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A Simple Model
of
Optimal Exploration for Oil
- Part I -
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Serie A - Nr. 151

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Short Abstract: Based on empirical evidence, a simple model of oil exploration and discovery is developed, which emphasizes the stochastics of the discovery process and its informational aspects. Open-loop-feedback, m-measurement-feedback and closed-loop optimal exploratory strategies are derived. In forthcoming Part II, the model behaviour when interfaced with a market mechanism will be analysed.

Keywords: Natural resources, oil exploration, stochastic control, Bayesian analysis.

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I. Introduction

As evidenced in DASGUPTA and HEAL (1979) for example, recent analytical models of optimal resource exploitation tend to relax the restrictive assumption of a known, fixed resource stock. Even in more advanced models, however, no explicit attention is given to the dynamics of resource discovery (an exception being the model of DERZKO and SETHI (1979/80)). The (expected) exploitable reserves are simply taken to be a smooth function of human effort (for example, of the exploration costs).

For most natural resources, however, the relationship between exploratory effort and discovery, as well as the discovery process itself are essentially discrete and probabilistic in nature. In general, resources are not homogeneously distributed on the surface of the earth. Their concentration may be high in a number of geographically well-delimited areas (i.e., in deposits) of varying sizes, negligible elsewhere. Thus, at a given moment of time and for a wide range of relative prices and techniques, only those resources contained in known deposits constitute the exploitable reserves. If successful, additional exploratory effort will cause a discrete increase (jump) of random size in the known reserves at the moment of the discovery of a new deposit. Between two discoveries and when there is no exploration, the reserves remain unchanged.

There are, of course, special situations when the assumption of a smooth relationship between human effort and exploitable

reserves will be an appropriate working hypothesis. In general, this assumption appears to be more justified in a macroeconomic or global context than in a microeconomic or regional one. When exploratory activity is simultaneously conducted in a number of areas and when the size of single deposits is small relative to the total resource base, discovery can on the average be realistically approximated as a continuous, deterministic process, although the discovery process in a single area or the discovery pattern for a single operator may definitively be discrete and random.

In many cases, however, the dynamics of discovery may be central to an economic analysis of resource exploitation. Indeed, since short-run phenomena can influence both the economy's short-run dynamic behaviour and its long-run equilibrium, the approximation of the actual discovery process by a continuous relationship may lead to completely invalid analytical results. This point is strikingly demonstrated by a recent paper of DERZKO and SETHI (1979/80), which uses, however, a rather rudimentary description of the discovery process.

In this note we derive and analyse an optimal exploration strategy for oil and the resulting (random) discovery process. Specifically, we try to shed some light on the way past discoveries can be used to update our probabilistic knowledge of future discovery patterns. Furthermore, we demonstrate the value and use of expected future information in closed-loop optimization. In the present Part I, it is assumed that the relative prices of oil and of exploratory

effort are known and exogenously given. In the follow-up paper Part II we interface the optimal exploration/discovery process described in Part I with a model of the oil market.

The main purpose of this note being to demonstrate a line of attack more than to derive results directly applicable to policy analysis, we kept the model simple enough to allow analytical treatment. In spite of its simplicity, however, we believe that the model renders fairly well the central features and statistical regularities of the actual oil discovery process. Nevertheless, there is no doubt that an actual policy model along these lines would be incomensurably more complex and tractable only through numerical analysis.

Acknowledgements: Most of the empirical evidence used in this note was found in BAROUCH and KAUFMAN (1977), which also suggests the approach followed here, and in the literature quoted there. Important references are also GILBERT (1976) and GRAYSON (1960).

II. The stochastics of oil exploration: Empirical evidence

Oil exploration typically proceeds in the following manner. A preliminary phase of information gathering through geophysical surveying aims at identifying prospects, i.e. geological configurations presumed to contain exploitable reserves of hydrocarbons. Given a prospect, the only way to get definitive information about the spatial distribution of oil within the prospect (i.e. in particular: the only way to know for certain if there is exploitable oil at all) is to drill exploratory wells or wildcats into it. A successful wildcat leads to the discovery of a pool or field of finite size. Once a field is discovered, however, limited additional information gathering in general permits a fast and fairly accurate delimitation of the field's size and geographical location. Further exploratory activity accordingly concentrates on the part of the prospect where no fields have been discovered to date.

Empirical evidence suggests that the size distribution of the fields and the discovery sequence possess certain fundamental statistical regularities which can be approximated as follows:

Property I: Each prospect contains a finite number N of fields of size g^1, \dots, g^N . The sizes g^1, \dots, g^N are values of mutually independent, identically distributed, lognormal random variables.

Property II: Discovery may be viewed as sampling without replacement and proportional to size.

Application of exploratory drilling effort to a prospect yields an unfolding sequence of discoveries of randomly varying sizes, at irregularly spaced points of time, which can be viewed as the realisation of a discrete random process. On the average, large pools will be discovered first; continued exploration in a given prospect will lead to increasingly smaller expected discovery sizes.

It should be noted that more than half of the world's proven oil reserves are contained in 37 giant fields out of a total of approximately 5000 known fields. There is evidence that the traditionally exploited prospects still yield a large number of undiscovered fields (none of them presumably large), as shown by the results of renewed exploratory activity on these prospects following the first oil shock of 1974.

III. The model

Consider the following concrete situation. A firm or decision-maker DM possesses exclusive operating rights on a given prospect. Based on the result of previous geophysical surveys, DM has a probabilistic conception of the number of fields in the prospect and of the field's size distribution, expressed by an a priori probability density function. At moment t , $t = 0, 1, 2, \dots$, DM can decide to drill or not to drill an exploratory well. If the wildcat is successful, it will be followed by subsequent exploration

to determine the size and geographical limits of the field discovered. The information generated by a wildcat (successful or not) and, if appropriate, by the subsequent exploration can be used to update the DM's probabilistic knowledge of the prospect. At $t + 1$, DM must again decide whether to drill or not, etc. As previously stated, we are interested in defining an optimal strategy for exploratory drilling and the informational aspects of this strategy.

For modelling purposes we assume that the size and geographical limits of a pool are exactly observed immediately after the pool is discovered. Further exploratory activity therefore takes place exactly on the surface area of the prospect which does not lie "over" already discovered fields, i.e. on the not-yet-explored part of the prospect. On the average, the size of a field (i.e. its content in oil units) is supposed to be one-to-one proportional to its surface area after proper norming. The distribution of oil fields over the prospect is homogeneous, i.e. a wildcat has the same probability of success no matter where the drilling occurs within the prospect.

The above assumptions are reasonable first-order approximations of the real world and can be easily relaxed. However, in order to obtain an analytical solution we make two less justifiable assumptions:

- a) The total number of fields in the prospect, N , is known a priori.
- b) The subjective a priori size distribution is given by a β -density function.

Assumption b) implies that the size of every field in the prospect lies between the (normed) values 0 and 1. Together with the assumption of direct proportionality between field's surface and field's size, it implies that the total surface area of the fields within a prospect do not exceed the prospect's surface area.

We can now state the formal model. Let

$t = 0, 1, 2, \dots$ = time index

S_t = surface area of prospect relevant for exploration at time t (not-yet-explored surface area in t) = S_0 less the cumulated surface area of discoveries up to t , S_0 given

N_t = number of pools in S_t , N_0 given

g^j = size of pool j , $j = 1, \dots, N_0$, $0 < g^j < 1$, $\sum_{j=1}^{N_0} g^j \leq S_0$.

c_t = relative cost of drilling a wildcat at time t ,

$c_t > 0$, given

u_t = decision in t , $u_t = \begin{cases} 1 & \text{drill} \\ 0 & \text{do not drill} \end{cases}$

h_t = success of a drill in t , $h_t = \begin{cases} 1 & \text{oil discovered} \\ 0 & \text{no oil} \end{cases}$

(by convention $h_t = 0$ whenever $u_t = 0$)

d_t = size of discovery in t , $0 \leq d_t < 1$

r = discount factor, given (may be time-dependent)

$I_t = \{(u_0, h_0, d_0), (u_1, h_1, d_1), \dots, (u_{t-1}, h_{t-1}, d_{t-1})\}$ = information state in t

J_t = joint distribution or "statistics" of the future observations $(u_{t+i}, h_{t+i}, d_{t+i})$, $i=0, \dots, T-t$, conditional on I_t

T = planning horizon, $T < \infty$.

$f_{g^j}^t(\cdot / \theta_t)$ = density of g^j given I_t

The given a priori densities $f_{g^j}^0$ are independent, identical β -densities with parameter $\theta_0 = (a_0, b_0)$,

$$f_{g^j}^0(\cdot) = f_g^0(\cdot) = \begin{cases} \rho(a_0, b_0) g^{a_0-1} (1-g)^{b_0-1}, & 0 < g < 1, \\ 0 & \text{otherwise,} \end{cases}$$


with $a_0 > 0$, $b_0 > 0$, $\rho(a_0, b_0)$ a normalizing constant.

All probabilities are subjective unless otherwise stated. The variables c_t , d_t , g^j are properly normed and defined in "oil units." As previously noted, we assume that g^j is directly (one-to-one) proportional to the surface area of pool j , and that size and location of pool j are exactly measured immediately after pool j is discovered. Without loss of generality we use the following notational convention: Through reordering of the index set $\{j\}$, the field discovered at the n -th successful wildcat will be labeled after it is discovered as field $N_0 - n + 1$, i.e. equivalently as field N_t . The $N_0 - n$ not-yet-discovered fields are accordingly indexed with $j=1, \dots, N_0 - n$.

We are interested in the sequential exploratory drilling strategy which maximizes the expected revenue W_0 defined by

$$W_t = E \left[\sum_{\tau=t}^T r^{\tau-t} u_{\tau} \cdot (d_{\tau} - c_{\tau}) \right] := E \left[\sum_{\tau=t}^T r^{\tau-t} U_{\tau} \right],$$

where U_{τ} designates the undiscounted period revenue $u_{\tau} \cdot (d_{\tau} - c_{\tau})$.

The possible process behaviour between $t-1$ and t , $0 < t \leq N$, is described graphically in Figure 1, where  indicates a probabilistic transition.

IV. Basic properties and updating equations.

Objective probabilities and process behaviour

At time t , the not-yet-explored part of the prospect, S_t , contains N_t fields of given sizes g^1, \dots, g^{N_t} . The objective probability of discovering one of these fields, say field j , $1 \leq j \leq N_t$, with a wildcat in period t is

$$\bar{\alpha}_t^j = \frac{g^j}{S_t} .$$

By the homogeneity and independence assumptions, the probability $\bar{\alpha}_t^j$ does not depend on where within the not-yet-explored surface area the wildcat is drilled.

The objective probability $\bar{\alpha}_t$ of hitting a pool at all with a wildcat in period t is accordingly

$$\bar{\alpha}_t = \frac{\sum_{j=1}^{N_t} g_j}{S_t} .$$

The updating equations for $\bar{\alpha}_t^j$ and $\bar{\alpha}_t$ are given by

$$\bar{\alpha}_{t+1}^j = \frac{g^j}{S_t - d_t} , \quad j < N_t , \quad \bar{\alpha}_{t+1} = \frac{\sum_{j=1}^{N_t} g^j - d_t}{S_t - d_t} .$$

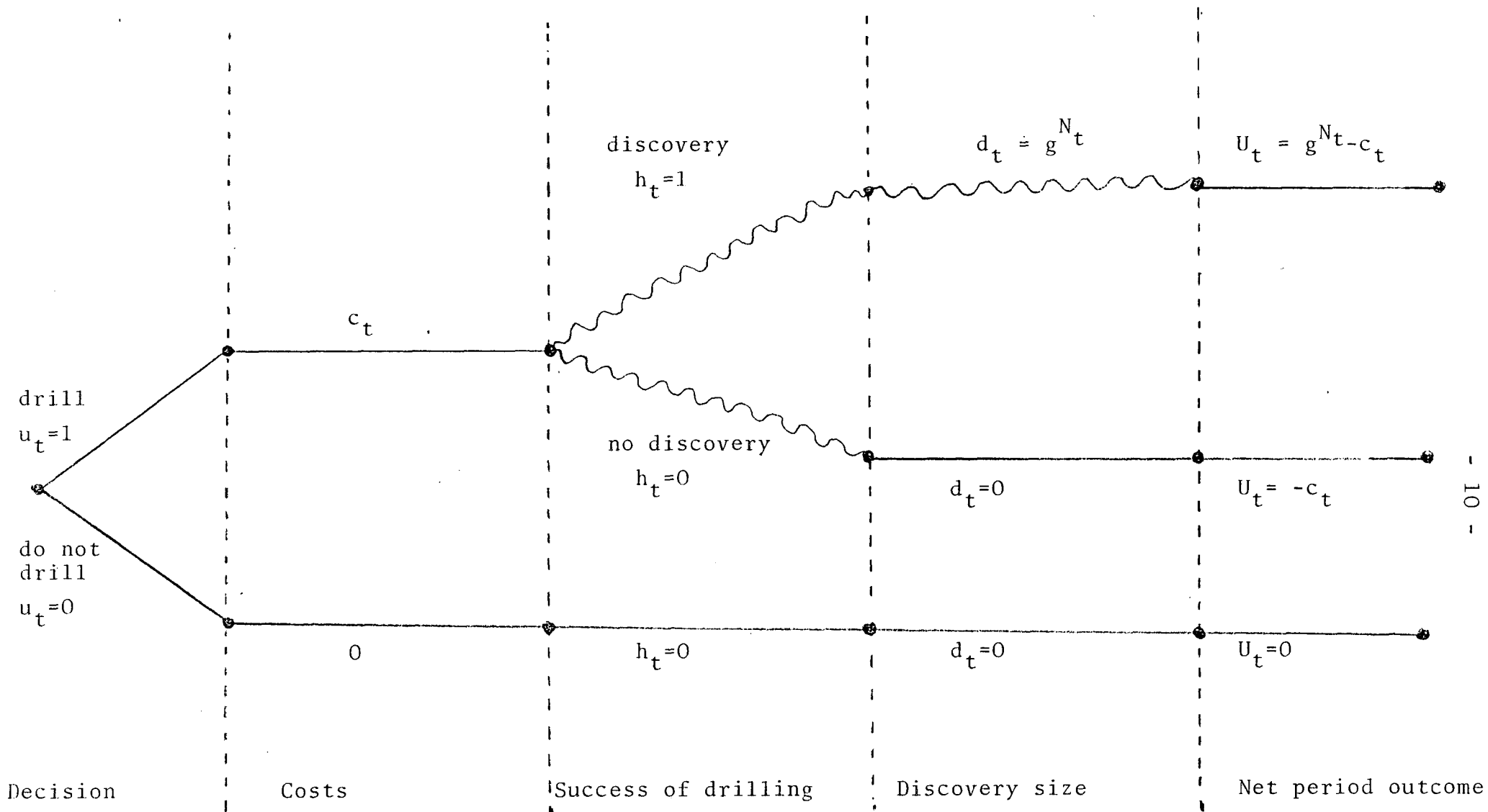


Figure 1

In the case of an unsuccessful wildcat or no wildcat, $d_t=0$ thus $\bar{\alpha}^j$ and $\bar{\alpha}$ remain unchanged,

$$h_t = 0 \Rightarrow \bar{\alpha}_{t+1}^j = \bar{\alpha}_t^j, \quad \bar{\alpha}_{t+1} = \bar{\alpha}_t.$$

When the wildcat is successful, $d_t = g^{N_t} > 0$ by convention.

Thus, except in the

border case $\sum_{j=1}^{N_t} g^t = S_t$, the probability of discovering a

new pool decreases with every past success. The probability of discovering a given not-yet-discovered pool, however, increases.

Let's assume $h_{t-1} = 1$. The objective behaviour of h_τ , $\tau \geq t$, until the next discovery ($h_\tau=1$) follows a Bernoulli process with (objective) probability mass function

$$p_h(h_\tau) = \begin{cases} 1 - \bar{\alpha}_t & , & h_\tau = 0 & , \\ \bar{\alpha}_t & , & h_\tau = 1 & , \end{cases}$$

whose properties are well known from elementary probability theory. In particular the random variable k_t which describes the number of wildcats between a discovery in $t-1$ and the next discovery, i.e. the interarrival time, is defined by the geometric probability mass function

$$p_k(k_t) = \bar{\alpha}_t (1 - \bar{\alpha}_t)^{k_t}, \quad k_t = 1, 2, \dots$$

The objective discovery pattern over $t=1, 2, \dots$ is described by a sequence of interconnected Bernoulli processes with non-increasing success probability. An analysis of the statistical properties of this sequence remains to be done.

Subjective probabilities

The g^j are not known to DM, who has only a probabilistic perception of the size distribution in the form of a subjective density function. In $t=0$, the subjective density function is by hypothesis β with parameter $\theta_0 = (a_0, b_0)$. The subjective probability α_0 of discovering a pool with the first wildcat is therefore

$$\alpha_0 = \frac{E \left[\sum_{j=1}^{N_0} g^j \right]}{S_0} = N_0 \cdot \left[\frac{a_0}{a_0 + b_0} / S_0 \right].$$

The expected discovery size at the first wildcat, given that the wildcat is successful, is

$$\gamma_0 = E \left[g^{N_0} / \theta_0, u_0=1, h_0=1 \right] = \frac{a_0}{a_0 + b_0}$$

and the expected discovery size before it is known if the wildcat is successful or not is

$$\kappa_0 = E \left[g^{N_0} / \theta_0, u_0=1 \right] = \alpha_0 \cdot \frac{a_0}{a_0 + b_0} = N_0 \cdot \left[\left\{ \frac{a_0}{a_0 + b_0} \right\}^2 / S_0 \right].$$

The fact that a wildcat is successful or not represents useful information in itself that can be used to revise the a priori probability density function f_g^0 . We consider here the case of a Bayesian updating of f_g^0 . Note that from DM's point of view the outcome of the first trial corresponds to the outcome of a Bernoulli trial with success probability α_0 . Since β -density and Bernoulli process are natural conjugates, one obtains immediately for the updated densities $f_{g^j}^t$, $t=1,2$,

The f_j^t , $j=1, \dots, N_t$, are independent, identical β -densities f_g^t with parameter $\theta_t = (a_t, b_t)$ given by

$$a_t = a_{t-1} + h_{t-1}, \quad b_t = b_{t-1} + u_{t-1} - h_{t-1},$$

see e.g. RAIFFA and SCHLAIFER (1961), p. 53. The simplicity of this result is indeed the one and only reason why we assumed f_g^0 to be β -densities. Note that the probabilities α do not play any role in the updating process.

In our model, a successful wildcat also supplies additional information on the size and location of the field discovered. This information has no influence on the updating of f_g^t . However, it permits us to update S_t and N_t according to the formula

$$S_{t+1} = S_t - d_t, \quad N_{t+1} = N_t - h_t.$$

These are none other than the definitory equations for S_t and N_t . The probabilities and expectations α , γ , κ are updated by using the new values of S , N and f_g in the corresponding formulas.

Summarizing, the updating process can be verbally characterized as follows for $t=0,1,2, \dots$:

- no wildcat ($u_t=0$, $h_t=0$): no new information, no updating.
- wildcat ($u_t=1$) but unsuccessful ($h_t=0$): only the size distribution is affected, $b_{t+1} = b_t + 1$.
- successful wildcat ($u_t = 1$, $h_t = 1$):
 - a) the size distribution is updated, $a_{t+1} = a_t + 1$.
 - b) the surface area of the discovered field is "subtracted" from the not-yet-explored part of the prospect,

$S_{t+1} = S_t - d_t$ (no exploratory drilling will be made anymore on the surface area of the field discovered)

- c) The probabilities and expectations α , γ and κ are updated according to the new values of f_g , S and N .

Following an unsuccessful wildcat, the probabilities and expectations α , γ and κ are always revised downwards. A successful wildcat always leads to an upward revision of γ . At the same time, however, it decreases the not-yet-explored surface area by d , $0 < d < 1$, and the remaining number of fields by 1. If d is small enough, $d < \bar{d}(\Delta\gamma)$, κ will be revised downwards. Similarly, a success leads to a downward updating of α for $d < \bar{d}(\Delta\gamma)$, $\bar{d} < \bar{d}$.

Note that the updated densities f_g , probabilities α and expectations γ and κ form the "best approximation" of the real size distributions, probabilities and expectations available to DM at any moment of time.

V. Optimal Control Solutions

We now consider DM's problem:

"Determine the exploratory strategy which maximizes the revenue W_0 as defined in section III, subject to the oil exploration model developed in sections III and IV."

For the solution of this stochastic control problem only the subjective probabilities are of relevance, since the objective ones are unknown to DM.

Closed-loop-optimal control

The "true" solution of DM's problem is known as the closed-loop-optimal (CLO) solution of the problem.

(This solution may not be computable in practice thus making it necessary to introduce other solution concepts).

The CLO control in t , u_t^{CLO} , is a function of the statistics J_t of the future observations as well as of the information state I_t . In other words, in the determination of u_t^{CLO} the fact is taken into account that an optimal exploratory program will be pursued over $t = t, \dots, T$. The CLO control anticipates in a statistical sense the results of the future exploratory program; it assumes that "the observation loop will remain closed" over the planning horizon. The optimal value of the corresponding functional equation of dynamic programming, which yields the (closed loop) optimal revenue and defines the corresponding optimal policy over $t = t, \dots, T$, is given by

$$V_t^{CLO} = \max_{u_t} \left\{ E \dots \max_{u_{T-1}} E \left\{ \max_{u_T} E[W_t/I_T]/I_{T-1} \right\} \dots / I_t \right\},$$

see BELLMAN (1961). The nested expectations and minimizations assure that the future exploratory program is taken into consideration. Whenever a control is computed, the expectation of the reward conditioned on the available information state is first obtained: the expectation is over the subsequent observations which are "averaged out." Since this is done at every step, the resulting control depends, as required,

on the statistics of the future exploratory outcome.

As first shown by JOSEPH and TOU (1961) in the case of an additive revenue function, V_t^{CLO} can be rewritten in the form

$$V_t^{CLO} = \max_{u_t} E \left[U_t + r V_{t+1}^{CLO} / I_t \right],$$

i.e. for our particular problem

$$V_{T+1}^{CLO} = 0$$

$$V_T^{CLO} = \max_{u_T} E[u_T(d_T - c_T) / I_T] = \max \begin{cases} 0 \\ \kappa_{T-1}^{-c_T} \end{cases},$$

$$V_{T-1}^{CLO} = \max_{u_{T-1}} E[u_{T-1}(d_{T-1} - c_{T-1}) + r V_{T-1}^{CLO} / I_{T-1}] =$$

$$= \max \begin{cases} 0 + \max r \begin{cases} 0 \\ \kappa_{T-1}^{-c_T} \end{cases} \\ \kappa_{T-1}^{-c_{T-1}} + r \left[\gamma_{T-1} \max \begin{cases} 0 \\ \kappa_{T-1}^{+c_T} \end{cases} + (1 - \gamma_{T-1}) \max \begin{cases} 0 \\ \kappa_{T-1}^{-c_T} \end{cases} \right] \end{cases}$$

where

$$\kappa_{T-1}^{+} = N_{T-1}^{-1} \left(\frac{a_{T-1} + 1}{a_{T-1} + 1 + b_{T-1}} \right)^2 / \left(S_{T-1} - \frac{a_{T-1}}{a_{T-1} + b_{T-1}} \right),$$

$$\kappa_{T-1}^{-} = N_{T-1} \left(\frac{a_{T-1}}{a_{T-1} + b_{T-1} + 1} \right)^2 / S_{T-1},$$

and so on for T-2, T-3, etc. The general structure of V_t^{CLO} is presented in Figure 2.

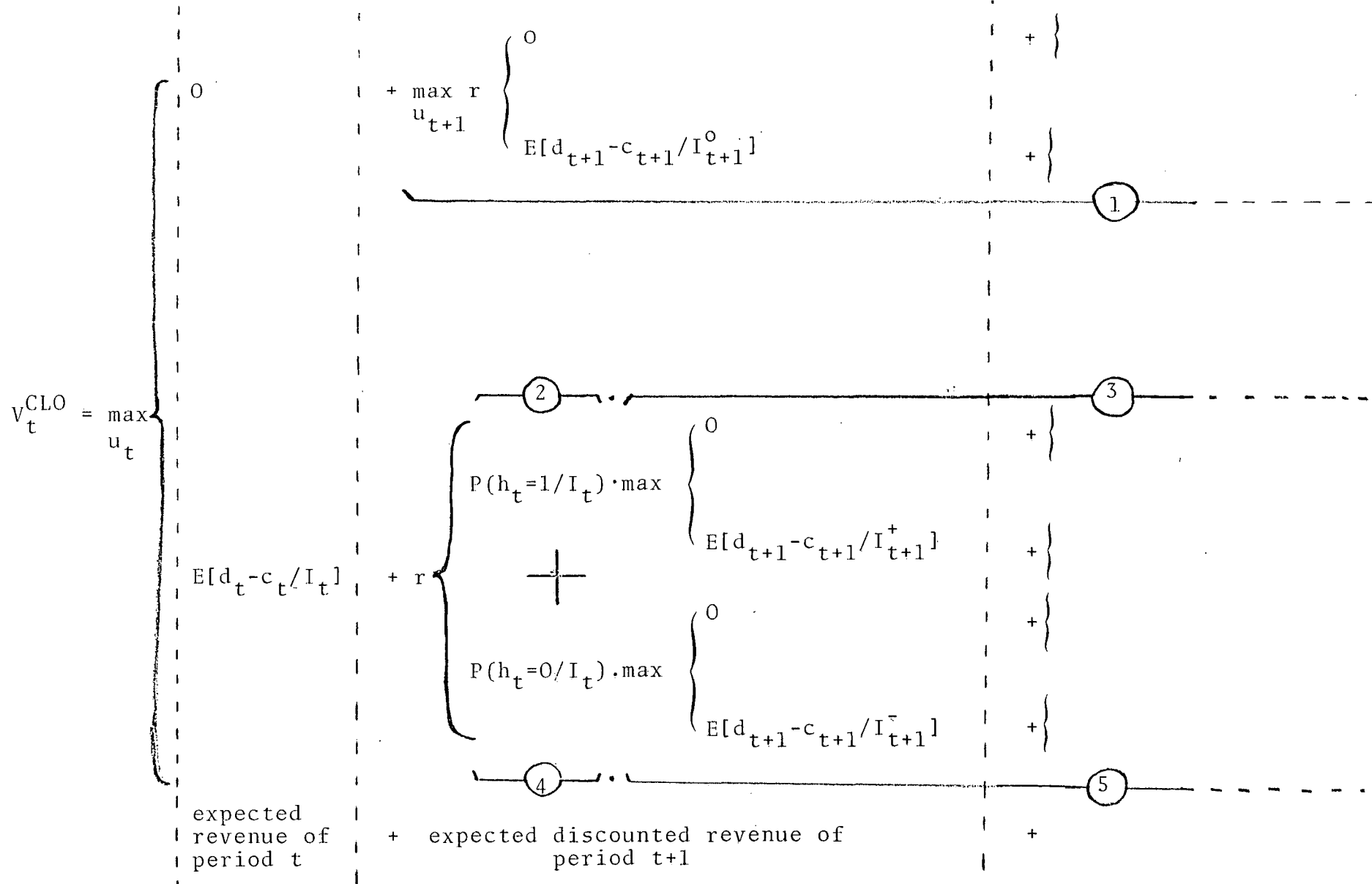


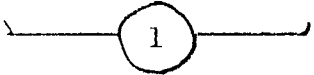




Figure 2

where

$$I_{t+1}^0 = I_t \text{ since } u_t = 0$$

$$I_{t+1}^+ = I_t U\{u_t = 1, h_t = 1, d_t = E\{g/I_t\}\}$$

$$I_{t+1}^- = I_t U\{u_t = 1, h_t = 0\}$$

-  = expected value of an optimal exploration program over $t=t+1, \dots, T$ given $h_t = 0$
-  = probability that a wildcat in t will be successful
-  = expected value of an optimal exploration program over $t=t+1, \dots, T$ given an "average" success of the wildcat in t
-  = probability that a wildcat in t will be unsuccessful
-  = expected value of an optimal exploration program over $t=t+1, \dots, T$ given that a wildcat in t was unsuccessful

OLFO and mFO control

The expression V_t^{CLO} obviously becomes rapidly unmanageable with increasing futurity $T-t$; that is, the CLO solution is impractical for large planning horizons T . We are therefore led to consider two other solution concepts:

- The open-loop-feedback-optimal (OLFO) decision in t , u_t^{CLO} , assumes that no exploration program will be conducted after $t-1$. The optimal decision u_t^{OLFO} is thus independent of J_t and defined as a function of I_t only,

$$u_t^{OLFO} = u(I_t, t) \quad .$$

- The m -measurement-feedback-optimal (mFO) decision in t , u_t^{mFO} , is based on the assumption that an optimal exploration program will be conducted over exactly the m next periods. Accordingly, it is defined as a function of I_t and of the common distribution J_t^m of (u_τ, h_τ, d_τ) , $\tau = t, \dots, t+m-1$, conditional on I_t , only:

$$u_t^{mFO} = u(I_t, J_t^m, t) \quad .$$

In general, of course, OLFO optimization leads to a "worse" solution than CLO optimization. The concept of mFO optimization, first introduced by CURRY (1969), aims at defining "best computable approximations" of the CLO solution.

The decisions u_t^{OLFO} , $t=0, \dots, T$, are obtained by solving

$$\max_{u_t} E \left[\sum_{\tau=t}^T r^{\tau-t} U_{\tau} / I_{\tau} \right],$$

i.e. here

$$\max_{u_t} E[U_t / I_t] = \max \begin{cases} 0 & (u_t=0) \\ \alpha_t \cdot \frac{a_t}{a_t + b_t} - c_t & (u_t=1) \end{cases}$$

since u_t does not influence the future of the process except for the updating of α and f_g , which is not anticipated in the case of OLFO control. By the same independence argument one easily recognizes that u_t^{mFO} follows from the maximization of the first m additive terms in V_t^{CLO} for $m \geq T-t$, of all its terms for $m < T-t$.

Direct comparison of the three types of solution is fully sufficient to reveal the informational advantages (in the increase in the solution's complexity) resulting from taking into consideration the expected results of a future exploratory program. From the non-negativity and separability of the additive terms in V_t^{CLO} one derives immediately $u_t^{CLO} = 0 \Rightarrow u_t^{mFO} = 0 \Rightarrow u_t^{OLFO} = 0$. In other words, increasing anticipation of future exploration activity can only "encourage" present exploration.

Basic properties

In the follow-up article Part II we shall study the properties of the optimal sequence $\{u_t^{mFO}\}$ when previous discoveries influence the relative oil price over a market mechanism (i.e. equivalently, the relative exploration costs c_t). At this point we would like to point out two basic properties of $\{u_t^*\}$ which can be derived by direct examination of figure 2

- For all types of optimal solutions $*$ (OLFO, mFO or CLO),
 $u_t^* = 0 \Rightarrow u_{t+\tau}^* = 0$ for all $\tau > 0$ whenever $c_{t+\tau} \geq c_t$.
- Let's assume $u_t^* = 0$, $c_t > 0$ and $N_t > 0$. Then there exists $0 < \bar{c} < c_t$ such that $u_\tau^* = 0$ in every period τ with $c_\tau > \bar{c}$, and $u_\tau^* = 1$ in at least one period τ with $c_\tau \leq \bar{c}$, $t < \tau < T$.

That is, as long as there is at least one non-discovered field, there exists a relative price level \bar{c} at which resuming exploratory activity is optimal, whatever the past process history may be. Note that this does not imply that at price level \bar{c} exploratory activity will be maintained until a field is discovered.

Choice of the planning horizon

In the formulation of the DM's optimization problem we assumed a given, finite planning horizon T . The optimization's result, however, obviously depends on the choice of T . An arbitrary T might be economically unjustified since, for example,

continued exploration after T could lead to an increase in the total expected revenue.

The problem, however, is not as serious as one might think. Consider first mFO optimization. Then, we obviously have $u_t^{\text{mFO}}(T) = u_t^{\text{mFO}}(\bar{T})$, $T \leq \bar{T} < \infty$, when $T - t \geq m$. In other words, the solution obtained by using mFO optimization to solve a sequence of DM's problems with starting times $t_0 = 0, 1, 2, \dots$, and planning horizons $t_f = t + m$ is not dominated by any mFO solution of the original DM's problem. Thus, in mFO optimization there is a "naturally optimal" T which can be trivially determined.

Consider now CLO optimization. It is easy to show the following. If $c_t = \bar{c} > 0$ for $t \geq \bar{t}$, then there exists $\bar{T} < \infty$ such that $W_0^{\text{CLO}}(T) \geq W_0^{\text{CLO}}(T')$ for $T \geq \bar{T}$, T' arbitrary. That is, although the optimal stopping time T_f (the time, when exploration should stop forever, when W_0 is to be maximized with regard to $\{u_t\}$ and T_f) is a random variable, we can assume without loss of performance in the optimizations a deterministic planning horizon T , when T is "large enough." (This result implies that no realisation of T_f can be greater than \bar{T}). To prove the proposition, let's assume without loss of generality $\bar{t} = 0$. The additive term in V_0^{CLO} corresponding to the period t is of the form

$$P_t \cdot \frac{a_0 + n_t}{a_0 + b_0 + n_t + r_t} - \bar{c} := P_t Z_t \quad \text{for } Z_t > 0.$$

(In this formula n_t designates the number of discoveries and r_t the number of unsuccessful wildcats. Since in the present case, $c_t = \bar{c}$ being a constant, $u_t^* = 0 \Rightarrow u_{t+\tau}^* = 0$, $\tau > 0$, we can assume $n_t + r_t = t + 1$). For $Z_t < 0$, the corresponding additive term

is 0. (The trivial case $Z_t=0$ will not be separately discussed here). By assumption $n_t \leq N_0 < \infty$ for all t . Obviously Z_t reaches its maximum $Z_t^{\max}(r_t)$ for $n_t=N_0$ and there exists a value $\bar{r} < \infty$ of r_t such that $Z_t^{\max}(\bar{r}) < 0$. In other words, $Z_t < 0$ for $t > \bar{r} + N_0$. It is clear that whatever the process behaviour has been between 0 and τ , τ arbitrary, the additive term in V_τ^{CLO} corresponding to periods $t > \bar{r} + N_0$ are also 0. Thus, $u^* = 0$ with certainty from period $\bar{r} + N_0 + 1$ on, i.e. $\bar{T} = \bar{r} + N_0 + 1$.

The validity of the proposition crucially depends on the assumption of a constant value \bar{c} for c_t from a finite point of time \bar{t} on. If one allows e.g. c_t to rise for arbitrarily long periods of time, exploration could stop for equally arbitrarily long periods of time before possibly resuming. Much less trivial is the behaviour of $\{u_t^*\}$ when $\{c_t\}$ is slowly decreasing. A theorem on general conditions which assure the existence of \bar{T} remains yet to be derived.

VI. Conclusions

The model presented here and its optimal solution possess a very simple formal structure, thus making them amenable for analytical treatment. Nevertheless, all the properties of the model and the solution are plausible ones. They appear to reflect the real process of oil discovery within a given prospect well, at least qualitatively. In particular, the model leads to the conclusion that a sufficient increase in

the relative price of oil makes resuming exploration in old prospects economically viable, unless the decision-maker believes the prospect is completely exhausted.

The arbitrary assumption, the DM's a priori density function is β , does not appear to be damaging with respect to the plausibility of the results. Indeed, it is common in pre-posterior analysis to approximate an empirically more justified distribution by another one in order to gain the computational advantages presented by conjugate distributions. In general, this does not mean any serious loss of performance since the empirical evidence and DM's a priori knowledge are characterized by a great degree of uncertainty. Nevertheless, we must admit that we did not study the robustness of the results with respect to the choice of an a priori distribution.

Similarly, the assumption of a known number of fields, while obviously unsatisfactory, does not seriously impair the practical value of the model. This may reflect the fact that in reality the decision-maker's a priori knowledge encompasses more than just the size distribution and certainly extends in some way to the number of fields. Here again, however, we did not study the way in which the assumption influences the results. Furthermore, we do not know how to relax it and still obtain an analytical solution.

Numerous elaborations can be thought of to increase the model's plausibility and its value for policy analysis. One can, for example, weaken the assumption of an exact and immediate measurement of the fields discovered; allow the

discovery probability to vary within the prospect, reflecting differentiated information from geographical surveys; explicitly consider optimal exploitation patterns, stockage possibilities and different oil qualities; allow more than one wildcat to be drilled at any moment of time and introduce a "production function" for wildcats; consider several prospects at once; finally, introduce adjustment costs reflecting the fact that it is costly to build up and, in inactivity periods, to maintain an exploration team. Most of these extensions, however, are either rather trivial and should not significantly change the qualitative results, or will be amenable only by numerical analysis.

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