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by

Matthias Kräkel

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Bonn Graduate School of Economics
Department of Economics
University of Bonn
Adenauerallee 24 - 42
D-53113 Bonn

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Matthias Kräkel, *University of Bonn***

Abstract

In practice, two types of tournaments can be distinguished – U-type and J-type tournaments. In U-type tournaments, workers receive prizes that have been fixed in advance. In J-type tournaments, the employer fixes an aggregate wage bill in advance, which is then shared among the workers according to their relative performance. The results of the paper show that the outcomes of the two tournament types substantially differ. Especially, an employer will prefer J-type to U-type tournaments if the number of workers is large, whereas the opposite holds for small numbers of workers.

JEL classification: J3, M1.

Key words: collusion, human capital, relative deprivation, tournaments.

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**Matthias Kräkel, Department of Economics, BWL II, University of Bonn, Adenauerallee 24-42, D-53113 Bonn, Germany, phone: +49-228-739211, fax: +49-228-739210, e-mail: m.kraekel@uni-bonn.de.

1 Introduction

Workers' outputs and human capital investments often cannot be verified by a third party (e.g., a court) in practice. These outputs or human capital investments are called unverifiable or non-contractible, because explicit labor contracts cannot be made contingent on them. For this reason, labor contracts are often incomplete leaving room for opportunistic behavior. For example, an employer can promise a worker to pay high bonuses for high outputs. But if these outputs are unverifiable, a rational employer will save labor costs by claiming that the worker's output was low in case of a high output. Anticipating the employer's opportunistic behavior the worker will have no incentives to exert any effort.

Malcomson (1984, 1986) offers a practical solution to this unverifiability problem. He shows that tournament compensation schemes will be contractible, even if the workers' outputs are unverifiable.[1]In tournaments workers are compensated according to the ordinal ranks of their realized outputs. The tournament prizes are fixed in advance, before the tournament starts. Since the fixed tournament prizes are verifiable by court, the employer cannot save labor costs by understating the workers' realized outputs (i.e., if the employer does not declare the worker with the highest output the winner of the tournament, the winner prize will be given to another worker). Thus, the employer will correctly allocate the workers to their ranks. By anticipating this, workers will have strong incentives for exerting effort to attain the winner prize in the tournament. There are a lot of examples for this tournament solution in practice, e.g. job promotion tournaments within a corporate hierarchy or tournaments between salesmen.

Kanemoto and MacLeod (1989, 1992) argue that in the evolutionary process of labor market institutions two different types of tournaments have evolved as alternative solutions to the unverifiability problem. The first type is a kind of job promotion tournament and can be observed in U.S. firms (see, e.g., Baker, Gibbs, and Holmström 1994a, 1994b). Here, the tournament prizes are wages that are attached to jobs along a firm's hierarchy. The wages rise with increasing hierarchy level. On each level of the hierarchy, workers compete in tournaments against each other to win a promotion to the next level. As the hierarchical wage structure of a firm is verifiable, this type of tournament prevents employers from opportunistic behavior and, therefore, gives effort incentives to workers. Such job promotion tournaments can be called (U.S. or) *U-type tournaments*.

The second type of tournament can be found in Japanese firms and will be called *J-type tournaments*.[2] Here, the employer agrees on an aggregate wage bill for his workers that is specified by an explicit contract between the firm and the local union. For example, this wage bill can be the amount of bonuses which are biannually payed by large Japanese firms to their workers.[3]When the employer has signed the collective agreement, he can no longer save labor costs by opportunistically rating his workers' outputs, because the aggregate wage bill is verifiable. By using a tournament the wage bill is then shared among the workers.[4]For this purpose, each worker takes place in a rating or assessment process (”satei”), in which the workers are subjectively evaluated by their supervisors (e.g., Itoh 1991; Endo 1994). This assessment process can be called a tournament, because the more merit points a worker has made the larger is his individual share in the aggregate wage bill compared to the other workers. If, for example, in a firm with two workers A and B the supervisor claims that A's unverifiable output has been three times as high as B's, then A will receive 75% of the wage bill and B only 25%.

This paper will discuss the questions, whether U-type and J-type tournaments substantially differ, and which tournament type is preferable from an employer's viewpoint. I will focus on the central issue that tournaments will lead to contractible incentive schemes even if workers' outputs are unverifiable. For this purpose, incentive problems due to hidden action are neglected. Therefore, this paper assumes a deterministic production technology for the workers (i.e., the output only depends on a worker's effort and not on an additional noise term or exogenous random variable). Hence, the model refers to situations where output is mainly determined by a worker's behavior and where the monitoring precision of supervisors is high.[5]Such situations can be often found in industrial production where the technical context and production breakdowns are usually observed by the employer. The model fits less well with the situation of salespersons, whose results are also determined by exogenous market conditions (especially, the customers). The results of the model will show that the two tournament types clearly differ. Especially, we will see that from an employer's viewpoint J-type tournaments have comparative advantages when discussing horizontal collusion between the workers, human capital investments in a previous stage prior to the tournament, and tournaments with many workers.

In Section 2, the basic model is introduced. Section 3 compares the outcomes of U-type and J-type tournaments. Section 4 concludes.

2 The Basic Model

A (U-type or J-type) firm is considered which employs two risk neutral workers A and B. Each worker has the same production function $q_i = e_i$ where e_i denotes worker i's $(i = A, B)$ observable but unverifiable effort.[6]Effort entails costs to a worker, which are described by $c(e_i)$ with $c(0) = 0$, $c'(e_i) > 0$, and $c''(e_i) > 0$. Output and costs are both measured in monetary terms. The reservation utility of each worker is given by $\bar{u} \geq 0$. Both workers want to maximize their expected net income, i.e. their expected wages net off the expected costs. It is assumed that the workers are not able to pay up-front fees because of restricted wealth. The employer is assumed to be risk neutral, too. He wants to maximize expected profits, i.e. expected outputs minus wages.

In a U-type tournament the employer specifies a winner prize w_1 and a loser prize w_2 (with $0 \leq w_2 < w_1$) to induce work incentives and to make the two workers participate in the tournament. Let $\Delta w = w_1 - w_2$ be the prize spread. Worker i will win the tournament if $q_i > q_j$ ($i, j = A, B$; $i \neq j$). In the case of $q_i < q_j$ worker i loses. If $q_i = q_j$ the employer will determine the winner by flipping a fair coin so that each worker's probability of winning is 1/2.

In a J-type tournament the employer chooses an aggregate wage bill w before the tournament starts. Worker i gets the fraction $q_i / (q_i + q_j)$ of the wage bill w given that at least one worker exerts positive effort ($i, j = A, B$; $i \neq j$). If $q_i = q_j = 0$, each worker will receive the wage $w/2$. The timing structure is identical in both tournaments. On the first stage, the employer chooses (w_1, w_2) in the U-type tournament and w in the J-type tournament. On the second stage, given that the two workers A and B decide to participate in the tournament they simultaneously choose their efforts e_A and e_B.

In the following section, the U-type and the J-type tournament are compared. First, the basic model is considered. Then, this model will be extended to discuss the possibility of collusions, relative deprivation, human capital investments, and tournaments between more than two workers.

3 Results

3.1 Comparison with the First-Best Solution

As the employer and the two workers are all risk neutral it makes sense to compare the outcomes of the U-type and the J-type tournament to the first-best outcome as a reference solution. Under first-best conditions there are no observability and contracting problems. Therefore, the employer would choose the effort level that maximizes $e_i - c(e_i)$ for each worker i $(i = A, B)$ and would pay each worker a wage that just compensates him for the costs $c(e_i)$ and the reservation utility $\bar{u}$. This leads to the following definition:

Definition *The outcome where each worker exerts effort* $e^{FB} = c'^{-1}(1)$ *and the total welfare of the employer and the two workers amounts to* $2e^{FB} - 2c\left(e^{FB}\right)$ *will be called first-best solution.*[7]

Deriving the subgame perfect equilibrium of the U-type and the J-type tournament we obtain the following result:

Proposition 1 *Neither the U-type nor the J-type tournament yields the* *first-best solution.*

Proof. See the Appendix. ■

The proof of Proposition 1 shows that in both tournament types the employer is able to implement first-best efforts (on average), but not at first-best costs. In the J-type tournament, the employer does not want to implement first-best efforts, because e^{FB} would imply negative profits. The result concerning the U-type tournament seems to be even more surprising. The seminal article by Lazear and Rosen (1981) has already shown that the U-type tournament with two homogeneous and risk neutral workers and stochastic production generates the first-best solution. The intuition for this result is clear: Since both workers are risk neutral there is no trade-off between incentives and risk sharing. Thus, the employer is able to induce optimal incentives without risk premia. In this paper, production is deterministic. At first sight, we might suppose that this would make things even better. But the proof of Proposition 1 shows that the opposite is true. Because of deterministic production there only exists a Nash equilibrium in mixed strategies on the tournament stage of the game. Theoretically, the employer may be able to choose the tournament prizes w_1 and w_2 so that a worker's expected effort equals e^{FB}.[8]Nevertheless, the first-best solution cannot be obtained, because the corresponding expected costs are higher than $c\left(e^{FB}\right)$ due to the convexity of $c(\cdot)$ and Jensen's inequality.

This surprising result has a direct implication for the design of U-type tournaments in practice. In the model of Lazear and Rosen (1981) stochastic outputs are described by $q_i = e_i + \varepsilon_i$ $(i = A, B)$ where the noise terms ε_A and ε_B are identically and independently distributed. These noise terms can be interpreted as measurement errors due to imperfect monitoring. In analogy, we can argue that the deterministic production function $q_i = e_i$ is achieved by perfect monitoring. But then Proposition 1 makes clear that imperfect monitoring is strictly better, since it leads to first-best efforts and a reduction of monitoring costs. According to Proposition 1, the employer would even prefer the existence of measurement errors to perfect monitoring if monitoring costs were zero.

Unfortunately, the general form of the cost function $c(e_i)$ does not allow a direct comparison between the efforts and the employer's expected profits in the U-type and the J-type tournament. Both tournament types have symmetric subgame perfect equilibria. Let $E\left[e^U\right]$ denote a worker's expected equilibrium effort in a U-type tournament and e^J the equilibrium effort of a worker in the J-type tournament. Furthermore, let $E\Pi^U$ $(E\Pi^J)$ be the employer's expected profits per worker in a U-type tournament (J-type tournament) and $E\Pi^{FB}$ the expected profits per worker with first-best effort. Using a quadratic cost function the following result can be obtained:

Corollary 1 *If* $c(e_i) = (k/2) \cdot e_i^2$ *with* $k > 0$ *and* $\bar{u} = 0$*, then* $e^{FB} >$

$E\left[e^U\right] > e^J$ *and* $E\Pi^{FB} > E\Pi^U > E\Pi^J$.

Proof. See the Appendix. ■

The proof of the corollary shows that the participation constraint is binding in the U-type tournament, whereas workers receive more than their reservation utilities in the J-type tournament. But, more importantly, workers exert less effort in the J-type than in the U-type tournament on average. In addition, the employer's expected profits per worker are larger in the U-type than in the J-type tournament. Thus, the corollary indicates that the U-type tournament seems to dominate the J-type tournament from the employer's viewpoint. The following results will show that this conclusion does not hold.

3.2 Collusion between the Workers

In the literature also some problems of tournaments have been mentioned. Many authors suppose that collusion between the workers will be one of the major problems of tournaments.[9]Obviously, collusion in which all workers agree to exert zero effort will be Pareto efficient from the workers' viewpoint. Consider, for example, the above two-person U-type or J-type tournament. In both cases, there exists a unique Nash equilibrium on the tournament stage which is symmetric and in which the workers exert positive efforts. Thus, the two workers are strictly better off by both choosing zero effort.

Their expected wages are still $(w_1 + w_2)/2$ or $w/2$, respectively, but the workers will save their (expected) effort costs. Therefore, collusion would lead to a Pareto improvement for the workers. But since there is a unique equilibrium in each type of tournament, a collusive agreement cannot be stable in a one-shot game.[10] If the tournament between the same two workers is repeated several times, a collusive agreement may become self-enforcing. A finitely repeated tournament cannot lead to stable collusion because of the backward-induction argument. But as we know from the Folk Theorem, a collusion equilibrium can be achieved in a tournament supergame with an infinite number of repetitions (or with an unknown endgame, respectively). Let $\delta = 1/(1+r)$ be the workers' discount factor for future payoffs with r as interest rate. In addition, assume that both workers use the grim strategy to enforce their collusive agreement.[11] Then, the following result can be derived:

Proposition 2 *There exist lower bounds δ^U and δ^J for U-type and J-type tournament supergames so that collusion will be stable if $\delta \geq \delta^U$ and $\delta \geq \delta^J$, respectively, with $\delta^J \geq \delta^U$.*

Proof. See the Appendix. ■

The proposition states that in a tournament supergame stable collusion may be possible in U-type as well as in J-type tournaments. But as $\delta^J \geq \delta^U$ hold for the two threshold levels, stable collusion is more likely in U-type than in J-type tournaments. Thus, concerning possible collusion an employer prefers J-type to U-type tournaments. The intuition for the result of Proposition 2 is based on two facts: First, if a worker deviates from the collusive agreement in the J-type tournament by choosing an arbitrarily small but positive effort he will receive the total aggregate wage bill. If a worker deviates from collusion in the U-type tournament he will only get the winner prize whereas the other worker receives the loser prize. Altogether, incentives for deviating should be greater in the J-type than in the U-type tournament.

Second, note that the participation constraint of the one-shot game is always binding in U-type tournaments (i.e., each worker receives his reservation utility $\bar{u}$), whereas this is not clear for J-type tournaments. Especially, the proof of Corollary 1 shows that participants of J-type tournaments get more than $\bar{u}$. The proof of Proposition 1 shows that in U-type tournaments there is always complete rent dissipation on the tournament stage, i.e. each worker's expected utility will always equal the loser prize w_2, no matter whether w_2 is high or low. Hence, the best the employer can do is to choose $w_1 = \bar{u}$ to minimize labor costs, which makes the participation constraint binding. Therefore, workers' incentives to deviate from a collusive agreement will be greater in J-type than in U-type tournaments: When cooperation breaks down and the workers play their one-shot Nash equilibrium strategies, workers' expected utilities are larger in J-type tournaments in each round. Hence, the favourable property of a U-type tournament − a binding participation constraint − becomes an unfavourable one in a dynamic context from the employer's viewpoint.

Besides, the proof of Proposition 2 shows that, contrary to J-type tournaments, the condition for a collusive equilibrium in U-type tournaments (i.e., $\delta \geq \delta^U$) is independent of the tournament prizes w_1 and w_2. Thus, the employer cannot adjust the two prizes to make collusion in the U-type tournament supergame more difficult.

3.3 Relative Deprivation

In practice, people sometimes do not only care for their absolute but also for their relative incomes, compared to a certain reference group. Such preferences are described by the concept of relative deprivation (RD).[12]A person will be relatively deprived, if his income is lower than the income of a chosen reference group. The larger this income difference and the group size are, the larger is the individual's RD. Combining RD with tournaments makes some sense, because the participants of a tournament naturally compare themselves to each other, and the losers may feel relatively deprived compared to the winners of the tournament. The workers who compete against each other in the tournament will exert effort to minimize their (expected) RD.[13]Let $E\left[e_{RD}^U\right]$ and e_{RD}^J denote the workers' (expected) efforts in the U-type or J-type tournament with RD workers, respectively, and $E\Pi_{RD}^{U}$ and $E\Pi_{RD}^{J}$ the corresponding values for the employer's expected profits per worker. Then we have the following results:

Proposition 3 *In U-type and J-type tournaments with RD workers*

(i) $E\left[e_{RD}^{U}\right] > E\left[e^{U}\right]$ *for a given prize spread* Δw, *and* $e_{RD}^{J} > e^{J}$ *for a given aggregate wage bill* w,

(ii) $E\Pi_{RD}^{U} > E\Pi^{U} > E\Pi_{RD}^{J} > E\Pi^{J}$ *when costs are quadratic,*

(iii) *in neither type of tournament there is profitable collusion.*

Proof. See the Appendix. ■

Result (i) is similar to the results for tournaments with noise (see Kräkel 2000). Minimizing the relative income difference instead of maximizing absolute income leads to higher efforts for given wages. Here, the competition between only two workers suffices to generate this effort effect. Result (ii) describes a ranking for the employer's expected profits per worker. Given RD workers the employer again prefers the U-type to the J-type tournament. The result also shows that the employer prefers organizing a tournament between RD workers to a competition between non-RD workers for each type of tournament (i.e., $E\Pi_{RD}^{i} > E\Pi^{i}$ with $i = U, J$). This is not surprising, because RD workers weight their income differences with the relative size of the reference group, and because the employer need not compensate the RD workers for their absolute effort costs. Result (iii) becomes clear by the fact that in either type of tournament there is a symmetric equilibrium. Since workers only care about the relative income difference which is zero in a symmetric equilibrium, they cannot profitably deviate from this equilibrium by both exerting zero effort. The only way to gain by a collusive agreement would be to choose a slightly higher effort than the other worker, but this worker would never agree to such collusion.

3.4 Human Capital Investment

Often workers do not only compete against each other by exerting effort, but also by accumulating human capital. Therefore, the basic model will be extended by introducing human capital investment of the workers before the tournament starts. Now, it is assumed that each type of tournament consists of three stages. On the first stage, the employer chooses (w_1,w_2) or w, respectively. On the second stage, each worker makes an investment in his human capital. He chooses an ability parameter a_i ($i = A, B$) with $a_i \in \{0, 1\}$.[14]This means that a worker can either agree ($a_i = 1$) or disagree ($a_i = 0$) to take part in training activities. The low ability level $a_i = 0$ entails zero costs to the worker. The high ability level $a_i = 1$ leads to costs $\bar{c} > 0$ which are sufficiently small so that human capital investment may be rational. On the the third stage, the two workers know a_A and a_B, and choose their effort levels to become the winner of a U-type or a J-type tournament, respectively. The production function of worker i ($i = A, B$) is now characterized by $q_i = a_i e_i$. This means that if worker i has dropped out of the competition on the previous stage by choosing $a_i = 0$, he would now rationally choose $e_i = 0$ to minimize his costs $c(e_i)$. If worker i has chosen $a_i = 1$, he would have the same production function $q_i = e_i$ as described in Section 2. For this modified game the following results can be derived:

Proposition 4a *In the three-stage U-type tournament there are two subgame perfect equilibria:* (i): $w_1^* = w_2^* = 0$, *and the workers do not participate.* (ii): *On the first stage,* $w_2^* = \bar{u}$ *and*

$$w_1^* = \underset{w_1}{\operatorname{argmax}} \left\{ \frac{(\Delta w - \bar{c})^2}{\Delta w^2} \left(c^{-1}(\Delta w) - \int_0^{c^{-1}(\Delta w)} \frac{c(x)}{\Delta w} dx \right) - \frac{w_1 + \bar{u}}{2} \right\}$$

subject to $\Delta w \geq \bar{c}$ *with* $\Delta w = w_1 - \bar{u}$. *On the second stage, worker* $i\,(i = A, B)$ *chooses* $a_i^* = 0$ *with probability* $\bar{c}/\Delta w$, *and* $a_i^* = 1$ *with probability* $(\Delta w - \bar{c})/\Delta w$. *On the third stage, worker* i $(i = A, B)$ *will choose*

$$e_i^* = \begin{cases} 0, & \text{if} \quad a_i^* = 0 \\ \varepsilon, & \text{if} \quad a_i^* = 1 \text{ and } a_j^* = 0 \\ x, & \text{if} \quad a_i^* = a_j^* = 1, \end{cases}$$

where x *is a random variable with cdf* $G(x) = c(x)/\Delta w$ *over the interval* $[0, c^{-1}(\Delta w)]$, *and* ε *being an arbitrarily small but positive number.*[15]

Proof. See the Appendix. ■

The results of Proposition 4a indicate that U-type tournaments become highly problematic when workers should invest in human capital prior to the tournament. In equilibrium (i) the employer decides to shut down production. The proof of Proposition 4a shows that otherwise the employer would make negative profits, because on the second stage one of the workers would drop out of the tournament (i.e., he chooses $a_i = 0$) which would imply zero incentives for both workers on the third stage.

Equilibrium (ii) has also some disadvantages from the employer's viewpoint compared to the results for the basic model without human capital investment. In equilibrium (ii), the collective output of the two workers is only strictly positive with probability $(\Delta w - \bar{c})^2 / \Delta w^2$. Then, both workers realize $a_i^* = a_j^* = 1$ on the second stage and choose mixed strategies according to $G(x)$ on the third stage, which lead to the same expected efforts as in the basic model (see the proof of Proposition 1). Of course, the employer can raise the probability $(\Delta w - \bar{c})^2 / \Delta w^2$ by increasing w_1,[16]but this would lead to high labor costs and, therefore, to low profits. Thus, we have an additional trade-off which becomes clear by looking at the employer's objective function on the first stage. In addition, the part $\left(c^{-1}(\Delta w) - \int_0^{c^{-1}(\Delta w)} \frac{c(x)}{\Delta w} dx\right) - \frac{w_1 + \bar{u}}{2}$ of the employer's objective function is identical with the objective function in the basic model.[17]Hence, the employer's expected profits are strictly less in the case with human capital investment because of the multiplication with the joint probability $(\Delta w - \bar{c})^2 / \Delta w^2$.

At last, we can speculate which of the two subgame perfect equilibria is the most plausible one. There are good reasons why the equilibrium (ii) should be expected as outcome of the three-stage game. We see that the two workers receive their reservation utility in each of the two equilibria,[18]but the employer's expected profits are only positive in equilibrium (ii). Thus, Pareto efficiency (or, alternatively, coalition proofness in the sense of Bernheim, Peleg, and Whinston 1987) would lead to equilibrium (ii) as the outcome of the game.

Proposition 4b *In the three-stage J-type tournament there exists the following subgame perfect equilibrium: On the first stage,* $w^* = \underset{w}{\operatorname{argmax}} \left\{ e - \frac{w}{2} \right\}$ *subject to* $\frac{w}{4e} = c'(e)$ *and* $\frac{w}{2} - c(e) - \bar{c} \geq \bar{u}$. *On the second stage,* $a_i^* = 1$ $(i = A, B)$. *On the third stage, each worker chooses* e_i^* *with* $\frac{w^*}{4e_i^*} = c'(e_i^*)$ $(i = A, B)$.

Proof. See the Appendix. ■

Proposition 4b shows that the results for the J-type tournament are nearly identical to the results in the basic model without human capital investment. This becomes obvious by comparing Proposition 4b with the proofs of Proposition 1 and Corollary 1. The only difference is that now the employer has to compensate the two workers for their human capital investment. Therefore, we have the same optimization problem for the employer with the exception that the workers' adjusted reservation utility in the participation constraint is now $\bar{u} + \bar{c}$.

The analysis of human capital investment prior to the tournament has been very specific, because there are only two possible investment levels. In general, we have more than two levels. This yields the following result:

Corollary 2 *If there are more than two − discrete or continuous − investment levels, the two workers will choose mixed strategies on each of the last two stages in the U-type tournament.*

Proof. See the Appendix. ■

The corollary claims that in general the two workers will randomize on both stages in the U-type tournament. But the more investment levels exist the higher is the probability that the workers have different levels of human capital when the tournament starts.[19]However, such tournaments with heterogeneous workers are not desirable from the employer's viewpoint. The worker with more human capital is always able to win the tournament with certainty, which would completely discourage the other worker. In equilibrium, both workers still choose mixed strategies over the interval $[0, c^{-1}(\Delta w)]$ on the third stage of game, but the worker with less human capital will drop out of the tournament with a certain probability. This drop-out probability raises in the difference of the human capital investment of the two workers. This has also important implications for situations in practice where workers do not simultaneously choose their human capital investment on the second stage of the game. Then, each worker may have an incentive to realize a first-mover advantage by choosing a high amount of human capital before the investment decision of the other worker to discourage this worker. On the other hand, regardless whether the workers choose their human capital investment simultaneously or sequentially, the employer will have some interest in concealing the information about the realized investment levels so that the workers do not know who is the stronger player.

3.5 n-Person Tournaments

All previous results deal with the case of a two-person tournament. Now, it will be discussed whether the major findings also hold for tournaments with $n > 2$ workers.[20]Since in the standard U-type tournament there is only one vacant job for which the workers compete, we have one winner prize w_1 and $n-1$ loser prizes w_2. Let $E[e_n^U]$ denote a worker's expected effort. Then we get the following proposition:

Proposition 5a *In the U-type tournament with $n > 2$ workers there exists a symmetric subgame perfect equilibrium where the employer chooses $w_2^* = \bar{u}$ and*

$$w_1^* = \arg\max_{w_1} \left\{ c^{-1}(\Delta w) - \int_0^{c^{-1}(\Delta w)} \left(\frac{c(x)}{\Delta w} \right)^{\frac{1}{n-1}} dx - \frac{w_1 + (n-1)\bar{u}}{n} \right\}$$

with $\Delta w = w_1 - \bar{u}$ *on the first stage, and the workers choose an effort* x *according to the cdf*

$$G(x) = \left(\frac{c(x)}{\Delta w}\right)^{\frac{1}{n-1}}$$

over the interval $[0, c^{-1}(\Delta w)]$ *on the second stage. In equilibrium* $dw_1^*/dn > 0$*, whereas the sign of* $dE[e_n^U]/dn$ *is ambiguous. Furthermore, there exists a continuum of asymmetric equilibria.*

Proof. See the Appendix. ■

The result of Proposition 5a is similar to the results for rent-seeking contests (see Baye, Kovenock and de Vries 1996, p. 293). The existence of asymmetric equilibria is also intuitively plausible. In the symmetric equilibrium, each worker's expected utility equals the loser prize w_2 (i.e., there is complete rent dissipation among the workers). Thus, a worker will be indifferent between choosing an effort level according to $G(x)$ or dropping out of the tournament by choosing zero effort and getting w_2 with certainty. Therefore, $n-1$ workers competing actively with cdf $G(x)$ and one worker dropping out will also be an equilibrium. Especially, there is an equilibrium where $n-2$ workers drop out and only two workers remain active. This equilibrium is the least preferred one by the employer, because he pays each of the $n-2$ passive workers $w_2 = \bar{u}$ for doing nothing so that expected profits may become negative.

Comparative statics for the symmetric equilibrium show that the winner prize w_1^* and, therefore, the prize spread Δw raise with increasing n. Thus, an increase in worker competition will result in a higher hierarchical wage differential. However, the effect of n on a worker's expected effort is ambiguous. Whether increased competition encourages or discourages a worker depends on the tournament size n and the shape of the cost function $c(e_i)$.

Proposition 5b *In the J-type tournament with $n > 2$ workers there exists a unique and symmetric subgame perfect equilibrium where the employer chooses $w^* = \underset{w}{\operatorname{argmax}} \left\{e - \frac{w}{n}\right\}$ subject to $w = \frac{n^2}{n-1} ec'(e)$ and $\frac{w}{n} - c(e) \geq \bar{u}$ on the first stage, and the workers choose $e_n^J < e^{FB}$ with $e_n^J c'\left(e_n^J\right) = w^* \frac{n-1}{n^2}$ on the second stage, where $de_n^J/dn > 0$.*

Proof. See the Appendix. ■

Proposition 5b about J-type tournaments is surprising. Although there are many workers in the tournament with many possible effort combinations there only exists a unique and symmetric equilibrium. The workers' equilibrium efforts will always be smaller than the first-best effort regardless whether n is small or large. In contrast to U-type tournaments, comparative statics show that in J-type tournaments increased competition unambiguously leads to higher equilibrium efforts.

Proposition 6 *There exist lower bounds δ_n^U and δ_n^J for U-type and J-type tournament supergames with n workers using the grim strategy so that* *collusion will be stable, if* $\delta \geq \delta_n^U$ *and* $\delta \geq \delta_n^J$*, respectively, with* $\delta_n^J \geq \delta_n^U$*. Moreover,* δ_n^U *is independent of* w_1 *and* w_2*, whereas* δ_n^J *depends on* w*; whether an increase of* w *raises or lessens the possibility of stable collusion crucially depends on the shape of* $c(e_i)$*. In the U-type tournament collusion becomes less stable with increasing* n*, whereas this is not clear for the J-type tournament.*

Proof. See the Appendix. ■

The result of Proposition 6 shows that concerning possible collusion the major findings for two-person tournaments also hold for n-person tournaments: The possibility of stable collusion is higher in U-type than in J-type tournaments. Again, the employer cannot influence this possibility by suitably choosing w_1 and w_2 in the U-type tournament, whereas the possibility of stable collusion in J-type tournaments depends on the aggregate wage bill w. Stability of collusion in U-type tournaments decreases in n. This finding is very plausible. In practice, collusion typically emerges in small groups between persons that trust each other. In this context, the result is caused by the fact that each worker's expected utility from colluding decreases in n and the expected utility from deviating from the collusion is independent of n.[21]There is no clear result for the J-type tournament, because here both expected utilities depend on n.

At last, we can examine which tournament type is more profitable from the employer's viewpoint. Let $E\Pi_n^U$ $\left(E\Pi_{n,RD}^U\right)$ denote the employer's expected profits per worker from a n-person U-type tournament (with RD workers), and $E\Pi_n^J$ $\left(E\Pi_{n,RD}^J\right)$ the expected profits per worker from a n-person J-type tournament (with RD workers). In addition, let $e_{n,RD}^J$ be a worker's equilibrium effort in a J-type tournament between n RD workers. Then we can derive the following results:

Proposition 7 *Comparing the symmetric equilibria of the two tournament types yields:*

(i) $E\Pi_n^U < E\Pi_n^J$ *and* $E\Pi_{n,RD}^U < E\Pi_{n,RD}^J$ *as* n *becomes sufficiently large,*

(ii) $e_{n,RD}^J$ *and* $E\Pi_{n,RD}^J$ *are independent of* n.

Proof. See the Appendix. ■

The results of Proposition 7 are restricted to symmetric equilibria to make the outcomes of the two tournament types comparable. Moreover, the asymmetric equilibria in the U-type tournament between non-RD workers are strictly worse than the symmetric one from the employer's viewpoint (e.g., the discussion of Proposition 5a has shown that in U-type tournaments there exists an asymmetric equilibrium where $n-2$ workers choose zero effort). In addition, symmetric equilibria seem to be more plausible than asymmetric equilibria in this context, since all workers are homogeneous. Result (i) shows that the employer's expected profits per worker will be higher in J-type than in U-type tournaments if the number of workers is large. This result holds for non-RD as well as for RD workers. Thus, the special result of Corollary 1 with two workers and quadratic costs cannot be generalized. On the contrary, an employer should organize a tournament between a large number of workers as a J-type tournament. The intuition for this result will be discussed below in connection with Corollary 3. Result (ii) shows that the workers' equilibrium efforts and the employer's expected profits per worker in a J-type tournament with RD workers do not change with the tournament size. The proof of Proposition 7 makes clear that this result arises because the workers weight their income differences with the relative size of the reference group, $1/n$, so that n cancels out in the employer's objective function when substituting for the workers' reaction function.

The intuition for the profitableness of J-type tournaments becomes clearer when considering the special case of quadratic costs $c(e_i) = \frac{k}{2}e_i^2$ with $k > 0$. Let $E\left[e_{n,RD}^U\right]$ be a worker's expected effort in a U-type tournament with n RD workers:

Corollary 3 *In the symmetric equilibria of the n-person tournaments*
(i) *given quadratic costs and $\bar{u} = 0$ there exists a $\bar{n}$ so that $E\left[e_n^U\right] \gtrless e_n^J$ and $E\Pi_n^U \gtrless E\Pi_n^J$ for $n \lesseqgtr \bar{n}$; furthermore $dE\left[e_n^U\right]/dn < 0$ and $de_n^J/dn > 0$,*
(ii) *given quadratic costs and met participation constraints there exists a $\hat{n}$ so that $E\left[e_{n,RD}^U\right] \gtrless e_{n,RD}^J$ and $E\Pi_{n,RD}^U \gtrless E\Pi_{n,RD}^J$ for $n \lesseqgtr \hat{n}$ where $\hat{n} > \bar{n}$; moreover $dE\left[e_{n,RD}^U\right]/dn < 0$ and $de_{n,RD}^J/dn = 0$.*

Proof. See the Appendix. ■

The results of Corollary 3 show that the (expected) effort and the employer's expected profits per worker are only for small tournaments greater in U-type than in J-type tournaments, whereas J-type tournaments are preferred by the employer when the tournament size surpasses a critical value. The intuition for this becomes clear by the derivatives of the (expected) equilibrium efforts with respect to the tournament size: Increased competition lowers incentives in U-type tournaments for non-RD as well as for RD workers. But the same does not hold for J-type tournaments: RD workers' equilibrium efforts are independent of the tournament size, and the efforts of non-RD workers even raise with increased competition. In addition, we can compare the two critical values of the tournament size for non-RD and RD workers. Since $\hat{n} > \bar{n}$ there exist tournament sizes for which the employer prefers the U-type tournament when the workers are RD workers, whereas he prefers the J-type tournament when the workers only care for absolute incomes.

4 Conclusions

In this paper, the characteristics of U-type and J-type tournaments are compared and discussed from an employer's viewpoint. The major findings can be summarized as follows: (1) Neither U-type nor J-type tournaments yield the first-best solution although the workers are risk neutral. (2) Stable collusion is more likely in U-type than in J-type tournaments. (3) When workers have to invest in their human capital prior to the tournament, U-type tournaments are more problematic than J-type tournaments because of possible heterogeneity among the workers. If, for example, the workers choose different investment levels, all incentives in the subsequent tournament may break down. (4) When the tournament size becomes large, the employer's expected profits are higher in J-type than in U-type tournaments. This result will also hold qualitatively if workers feel relative deprivation.

Appendix

Proof of Proposition1:

First, the U-type tournament is considered, which is similar to (rent-seeking) contests (see, e.g., Hillman and Riley 1989; Hillman 1989, pp. 62-67; Hirshleifer and Riley 1992, pp. 369-404).[22]In analogy to these contests, there do not exist equilibria in pure strategies on the tournament stage: Let $p_i(e_i, e_j)$ be worker i's winning probability $(i, j = A, B;\ i \neq j)$ so that his expected utility can be written as

$$EU_i(e_i) = w_2 + \Delta w \cdot p_i(e_i, e_j) - c(e_i)\,. \tag{1}$$

Worker i $(i = A, B)$ at most exerts effort $\bar{e}$ with $\Delta w = c(\bar{e}) \Leftrightarrow \bar{e} = c^{-1}(\Delta w)$, because higher efforts would result in $EU_i(e_i) < w_2$ in case of winning the tournament. This outcome cannot be rational, since worker i can achieve $EU_i(e_i) = w_2$ with $e_i = 0$. We can distinguish three cases: $e_i > e_j$ $(i, j = A, B;\ i \neq j)$ cannot be an equilibrium, because worker i can reduce e_i and will still win the tournament. $e_i = e_j < \bar{e}$ cannot be an equilibrium, too, since i can win with certainty by marginally increasing e_i. $e_i = e_j = \bar{e}$ cannot be an equilibrium, because i would prefer to choose $e_i = 0$. Thus, there can only exist equilibria in mixed strategies. From the literature on rent-seeking contests it is also known that competition will result in complete rent dissipation (see, e.g., Hillman 1989, p. 59; Baye, Kovenock and de Vries 1996, p.

293). In this context, it means that $EU_i(e_i) = w_2$ $(i = A, B)$ in equilibrium. Let $G_j(e_i)$ $(i, j = A, B;\ i \neq j)$ be the probability that j chooses e_i or less effort. Then, (1) can be rewritten as $EU_i(e_i) = w_2 + \Delta w G_j(e_i) - c(e_i)$. The equilibrium condition $EU_i(e_i) = w_2$ results in $G_j(e_i) = c(e_i)/\Delta w$. Hence, on the tournament stage each worker chooses his effort according to the cumulative distribution function (cdf) $G(x) = c(x)/\Delta w$ over the interval $[0, c^{-1}(\Delta w)]$. On the first stage, the employer has to choose w_1 and w_2. Since incentives only depend on the prize spread Δw and not on the absolute prizes and since $EU_i(e_i) = w_2$, the participation constraint $EU_i(e_i) \geq \bar{u}$ will always be binding, i.e. the employer minimizes labor costs and optimally chooses $w_2^* = \bar{u}$. The incentive compatibility constraint is given by $G(x)$. As both workers behave symmetrically, the employer can consider one of the workers for his objective function. Thus, the employer wants to maximize $E[q_i] - \frac{w_1+w_2}{2} = E[x] - \frac{w_1+\bar{u}}{2}$ and chooses

$$w_1^* = \underset{w_1}{\operatorname{argmax}} \left\{ \int_0^{c^{-1}(\Delta w)} x \cdot \frac{c'(x)}{\Delta w} dx - \frac{w_1 + \bar{u}}{2} \right\} \tag{2}$$

Integrating by parts gives

$$w_1^* = \underset{w_1}{\operatorname{argmax}} \left\{ c^{-1}(\Delta w) - \int_0^{c^{-1}(\Delta w)} \frac{c(x)}{\Delta w} dx - \frac{w_1 + \bar{u}}{2} \right\}. \tag{3}$$

Applying Leibniz' rule yields the following first-order condition:

$$\int_0^{c^{-1}\left(w_1^* - \bar{u}\right)} \frac{c(x)}{\left(w_1^* - \bar{u}\right)^2} dx - \frac{1}{2} = 0. \tag{4}$$

From Eq. (2) it becomes clear that theoretically the employer can choose w_1 so that the expected effort $E\left[e_i\right] = \int_0^{c^{-1}(\Delta w)} x \cdot \frac{c'(x)}{\Delta w} dx$ equals first-best effort e^{FB} and that, in addition, this w_1 may solve the first-order condition (4). But in this case expected costs would be $E\left[c\left(e_i\right)\right] = c\left(E\left[e_i\right]\right) + \gamma = c\left(e^{FB}\right) + \gamma$ with $\gamma > 0$ according to Jensen's inequality. Expected costs can be written as

$$E\left[c\left(e_i\right)\right] = \int_0^{c^{-1}(\Delta w)} c\left(x\right) \cdot \frac{c'\left(x\right)}{\Delta w} dx = \frac{\Delta w}{2}, \tag{5}$$

where the last equality follows from integrating by parts. Thus, in the case $E\left[e_i\right] = e^{FB}$ the employer's welfare would be $2e^{FB} - \Delta w - 2w_2 = 2e^{FB} - 2E\left[c\left(e_i\right)\right] - 2\bar{u} = 2e^{FB} - 2c\left(e^{FB}\right) - 2\gamma - 2\bar{u}$ and the welfare of the two workers would be $2\bar{u}$ so that total welfare would amount to $2e^{FB} - 2c\left(e^{FB}\right) - 2\gamma$, which is smaller than first-best welfare.

Next, the J-type tournament is considered.[23]Worker i's expected utility is described by

$$EU_i\left(e_i\right) = \frac{e_i}{e_i + e_j} w - c\left(e_i\right), \qquad i, j = A, B;\ i \neq j. \tag{6}$$

From the workers' first-order conditions we obtain[24]

$$\frac{w}{\left(e_i + e_j\right)^2} = \frac{c'\left(e_i\right)}{e_j} = \frac{c'\left(e_j\right)}{e_i} \Rightarrow e_i c'\left(e_i\right) = e_j c'\left(e_j\right). \tag{7}$$

Since $c\left(\cdot\right)$ is monotonely increasing, we have $e_i = e_j$. Substituting into the first-order condition yields

$$\frac{w}{4e_i} = c'\left(e_i\right). \tag{8}$$

The employer's objective function is given by his profits $2e_i - w$. Substituting for w according to (8) leads to $2e_i - 4e_i c'(e_i)$. If the employer wants to implement first-best effort e^{FB}, which is defined by $c'\left(e^{FB}\right) = 1$, then his profits will be $2e^{FB} - 4e^{FB} < 0$. But this cannot be optimal from the employer's viewpoint, because he can ensure himself zero profits by choosing $w = 0$.

Proof of Corollary 1:

In the U-type tournament with quadratic costs $c(e_i) = (k/2)\, e_i^2$ the first-order condition (4) gives

$$\int_0^{\sqrt{\frac{2\Delta w}{k}}} \frac{kx^2}{2\Delta w^2} dx = \frac{1}{2} \Rightarrow \Delta w = \frac{8}{9k}, \tag{9}$$

which results in $E\left[e^U\right] = 8/(9k)$. In addition, we have $E\Pi^U = E\left[e^U\right] - (w_1^* + w_2^*)/2 = 4/(9k)$, because $w_2^* = \bar{u} = 0$ and $w_1^* = \Delta w = 8/(9k)$. In the J-type tournament the two workers behave identically so that it suffices to consider one of the workers. The employer wants to maximize $e_i - w/2$ subject to the incentive compatibility constraint (8) and the participation constraint $w/2 - c(e_i) \geq \bar{u} = 0 \Leftrightarrow w/2 - (k/2)\, e_i^2 \geq 0 \overset{(8)}{\Leftrightarrow} w/2 - w/8 \geq 0$. Hence, the participation constraint is non-binding. The incentive compatibility constraint (8) leads to $e_i = \sqrt{\frac{w}{4k}}$. Substituting in the employer's objective function and differentiating with respect to w yields the first-order condition $1/\left(4\sqrt{kw}\right) - \frac{1}{2} = 0 \Rightarrow w = 1/(4k)$. We obtain $e^J = 1/(4k) < E\left[e^U\right] <$

$1/k = e^{FB}$ and $E\Pi^J = e^J - w/2 = 1/(8k) < E\Pi^U < E\Pi^{FB} = 1/(2k)$.

Proof of Propositon 2:

If the two workers collude in the U-type tournament supergame, they will choose $e_A = e_B = 0$ and receive the expected utility

$$\begin{aligned} EU_i\left(e_i = 0\right) &= \frac{w_1 + w_2}{2}\delta + \frac{w_1 + w_2}{2}\delta^2 + \frac{w_1 + w_2}{2}\delta^3 + \cdots \\ &= \frac{w_1 + w_2}{2} \cdot \frac{\delta}{1-\delta}, \qquad i = A, B, \end{aligned} \tag{10}$$

with δ as discount factor. If worker i deviates from the collusive agreement, he will exert an arbitrarily small effort ε. In this period he wins w_1 with certainty. But then because of the grim strategy cooperation breaks down and the two workers will choose their mixed strategies $G(x)$ and get an expected utility w_2 (see the proof of Proposition 1) in each of the subsequent periods. Thus, worker i's expected utility from deviating is approximately

$$\begin{aligned} EU_i\left(e_i = \varepsilon\right) &= w_1\delta + w_2\delta^2 + w_2\delta^3 + \cdots \\ &= w_1\delta + w_2\frac{\delta^2}{1-\delta}. \end{aligned} \tag{11}$$

The collusive agreement will be stable, if $EU_i\left(e_i = 0\right) \geq EU_i\left(e_i = \varepsilon\right) \Leftrightarrow \Delta w\left(\delta - 2\delta^2\right) \leq 0 \Rightarrow \delta \geq \frac{1}{2} = \delta^U$.

Collusion in the J-type tournament yields the expected utility

$$EU_i\left(e_i = 0\right) = \frac{w}{2}\delta + \frac{w}{2}\delta^2 + \cdots = \frac{w}{2}\frac{\delta}{1-\delta}. \tag{12}$$

Deviating from the collusive agreement by choosing $e_i = \varepsilon$ gives

$$\begin{aligned} EU_i\left(e_i = \varepsilon\right) &= w\delta + \left(\frac{w}{2} - c\left(e^*\right)\right)\delta^2 + \left(\frac{w}{2} - c\left(e^*\right)\right)\delta^3 + \cdots \\ &= w\delta + \left(\frac{w}{2} - c\left(e^*\right)\right)\frac{\delta^2}{1-\delta}, \end{aligned} \tag{13}$$

where e^* denotes the workers' one-shot Nash equilibrium strategy characterized by (8). The collusion will be stable, if $EU_i\left(e_i = 0\right) \geq EU_i\left(e_i = \varepsilon\right)$, which can be rearranged to

$$\frac{w}{2} - \delta\left[\frac{w}{2} + c\left(e^*\right)\right] \leq 0. \tag{14}$$

The participation constraint $w/2 - c\left(e^*\right) \geq \bar{u} \geq 0$ implies $c\left(e^*\right) \leq w/2$. Thus, $\left[\frac{w}{2} + c\left(e^*\right)\right] \leq w$ and the lower bound for the discount factor that meets (14) is given by $\delta^J \geq \frac{1}{2} = \delta^U$.

Proof of Proposition 3:

Let w_i denote worker i's wage ($i = A, B$). Then i's relative deprivation can be written as

$$R_i = ([w_j - c(e_j)] - [w_i - c(e_i)]) \cdot \frac{1}{2} \to \min_{e_i} \tag{15}$$

($i, j = A, B;\ i \neq j$). Thus, i wants to minimize the net income difference to the other worker weighted with the relative size of the reference group.[25]In the U-type tournament, i's expected RD is described by

$$\begin{aligned} ER_i(e_i) &= (\Delta w \cdot \text{prob}\{e_i < e_j\} - c(e_j) - \Delta w \cdot \text{prob}\{e_j < e_i\} + c(e_i)) \cdot \frac{1}{2} \\ &= \frac{\Delta w}{2} - \Delta w \cdot G_j(e_i) - \frac{c(e_j)}{2} + \frac{c(e_i)}{2}, \end{aligned} \tag{16}$$

where $G_j(e_i)$ denotes the probability that j exerts e_i or smaller effort. In analogy to the proof of Proposition 1, it can easily be checked that there does not exist an equilibrium in pure strategies on the second stage: To see this we can distinguish the same three cases with a new upper bound $\bar{e} = c^{-1}(2\Delta w)$. From Eq. (16) i's first-order condition yields $-\Delta w G_j'(e_i) + c'(e_i)/2 = 0 \Rightarrow G_j(e_i) = c(e_i)/(2\Delta w) + \alpha$ with α being a constant. Since $G_j(0) = 0$ and $G_j(c^{-1}(2\Delta w)) = 1$, each worker's mixed strategy in equilibrium is characterized by the cdf $G(x) = c(x)/(2\Delta w)$ over the interval $[0, c^{-1}(2\Delta w)]$.

In the J-type tournament, worker i wants to minimize

$$R_i(e_i) = \left(\frac{we_j}{e_i + e_j} - c(e_j) - \frac{we_i}{e_i + e_j} + c(e_i)\right) \cdot \frac{1}{2}. \tag{17}$$

The first-order conditions imply that $e_A = e_B = e_{RD}^J$ with

$$\frac{w}{2e_{RD}^J} = c'\left(e_{RD}^J\right). \tag{18}$$

Result (i) can be proved as follows. $e_{RD}^J > e^J$ for a given w is obvious by comparing Eqs. (18) and (8). $E\left[e_{RD}^U\right] > E\left[e^U\right]$ means that

$$\begin{aligned} \int_0^{c^{-1}(2\Delta w)} x\frac{c'(x)}{2\Delta w}dx &> \int_0^{c^{-1}(\Delta w)} x\frac{c'(x)}{\Delta w}dx \Leftrightarrow \\ \int_{c^{-1}(\Delta w)}^{c^{-1}(2\Delta w)} x\frac{c'(x)}{2\Delta w}dx &> \int_0^{c^{-1}(\Delta w)} x\frac{c'(x)}{2\Delta w}dx \end{aligned} \tag{19}$$

which holds, because $xc'(x)$ is monotonely increasing and $c^{-1}(2\Delta w) > 2c^{-1}(\Delta w)$ due to the convexity of the cost function (i.e., the range of the interval on the left-hand side of (19) is greater than the one on the right-hand side).

For result (ii) first note that in both types of tournaments there is a symmetric equilibrium. Thus, (expected) RD of each worker is always zero in both tournaments. So we can neglect the participation constraint for both tournaments or assume a reservation value $\bar{R} \geq 0$ so that the constraint always holds. The employer's expected profits per worker in the U-type tournament is given by

$$E\Pi_{RD}^U = \int_0^{c^{-1}(2\Delta w)} x\frac{c'(x)}{2\Delta w}dx - \frac{w_1 + w_2}{2} = \frac{2}{3}\sqrt{\frac{4\Delta w}{k}} - \frac{w_1 + w_2}{2} \tag{20}$$

using the assumption of quadratic costs $c(e_i) = \frac{k}{2}e_i^2$. The employer optimally chooses $w_2^* = 0$ because incentives only depend on Δw, and $w_1^* = 16/(9k)$ to maximize (20). Therefore, $E\left[e_{RD}^U\right] = 16/(9k)$ and $E\Pi_{RD}^U = 8/(9k)$.

In the J-type tournament, the employer wants to maximize

$$E\Pi_{RD}^{J} = e_{RD}^{J} - \frac{w}{2} \tag{21}$$

subject to Eq. (18). Using the quadratic cost function we obtain $w = e_{RD}^{J} = 1/(2k)$ and $E\Pi_{RD}^{J} = 1/(4k)$.

The proof of result (iii) is already sketched in the text and therefore obmitted here.

*Proof of Proposition 4*a *and Corollary 2:*

Let the starting point be the scenario of Corollary 2 where a_A and a_B can be chosen out of a wide range of possible values. Furthermore, let $a_A > 0$ and $a_B > 0$ for a moment. Two cases have to be distinguished for the third stage. If $a_A = a_B$, then prob $\left\{a_A e_A \gtreqless a_B e_B\right\} \equiv$ prob $\left\{e_A \gtreqless e_B\right\}$ and we obtain the symmetric solution with mixed strategies described in the proof of Proposition 1. If $a_A \neq a_B$, this symmetric solution will no longer hold. Let $a_i < a_j$ $(i, j = A, B;\ i \neq j)$. Again, no equilibria in pure strategies are possible for the third stage because of the arguments given in the proof of Proposition 1 which, in analogy, hold for the asymmetric case, too. The maximum effort worker i is willing to choose is characterized by $\Delta w = c\left(\bar{e}_i\right) \Leftrightarrow \bar{e}_i = c^{-1}\left(\Delta w\right)$. Worker j's maximum effort $\bar{e}_j$ is lower and meets $a_j \bar{e}_j = a_i \bar{e}_i \Leftrightarrow \bar{e}_j = \frac{a_i}{a_j}\bar{e}_i = \frac{a_i}{a_j}c^{-1}\left(\Delta w\right)$. Any effort higher than $\bar{e}_j$ would only increase j's costs $c\left(e_j\right)$ without raising j's winning probability, because $\text{prob}\{a_i e_i \leq a_j \bar{e}_j\} = 1$ given the workers' continuous mixed strategies. It is obvious that worker j could always win with certainty by exerting effort $e_j = \bar{e}_i + \varepsilon$ with $\varepsilon \to 0$. But this cannot be an equilibrium, since i's best response would be $e_i = 0$ which would then imply $e_j = \varepsilon$ with $\varepsilon \to 0$ as j's best response and so on. In equilibrium, j's expected utility will meet

$$\begin{aligned} w_2 + \Delta w \,\text{prob}\{a_i e_i \leq a_j e_j\} - c(e_j) &= w_2 + \Delta w - c\left(\frac{a_i}{a_j}\bar{e}_i\right) \Leftrightarrow \\ \Delta w G_Y(e_j) - c(e_j) &= \Delta w - c\left(\frac{a_i}{a_j}\bar{e}_i\right) \Leftrightarrow \\ G_Y(e_j) &= 1 - \frac{c\left(\frac{a_i}{a_j}\bar{e}_i\right)}{\Delta w} + \frac{c(e_j)}{\Delta w} \end{aligned} \tag{22}$$

with $Y = \frac{a_i}{a_j}e_i$ being distributed over the interval $\left[0, \frac{a_i}{a_j}\bar{e}_i\right]$ with cdf $G_Y(\cdot)$. Eq. (22) describes the cdf according to which worker i chooses $\frac{a_i}{a_j}e_i$ in equilibrium. Hence, i's mixed strategy for his effort is characterized by the cdf

$$G_i(x) = 1 - \frac{c\left(\frac{a_i}{a_j}\bar{e}_i\right)}{\Delta w} + \frac{c\left(\frac{a_i}{a_j}x\right)}{\Delta w} \tag{23}$$

with $x \epsilon [0, \bar{e}_i]$. On the other hand, in equilibrium the expected utility of worker i has to be w_2 for the same argument given above concerning worker j. Thus

$$w_2 + \Delta w \,\text{prob}\{a_j e_j \leq a_i e_i\} - c(e_i) = w_2 \Leftrightarrow G_Z(e_i) = \frac{c(e_i)}{\Delta w} \tag{24}$$

with $Z = \frac{a_j}{a_i}e_j \in [0, \bar{e}_i]$ and $G_Z(\cdot)$ as the cdf of Z. Worker j's mixed strategy for his effort is therefore described by the cdf

$$G_j(x) = \frac{c\left(\frac{a_j}{a_i}x\right)}{\Delta w} \qquad \text{with} \quad x \in \left[0, \frac{a_i}{a_j}\bar{e}_i\right]. \tag{25}$$

Let $\tilde{c}(a_i)$ and $\tilde{c}(a_j)$ be the costs for the workers' human capital investments a_i and a_j on the second stage. Then their total expected utilities for the symmetric ($a_A = a_B$) and the asymmetric case ($a_i < a_j;\ i,j = A,B;\ i \neq j$) can be calculated as follows: If $a_A = a_{B,}$ then

$$EU_A = EU_B = w_2 - \tilde{c}(a_i) = w_2 - \tilde{c}(a_j). \tag{26}$$

If $a_i < a_j$, then

$$EU_i = w_2 + \int_0^{\bar{e}_i} \left[\Delta w G_Z(e_i) - c(e_i)\right] G_i'(e_i)\, de_i - \tilde{c}(a_i) = w_2 - \tilde{c}(a_i) \tag{27}$$

$$\begin{aligned} \text{and} \quad EU_j &= w_2 + \int_0^{\frac{a_i}{a_j}\bar{e}_i} \left[\Delta w G_Y(e_j) - c(e_j)\right] G_j'(e_j)\, de_j - \tilde{c}(a_j) \\ &= w_2 + \Delta w - c\left(\frac{a_i}{a_j}\bar{e}_i\right) - \tilde{c}(a_j). \end{aligned} \tag{28}$$

Note that $\Delta w - c\left(\frac{a_i}{a_j}\bar{e}_i\right) > 0$, because $\frac{a_i}{a_j} < 1$ and therefore $c\left(\frac{a_i}{a_j}\bar{e}_i\right) < c(\bar{e}_i) = \Delta w$.

Now, Proposition 4a can be proved as follows: On the second stage, the workers have to choose a_A, $a_B \in \{0,1\}$ at costs $\tilde{c}(0) = 0$ and $\tilde{c}(1) = \bar{c}$. Given that worker i chooses $a_i = 0$, worker j yields $EU_j = w_2$ with $a_j = 0$ according to (26) and $EU_j = w_1 - \bar{c}$ with $a_j = 1$ according to (28) ($i,j = A,B;\ i \neq j$). Given that worker i chooses $a_i = 1$, worker j obtains $EU_j = w_2$ with $a_j = 0$ according to (27) and $EU_j = w_2 - \bar{c}$ with $a_j = 1$ according to (26). Hence, if $\Delta w \geq \bar{c}$ the subgame consisting of the second and the third stage will have two equilibria (I) and (II): The workers choose $a_i^* = 0$ and $a_j^* = 1$ $(i, j = A, B;\ i \neq j)$ on the second stage, and $e_i^* = 0$ and $e_j^* = \varepsilon$ with $\varepsilon \to 0$ on the third stage. On the first stage, the employer chooses $w_1^* = w_2^* = 0$ to avoid negative profits. Thus, the two cases (I) and (II) coincide to the subgame perfect equilibrium (i) where the employer makes the two workers not to participate in the tournament.

It can easily be seen that there is a second subgame perfect equilibrium (ii) where the two workers randomize between the two cases (I) and (II) considered above. Let p_{i0} be the probability with which worker i chooses $a_i = 0$ and $1 - p_{i0}$ the probability for choosing $a_i = 1$. Define p_{j0} and $1 - p_{j0}$ analogously for worker j. Then i's and j's expected utilities on the second stage can be written as

$$EU_i = p_{i0} w_2 + (1 - p_{i0})\, p_{j0}\, (w_1 - \bar{c}) + (1 - p_{i0})\,(1 - p_{j0})\,(w_2 - \bar{c}) \quad (29)$$

$$EU_j = p_{j0} w_2 + (1 - p_{j0})\, p_{i0}\, (w_1 - \bar{c}) + (1 - p_{j0})\,(1 - p_{i0})\,(w_2 - \bar{c})\,. \quad (30)$$

Therefore, the optimal mixed strategies for the two workers on the second stage are

$$p_{i0} = p_{j0} = \frac{\bar{c}}{\Delta w}; \qquad 1 - p_{i0} = 1 - p_{j0} = \frac{\Delta w - \bar{c}}{\Delta w}. \quad (31)$$

On the third stage, the workers' choices depend on the realizations of the mixed strategies: $e_i^* = 0$ if $a_i^* = 0$, and $e_i^* = \varepsilon$ if $a_i^* = 1$ and $a_j^* = 0$

$(i, j = A, B;\ i \neq j)$, which directly follows from the two asymmetric cases (I) and (II). If, however, $a_i^* = a_j^* = 1$ have been realized, then we will have the symmetric solution of the basic model without human capital investment (see the proof of Proposition1): Both workers choose their efforts randomly out of the interval $[0, c^{-1}(\Delta w)]$ according to the cdf $G(x) = c(x)/\Delta w$. On the first stage, the employer has to choose w_1 and w_2. Irrespective of the realizations of the workers' mixed strategies, at most one of the workers will have the expected utility $EU_i = w_2 \geq \bar{u}$. Since incentives are only created by Δw and not by the absolute value of w_2, the employer optimally chooses $w_2^* = \bar{u}$. Furthermore, he knows that the two workers will only realize strictly positive expected efforts with joint probability $(\Delta w - \bar{c})^2 / \Delta w^2$. In this state, we have the symmetric case of the basic model without human capital investment and the employer chooses w_1 to maximize $(\Delta w - \bar{c})^2 / \Delta w^2$ times the expression given by (3) (see the proof of Proposition 1) subject to $\Delta w \geq \bar{c}$ which ensures that $a_i = a_j = 1$ is rational from the workers' viewpoint.

Corollary 2 can be proved as follows: Let a_i and a_j be continuous variables or discrete with more than two realizations. Then there cannot exist an equilibrium in pure strategies on the second stage of the U-type tournament game. To see this, Eq. (28) has to be considered: We can start, for example, with $a_i = 0$. j's best response to this would be $a_j = \varepsilon$ with $\varepsilon \to 0$. Then i's best response would be $\hat{a}_i$ with

$$\hat{a}_i > \varepsilon \text{ and } \hat{a}_i = \underset{a_i}{\operatorname{argmax}} \left\{ \Delta w - c\left(\frac{\varepsilon}{a_i} c^{-1}(\Delta w)\right) - \tilde{c}(a_i) \right\}. \tag{32}$$

But then j's best response would be $\hat{a}_j$ with

$$\hat{a}_j > \hat{a}_i \text{ and } \hat{a}_j = \underset{a_j}{\operatorname{argmax}} \left\{ \Delta w - c\left(\frac{\hat{a}_i}{a_j} c^{-1}(\Delta w)\right) - \tilde{c}(a_j) \right\} \tag{33}$$

and so on. Thus, each of the two workers wants to be the stronger player with the higher amount of human capital to get EU_j according to Eq. (28) instead of EU_i according to Eq. (27). But since $\tilde{c}(a_i)$ and $\tilde{c}(a_j)$ increase in a_i and a_j, respectively, there will exist some level of human capital investment so that overbidding is not optimal for the other worker any longer, who therefore chooses a zero investment level as his best response. But then the first worker would choose ε as a best response and so on. Altogether, both workers will choose $\hat{a}_i$ and $\hat{a}_j$ as mixed strategies over the interval from zero to a certain upper bound which is identical for both workers. Note, that even in the case of continuous variables $\hat{a}_i$ and $\hat{a}_j$ the two workers would not marginally increase their investment levels but would choose discrete jumps. This becomes clear from Eq. (28): If $\hat{a}_i$ and $\hat{a}_j$ only marginally differ from each other, then the worker with the higher investment level – for example worker j – will approximately obtain $EU_j = w_2 + \Delta w - c(\bar{e}_i) - \tilde{c}(a_j) = w_2 - \tilde{c}(a_j)$, which is less than the expected utility w_2 when dropping out of the tournament at the beginning.

*Proof of Proposition 4*b:

The proposition states that in the J-type tournament there exists a symmetric equilibrium in pure strategies. The workers' first-order conditions for their optimal strategies on the third stage yield

$$\frac{wa_ia_j}{(a_ie_i + a_je_j)^2} = \frac{c'(e_i)}{e_j} = \frac{c'(e_j)}{e_i}. \tag{34}$$

In analogy to the proof of Proposition 1, we get $e_i = e_j = e^*$ which will be characterized by

$$\frac{wa_ia_j}{(a_i + a_j)^2 e^*} = c'(e^*), \tag{35}$$

if a_i and a_j are both different from zero. Now, the subgame consisting of the last two stages is considerd. We have to distinguish four cases: (a) If $a_i = a_j = 0$, then $q_i = q_j = 0$ and $EU_i = \frac{w}{2} - c(e_i)$ and $EU_i = \frac{w}{2} - c(e_j)$. On the third stage, the two workers choose $e_i^* = e_j^* = 0$ and get $EU_i = EU_j = \frac{w}{2}$. (b), (c) If $a_i = 0$ and $a_j = 1$ (or, in analogy, $a_i = 1$ and $a_j = 0$), then $q_i = 0$ and $q_j = e_j$. On the second stage, the workers' expected utilities are $EU_i = -c(e_i)$ and $EU_j = w - c(e_j) - \bar{c}$. Given that $w > \bar{c}$, on the third stage the workers choose $e_i^* = 0$ and $e_j^* = \varepsilon$ with $\varepsilon \to 0$ and receive $EU_i = 0$ and $EU_j = w - \bar{c}$.[26] (d) If $a_i = a_j = 1$, then we have the basic model of Proposition 1 and (8) and (35) are identical: $w/(4e^*) = c'(e^*)$. Both workers receive $EU_i = EU_j = \frac{w}{2} - c(e^*) - \bar{c}$. On the first stage, the employer anticipates the four cases (a)-(d) and wants to implement (d), which is the only case where the workers exert strictly positive efforts. Therefore, the employer chooses w to maximize $e - w/2$ subject to $w/(4e) = c'(e)$ and $w/2 - c(e) - \bar{c} \geq \bar{u}$.

*Proof of Proposition 5*a*:*

The symmetric equilibrium can be proved in analogy to the proof of Proposition 1 (see especially Eq. (3)) and by using the general result from rent seeking contests that there is complete rent dissipation. In this context, it means that each worker's mixed strategy is given by $G(x)$ over the interval $[0, c^{-1}(\Delta w)]$ with $w_2 + \Delta w G^{n-1}(x) - c(x) = w_2$. For the claim of a continuum of asymmetric equilibria see, in analogy, Baye, Kovenock and de Vries (1996), p. 293. An intuition is sketched in the text.

Comparative statics for the symmetric equilibrium can be derived from the first-order condition for w_1^*, which – in analogy to Eq. (4) – gives:

$$F^U(w_1^*, n) := \int_0^{c^{-1}\left(w_1^* - \bar{u}\right)} \frac{1}{n-1} \frac{c(x)^{\frac{1}{n-1}}}{(w_1^* - \bar{u})^{\frac{n}{n-1}}} dx - \frac{1}{n} = 0. \tag{36}$$

Thus, $dw_1^*/dn = -[\partial F^U/\partial n]/[\partial F^U/\partial w_1^*]$ by the implicit-function rule, where $\partial F^U/\partial w_1^* < 0$ must hold as second-order condition. After some calculations we get

$$-\frac{\Delta w^{-1}}{(n-1)^2} \int_0^{c^{-1}(\Delta w)} \left(\frac{c(x)}{\Delta w}\right)^{\frac{1}{n-1}} \left[1 + \ln\left[\left(\frac{c(x)}{\Delta w}\right)^{\frac{1}{n-1}}\right]\right] dx + \frac{1}{n^2} > 0$$

with $\Delta w = w_1^* - \bar{u}$. The integral is negative, because the logarithm of the cdf, $\ln[(c(x)/\Delta w)^{\frac{1}{n-1}}]$, takes only negative values. Altogether, we obtain

$dw_1^*/dn > 0$. The second comparative static result deals with $E[e_n^U]$ with

$$E[e_n^U] = c^{-1}(\Delta w) - \int_0^{c^{-1}(\Delta w)} \left(\frac{c(x)}{\Delta w} \right)^{\frac{1}{n-1}} dx, \tag{37}$$

where $\Delta w = w_1^*(n) - \bar{u}$ with $dw_1^*/dn > 0$. Applying Leibniz' rule the derivative of $E[e_n^U]$ with respect to n yields

$$\int_0^{c^{-1}(\Delta w)} \left[\frac{1}{n-1} \left(\frac{c(x)}{\Delta w} \right)^{\frac{1}{n-1}} \right] \left[\frac{1}{\Delta w} \frac{dw_1^*}{dn} + \ln \left[\left(\frac{c(x)}{\Delta w} \right)^{\frac{1}{n-1}} \right] \right] dx.$$

The sign of this expression is ambiguous because of the second term in brackets which contains the logarithm of the cdf, and crucially depends on n and the shape of the cost function $c(e_i)$ which both determine Δw and dw_1^*/dn in equilibrium.

Proof of Proposition 5b*:*

In the J-type tournament, each worker i ($i = 1, ..., n$) wants to maximize $(we_i) / \left(\sum_{j=1}^n e_j \right) - c(e_i)$ on the second stage. The first-order conditions yield

$$\frac{w}{\left(\sum_{j=1}^n e_j \right)^2} = \frac{c'(e_i)}{\sum_{j \neq i} e_j} = \frac{c'(e_k)}{\sum_{j \neq k} e_j} = \cdots. \tag{38}$$

Let, for example, $i = 1$ and $k = 2$ so that Eq. (38) implies

$$c'(e_1)(e_1 + e_3 + e_4 + \cdots + e_n) = c'(e_2)(e_2 + e_3 + e_4 + \cdots + e_n). \tag{39}$$

Since $c'(\cdot)$ is monotonely increasing, it must hold that $e_1 = e_2$. This can be done in analogy with any pair $\{i, k\}$ ($i, k = 1, \cdots, n;\ i \neq k$) so that we have

$e_1 = e_2 = \cdots = e_n^J$ and Eq. (38) can be rewritten as

$$w\frac{n-1}{n^2} = e_n^J c'\left(e_n^J\right). \tag{40}$$

The employer's optimization problem on the first stage is then similar to the one in Proposition 4b. The claim that $e_n^J < e^{FB}$ is obtained from the employer's objective function $e_n^J - \frac{w}{n} = e_n^J - \frac{n}{n-1} e_n^J c'\left(e_n^J\right)$. If the employer wants to implement first-best effort e^{FB} with $c'\left(e^{FB}\right) = 1$, his objective function will become $e^{FB} - \frac{n}{n-1} e^{FB} < 0$. Thus, since $c'(\cdot)$ is monotonely increasing, the employer always wants to implement an $e_n^J < e^{FB}$.

For the comparative static result we have to consider the employer's objective function mentioned in the last paragraph. The first-order condition with respect to e_n^J gives

$$F^J(e_n^J, n) := 1 - \frac{n}{n-1} c'\left(e_n^J\right) - \frac{n}{n-1} e_n^J c''\left(e_n^J\right) = 0.$$

From the implicit function rule we obtain

$$\frac{de_n^J}{dn} = -\frac{\frac{1}{(n-1)^2}\left[c'\left(e_n^J\right) + e_n^J c''\left(e_n^J\right)\right]}{\partial F^J / \partial e_n^J} > 0.$$

Note that the denominator has to be negative as second-order condition for e_n^J.

Proof of Proposition 6:

Proposition 6 can be proved in analogy to Proposition 2. In the U-type tournament a worker's expected utility will be $\left[w_1 + (n-1)\, w_2\right] / n \cdot \delta / (1-\delta)$

if keeping the collusive agreement, and $w_1\delta+(w_2\delta^2)/(1-\delta)$ if deviating from collusion. Thus, collusion will be stable if $\delta \geq (n-1)/n = \delta_n^U$. In the J-type tournament, a worker's expected utility will be $w/n \cdot \delta/(1-\delta)$ if keeping the collusive agreement and $w\delta + \left[w/n - c\left(e_n^J\right)\right] \cdot \delta^2/(1-\delta)$ if breaking it. Hence, stable collusion requires

$$\delta \geq \frac{w}{w + \frac{n}{n-1} c\left(e_n^J\right)} \tag{41}$$

to hold. Since the participation constraint ensures $c\left(e_n^J\right) \leq w/n$ the lower bound δ_n^J meets $\delta_n^J \geq \delta_n^U$. The claim about the connection of the influence of w and the shape of the cost function can be shown as follows: Differentiating the right-hand side of (41) with respect to w and substituting for w according to (40) yields

$$\frac{\frac{n}{n-1}}{D^2}\left(c\left(e_n^J\right) - \frac{n^2}{n-1} e_n^J \left[c'\left(e_n^J\right)\right]^2 \frac{de_n^J}{dw}\right), \tag{42}$$

where D denotes the denominator of the right-hand side of (41). By implicitly differentiating (40) we get $de_n^J/dw = \left[(n-1)/n^2\right]/\left[c'\left(e_n^J\right) + e_n^J c''\left(e_n^J\right)\right] > 0$ so that (42) can be rewritten as

$$\frac{\frac{n}{n-1}}{D^2}\left(c\left(e_n^J\right) - \frac{e_n^J\left[c'\left(e_n^J\right)\right]^2}{c'\left(e_n^J\right) + e_n^J c''\left(e_n^J\right)}\right). \tag{43}$$

Proof of Proposition 7:

First, non-RD workers are considered. As we know from Proposition 5a expected profits per worker in the U-type tournament can be written as $E\Pi_n^U(w_1^*(n), n) = E[e_n^U] - [w_1^* + (n-1)\bar{u}]/n$. According to the Envelope Theorem, $dE\Pi_n^U/dn = dE\Pi_n^U/dw_1^* \cdot dw_1^*/dn + \partial E\Pi_n^U/\partial n = \partial E\Pi_n^U/\partial n$ with (using (37))

$$\begin{aligned}
\frac{\partial E\Pi_n^U}{\partial n} &= \frac{\partial}{\partial n}\left\{c^{-1}(\Delta w) - \int_0^{c^{-1}(\Delta w)} \left(\frac{c(x)}{\Delta w}\right)^{\frac{1}{n-1}} dx\right\} + \frac{\Delta w}{n^2} \\
&= \int_0^{c^{-1}(\Delta w)} \frac{1}{n-1}\left(\frac{c(x)}{\Delta w}\right)^{\frac{1}{n-1}} \ln\left[\left(\frac{c(x)}{\Delta w}\right)^{\frac{1}{n-1}}\right] dx + \frac{\Delta w}{n^2} \quad (44)
\end{aligned}$$

with $\Delta w = w_1^*(n) - \bar{u}$. The first-order condition (36) can be rearranged to

$$\frac{\Delta w}{n^2} = \int_0^{c^{-1}(\Delta w)} \frac{1}{n}\frac{1}{n-1}\left(\frac{c(x)}{\Delta w}\right)^{\frac{1}{n-1}} dx. \quad (45)$$

Substituting for $\Delta w/n^2$ in (44) gives

$$\frac{\partial E\Pi_n^U}{\partial n} = \int_0^{c^{-1}(\Delta w)} \frac{1}{n-1}\left(\frac{c(x)}{\Delta w}\right)^{\frac{1}{n-1}} \left[\frac{1}{n} + \ln\left[\left(\frac{c(x)}{\Delta w}\right)^{\frac{1}{n-1}}\right]\right] dx. \quad (46)$$

Since $1/n$ decreases in n and the logarithm of the cdf, $\ln[(c(x)/\Delta w)^{\frac{1}{n-1}}]$, takes only negative values, $E\Pi_n^U$ is a decreasing function of n for sufficiently large n. From the proof of Proposition 5b we know that in J-type tournaments $E\Pi_n^J(e_n^J(n), n) = e_n^J - \frac{n}{n-1}e_n^J c'\left(e_n^J\right)$. Applying the Envelope Theorem again yields $dE\Pi_n^J/dn = \partial E\Pi_n^J/\partial n = \frac{1}{(n-1)^2}e_n^J c'\left(e_n^J\right) > 0$. Thus, for sufficiently large n we have $E\Pi_n^U < E\Pi_n^J$.

Second, n-person tournaments with RD workers are considered. In the

U-type tournament, expected RD of worker i will be in analogy to Eq. (16)

$$ER_i(e_i) = \frac{\Delta w}{n} - \frac{2\Delta w}{n} G_j^{n-1}(e_i) - \frac{c(e_*)}{n} + \frac{c(e_i)}{n} \tag{47}$$

where e_* denotes the effort of one of the other workers who – according to the symmetry assumption – behave identically. Then, in analogy to Proposition 3 we get the result that each worker's mixed strategy is described by the cdf $G(x) = (c(x)/[2\Delta w])^{\frac{1}{n-1}}$ over the interval $[0, c^{-1}(2\Delta w)]$. In analogy to the case of non-RD workers, we have

$$E\Pi^U_{n,RD} = c^{-1}(2\Delta w) - \int_0^{c^{-1}(2\Delta w)} \left(\frac{c(x)}{2\Delta w}\right)^{\frac{1}{n-1}} dx - \frac{w_1^* + (n-1)\bar{w}_2}{n}, \tag{48}$$

where $\bar{w}_2$ is the lowest possible loser prize satisfying the participation constraint. According to the Envelope Theorem $dE\Pi^U_{n,RD}/dn =$

$$\frac{\partial E\Pi^U_{n,RD}}{\partial n} = \int_0^{c^{-1}(2\Delta w)} \frac{1}{n-1}\left(\frac{c(x)}{2\Delta w}\right)^{\frac{1}{n-1}} \ln\left[\left(\frac{c(x)}{2\Delta w}\right)^{\frac{1}{n-1}}\right] dx + \frac{\Delta w}{n^2}. \tag{49}$$

Using Leibniz' rule and deriving the first-order condition for w_1^* from Eq. (48) gives

$$\frac{\Delta w}{n^2} = \int_0^{c^{-1}(2\Delta w)} \frac{1}{n-1}\left(\frac{c(x)}{2\Delta w}\right)^{\frac{1}{n-1}} \frac{1}{n} dx \tag{50}$$

after rearranging. By substituting into Eq. (49) we obtain

$$\frac{\partial E\Pi^U_{n,RD}}{\partial n} = \int_0^{c^{-1}(2\Delta w)} \frac{1}{n-1}\left(\frac{c(x)}{2\Delta w}\right)^{\frac{1}{n-1}} \left[\frac{1}{n} + \ln\left[\left(\frac{c(x)}{2\Delta w}\right)^{\frac{1}{n-1}}\right]\right] dx \tag{51}$$

which becomes negative for large n. In the J-type tournament, worker i

wants to minimize

$$R_i(e_i) = \left(w \frac{e_* - e_i}{\sum\limits_j e_j} - c(e_*) + c(e_i) \right) \cdot \frac{1}{n}. \tag{52}$$

Differentiating with respect to e_i and using the symmetry assumption $e_1 = \cdots = e_n = e^J_{n,RD}$ the first-order conditions yield

$$w = nc'\left(e^J_{n,RD}\right) e^J_{n,RD}. \tag{53}$$

Substituting into $E\Pi^J_{n,RD} = e^J_{n,RD} - \frac{w}{n}$ yields

$$E\Pi^J_{n,RD} = e^J_{n,RD} - c'\left(e^J_{n,RD}\right) e^J_{n,RD}. \tag{54}$$

Hence, $dE\Pi^J_{n,RD}/dn = 0$ and $de^J_{n,RD}/dn = 0$ in equilibrium, which completes the proof of Proposition 7(i)-(ii).

Proof of Corollary 3:

Result (i) can easily be proved by explicitly calculating the subgame perfect equilibria for the U-type and the J-type tournament using quadratic costs $c(e_i) = \frac{k}{2}e_i^2$. We obtain $w_1^* = [2/k] \cdot \left[n^2/(n+1)^2\right]$ and $E\left[e_n^U\right] = [4n]/\left[k(n+1)^2\right]$ and $E\Pi_n^U = [2n]/\left[k(n+1)^2\right]$ for the U-type tournament, and $w^* = (n-1)/(4k)$ and $e_n^J = (n-1)/(2kn)$ and $E\Pi_n^J = (n-1)/(4kn)$ for the J-type tournament. The claims of result (i) immediately follow from these expressions.

Result (ii) deals with tournaments between n RD workers. With quadratic costs we obtain the following outcomes for the symmetric equilibria: $w_1^* =$

$(4n^2) / \left[k(n+1)^2\right]$ and $E\left[e^U_{n,RD}\right] = (8n) / \left[k(n+1)^2\right]$ and $E\Pi^U_{n,RD} = (4n) / \left[k(n+1)^2\right]$ in the U-type tournament, and $w^* = n/(4k)$ and $e^J_{n,RD} = 1/(2k)$ and $E\Pi^J_{n,RD} = 1/(4k)$ in the J-type tournament. The claims of Corollary 3(ii) directly follow from these expressions.[27]

Notes

1. The following arguments also hold for human capital investments, but for brevity only unverifiable outputs are considered here.

2. The labels "U-type" and "J-type" are used here, because the two types of tournaments can also be found in other countries, and because the following analysis does not offer a complete comparison of typical U.S. and Japanese incentive systems.

3. Such bonuses can make up 18-30% of a worker's yearly income; see Itoh (1991, pp. 348-350), Kanemoto and MacLeod (1992, p. 145), Ito (1992, pp. 231-239).

4. In the traditional Japanese firm, there are no definite demarcation lines between "jobs". Thus, the U-type solution by attaching wages to jobs cannot be used here.

5. Of course, assuming deterministic production describes a highly stylized situation. But this simplification is comparable to the assumption of a risk neutral principal instead of a less risk averse principal (compared to the agent) in principal agent models. Technically, the mixed strategy equilibria in the U-type tournament sketch the situation of Lazear-Rosen tournaments with too less noise so that the second-order

condition for a pure strategy equilibrium does not hold. In the J-type tournament below, $e_i/(e_i + e_j) = q_i/(q_i + q_j)$ is a good approximation when there is low noise (see Theorem 4 in Mood, Graybill and Boes 1974, p. 181), since the error terms are usually i.i.d. in tournaments.

6. Tournament models often use linear production functions; see, e.g., the seminal paper of Lazear and Rosen (1981). Here, I will focus on the unverifiability problem and do not add an exogenous noise component to e_i.

7. $c^{'-1}(\cdot)$ denotes the inverse of the workers' marginal cost function.

8. But even in this case there is a welfare loss, because realized effort will usually differ from e^{FB}.

9. See, e.g., Dye (1984), p. 148; McLaughlin (1988), p. 248; Milgrom and Roberts (1992), p. 369.

10. Since collusion is illegal, such an agreement cannot be enforced by contract, but has to be self-enforcing.

11. Using the grim strategy (or trigger strategy) worker i begins playing his collusive effort $e_i = 0$. Worker i remains playing $e_i = 0$ as long as the other worker j has chosen zero effort in the last round. But if the other worker j chooses an effort different from zero, worker i will switch to

his Nash equilibrium strategy of the one-shot tournament and will keep on playing it in each of the subsequent rounds. Of course, according to the Folk Theorem there may be a wide range of equilibria which lead to stable collusions. But for simplicity, the analysis is restricted to possible equilibria in grim strategies. Moreover, grim strategies have the desirable property that they can result in subgame perfect equilibria.

12. Stark (1987, 1990) introduces the RD concept in the discussion of tournaments. For a modelling of tournarments with RD based on net income see Kräkel (2000).

13. For a formal description of the workers' objective functions see Eq. (15) in the Appendix.

14. The following results will qualitatively hold, if we assume $a_i \in \{a_L, a_H\}$ with $a_L < a_H$ and costs c_L for a_L and c_H for a_H.

15. Since $\varepsilon \to 0$, the terms with ε and $c(\varepsilon)$ have been neglected for simplicity when deriving the optimal strategies for the first and the second stage.

16. Note that Δw cannot be increased by lowering w_2, as $w_2^* = \bar{u}$ is fixed.

17. See Eq. (3) in the Appendix.

18. This is obvious for (i). For (ii) see the proof of Proposition 4a in the Appendix.

19. For the following considerations see the proofs of Proposition 4a and Corollary 2. Especially, note that for $a_j > a_i$ worker i's mixed strategy on the third stage is described by the cdf $G_i(x) = 1 - c\left(\frac{a_i}{a_j}\bar{e}_i\right)/\Delta w + c(\frac{a_i}{a_j}x)/\Delta w$ over the interval $[0, \bar{e}_i]$ with $\bar{e}_i = c^{-1}(\Delta w)$. Thus, worker i's drop-out probability mentioned below (i.e., $G_i(0)$) tends to 1 for $a_j \to \infty$ or $a_i \to 0$.

20. The three-stage tournament with human capital investment will not be discussed for $n > 2$ workers, because there will not be new insights from this modeling. For example, in the U-type tournament with many investment levels we will have competition between n heterogeneous workers. In analogy to the contest literature, we should then expect an asymmetric equilibrium, where the two strongest players compete against each other and the $n-2$ other workers drop out (see, e.g., Baye, Kovenock and de Vries 1996, p. 297).

21. Of course, the expected utilities depend on w_1 and w_2 which may depend on n, but the condition for stable collusion is independent of w_1 and w_2.

22. The main differences to rent-seeking contests are that in tournaments workers have convex cost functions and the prizes are optimally chosen by an employer.

23. J-type tournaments are similar to logit-form contests (see, e.g., Dixit 1987, p. 893). The major differences are the convex cost function and the endogeneous wage bill w.

24. The second-order conditions always hold in the J-type tournament.

25. Note that this formulation of RD is more general than the definition in Kräkel (2000), where a worker only feels relatively deprived when becoming a loser in the tournament.

26. $c(\varepsilon)$ is neglected because of $c(\varepsilon) \to 0$.

27. Note that again $w_2^* = 0$ in the U-type tournament.

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