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INCLUDING A GOVERNMENT BUDGET
RESTRAINT IN STANDARD
MACROECONOMIC ANALYSIS:
SOME FISCAL AND MONETARY
POLICY IMPLICATIONS

HANS G. MONISSEN

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Including a Government Budget Restraint in
Standard Macroeconomic Analysis:
Some Fiscal and Monetary Policy Implications*

Recently, some economists have argued that traditional (Keynesian) macro-analysis has not treated the bond-finance and new money-finance cases of government deficits appropriately, which has led to incorrect conclusions regarding the multiplier effects of government spending activities.¹⁾ The argumentation pertains to two different, but, nevertheless, related issues: Firstly, it is the analytical consequence of the valid observation that the world has generally refused to behave according to the guidelines provided by the standard Keynesian paradigm. The failure of the

* The author is indebted to his colleague Nikolaus Läufer for helpful comments.

1) See, for example, Carl F. Christ, "A Short-Run Aggregate Demand Model of the Interdependence and Effects of Monetary and Fiscal Policies with Keynesian and Classical Interest Elasticities," *American Economic Review, Papers and Proceedings*, Vol. 57, 1967, pp. 434-443; Carl F. Christ, "A Simple Macroeconomic Model with a Government Budget Restraint," *Journal of Political Economy*, Vol. 76, 1968, pp. 53 - 67; David J. Ott and Attiat F. Ott, "Budget Balance and Equilibrium Income," *Journal of Finance*, Vol. 20, 1965, pp. 71-77; William L. Silber, "Fiscal Policy in IS-LM Analysis: A Correction", *Journal of Money Credit and Banking*, Vol. 2, 1970, pp. 461-472. Our discussion will be mainly related to these papers. For further references to previous efforts to account for the impact of the budget balance compare especially the papers by the Ottts and Silber.

Keynesian model is attributed to a faulty specification of the equation system describing the government sector behavior and, specifically, to a general omission of the different impacts of both level and interest induced wealth effects related to the financing of the government budget balance. Secondly, it is the consequence of an effort to correct a longstanding textbook tradition trying to introduce the student into the analytics of the Keynesian system. Under this second issue, it is an open question whether the frame per se provides some useful guidelines for a solution of certain key-policy problems. The effort aims at a logically consistent and analytically coherent presentation of the basic language system.

Unfortunately, the proposals for a correction and reformulation of standard macro-analysis differ fundamentally in the specification of the stock-flow mechanism linking the short-run solution to the dynamic adjustment path over time. These striking analytical differences pose important questions which require clarification. Only one of the studies, the most recent one by Silber, has tried to integrate the financial aspects of a budget balance into the simple textbook analysis of the IS-LM framework. However, Silber's analysis involves a serious stock-flow confusion which invalidates his results. One major concern, therefore, will be to correct Silber's position and to integrate properly the different financial impacts of a government budget balance into the IS-LM analysis.

The following section presents a simple Keynesian model including a government budget restraint and states the major structural properties of the model. We will compare our procedure with those proposed in the literature.

Without a satisfactory theory for the determination of the absolute price level in the context of an aggregate supply theory, we are restricted in the specification of our model by the two Keynesian options: We could assume either that the price level is rigid, an assumption which would correspond to the procedure of the more orthodox Keynesian approach, or we could postulate a given full employment income, an assumption more in accordance with the neoclassical procedure. Relying on these options, we shall analyse in section II and III the major fiscal and monetary policy implications of our model. A short summary is given in section IV.

Two mathematical appendices will complete the paper. In the first, we state the mathematical derivatives on which our analysis is based; in the second, we briefly outline the dynamic implications of our model by solving for the behavior of income over time.

Our model including a government budget restraint is presented in equations (1) - (5) below:

$$(1) \quad \frac{W_0}{p} = \frac{\lambda Y}{i} + \frac{M_0}{p} + \frac{N_0}{pi} - \xi \frac{T}{pi} \quad \begin{array}{l} \text{wealth at the beginning} \\ \text{of the period } 2) \end{array}$$

$$(2) \quad Y = C\left(\frac{W_0}{p}, Y + \frac{N_0}{p} - \frac{T}{p}, \lambda\right) + I\left(i, \frac{W}{p}\right) + \alpha + \frac{G}{p}$$

commodity-market equilibrium

$$0 < C_1 < 1; 0 < C_2 < 1; C_3 < 0; I_1 < 0; I_2 < 0;$$

$$(3) \quad \frac{M_1}{p} = m\left(\frac{W_0}{p}, Y, i, \lambda\right) \quad \text{money-market equilibrium}$$

$$0 < m_1 < 1; 0 < m_2 < 1; m_3 < 0; m_4 < 0;$$

$$(4a) \quad \frac{T}{p} = c + t \frac{N_0}{p} + t Y \quad \text{resp.}$$

$$0 < t < 1; \quad \text{tax function}$$

2) We use the term "period" as a theoretical term analogously to the conception of the Hicksian week as applied for instance in Don Patinkin, Money, Interest, and Prices, second edition, New York 1965, passim.

$$(4b) \frac{T}{p} = \frac{c^*}{p} + t \frac{N_0}{p} + t Y$$

$$(5) \frac{G}{p} + \frac{N_0}{p} - \frac{T}{p} = \frac{M_1}{p} - \frac{M_0}{p} + \frac{N_1}{pi} - \frac{N_0}{pi} \quad \text{government bud-} \\ \text{get restraint}$$

List of symbols

C	consumption expenditure in real terms
$c, \frac{c^*}{p}$	autonomous tax payments in real terms
I	investment expenditure in real terms
i	market interest rate (nominal rate equal to real rate)
M_0	supply of money at the beginning of the period in nominal terms
M_1	supply of money at the end of the period in nominal terms
m	demand for real cash balances
N_0	number of government bonds outstanding at the beginning of the period (equal to current coupon payments in nominal terms)
N_1	number of government bonds outstanding at the end of the period (equal to future coupon payments in nominal terms)
p	absolute price level
T	current tax payments in nominal terms
t	marginal tax parameter
W_0	wealth at the beginning of the period in nominal terms
w	nominal wage rate
Y	gross national product in real terms
$Y + \frac{N_0}{p} - \frac{T}{p}$	disposable income in real terms
α	autonomous investment in real terms
λ	distribution parameter: fraction of nonhuman income to total income
ξ	parameter describing the degree of capitalization of current and future tax liabilities

We introduced our model as of the "Keynesian" type. This classification should be specified. The standard Keynesian position is expressed by an array of income expenditure models which all share some common characteristic features:

The crucial linkage transmitting monetary impulses on the path of economic activity is based on the positions and slope properties of two reduced-form equations, the familiar IS-LM curves. Apart from the minor role of price- and interest-induced wealth effects in the expenditure functions, the standard model connects the monetary sector with the real sector by a single variable, the long term rate of interest. Because it is argued that the magnitude of the relative borrowing cost is a major factor determining the slope of the investment function, this linkage is referred to as "the borrowing cost conception of the transmission mechanism". The second common denominator is defined by the set of assets included. The standard Keynesian model includes several assets, but introduces only two different yields: the rate of return on money, which is set equal to zero, and the long term interest rate. Real capital, government bonds, bank loans, and other private debts are thus regarded as perfect substitutes. ³⁾

The two-asset model is formally introduced in equation (1). The wealth definition includes the market value of all productive units, i.e. firms, - measured as the capitalized value of nunhuman income, the money stock, the government debt outstanding, and the discounted value of current and future tax liabilities. The parameter ξ measures the degree to which the tax liabili-

3) See James Tobin, "Money, Capital and Other Stores of Value," American Economic Review, Papers and Proceedings, Vol. 51, 1961, pp.26-37. For an excellent discussion of standard "Keynesian" economics as distinct from Keynes' own theory compare Axel Leijonhufvud, On Keynesian Economics and the Economics of Keynes: A Study in Monetary Theory, New York 1968.

ties will affect the current wealth position. Our wealth definition, however, can only be accepted as a short-run approximation. Specifically, we should subtract the market equivalent of the "productive" part of the money stock, which is already included in $\frac{\lambda Y}{i}$, from the total money stock. To account for this part would presuppose a much more complicated model, an approach, which is not justified by our further analysis. Because we exclude human wealth from the wealth restraint, it follows that the capitalized value of the current and future tax liabilities is always less than the sum of the capital values of nonhuman income and government coupon payments. A more complete approach would introduce ξ as a variable depending on the tax parameters, coupon payments, total income, and resource components. By assigning zero values to both λ and ξ , we can transform the wealth definition to a form more common in economic analysis.

Equation (2) describes the familiar commodity-market equilibrium and equation (3) the corresponding money market equilibrium. Contrary to the usual Keynesian procedure, we have included wealth-effects into both the consumption function and the demand for money function. This procedure is more general because we can retain the orthodox Keynesian results simply by setting the marginal response coefficients C_1 in the consumption function and m_1 in the money-demand function equal to zero. We should note that the general omission of wealth effects in the standard paradigm precludes any systematic assessment of different operations leading to an increase in the money supply, i.e. increases brought about by fiscal deficits, open-market operations, or pure wealth changes.

The implicitly written investment function in equation (2) includes, in addition to the market interest rate, the real wage rate. This variable clearly affects the marginal efficiency calculus. The usual omission of this variable, even in models including a labor market,

is in no way sensible and can only be rationalized as a consequence of the general downgrading of relative price theory in Keynesian economics.

Because any change in the money supply related to the current financial transactions of the central authorities has to be absorbed into the portfolios of the wealth owners at the end of the period, the demand for money has to absorb the supply magnitude M_1 which includes the money-financed part of the governmental budget operations.

Equations (4a) and (4b) specify two different tax functions. The formulation in (4a) differs from the one in (4b) insofar as the fixed autonomous part will be adjusted according to a variation of the absolute price level.

Finally, equation (5) describes what is referred to as the government budget restraint. Government expenditure for final output and current coupon payments must be covered either by raising taxes, by printing new money, or by issuing new debt.

We shall see that with an appropriate interpretation the government budget restraint operates as a simple money supply process.

If we classify \bar{G} , \bar{M}_0 , \bar{M}_1 , \bar{N}_0 , $\bar{\alpha}$, \bar{c} , \bar{t} , $\bar{\xi}$, $\bar{\lambda}$ and $\left(\frac{\bar{W}}{\bar{p}}\right)$ as exogenous variables,⁴⁾ we are left with six endogenous variables - Y , p , i , W_0 , T and N_1 . But we have available only five independent equations. We mentioned that without a satisfactory theory for the determination of the absolute price level, we have no other choice than to assume either a rigid absolute price level or a given full employment income level. The first model interpretation is more relevant for our mainly expository purposes. A discussion of the second version is therefore postponed until a later section.

4) Footnote 4: see page 8

The assumption of a given price level allows a reclassification of our exogenous variables into

$$\left(\frac{\bar{G}}{p}\right), \left(\frac{\bar{M}_0}{p}\right), \left(\frac{\bar{M}_1}{p}\right), \left(\frac{\bar{N}_0}{p}\right), \bar{\alpha}, \bar{c}, \bar{t} \text{ and } \bar{\xi}$$

After substitution of equation (1) und (4a) into (2) and (1) and (4a) into (3), we are left with two implicit functions in Y and i which can be solved for either one of the variables. The graphs of these two semi-reduced form equations depict the familiar IS-LM curves determining the equilibrium values of Y and i (Fig. 1).⁵⁾

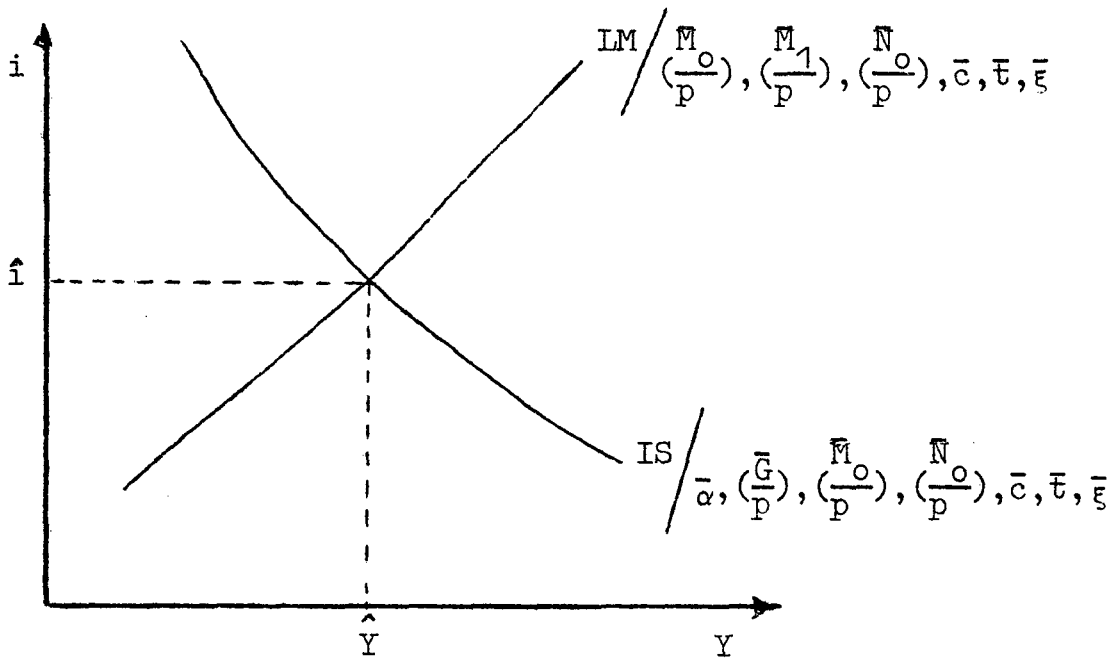


fig. 1

-
- 4) The exogenous variables and the policy instruments are marked by a bar. In the context of our model, we are not interested in the effects of changes of the distribution parameter λ and the real wage rate $\frac{w}{p}$. We introduce these magnitudes as given constants.
- 5) Arrendix I at the end of the paper presents the mathematical syntax underlying our discussion.

Because we include N_1 , the number of bonds outstanding at the end of the period, among the set of endogenous variables, equation (5) does not impose any restriction on the remaining equation system. This is an important property which justifies, after a proper interpretation of the underlying model structure, most of the present results of standard macroeconomic analysis. The budget identity restricts the policy choices of the central authorities: Given the predetermined values of the variables \bar{M}_0 and \bar{N}_0 , the government can only choose three variables independently out of the available set containing the four policy instruments \bar{G} , \bar{M}_1 , \bar{N}_1 and \bar{T} , which is indirectly controlled by the instruments \bar{c} and \bar{t} . We introduced N_1 as an endogenous variable which resolves the choice problem. In addition to this, this procedure allows a simple reinterpretation of the standard approach which generally ignores the government budget restraint.

Focusing; on our target variables Y and i , we can write down the qualitative results of the mutatis-mutandis effects of all exogenous variables including the policy instruments in form of a sensitivity matrix. This is done in Table 1. The signs are those of the partial derivatives of the respective reduced form equation for Y and i .⁶⁾ The signs under the variable $(\frac{\bar{T}}{p})$ will be explained in a moment.

Table 1

Sensitivity analysis of various mutatis-mutandis effects of fiscal and monetary parameters on the target variables Y and i .

Effect on		Change in the exogenous variables								
		$\bar{\alpha}$	$(\frac{\bar{G}}{p})$	$(\frac{\bar{M}_0}{p})$	$(\frac{\bar{M}_1}{p})$	$(\frac{\bar{N}_0}{p})$	\bar{c}	\bar{t}	\bar{s}	$(\frac{\bar{T}}{p})$
		1	2	3	4	5	6	7	8	9
Y	1	+	+	?	+	?	?	?	?	?
i	2	+	+	+	-	+	-	-	-	-

6) The mathematical computations underlying our qualitative propositions are stated in Appendix I.

These signs follow from a simple addition of the shift directions of the IS and LM curve, measured either along the Y-axis or along the i-axis. These shifts follow as a consequence of a change in one or more of the exogenous variables (see Table 2).

Table 2

Sensitivity analysis of various ceteris-paribus effects of fiscal and monetary parameters on Y resp. i in IS-LM analysis.

Effect on		Slope property		Change in the exogenous variables								
		Y	i	α	$(\frac{\bar{G}}{P})$	$(\frac{\bar{M}_0}{P})$	$(\frac{\bar{M}_1}{P})$	$(\frac{\bar{N}_0}{P})$	\bar{c}	\bar{t}	\bar{s}	$(\frac{\bar{\pi}}{P})$
		1	2	3	4	5	6	7	8	9	10	11
Y/i-IS curve	1	.	-	+	+	+	0	+	-	-	-	-
i/Y-IS curve	2	-	.	+	+	+	0	+	-	-	-	-
I/i-LM curve	3	.	+	0	0	-	+	-	+	+	+	+
i/Y-LM curve	4	+	.	0	0	+	-	+	-	-	-	-

Columns 1 and 2 measure the slopes of the IS-LM curves in the Y and i directions. Columns 3 - 10 indicate the shift direction of the IS and LM curves measured either in the Y-direction (rows 1 and 3) or in the i-direction (rows 2 and 4). Given the opposite slopes of the IS and LM curve, we can usually determine the direction of the total effect, simply by adding row 1 to row 3 and row 2 to row 4.

Referring back to Table 1, we see that only three of the nine policy effects on real income are determined. This is a direct consequence of the inclusion of wealth-effects into the standard Keynesian behavior functions

which lead to an interaction of the IS and LM curves. It is a striking result that all effects of changing tax parameters on real income are undetermined.⁷⁾ We recall from standard Keynesian economics that all shifts of the IS-curve are linked to changes in fiscal parameters and that the shifts of the LM-curve are linked to change in monetary parameters. We should note, however, that the textbook results only follow, if we include gross national income and not disposable income into the money-demand function. This procedure is, of course, correct, if we only want to include a variable summarizing the transactions volume. The wealth variable relates this behavior function to fiscal influences, with the consequence that the total effect on income remains dubious.

A change of the initial money endowment is equivalent to an increase in the initial wealth endowment. This leads both to an increase in the demand for final output, via the wealth effect in the consumption function, and to an increase in the demand for money, based on the same effect. But the supply quantity \bar{M}_1 is given which leads to an increase in the market interest rate. This, in turn, will depress the demand for new capital goods. From this, we conclude that the total effect on income is indeterminate. A similar reasoning applies to a change in the initial endowment of government bonds. It is interesting to note that these two effects are de-

7) Contrary to the usual Keynesian policy conception, this is generally true for more sophisticated models. For an analysis of tax effects in the context of a two-sector model compare Hans G. Monissen, "Some Theoretical Issues in Keynesian Stabilization Policy", an, as yet, unpublished paper prepared for the Second Konstanz Seminar on Monetary Theory and Policy, June 1971.

defined only if we include wealth effects into one of the two behavior functions.

Until now, we disregarded the signs under column 9 of Table 1 and column 11 of Table 2. These signs correspond to a simple system excluding the built-in-stabilization effects of the government budget. This system does not contain equation (4a), the tax function, which means that we have to introduce T as an exogenous variable and to disregard the tax parameters \bar{c} and \bar{t} . This new system, which allows a simpler mathematical presentation, leads to the same qualitative results. In addition to this, we gain the theoretical advantage of focusing directly on the critical variables entering the government budget restraint.

Let us ignore for the moment all wealth effects in both the consumption function and the money-demand function. Given the initial values of \bar{M}_0 , \bar{N}_0 and $\bar{\xi}$, the effect of a change of all remaining exogenous variables on Y is defined as follows:⁸⁾

$$(6) \quad dY = \frac{1}{k} \left\{ d\bar{\alpha} + d\left(\frac{\bar{G}}{p}\right) - c_2 (d\bar{c} + Y d\bar{t}) + \frac{I_i}{m_3} d\left(\frac{\bar{M}_1}{p}\right) \right\}$$

If we exclude the tax function, the formula simplifies to

$$(7) \quad dY = \frac{1}{k^*} \left\{ d\bar{\alpha} + d\left(\frac{\bar{G}}{p}\right) - c_2 \left(\frac{\bar{T}}{p}\right) + \frac{I_i}{m_3} d\left(\frac{\bar{M}_1}{p}\right) \right\}$$

8) Once more, the reader is referred to the mathematical appendix. $\frac{1}{k}$ and $\frac{1}{k^*}$ are defined as $\frac{1}{\Delta} \frac{1}{i}(-m_3)$ and $\frac{1}{\Delta T} \frac{1}{i}(-m_3)$, respectively. Any set of observed values satisfying equation (6) will necessarily satisfy equation (7), too. If we subtract in equation (6) the effect of a change of autonomous investment, $d\bar{\alpha}$, from the total effect, we get the effects of all discretionary fiscal measures. Similarly, the total effect of the budget is defined as the difference of the actual income change and the change, which would result if no change in the budget items would have taken place. This

There is now a general agreement that the national income account budget balance, $\frac{G}{p} + \frac{N_o}{p} - \frac{T}{p}$, is not a reliable indicator of the strength and direction of fiscal policy. This inadequacy is mainly attributed to the fact that the budget position affects as well as reflects the level of national income. But this problem is only of minor importance. The basic question is whether it is possible to construct a one dimensional cardinal index which summarizes miscellaneous policy items working in different directions and affecting the economy in varying degrees. In our simple model, these different policy items are government expenditures and tax revenues which enter equations (6) and (7) with different numerical weights.

Recently, two overall fiscal measures were proposed which should resolve these problems: The Full Employment Budget Surplus and the Initial Fiscal Stimulus.¹⁰⁾ Without going into details, we should note that the Full

footnote 8) continued:

measure is related to equation (7). The difference between the two effects defines the automatic effects of the budget. For an elaboration of these measures see Bent Hansen and W. Snyder, Fiscal Policy in Seven Countries, Paris 1969, chap. I.

- 9) The index should have cardinal properties because it is used in standard regression analysis and subjected to the usual test procedures.
- 10) These measures are extensively discussed in William H. Oakland, "Budgetary Measure of Fiscal Performance," Southern Economic Journal, Vol. 35, 1968, pp.347-358; and E. Gerald Corrigan, "The Measurement and Importance of Fiscal Policy Changes," Federal Reserve Bank of New York, Monthly Review, June 1970, pp.133-145. For a good account of the measurement problems related to the Full Employment Budget Surplus compare Michael E. Levy, Fiscal Policy, Cycles and Growth, National Industrial Conference Board, Studies in Economics, No. 81, 1963.

Employment Budget Surplus does not solve the index problem. This measure only defines a new variable and thus does not reduce the degrees of freedom associated with our problem. The Initial Fiscal Stimulus seems a more promising measure. This measure is defined in equations (6) and (7) as the weighted sum of the first three terms in the brackets. But this measure is defective, too. Firstly, it is not uniquely defined. The weights depend on the underlying model structure, which is easily verified by inspecting our mathematical formulas in Appendix I. Secondly, it ignores the government budget restraint and thus the financing of the budget balance. This objection, however, can be corrected by including the proper finance assumptions as done in our model.

Our analysis is directly related to some recent investigations by Andersen and Jordan who try to measure the relative importance of monetary and fiscal actions.¹¹⁾ The theoretical form of their proposed estimating equation is stated as

$$(8) \quad \Delta(Y_p) = a + b \Delta(\quad) + c \Delta M$$

where $\Delta(\quad)$ refers to the change of the Full Employment Budget Surplus, as a measure of fiscal performance, and ΔM to the change of the money stock or the adjusted monetary base, as alternative measures of the monetary impulse. We note that all variables are expressed in nominal terms, but this is not important for our argumentation.

Andersen-Jordan have been criticized partly for statistical and econometric reasons, partly for a misspecification of the proper fiscal and monetary variables. However, one important aspect seems to be consistently

11) See especially Leonall C. Andersen and Jerry L. Jordan, "Monetary and Fiscal Actions: A Test of Their Relative Importance in Economic Stabilization," Federal Reserve Bank of St. Louis, Vol. 50, November 1968, pp. 11-23; and the critique by E. Gerald Corrigan, op.cit.

overlooked by the critics. If we refer back to equation (6) and (7), we see that $d\left(\frac{\bar{M}_1}{p}\right)$ is included as a variable of the reduced form in addition to the Initial Fiscal Stimulus. Written explicitly,

$d\left(\frac{\bar{M}_1}{p}\right)$ is equal to $d\left(\frac{\bar{M}_0 + \Delta\bar{M}}{p}\right)$, thus $d\left(\frac{\bar{M}_1}{p}\right)$ corresponds to

$\Delta\left[\Delta\left(\frac{\bar{M}}{p}\right)\right]$, the change of the new-money finance part of the budget, and not to $\Delta\left(\frac{\bar{M}}{p}\right)$, the change of the money stock or the adjusted monetary base. Andersen-Jordan were never explicit about the hypotheses which lead to their proposed test equation. But there is some textual evidence that their analysis is directed either towards a test of a class of so-called crowding-out effects of private expenditures by fiscal actions or a comparative assessment of the following effects: 1. The income effect of a fiscal budget surplus financed by issuing new interest bearing debt, 2. the income effect of a new money financed budget balance, 3. the income effect of tax financed government expenditure, and 4. the income effect of open-market operations. The Andersen-Jordan test equation is misspecified, if it is used to test either one of these effects. Judged on a priori grounds, this invalidated all their derived conclusions. The misspecification is a failure to account properly for the restriction imposed by the government budget restraint. It remains, of course, an open question, how we should formulate the statistical test equation, if we want to include for instance lagged adjustments or the initial conditions, or how we should interpret a set of generated time series observations, if our theoretical argument is based on some simple comparative-static experiments.

Before we analyze the fiscal and monetary policy implications of our model, we should compare our proposal for incorporating the government budget restraint with those stated in the literature. But first of all, some points of interpretation should be noted:

The static Keynesian model becomes dynamic when we include a government budget restraint. Thus the model has both a one-period or short-run equilibrium solution from which we can derive the impact multipliers based on comparative-static experiments, as well as a long-run equilibrium solution describing the time path of the explained variables. Some dynamic implications of our model are described in Appendix II.

Keynesian economics usually abstracts from the existence of a banking system. We extend the scope of our model by assuming that the money demand function specifies the joint demand function of both commercial banks and the non-bank community for base money (high-powered money) or, more specifically, for adjusted base money. This latter magnitude is computed by adding bank reserves and privately held currency, i.e. the monetary base, and subtracting the discounts and advances of the commercial banks. For the sake of simplicity, we assume that the government does not hold deposits with private banks. The wealth definition now includes the adjusted monetary base instead of the money stock. The activities of the central fiscal and monetary authorities are restricted by an overall government budget restraint. The behavior of the government determines the stock of adjusted base money held by banks and other private agents. ¹²⁾

To compare the different proposals to account for the impact of the budget balance, we simply concentrate on the equations describing the government budget restraint, thus ignoring other differences in the specification of the models.

12) Such a reduced form of a money supply process is described in Hans G. Monissen, "Some Theoretical Issues in Stabilization Policy," op. cit.

Ott and Ott propose the following restraint

$$(9) \quad \frac{M_t}{p} = \frac{M_{t-1}}{p} + \beta \left(\frac{G_{t-1}}{p} - \frac{T_{t-1}}{p} \right)$$

The Otts include only level induced wealth effects.

The parameter β describes the proportion of the budget balance financed by issuing new-base money. The stock of private wealth, which enters both the consumption function and the money-demand function is determined by the history of the government's budget behavior:

$$(10) \quad \frac{W_t}{p} = \sum_{i=1}^{\infty} \left(\frac{G_{t-i}}{p} - \frac{T_{t-i}}{p} \right)$$

We note that the short-run solution is not affected by the current financial decisions. These decisions can only be incorporated during the next period. The proposal of the Otts thus cannot be related to standard short-run macroeconomic analysis. In addition to this, the underlying stock-flow adjustment is faulty because there are no market clearing functions defined which relate to the current period.

Both Silber and Ohrist write the government budget restraint as follows: ¹³⁾

$$(11) \quad \frac{G}{p} - \frac{T}{p} = \lambda \frac{M}{p} + \lambda \frac{N}{ip}$$

This is the same restraint as included in our model. But Silber and Christ introduce another stock-flow confusion. Instead of relating the behavior functions to the initial or endowed values of the wealth components, they

13) All three authors ignore the current coupon payments. We shall see that this item raises some special difficulties.

write the functions as depending on the end of period stocks. ¹⁴⁾ The consequence of this incorrect specifications will be demonstrated in the next section.

If we hold \bar{M}_1 and \bar{T} constant, ¹⁵⁾ a change in government expenditure for final output will be matched by an equivalent change in the value of government bonds outstanding at the end of the period. Because our representative wealth owner regards all debt instruments as homogenous goods, it will be a matter of indifference to him whether an increase in his wealth is brought about by an increase in the number of government bonds or an equal increase in privately generated wealth, i.e. private investment. The new issued debt will pay the current market rate beginning with the next period. We abstract for a moment from the fact that the coupon payments of the changed government debt has to be financed in one way or other, which could lead to a change in the discounted value of future tax payments. This effect can be easily incorporated into our frame.

14) This is especially true for the Silber paper. Christ's paper (1967) is on a mixed basis, because he includes the beginning stocks into the consumption function and the end of period stocks into the money-demand function. When he linearizes his model, he drops the lagged first differences of all variables. But this simplification retains the incorrectly specified wealth argument in the money-demand function. In a later paper (1968), Christ is only interested in the dynamic consequences of the model. To simplify, he only analyses the new-money finance part of the government balance and excludes all wealth effects from the behavior equations. But these simplifications bury the basic issue. More recently, Christ has extended his analysis but without resolving the basic problems raised by his previous efforts; see his " A Model of Monetary and Fiscal Policy Effects in the Money Stock, Price Level, and Real Output, " Journal of Money, Credit and Banking, Vol. 1, 1969, pp. 683-705.

15) Footnote 15): see page 19

Given a stable money-demand function and an unchanged money supply, it follows from Walrs law that the demand for investment goods can be interpreted as a change in the supply of new private securities. The theoretical argument backing the assumption that the current market value of the capital stock will not be changed by the way of financing the stock is provided by Modigliani-Miller. ¹⁶⁾ However, the logical structure of the stock-flow mechanism is easier to understand, if we rely on the Metzlerian assumption that the capital stock is financed simply by issuing common stock. ¹⁷⁾

This, a fortiori, rationalizes the procedure for measuring the market value of the capital stock by capitalizing nonhuman income at the current interest rate.

From this discussion we expect that

$$(12)-(13) \quad \frac{\delta Y}{\delta \bar{\alpha}} = \frac{\delta Y}{\delta \left(\frac{\bar{G}}{P}\right)} > 0 \quad \text{and} \quad \frac{\delta i}{\delta \bar{\alpha}} = \frac{\delta i}{\delta \left(\frac{\bar{G}}{P}\right)} > 0$$

15) To simplify the analysis, we introduce the current tax payments as an exogenous variable. We could reach the same conclusions by varying the tax parameters \bar{c} and \bar{t} . But this would unnecessarily complicate the mathematical structure of our arguments.

16) Merton H. Miller and Franco Modigliani, "Dividend Policy, Growth, and the Valuation of Shares," in Stephen H. Archer and Charles A. d'Ambrosio, eds., *The Theory of Business Finance, A Book of Readings*, New York 1967, pp. 339-366

17) Lloyd A. Metzler, "Wealth, Saving, and the Rate of Interest," *Journal of Political Economy*, Vol. 59, 1951, pp. 95-110. Metzler's article is the standard reference for the type of model presented in equations (1) - (5).

We recall that N_1 , the number of bonds outstanding at the end of the period, is included as an endogenous variable. N_1 will increase during the operating as the consequence of both a change of government expenditure and an increase of the interest rate depressing the market value of the standard bond. This result was previously stated without further comment.

If we eliminate the interest-induced wealth-effects in both the consumption function and the money-demand function, equations (12) - (13) state the familiar textbook results. Thus the standard model correctly indicates the direction of change of Y and i as a consequence of a bond-financed increase in government expenditure (compare fig. 2). The multiplier values of this change are the same as the ones which describe the effects of a change of private autonomous investment expenditure. The textbook results are at most stated implicitly without any explicit discussion of the underlying theoretical structure and without any reference to the overall budget restraint of the central authorities. The familiar results are verified by our extended analysis.

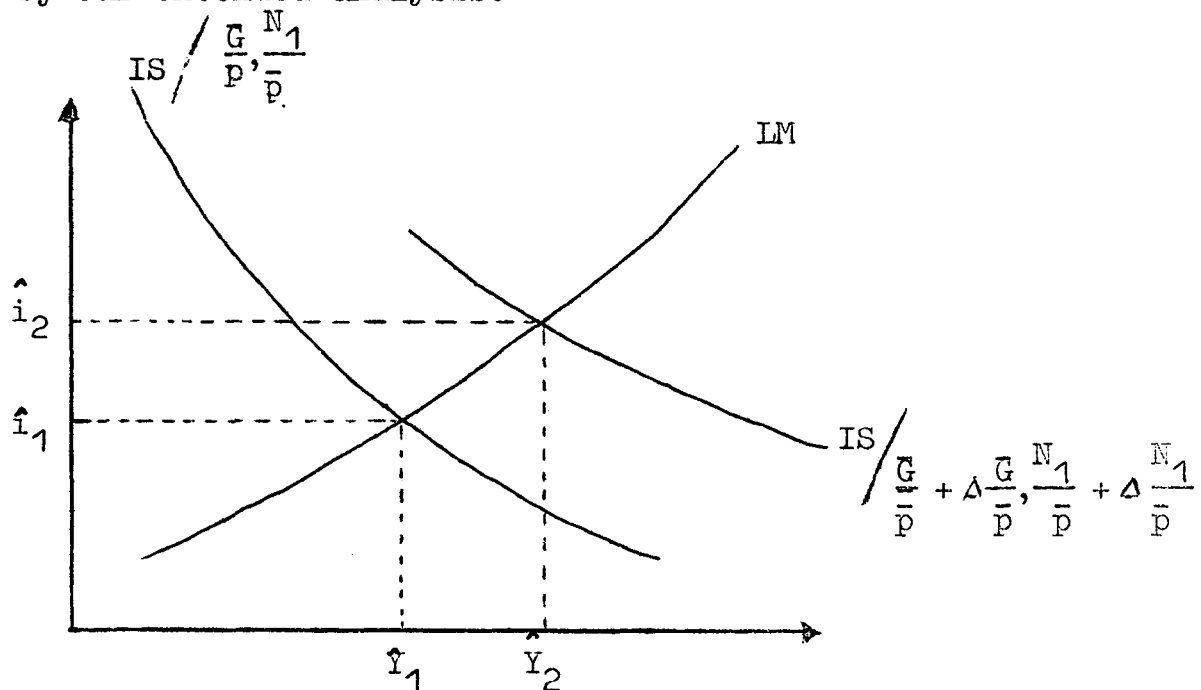


fig. 2

It is especially Silber who argues that in the case of a bond financed government deficit the standard procedure fails to account properly for the shift directions of national income and the market interest rate: "If G increases, this causes the IS curve to shift to the right... If the deficit is financed by new money creation, the LM shifts to the right.... If the deficit is financed by bond sales to the public, the LM curve shifts to the left. Most previous discussions of the bond-finance case maintain that the LM curve remains fixed. This clearly cannot be the case, when the effects on the stocks of financial assets outstanding, both money or bonds, are treated symmetrically." ¹⁸⁾ His conclusions can be criticised on the following basis: Firstly, his results are derived from a model which includes wealth effects - his incorrect formulation notwithstanding. The standard model does not include any wealth effect. This may be a serious omission judged by empirical standards. This is, however, not an argument sustained on the formal level that the system is formulated inconsistently. Secondly, we have shown that the number of bonds is implicitly included. It is not clear what Silber means when he charges the standard approach for failing to treat the effects on the stocks of financial assets symmetrically. If it means that every change should produce a shift of the curve describing the "financial" sector, i.e. the LM curve, we can only refer to Walras' law and to the formal procedure by which the two semi-reduced equations are derived. Thirdly, the alleged shift of the LM curve is based on a stock-flow confusion. This is easily explained when we describe Silber's argumentation in terms of our model.

18) Compare William L. Silber, op.cit., p. 465.

He analyses the bond-finance case of government expenditure as follows:

$$(14) \quad dY = \frac{\delta Y}{\delta(\frac{\bar{G}}{p})} d(\frac{\bar{G}}{p}) + \frac{\delta Y}{\delta(\frac{\bar{N}_0}{p})} d(\frac{\bar{N}_0}{p})$$

$$(15) \quad di = \frac{\delta i}{\delta(\frac{\bar{G}}{p})} d(\frac{\bar{G}}{p}) + \frac{\delta i}{\delta(\frac{\bar{N}_0}{p})} d(\frac{\bar{N}_0}{p})$$

where we have to restrict $d(\frac{\bar{N}_0}{p})$ in such a way that $d(\frac{\bar{N}_0}{p}) = i d(\frac{\bar{G}}{p})$. This allows a transformation of the two differentials into derivatives. Because $\frac{\delta i}{\delta(\frac{\bar{N}_0}{p})}$ is

definitely positive, this procedure leads to an over-estimation of di in the above equation. The sign of $\frac{\delta Y}{\delta(\frac{\bar{N}_0}{p})}$ is undetermined so that the total effect of a bond financed increase in government expenditure on gross national product is undetermined. We recall that in our analysis the sign of this critical could be uniquely inferred. ¹⁹⁾ The indeterminacy of the income effect in Silber's analysis is the consequence of including a kind of a windfall gain or gift into the budget restriction, i.e. the unjustified variation of \bar{N}_0 , or to put it differently, the consequence of a failure to separate balance sheet items from the flow concepts of the in-

19) We should point out that it does not follow from our analysis that this effect is empirically important. A major hypothesis of the monetarist position, particularly in the version of Milton Friedman, states that the crowding-out effect of bond-financed government expenditure on private investment expenditure is so strong that the total effect is approximately zero, or may be even negative, if we take a broader spectrum of assets and liabilities into consideration. For a dis-

come statement. 20)

Before we compute the effects of a new-money financed budget balance in terms of our model, we have to decide how the interest induced wealth change of the already projected bond-finance part should be covered. We have two interesting options: Firstly, we can keep the market value of the bond-finance part constant, which means that we allow a variation of N_1 . Secondly, we could fix the coupon payments N_1 , a policy which is realistic if the government is more concerned about the long-run consequences of paying for the government debt.

The general approach for analysing the new-money finance case of government expenditure is described as follows:

$$(16) \quad dY = \frac{\delta Y}{\delta \left(\frac{\bar{G}}{p}\right)} d\left(\frac{\bar{G}}{p}\right) + \frac{\delta Y}{\delta \left(\frac{\bar{M}_1}{p}\right)} d\left(\frac{\bar{M}_1}{p}\right)$$

$$(17) \quad di = \frac{\delta i}{\delta \left(\frac{\bar{G}}{p}\right)} d\left(\frac{\bar{G}}{p}\right) + \frac{\delta i}{\delta \left(\frac{\bar{M}_1}{p}\right)} d\left(\frac{\bar{M}_1}{p}\right)$$

If we keep the market value of the bond financed part constant, we have to restrict

$d\left(\frac{\bar{M}_1}{p}\right)$ such that

$$d\left(\frac{\bar{G}}{p}\right) = d\left(\frac{\bar{M}_1}{p}\right)$$

footnote 19) continued:

cussion of the crowding-out effects in the literature see Roger W. Spencer and William P. Yohe, "The 'Crowding-Out' of Private Expenditures by Fiscal Actions," Federal Reserve Bank of St. Louis, October 1970, pp. 12-24.

20) The same unjustified wealth variation is included, when Silber analyzes the new-money finance case of a government budget deficit.

If the compensation is such that the number of debt instruments at the end of the period will remain constant, we have to apply the following restriction:

$$d\left(\frac{\bar{M}_1}{p}\right) = - \left(\begin{array}{c} \frac{\delta\left(\frac{N_1}{p}\right)}{\delta\left(\frac{\bar{G}}{p}\right)} : \frac{\delta\left(\frac{N_1}{p}\right)}{\delta\left(\frac{\bar{M}_1}{p}\right)} \end{array} \right) d\left(\frac{\bar{G}}{p}\right) = \mu_1 d\left(\frac{\bar{G}}{p}\right)$$

This restriction may be verified upon substitution into the differential for $d\left(\frac{N_1}{p}\right)$. The term in brackets is negative. From this follows that the direction of change according to both restrictions is the same. dr in equation (17) is indeterminate, dY in equation (16) is definitely positive. These later results can be immediately derived from Table 1.

Our result corresponds with the Keynesian analysis: The change in government expenditure shifts the IS-curve to the right, the following rightward shift of the LM-curve reinforces the income raising effect, but offsets the increase of the interest rate by weakening the crowding-out effects on private investment expenditure.

A third way to finance a change in government expenditure is to raise taxes. This case can be analysed as follows:

$$(18) \quad dY = \frac{\delta Y}{\delta\left(\frac{\bar{G}}{p}\right)} d\left(\frac{\bar{G}}{p}\right) + \frac{\delta Y}{\delta\left(\frac{\bar{T}}{p}\right)} d\left(\frac{\bar{T}}{p}\right)$$

$$(19) \quad di = \frac{\delta i}{\delta\left(\frac{\bar{G}}{p}\right)} d\left(\frac{\bar{G}}{p}\right) + \frac{\delta i}{\delta\left(\frac{\bar{T}}{p}\right)} d\left(\frac{\bar{T}}{p}\right)$$

Analogously to our discussion of the money-finance case, we have two ways to define a restriction on $d\left(\frac{\bar{T}}{p}\right)$. The first restriction is

$$d\left(\frac{\bar{G}}{p}\right) = d\left(\frac{\bar{T}}{p}\right)$$

This leaves the market value of the projected bond-finance part of the budget deficit constant.

The second restriction is

$$d\left(\frac{\bar{T}}{p}\right) = - \left(\frac{\delta\left(\frac{N_1}{p}\right)}{\delta\left(\frac{\bar{G}}{p}\right)} : \frac{\delta\left(\frac{N_1}{p}\right)}{\delta\left(\frac{\bar{T}}{p}\right)} \right) d\left(\frac{\bar{G}}{p}\right) = \mu_2 d\left(\frac{\bar{G}}{p}\right)$$

This restriction fixes the number of bonds outstanding at the end of the period, i.e. $d\left(\frac{N_1}{p}\right) = 0$. The signs of the effects of a change in real taxes on both the number of government bonds outstanding at the end of the period and real income are undetermined, thus the signs of dY and di in equations (18) and (19) cannot be derived. This result once more illustrates the fact that tax policy is a very unreliable stabilization instrument. In addition, it contradicts the famous balanced budget theorem which would assign the value one to our first derivative. The balanced budget theorem is restricted to the simple 45°-diagram and is not even valid in the complete IS-LM analysis excluding wealth effects.

Let us summarize the effects and try to derive some order conditions.

Government expenditure financed by borrowing from the private sector:

$$(12) \quad dY = \frac{\delta Y}{\delta\left(\frac{\bar{G}}{p}\right)} d\left(\frac{\bar{G}}{p}\right) > 0$$

$$(13) \quad di = \frac{\delta i}{\delta\left(\frac{\bar{G}}{p}\right)} d\left(\frac{\bar{G}}{p}\right) > 0$$

Government expenditure financed by issuing new money:

$$(16a) \quad \cancel{\frac{dY}{\bar{p}i}} - \frac{N_0}{\bar{p}i} = c = \frac{\delta Y}{\delta \left(\frac{G}{p}\right)} d\left(\frac{G}{p}\right) + \frac{\delta Y}{\delta \left(\frac{M_1}{p}\right)} d\left(\frac{G}{p}\right) > 0$$

$$(16b) \quad \cancel{\frac{dY}{\bar{p}}} = c = \frac{\delta Y}{\delta \left(\frac{G}{p}\right)} d\left(\frac{G}{p}\right) + \frac{\delta Y}{\delta \left(\frac{M_1}{p}\right)} \mu_1 d\left(\frac{G}{p}\right) > 0$$

where $\mu_1 > 0$.

$$(17a) \quad \cancel{\frac{di}{\bar{p}i}} - \frac{N_0}{\bar{p}i} = c = \frac{\delta i}{\delta \left(\frac{G}{p}\right)} d\left(\frac{G}{p}\right) + \frac{\delta i}{\delta \left(\frac{M_1}{p}\right)} d\left(\frac{G}{p}\right) \geq 0$$

$$(17b) \quad \cancel{\frac{di}{\bar{p}}} = c = \frac{\delta i}{\delta \left(\frac{G}{p}\right)} d\left(\frac{G}{p}\right) + \frac{\delta i}{\delta \left(\frac{M_1}{p}\right)} \mu_1 d\left(\frac{G}{p}\right) \geq 0$$

Government expenditure financed by raising taxes

$$(18a) \quad \cancel{\frac{dY}{\bar{p}i}} + \frac{N_0}{\bar{p}i} = c = \frac{\delta Y}{\delta \left(\frac{G}{p}\right)} d\left(\frac{G}{p}\right) + \frac{\delta Y}{\delta \left(\frac{T}{p}\right)} d\left(\frac{G}{p}\right) \geq 0$$

$$(18b) \quad \cancel{\frac{dY}{\bar{p}}} = c = \frac{\delta Y}{\delta \left(\frac{G}{p}\right)} d\left(\frac{G}{p}\right) + \frac{\delta Y}{\delta \left(\frac{T}{p}\right)} \mu_2 d\left(\frac{G}{p}\right) \geq 0$$

where $\mu_2 \geq 0$.

$$(19a) \quad \frac{di}{\bar{p}_i} - \frac{\bar{N}_0}{\bar{p}_i} = c = \frac{\delta i}{\delta(\frac{\bar{G}}{p})} d(\frac{\bar{G}}{p}) + \frac{\delta i}{\delta(\frac{\bar{T}}{p})} d(\frac{\bar{T}}{p}) \geq 0$$

$$(19b) \quad \frac{di}{\bar{p}} = c = \frac{\delta i}{\delta(\frac{\bar{G}}{p})} d(\frac{\bar{G}}{p}) + \frac{\delta i}{\delta(\frac{\bar{T}}{p})} \mu_2 d(\frac{\bar{T}}{p}) \geq 0$$

We know that μ_1 is greater than zero. This implies that the income change defined by equations (16 a) and (16b) is greater than the change defined by equation (12). The other results can only be compared by imposing special order conditions on the parameter values of the behavior equations.

Table 3

Fiscal and Monetary Policy: A Classification Related to the Exogenous Budget Items.

	.	G	T	M ₀	M ₁	N ₀
.	.	F	F	M	M	F
G	F	.	FF	FM	FM	FF
T	F	FF	.	FM	FM	FF
M ₀	M	FM	FM	.	MM	FM
M ₁	M	FM	FM	MM	.	FM
N ₀	F	FF	FF	FM	FM	.

Before we continue our discussion of the effects of various fiscal and monetary policy measures, we introduce a classification scheme related to the exogenous budget items (Table 3). F designates a pure fiscal policy measure, M a monetary measure, FM a mixed policy measure,

and FF resp. MM refers to a composite fiscal or monetary operation. The labeling of a certain element in the matrix specifies the exogenous variables which are varied during the operation. The table should be read from the left to the right. A policy operation is always related to the first variable. The restriction is always defined on the second variable. An operation is classified as a monetary one, if it involves a change in the money stock; and classified as a fiscal one, if either government expenditure for final output, the current coupon payments, or the tax payments change.

Our previous discussion was centered on G, a pure fiscal operation, on GM₁, a mixed operation, and on GT, once more, a pure fiscal policy operation.

Especially, the second measure has brought about a controversy on the question of defining the effects of fiscal and monetary policy, if both move in the same direction. Under our scheme, this operation is equivalent to a combination of open-market operations and bond-financed budget balance, with the net result that either the number of outstanding government bonds or the market value of the projected bond-financed part remains constant.

One advantage of including N₁ among the list of endogenous variables is that open-market operations, i.e. a pure monetary measure, can be simply described by varying

$\frac{\bar{M}_1}{p}$, over and above the provision for the financing of the budget balance. The effects of this policy operation are defined as

$$(20) \quad dY = \frac{\delta Y}{\delta \left(\frac{\bar{M}_1}{p} \right)} d\left(\frac{\bar{M}_1}{p} \right)$$

$$(21) \quad di = \frac{\delta i}{\delta \left(\frac{\bar{M}_1}{p} \right)} d\left(\frac{\bar{M}_1}{p} \right)$$

The signs of the partial derivatives are stated in Table 1. In the case of open-market purchases, i.e. increases in

$d\left(\frac{\bar{M}_1}{p}\right)$, gross national product will increase and the market interest rate will decrease. The expansive effect on gross national product is less than the effect of a change in government expenditure financed by issuing new money. The advantage of the open-market operation lies in the decrease of the market interest rate which affects the investment component of the total product absorption.

We should now compare the effect of open-market operations on gross national product with the one defined by a change of government expenditure financed by private borrowing. If we compare the dollar by dollar change of \bar{M}_1 and \bar{G} , we have to compare the two partial derivatives

$$\frac{\delta Y}{\delta \left(\frac{\bar{G}}{p}\right)} \quad \text{and} \quad \frac{\delta Y}{\delta \left(\frac{\bar{M}_1}{p}\right)}$$

The first effect crucially depends on the coefficients of the money-demand function, the second on the marginal propensities to consume out of wealth and the responsiveness of investment with regard to the interest rate. It is not possible, however, to derive an a priori order condition.

In judging these results we should not disregard possible differences in the allocative effects of the two operations. These stand on an equal footing, as we have demonstrated elsewhere. ²²⁾

22) Compare Hans G. Monissen, "Some Theoretical Issues in Keynesian Stabilization Policy," op.cit.

Given the structure of our model, we should note that open-market operations cannot be described by the following operation:

$$(22) \quad dY = \frac{\delta Y}{\frac{\bar{M}_0}{p}} d\left(\frac{\bar{M}_0}{p}\right) + \frac{\delta Y}{\frac{\bar{N}_0}{p}} d\left(\frac{\bar{N}_0}{p}\right)$$

under the restriction

$$d\left(\frac{\bar{N}_0}{p}\right) + d\left(\frac{\bar{M}_0}{p}\right) i = 0.$$

If we would ignore the change of the flow magnitude \bar{N}_0 , which enters the consumption function, this effect would be completely neutral, describing only an intermezzo in the equilibrium process. This follows from the fact that \bar{M}_1 is given and the demand for bonds can adjust to the previous level N_1 .

A second, more relevant alternative is to describe open-market operation as follows:

$$(23) \quad dY = \frac{\delta Y}{\frac{\bar{M}_0}{p}} d\left(\frac{\bar{M}_0}{p}\right) + \frac{\delta Y}{\frac{\bar{M}_1}{p}} d\left(\frac{\bar{M}_1}{p}\right) + \frac{\delta Y}{\frac{\bar{N}_0}{p}} d\left(\frac{\bar{N}_0}{p}\right)$$

$$\text{where } d\left(\frac{\bar{M}_0}{p}\right) = d\left(\frac{\bar{M}_1}{p}\right) \text{ and } d\left(\frac{\bar{M}_0}{p}\right) i + d\left(\frac{\bar{N}_0}{p}\right) i = 0$$

This operation will lead, as it should, to the same result as the operation defined in equation (20), if we disregard the effect of a change in disposable income based on the change in the current coupon payments.

Until now, we by-passed the problem of the future coupon payments related to a change in N_1 . Coupon payments may be financed either by raising taxes, by issuing new debt, or by printing new money. All three cases can be analyzed in the context of our model.

Because the payments for the increased debt are first due during the next period, they do not directly enter the current period decisions. The economic agents may anticipate now that the current change of N_1 will lead to a future change of \bar{T} . If we assume that the economic agents do not anticipate the future change in their tax liabilities, our previous discussion is correct.

Because wealth owners are uncertain about the future financing decisions of the government, we can approximately assume that there is no opposite liability effect.

Suppose, however, that it is anticipated, whether, correctly or not does not matter, that an increase of the government debt at the end of the period,

$\frac{N_1}{p_i} - \frac{\bar{N}_0}{\bar{p}_i}$, will lead to an increase in tax obligations

during the next period to finance the increased coupon obligations.

The structure of our model does not allow us to compensate for this effect by changing \bar{T} , because \bar{T} is included as the current tax liability both in the computation of disposable income and current government expenditure. But we can incorporate this effect by an appropriate variation of the parameter $\bar{\xi}$.

The total effect on income of a variation of $\left(\frac{\bar{G}}{p}\right)$ and $\bar{\xi}$ is defined as

$$(24) \quad dY = \frac{\delta Y}{\delta \left(\frac{\bar{G}}{p}\right)} d\left(\frac{\bar{G}}{p}\right) + \frac{\delta Y}{\delta \bar{\xi}} d\bar{\xi}$$

The same variation will affect the number of bonds outstanding:

$$(25) \quad d\left(\frac{N_1}{p}\right) = \frac{\delta\left(\frac{N_1}{p}\right)}{\delta\left(\frac{\bar{G}}{p}\right)} d\left(\frac{\bar{G}}{p}\right) + \frac{\delta\left(\frac{N_1}{p}\right)}{\delta\bar{\xi}} d\bar{\xi}$$

Because the increase in tax payments is first due during the next period, we introduce the following transformation:

$$d\left(\frac{N_1}{p}\right) = \frac{d\left(\frac{\bar{\pi}}{p}\right)}{1+i}$$

The change of $\left(\frac{\bar{\pi}}{p}\right)$ has to be expressed in terms of $\bar{\xi}$:

$$d\bar{\xi} = \frac{\bar{\xi}}{\left(\frac{\bar{\pi}}{p}\right)} d\left(\frac{\bar{\pi}}{p}\right)$$

Equations (24) and (25) and the two restrictions define a set of four equations in terms of five variables. This allows us to solve for dY as a function of $d\left(\frac{\bar{G}}{p}\right)$. It can be verified that the change of Y will be smaller when compared with our previous uncompensated operation.

III.

Our preceding discussion was exclusively based on the more orthodox interpretation of the model. We now reverse the Keynesian adjustment velocities by assuming a given full employment income level and introducing a variable price level. Apart from the government budget restraint and a more complete description of the price and interest included wealth effects, our model corresponds with the one introduced by Don Patinkin in his now classical book on "Money, Interest, and Prices". Our extended model not only offers

of possible fiscal and monetary policy measures, but also justifies some of his more implicitly derived conclusions.

Once more, we substitute equations (1) and (4a) into the commodity-market equilibrium condition. Remembering that we now fix real gross national product instead of the absolute price level, we derive our first semi-reduced form equation in the variables p and i . This is Patinkin's CC-curve. The CC-curve depicts the set of all values of p and i for which the commodity market is in an equilibrium state. If we analyze the slope properties, we have to decide which variables will be affected by a change in p and i . We already decided to include equation (4a) and not (4b). We postpone a short description of this "money illusion" case and first discuss the model under neo-classical assumptions. To complete this picture we introduce government expenditure for final output and coupon payments in real terms. Computing the derivative of p with respect to i we see that this slope is negative, a consequence of the real balance effect .²³⁾

We should note, however, that this slope is not uniquely determined, if we include the real wage rate into the investment function and allow a change of p relative to w . A decrease of the absolute price level will stimulate private consumption expenditures but depress expenditure for new capital. To resolve this indeterminacy, we follow the neo-classical analysis and assume that the real wage is determined on the labor market.

A second semi-reduced form equation, the LL-curve, is similarly derived from the money-market equilibrium condition and equations (1) and (4a). This curve has a positive slope if \bar{M}_1 is greater than $m_1 \bar{M}_0$.

23) Once more, the reader may consult Appendix I for the underlying mathematical derivations.

If we wish, we could introduce a BB-curve, specifying the equilibrium condition for interest bearing claims. This curve is defined on a broader range of assets than the one described in Patinkin analysis. Our curve includes the equilibrium stocks of both government securities and privately issued liabilities, i.e. claims representing the market value of the physical capital stock.

Table 4 states the qualitative properties of the model in form of a sensitivity matrix.

Table 4

Sensitivity analysis of various ceteris-paribus effects of fiscal and monetary parameters on p and i in CC-LL analysis.

Effect on	Slope Property		Change in the exogenous variables									
	p	i	$\bar{\alpha}$	$\left(\frac{G}{p}\right)$	M_0	M_1	$\left(\frac{N_0}{p}\right)$	\bar{c}	\bar{t}	\bar{s}	$\left(\frac{T}{p}\right)$	
	1	2	3	4	5	6	7	8	9	10	11	
p/i-CC curve	1	.	-	+	+	+	0	+	-	-	-	-
i/p-CC curve	2	-	.	+	+	+	0	+	-	-	-	-
p/i-LL curve	3	.	+	0	0	-	+	-	+	+	+	+
i/p-LL curve	4	+	.	0	0	+	-	+	-	-	-	-
p (directional change)	5	.	.	+	+	?	+	?	?	?	?	?
i (directional change)	6	.	.	+	+	+	-	+	-	-	-	-

The analytics of the model are completely analogous to our previous discussion. But two operations should be singled out. A proportional increase in M_0 and M_1 will produce the classical neutrality results, namely a proportionate increase in the absolute price level without affecting the market interest rate.

$$(26) \quad d \log p = \frac{\delta \log p}{\delta \log \bar{M}_0} d \log \bar{M}_0 + \frac{\delta \log p}{\delta \log \bar{M}_1} d \log \bar{M}_1 \stackrel{!}{=} 0$$

$$\text{where } d \log \bar{M}_0 = d \log \bar{M}_1 \stackrel{!}{=} 0$$

$$(27) \quad di = \frac{\delta i}{\delta \bar{M}_0} d\bar{M}_0 + \frac{\delta i}{\delta \bar{M}_1} d\bar{M}_1$$

$$\text{where } d\bar{M}_0 = d\bar{M}_1$$

The first statement can immediately be verified by inspection of the system (1) - (5). This first result proves the second statement. A necessary condition for the second statement is that the directions of change of i as a consequence of changes of M_0 and M_1 are opposite to each other, which can be verified too.

The effect of open-market operations are simply computed by analysing the effects of a change of M_1 given the values of all the other parameters. The results can be read from the table. The price level will increase and the interest rate will decrease, if we consider open-market purchases. Open-market sales will produce the opposite results.

We should note that it was mainly from the writings of Metzler, Patinkin, a.o. that we learned how to analyse open-market operations in the context of a macro-model. But it was only by restricting the activities of the government or introducing extraneous assumptions that this operation could be carried out. The authors never succeeded in incorporating a realistic picture of the

government spending and taxing activities; or, to put it differently, they never succeeded in incorporating the government budget restraint into their formal analysis.

If we introduce both \bar{N}_0 and \bar{T} and not their real values as dependent variables and substitute equation (4b), we have formally constructed a model with "money illusion". Under these assumptions we cannot infer definite slope properties without specifying a priori order conditions. The money-illusion case is usually explained by referring to irrational behavior. But this notion is mistaken. The "New Microeconomics" have forcibly reminded us that information is not a free good and adjustment costs, both average and marginal, are not equal to zero. Under a regime of positive information and adjustment costs, the "money-illusion" case may well be the "normal" one. ²⁴⁾

IV.

Only a few articles in the professional literature have stressed the importance of including a government budget restraint in standard macroeconomic analysis. The budget restraint defines a functional relationship on the available set of policy instruments and thus effectively limits the policy choices of the central monetary and fiscal authorities. However, the proposals offered differ fundamentally in the way they specify the stock-flow mechanism linking the short-run solution to the dynamic adjustment path over time. We were able to demonstrate that none of these proposals were analytically acceptable, because they either completely ignore the short-run equilibrium process by focusing only on the long-run solution properties of the model or they introduce an incorrectly specified wealth effect which causes a serious stock-flow confusion.

24) For a summary of the "New Microeconomics" compare Edmund S. Phelps et al. *Microeconomic Foundations of Employment and Inflation Theory*, New York 1970.

In our paper we therefore tried to present what we regard as an appropriate incorporation of the government budget restraint into macroeconomic analysis. The fiscal and monetary policy implications of the model were described in two major sections. Our results, stated both in the language of the Keynesian IS-LM frame and Patinkin's CC-LL apparatus, were confronted with those derived from the standard model. It could be shown that most of the familiar results valid after a careful reinterpretation of the underlying model structure.

Our analysis may help to bridge some of the communication and interpretation gaps in recent stabilization policy, especially the one bearing on the relative importance of monetary and fiscal policy in realizing overall economic targets. It was shown that the estimating equations of some of the recent tests on the relative income effects of monetary and fiscal policy are either misspecified or incompletely formulated. This could be explained by introducing the government budget restraint into the analysis.

Appendix I: Summary of the underlying mathematical derivatives

The effects of various policy measures on Y are stated inequations (I.1) to (I.7). The underlying model includes the tax function defined in equation (4a).

$$(I.1) \quad \frac{\delta Y}{\delta \bar{\alpha}} = \frac{\delta Y}{\delta \left(\frac{G}{p}\right)} = \frac{1}{\Delta} \frac{1}{i} \left[m_1 \left(\frac{\lambda Y}{i^2} + \frac{N_o}{\bar{p}i} - \bar{\epsilon} \frac{T}{2} \right) - m_3 \right] > 0$$

$$\frac{\lambda Y}{i^2} + \frac{N_o}{\bar{p}i} - \bar{\epsilon} \frac{T}{2} > 0$$

as explained in the text.

$$(I.2) \quad \frac{\delta Y}{\delta \left(\frac{M_o}{p}\right)} = \frac{1}{\Delta} \frac{1}{i} (m_1 I_1 - m_3 C_1) \geq 0$$

$$(I.3) \quad \frac{\delta Y}{\delta \left(\frac{M_1}{p}\right)} = \frac{1}{\Delta} \frac{1}{i} \left[C_1 \left(\frac{\lambda Y}{i^2} + \frac{N_o}{\bar{p}i} - \bar{\epsilon} \frac{T}{2} \right) - I_i \right] > 0$$

$$(I.4) \quad \frac{\delta Y}{\delta \left(\frac{N_o}{p}\right)} = \frac{1}{\Delta} \frac{1}{i} \frac{1}{i} \left\{ (m_1 I_i - m_3 C_1) - C_2 [m_3 - m_1$$

$$\left(\frac{\lambda Y}{i^2} + \frac{N_o}{\bar{p}i} - \bar{\epsilon} \frac{T}{2} \right) \right\} - t \left[m_1 \left(C_2 \left(\frac{\lambda Y}{i^2} + \frac{N_o}{\bar{p}i} - \bar{\epsilon} \frac{T}{2} \right) + I_i \bar{\epsilon} \frac{1}{i} \right) - m_3 \left(C_2 + C_1 \bar{\epsilon} \frac{1}{i} \right) \right] \geq 0$$

$$(I.5) \quad \frac{\delta Y}{\delta \bar{c}} = \frac{1}{\Delta} \frac{1}{i} \left[m_3 \left(C_2 + C_1 \bar{\epsilon} \frac{1}{i} \right) - m_1 C_2 \left(\frac{\lambda Y}{i^2} + \frac{N_o}{\bar{p}i} - \bar{\epsilon} \frac{T}{2} \right) - m_1 I_i \bar{\epsilon} \frac{1}{i} \right] \geq 0$$

$$(I.6) \quad \frac{\delta Y}{\delta \bar{c}} = \left(Y + \frac{\bar{N} \cdot 0}{\bar{p}} \right) \frac{\delta Y}{\delta \bar{c}} \gtrless 0$$

$$(I.7) \quad \frac{\delta Y}{\delta \bar{c}} = - \frac{T}{\bar{p} i} \frac{\delta Y}{\delta \left(\frac{\bar{M}}{\bar{p}} \right)} \gtrless 0$$

Excluding the tax function will only change the value of the determinant of the coefficient matrix, Δ . This determinant is denoted as Δ^T , Both Δ and Δ^T are positive under economically reasonable order conditions.²⁵⁾ It can be demonstrated that $\Delta > \Delta^T$.

The effects on i are similarly defined:

$$(I.8) \quad \frac{\delta i}{\delta \bar{\alpha}} = \frac{\delta i}{\delta \left(\frac{\bar{G}}{\bar{p}} \right)} = \frac{1}{\Delta} \frac{1}{i} \left[m_2 + m_2 \frac{1}{i} (\lambda - \bar{t} \bar{c}) \right] > 0$$

We assume that the term in brackets is positive which simply means that the partial response coefficient $\frac{\delta m}{\delta Y}$ is positive:

$$\frac{\delta m}{\delta Y} = m_1 \frac{\lambda}{i} - m_1 \bar{t} \bar{c} \frac{1}{i} + m_2 > 0$$

$$(I.9) \quad \frac{\delta i}{\delta \left(\frac{\bar{C}}{\bar{p}} \right)} = \frac{1}{\Delta} \frac{1}{i} \left[m_2 C_1 - m_2 (C_2 (1 - \bar{t}) - 1) \right] > 0$$

$$(I.10) \quad \frac{\delta i}{\delta \left(\frac{\bar{M}_1}{\bar{p}} \right)} = \frac{1}{\Delta} \frac{1}{i} \left[C_2 (1 - \bar{t}) + C_1 \frac{1}{i} (\lambda - \bar{c} \bar{t}) - 1 \right] < 0$$

The term in brackets is negative, because we assume that the marginal propensity to consume income is less than one:

25) The operation described in equation (I.6) is not defined, if we exclude the tax function. Under this simpler system, equation (I.5) corresponds to an autonomous change of the exogenous variable

$\left(\frac{\bar{M}}{\bar{p}} \right)$.

$$\frac{\Delta C}{\delta Y} = \frac{\lambda C_1}{i} - C_1 \frac{1}{i} \bar{\xi} \bar{\tau} + C_2 (1 - \bar{\tau}) < 1$$

$$(I.11) \quad \frac{\delta i}{\delta \left(\frac{N_0}{p} \right)} = \frac{1}{\Delta} \frac{1}{i} \left\{ \frac{1}{i} \left[m_2 C_1 + m_1 (1 - C_2 (1 - \bar{\tau})) \right] + C_2 \right. \\ \left. \left[m_2 + m_1 \frac{1}{i} (\lambda - \bar{\tau} \bar{\xi}) \right] - \bar{\tau} \left[m_1 \left(\bar{\xi} \frac{1}{i} + C_2 \frac{1}{i} \right. \right. \right. \\ \left. \left. \left. (\lambda - \bar{\xi}) \right) + m_2 (C_2 + C_1 \bar{\xi} \frac{1}{i}) \right] \right\} > 0$$

$$(I.12) \quad \frac{\delta i}{\delta \bar{c}} = \frac{1}{\Delta} \frac{1}{i} \left[- m_2 C_2 - m_2 C_1 \bar{\xi} \frac{1}{i} - m_1 \left(\bar{\xi} \frac{1}{i} + C_2 \frac{1}{i} \right. \right. \\ \left. \left. (\lambda - \bar{\xi}) \right) \right] < 0$$

$$(I.13) \quad \frac{\delta i}{\delta \bar{\tau}} = \left(Y + \frac{N_0}{p} \right) \frac{\delta i}{\delta \bar{c}} < 0$$

$$(I.14) \quad \frac{\delta i}{\delta \bar{\xi}} = - \frac{T}{\bar{p} i} \frac{\delta i}{\delta \left(\frac{M_0}{p} \right)} < 0$$

To derive the effects for the system excluding the tax function, we have to substitute for Δ the determinant Δ^T and to set $\bar{\tau}$ equal to zero.

From equations (I.8) and (I.10) and the government budget restraint we can infer the following results:

$$(I.15) - (I.16) \quad \frac{\delta \left(\frac{N_1}{p} \right)}{\delta \left(\frac{G}{p} \right)} > 0 \quad \text{and} \quad \frac{\delta \left(\frac{N_1}{p} \right)}{\delta \left(\frac{M_1}{p} \right)} < 0$$

These results underly equations (16b) and (17b) stated in the text. It is obvious that it is not possible to derive an a priori order restriction which would determine the magnitude of the quotient of the two expressions.

This quotient defines μ_1 , a negative expression. The sign of μ_2 cannot be determined, because the

sign of $\frac{\delta(\frac{N_1}{p})}{\delta(\frac{\pi}{p})}$ cannot be inferred from the

structural properties of the model.

To derive the IS-curve and the CC-curve we substitute model equations (1) and (4a) into model equation (2):

(I.17)

$$Y - C \left[\left[\frac{\lambda Y}{i} + \left(\frac{\bar{N}_0}{p} \right) \frac{1}{i} + \left(\frac{\bar{M}_0}{p} \right) - \bar{\xi} \frac{1}{i} (\bar{c} + \bar{\tau} Y + \bar{\tau} \left(\frac{\bar{N}_0}{p} \right)) \right], \right.$$

$$\left. Y + \left(\frac{\bar{N}_0}{p} \right) - \bar{c} - \bar{\tau} Y - \bar{\tau} \left(\frac{\bar{N}_0}{p} \right) \right] - I \left[i, \left(\frac{\bar{w}}{p} \right) \right] - \bar{\alpha} - \left(\frac{\bar{G}}{p} \right) = 0$$

If we arbitrarily fix the price level, the equation defines the implicit form of the IS-curve. If we assume a given real income level, the equation describes the CC-curve. Both slope and shift properties can be determined by simple partial differentiation.

The implicit form of the LM-curve and the LL-curve is specified after substituting equations (1) and (4a) into equation (3).

$$(I.18) \quad \frac{\bar{M}_1}{p} - m_1 \left[\frac{\lambda Y}{i} + \left(\frac{\bar{N}_0}{p} \right) \frac{1}{i} + \frac{\bar{M}_0}{p} - \bar{\xi} \frac{1}{i} (\bar{c} + \bar{\tau} Y + \bar{\tau} \left(\frac{\bar{N}_0}{p} \right)) + \bar{\tau} \left(\frac{\bar{N}_0}{p} \right), Y, i \right] = 0$$

The derivation of the LL-curve poses some special problems. We have to assume that the nominal stock \bar{M}_1 is

given, otherwise the slope would be negative. But even with this assumption, it may happen that we derive the inconvenient negative slope property. This is the case if \bar{M}_1 is small relative to \bar{M}_0 .

The "money illusion" case follows after substituting equation (4b) instead of equation (4a) and redefining the proper exogenous variables.

Appendix II: Some simple dynamic implications

The discussion is based on the orthodox interpretation of our Keynesian model which means that we assume a given price level. To study the dynamic path of our endogenous variables, we simplify the structure as follows:

- (1) We ignore the effects of capital accumulation on income and the growth of private wealth.
- (2) We ignore the interest-induced wealth effect.
- (3) We assume a fixed proportion β of bond financed and money financed government deficits.
- (4) We linearize the model and drop the price variable.

$$(II.1) \quad W_t = \bar{F}_t$$

$$(II.2) \quad Y_t = C_1 \bar{F}_t + C_2 Y_t - C_2 T_t + I_i i_t + \bar{\alpha}_t + \bar{G}_t$$

$$(II.3) \quad \bar{\beta} F_{t+1} = m_1 \bar{F}_t + m_2 Y_t + m_3 i_t$$

$$(II.4) \quad T_t = \bar{c}_t + \bar{\tau} Y_t$$

$$(II.5) \quad \bar{G}_t - T_t = F_{t+1} - \bar{F}_t$$

\bar{F}_t is the predetermined stock of financial assets, depending on the past history of government spending and taxing behavior. The linearity assumption allows a solution for the endogenous variables W_t , Y_t , i_t , T_t , F_{t+1} . Because these solutions are rather involved, we display them only partially.

F_{t+1} is determined by the following reduced form equation:

$$(II.6) \quad F_{t+1} = \frac{\delta F_{t+1}}{\delta \bar{F}_t} \bar{F}_t + \frac{\delta F_{t+1}}{\delta \bar{G}_t} \bar{G}_t + \frac{\delta F_{t+1}}{\delta \bar{c}_t} \bar{c}_t + \frac{\delta F_{t+1}}{\delta \bar{\alpha}_t} \bar{\alpha}_t$$

We get similar expressions for the remaining variables. The linearity assumption assures that the partial derivatives in the above equations are constants.

If we assign arbitrary values to the policy parameters \bar{F}_t , \bar{G}_t , \bar{c}_t and $\bar{\alpha}_t$, equation (II.6) specifies a first order linear difference equation with constant coefficients.

Given arbitrary initial conditions, the general solution is defined as follows:

$$(II.7a) \quad F_t = \left(\frac{\delta F_{t+1}}{\delta \bar{F}_t} \right)^t \bar{F}_0 + \left[\frac{\delta F_{t+1}}{\delta \bar{G}_t} \bar{G}_0 + \frac{\delta F_{t+1}}{\delta \bar{c}_t} \bar{c}_0 + \frac{\delta F_{t+1}}{\delta \bar{\alpha}_t} \bar{\alpha}_0 \right] \frac{1 - \left(\frac{\delta F_{t+1}}{\delta \bar{F}_t} \right)^t}{1 - \frac{\delta F_{t+1}}{\delta \bar{F}_t}}$$

for $\frac{\delta F_{t+1}}{\delta \bar{F}_t} \neq 1$

$$(II.7b) \quad F_t = \bar{F}_0 + \left[\frac{\delta F_{t+1}}{\delta \bar{G}_t} \bar{G}_0 + \frac{\delta F_{t+1}}{\delta \bar{c}_t} \bar{c}_0 + \frac{\delta F_{t+1}}{\delta \bar{\alpha}_t} \bar{\alpha}_0 \right] t$$

for $\frac{\delta F_{t+1}}{\delta \bar{F}_t} = 1$

Given the initial conditions, the solution sequence is completely determined by the magnitude $\frac{\delta F_{t+1}}{\delta \bar{F}_t}$:

$$(II.8) \quad \frac{\delta F_{t+1}}{\delta \bar{F}_t} = \frac{-m_3 (1 - C_2 + \bar{c} C_2) + m_3 \bar{c} C_1 - m_2 I_i - \bar{c} I_i m_1}{-m_3 (1 - C_2 + \bar{c} C_2) - m_2 I_i - \bar{c} I_i \bar{\beta}}$$

This magnitude can be influenced by appropriate variations of $\bar{\beta}$ and $\bar{\tau}$. $\bar{\beta}$ is a monetary parameter and $\bar{\tau}$ a fiscal parameter. The value of F_t over time depends both on fiscal and monetary parameters. The same is generally true for the variable i_t . But if the sequence converges, the solution value for both Y_t and T_t depends only on fiscal parameters. The value of $\frac{\delta F_{t+1}}{\delta \bar{F}_t}$ determines whether the sequence converges or diverges. This remains true after substitution of the solution value for \bar{F}_t into the remaining reduced form equations.

If we substitute equation (II.7a) into the reduced form equation for Y_t , the result is:

$$(II.9) \quad \delta Y_t = \frac{\delta Y_t}{\delta \bar{F}_t} \left\{ \left(\frac{\delta F_{t+1}}{\delta \bar{F}_t} \right)^t \bar{F}_0 + \left[\frac{\delta F_{t+1}}{\delta \bar{G}_t} \bar{G}_0 + \frac{\delta F_{t+1}}{\delta \bar{c}_t} \bar{c}_0 + \frac{\delta F_{t+1}}{\delta \bar{\alpha}_t} \bar{\alpha}_0 \right] \frac{1 - \left(\frac{\delta F_{t+1}}{\delta \bar{F}_t} \right)^t}{1 - \frac{\delta F_{t+1}}{\delta \bar{F}_t}} \right\} + \frac{\delta Y_t}{\delta \bar{G}_t} \bar{G}_0 + \frac{\delta Y_t}{\delta \bar{c}_t} \bar{c}_0 + \frac{\delta Y_t}{\delta \bar{\alpha}_t} \bar{\alpha}_0$$

The term in brackets is equal to the variable \bar{F}_t , which is now endogenously determined.

The effect of a change in \bar{G}_0 on Y_t can be calculated by a simple differentiation:

$$(II.10) \quad \frac{\delta Y_t}{\delta \bar{G}_0} = \frac{\delta Y_t}{\delta \bar{F}_t} \frac{\delta F_{t+1}}{\delta \bar{G}_t} \frac{1 - \left(\frac{\delta F_{t+1}}{\delta \bar{F}_t} \right)^t}{1 - \frac{\delta F_{t+1}}{\delta \bar{F}_t}} + \frac{\delta Y_t}{\delta \bar{G}_t}$$

If we assume that the sequence converges, the result for $t \rightarrow \infty$ is:

$$(II.11) \quad \frac{\delta Y_t}{\delta \bar{G}_0} = \frac{\delta Y_t}{\delta \bar{F}_t} \frac{\delta F_{t+1}}{\delta \bar{G}_1} \frac{1}{1 - \frac{F_{t+1}}{\bar{F}_t}} + \frac{\delta Y_t}{\delta \bar{G}_t}$$

The unknown terms are:

$$(II.12) \quad \frac{\delta Y_t}{\delta \bar{F}_t} = \frac{-m_3 C_1 + m_1 I_i \bar{\beta}}{\Delta} \begin{matrix} > \\ = \\ < \end{matrix} 0$$

$$(II.13) \quad \frac{\delta F_{t+1}}{\delta \bar{G}_t} = \frac{-m_2 I_i - m_3 (1 - C_2 + \bar{\tau} C_2 - \bar{\tau})}{\Delta} > 0$$

because $(1 - C_2 + \bar{\tau} C_2 - \bar{\tau})$ is assumed to be positive.

(II.14)

$$\frac{\delta F_{t+1}}{\delta \bar{F}_t} = \frac{-m_3 (1 - C_2 + \bar{\tau} C_2 - \bar{\tau} C_1) - m_2 I_i - \bar{\tau} I_i m_1}{\Delta} > 0$$

Δ is the determinant of the coefficient matrix. This determinant is positive.

After substitution the result is:

$$(II.15) \quad \frac{\delta Y_t}{\delta \bar{G}_0} = \frac{1}{\bar{\tau}}$$

and analogously:

$$(II.16) \quad \frac{\delta T_t}{\delta \bar{G}_0} = 1$$

In the convergent case the change is completely independent of monetary factors, i.e. independent of the value of the parameter $\bar{\beta}$.²⁶⁾ The result is certainly obvious from simple economic reasoning. The simplifying assumptions, however, stated at the beginning, should warn us not to overrate this conclusion.

Thus we conclude that income can only change if the budget is in a deficit or surplus position.

The time sequence or the other variables may be determined similarly.

26) The same long-run result is derived by Silber, Christ, and the Otts. We recall that their short-run model differs from the one analyzed in our paper.