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Unemployment, Growth, and Complementarities between Innovation and Knowledge Diffusion

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Working Paper No. 32

**Unemployment, Growth, and Complementarities between
Innovation and Knowledge Diffusion**

von

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Unemployment, Growth, and Complementarities between Innovation and Knowledge Diffusion

Pascal Hetze*

Abstract

This paper analyzes the relationship between endogenous growth and unemployment. It provides knowledge diffusion as the link between innovation-based growth through creative destruction and the labor market outcome. Three dimensions of knowledge are considered: human capital (general skills), know-how gained through learning-by-using, and codified knowledge accumulated by research activities. Output growth is driven by innovations. However, the implementation of technological progress into a vintage-type production process requires the know-how that is only acquirable by the diffusion of knowledge through learning-by-using. A mutual feedback between research and the employment level thus arises, based on the complementary relationship between the input of labor in R&D and manufacturing. Inadequate knowledge diffusion causes limited growth and mismatch unemployment.

JEL-classification: E24, J63, O33

Keywords: mismatch unemployment, innovation-based growth, knowledge diffusion

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1. Introduction

Two key measures of the performance of an economy are output growth and the development of unemployment. In economic theory they are usually treated in different strings of the literature. On the one hand, the business cycle literature¹ – be it Walrasian or Keynesian – analyzes the labor-input and goods-output relationship, but ignores lasting non-erratic effects, as the role which technological progress plays for growth and employment. Instead, growth and variations in the employment level are typically regarded as autonomous in the long run. On the other hand, despite the fact that unemployment is a significant problem in most industrialized countries, growth theory usually assumes a cleared labor market. This view demands that growth and unemployment are non-related, which can only be true if unemployment is transitory. However, the data² show that unemployment is persistent, suggesting a possible link between labor market outcome and growth. Analyzing this link should permit the identification of some of the sources of the so-called jobless growth phenomenon, when positive growth rates are accompanied by a persistently high unemployment rate. The aim of the present paper is to present a model, which centers around the idea that knowledge, is the key for a successful economy and hence analyzes the dynamics of knowledge supply and demand. As will be shown, this dynamics implies an interaction between employment and innovation based growth.

There are a few approaches in growth theory, which include the analysis of unemployment. An early one is the Post-Keynesian approach of Harrod (1939) and Domar (1946). Driven by a Leontief technology, the economy in their model faces increased unemployment in the course of growth if capital accumulation is insufficient. The subsequent neo-classical growth models ignored this possibility by assuming away all imperfections in any market. For the most part, new growth theory has taken over the assumption of a cleared labor market. However, Aghion and Howitt (1994) offer a remarkable exception, analyzing the effects of innovation on matching unemployment. They argue that an accelerating rate of innovation raises the job-turnover rate whenever technological progress increases the number of market entries and exits simultaneously within a period. If the search for a new job is time-consuming, a higher rate of job-turnover produces more unemployment.

¹ Kydland (1995) provides among others an overview of the real business cycle theory.

² OECD (1992) and Layard, Nickel and Jackman (1991) emphasize that in a lot of countries the efficiency of the job-turnover has weakened inducing higher unemployment in the course of time.

In the new growth theory, the relationship between labor and knowledge rather than capital has become the focus of the analysis, because capital accumulation cannot by itself sustain growth because of its diminishing rate of return. The interaction between labor and knowledge is considered in two ways in the literature³. Firstly, learning by doing or education increases the productivity of labor, which corresponds to an increase in the human capital stock. Major contributions originate from Romer (1986) and Lucas (1988). Secondly, a number of authors - e.g. Romer (1990), Grossman and Helpman (1991), and Aghion and Howitt, (1992) - have regarded the input of labor in productivity-increasing innovation activities. However, the knowledge base in these models is not fully specified. Human capital models implicitly assume that an unlimited demand for an increase in knowledge exists, without identifying the source of knowledge demand, e.g. new technologies. In contrast, in the innovation models, skill supply is not specified, presupposing that workers are endowed with the necessary skills without any kind of restrictions or costs. Some technological boundaries exist, however, like when workers lack the knowledge to make use of the latest innovation. The literature on skill-biased technological change has pointed out this idea while modeling unemployment, in particular for less skilled workers, as the result of new innovative technologies. But these innovations either arise as a shock (Agénor and Aizenman, 1995) or at a constant rate (Gregg and Manning, 1997), such that no feedback from the labor market on growth can be considered.

In contrast to the existing literature, in this paper we consider both the effects of knowledge on individual employment performance and on the economy's ability to innovate at the aggregate level. We will argue that limited skill supply facing an increasing knowledge demand driven by the pace of innovation, can be identified as a source of the jobless growth phenomenon. For this purpose, we modify the vintage approach of Aghion and Howitt (1992 and 1994), by introducing learning-by-using to derive the necessity of skill match in the process of job-turnover. Moreover, the diversity of knowledge is explored. The distinction between codified and tacit knowledge on the one side, and the distinction between know-why and the know-how on the other becomes important in our analysis. On one hand, knowledge can either be embodied in individuals in the form of skills, i.e. tacit knowledge, or it can be codified in a knowledge carrier to be reached by a large number of users. On the other hand, skills may contain general knowledge, namely the basic ability to learn why something works, or specific skills to get to know how a particular technology works. The importance of the

³ Barro and Sala-I-Martin (1990) provide an overview.

knowledge base for the growth and employment outcome arises from a mutual influence of developing innovative codified knowledge and having adequate tacit knowledge at the user's disposal. For example, just a few bookkeepers really know the mechanisms by which the accountancy software operates, and most of the programmers have no idea of how the bookkeeping is done. But the combination of both realms of knowledge is necessary in order to replace the books with a computer. For this reason, we assume that the output enhancing effect of technological progress has two strongly linked steps: firstly, an invention, made in a R&D facility, offers the blueprint for an improved technology; secondly, a worker in the manufacturing sector becomes familiar with the blueprint so as to implement the new idea in the manufacturing firms. The former represents innovation activities, the latter represents knowledge diffusion. In addition to these technological necessities, there are also pecuniary restrictions, when increased knowledge diffusion raises the demand for and the gain from innovations, and this, in turn, strengthens the incentives to do research. Hence, innovation activities and knowledge diffusion encourage each other, depicting a complementary relationship for growth.

The paper is organized as follows: Section two presents the basic model. It addresses the knowledge base of the economy, introduces the basic assumptions and the distinction between the different concepts of knowledge and closes with the derivation of the steady state conditions. Section three illustrates the equilibrium and discusses some comparative static effects of policy variables. Section four concludes the analysis.

2. The model

2.1 The knowledge base

We assume that the variety of knowledge is the main cause for the different outcomes in the economy, in particular concerning growth performance and the employment situation. Specifically, the distinction between tacit and codified knowledge and know-why and know-how should be considered in the following way: *codified knowledge* implies that former knowledge is transformed into a set of information. Thus it is independent of the individual's abilities. In the model, codified knowledge arises in the form of blue prints or collected information -e.g. databases or software-, which are developed by R&D departments. Tacit knowledge can be classified as either *human capital* or *know-how*. These two kinds of tacit

knowledge represent different mental capabilities corresponding to special skills. Workers physically embody both of these skills. In this sense, human capital refers to the general abilities belonging to a theoretical base, like basic mathematical or reading skills, but it may also contain additional nonspecific abilities like social competence, basic technology or language knowledge, etc. Know-how, on the other hand, corresponds to application skills, which allow for the use of a particular system without having knowledge about the underlying channels and connections. The system, e.g. a special technology, and the knowledge of how to use it are strongly linked; that is, with the disappearance of a system the corresponding knowledge is lost.

2.2 Basic assumptions and the model's set up

General assumptions

The economy is populated by a continuous mass L of infinitely living individuals with linear intertemporal preferences. Their utility v is generated by the individual amount of the consumption good, y , which is used during an infinite time horizon, with t denoting the real time: $v(y) = \int_0^\infty y_t e^{-rt} dt$, where r is the interest rate, which also equals the time preference. During a time unit interval, each individual is uniformly endowed with one unit of labor and h units of general abilities or human capital, so L is equivalent to the flow of labor supply and $H=hL$ is equal to the aggregate human capital stock. Furthermore, workers differ in their embodiment of know-how. According to this, an individual i can supply specific technological skills corresponding to its individual stock of know-how of A_i . Labor, and with it the related abilities, are divided into the two sectors R&D, with L^R denoting the concerned amount of labor and manufacturing, L^M . We consider a non-cleared labor market; hence, unemployment is taken into account in the following labor market equation, with u denoting the unemployment rate:

$$L = L^R + L^M + uL \tag{1}$$

The R&D sector

Two activities belong to the R&D sector: the production of a flow of an intermediate good, and the development of an innovative form of the intermediate good. The intermediate good is the physical embodiment of codified knowledge, and each stage of development represents a technology linked with a certain level of productivity. The production of a flow of the intermediate good needs no additional inputs, after a certain amount of fixed costs have been paid for. We assume that costs induced by the production of one unit of x can be neglected and, therefore, should be equal to zero. Instead, for the development of the next level of codified knowledge the input of human capital and labor becomes necessary. This is a common input-structure for many research-intensive products supplied by pharmaceutical, chemical, or information technology industries, in which a skilled-labor-intensive development faces lower input using production of knowledge carriers as pills, special synthetic material, or disks. The productivity level⁴ A_τ of the intermediate good reveals in its stage of development τ , which may arise in the interval from zero to infinity. Then, τ also measures the number of innovations that increased the level of codified knowledge. The productivity level rises due to successfully terminated R&D activities, where the efficiency parameter λ determines the productivity difference that occurs due to an innovation, i.e. the step from τ to $\tau+1$:

$$A_{\tau+1} = \lambda A_\tau, \quad \text{with } \lambda > 1 \quad (2)$$

While the size of an innovation is fixed at λ , the frequency in which innovations occur depends on the average human capital h and the number of workers in the R&D sector L^R . Specifically, the probability that an innovation appears is Poisson-distributed, with the R&D related human capital stock hL^R times the efficiency parameter ε denoting the arrival rate of an innovation during a period. Hence, the time dependent human capital stock in the R&D sector determines the number of innovations in a period, and an increase in the human capital stock results in an exponential increase in the productivity level. Hence, with the initial value of codified knowledge A_0 , transforming (2) into real time units yields the rate of innovation $g_A = \dot{A}_t / A_t$:

⁴ We use the denotation A for both the level of codified knowledge and the skills of a particular worker to show the common base: skills are the knowledge of a certain technology represented by a particular level of codified knowledge.

$$A_t = \lambda^{\varepsilon h L^R t} A_0$$

$$g_A = \varepsilon h L^R \ln(\lambda), \quad (3)$$

Generally, L^R depends on time, but it is constant in the steady state.

The manufacturing sector

Two different activities belong to the manufacturing sector: the production of the consumption good, and the process of endowing workers with know-how.

The individual know-how is gained through learning-by-using, i.e. working with a particular level of the codified knowledge. This process demands no market activities and, therefore, corresponds to the idea of knowledge diffusion.⁵ The probability that a worker becomes endowed with $A_t = A_\tau$ in the time interval from t to $t+1$, where τ is the current technology at this point, depends on the human capital in the manufacturing sector hL^M and a productivity parameter μ . Considering the total amount of labor in manufacturing indicates a linear externality, where knowledge diffusion is the more efficient the more employees work together. On the other side, we assume that knowledge diffusion is not restricted to a certain sector, so that the total labor force is involved in the process of knowledge diffusion. Public institutions offer training seminars for the unemployed, and certainly workers in the R&D sector should get access to the applications skills, too. Hence, the expected value of workers endowed with A_τ , i.e. $E(L_{A_\tau}^M)$, equals $\mu h L L^M$. Let D_t denote the share of workers, who acquire the current knowledge per period. Then the extent of knowledge diffusion, measured in the number of skilled workers in a period, becomes:

$$D_t = \frac{E(L_{A_\tau}^M)}{L} = \mu h L^M \quad (4)$$

⁵ Early on, Nelson and Phelps (1966) emphasized the role of knowledge diffusion for innovation-based growth. However, the present modeling of knowledge diffusion is similar to some learning by doing models. In both, the increasing use of technologies in the production sector has in turn a productivity-increasing effect. But in contrast to most of the learning by doing models, e.g. Romer (1986), the learning curve is bounded when no further innovations arise. Hence, innovation activities and knowledge diffusion are rather complements in the implementation of new technologies. However, the introduction of complementarities in learning by doing models is not innovative, because Arrow's (1962) model uses such a technology.

The consumption good is produced by a number of firms using different productivity levels. The pattern of producing firms depends on the relationship between real time, measured in time steps t , and technological time, measured in innovation steps: several levels of codified knowledge, each representing one vintage τ , in the interval τ^{\min} to τ^{\max} occur simultaneously at one point of t , and one particular stage $\bar{\tau}$ is present during a period from $t = \bar{t}$ to $t = T$. This two times overlapping is the result of the assumed process of creative destruction in the manufacturing sector that allows the existence of a restricted number of vintages in a period and a limited survival time of a certain vintage. In this sense, τ^{\min} and τ^{\max} are the boundaries of different vintages working in the same period, but with different stages of the intermediate good, whereas the time between T and \bar{t} is the horizon, in which a particular vintage can maintain its production. For a particular vintage $\bar{\tau}$, this means a move from the most productive level $\bar{\tau}^{\max}$ at \bar{t} to the minimum productivity level $\bar{\tau}^{\min}$ at point T , and a market exit afterward. Figure 1 illustrates this relationship. To make use of the productivity determining level of codified knowledge that is linked with the intermediate good, a worker endowed with the equivalent skills is necessary. This, and additional requirements yield the model's assumptions on the knowledge base:

- (a) Codified knowledge requires the presence of tacit knowledge in order to be beneficial.
- (b) The current technology is compatible backwards, i.e. $A_{\tau^{\max}}$ also the previous sets of A_{τ} with $\tau \leq \tau^{\max}$.
- (c) A firm chooses a particular technology that equals the current one at the date of the firm's implementation. Afterward no upgrade is possible.
- (d) The productivity level cannot exceed the skill level of the concerned worker.

Hence, (c) and (d) imply that the productivity is limited in two ways. The minimum of both tacit knowledge and codified knowledge -the first is equal to the worker's skills and the second to the technology- determines the productivity level. Let \tilde{A}_{τ} denote the productivity level that can be currently used by a particular vintage, then from (a) to (d) follows:

$$A_{\tau} \geq \tilde{A}_{\tau} \leq A_i \tag{5}$$

The amount produced in a single vintage τ depends on the input of the intermediate good x and the level of codified knowledge \tilde{A}_τ that is embodied in x . We assume a one to one ratio between x and the relevant skilled workers employed in the manufacturing sector, L^M , i.e. one worker uses one software license, and therefore $L^M = x$. According to these assumptions, the production function of a vintage can be written as:

$$Y_\tau = \tilde{A}_\tau L_\tau^{M^\alpha}, \quad (6)$$

where $0 < \alpha < 1$.

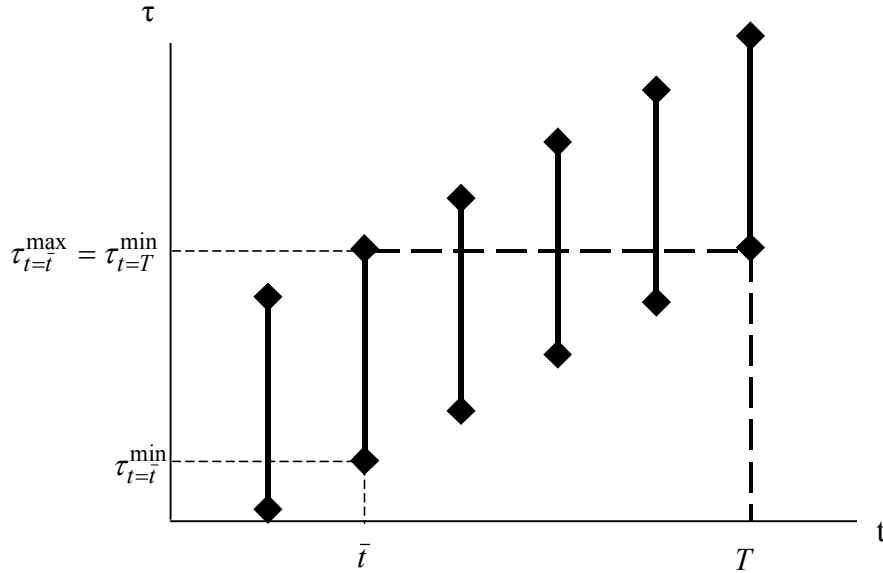


figure 1: *real time technological time relationship*

Steady state conditions

Steady state growth requires a constant amount of labor in the three sectors, in particular in R&D and manufacturing. This becomes true if the labor market outcome satisfies the no-arbitrage and the labor market conditions. The no-arbitrage condition requires that a move from the manufacturing to the R&D sector, and vice versa, allows no monetary advantage. The labor market condition demands a labor allocation, which guarantees a balance between R&D and learning-by-using, in that no mismatch between skills-supply and skills-demand produces additional unemployment. These conditions are derived in the two following sections.

2.3 The no-arbitrage condition

Labor can be freely allocated between manufacturing and R&D.⁶ This implies that the income from working in the different sectors has to be the same in order to guarantee a constant fraction of the labor force working in the different sectors. Otherwise, if there were a higher income in one sector, workers would change to this option. A constant fraction of workers in the different sectors is one of the steady state conditions. Hence, to develop an arbitrage equation, which depicts the different combinations of L^M and L^R satisfying this condition, we determine the income possibilities from working in manufacturing and in R&D.

The technology depending wage in the manufacturing sector during a unit time interval, w_τ , and the discounted expected income in the R&D sector are simultaneously determined. Furthermore, we attribute the general income level to the labor market equilibrium indicating the number of researchers, which, in turn, determines the profitability of R&D and in turn, the wage in manufacturing. Hence, to identify the no-arbitrage condition it becomes necessary to determine the expected income from R&D, namely $\varepsilon h V_{\tau+1}$, when τ innovations occurred so far. The expected income consists of the two properties' probability of being the next innovator, εh , and the gain from the successfully developing stage $(\tau+1)$, $V_{\tau+1}$, that gives the opportunity to sell the intermediate good as a monopolist. Therefore, no-arbitrage between working in R&D and manufacturing requires equality of the current wage and expected income from R&D at this point:

$$w_\tau = \varepsilon h V_{\tau+1} \tag{7}$$

where the left-hand side refers to the value per period of one unit of labor in manufacturing, whereas the right-hand side is the expected value of one unit of labor in R&D. The value $V_{\tau+1}$ is the expected present value of a flow of monopoly profits $\pi_{\tau+1}$ generated by the $(\tau+1)^h$ innovation over an interval whose length is exponentially distributed with parameter $\varepsilon h L_{\tau+1}^R$, namely the arrival rate of the $(\tau+2)^h$ innovation.

⁶ No frictionless move between the sectors is necessary to satisfy the stability of the equilibrium. Hence, steady state conditions do not demand the immediate change from being a manufacturing worker to becoming a researcher. See Appendix C details.

$$V_{\tau+1} = \frac{\pi_{\tau+1}^R}{r + \varepsilon h L_{\tau+1}^R} \quad (8)$$

The profit per period of the innovator, $\pi_{\tau+1}^R$, arises from producing a flow x of an intermediate good at price p_τ . To keep the analysis simple, we assume the last innovator to be a monopoly.⁷ Because of this monopolistic power, the amount of produced x can be attributed to the profit-maximization problem:

$$\max_x \pi_\tau^R = p_\tau(x)x - C_\tau, \quad (9)$$

where just fixed costs of C_τ are necessary to enable the production. We assume the consumption good sector to be competitive⁸. Hence, the first order condition of the maximization of profits per period of a manufacturing vintage τ , namely π_τ^M , according to

$$\max_x \pi_\tau^M = Y_\tau - (p_\tau + w_\tau)x_\tau \quad (10)$$

yields the inverse demand function for x . Equation (10) indicates that profit arises by selling the amount of Y_τ of the consumption good, accompanied by costs for the intermediate good, $p_\tau x$, and for labor, $w_\tau L_\tau^M$. Due to the one to one relationship of x and L^M , the use of one unit of x induces costs of $(p_\tau + w_\tau)$. With substituting Y_τ by (6) and the consumption good as numéraire the inverse demand function

$$p_\tau = \tilde{\alpha} \tilde{A}_\tau L_\tau^{M^{\alpha-1}} - w_\tau \quad (11)$$

produces together with (9) the profit maximizing amount of the intermediate good:

⁷ From (9) it is straightforward to see that sufficiently high fixed costs in the production of x prevent previous R&D firms from selling the intermediate good.

⁸ In the next section we discuss the market structure of the consumption good sector. There, we assume that one firm monopolizes a single vintage. However, the number of vintages supplying the identical consumption good is high enough to guarantee competition between the different vintage firms.

$$x_\tau = \left(\frac{\alpha^2}{\omega_\tau} \right)^{\frac{1}{1-\alpha}}, \quad (12)$$

with $\omega_\tau = w_\tau / A_\tau$ denoting the productivity-adjusted wage. For simplicity we assume that the R&D firms have no information about the distribution of the different technology levels used in the consumption good sector. Hence, according to profit-maximization, the monopolistic supplier of x only considers the technological level of his own product, namely A_τ , and (12) represents the total supply, including the interval $[\tau^{\min}, \tau^{\max}]$.⁹

The arbitrage equation is almost specified now. The inverse demand-function of (11) and the optimal amount produced from (12) generate the profit from innovating. Considering that the no-arbitrage condition (7) requires future values, namely $\pi_{\tau+1}^R$, the profit from being the $(\tau+1)^h$ innovator is:

$$\pi_{\tau+1}^R = A_{\tau+1} \omega_{\tau+1}^{-\frac{\alpha}{1-\alpha}} \alpha^{\frac{2}{1-\alpha}} \left(\frac{1}{\alpha} - 1 \right) - C_{\tau+1} \quad (13)$$

The profit of (13) under the consideration of both the expected time horizon for profits, shown in the obsolescence adjusted interest rate in (8), and the probability of inventing within one time unit in (3), produces the value of one unit of labor in R&D. The result, in turn, determines also the wage in the manufacturing sector. We thus attain a specification of the no-arbitrage condition in (7):

$$w_\tau = \varepsilon h \frac{w_{\tau+1} L_{\tau+1}^M \left(\frac{1}{\alpha} - 1 \right) - C_{\tau+1}}{r + \varepsilon h L_{\tau+1}^R}, \quad (14)$$

where the amount of labor in manufacturing, L^M , substitutes expression of (13) using (12) and taking the one-to-one relationship between L^M and x into account. Next, we develop the arbitrage equation, depicting different equilibrium labor allocations between R&D and manufacturing. The different solutions of the arbitrage equation yield the arbitrage curve,

⁹ Otherwise, a repeated technology-induced shift in the x -demand-function produces infinite demand for the intermediate good, when $\tau \rightarrow \infty$, and prevents the model from attaining a steady state. Furthermore, this assumption avoids, for a sufficient high labor force, the possibility of a non-continuous demand function if the labor demand, equal to the x -demand, exceeds total labor supply, when $\tau \rightarrow \infty$.

where each of them refers to the steady state. For this reason the labor allocation remains constant, i.e. $L^M_\tau = L^M$ and $L^R_\tau = L^R$. This holds equal for the technology adjusted wage, namely $\omega_t = \omega$, indicating that the wage increases with the rate of the technological level: $w_{\tau+1} = \lambda w_\tau$. Hence, solving (14) with respect to L^R yields the arbitrage equation (AE):

$$L^R = \lambda L^M \left(\frac{1}{\alpha} - 1 \right) - \frac{r}{\varepsilon h} - \frac{C}{w} \quad (\text{AE})$$

The resulting arbitrage curve has a positive slope in the (L^M, L^R) -space. This outcome refers to pecuniary complementarities between the input of labor in the two sectors, if labor can be reallocated between manufacturing and R&D, whenever the payments in the sectors diverge. Suppose that the labor force is not fully in charge. A somehow induced increase in L^M induces a higher demand of x and, therefore, rising profits in the R&D sector. To eliminate the arisen income differences, L^R also has to increase in order to again reduce the expected profit in R&D. This comes about because a higher innovation rate, caused by the raised number of researchers, reduces the time interval in which the monopoly of the current innovation can be maintained. On the other hand, an increase in L^R produces more technological progress, raising the productivity and profitability in the consumption good sector. Consequently, L^M will rise.

2.4 The labor market condition

In contrast to standard innovation based growth models, the occurrence of an innovation is not sufficient to increase productivity and enlarge output. Instead, a production unit has to employ a worker, endowed with the know-how of the current technology. Afterward, the current level of codified knowledge can be implemented in the production process of manufacturing.¹⁰ This refers to the technological complementary relationship of know-how and codified knowledge. If the supply of know-how, produced by the knowledge diffusion according to (4), is insufficient, unemployment will occur. Therefore, we subsequently derive the conditions for arising unemployment.

To receive the labor market equation, we rewrite (1):

¹⁰ See equation (5).

$$L^R = (1-u)L - L^M, \quad (15)$$

and determine in the following the rate of unemployment depending on the fraction of labor that is used in the different sectors.

For simplicity, we assume that the codified knowledge that a firm uses is fixed at the level it attained at the date of the firm's set up. Consequently, innovations restrict the surviving time of a manufacturing firm.¹¹ Producing the consumption good is characterized by the arising of fixed costs when the manufacturing firm implements an innovation into the production process. Hence, due to increasing returns to scale, a monopolistic supplier rules out a vintage production. The vintage production $Y_{\bar{\tau}}$ corresponds to the firm's output that uses the level of codified knowledge $A_{\bar{\tau}}$. However, competition occurs due to the number of vintages, turning out the same output, demanding the same inputs, but producing with different technologies. See Appendix A for details. Suppose a particular vintage $\bar{\tau}$ enters the market at date \bar{t} , hence $\bar{\tau} = \tau_{t=\bar{t}}^{\max}$, and with the production's shut down at date T , when $\bar{\tau} = \tau_{t=T}^{\min}$. The deterring of the market entry of a competitor requires zero-profits for the incumbent in the dynamic perspective, i.e. during the time horizon $S_{\bar{\tau}} = T - \bar{t}$, in which the used technology is productive enough to continue the production:

$$\int_{t=\bar{t}}^{t=T} e^{-r(t-\bar{t})} \left(A_{\bar{\tau}} x_{\bar{\tau},t}^{\alpha} - (p_{\tau,t} + w_{\tau,t}) x_{\bar{\tau},t} \right) dt = F_{\bar{\tau}}, \quad (16)$$

with $F_{\bar{\tau}} = \theta A_{\bar{\tau}}$ denoting the fixed costs of the implementation, that increase equal to the level of codified knowledge. While level $A_{\bar{\tau}}$ is fixed, the other values depend on time.

In this setting, the fixed technology and an increasing wage induce the market exit of a firm when it becomes unprofitable. From (12) we can see that the steady state demands a constant productivity-adjusted wage $\omega_t = w_t / A_t$. Hence, the nominal wage w_t has to increase

¹¹ For a reminder of the vintage structure in the manufacturing sector, see again section 2.2 and figure 1, in particular.

with the rate of innovation g_A . Wages increase for all vintages, because of the outside option moving in the R&D sector and earning $V_{\tau+1}$.¹²

Increasing wages induce a market exit after date T , when the firm arrives at the break-even point. A firm becomes unprofitable if costs per period exceed the revenues. Hence, at date T , expenditures $q_{\bar{\tau},T} = (p + w_T)x_{\bar{\tau},T}$ equal revenues $Y_{\bar{\tau}} = A_{\bar{\tau}}x_{\bar{\tau},T}^\alpha$. At this point, $w_{\bar{\tau},t}$ attains its maximum value $w_{\bar{\tau}}^{\max}$, and the input of x is at its minimum. A further increase above the level $q_{\bar{\tau}}(w_{\bar{\tau}}^{\max})$ leads to an immediate market exit. A firm produces less and less, until production becomes unprofitable. The production approaches zero asymptotically, but due to $x_{\bar{\tau}} = L_{\bar{\tau}}^M$ a definite point in time for market exit is determined, when demand goes below the minimum labor input of one unit. The rate of innovation g_A determines the speed of the increase in $q_{\bar{\tau},t}$. Hence, innovation activities cause the dimension of creative destruction.

For vintage $\bar{\tau}$, the wage equals $w_{\bar{\tau}}^{\max}$ at $t=T$. Hence, with the wage rising with g_A beginning at the initial value, w_0 , the maximum wage is:

$$w_{\bar{\tau}}^{\max} = w_T = w_0 e^{g_A T} = w_0 e^{g_A (\bar{t} + S_{\bar{\tau}})} = w_{\bar{t}}^{g_A S_{\bar{\tau}}} \quad (17)$$

Because of the symmetry in the steady state the time horizon is fixed, $S_{\bar{\tau}} = S$. To yield an expression for S , we take the logarithm from (17) and substitute g_A by (3):

$$S = \frac{\Gamma}{g_A} = \frac{\Gamma}{\varepsilon h L^R \ln(\lambda)}, \quad (18)$$

where Γ denotes the production horizon in technological time with $\Gamma = \ln w_{\bar{\tau}}^{\max} - \ln w_{\bar{t}}$, measuring the amount of productivity gains that has to arise, until an old technology becomes obsolete, whereas g_A shows the increase of productivity per period. As a result, speeding up technological change cuts the surviving time of a firm and, thereby, reinforces creative destruction.

¹² The wage is an input that produces increasing costs. The price p_t for the intermediate good is constant in the equilibrium. This result is straightforward to see from (11), when the technological level and wage increase at the same rate.

The limited survival time when S is finite for $g_A > 0$ produces permanent job turnover. The number of workers who are dismissed because of creative destruction, consist of those who worked in the closed vintage τ^{min} , and those who lose their jobs due to the reduced labor demand in other vintages. But simultaneous to these lay-offs, new labor demand arises in the current vintage τ^{max} .

If an innovation occurs, existing production vintages reduce the demand for x according to (12). According to the one to one relationship of x and L^M , the number of dismissed workers per vintage corresponds to the difference between the demand for the intermediate good of two subsequent vintages in a point in time, namely $x_{\tau,t}$ and $x_{\tau-1,t}$, and can be attributed to the difference in the technology adjusted wage caused by the fixed technology, namely $\omega_\tau = w_\tau / A_\tau$ and $\omega_{\tau-1} = w_\tau / A_{\tau-1}$. Calculating $x_{\tau,t}$ from (10) and using (2) produces the loss of employment per vintage, that equals:

$$x_{\tau,t} - x_{\tau-1,t} = \left[(\lambda - 1) \left(\frac{\alpha^2}{\omega_\tau \lambda} \right) \right]^{\frac{1}{1-\alpha}} \quad (19)$$

Besides to the number in (19), the worker of vintage τ^{min} goes into unemployment.

The number of vintages follows from the multiplication of the arrival rate of an innovation, $\varepsilon h L^R$, indicating the innovations per period, and the periods that a technology is able to survive. With (18), the number of present producing vintages becomes:

$$\tau^{max} - \tau^{min} = \varepsilon h L^R S = \frac{\Gamma}{\ln(\lambda)} \quad (20)$$

If $\varepsilon h L^R$ innovations arise between t and $t+1$, the steady state flow of workers in manufacturing going into unemployment per period, $U_{L^M}^+$, is given as the number of innovations multiplied by the number of dismissals per innovation:

$$U_{L^M}^+ = \varepsilon h L^R \left[(\tau^{max} - \tau^{min}) (x_\tau - x_{\tau-1}) + x_{\tau^{min}} \right], \quad (21)$$

The expression in the square brackets indicates the number of lay-offs that are produced by an innovation, and which, in turn, consists of the first term, indicating the number of vintages

times the number of dismissed workers per present producing vintages, and the second term, that is the former labor demand of τ^{\min} . From (19), (20) and $x_{\tau^{\min}} = 1$, we can see that the number of lay-offs per period is fixed in the steady state. Therefore, we can simplify the analysis when we consider a constant fraction ϕ of the labor force in manufacturing, namely ϕL^M , that becomes unemployed per innovation, where according to (19) and (20) ϕ is a function of α , λ , ω , and Γ . To complete the flow into unemployment, we assume that the same share of workers in both the R&D and manufacturing sectors, lose their jobs. Hence, the total flow into unemployment, U^+ , is:

$$U^+ = \varepsilon h L^R \phi (L^M + L^R) \quad (22)$$

The number of dismissed workers per period depends on the total employment in manufacturing and R&D, so that the current unemployment rate has some influence on the outcome. Using (1) yields:

$$U^+ = \varepsilon h L^R \phi (1 - u) L \quad (23)$$

Now we can distinguish between two channels a change in the innovation rate influences the flow into unemployment. The first effect occurs when an increase in one argument of the innovation's arrival rate, namely ε , h , or L^R , reduces the time in which manufacturing offers profits. The increase in the frequency of innovations indicates an increase in creative destruction accompanied by an increase in the job turnover and a rise in U^+ . The second effect is concerned with an increase in the size of innovations, λ . If the size of an innovation grows, only a few innovations are necessary to bring a technology to obsolescence. The interval from τ^{\min} to τ^{\max} cuts down while the output and therefore the labor demand of a single vintage increases. Hence, two opposite outcomes concerning U^+ arise from an increase in λ .

The flow out of unemployment can be attributed to the success of the recruiting efforts of a firm, as shown in (4). This kind of a matching technology¹³ depends on the supply of

¹³ A standard matching technology is given by the probability of a worker to find a new job, and the probability of a firm to fill a vacancy. The output of this matches-producing function depends on the rate of unemployment and the number of vacancies. See e.g. Blanchard and Diamond (1989) and Pissarides (1990).

know-how endowed labor, if the occurrence of the τ^{th} innovation produces a sufficiently high labor demand in manufacturing. However, just a small number of $D_t L$ workers are able to fill the vacancies arising in vintage τ^{max} , because they achieved the demanded skills by learning-by-using. Hence, substituting D_t by (4) yields the flow out of unemployment, U^- , after the τ^{th} innovation:

$$U^- = \mu h L^M L \quad (24)$$

If knowledge diffusion is inadequate, then U^+ exceeds U^- during the transition to the equilibrium unemployment. This process represents the mismatch character of labor reallocation, because unemployment does not occur due to insufficient labor demand caused by non-clearing wages, but due to a non-matching labor supply. In this case, the matching technology creates excess labor demand in vintage τ^{max} and excess labor supply in the vintages $\tau < \tau^{min}$. Equilibrium unemployment, with a constant diffusion rate $D_\tau = D$, follows from the equality of the flow into and out of unemployment. Hence, u ensues from the equality of (23) and (24):

$$u = \begin{cases} 1 - \frac{\mu L^M}{\varepsilon \phi L^R} & \text{if } u > 0 \\ 0 & \text{otherwise} \end{cases} \quad (25)$$

A necessary condition for the existence of unemployment is the inadequate endowment with skills of the labor force. According to the first expression in (25) unemployment occurs, if:

$$\frac{L^M}{L^R} < \frac{\varepsilon \phi}{\mu}, \quad (\text{UC})$$

As demonstrated by the unemployment condition (UC), mismatch unemployment is the result of an inappropriate labor allocation. In particular, too few workers in manufacturing compared with the number of researchers indicate inadequate knowledge diffusion and thereby mismatch unemployment. R&D induced innovations drive the level of codified knowledge, but the complementary tacit knowledge is just acquired by an insufficient

minority of workers. The reason is, the fewer workers are employed in manufacturing, the fewer workers can be part of knowledge diffusion, where technological skills are acquirable.

With the specification of the unemployment rate, we can now devise the labor-market equation by inserting (25) in (1):

$$L^R = \begin{cases} \left(\frac{\mu L}{\varepsilon \phi L^R} - 1 \right) L^M & \text{if } u > 0 \\ L - L^M & \text{if } u = 0 \end{cases} \quad (\text{LE})$$

The labor-market equation represents all combinations of labor allocations between R&D and manufacturing which satisfy the technological condition. This means that the relevant L^R/L^M combinations produce a balance between innovation and knowledge diffusion, so that the unemployment rate remains at a constant level. The different solutions of (LE) produce the labor market curve. Besides a regular downward-sloped labor market curve in the case of full employment, i.e. the second expression of (LE), we get an upward-sloped labor market curve in the (L^M, L^R) -space for a positive unemployment rate. This indicates that different equilibriums in the labor market will produce either more or less employment in both the manufacturing and R&D sectors. High employment rates in manufacturing cause an appropriate rate of learning-by-using, i.e. knowledge diffuses at a fast pace. This allows a high rate of innovations in a time unit without producing a lack of application skills, so that a high number of researchers can work in R&D. Because the number of lay offs, U^+ , is a quadratic function of L^R , whereas the number of new occupations, U^- , is linear with respect to L^M , the labor market function's slope decreases.¹⁴

3. The equilibrium

Steady state conditions are given by the system characterized by the no-arbitrage (AE) and the labor market condition (LE). We can derive the unemployment rate from innovation activities

¹⁴ See Appendix B for the comprehensive solution.

and the degree of learning-by-using. The growth rate is endogenous because of the feedback that the labor market produces on R&D.

The steady state for a constant amount of human capital requires a fixed allocation of labor between manufacturing and R&D. Therefore, we demand that the ratio L^M to L^R satisfies both the no-arbitrage and the labor market condition. Figure 2 depicts the solution of the system, given by the no-arbitrage (AE) and labor market (LE) curve and the intersection point as equilibrium allocation of L^M and L^R . The points Q and Q' represent a stationary equilibrium for this economy. The equilibrium exists and is unambiguous because of the concavity of (LE) and the linearity of (AE) intersecting the abscissa right from the origin¹⁵. For a sufficient high adjustment to the no-arbitrage condition and a sufficient small number of researchers the equilibrium is a stable focus.¹⁶ The solution to the system of (AE) and (LE) can be used together with the innovation function (3) to determine the steady state growth rate of codified knowledge, g_A . Furthermore, g_A equals the growth rate of the aggregate output. This result is straightforward to see from the production function of a single vintage in (6). The steady state input of L^M is fixed and therefore vintage production increases with the rise of the productivity parameter, namely g_A . Furthermore, according to (20) a constant intensity in the R&D sector, i.e. a constant amount of L^R , leaves the number of vintages unchanged, so that aggregate output is just driven by an increase in the vintage production.

In addition to the determination of the growth rate, the system of (AE) and (LE) decides on the employment site in the economy. According to (25) the possibility of full employment is not ruled out. However, unemployment occurs if workers are deficient in skills demanded by a certain level of codified knowledge used in manufacturing. In the case of $u > 0$ the labor market curve equals the first equation of (LE). Comparing the solution with the full employment case yields the amount of unemployment in the economy. This outcome corresponds to figure 2a. Instead, for $u = 0$ the second equation of (LE) represents the labor market condition, corresponding to point Q' in figure 2b. Which of the two settings characterizes the economy's situation depends on the unemployment condition (UC).

¹⁵ R&D does not occur, until a minimum number of workers are employed in the production sector. The level of demand for the R&D-based intermediate good corresponding to L^M_{min} is necessary to generate sufficiently high profits to implement research incentives. This is a usual result for innovation models. New products need a sufficiently high demand, or the private incentives for costly research are too low. See, e.g., Grossman and Helpman (1991), Romer (1990) and Young (1993).

¹⁶ For a detailed solution see Appendix C.

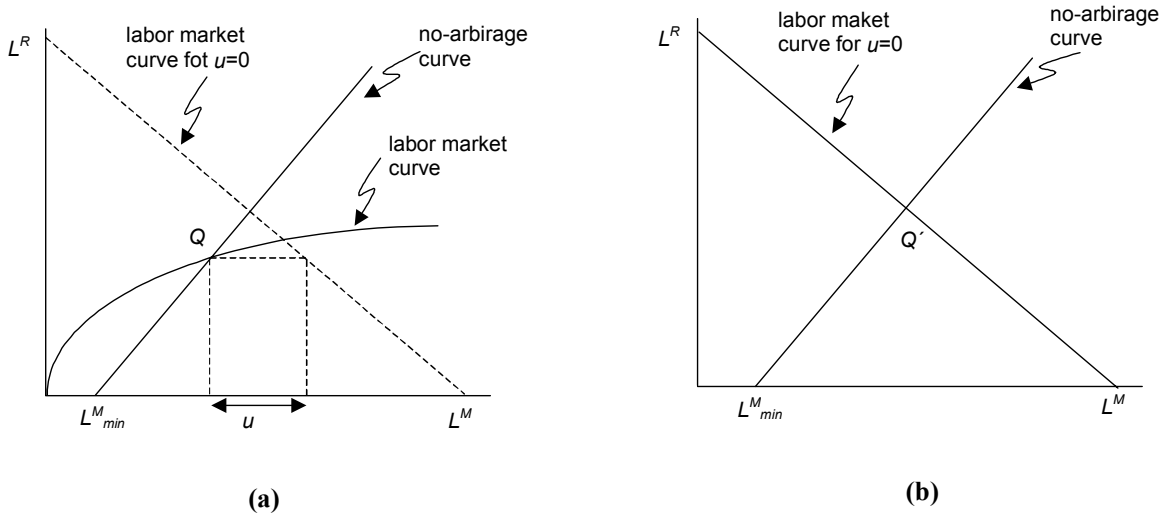


figure 2: *steady state solutions for $u > 0$ and $u = 0$*

Comparative static analysis confirms, in general, the common belief of an inverse relationship between growth and unemployment, when a parameter variation induces an acceleration of growth that is accompanied by a reduction of unemployment. However, some parameters affect growth and unemployment ambiguously, when changes in the no-arbitrage and the labor market condition have opposite effects. Table 1 summarizes the results. Of outstanding interest seems to be the result of an increase in average human capital, h . Despite the tendency toward a positive effect on the accelerated growth rate, where higher educated researchers are accompanied by a higher number of researchers, rising unemployment may contradict the positive result. An increase in h reduces the time in which research is necessary to develop an innovation. This equals a cost cut, encouraging entrance into the R&D sector. On the other hand, unemployment arises due to the acceleration of the number of innovation-based shut downs of firms. If unemployment rises, the positive employment effect for R&D becomes negative and makes the growth effect ambiguous when fewer researchers work with an increased productivity. Because of the complementary relationship with tacit knowledge, an increase in just the research productivity, namely ε and λ , so the level of codified knowledge can be increased more easily, produces at least ambiguous effects for growth and employment. Stronger monetary incentives to research, i.e. a decrease in r and C , strengthen the creative destruction, and total employment as well as employment in the research sector diminish. Still, it remains indefinite whether policy should prefer subsidies for the productivity of knowledge diffusion to innovation because the model can say nothing about the quantity of the different growth effects. While the growth effect of increasing research productivity is ambiguous, an improvement in the process of imparting workers with the tacit

knowledge in the form of application skills, namely an increase in μ , is definitely growth enhancing and job creating.

increase in	shift of the		total effect on	
	arbitrage curve	labor market curve	L^R	u
λ	LHS	RHS if $\phi_\lambda > 0$	-	+
		LHS if $\phi_\lambda < 0$?	?
α	RHS	RHS	?	?
h	LHM	0	-	+
ε	LHM	RHS	-	+
r	RHS	0	+	-
C	RHS	0	+	-
ω	LHS	LHS	?	?
μ	0	LHS	+	-
Γ	0	RHS if $\phi_\Gamma > 0$	-	+
		LHS if $\phi_\Gamma < 0$	+	-

LHS = left-hand shift; RHS = right-hand shift; 0 = unaffected; + = positive effect; - = negative effect; ? = ambiguous effect

table 1

4. Conclusion

This paper has analyzed a model linking innovation-based growth and unemployment, whereby knowledge is an input used in various ways. Specifically, three dimensions of knowledge were considered: codified knowledge, i.e. former knowledge transformed into a set of information that can be used for production, as software, databases, etc; know-how, i.e. the skills to deal with a particular level of codified knowledge, e.g. the ability to use a current software; and finally human capital, i.e. general skills that have a widespread use as mathematical or reading capabilities. Growth results exclusively from an increase in the level of codified knowledge, arising as an innovation that drives technological progress. Each innovation is embodied in an intermediate good that can be used in a manufacturing sector to produce the consumption good more efficiently than was previously possible. The use of codified knowledge is strongly connected with the presence of know-how embodied in the

workers. We argue that mismatch unemployment arises when the pace of innovation exceeds the rate at which workers can achieve the relevant know-how. Furthermore, unemployment is a loss of knowledge that feeds back negatively on innovation activities.

As a result, some common beliefs regarding the consequences of policy measures cannot be verified. For example, subsidies in R&D just reinforce research efficiency, whereas the know-how in the labor force remains unaffected. Hence, an increasing knowledge mismatch will raise unemployment and lower the innovation rate. Furthermore, we derived ambiguous effects of employment and growth for human capital accumulation. Because basic skills improve both research efficiency and the diffusion of know-how, the overall impact depends on which one of the two activities is more affected by an increased human capital stock.

References

- AGÉNOR, P. R. and J. AIZENMAN, 1995, Technological Change, Relative Wages, and Unemployment, *European Economic Review*, 41, 187 -205
- AGHION, P. and P.HOWITT, 1991, Unemployment-A Symptom of Stagnation or Side Effect of Growth, *European Economic Review*, 35, 535-541
- AGHION, P. and P. HOWITT, 1992, A Model of Growth through Creative Destruction, *Econometrica*, 60, 323-351
- AGHION, P. and P. HOWITT, 1994, Growth and Unemployment, *Review of Economic Studies* 61, 477-94
- ARROW, K. J., 1962, The Economic Implications of Learning by Doing, *Review of Economic Studies*, 29, 155 -173
- BARRO, R. J. and X. SALA-I-MARTIN, 1995, *Economic Growth* (McGraw-Hill, New York)
- BLANCHARD, O. and P. DIAMOND, 1989, The Beveridge Curve, *Brooking Papers of Economic Activity*, 1, 1-60

- DOMAR, E. D., 1946, Capital Expansion, Rate of Growth, and Employment, *Econometrica*, 14, 137 – 147
- GREGG, P. and A. MANNING, 1997, Skill-biased Change, Unemployment and Wage Inequality, *European Economic Review*, 41, 1173-1200
- GROSSMAN and E. HELPMAN, 1991, *Innovation and Growth* (MIT Press, Cambridge)
- HARROD, R. F., 1939, An Essay in Dynamic Theory, *Economic Journal*, 49, 14 - 33
- KYDLAND, F.E., 1995, *Business Cycle Theory*, (Elgar, Aldershot)
- LAYARD, R., S. NICKELL and R. JACKMAN, 1991, *Unemployment: Macroeconomic Performance and the Labor Market*, Oxford University Press, Oxford
- LUCAS, R., 1988, On the Mechanics of Development Planning, *Journal of Monetary Economics*, 22, 3-24
- NELSON R. R. and E. S. PHELPS, 1966, Investment in Humans, Technological Diffusion, and Economic Growth, *American Economic Review*, 56, 69-74
- OECD, 1992, *Employment Outlook* (OECD, Paris)
- OLECH, C., 1963, On the Global Stability of an Autonomous System on the Plane, in: J.P. LaSalle, J.B. Diaz et al., eds, *Contributions to Differential Equations*, Vol. I, (Interscience Publisher, New York) 389 - 400
- PISSARIDES, C.A., 1990, *Equilibrium Unemployment Theory* (Basil Blackwell, Oxford)
- ROMER, P., 1986, Increasing Returns and Long-Run Growth, *Journal of Political Economy*, 94, 1002 -1037
- ROMER. P., 1990, Endogenous Technological Change, *Journal of Political Economy*, 98, 101
- YOUNG, A., 1993, Invention and Bounded Learning by Doing, *Journal of Political Economy*, 101, 443 – 472

Appendix

A Market structure in the consumption good sector

Due to fixed costs, sunk after the firm's implementation, the innovative vintage τ cannot monopolize the manufacturing sector.

Proof:

To induce market exit of vintage $(\tau-1)$ the incumbent sets a price $(p+w)^{\max}$, so that vintage $(\tau-1)$ produces no profit:

$$A_{\tau-1}x_{\tau-1}^{\alpha} - (p+w)^{\max}x_{\tau-1} \stackrel{!}{\leq} 0 \quad (\text{A1})$$

Hence, by (2):

$$(p+w)^{\max} \stackrel{!}{\geq} \frac{A_{\tau}}{\lambda} x_{\tau-1}^{\alpha-1}, \quad (\text{A2})$$

where equality holds, if profit maximization is valid. Present value profit during the time horizon S of the monopolistic incumbent should be non-negative:

$$\int_{t=0}^{t=S} e^{-rt} \bar{\pi}^M \stackrel{!}{\geq} F_{\tau} \quad (\text{A3})$$

Revenues per period, $\bar{\pi}^M$, are constant, because the monopoly sells the consumption good at the same price until an innovation replaces the incumbent after S periods. Then, we can write:

$$\bar{\pi}_{\tau}^M \int_{t=0}^{t=S} \frac{1}{(1+r)^t} \stackrel{!}{\geq} F_{\tau} \quad (\text{A4})$$

Hence,

$$\pi_{\tau}^M \left[\frac{1}{\ln(1+r)} \left(1 - \frac{1}{(1+r)^S} \right) \right] \stackrel{!}{\geq} F_{\tau} \quad (\text{A5})$$

By (6):

$$\left[A_\tau x_\tau^\alpha - (p+w)^{\max} x_\tau \right] \left[\frac{1}{\ln(1+r)} \left(1 - \frac{1}{(1+r)^S} \right) \right] \stackrel{!}{\geq} F_\tau \quad (\text{A6})$$

Substituting $(p+w)^{\max}$ by (A2):

$$A_\tau x_\tau^\alpha - \left(\frac{A_\tau}{\lambda} x_{\tau-1}^{\alpha-1} \right) x_\tau \stackrel{!}{\geq} \frac{F_\tau}{\left[\frac{1}{\ln(1+r)} \left(1 - \frac{1}{(1+r)^S} \right) \right]} \quad (\text{A7})$$

$$\ln(1+r) \frac{F_\tau}{Y_\tau \left(1 - \frac{1}{(1+r)^S} \right)} \stackrel{!}{\leq} 0 \quad (\text{A8})$$

However, the left-hand side of (A8) is positive. No monopoly will arise in the manufacturing sector, if just a loss is produced.

B The qualities of the labor market curve

Solving (LE) for L^R yields:

$$L^R = \frac{1}{2} \left[\frac{\sqrt{\left(L^M \varepsilon \phi \right)^2 + 4 \varepsilon \phi \mu L L^M}}{\varepsilon \phi} - L^M \right] \quad (\text{B1})$$

$$\frac{\partial L^R}{\partial L^M} = \frac{2 L^M \phi^2 \varepsilon^2 + 4 \phi \varepsilon \mu L}{4 \phi \varepsilon \sqrt{\left(L^M \phi \varepsilon \right)^2 + 4 \phi \varepsilon \mu L L^M}} - \frac{1}{2} \quad (\text{B2})$$

The (LE)-curve has a positive slope in the (L^R, L^M) -space, if $\partial L^R / \partial L^M > 0$; Hence, according to (B2):

$$L^M \phi^2 \varepsilon^2 + 2 \phi \varepsilon \mu L \stackrel{!}{>} \phi \varepsilon \sqrt{\left(L^M \phi \varepsilon \right)^2 + 4 \phi \varepsilon \mu L L^M} \quad (\text{B3})$$

$$4\mu^2 L^2 > 0 \quad \text{q.e.d.} \quad (\text{B4})$$

The concavity of the (LE)-curve requires $\partial^2 L^R / \partial L^M{}^2 < 0$. Hence,

$$\frac{\phi \varepsilon}{2\sqrt{(L^M \phi \varepsilon) + 4\phi \varepsilon \mu L L^M}} < \frac{(2L^M \phi^2 \varepsilon^2 + 4\phi \varepsilon \mu L)^2}{8\phi \varepsilon \left[(L^M \phi \varepsilon)^2 + 4\phi \varepsilon \mu L L^M \right]^{\frac{3}{2}}}, \quad (\text{B5})$$

respectively:

$$\frac{L^M}{L} < \frac{\mu}{\phi \varepsilon} \quad (\text{B6})$$

C Equilibrium Analysis

The equilibrium of the arbitrage curve (AE) implies that $\dot{L}^R < 0$, when $L^R > L^R|_{AE}$, because more researchers reduce profits from research and make production for the better alternative. Let φ_1 denote the speed of this adjustment to write:

$$\dot{L}^R = \varphi_1 \left[\lambda L^M \left(\frac{1}{\alpha} - 1 \right) - \frac{r}{\varepsilon h} - \frac{C}{w} - L^R \right] \quad (\text{C1})$$

The labor market condition (LE) implies that $\dot{L}^M > 0$, when $L^M > L^M|_{LE}$, because more employment in manufacturing increases knowledge diffusion and reduces unemployment in both sectors. Let φ_2 denote the speed of this move to write:

$$\dot{L}^M = \varphi_2 \left[\frac{\varepsilon \phi}{\mu L - \varepsilon \phi L^R} L^{R2} - L^M \right] \quad (\text{C2})$$

According to Olech's theorem¹⁷ the equilibrium of the system of the two differential equations (C1) and (C2) is asymptotically stable; if:

¹⁷ See Olech (1963)

$$\text{i)} \quad \frac{\partial \dot{L}^M}{\partial L^M} + \frac{\partial \dot{L}^R}{\partial L^R} < 0 \quad (\text{C3})$$

By (C1) and (C2):

$$-\varphi_1 - \varphi_2 < 0 \quad \text{q.e.d.} \quad (\text{C4})$$

And secondly:

$$\text{ii)} \quad \frac{\partial \dot{L}^M}{\partial L^M} \frac{\partial \dot{L}^R}{\partial L^R} - \frac{\partial \dot{L}^M}{\partial L^R} \frac{\partial \dot{L}^R}{\partial L^M} > 0 \quad (\text{C5})$$

satisfies, if:

$$\varepsilon L^R < \frac{\mu}{\mu \left[\lambda \left(\frac{1}{\alpha} - 1 \right) + 1 \right] - 1} \quad (\text{C6})$$

The term εL^R is part of the arrival rate of an innovation. Hence, the equilibrium is stable for a moderate rate of technological progress.

Because of $\partial \dot{L}^R / \partial L^R = -\varphi_1 < 0$ and $\partial \dot{L}^M / \partial L^M = -\varphi_2 < 0$ the solution of the differential system is a stable focus, shown in figure A1.

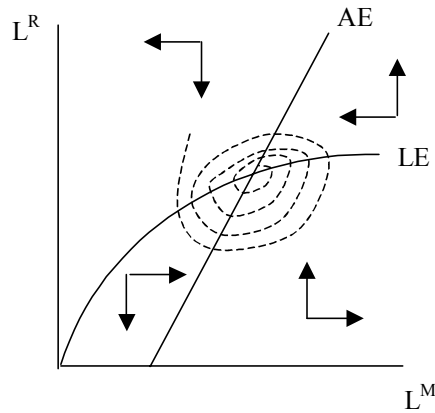


figure A1

Notation

α	production elasticity with respect to x
Γ	surviving time of a manufacturing vintage firm in technological units
ε	number of innovations per period
ϕ	ratio of dismissed workers to total labor force induced by an innovation
λ	size of an innovation in technological units
μ	productivity-parameter of knowledge diffusion
π^M	profits of a manufacturing vintage firm per period
π^R	profits of an innovator per period
$\tau \in [0, \tau^{\max}]$	vintage indicating the number of previous innovations
$\bar{\tau} \in [0, \tau^{\max}]$	a particular vintage
τ^{\max}	innovation indicating the current vintage
τ^{\min}	innovation indicating the vintage producing with the minimum level of knowledge
ω	technology-adjusted wage
\tilde{A}_τ	productivity-parameter of a vintage manufacturing firm
A_i	level of know-how of individual i
A_τ	level of codified knowledge
C	fixed costs of the production of the intermediate good
$D_i \in [0, 1]$	share of workers endowed with the maximum level of know-how
$F_{\bar{\tau}}$	fixed costs in the manufacturing sector for a particular vintage
g_A	rate of technological progress
$i = 1, 2, \dots, L$	index of individuals
h	average level of human capital
H	aggregate stock of human capital
L	total labor force
L^M	labor in the manufacturing sector
L^M_{\min}	minimum employment in manufacturing to enforce R&D activities
L^R	labor in the R&D sector
p	price for the intermediate good

q	total costs per period in a vintage manufacturing firm
r	interest rate
S	surviving time of a vintage manufacturing firm in time units
t	time index
\bar{t}	time, when a particular vintage enters the market
T	time, when a particular vintage leaves the market
U^+	number of dismissals per period
U^-	number of jobs created per period
u	unemployment rate
v	individual utility
V	expected value of a research facility
w	nominal wage
w^{max}	maximum level of the wage rate inducing the market exit for a particular vintage manufacturing firm
x	intermediate good
y	output per capita
Y_τ	vintage output
A_0, w_0	initial values of the arguments
g_A^*, L^{M*}, L^{R*}	any equilibrium values
ϕ_h, ϕ_Γ	first derivatives of the dismissal's rate with respect the arguments