(a)$ , and verify that any such effort level is smaller than  $a^{FB}$ . By the arguments given in the proof of Lemma 1, such a deviation



cannot be profitable because  $B$  will then expend  $b^*(a)$  and seize the asset as a collateral. Next, consider investment levels with the property  $\tilde{b}(a, R) < b^*(a)$ .  $B$  will then either invest  $b^*(a)$  and exercise her option (in which case  $A$  deviates), or undertake the threshold investment  $\tilde{b}(a, R)$  and insist on  $R$  in which case  $A$ 's surplus is smaller than  $S(a^{FB}, b^{FB}) - (R - L)$  and he will not deviate. At strike price  $p^{**}$ ,  $B$  pursues the former strategy iff

$$v(a, b^*(a)) - b^*(a) - [v(a^{FB}, b^{FB}) - b^{FB} - \tilde{b}(a^{FB}, R)] + \tilde{b}(a, R) \geq 0. \quad (7)$$

This condition holds with equality if  $a = a^{FB}$  so that  $B$  will invest efficiently and exercise her option if  $A$  indeed expends  $a^{FB}$ . Since  $A$  will never invest more than  $a^{FB}$ , he will not deviate if (??) is violated for any  $a < a^{FB}$ . To see that this condition is satisfied, consider the derivative of the left-hand side of (??) with respect to  $a$ , which yields (by the implicit function theorem)

$$v_a(a, b^*(a)) - \frac{d\tilde{b}(a, R)}{da} = v_a(a, b^*(a)) - \frac{v_a(a, \tilde{b}(a, R))}{v_b(a, \tilde{b}(a, R))}. \quad (8)$$

Since  $\tilde{b}(a, R) < b^*(a)$ , we have  $v_a(a, b^*(a)) \geq v_a(a, \tilde{b}(a, R))$  and  $v_b(a, \tilde{b}(a, R)) > v_b(a, b^*(a)) = 1$  if investments are weakly complementary,  $v_{ab}(\cdot) \geq 0$ . Hence, the left-hand side of (??) is strictly increasing in  $a$ . Since  $B$  will not exercise her option when  $a < a^{FB}$  and  $A$ 's payoff is decreasing in  $a$  for any  $a \geq a^{FB}$ , a contractual arrangement  $(L, R, p^{**})$  will thus implement efficient investments.

Finally, we show that any arbitrary distribution of the social surplus  $S(a^B, b^{FB})$  can be achieved by proper choice of  $R$  and  $L \geq K$ . To see this, notice that  $B$ 's equilibrium payoff  $U^B = R - L - \tilde{b}(a^{FB}, R)$  strictly increases in  $R$  as long as  $\tilde{b}(a^{FB}, R) < b^*(a^{FB}) = b^{FB}$  so that  $dU^B/dR = 1 - 1/v_b(a^{FB}, \tilde{b}(\cdot)) > 0$ . Hence, we must show that - for any distribution of total surplus - the accompanied repayment level  $R$  indeed satisfies the property  $\tilde{b}(a^{FB}, R) < b^{FB}$ . To verify this, notice that  $\tilde{b}(a^{FB}, R) < b^{FB}$  for any  $R \leq \bar{R} \equiv v(a^{FB}, b^{FB}) + (L - K)$  by the definition of  $\tilde{b}(\cdot)$ . Inserting  $\bar{R}$  in  $U^B$ , we obtain  $U^B(R = \bar{R}) = v(a^{FB}, b^{FB}) - b^{FB} - K > S(a^{FB}, b^{FB})$ . For  $R = \bar{R}$ , agent  $B$  appropriates more than the entire social surplus so that  $R \geq \bar{R}$  violates the participation constraint of agent  $A$ . Conversely, for any  $R < \bar{R}$ ,  $\tilde{b}(a, R) < b^{FB}$  is satisfied which completes the proof.  $\square$

## Appendix C: Proof of Proposition 4

Proposition 4(a) follows immediately from Proposition 1 and results in Nöldeke and Schmidt (1998). If  $A$  expends  $a \geq a^{FB}$ ,  $B$  has no credible threat to invest less than  $b^{FB}$  at date 3 so that renegotiation does not arise. Also,  $A$  will never invest less than  $a^{FB}$  because  $B$  can assure herself a continuation payoff  $R$  by not investing and insisting on the repayment at date 5, and because date-2 renegotiations will only raise this payoff. Since  $A$  reaps  $S(a^{FB}, b^{FB}) - (R - L)$  when investing efficiently, any deviation reduces his payoff and hence cannot be part of an equilibrium strategy.

To prove Proposition 4(b), observe first that  $B$  cannot credibly insist on renegotiation if  $A$  chooses an investment  $a \geq a^{FB}$  (see Lemma 1). If  $A$  invests  $a < a^{FB}$ ,  $B$ 's default response under an option contract with strike price  $p^{**}$  is (a)  $b^*(a)$  if  $b^*(a) \leq \tilde{b}(a, R)$ , and (b)  $\tilde{b}(a, R)$  if  $b^*(a) > \tilde{b}(a, R)$  (see Proposition 2). In the former case, no renegotiation occurs and  $B$  seizes the asset as a collateral at date 5 because  $A$  cannot repay. Hence,  $A$  reaps a non-positive payoff and a deviation cannot be optimal. In the latter case,  $B$ 's default investment  $\tilde{b}(a, R)$  is strictly smaller than the conditionally efficient level. Accordingly, the parties renegotiate at date 2, and  $B$  acquires the asset in order to ensure the conditionally efficient investment  $b^*(a)$ . In these negotiations,  $A$  appropriates a payoff  $\gamma[v(a, b^*(a)) - b^*(a) - (R - \tilde{b}(a, R))]$  under Nash bargaining. He therefore deviates from  $a^{FB}$  if and only if there exists some  $a$  with  $\tilde{b}(a, R) \in [0, b^*(a)]$  for which

$$p^{**} - a^{FB} = v(a^{FB}, b^{FB}) - b^{FB} - R + \tilde{b}(a^{FB}, R) < \gamma[v(a, b^*(a)) - b^*(a) - (R - \tilde{b}(a, R))] - a$$

is satisfied (condition (C3)). In the main text, we show that a deviation can indeed occur for  $\gamma$  sufficiently large. Finally, one can verify that  $A$  will underinvest for any strike price  $p < p^{**}$  because  $B$  then exercises her option in some range where  $a < a^{FB}$ . Conversely,  $B$  never exercises for  $a = a^{FB}$  and  $p \geq p^{**}$  by construction of  $p^{**}$ .

To prove proposition 4(c), notice again that  $A$  will never expend  $a > a^{FB}$ . Hence, consider a deviation  $a < a^{FB}$ . For any such deviation,  $B$  will expend zero effort if renegotiation fails. Also, notice that  $A$  will never deviate in a way that  $\tilde{b}(a, R^C) > 0$ , i.e.,  $v(a, 0) < R^C$ , because  $C$  can then insist on a repayment  $R^C$  at date 5. Hence, consider deviations which satisfy  $v(a, 0) < R^C$  and suppose date-2 renegotiations are unsuccessful. Then,  $A$  cannot repay  $R^C$  at date 5, and  $C$  seizes the asset and obtains a continuation payoff  $v(a, 0)$ . In equilibrium, renegotiation is successful and allows to

realize a bargaining gain  $[v(a, b^*a) - b^*(a) - v(a, 0)]$ . Assuming that the outcome of renegotiation is described by the 3-persons Nash-bargaining rule, agent  $A$  appropriates a fraction  $\gamma_A$  of this bargaining surplus. Therefore, he will deviate from  $a = a^{FB}$  if and only if there exists some  $a$  which satisfies  $v(a, 0) < R^C$  and

$$p^* - a^{FB} = v(a^B, b^{FB}) - a^{FB} - b^{FB} - R^C < \gamma_A[v(a, b^*a) - b^*(a) - v(a, 0)] - a \quad (C4).$$

To give an example, suppose the asset value is  $v(a, b) = 2a^{1/2} + 2b^{1/2}$ . For this functional form, one obtains  $a^{FB} = b^*(a) = b^{FB} = 1$  and  $S(a^{FB}, b^{FB}) = 2 - K$ . Noting that  $A$  will choose  $a = 0$  when he deviates so that  $v(a, 0) = 0$ , condition (C4) reads  $2 - R^C \geq \gamma_A[2(b^{FB})^{1/2} - b^{FB}] = \gamma_A$  which is violated (at least) if  $K > 2 - \gamma_A$ . Hence, external financing cannot implement an efficient outcome if  $K \in (2 - \gamma_A, 2)$ .  $\square$

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