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Prices and selling effort when resources are limited: A geometrical note

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PRICES AND SELLING EFFORT
WHEN RESOURCES ARE
LIMITED: A GEOMETRICAL NOTE *)

DIETER SPAETLING

Dezember 1971

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Diskussionsbeiträge
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Nr. 13

1. The purpose of this note is to show graphically the solution of a decision problem of a producer and supplier whose resources are limited. The analysis focuses on the solution of the problem of how to distribute and to use the scarce resources in producing physical output and/or making selling outlays in order to increase the enforceable selling price, and thereby realizing maximum profit. For the sake of analytical simplicity the assumption of certainty is made throughout this note.
2. Let us assume a single firm which is provided with limited resources, R , within a given period of time. Two activities are open to this firm: The firm can use a fraction, S , of its resources at the beginning of the period in order to realize selling outlay, a , during this period. From the selling effort the firm expects - with certainty - a selling price p^* for its product at the end of the period which is a function $p^*(a, y)$ of both selling outlay and output produced, y , of good Y . The rest X of R which is not used for 'producing' selling effort is employed in order to produce physical output y during the period. It is further assumed that there exists a demand function facing the firm and that the firm knows this demand function. The demand function indicates those quantities of Y the consumers would buy at alternatively given selling prices if no selling effort is made by the firm. The effect of selling outlay on demand is assumed to be a shift of the demand curve to the right.

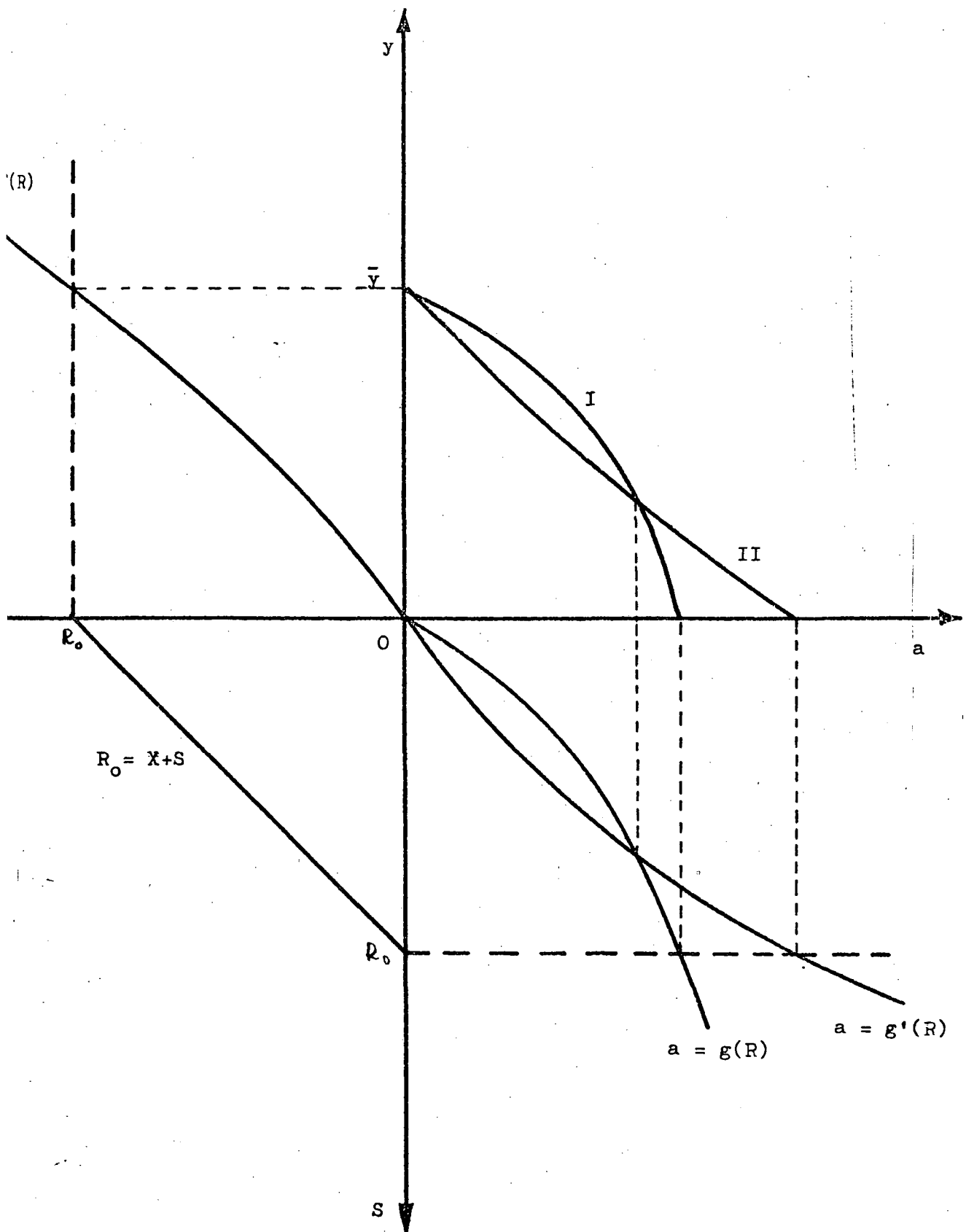


Figure 1

3. The decision problem of the firm is to determine the optimal distribution of R between S and X . The nature of this decision problem can be shown graphically as in Figure 1 : If $R = R_0$ is given, production-possibility-curves can be drawn showing all physical output/selling effort combinations which can be realized with given resources R_0 . Two possible production-possibility-curves are shown in Figure 1. The curves $y = f(R)$ and $a = g(R)$ and $a = g'(R)$ are production functions. Production-possibility-curve I is obtained if both, production output and selling effort, show decreasing returns to scale, or in other words : if increasing marginal costs are assumed in both directions, production of physical output y and accomplishing selling effort, respectively. Production-possibility-curve II illustrates the case of increasing returns to scale or decreasing marginal costs of selling effort 'production'.

Decreasing marginal costs of selling effort can simply stem from the fact of trade discount. Another cause for decreasing marginal costs of selling effort may be given by the effect of volume on total cost at a given rate of production, as has been shown by Armen A. Alchian [1, p. 162], [2, pp. 228 - 232]. If we assume that the firm's objective is maximization of period profit the task of the decision maker is to find that combination of X and S , at a given R_0 , which maximizes this profit.

4. Whereas the firm has the options of producing either physical output or selling effort or both, we assume that the firm can get proceeds only by selling physical output to consumers. At the one hand all resources spent in selling

outlay reduce potential output; at the other hand, however, the price p^* to be realized in selling the physical output produced at the end of the period will be higher if selling effort is made. So the problem of the decision maker is to determine how much of resources R_0 should be devoted to produce selling effort, or in other words: how much of potential physical output - whose maximum \bar{y} is given if all resources R_0 are employed in producing Y - should be sacrificed in order to achieve a maximum profit. Maximum profit will be obtained if marginal revenue and marginal costs of sacrificing potential physical output are equal. For as long as marginal revenue from sacrificing output is higher than marginal costs of foregone physical output it pays to sacrifice more of potential physical output. Thus the optimal, profit-maximizing combination of X and S - given R_0 - is that for which marginal revenue and marginal costs of potential physical output sacrificed are equal.

5. In order to solve the decision problem let us first determine the marginal revenue from sacrificing potential physical output of Y . Let the demand curve facing the firm if no selling effort is made be given by curve dd in Figure 2. If maximum potential physical output is given by \bar{y} , price \bar{p} shows the average revenue expected with certainty if all resources are devoted to production of physical output. Or in other words: if no potential output is sacrificed to undertake selling effort the average revenue to be

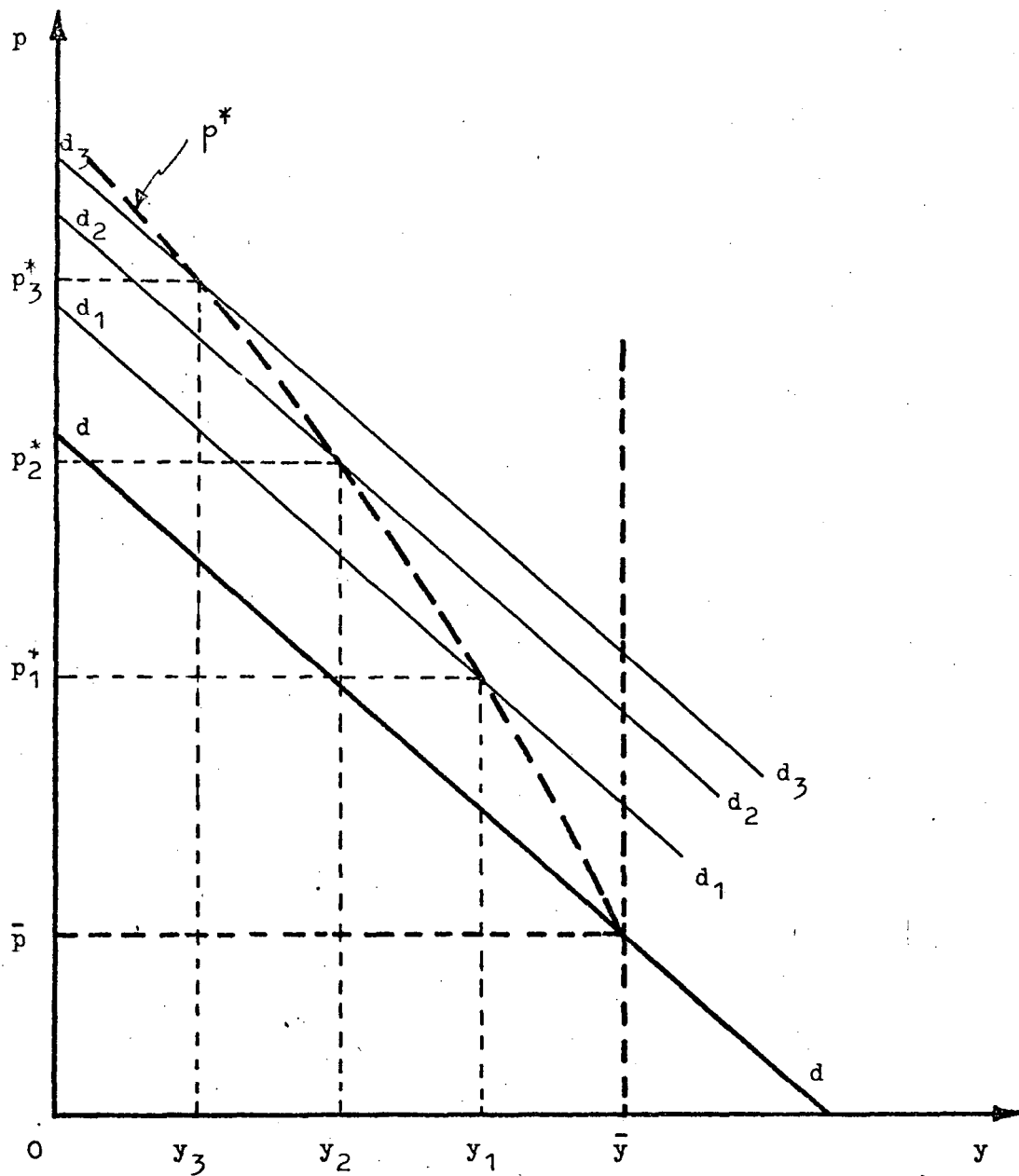


Figure 2

achieved at the end of the period is \bar{p} . We assume that the effect of selling outlay on demand is shifting the demand curve to the right as is shown in Figure 2. Thus if a fraction $(\bar{y} - y_1)$ of potential physical output is sacrificed, the new relevant demand curve facing the firm is d_1d_1 and the expected average revenue will be p_1^* . If a fraction $(\bar{y} - y_2)$ is given up in order to undertake selling effort the average revenue will be p_2^* , and so on. In order to obtain average revenue as a function of potential physical output sacrificed, we only need to reverse the quantities on the y- axis in Figure 2. In Figure 3 the curve p_S^* indicates average revenue from selling $(\bar{y} - y_S)$ as a function of y_S , where y_S is potential output sacrificed. Total revenue from selling $(\bar{y} - y_S)$ is given by

$$(1) \quad TR = p_S^*(\bar{y} - y_S)$$

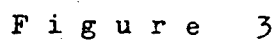
where p_S^* is a function of y_S . From Eq. (1) marginal revenue with respect to y_S is derived as:

$$(2) \quad MR_S = \frac{\partial p_S^*}{\partial y_S} (\bar{y} - y_S) - p_S^*$$

From Eq. (2) we obtain

$$(3) \quad MR_S = \frac{p_S^*}{\varepsilon(y_S, p_S^*)} \cdot \frac{(\bar{y} - y_S)}{y_S} - p_S^*$$

where $\varepsilon(y_S, p_S^*)$ is the elasticity of y_S with respect to p_S^* .



6. Eq. (3) is a formula for the marginal revenue of potential physical output sacrificed belonging to average revenue p_S^* with respect to potential output sacrificed. From Eq. (3) a simple method of plotting marginal revenue with respect to potential output sacrificed is obtained.

Let us plot point W on the MR_S -curve belonging to point P on the p_S^* -curve in Figure 3 : A line tangent to p_S^* through point P intersects with the ordinate at point S, with the abscissa at point T, and with the line vertical on \bar{y} at point Q. Price p_S^* in Eq. (3) can be represented by distance \overline{PA} in Figure 3, and the elasticity $\varepsilon(y_S, p_S^*)$ is given in this case by means of the well-known Marshallian method of graphically measuring point elasticities [3, p.86] by ratio $\overline{PT}/\overline{PS}$ of distances \overline{PT} and \overline{PS} . Since triangles PCS and PAT in Figure 3 are similar, the ratios $\overline{PA}/\overline{PT}$ and $\overline{PC}/\overline{PS}$ are equivalent. Considering this fact we obtain:

$$(4) \quad \frac{p_S^*}{\varepsilon(y_S, p_S^*)} = \frac{\overline{PA} \overline{PS}}{\overline{PT}} = \frac{\overline{PC} \overline{PS}}{\overline{PS}} = \overline{PC}$$

Since $(\bar{y} - y_S)$ can be represented by distance \overline{PV} in Figure 3 and y_S by distance \overline{CS} we have from Eqs. (3) and (4) :

$$(5) \quad MR_S = \overline{PC} \frac{\overline{PV}}{\overline{CS}} - \overline{PA}$$

Now triangles PQV and SPC are similar so that ratios

$\overline{PC}/\overline{CS}$ and $\overline{QV}/\overline{PV}$ are equivalent. If this is taken into consideration we find from Eq. (5) :

$$(6a) \quad MR_S = \frac{\overline{QV} \overline{PV}}{\overline{PV}} - \overline{PA} = \overline{QV} - \overline{PA} \quad \text{or}$$

$$(6b) \quad -MR_S = \overline{PA} - \overline{QV}$$

Thus point W on the MR_S -curve is found by first drawing a line parallel to line \overline{QT} through point V which intersects with line \overline{PA} at point W'. If point W' is reflected at the y - axis point W on the marginal revenue curve is obtained. Distance \overline{AW} gives the marginal revenue belonging to point P. It is obvious that the preceding method can be applied to every point on the p_S^* -curve. The result is the dotted curve MR_S in Figure 3.

7. We now have to ask for the marginal cost of sacrificing potential physical output. This marginal cost of potential output foregone is assumed to be equal to \bar{p} in Figure 2 and is independent of y_S , i.e. the opportunity cost of sacrificing potential output is equal to foregone potential revenue. If nothing out of R_0 is directed into selling effort the average price which could be realized with certainty if all resources were used to produce \bar{y} is \bar{p} . Thus the opportunity cost of sacrificing a quantity y_S of potential output is $\bar{p}y_S$. In this case average cost and marginal cost of sacrificing potential output are the same and equal to \bar{p} .

8. Now the intersection of the MR_S -curve and the \bar{p} -curve gives the optimal output sacrificed, \hat{y}_S , as indicated in Figure 3. In order to determine the profit-maximizing distribution of R_0 between X and S we use the production functions $y = f(R)$ and $a = g(R)$. In Figure 4 the preceding discussion and graphical derivations are summarized: \hat{A} and \hat{y} show the profit-maximizing selling outlay and physical output, respectively, whereas by \hat{S} and \hat{X} the optimal distribution of R_0 between S and X is indicated. In parts (d) through (g) of Figure 4 the assumptions concerning the production functions are summarized. Two production-possibility-curves are shown: curve I is drawn under the assumption of increasing returns to **scale** and curve II under the assumption of decreasing returns, respectively, in producing selling effort. Part (a) of Figure 4 shows average price and marginal revenue with respect to potential physical output sacrificed, and \bar{p} is marginal cost of foregoing potential output.
9. Up to now the analytical and graphical framework had been developed on the basis of selling effort generally. Under the heading 'selling effort', however, several things can be summarized. So, especially, the problem of product variation can be treated by the preceding method as well. In this case the alternative to producing physical output of the good in question is product variation by changing the shape and the appearance of the product in the eyes of the consumers.

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