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**Thünen-Series of Applied Economic Theory**  
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Working Paper No. 35

**Endogenous Growth with Endogenous Fertility  
and Social Discrimination in Education**

von

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# Endogenous Growth with Endogenous Fertility and Social Discrimination in Education

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## Abstract

We simulate a two-sectoral, three-period OLG-model with endogenous fertility and endogenous education. Parents receive utility from quantity and quality (education) of their offspring, generating a trade-off between the former and the latter. Since education governs efficiency in production, hence wage income and growth, labour supply and education per child increase and fertility declines during the process of economic development. Therefore, the model is able to explain the recent fertility decline, in all developed countries, and to single out the determinants for long-run growth in per capita terms. The transition speed towards the steady state is governed by the growth rate of the wage rate, which in turn depends on education investments and fertility of previous generations, leading to an intergenerational persistence in these variables. Due to this effect, social discrimination in the education sector leads to high fertility and low education in the low-income percentiles, while the opposite is true for the upper ones. Hence, the average level of education declines and hinders growth.

JEL: J1, I2, O0

*Keywords:* OLG-model, endogenous growth, endogenous fertility, education, intergenerational persistence, discrimination

# 1 Introduction

The aim of this paper is to put up a framework in which the fall of the fertility rate is a transitory and endogenous phenomenon. The intend is to contribute to a better understanding of why the total fertility rate has declined in all developed countries and what economic policies could influence this demographic process. Specifically, we analyse the transitional dynamics and the equilibrium outcome of the interaction between fertility and education in the process of economic growth.

Traditional growth theory is based on a positive relationship between capital per worker  $k$  and output per capita  $y$ :  $y = f(k)$  with  $f'(k) > 0$ . The endogenous variables are  $k$  and per capita consumption whereas fertility  $n$  is a given constant. An increase in  $n$  leads to a decrease in  $k$  (an effect called capital dilution) and therefore to a decrease in  $y$  and an increase in the marginal product of capital. In modern growth theory, the population is partly taken to be constant. The consequence is a so called scale effect (big countries grow faster) on which opinions diverge. In approaches labelled semi-endogenous, the fertility rate is, again, given and constant.

The first macroeconomic models with endogenous fertility employed the microeconomic approach of Becker (1960) in which children show up in their parents' utility function as durable goods. In Barro/Becker (1988, 1989), Becker/Murphy/Tamura (1990) and Barro/Sala-i-Martin (1995) fertility is endogenized by means of a dynastic utility function. Whereas Barro/Becker (1988) is partial equilibrium in nature, Becker/Barro (1989) and Barro/Sala-i-Martin (1995) are one-sectoral macroeconomic models with exogenous technological progress in which time and capital costs of child rearing are taken into account. However, fertility decisions are consumption oriented in these approaches, ignoring the investment efforts parents undertake when providing education for their children.

As soon as education is considered, the question as to the relationship between quality and quantity of the offspring arises. There are two strands of the literature dealing with education in a growth-theoretic framework. The first is pioneered by Becker/Murphy/Tamura (1990) who implement the Becker/Lewis (1973) microeconomic quantity-quality approach into a macroeconomic context. They show that following an exogenous shock which triggers economic development, either all agents have positive investment in human capital with a decreasing fertility rate or investment in human capital is converging to zero, accompanied by an increasing fertility rate. Introducing heterogeneity (inequality) into this model would lead to zero income for some agents and exploding wealth for others. Kremer/Chen (2000) point out that there is

evidence for a positive correlation between inequality and fertility differentials. Taking the quality-quantity-trade-off into account, this correlation must imply education differentials as well. In the same line of argument, de la Croix/Doepke (2001) find empirical evidence for a growth-harming effect of fertility differentials which are based on educational differentials. There, as well as in Dahan/Tsiddon (1999) and Morand (1999), inequality is introduced by the initial distribution of human capital. In the steady state there is no growth in per capita terms and the distribution of human capital is equalized. Hence, there is no scope for redistributive policies. Social discrimination in the education sector is no issue in either of these models.

The second strand of the literature dealing with the macroeconomic role of educational investments of parents in their children follows the human capital approach of Lucas (1998). Thus, Glomm (1997) and Glomm/Ravikumar (1992, 1997) trace the human capital accumulation back to the parental decision to invest in the education of the next generation. Based on their work, Dessy (1998) investigates the relationship between fertility decisions and educational regimes on a balanced-growth path. On such a path, the population growth rate, the time allocation and the growth rate for human capital are all constant.

The present paper employs a three-period, two-sectoral, OLG framework in the spirit of Diamond (1965). Contrary to the above, there is human capital as well as real capital accumulation and the analysis encompasses both the equilibrium growth and the adjustment process. However, due to the complexity of the model, the transition dynamics is simulated. In each period the economy is populated by children, adults and old. During childhood each child consumes  $z$  units of time out of its parents' time budget and is endowed with education by its parents. In the next period agents participate in the labour market and take all relevant economic decisions: the supply of labour, fertility, education of the offspring, consumption and savings. Finally, agents retire and consume their savings including accrued interest payments. Similar to Galor/Weil (1996), the model generates decreasing fertility due to an increasing wage rate during the transition process to a steady state. But whereas their model only considers the link between transitory growth and fertility, we endogenize growth, fertility and education and can thus show that long-run growth does not cease.

One of our results is that the level of education of a child depends positively on the wage rate of its parents which in turn increases in the wage rate and educational attainment of their parents and so on, leading to an intergenerational persistence in fertility decisions, educational efforts and the wage rate. A low wage rate leads to high ferti-

ity and low education through generations. In general equilibrium, the allocation of labour to the production and the education sector is endogenously determined. The education technology depends positively on labour and the level of education of the parental generation. Fertility has instead a negative effect on per capita education and hence on output in the production sector. The transitional dynamics is characterized by an equal increase in capital per worker and in the wage rate, whereby the latter equals the product of the growth factors of education and its costs, controlled for the variation of the fertility rate. Hence, important for the transition speed is the relation between the growth factors of education and fertility. In the steady state all reallocations in the time budget of the households are closed. Wage rate and capital intensity are growing at an equal and constant rate, while the fertility rate, the labour supply and the factor distribution are constant. The simulations performed show that due to increasing wages, labour supply increases, fertility declines and education investments per child increase. Productivity in education depends negatively on fertility, hence the education sector becomes more productive, resulting in a higher allocation of labour to production, compared to earlier stages of development. The other way round, in earlier stages labour supply is low and fertility high. To equalize the marginal products of labour, a relative high amount of this low labour supply has to be allocated to education, which is compatible with the altruistic behaviour of the parental generation and the source for the wealth of future generations.

In a next step we consider agents which are heterogeneous in two respects: socially, due to different productivities in education and in the abilities to use their educational level in the production, hence intellectually. We show that intellectual discriminated dynasties have lower wages, lower savings, a higher fertility and lower investments in education per child. Furthermore, the convergence speed to the steady state depends negatively on the amount of discrimination. Despite the existence of intellectual heterogeneities, in the steady state all endogenous variables grow, again, with identical and equal rates, whereby the fertility rate is constant and equal for all social classes. However, due to different convergence speeds, income and education differences are preserved. Because of a higher fertility rate of the lower income percentiles during the transition process, redistributive pressure in a median voter system is maintained or eventually aggravated, too. This result is contrary to de la Croix/Doepke (2001), Dahan/Tsiddon (1999) and Morand (1999). Income transfers slow the convergence speed down, leading to higher fertility rates and lower investment in education per child in all income classes.

The existence of social discrimination does not only affect the convergence speed, but rather the long-run performances, too. Socially discriminated dynasties exhibit lower steady state growth rates and higher fertility rates. Here, redistributive pressure is maintained as well, but policy interventions in education are growth enhancing.

The paper is organized as follows. Section 2 introduces the basic framework with endogenous fertility, but without endogenous education. Section 3 endogenizes education and shows up the interactions between education and fertility with its consequences for the growth process. Section 4 extends the presented framework for intellectual and social heterogeneity, and Section 5 concludes.

## 2 Fertility during the Transition to the Steady State

### 2.1 The Households

We consider an economy, which is populated by a continuum of overlapping generations. Each generation lives exactly three periods. In the first period, childhood, each child consumes a fraction  $z$  out of the time budget of its parents. In the second period

of life, adulthood, agents make all their economic relevant decisions. Individuals are participating in the labour market and receive a labour income according to their offered time quantum. The remaining time is used for childrearing, hence the decision between labour supply  $l_t$  and fertility  $n_t$  is made simultaneously. If each household is endowed with one unit of time, a representative household faces the time budget restriction

$$zn_t + l_t = 1. \quad (1)$$

The labour income  $w_t(1 - zn_t)$  is used for consumption in the current period  $c_t$  and for savings  $s_t$ . Furthermore, individuals are endowed with a non-labour income  $V$ , which can be either consumed or saved, too. This leads to the following budget restriction

$$w_t + V = zn_tw_t + c_t + s_t. \quad (2)$$

In the third period, agents retire and consume their savings with accrued interest, so that

$$c_{t+1} = (1 + r_{t+1})s_t. \quad (3)$$



Individuals receive utility from the number of their offspring and current and future consumption. People are non-altruistic, so there is no bequest motive. Under a log-linear utility function a representative agent, born in period  $t-1$ , faces in period  $t$  the following optimization problem

$$\max_{\{n_t^{t-1}, c_t^{t-1}, c_{t+1}^{t-1}\}} u_t^{t-1} = \gamma \ln n_t^{t-1} + (1-\gamma) \left[ \ln c_t^{t-1} + \frac{1}{1+\rho} \ln c_{t+1}^{t-1} \right], \quad (4)$$

subject to (2) and (1), where  $\gamma$  and  $(1-\gamma)$  are the weights of the number of children and consumption in the utility function, respectively. Future consumption is discounted by the individuals rate of time preference  $\rho$ .

All this leads to optimal solutions

$$c_t^{t-1} = \left[ \frac{(1-\gamma)(1+\rho)}{2+\rho-\gamma} \right] [w_t^{t-1} + V], \quad (5)$$

$$n_t^{t-1} = \left[ \frac{\gamma(1+\rho)}{2+\rho-\gamma} \right] \left[ \frac{w_t^{t-1} + V}{zw_t^{t-1}} \right], \quad (6)$$

$$s_t^{t-1} = \frac{c_{t+1}^{t-1}}{1+r_{t+1}} = \left[ \frac{(1-\gamma)}{2+\rho-\gamma} \right] [w_t^{t-1} + V] \quad (7)$$

The growth rate of consumption between the second and the third period of life depends positively on the interest rate  $r_{t+1}$  and negatively on the discount rate, so that

$$\frac{c_{t+1}^{t-1}}{c_t^{t-1}} = \frac{1+r_{t+1}}{1+\rho}. \quad (8)$$

Obviously, consumption (5) depends positively on its weight in the utility function  $(1-\gamma)$ , on the rate of time preference  $\rho$  and on the sum of the wage rate and non-labour income  $(w_t^{t-1} + V)$ . Contrary, saving (7) depends negatively on the rate of time preference  $\rho$ , but positively on  $(1-\gamma)$  and  $(w_t^{t-1} + V)$ . The number of children (6) depends positively on  $\gamma$  and  $\rho$ , but negatively on the opportunity costs per child priced by the wage rate  $(zw_t^{t-1})$ .

## 2.2 Production

The aggregated output  $Y_t$  is produced with labour  $L_t$  and capital  $K_t$ , where  $L_t$  is not the number of employees, but the aggregated quantum of labour offered by the households, so that

$$L_t = l_t^{t-1} N_t = (1 - zn_t^{t-1}) N_t. \quad (9)$$

Furthermore, production is subject to a linear-homogenous technology of Cobb-Douglas type

$$Y_t = BK_t^\alpha L_t^{1-\alpha} = BK_t^\alpha \left( (1 - zn_t^{t-1}) N_t \right)^{1-\alpha}, \quad (10)$$

which leads to a relation expressing the output per worker

$$y_t = Bk_t^\alpha \left( l_t^{t-1} \right)^{1-\alpha} = Bk_t^\alpha \left( 1 - zn_t^{t-1} \right)^{1-\alpha}. \quad (11)$$

Assuming perfect competition on goods and factor markets, factor prices are equal to their marginal products

$$w_t^{t-1} = (1 - \alpha) B \tilde{k}_t^\alpha = (1 - \alpha) B k_t^\alpha \left( 1 - zn_t^{t-1} \right)^{-\alpha}, \quad (12)$$

$$r_t = \alpha B \tilde{k}_t^{\alpha-1} = \alpha B k_t^{\alpha-1} \left( 1 - zn_t^{t-1} \right)^{1-\alpha}. \quad (13)$$

The factor prices are a function of capital per employed unit of labour  $\tilde{k}_t = \frac{K_t}{l_t N_t}$ , so that

the wage rate depends positively on capital per worker  $k_t = \frac{K_t}{N_t}$  and on the time fraction allocated to child-rearing  $zn_t$  e.g. due to an increasing fertility, hence decreasing labour supply. On the other hand, a higher wage rate leads according to (6) to a reduction of the number of children. Hence with (6) and (12), in a general equilibrium, the following relationship holds, which must be unambiguously, due to the monotony of the production function

$$n_t^{t-1} = \frac{\gamma}{z} \frac{\gamma(1+\rho)}{2+\rho-\gamma} \frac{(1-\alpha)Bk_t^\alpha \left( 1 - zn_t^{t-1} \right)^{-\alpha} + V}{(1-\alpha)Bk_t^\alpha \left( 1 - zn_t^{t-1} \right)^{-\alpha}}. \quad (14)$$

Equation (14) determines the number of children per household and plays a key role for the development of the economy, which will be explored in the following section. Unfortunately, solutions for  $n_t$  are not obtainable until exact values for the parameters of the model are given. This seems to reduce the generality of the analysis, but one has to note, that the qualitative behaviour of the economy is not affected by the

choice of any feasible set of parameter values. A feasible parameter set is such, that only economic reasonable values of the endogenous variables are generated, so that any variable is an element of  $\mathbf{R}_+$ . In addition, the parameter set must guarantee the

convergence to a steady state (growth) path. But, once again, these requirements have to be taken into account for any analytical analysis as well. Hence, we argue without further loss of generality under a given parameter constellation.

## 2.3 The Dynamics of the Economy

The stock of capital in the following period  $t+1$  ( $K_{t+1}$ ) is given by the sum of savings per household of the working generation in the current period  $t$  ( $s_t^{t-1} N_t$ ), hence

$$K_{t+1} = s_t^{t-1} N_t. \quad (15)$$

Production in  $t+1$  takes place by the then working generation renting capital from the retired generation by an interest rate  $r_{t+1}$ . The number of working households in  $t+1$  equals the number of children in period  $t$ , so that

$$N_{t+1} = n_t^{t-1} N_t. \quad (16)$$

If  $k_{t+1}$  represents the capital stock per worker, it follows

$$k_{t+1} = \frac{K_{t+1}}{N_{t+1}} = \frac{s_t^{t-1}}{n_t^{t-1}}. \quad (17)$$

Substituting for  $s_t^{t-1}$  and  $n_t^{t-1}$  in (17) by (7) and (6) or rather (14), leads to

$$k_{t+1} = \frac{z}{\gamma} \frac{1-\gamma}{1+\rho} w_t^{t-1}. \quad (18)$$

Evidently, capital per worker in the future period depends positively on the wage rate in the current period, because a higher wage rate increases savings and lowers fertility. Furthermore,  $k_{t+1}$  increases with the weight of future consumption in the utility function  $\left(\frac{1-\gamma}{1+\rho}\right)$  making savings more attractive. Increasing time costs of childrearing  $z$  or a decreasing weight of children  $\gamma$  lowers fertility and increases  $k_{t+1}$ . Obviously, the economy exhibits positive growth  $\left(\frac{k_{t+1}}{k_t} > 1\right)$  as long as the wage rate is growing. Because of diminishing marginal returns to all production factors capital accumulation leads to increasing wage rates, increasing labour supply and decreasing fertility, although with

declining rates. Hence, the steady state exhibits zero growth in per-worker terms with a constant fertility

$$\hat{k}_{t+1} = \left( \frac{k_{t+1}}{k_t} \right)^* = \left( \frac{w_t^{t-1}}{w_{t-1}^{t-2}} \right)^* = 1 \quad \text{and} \quad n_t^{t-1} = n^* \quad \forall \quad t.^1 \quad (19)$$

## 2.4 Simulation

As commented in section 2.2 equation (14) can only be solved numerically for given parameter values. The solution determines the labour supply and the wage rate in the considered period. The latter on its part governs via (18) the value of  $k_{t+1}$ . Hence, the simulation is programmed as a loop, which repeats these process until the system is sufficiently close to the steady state. <sup>2</sup>

**Parameter:**  $\alpha = 0.75; B = 10; z = 0.5; \rho = 0.5; \gamma = 0.6; V = 3$

**Initial value:**  $k_1 = 1$

$t$	$n_t$	$w_t$	$r_t$	$k_t$	$\tilde{k}_t$	$y_t$	$\tilde{y}_t$	$\hat{k}_{t+1}$
1	1.4052	6.2077	5.5385	1	3.3624	7.3847	24.8310	1.3795
2	1.3373	7.2871	5.2503	1.3795	4.1638	9.6571	29.1485	1.1738
3	1.3055	7.9342	5.1035	1.6193	4.6639	11.0193	31.7368	1.0887
4	1.2893	8.3114	5.0251	1.7631	4.9618	11.8135	33.2456	1.0475
..	.....	.....	.....	.....	.....	.....	.....	.....
37	1.2699	8.8118	4.9281	1.9581	5.3641	12.8670	35.2473	1
38	1.2699	8.8118	4.9281	1.9581	5.3641	12.8670	35.2473	1

Table 1: *Simulation*

The closed or numerical solutions, given the parameter values and an initial value for the first period  $k_1 > 0$ , are shown in Table 1. It confirms the above derived implications of the model. The wage rate is increasing with the capital stock per worker, leading to an increasing labour market participation and a lower fertility rate. The dynamics of the economy is plotted in Figure 1. Due to the properties of the production function the steady state is globally stable.

<sup>1</sup>  $x_t$  represent the change of  $x$  in period  $t$  compared to the previous period  $t-1$ . The star  $*$  indicates as usual steady state values.

<sup>2</sup> Sufficiently close means here and hence forth constant values from  $10^{-8}$  on.

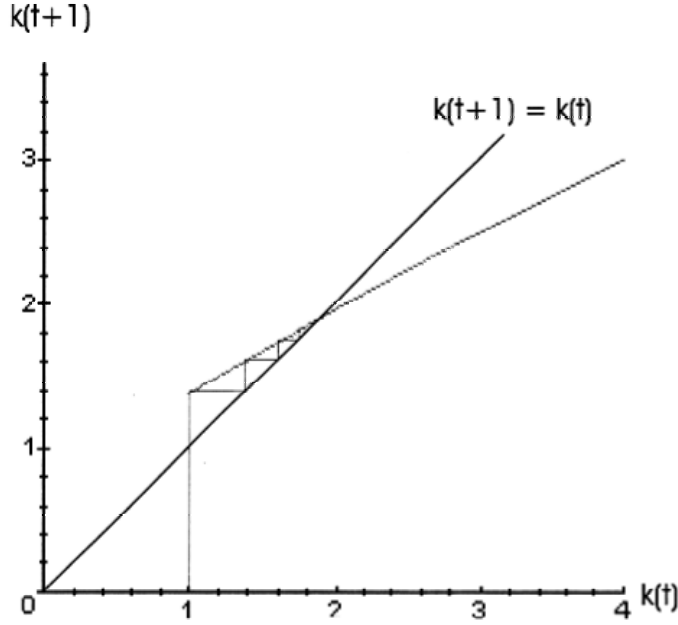


Figure 1: *Dynamics of  $k_{t+1}$  in Eq. (18)*

## 2.5 Long-run Growth with Exogenous Technological Progress

The previous section has shown that long-run growth ceases. The present section serves to implement exogenous technological progress in the model presented above. To be compatible with the properties of a steady state the technological progress has to be Harrod-neutral or rather labour augmenting. As a consequence, in the steady state the wage rate is increasing with the exogenously given rate of technological progress  $x$ . As stated in section 2.3 a permanent growth of the wage rate is the necessary condition for steady state growth. But, as was established in the previous section, a growing wage rate causes a decreasing fertility, thus increasing labour supply. Evidently, this situation is not compatible with the properties of an equilibrium, because reallocations are not closed. So, there seems to be an immanent contradiction between long-run growth caused by a permanent growth of the wage rate and the requirements of an equilibrium. Although King, Plosser and Rebello (1988) show, that this contradiction can be solved by applying utility functions of the following type

$$u(c, l) = \begin{cases} \frac{1}{1-\sigma} [c \cdot v(l)]^{1-\sigma} - \frac{1}{1-\sigma}, & \text{if } \sigma > 0 \text{ and } \sigma \neq 1 \\ \ln c + \ln v(l), & \text{if } \sigma = 1. \end{cases}$$

It is straight forward to show, that the steady state using this type of utility function exhibits the following properties:

- All variables in per worker terms grow with the rate of technological progress  $x$ .
- Labour supply  $l$  and fertility  $n$  are constant, such that  $l_t = l^*$  and  $n_t = n^*$
- Therefore capital per efficient unit of labour  $\tilde{k}_t^* = \frac{K_t}{(1+x)^t l^* N_t}$  is constant, too.

Obviously, the log-linear utility function applied in the present context (see Eq. (4)) corresponds to the special case with  $\sigma = 1$ .

The implementation of labour augmenting technological progress leads to the following expressions for the factor prices

$$r_t = \alpha B \tilde{k}^{\alpha-1}, \quad (20)$$

$$w_t^{t-1} = (1-\alpha) B \tilde{k}^\alpha (1+x)^t. \quad (21)$$

For the dynamics of the system Eq. (18) still holds, but it is governed by the permanent growing wage rate (21), so one can write again

$$k_{t+1} = \frac{z}{\gamma} \frac{1-\gamma}{1+\rho} w_t^{t-1},$$

where for the steady state must hold

$$\left( \frac{k_{t+1}}{k_t} \right)^* = \left( \frac{w_t^{t-1}}{w_{t-1}^{t-2}} \right)^* = (1+x) \quad \text{and} \quad n_t^{t-1} = n^* \quad \forall \quad t. \quad (22)$$

Due to spill-over effects from the production to the household sector it is assumed, that the non-labour income  $V$  is growing with the rate of technological progress, too. In order to explore the effect(s) of different rates of technological progress on fertility, two scenarios are examined,  $x = 0.1$  and  $x = 0.3$  (see Table 2 and 3).

**Parameter:**  $x = 0.1; \alpha = 0.75; B = 10; z = 0.5; \rho = 0.5; \gamma = 0.6; V = 3$

**Initial value:**  $k_1 = 1$

$t$	$n_t$	$w_t$	$r_t$	$k_t$	$\bar{k}_t$	$y_t$	$\tilde{y}_t$	$\hat{k}_{t+1}$	$\hat{w}_t$
1	1.4261	6.5303	5.6216	1	3.1681	7.4954	23.7467	1.4511	-
2	1.3662	8.2089	5.3770	1.4511	3.7851	10.4040	27.1369	1.2570	1.2570
3	1.3387	9.6664	5.2562	1.8242	4.1450	12.7847	29.0501	1.1775	1.1775
4	1.3249	11.0194	5.1941	2.1480	4.3471	14.8766	30.1057	1.1399	1.1399
..	.....	.....	.....	.....	.....	.....	.....	.....	.....
35	1.3098	220.3128	5.1240	44.5076	4.5899	304.0763	31.3585	1.1	1.1
36	1.3098	242.3441	5.1240	48.9584	4.5899	334.4840	31.3585	1.1	1.1

Table 2: *Simulation with  $x = 0.1$*

**Parameter:**  $x = 0.3; \alpha = 0.75; B = 10; z = 0.5; \rho = 0.5; \gamma = 0.6; V = 3$

**Initial value:**

$$k_1 = 1$$

$t$	$n_t$	$w_t$	$r_t$	$k_t$	$\bar{k}_t$	$y_t$	$\tilde{y}_t$	$\hat{k}_{t+1}$	$\hat{w}_t$
1	1.4633	7.1604	5.7638	1	2.8668	7.6850	22.0320	1.5912	-
2	1.4183	10.1975	5.5911	1.5912	3.2376	11.8622	24.1363	1.4241	1.4241
3	1.3984	13.8418	5.5112	2.2661	3.4294	16.6524	25.1012	1.3573	1.3573
4	1.3891	18.3736	5.4731	3.0759	3.5262	22.4467	25.7325	1.3274	1.3274
..	.....	.....	.....	.....	.....	.....	.....	.....	.....
32	1.380	29059.2557	5.4367	4967.3941	3.6213	36008.8749	26.2516	1.3	1.3
33	1.380	37777.0324	5.4367	6457.6123	3.6213	46811.5374	26.2516	1.3	1.3

Table 3: *Simulation with  $x = 0.3$*

Once  $n_t$  has converged to its steady state value  $n^*$ , capital per worker  $k$  is growing with the growth rate of the wage rate, which in turn is growing with the exogenously given rate of technological progress  $x$ . The technological progress accelerates the convergence speed towards the steady state and leads to a higher fertility in all periods. The latter result is quite implausible. Although there are signs indicating a correlation between population growth and technological progress in the long-run (Kremer (1993), Newton-effect see Jones (1998) or modern growth theory in the line of Grossman and Helpman (1995)), the causality applied here is a different. The exogenous technological progress is like an income effect, which weakens the substitution effect caused by a growing wage rate. The economy receives the technological progress costlessly and the increasing wage rate still raises the opportunity costs of children, but to a lower extent.

In the present context the use of exogenous technological progress seems to be dubious, especially if one takes into consideration that technological progress is strongly correlated with investments in human capital. Since the latter are negatively correlated with fertility the use of exogenous technological progress does not explain the relationship between fertility, human capital investments and the long-run growth performance of an economy. In the following section the growth process is endogenized by parental investments in education for their children.

### 3 Endogenous Growth with Endogenous Fertility and Endogenous Education

Agents are again assumed to be egoistic in that respect sense, that there is no bequest motive. Consequently, their wealth is consumed until the end of their third period of life. But they are assumed to be altruistic as far as investments in education for their offspring are concerned. Education is an accumulateable asset and all past investments in education are cumulated in the stock of human capital  $E$ , where  $E$  is used in the production sector and in the education sector. Since education improves the efficiency of agents in the use of production factors producing  $Y$ , the effect of fertility decisions on the economy is different to the previous sections. On the one hand fertility is similar to consumption and lowers, given the income, savings and hence capital per worker (capital dilution), on the other hand fertility has an investment character through education, which enhances efficiency in production, hence weakening the capital dilution effect.

#### 3.1 The Households

The costs of fertility are still priced by its opportunity costs  $zn_t w_t$ , but different to the previous setup education costs have to be taken into consideration. If  $E_t$  represents the aggregated amount of education undertaken by all households,  $\frac{E_t}{N_t} = e_t n_t$  is the education effort undertaken by one household. If further it is assumed, that one unit of  $e_t$  costs  $p_t$  units of the consumption good and the representative household chooses to have  $n_t$  children he faces in total costs of  $p_t e_t n_t$  for educating his offspring. Evidently, because of relative small opportunity costs a relative low wage rate leads to a relative high fertility, which in turn causes relative small expenditures for education per child and vice versa. In period  $t$ , the optimization problem of a representative household

born in period  $t - 1$  is as follows



$$(23) \quad \max_{\{n_t^{t-1}, e_t^{t-1}, c_t^{t-1}, c_{t+1}^{t-1}\}} u_t^{t-1} = \gamma \ln n_t^{t-1} + \beta \ln e_t^{t-1} + (1 - \beta - \gamma) \left[ \ln c_t^{t-1} + \frac{1}{1 + \rho} \ln c_{t+1}^{t-1} \right],$$

subject to

$$w_t^{t-1} + V = z n_t^{t-1} w_t^{t-1} + p_t e_t^{t-1} n_t^{t-1} + c_t^{t-1} + s_t^{t-1}, \quad (24)$$

$$l_t^{t-1} + z n_t^{t-1} = 1 \quad (25)$$

and it has to be taken into account, that

$$s_t^{t-1} = \frac{c_{t+1}^{t-1}}{1 + r_{t+1}} \quad \text{or} \quad c_{t+1}^{t-1} = (1 + r_{t+1}) s_t^{t-1}. \quad (26)$$

Solving the problem leads to optimal decisions

$$e_t^{t-1} = \frac{z w_t^{t-1}}{p_t} \frac{\beta}{\gamma - \beta}, \quad (27)$$

$$n_t^{t-1} = \frac{(\gamma - \beta)(1 + \rho)}{(2 + \rho)(1 - \beta) - \gamma} \frac{w_t^{t-1} + V}{z w_t^{t-1}}, \quad (28)$$

$$c_t^{t-1} = \frac{(1 - \beta - \gamma)(1 + \rho)}{(2 + \rho)(1 - \beta) - \gamma} \left[ w_t^{t-1} + V \right], \quad (29)$$

$$s_t^{t-1} = \frac{c_{t+1}^{t-1}}{1 + r_{t+1}} = \frac{(1 - \beta - \gamma)}{(2 + \rho)(1 - \beta) - \gamma} \left[ w_t^{t-1} + V \right], \quad (30)$$

with  $\gamma > \beta$ .

Interesting the relation between the education decisions of parents for their offspring (27) and their fertility decisions (28). Higher time costs of childrearing  $z$  reduce fertility  $n_t$  and increase education investments per child  $e_t$ . The effect of an increase of the wage rate works in the same direction. Remarkably, education efforts per child depend only on their wage rate  $w_t^{t-1}$  and not on their non-labour income  $V$  and therefore implicitly on the education efforts undertaken by their parents and so on. Hence, education efforts being the only altruistic element in the presented framework lead by its interdependency with the wage rate and fertility decisions to an intergenerational persistence in education levels, fertility levels and the wage rate.

From the dependency of the education decisions on the wage rate follow important implications for redistributive policies. An increase of income transfers, hence an increase of  $V$ , results in an income effect raising fertility, without providing any incentive to substitute fertility against education.

### 3.2 The Production Sector

As commented in the introduction in this section, education is assumed to enhance the efficiency of the production factors labour and capital, because the individual level of education determines the individual abilities in the use of the existing technology. Although it has to be taken into account that the educational level of a working agent in period  $t$  was a control variable of his/her parents in the previous period  $t - 1$ .

The offered labour quantum  $L_t = l_t^{t-1} N_t$  has to be distributed between production and education, hence the fraction of labour employed in the production sector is  $\phi_t l_t^{t-1} N_t$ , with  $0 < \phi_t < 1$ . Consequently, the production function is

$$Y_t = e_{t-1}^{t-2} B K_t^\alpha \left( \phi_t l_t^{t-1} N_t \right)^{1-\alpha}, \quad (31)$$

or in per-workers terms

$$y_t = e_{t-1}^{t-2} B k_t^\alpha \left( \phi_t l_t^{t-1} \right)^{1-\alpha}. \quad (32)$$

### 3.3 The Education Sector

Since education is compared to production less capital intensive, it is assumed that the education sector produces without capital. Education of the young is undertaken with the fraction of labour allocated to this sector  $(1 - \phi_t) l_t^{t-1} N_t$  and the educational level of the adult  $E_{t-1} = e_{t-1}^{t-2} N_t$ . Furthermore, productivity is negatively affected by the average number of children per household, representing a negative quality effect on the production side of education.

$$E_t = \left( 1 + \frac{A}{n_t^{t-1}} (1 - \phi_t^{t-1}) l_t^{t-1} \right) E_{t-1}, \quad \text{with } A > 0, \quad (33)$$

where  $A$  is a productivity parameter. The young generation inherits the existing aggregated level of human capital  $E_{t-1}$  from the adult generation. Because of the altruistic behaviour of the latter concerning the educational level per child, a fraction of labour is allocated to the education sector, leading to a higher stock of human capital in the

aggregate.<sup>3</sup> But more important for further investigations is the development of the educational level per child  $e_t^{t-1} = \frac{E_t}{N_{t+1}} = \frac{E_t}{n_t^{t-1} N_t}$

$$e_t^{t-1} = \left(1 + \frac{A}{n_t^{t-1}}(1 - \phi_t)l_t^{t-1}\right) \frac{E_{t-1}}{N_{t+1}} \quad (34)$$

$$e_t^{t-1} = \left(1 + \frac{A}{n_t^{t-1}}(1 - \phi_t)l_t^{t-1}\right) \frac{e_{t-1}^{t-2} N_t}{n_t N_t}$$

$$e_t^{t-1} = \left(1 + \frac{A}{n_t^{t-1}}(1 - \phi_t)l_t^{t-1}\right) \frac{e_{t-1}^{t-2}}{n_t^{t-1}} \quad (35)$$

Obviously, on the production side the educational level per child  $e_t^{t-1}$  depends positively on the educational level of its parents  $e_{t-1}^{t-2}$ , on the amount of labour allocated to this sector and also on the productivity parameter  $A$ . On the other hand,  $e_t^{t-1}$  depends negatively on the current fertility level  $n_t^{t-1}$ , because the existing inherited stock of human capital and the labour employed in the education sector must be spread over more children.

### 3.4 General Equilibrium Structure and the Dynamics

In a general equilibrium labour supply and factor distribution between sectors are endogenously determined by the equilibrium goods and factor prices. Hence, the production structure and the composition of the output bundle in the economy are completely described. At first an equilibrium requires, that the value of the marginal products of labour are equalized between the sectors, so that

$$w_t^{t-1} = (1 - \alpha) e_{t-1}^{t-2} k_t^a (\phi_t l_t^{t-1})^{-\alpha} = \frac{p_t A e_{t-1}^{t-2}}{n_t^{t-1}}. \quad (36)$$

Using (36), (35) and (27) the fraction of labour employed in the production sector  $u_t$  can be expressed as a function of labour supply  $l_t^{t-1}$  or rather of fertility  $n_t^{t-1}$

$$\phi_t = 1 - \frac{n_t^{t-1}}{l_t^{t-1}} \left[ \frac{z\beta}{\gamma - \beta} - \frac{1}{A} \right]. \quad (37)$$

---

<sup>3</sup>This behaviour is altruistic, because the amount of labour shifted from production to education leads to a decreasing output  $y_t$  and decreasing consumption  $c_t$  in the current period. Only the next generation can gain from higher human capital stock.

Due to different properties of the production functions in the two sectors, especially decreasing marginal returns to labour in production and constant returns in education, a reduction of labour supply e.g. caused by a high fertility leads other things equal to a lower fraction of labour allocated to production.

Substituting  $\phi_t$  in (36) by (37) yields with (28)

$$n_t = \frac{(\gamma - \beta)(1 + \rho)}{(2 + \rho)(1 - \beta) - \gamma} \frac{(1 - \alpha)e_{t-1}^{t-2} k_t^\alpha \left( l_t^{t-1} - \frac{n_t^{t-1}}{\gamma - \beta} + \frac{n_t^{t-1}}{A} \right)^{-\alpha} + V}{z(1 - \alpha)e_{t-1}^{t-2} k_t^\alpha \left( l_t^{t-1} - \frac{n_t^{t-1} z \beta}{\gamma - \beta} + \frac{n_t^{t-1}}{A} \right)^{-\alpha}}. \quad (38)$$

Similar to Eq. (14) the numerical solution of Eq. (38) determines the number of children in a general equilibrium and the static equilibrium values of all endogenous variables are obtainable. The dynamics of the system is still governed by  $k_{t+1} = \frac{s_t^{t-1}}{n_t^{t-1}}$ .

Hence the economy develops according to

$$k_{t+1} = \left[ \frac{1 - \beta - \gamma}{(\gamma - \beta)(1 + \rho)} \right] z w_t^{t-1}. \quad (39)$$

During the transition process towards the steady state, capital per worker and the wage rate are growing at the same rate, which equals the growth rate of the value of education divided by the change of the fertility rate

$$\left( \frac{k_{t+1}}{k_t} \right) = \left( \frac{w_t^{t-1}}{w_{t-1}^{t-2}} \right) = \frac{\left( \frac{p_t}{p_{t-1}} \right) \left( \frac{e_{t-1}^{t-2}}{e_{t-2}^{t-3}} \right)}{\left( \frac{n_t^{t-1}}{n_{t-1}^{t-2}} \right)} \quad (40)$$

or

$$\hat{k}_{t+1} = \hat{w}_t = \frac{\hat{p}_t \hat{e}_{t-1}}{\hat{n}_t}.$$

The economy has reached its steady, if, contrary to the transition, all reallocations in the time budget of the households and of labour between the sectors are closed compared to an arbitrarily chosen period  $t - 1$ . Wage rate and capital per worker grow not only at an equal rate, but rather at an equal and constant rate.

Thus, the following expression has to hold in the steady state

$$\left( \frac{k_{t+1}}{k_t} \right)^* = \left( \frac{w_t^{t-1}}{w_{t-1}^{t-2}} \right)^* = \left( \frac{p_t}{p_{t-1}} \right)^* \left( \frac{e_{t-1}^{t-2}}{e_{t-2}^{t-3}} \right)^*$$

or

$$\hat{k}_{t+1}^* = \hat{w}_t^* = \hat{p}_t^* \hat{e}_{t-1}^* \quad (41)$$

and

$$n_t^{t-1} = n^*, l_t^{t-1} = l^*, \phi_t = \phi^*, r_t = r^* \quad \forall \quad t.$$

According to Eq. (40) the relation between the variation of the value of education to the variation of fertility  $\frac{\hat{p}\hat{e}}{\hat{n}}$  is responsible for the convergence speed towards the steady state.

The simulation shows (see Table 4 ) like one might expect, that the transition process is characterized by an increasing wage rate, a decreasing fertility and raising investments in education. With a higher level of education per capita individuals are more efficient in production. Accompanied by a lower fertility, productivity in the education sector is enhanced, which allows for an increase of the fraction of labour employed in the production sector ( $\phi_t$  increases). Contrary, in low stages of development ( $k_t$  is relative small) an equalization of the value of marginal products of labour can only be reached by a high employment of labour in education ( $\phi_t$  is small). The latter lays the foundations for the wealth of future generations enabled by the altruistic behaviour of the parental generation.

**Parameter Values:**  $\alpha = 0.3; A = 8; B = 8; z = 0.35; \rho = 0.5; \gamma = 0.6; \beta = 0.3; V = 8$

**Initial Values:**  $k_1 = 0.1; e_0 = 0.8$

$t$	$n_t$	$w_t$	$r_t$	$\phi_t$	$k_t$	$e_{t-1}$	$p_t$	$y_t$
1	1.7319	11.6547	0.2062	0.0104	0.1	0.8	3.1539	0.6875
2	1.6882	20.2864	0.2806	0.0715	0.9064	1.2933	3.3100	0.8481
3	1.6629	35.2059	0.4187	0.1047	1.5778	2.1450	3.4117	2.2026
4	1.6310	62.9662	0.6125	0.1448	2.7382	3.6117	3.5544	5.5906
..	.....	.....	.....	.....	.....	.....	.....	.....
62	1.1180	$0.6490 \cdot 10^{33}$	7.3046	0.5867	$0.1359 \cdot 10^{32}$	$0.3889 \cdot 10^{23}$	$0.2332 \cdot 10^{10}$	$0.3311 \cdot 10^{33}$
63	1.1180	$0.2409 \cdot 10^{34}$	7.3046	0.5867	$0.5047 \cdot 10^{32}$	$0.9740 \cdot 10^{23}$	$0.3456 \cdot 10^{10}$	$0.1229 \cdot 10^{34}$

$t$	$\tilde{y}_t$	$\tilde{k}_t$	$\hat{k}_{t+1}$	$\hat{e}_t$	$\hat{p}_t$	$\hat{w}_t$
1	16.6496	24.2150	9.0648	1.6166	-	-
2	28.9805	30.9751	1.7406	1.6585	1.0494	1.7406
3	50.2942	36.0274	1.7354	1.6837	1.0307	1.7354
4	89.9517	44.0577	1.7885	1.7166	1.0418	1.7885
..	....	....	.....	.....	....	.....
62	$0.9271 \cdot 10^{33}$	$0.3807 \cdot 10^{32}$	3.7118	2.5044	1.4820	3.7118
63	$0.3441 \cdot 10^{34}$	$0.1413 \cdot 10^{33}$	3.7118	2.5044	1.4820	3.7118

Table 4: *Endogenous Education*

### 3.5 Parameter Analysis

This section serves to explore the transitory behaviour of all endogenous variables and the change of the steady state solutions, due to partial variations of the parameters, compared to the reference scenario in Table 4. Table 5 shows in which direction the endogenous variables are affected during the first four periods of the transition process to the steady state by a respective change of a parameter. In Table 6 the change of the steady state solutions and the convergence time are shown.

As described in the previous section, the convergence time is governed by the relation between  $\hat{p}_t \hat{e}_{t-1}$  and  $\hat{n}_t$  (see Eq. (40)). Consequently all parameter variations, which favour education per child and lower fertility accelerate the transition process to the steady state. E.g. a higher preference for fertility ( $\gamma = 0.62$ ) leads to a lower labour supply and a lower productivity  $\frac{A}{n_t}$  in the education sector. Under equal initial conditions productivity in production is not affected.

Due to the requirement of factor price equalization between the sectors (see Eq. (36)) a lower fraction of labour, but of a lower labour supply, is allocated to production ( $\phi_t$  decreases) accompanied by raising education costs ( $p_t$  increases). In the first period the wage rate is higher than in the reference scenario, but with a higher preference

Changed Parameter	$t$	$n_t$	$w_t$	$r_t$	$\phi_t$	$k_t$	$e_{t-1}$	$p_t$	$y_t$	$\tilde{y}_t$	$\tilde{k}_t$	$\hat{k}_{t+1}$	$\hat{e}_t$	$\hat{p}_t$	$\hat{w}_t$
$V = 6$	1	-	-	+	+	=	=	-	+	-	-	+	+		
	2	-	-	+	+	-	+	-	+	-	-	+	+	+	+
	3	-	-	+	+	-	+	-	+	-	-	+	+	+	+
	4	-	-	+	+	-	+	-	+	-	-	+	+	+	+
$V = 15$	1	+	+	-	-	=	=	+	-	+	+	+	-		
	2	+	+	-	-	+	-	+	-	+	+	-	-	-	-
	3	+	+	-	-	+	-	+	-	+	+	-	-	-	-
	4	+	+	-	-	+	-	+	-	+	+	-	-	-	-
$A = 6$	1	+	-	+	+	=	=	+	+	-	-	-	-		
	2	+	-	+	+	-	-	+	-	-	-	-	-	+	-
	3	+	-	+	+	-	-	+	-	-	-	-	-	-	-
	4	+	-	+	+	-	-	+	-	-	-	-	-	-	-
$A = 10$	1	-	+	-	-	=	=	-	+	+	+	+	+		
	2	-	+	+	-	+	+	-	+	+	+	+	+	-	+
	3	-	+	+	-	+	+	-	+	+	+	+	+	+	+
	4	-	+	+	-	+	+	-	+	+	+	+	+	+	+
$B = 6$	1	+	-	-	-	=	=	-	-	-	+	-	-		
	2	+	-	-	-	-	-	-	-	-	+	-	-	-	+
	3	+	-	-	-	-	-	-	-	-	+	-	-	-	+
	4	+	-	-	-	-	-	-	-	-	+	-	-	-	+
$B = 10$	1	-	+	+	+	=	=	+	+	+	-	+	+		
	2	-	+	+	+	+	+	+	+	+	-	+	+	+	+
	3	-	+	+	+	+	+	+	+	+	-	+	+	+	+
	4	-	+	+	+	+	+	+	+	+	-	+	+	+	+

Table 5: Variation of the transitory solutions compared to the reference scenario in Table 4 (continuation next page).

for fertility and a lower productivity in education the educational level of the young is lower. In the following period, the latter will affect productivity in education and production and aggravate the substitution of education against fertility caused by a now lower wage rate. Consequently, the transition process is prolonged. The steady state is characterized by a higher fertility and a lower growth rate  $\hat{k}_{t+1}^* = \hat{p}_t^* \hat{e}_t^*$ .

Table 6 reveals that there are three groups of parameter variations. The first group, changes in the households' parameters ( $\gamma, \beta, z$  and  $\rho$ ), alters the transitory solutions, hence the convergence speed and the steady state solutions, most important the long-run growth rate of the economy. The second group, changes in the productivity parameter in production  $B$  and the non-labour income  $V$ , only affects the transitory solutions and the convergence speed, but do not have any impact on the long-run performance of the economy. This is, because variations in  $B$  and  $V$  only affect the household income, given the preferences. Consequently, a higher productivity in production raises the wage income, leading to higher investments in education for each child and due to

Changed Parameter	$t$	$n_t$	$w_t$	$r_t$	$\phi_t$	$k_t$	$e_{t-1}$	$p_t$	$y_t$	$\tilde{y}_t$	$\tilde{k}_t$	$\hat{k}_{t+1}$	$\hat{e}_t$	$\hat{p}_t$	$\hat{w}_t$
$\gamma = 0.58$	1	—	—	+	+	=	=	—	+	—	—	+	+		
	2	—	+	+	+	+	+	—	+	+	—	+	+	+	+
	3	—	+	+	+	+	+	—	+	+	—	+	+	+	+
	4	—	+	+	+	+	+	—	+	+	—	+	+	+	+
$\gamma = 0.62$	1	+	+	—	—	=	=	+	—	+	+	—	—		
	2	+	—	—	—	—	—	+	—	—	+	—	—	—	—
	3	+	—	—	—	—	—	+	—	—	+	—	—	—	—
	4	+	—	—	—	—	—	+	—	—	+	—	—	—	—
$\beta = 0.28$	1	+	—	+	+	=	=	—	+	—	—	—	—		
	2	+	—	+	+	—	—	—	+	—	—	—	—	+	—
	3	+	—	—	+	—	—	—	—	—	—	—	—	—	—
	4	+	—	—	+	—	—	—	—	—	—	—	—	—	—
$\beta = 0.32$	1	—	+	—	—	=	=	+	—	+	+	+	+		
	2	—	+	—	—	+	+	+	—	+	+	+	+	—	+
	3	—	+	+	—	+	+	+	+	+	+	+	+	+	+
	4	—	+	+	—	+	+	+	+	+	+	+	+	+	+
$\rho = 0.4$	1	—	—	+	+	=	=	—	+	—	—	+	+		
	2	—	—	+	+	+	+	—	+	—	—	+	+	+	+
	3	—	—	+	+	+	+	—	+	—	—	+	+	+	+
	4	—	+	+	+	+	+	—	+	—	—	+	+	+	+
$\rho = 0.6$	1	+	+	—	—	=	=	+	—	+	+	—	—		
	2	+	+	—	—	—	—	+	—	+	+	—	—	—	—
	3	+	+	—	—	—	—	+	—	+	+	—	—	—	—
	4	+	—	—	—	—	—	+	—	—	+	—	—	—	—
$z = 0.33$	1	+	—	+	+	=	=	+	+	—	—	—	—		
	2	+	—	—	+	—	—	+	—	—	—	—	—	—	—
	3	+	—	—	—	—	—	+	—	—	+	—	—	—	—
	4	+	—	—	—	—	—	+	—	—	—	—	—	—	—
$z = 0.37$	1	—	+	—	—	=	=	—	—	+	+	+	+		
	2	—	+	+	—	+	+	—	+	+	+	+	+	+	+
	3	—	+	+	+	+	+	—	+	+	+	+	+	+	+
	4	—	+	+	+	+	+	—	+	+	+	+	+	+	+

Table 5: Variation of the transitory solutions compared to the reference scenario in Table 4 (continuation from the previous page).



Changed Parameter	$n^*$	$\phi^*$	$r^*$	$\hat{k}_{t+1}^*$	$\hat{e}_t^*$	$\hat{p}_t^*$	$t^*$
$V = 6$	=	=	=	=	=	=	61
$V = 15$	=	=	=	=	=	=	66
$A = 6$	=	+	-	-	-	-	96
$A = 10$	=	-	+	+	+	+	50
$B = 6$	=	=	=	=	=	=	66
$B = 10$	=	=	=	=	=	=	59
$\gamma = 0.58$	-	+	+	+	+	+	54
$\gamma = 0.62$	+	-	-	-	-	-	76
$\beta = 0.28$	+	+	-	-	-	-	78
$\beta = 0.32$	-	-	+	+	+	+	53
$z = 0.33$	+	+	-	-	-	-	72
$z = 0.37$	-	-	+	+	+	+	56
$\rho = 0.4$	-	+	-	+	+	+	61
$\rho = 0.6$	+	-	+	-	-	-	64

Table 6: Variation of the steady state solutions compared to the reference scenario in Table 4.

higher opportunity costs to a lower fertility. Convergence speed is accelerated and for given preference parameters the steady state properties remain unaffected. Interesting, a higher non-labour income  $V$  leads to a higher transitory fertility and lower transitory investments in education per child, because the substitution effect caused by the raising wage rate during the transition is weakened. Again, the long-run properties of the system remain unaffected. Finally, changes in the productivity parameter of the education sector  $A$  do not have any impact on the steady state fertility  $n^*$ , because of unchanged preference parameters, but on factor distribution and long-run growth rate. A higher productivity in education raises the wage income and opportunity costs of childrearing. Agents substitute fertility against education during the transition, which accelerates the convergence speed. Secondly, in the steady state, education remains to be more productive compared to production. The wage rate arbitrage condition (36) requires then, that more labour is allocated to education. The long-run growth rate  $\hat{k}_{t+1}^* = \hat{p}_t^* \hat{e}_t^*$  (see Eq. (41)) depends on the accumulation rate of human capital, which is higher, because  $\frac{A}{n}$  has increased. Consequently, policies, which improve productivity in education do not harm long-run fertility, but lead to a faster decline of fertility during the transition. Further more long-run growth is improved.

## 4 Heterogeneous Agents

The analysis in the previous section has revealed that the wage rate plays a crucial role for the dynamics of the economy. Therefore income heterogeneity seems to be of significant importance both for the transition process and for the long-run growth performance of the economy. In the present context two sources of heterogeneity are considered. Firstly, distinct intellectual ability to use the acquired level of education in production and secondly, social discrimination due to different productivity levels in the education sector.

### 4.1 Intellectual Heterogeneity

In the line of Loury (1981) and Benabou (1996) intellectual heterogeneity is modelled through an identical and independent distributed shock  $\varepsilon^i$  with a normalized mean, so that

$$E[\varepsilon] = 1. \quad (42)$$

Since we make a distinction between intellectual and social heterogeneity,  $\varepsilon^i$  does not represent the general access to human capital, but rather the ability to use the acquired human capital in production (e.g. other things equal, a more intelligent agent generates a higher degree of effectiveness with a high school degree in production than his classmates). Hence, the following expression has to hold for an arbitrarily chosen agent  $i$ , born in  $t-1$

$$y_t^i = \varepsilon_{t-1}^i e_{t-1}^{t-2,i} B k_t^{i\alpha} (\phi_t^i l_t^{t-1,i})^{1-\alpha}. \quad (43)$$

If two individuals with distinct abilities are considered, e.g.  $\varepsilon^i < \varepsilon^j$ <sup>4</sup> and if further it is assumed, that these abilities are of genetic nature and therefore hereditary, dynasty of type  $i$  is during the transition process characterized by a lower wage rate, a higher fertility, lower education efforts and lower savings, compared to dynasty  $j$ . Because of Eq. (40) it must be true, that

$$\hat{k}_{t+1}^i = \hat{w}_t^i < \hat{k}_{t+1}^j = \hat{w}_t^j. \quad (44)$$

Therefore dynasty  $j$  converges with a higher speed to the steady state equilibrium, accompanied by a lower fertility and higher education efforts. If all dynasties have reached the steady state growth path, fertility is constant and equal for all dynasties and all variables grow at equal and constant rates (see Eq. (41))

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<sup>4</sup>Setting  $\varepsilon^i = 0.75$  and  $\varepsilon^j = 1.25$  leads to an equivalent scenario shown in Table 5 and 6 with  $B = 8$  and  $B = 10$ . Therefore a particular simulation for this scenario is omitted here.

$$\hat{k}_{t+1}^i = \hat{w}_t^i = \hat{k}_{t+1}^j = \hat{w}_t^j, \quad \text{with:} \quad n^{i*} = n^{j*} = n^*. \quad (45)$$

Although intellectual heterogeneity has no long-run impact on growth rates and fertility, two points have to be taken into consideration. Firstly, due to different convergence speeds, differences in the income levels between dynasties are maintained and secondly, owing to a higher fertility of the lower income dynasties redistributive pressure and power has increased during the transition process and is maintained in the steady state. Consequently, the median voter will opt for income transfers lowering labour income in the upper income percentiles and raising non-labour income in the lower income percentiles. Thus, fertility raises in all income percentiles, but investments in education declines. The former is true, because of lower opportunity costs of child-rearing in the upper percentiles and higher household incomes in the lower ones (see Eq. (28)). Due to lower opportunity costs the taxed percentiles substitute education against fertility, whereas the lower percentiles are characterized by constant opportunity costs, but a higher non-labour income, which raises fertility accompanied by equal expenditures in education per child (see Eq. (27)). As a consequence policies like family allowances raise fertility, but are harmful for growth, since no incentive is provided to expand education per child (e.g. family allowances accompanied by education policies improving productivity in education).

## 4.2 Social Heterogeneity

The second source of heterogeneity in the economy is social discrimination in the education sector. To simplify matters, the society is assumed to be stratified into three classes (lower, middle and upper class) and discrimination works through the productivity parameter of the education sector  $A$  (e.g. due to different expenditures of the local community for school equipment), so that

$$A^{low} < A^{mid} < A^{up}. \quad (46)$$

Taking into account the simulation results in Table 5 and 6 (setting  $A^{low} = 6$ ,  $A^{mid} = 8$  and  $A^{up} = 10$ ), social heterogeneity does not only affect the transitory behaviour, but rather the long-run performance of the economy as well. Each of these three social classes converges to a different steady state, whereas, due to a higher fertility the

overall growth rate of the economy is limited by  $A^{low}$ .

In a regime with coexistence of intellectual and social heterogeneity the redistributive pressure with its growth harming effects is aggravated. The policy implications are the same. Income transfers from rich to poor would speed up fertility, but at the cost of a lower growth rate. Therefore the necessity of educational policy interventions has increased. This would eventually harm per capita growth in the upper classes, but enhance the overall long-run growth rate of the economy.

## 5 Concluding Remarks

In the present paper, we have shown that education of a child depends positively on the wage rate of its parents, which in turn depends positively on the wage rate and educational attainments of their parents and so on, leading to an intergenerational persistence in fertility decisions, educational efforts and wage rates. Contrary, a low wage rate leads to a high fertility and low education through the generations.

The transitional dynamics is characterized by an equal increase in capital intensity and wage rate, which depends positively on the relation between the variation of education and fertility. In the steady state wage rate and capital intensity are growing at an equal and constant rate, whereas fertility, labour supply and factor distribution between production and education are constant.

Because of the falling fertility rate during the transition process education and production becomes more productive, resulting in a higher allocation of labour to production, compared to earlier stages a development. Hence, a relative low allocation of labour to production in earlier stage of development is the source of the wealth for future generations.

Intellectual heterogeneity has no long-run impact as long as there is no income transfer system implemented, but due to higher fertility rates in lower income percentiles, during the transition redistributive pressure increases.

Social discrimination in the education sector however harm transitory and long-run growth. Income transfers speed up fertility and lower education efforts, resulting in a lower growth rate. Therefore in both scenarios (as well as in a regime with coexistence of intellectual and social heterogeneity) family allowances should be accompanied by educational policies.

Moreover, heterogeneity offers the possibility to endogenize the median voter system within a dynamical setting.

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## List of Notation

$Y_t$ :	aggregate output in period $t$
$A$ :	productivity parameter in the education sector
$B$ :	productivity parameter in the production sector
$N_t$ :	number of adult households in period $t$
$L_t$ :	aggregate labour supply in period $t$
$K_t$ :	aggregate stock of physical capital in period $t$
$E_t$ :	aggregate stock of human capital in period $t$
$y_t$ :	output per adult household in period $t$
$k_t$ :	capital intensity in period $t$
$e_t$ :	education per child in period $t$
$p_t$ :	price per unit education in units of the consumption good
$w_t$ :	wage rate in period $t$
$x_t$ :	exogenous rate of (labour-augmenting) technological progress
$V$ :	non-labour income
$r_t$ :	interest rate in period $t$
$n_t$ :	fertility (number of children) per household in period $t$
$l_t$ :	labour supply per adult household in period $t$
$\phi_t$ :	fraction of labour allocated to production
$z$ :	time costs per child
$c_t$ :	consumption per adult household in period $t$
$s_t$ :	savings per adult household in period $t$
$u_t$ :	utility per adult household in period $t$
$\alpha$ :	income share of physical capital
$\beta$ :	weight of education per child in the utility function
$\varepsilon$ :	ability shock
$\gamma$ :	weight of the offspring in the utility function